PARABOLA

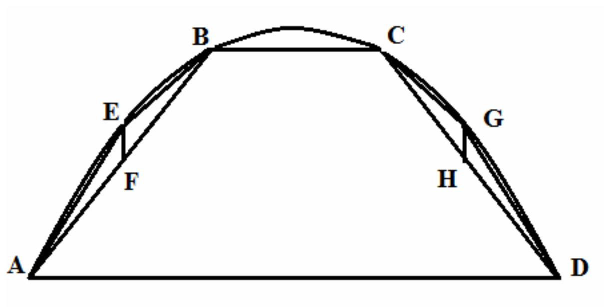
PART SIX

Parabolas and their parabolic segments will be prepared at first; in which figures of the maximum section will be inscribed.

PROPOSITION CCLIII.

Any two parallel lines AD, BC shall intersect the parabola ABC.
I say these segments to be taken as equal.

Demonstration.



The greatest triangles AEB, CGD of the sections AB, CD shall be inscribed; and which shall be equal to each other since BC and AD shall be parallel. Q.e.d.

PROPOSITION CCLIV.

In the same manner, it will be required to draw a right line from a given point C on the periphery, which shall produce a segment equal to the given segment AB.

Construction & Demonstration.

With BC joined, AD shall be drawn parallel to BC, and with CD joined, it is clear from the preceding, with DC drawn from the given point C, to be taken equal to the given segment AB. Q.e.d.

PROPOSITION CCLV.

With the segment ABC and the diameter GH given, it will be required to apply the ordinate line to that, which shall taken from a given equal segment.

Construction & Demonstration.

With AB bisected at F, the diameter FE shall be erected, which shall be made equal to GH: and the ordinate CD shall be put through H, I say what is required to be done : for AEB, CGD shall be joined ; because the diameters EF, GH are equal, also the triangles

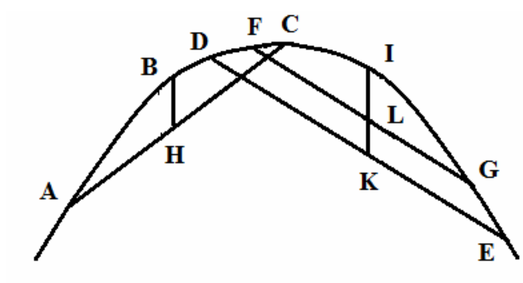
AEB, CGD are equal: which since the greatest of these segments AB, CD can be inscribed, also we can apply these to the equal segments AB, CD, &c. Q.e.d.

PROPOSITION CCLVI.

With the parabola ABC given, and with the segment AC therein, it will be required to draw some parallel line ED, which shall produce a segment equal to that given.

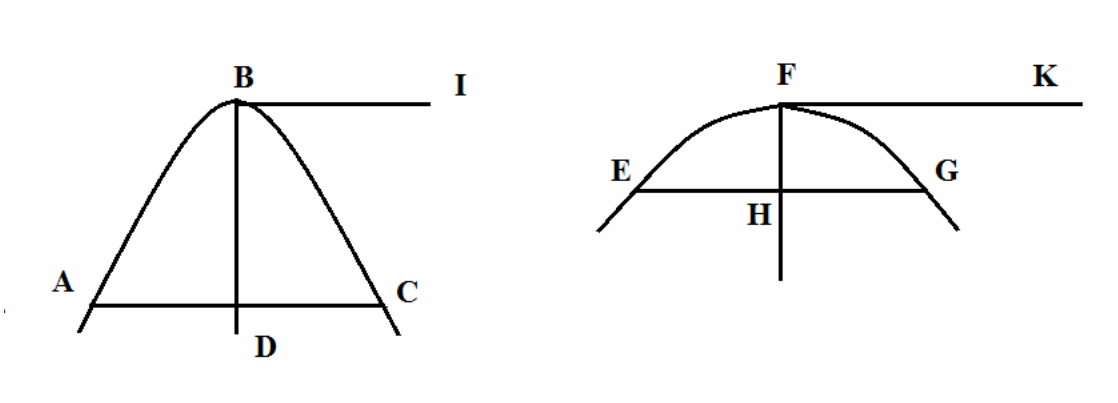
Construction & Demonstration.

With AC, DE bisected at H & K; the diameters HB, IK shall be erected: and FG shall be made parallel to DE with IL put through L, parallel to HB, it will be apparent from the preceding for the given segment FIG to be equal to the given segment AC; therefore we have drawn the parallel line DE from which the segment FIG will be drawn equal to the given segment AC. Which was required.



PROPOSITION CCLVII.

An ordinate AC shall be put in place for the axis BD of the parabola ABC of which the latus rectum is BI: moreover EFG shall be a parabola, the axis of which shall be FH; and the latus rectum FK; it will be required to take the segment of the parabola EFG to that of the parabola ABC, so that the ratio shall be had as BI to FK.



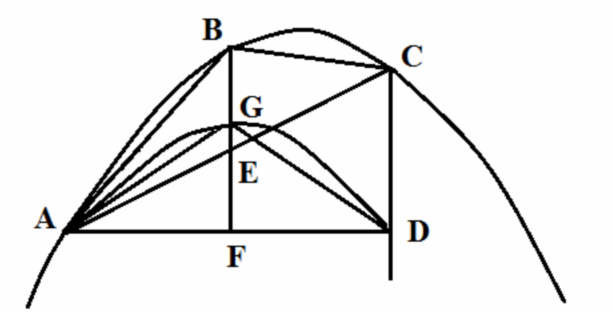
Construction & Demonstration.

As IB shall be to FK, thus there becomes FH to BD, and the ordinate EG may be put through H. I say what is required has been done: since indeed as IB to FK, thus FH to BD The rectangle on IB, BD is equal to the square AD; the rectangle on FK, FH, is equal to the square EH; from which AC, EG are equal to each other, therefore the maximum triangle of the segment EFG, is to the maximum triangle of the segment ABC, as FH to BD, that is, from the construction, as BI ad FK; and therefore as the segment EFG is to the segment ABC, thus as IB to FK; therefore so that we have removed the segment EFG from the parabola EFG which holds the same ratio to the segment ABC, as IB to FK; therefore we have removed that ratio from EFG which the latus rectum BI holds to the latus rectum FK. Q.e.d.

PROPOSITION CCLVIII.

AC shall be the ordinate put in place for the diameter BE of the parabola ABC: and with AD normal to the diameter sent down from C, BE,FG shall become equal lines; and a parabola shall be described through AGD, of which the axis shall be GF.
I say the segment AGD to be equal to the segment ABC.

Demonstration.

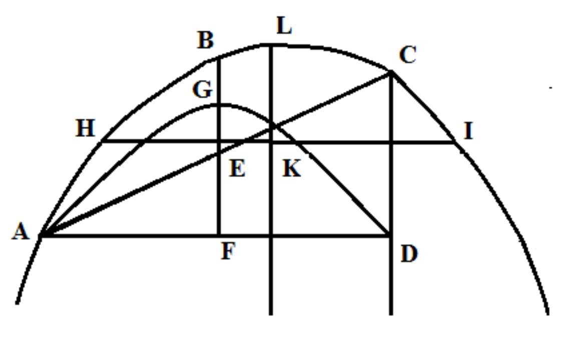


Because BE, GF are equal lines, not only the triangles BCE, GDF, but also BAE, GAF, and therefore ABC, AGD shall be triangles equal to each other, but also the greatest of these which can be described in the segments ABC, AGD ; therefore the segments ABC, AGD are equal. Q.e.d.

PROPOSITION CCLIX.

LK shall be put to be the axis of the parabola ABC, equal to the diameter BE, and the line HI put to be the ordinate through K.
I say that HI to become equal to the right line AD.

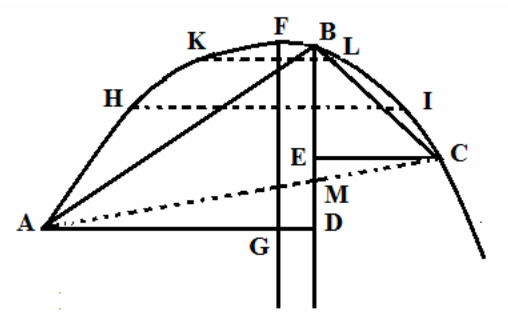
Demonstration.



Indeed from the preceding the parabola AGD shall be equal to the parabola ABC, that is, HLI: and therefore the maxima triangles AGD, HLI are equal [§. 240] : but from the hypothesis the altitudes of these LK, FG are equal, and therefore the bases AD, HI are equal to each other. Q.e.d.

PROPOSITION CCLX.

Some lines AB, BC shall cut the parabola ABC: and BD shall be a diameter dropped from B, to which AD, CE shall be the normals from A and C.
I say the segment AB to be to the segment BC in the triplicate ratio of the lines AD to CE.



Demonstration.

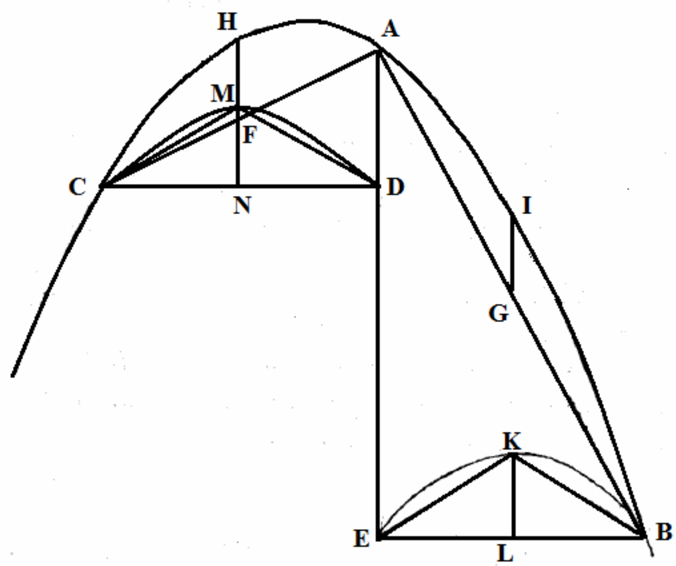
With the axis FG found, the ordinate lines HI, KL may be applied to that : and HI indeed shall be equal to AD; and KL truly to the right line CE: therefore the segment AB shall be equal to the segment HFI, and for the segment BC to be equal to the segment KFL, but the segment HFI to the segment KFL, has the triplicate ratio of the line HI to the line KL [§. 241]; and therefore the segment AB to the segment BC, has the triple ratio of the line HI to KL, that is from the hypothesis, AD to EC. Q.e.d.

Corollary.

Hence it follows with AC joined, which shall cross BD at M, the segments AB to BC to be in the triple ratio of AM to MC, it is clear, since as AD shall be to CE, thus AM to MC, therefore, &c.

PROPOSITION CCLXI.

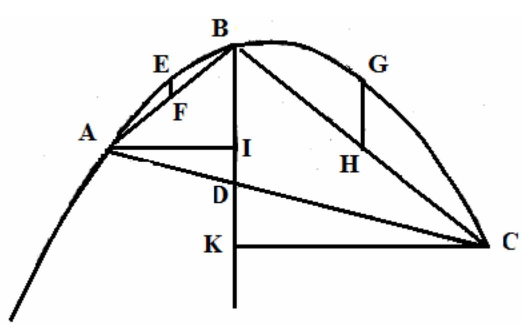
Any two lines AB, AC shall cut the parabola ABC : and with the diameter AD dropped from A, the normals BE, CE shall be put in place to that from B and C : then the lines AB, AC shall be bisected at F and G; FH, GI to be erected parallel to the diameter.
I say the segment AHC to the segment AIB, to have a ratio composed from the ratio FH to IG, & CE BE.



With the lines EB, CD bisected at L and N, the normals LK, NM shall be erected: and indeed LK is equal to IG ; NM is equal to HF; and likewise the points C, M, D of the parabola are described by E, K, B, of which the axes shall be LK, MN; and EKB, CMD shall be joined. Because LK is equal to IG, the segments EKB, AIB are equal : the segments AHC, DMC are equal by the same reason ; therefore the segment AHC is to the segment AIB as the segment DMC is to the segment EKB: but the segment DME is to the segment EKB as the triangle DMC is to the triangle EKB; and therefore the segment AHC to the segment AIB, as the triangle DMC to the triangle EKB, and by inverting so that as triangle DME is to triangle EKB, thus the segment AHC is to the segment AIB : but the ratio of triangle DMC to triangle IKB is composed from the ratio NM to LK, that is, FH to IG, and from DC to EB; therefore the ratio of the segment AHC to the segment AIB is composed from the ratio HF to IG, and DC to EB. Q.e.d.

PROPOSITION CCLXII.

Some line segments AB, BC shall be marked off, and with the diameter BD dropped from B, AC shall be put in place crossing the line BD at D: then AB, BC shall be bisected at F and H, and the diameters EF, GH shall be put in place through F & H.
I say EF to GH, to be the square of the ratio of that which AD has to DC.



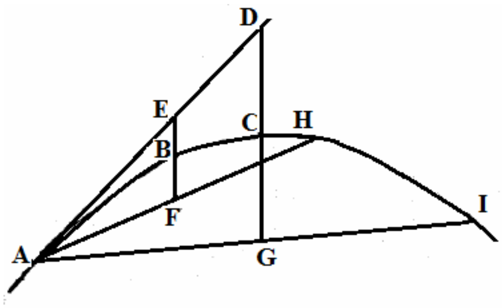
Demonstration.

The segment AB to the segment BC is in the ratio of AD to DC cubed [§.260]; but the ratio of the segment AB to the segment BC is composed from the ratio EF to GH, and from AD to DC, therefore the ratio EF to GH is the square of the ratio of that, which AD has to DC. Q.e.d.

Corollary.

Hence if follows, with the normals AI, CK drawn normal to the diameter BD ; the right line EF to GH also to be in the square ratio of that which the line AI hold to the line CK, as is evident from the demonstration.

PROPOSITION CCLXIII.



The line AD shall be a tangent to the parabola ABC at A: and with some lines AH, AI drawn from A, which shall be bisected at F and G, the diameters FB, GC shall be put in place, crossing the tangent AD at E and D.

I say the segment ABH to the segment ACI, to have the ratio composed from the ratio AE to AD, and EB to DC.

Demonstration.

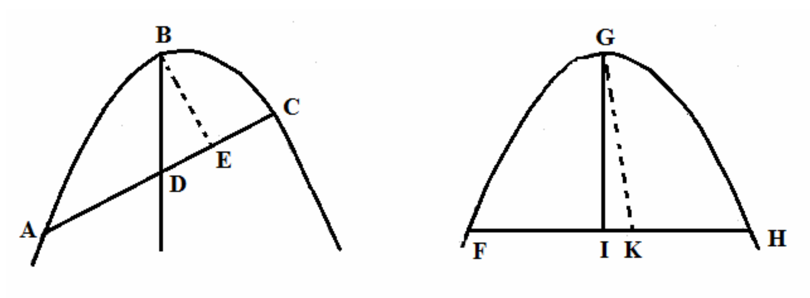
Because the ordinate lines AH, AI are put in place for the diameters BF, CG, and AD the tangent line, the lines EB, BF, likewise DC, CG are equal to each other; from which the triangle ABH is equal to the triangle AEF, and triangle ACI is equal to ADC; moreover as triangle ABH is to triangle ACL; thus the segment ABH is to the segment ACI ; and therefore segment ABH to segment ACI, as triangle AEF to triangle ADG : but the ratio of triangle AEF to triangle ADG, is composed from the ratio AE to AD, and from the ratio EF to DG, that is EB ad DC: and therefore segment ABH to segment ACI has a ratio composed from the ratio AE to AD, and EB to DC. Q.e.d.

PROPOSITION CCLXIV.

The two parabolas ABC, FGH subtend the right lines AC, FH giving rise to two equal segments ; moreover with the right lines AC, FH bisected at D and I, the diameters BD, GI shall be put in place, and from B and G, the normals BE, GK shall be send down to AC, FH.

I say that BE to be to GK, thus as FH to AC.

Demonstration.

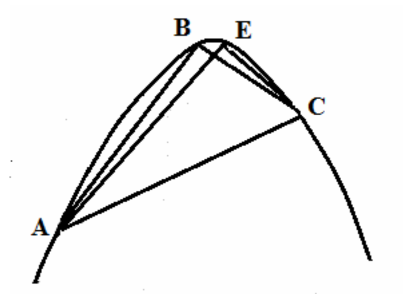


Indeed since the segments ABC, FGH shall be put equal, also the greatest triangles of these are equal to each other, so that as BE to GK, thus FH to AC. Q.e.d.

PROPOSITION CCLXV.

Two triangles shall be inscribed in the segment of the parabola ABC, and indeed ABC shall be the maximum of these triangles, which are able to be inscribed in the segment, truly ACE some other triangle.
I say the segments AE, EC taken together, to be greater than the segments AB, BC taken together.

Demonstration.

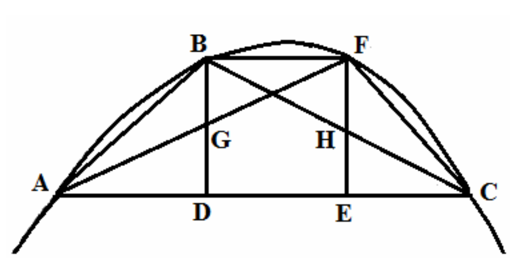


Since the triangle AEC shall be smaller than the triangle ABC, the remaining segments AE, EC are greater than the remaining segments AB, BC; and indeed the triangle ABC surpasses triangle AEC by the same excess, by which the above segments on the lines AE, EC exceed the segments on the lines AB, BC.

PROPOSITION CCLXVI.

The right line AC shall subtend the parabola ABC ; with which divided into some number of equal parts, at the points D, E ; the diameters DB, EF shall be erected ; and AB, BF, FC shall be joined.
I say the segments AB, BF, FC to be equal.

Demonstration.

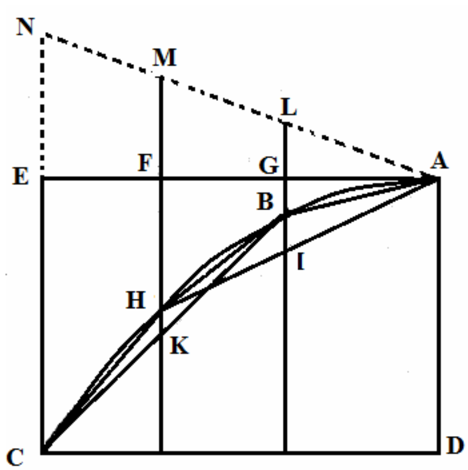


AF, BC shall be put in place and indeed AF shall cross BD at G; truly BC with the right line FE at H: so that as AD to DE, thus AG to GF, but AD, DE are equal by hypothesis ; and therefore AG, GF also shall be equal to each other, whereby ABC is the maximum triangle of these which are able to be inscribed for the segment ABF, and the segments AB, BF are equal; similarly the segments BF, FC are shown equally: therefore the segments AB, BF, FC, are equal.

PROPOSITION CCLXVII.

The line AE shall be a tangent at A to the parabola ABC of which AD is the diameter ; with which divided into equal parts by the points E, F, G, the diameters EC, FH, GB shall be drawn crossing the parabola at B, H, C; and with the points AB, BH, HC joined.
I say the segments AB, BH, HC to be equal to each other.

Demonstration.



AH, BC shall be drawn; and indeed AH shall cross the line GB produced at I; BC truly with EH at K. Because IG, FH shall be parallel and AG, GF shall be put equal, the right lines AI, IH are equal to each other [§.261-2.]: whereby the ordinate AH is put in place for the diameter IB, and ABH is the maximum triangle of these able to be described for the segment ABH: and thus the segments AB, BH are equal ; it is shown in the same manner the segment BH, HC are equal to each other ; therefore the segments AB, BH, HC are equal to each other. Q.e.d.

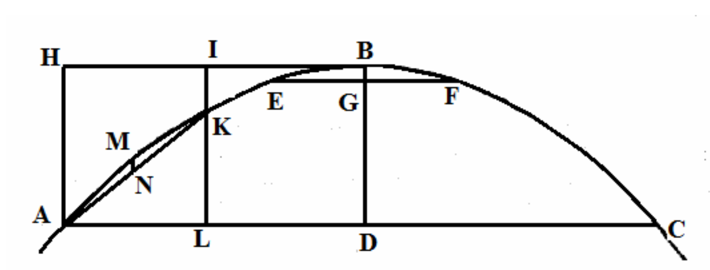
Corollary.

The proposition is true also, if the secant AN drawn from A shall be divided into equal parts at the points L and M : from which the lines LB, MH, NC parallel to the diameter AD shall be sent to the parabola. The demonstration to be apparent from the preceding.

PROPOSITION CCLXVIII.

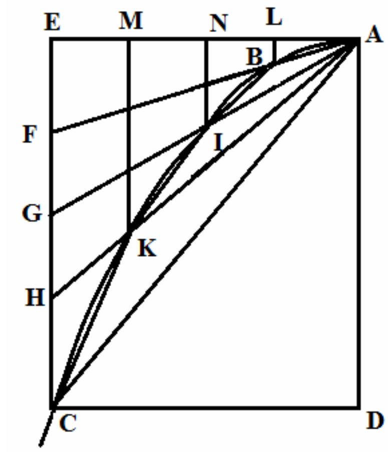
BD shall be the axis of the parabola ABC with the right ordinate EF put in place; and with the tangent BH acting through B, in that the part HI shall be taken equal to EF: and the diameters HA, IK shall be sent from H and I, crossing the parabola at A and K, and AK shall be joined.
I say the segment AK, to be equal to the segment EBF.

Demonstration.



With AK bisected at N, the diameter NM shall be drawn. Because the lines AL, EF are put equal and MN to BG is in the square ratio to AL to EF, the lines MN, BG are equal to each other ; but the ratio of the segment AK to the segment EBF is composed from the ratio MN to BG, and AL to EF; therefore the segment AK is equal to the segment EBF. Q.e.d.

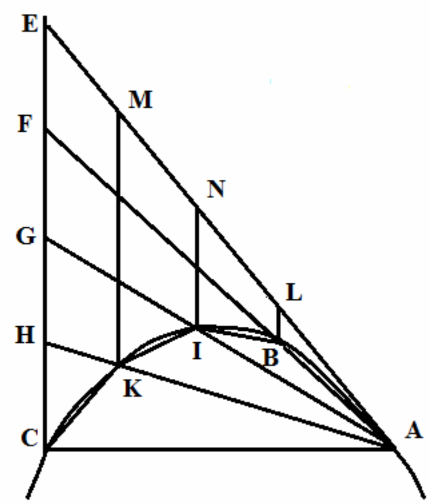
PROPOSITION CCLXIX.



The line AE shall be a tangent to the parabola ABC at A, of which AD shall be a diameter: on which with some point E assumed ; the diameter EC is put in place, which shall be cut into equal parts at F, G, H ; and with the lines AC, AH, AG, AF drawn ,which meet the parabola at B, I, K; AB, BI, IK, KC shall be joined.

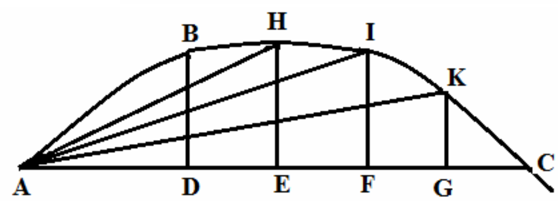
I say the segments AB, BI, IK, KC to be equal to each other.

Demonstration.



The diameters BL, IN, KM shall be erected from B, I, K: Because BL, IN, KM, CE shall be parallel to the axis, the right line AE is divided at L, N, M just as EC is divided at F, G, H: therefore the lines AL, LN, NM, ME, are equal and hence the segments AB, BI, IK, KC are equal to each other. Q.e.d.

PROPOSITION CCLXX.



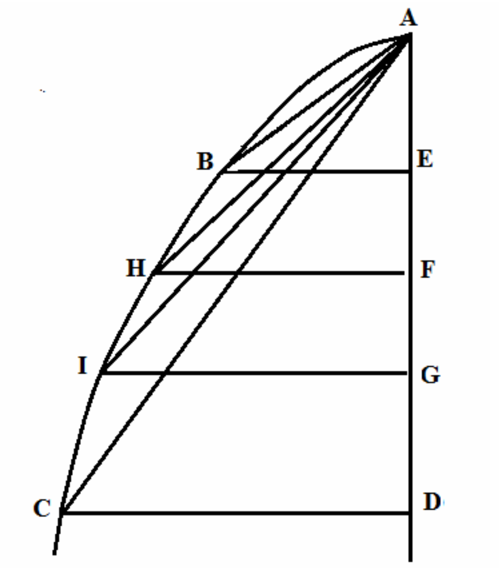
Some AC normal to the axis shall subtend the parabola ABC, which divided at D, E, F, G: so that AD, AE, AF, AG, AC shall be proportionals, the diameters shall be put DB, EH, FI, GK: and AB, AH, Al, AK shall be joined.

I say the segments AB, ABH, AHI, AIK, AKC to be in continued proportion.

Demonstration.

Segment AB to segment ABH has the ratio AD to AE cubed, [§. 267]: and segment AH to segment AE has the ratio AB to AF cubed, and thus for the rest; therefore since AD, AE, AF, AG, AC shall be continued proportionals. Q.e.d.

PROPOSITION CCLXXI.



AD shall be the diameter of the parabola ABC divided at the points E, F, G so that AE, AF, AG, AD shall be continued proportional lines: and with the ordinates EB, FH, GI, CD put in place ; AB, AH, AI, AC shall be joined.
I say the segments AB, ABH, ABI, ABC similarly shall be continued proportionals.

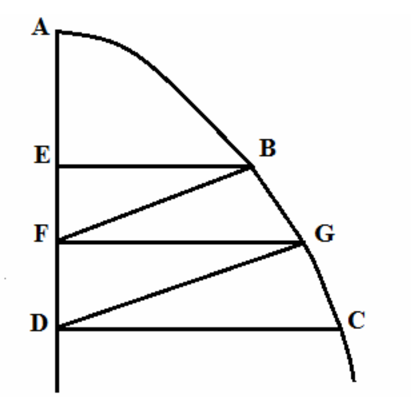
Demonstration.

The ratio of the segment AB to the segment AH is the cube of that ratio which BE has to HF ; again the segment ABH to the segment ABI has the cubic ratio of HF to IG, and thus for the others; but the lines EB, FH, GI, CD are continued proportionals, since AE, AF, AG, AD are put in continued analogous proportion ; and therefore AB, ABH, AHI, AIC are in a continued ratio. Q.e.d.

PROPOSITION CCLXXII.

AD shall be the diameter of the parabola ABC, divided at E and F, so that AE, AF, AD shall be proportionals, and the ordinates EB, FG; DC shall be put in place : and BG, GC shall be joined.
I say the segment BG to the segment GC to be in the triplicate ratio of that, which the line EB has to the line FG.

Demonstration.

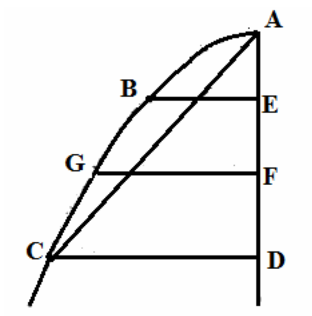


FB, DG shall be drawn. Because the lines AE, AF, AD are proportionals, EF is to FD, as AE is to AF, that is, as the square EB to the square FG. But the ratio of triangle FEB to triangle DFG is composed from the ratio EF to FD, and from the ratio EB to FG; therefore triangle FEB to triangle DFG, has the triplicate [i.e. cubic] ratio EB to FG. In the same way triangle FBG to triangle DGC, has the cubic ratio FG to DC, (since the following have composed the ratio, EF to FD, from the height to the height, and from FG to DC, that is EB to FG; (since EB, FG, DC shall be proportional); therefore the whole rectangle EBGF is to the whole rectangle FGCD in the triplicate ratio EB to FG; moreover the curvilinear EBGF is to the curvilinear DFGC in the triplicate EB to FG, for since the curvilinear EBGF is to the curvilinear DFGC in the triplicate ratio EB to FG; for since EB, FG, DC shall be proportionals of the parabola, also EAB, FAC, DAC are in continued proportion : and thus so that the parabola ABE is to the parabola FAG, thus the curvilinear EBGF is to the curvilinear FGCD, and therefore the remaining segment BG is to the remaining segment GC, in the cubic ratio EB to FG. Q.e.d.

PROPOSITION CCLXXIII.

Again the proportionals AE, AF, AD, and AF shall be equal to the right side of the diameter AD, and AB, AC shall be joined.
I say the segment AB to be to the segment AC as the square AB to the square AC.

Demonstration.



The segment AB is to the segment AC in the three-fold ratio EB to DC, that is in the six-fold ratio EB to FG, since EB, FG, DC shall be proportionals: but the square AB to the square AC is in the six-fold ratio of the line EB to FG; therefore as the square AB to the square AC, thus the segment AB to the segment AC. Q.e.d.

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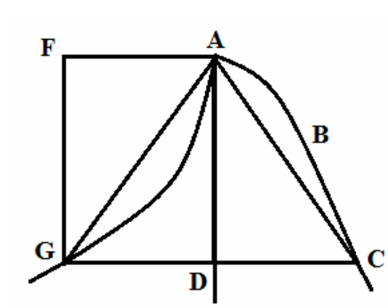
PROPOSITION CCLXXIV.

The line AF shall be a tangent at A to the parabola ABC of which the diameter is AD; then the parabola FAG shall be described through A, of which the diameter shall be AF and the tangent AD, and the ordinate line GC shall be drawn for the parabola ABC, meeting the parabola FAG at G, and AC, AG shall be drawn.
I say the segment AG to be to the segment AC, as the line GD to the line DC.

Demonstration.

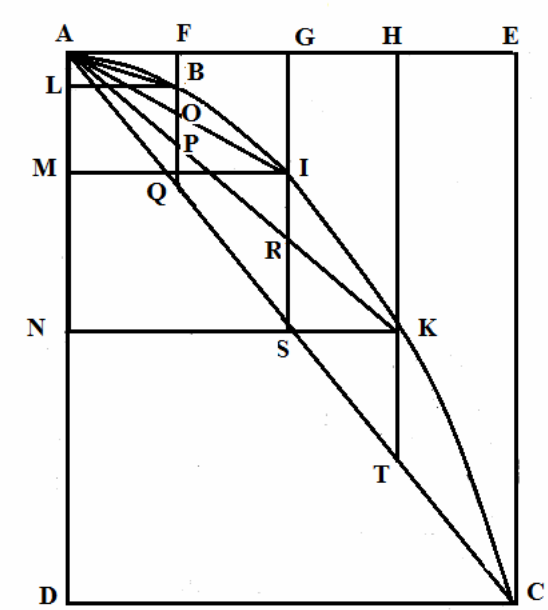


The line GF is erected from G parallel to the tangent AD, therefore the segment GA, to the segment ABC, shall be as the triangle FAG, that is triangle GDA; to triangle DAC: but GD is to DC, as triangle GDA to triangle DAC: therefore the segment AG is to the segment AC, as the line GD to the line DC. Q.e.d.

Scholium.

It pleases that in place of the 267th proposition, to consider the following proportions as if arithmetical, without doubt the lines and segments of which, just as the increment both of the convex as well as concave sections, shall be in an arithmetical progression.

AD *shall be the diameter of the parabola* ABC, *and the tangent* AE: *which is divided into some number of equal parts at the points* F, G, H, *the diameters* FB, GI, HK, EC *shall be sent crossing the parabola at* B, I, K, C: *for which the ordinates* BL, IM, KN, CD *shall be put in place* : *and with* AB, AI, AK, AC *drawn*, BI, IK, KC *shall be drawn and the lines* FB, GI, HK *shall be produced, then* AI, AK, AC *will be crossing the lines at the points* O, P, Q, R, S, T.



Therefore the segments AB, BI, IK, KC *will be equal to each other: likewise the lines* FB, BO, OP, PQ *shall be equal to each other* : *since the line* FQ *shall be divided in the same manner as* AE.

In the first place, thus the square AF *shall be to the square* AG, *as the line* FB *is to the line* GI: *but the square* AG *is four times the square* AF, *because* AG *is twice AF; and therefore* GI *is four times the line* FB, *that is,* AM *is four times* AL; *again the square* AH *is to the square* AF, *as* 9 *to* 1 *since AH shall be three times* AF, *and therefore the line* HK *is to the line* AF, *that is* AN *to* AL, *as nine to one: again the square* AE *is to the square* AF *as* 16 *to* 1, *and therefore* EC *is to* FB, *that is* AD *to* AL, *as* 16 *to* 1 *and thus likewise with regard to the remaining: therefore* AL *gives one;* AM *four*; AN *nine*; AD *sixteen, etc*.

Again since IR *shall be to* OP, *as* AI *to* AO, *that is* AG *to* AF, *moreover* AG *shall be put equal to double* AF, *and* IR *will be double* OP: *in the same manner* KT *is shown to be three times* PQ *or* OP, *and one and a half of* IR; *from which the increment of the lines* BO, IR, KT *becomes known*.

In the second case of the convex segments AB, AI, AK, AC *thus can be put in place in arithmetic proportion. Triangle* ABO *is equal to triangle* AFB, (*since* FB, BO *shall be shown to be equal*) *and thus three times the segment AB: whereby the whole triangle* ABI, *is six times the segment* AB; *therefore with the equal* BI *segments added to* AB, *the whole segment* ABI *shall be to the segment* AB *as* 8 *to* 1. *Again the triangle* AIR (*having the base* IR *double that of* OP, *and the height* IM *twice the height of* LB) *is four times triangle* ABO: *moreover triangle* IKR *is double triangle* ABO, *since they have the same height, and the base of* IR *is twice the base of* OB, *therefore the whole triangle* AIK, *is six times triangle* ABO, *so that it is as* 18 *to* 1; *therefore with the segment* IK *added, equal to the segment* AB, *and with the segment* AI, *which is eight times the segment* AB, *the whole segment* AK *shall be to the segment* AB, *as* 27 *to* 1: *again, since the triangle* AKT *shall have the base* TK *three time the base* OB, and the height NK *three times the height* LB, AKT *will be nine times triangle* ABO: *moreover triangle* KCT *is three times triangle* ABO, *and therefore the whole triangle* AKC, *will be twelve times* ABO: *and whereby shall be as* 36 *to* 1 *to the segment* AB *; therefore with the segment* KC *added, to be equal to* AB *and with the section* AIK, *which to the section* AB *is as* 27 *to* 1; *the section* AKC *to segment* AB, *as* 64 *to one ; and thus for the remaining.*

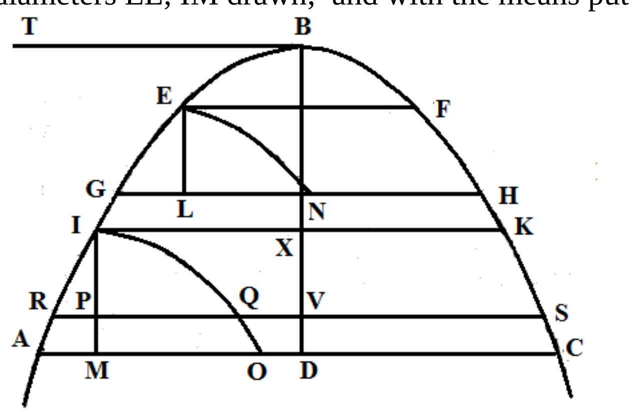
In the third place the parabola AIM *is to the parabola* ABI, *as the triangle* AIM *is to the triangle* ABL, *but triangle* AIM *is eight times triangle* ABL, (*since the base* AM *shall be shown to be four times the base* AL, *and the height* MI *twice* LB;) *therefore the parabola* AIM *is eight times* ABL: *in the same manner since* AN *shall be nine times* AL, *and* NK *three times* LB, *the triangle* AKN *shall be to the triangle* ABL *as* 27 *is to* 1; *and from which the whole parabola* AKN *will contain the parabola* ABL *and will be had by proceeding in the same manner.*

But also the same excess in the same ratio is observed of the concave segments AFB, AGI, AHK, etc., *as with the triangles* AFB, AGI, AHK, *of which the proportion is known*.

PROPOSITION CCLXXV.

BD shall be the axis of the parabola ABC, to which the ordinates EF, GH, & IK, RS shall be put in place : and then with the diameters EL, IM drawn; and with the means put in place LN, MC between GL, LH, and AM, ME, the parabolas ELN, IPQ shall be described about the axis EL, IM and the points N and Q.

I say these parabolas to be equal.



Demonstration.

BT shall be the latus rectum of the axis BV; therefore the rectangle VBT will be equal to the square VR, and the rectangle XBT equal to the square XI, that is, to the square VP: and thus with the rectangle VBT taken from the rectangle XBT, the rectangle VXBT will remain, equal to the rectangle RPS, which remains from the square VR, by 5.2, with the square VP taken away; but since from the construction RP, PQ, PS shall be continued, the rectangle RPS shall be equal to the square PQ; therefore the latus rectum BT is of the parabola IQP. And clearly by the same discussion BT is the latus rectum of parabola ENL; therefore the parabolas are equal. Q.e.d.

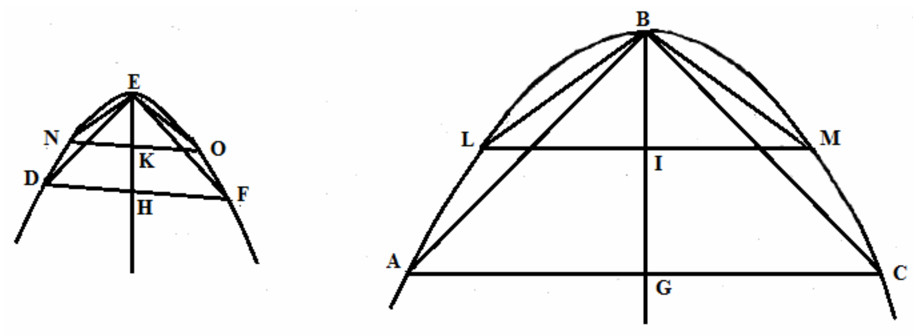
PROPOSITION CCLXXVI.

All parabolas are similar to each other.

To be observed:

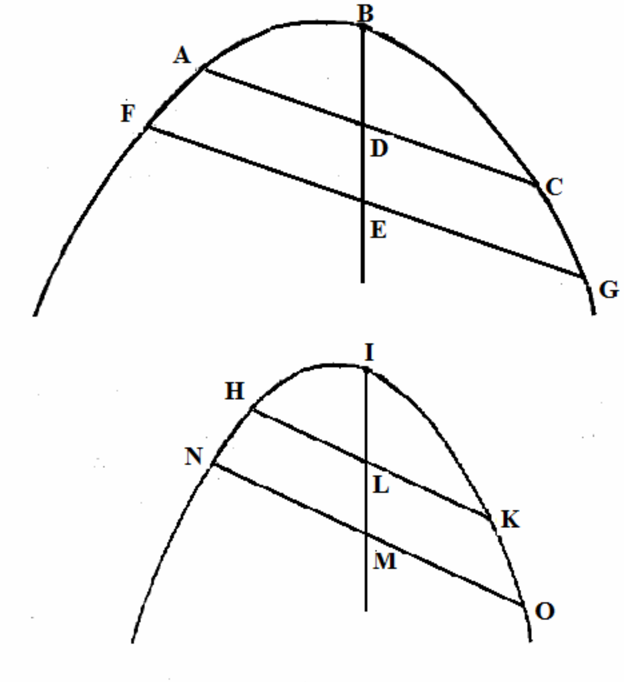
Two curved surfaces are said to be similar in a twofold manner: In the first place, when an infinite number figures similar to these can be described: and in this sense both Archimedes and Euclid showed certain curvilinear to be similar. Secondly curvilinear figures are said to be similar of which the essential properties are the same. We ourselves will demonstrate this similitude of parabolas to be present.

Demonstration.



The axes BG, EH of the parabolas ABC, DEF with their equal sides shall be put in place; and the ordinates AC, DF through G and H : and ABC, DEF shall be joined. Because the lines EH, BG are put in place with their equal right sides and AC, DF shall be the applied ordinates, the angles DHE, AGB are right: and because the triangles DHE, AGB, and hence the whole triangles DEF, ABC are similar : Again with the lines AG, EH divided proportionally at I and K, and the ordinates LM, NO shall be put through I and K : LBM, NEO shall be joined . Therefore since as BI to BG, thus EK is to EH, so that as the square LI is to the square AG, thus the square NI is to the square DH, so that the line LI to the line AG, thus the line NK to DH; and on permutating and inverting, so that as AG to DH, thus LI ad NK: But so that as AG to DH, that is BG to EH, (since AG, GB likewise DH, HE are equal) thus BI to NK; therefore by construction therefore as LI to NK, thus BI is to EK, and on permutating so that as LI ad BI, thus NK to EK: whereby since the angles LIB, NKE contained by the proportional sides shall be right, the triangles LIB, NKE, and thus the whole LBM, NEO are similar to each other: likewise if ND, OF, LA, MC shall be joined, the triangles DNE, EOF shall be shown to be similar to the triangles BLA, BMC : and thus the whole figure DNEOF to the similar figure ALBMC, which operation shall be able to be continued without end: thus, the parabolas ABC, DEF shall be similar, according to the first way.

The parabolas will be considered similar according to the second manner, thus it shall be shown, some diameter AD of the parabola ABC to be divided in some manner at the points D & E, through which the ordinates AC, FG shall be put in place ; but the diameter IL of the parabola HIK shall be divided proportionally at L & M, and the ordinates HK, NO put through L & M. Because BD shall be to BE, thus as IL to IM, as the square AC shall be to the square NO, in the same manner if again the diameters BD, IL shall be divided proportionally, and the ordinates shall be put in place at the points of division of the line, it will be shown the squares of the ordinates put in place in one parabola, to be proportional to the squares of the ordinates in the other parabola. So that since it shall always be able to be done indefinitely, it is evident the parabolas ABC, HIK to be similar according to the second manner. Q.e.d.



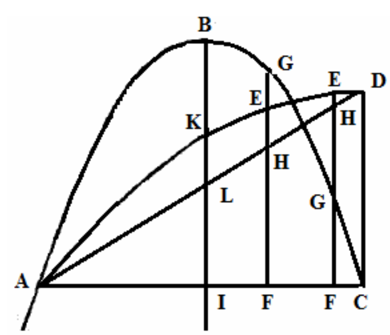
PROPOSITION CCLXXVII.

The ordinate AC shall be put in place for the axis BI of the parabola ABC, and with the diameter CD erected, some point D may be taken, and a parabola described through A and D, the diameter of which shall be DC, and with AD joined: since the diameter EF shall be put in place crossing the parabola ABC at G, and AED at E, truly the right line AD at H.

I say the parabola ABC to be to the segment AED, as the line FG to the line EH.

Demonstration.

The parabola AED shall cross the axis BI of the parabola ABC at K, and AD at L, therefore since AC is bisected at I, it shall be divided and CD shall be parallel to BI, and AD also bisected at in L and the ordinate to the diameter LK put in place: truly because AC is normal to CD, and is common to each parabola, the parabola ABC will be to the segment AED; as BI to LK, (since they shall have a ratio composed from the ratio BI to LK, and AC to AC), but as BI to LK, thus GF to EH, because LK is to EH, as BI to FG, that is as the rectangle ALD to the rectangle AHD, as the rectangle



AIC to the rectangle AFC ; therefore as FG to EH, thus the parabola ABC is to the segment AED. Q.e.d.

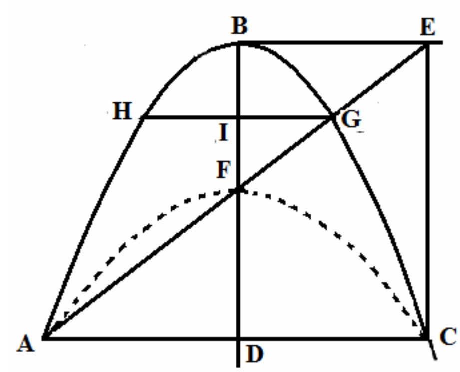
PROPOSITION CCLXXVIII.

BD shall be the axis of the parabola ABC, with the ordinate AC put in place, truly the tangent BE, shall intersect the diameter erected from C at E, moreover it shall interact AE which shall cut the axis at F: and the parabola at G, which is described through the points A, F, C, having the apex at F, and the ordinate GH shall be put in place. crossing the axis BD at I.

I say the parabola HBG to the parabola AFC to be twice the ratio of HG to AC.

Demonstration.

Since EB shall be parallel to AC, so that as AF to FE, thus FD is to FB, and EB to AD: but since AF, FE are equal, therefore EB, AD, and BF, FD are equal also: truly since EF is twice as much as GF, BI is twice IF, and EB will be twice GI ; from which the whole length HG is equal to EB, that is AD is equal to half the right line AC; whereby so that BI is to BF, that is to FD, thus as HG is to AC, but the ratio of the parabola HBG to the parabola AFC is composed from the ratio BI to FD, and HG to AC : therefore the ratio of parabola HBG to parabola AFC is the square of the ratio HG ad AC. Q.e.d.



PROPOSITION CCLXXIX.

With the same in place:

I say the parabola ABC to be eight times the parabola HBG.

Demonstration.

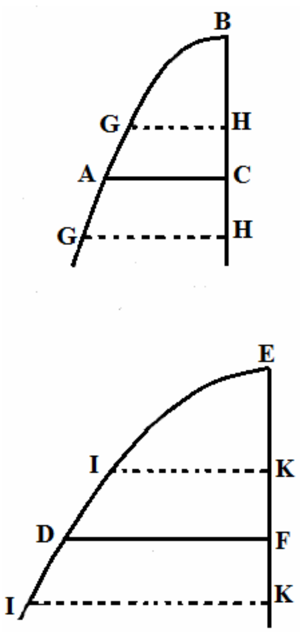
Because the lines AD, HG have been shown to be equal, and BD the quadruple of BI, triangle ABC is eight times triangle HBG, but parabola ABC is to parabola HBG, as triangle ABC to triangle HBG; therefore parabola ABC to be eight times parabola HBG. Q.e.d.

PROPOSITION CCLXXX.

BC, EF shall be the axes of the parabolas ABC, DEF, and BC shall be divided proportionally in some manner at H, and EF at K, and the ordinates HG, IK shall be put in place.

I say the parabola GBH to be to the parabola IEK, as the parabola ABC is to the parabola DEF.

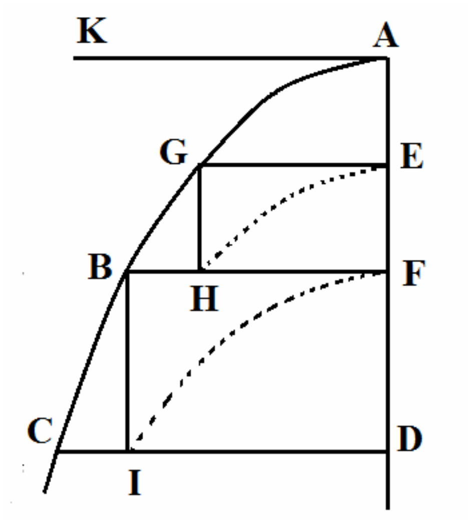
Demonstration.



The ordinates CA, FD shall be put in place: so that as BH to BC, thus EK is to EF, therefore so that as the square GH to the square AC, thus the square IK to the square DF, and on inverting and interchanging, so that as the square AC shall be to the square DF, thus the square GH shall be to the square IK, and as AC to DF, thus GH to IK, but the parabola ABC is to the parabola GBH in the cubic ratio AC to GH ; and the parabola DEF to the parabola IEK, in the cubic ratio DF to IK, therefore so that the parabola ABC shall be to the parabola in GBH, thus as the parabola DEF is to the parabola IEK: and on interchanging so that the parabola ABC shall be to the parabola DEF, thus the parabola GBH to the parabola IEK; q.e.d.

Truly if the given right sides BC, EF were equal the parabola GBH will be to the parabola IEK in the square ratio GH to IK: since the parabola ABC is to the parabola DEF in the square ratio AC to DF, since the lines AC, CB, likewise DF, FE shall be put to be equal, from which they have the composed ratio.

PROPOSITION CCLXXXI.



The axis AD of the parabola ABC shall be divided at E and F, so that AE, AF, AD shall be proportionals, and with the ordinates EG, FB, DC put in place from G and B, the diameters GH, BI shall be dropped crossing the lines FB, DC at H and I: and the parabolas having the apexes at E and F shall be described through E, H and F, I.

I say the parabola FEH to the parabola DFI to be in the cubic ratio FH to DI.

Demonstration.

Since AE, AF, AD are continued proportionals, as AE to AF, thus EF is to FD: but as AE to AF, thus the square EG is to the square FB, that is the square FH to the square DI; therefore as EF to FD, thus the square FH is to the square DI: because truly the ratio of the parabola FEH to the parabola DFI is composed from the ratio EF to FD, that is from the squared ratio FH to DI, and again from the ratio FH to DI, the parabola FEH is to the parabola DFI, in the cubed ratio FH to DI. Q.e.d.

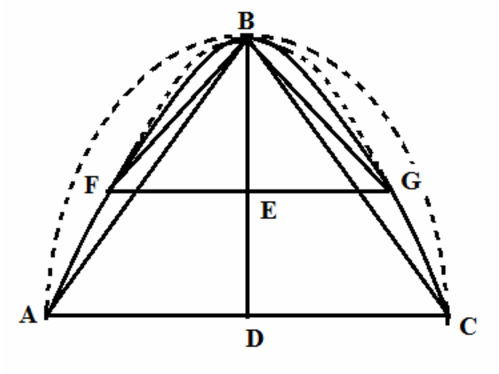
Corollary.

Hence it follows, the curvilinear figure HGB, to be to the figure IBC in the cubic ratio HB ad IC, since indeed EG, FB, DC shall be proportionals, the parabolas EAG, FAB, DAC also shall be in continued analogous proportion: and whereby the curvilinear FEGB to the curvilinear figure DFBC is as the parabola EAG to the parabola FAB, that is in the cubic ratio EG to FB, that is FH to DI: but the rectangle FG, to the rectangle DB is in the cubic ratio of the line FH to the line DI, since the ratio has been composed from the ratio EF to FD, and FH to DI. And therefore the remaining HGB to the remaining IBC is in the cubic ratio FH ad DI; truly since GE is to FB, that is as FH to DI, as FB to DC, the right line HB is to IC; the remainder to the remainder as FH to DI : therefore the figure HGB to the figure IBC, is in the cubic ratio HB to IC. Q.e.d.

PROPOSITION CCLXXXII.

BD will be the diameter of the parabola ABC, divided in some manner at D and E, thus so that neither BE nor BD shall be equal to the latus rectum of the diameter BD; and with the ordinates AC, FG described through D and E, with the points of the ellipses describe by A, B, C, and F, B, G, of which the conjugate diameters shall be AC, BD, FG, BE.

I say the parabola ABC to be to the parabola FBG, as the ellipse ABC to the ellipse FBG.



Demonstration.

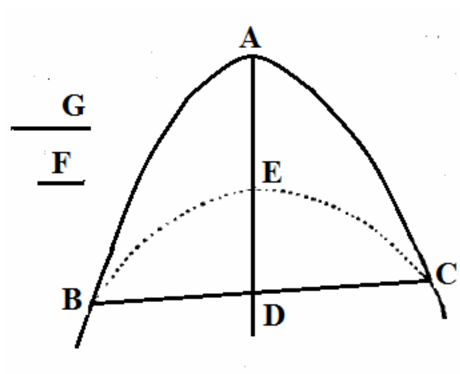
ABC, FBG shall be joined, so that as triangle ABC to triangle FBG, thus the parabola ABC is to the parabola FBG: but the ellipse ABC is to the ellipse FBG, as triangle ABC to triangle FGB; as the parabola ABC is to the parabola FBG, etc. Q.e.d.

PROPOSITION CCLXXXIII.

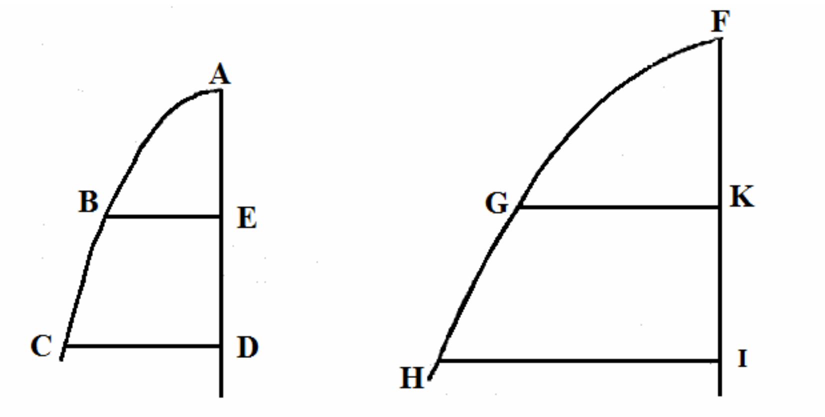
Some right line BC shall subtend the parabola ABC, it shall be required to describe a parabola on that which shall have the given parabola ABC in the ratio F to G.

Construction and demonstration.

With BC bisected at D, the diameter shall be erected DA, which shall be divided at E, so that AD shall be to DE, thus as G is to F; then a parabola shall be described through the points B, E, C, of which the diameter shall be DE, and the ordinate BC applied to that, I say what is sought to be done. Because the parabolas ABC, BEC have the common subtending chord BC, the parabola BEC is to the parabola ABC, as the line ED to the line AD, that is as F to G.



PROPOSITION CCLXXXIV.



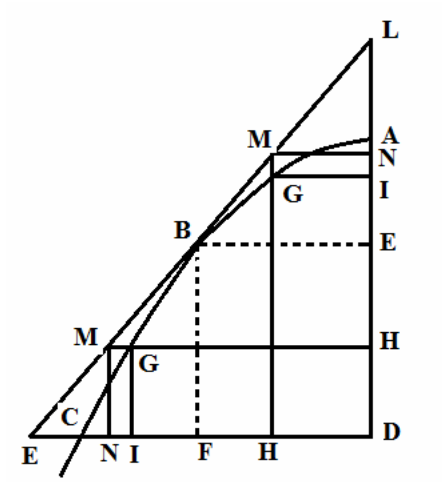
AD will be the diameter of the parabola ABC, divided in some manner at E and D, and the ordinates BE, CD put in place. Moreover FI shall be the diameter of the parabola FGH, divided at some point I and with the ordinate HI put in place ; it will be necessary to divide the parabola FHG again, just as the parabola ABC is divided.
.

Construction and demonstration.

It shall be done so that as AD shall be to AE, thus FI shall be to FK, and the ordinate KG shall be put in place from K : I say what is required to be done. Because as AE shall be to AD, thus FK shall be to FI, and there will be GK to HI, as BE to CD: but the parabola BAE to the parabola CAD is in the threefold ratio of the line BE to CD, and the parabola GFK to the parabola HFI to be in the threefold ratio GK to HI, therefore as the parabola BAE is to the parabola CAD, thus the parabola GFK is to the parabola HFI.

Q.e.d.

PROPOSITION CCLXXXV.



DC shall be the ordinate put in place of the diameter AD of the parabola ABC ; and with AD divided at E, so that ED doubled shall be AE, the ordinate EB shall be put in place from E, and the diameter BF shall be dropped from B, crossing the right line DC at F.

I say the parallelogram DEBF to be the greatest of these, which can be inscribed in the angle EBF at the ends of the parabolas ABCD.

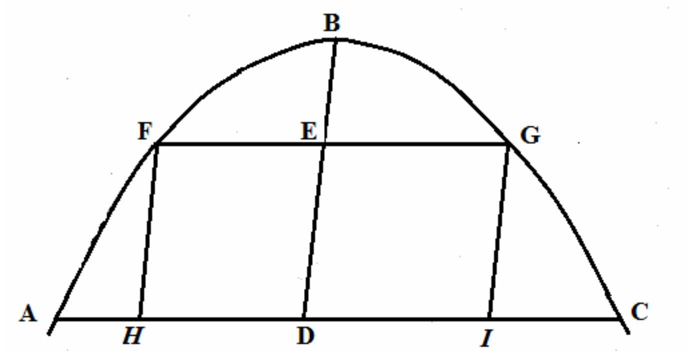
Demonstration.

For some parallelogram IGH shall be inscribed having the angle IGH equal to the angle EBF : and it shall be acting through the contact point B of the tangent LK, crossing the diameter AD at L, and the line DC at K, and truly the right line HG at M ; from which the right line MN shall be put parallel to GI. Because BL is a tangent, and EB the ordinate put in place, the right lines LA, AE are equal ; and thus the whole length LE, is equal to ED which is put to be double AE: from which LK also is bisected at B: therefore the parallelogram DB is greater than the parallelogram DM: but the parallelogram DM is greater than the parallelogram DG: since the point M falls outside the parabola; therefore the parallelogram DB, is much greater than the parallelogram DG: it is shown likewise from any other parallelogram; therefore DB is the maximum of these parallelograms which can be inscribed in the angle EBF at the ends of these parabolas ABC. Q.e.d.

PROPOSITION CCLXXXVI.

To inscribe the maximum parallelogram of a given terminated parabola.

Construction and demonstration.

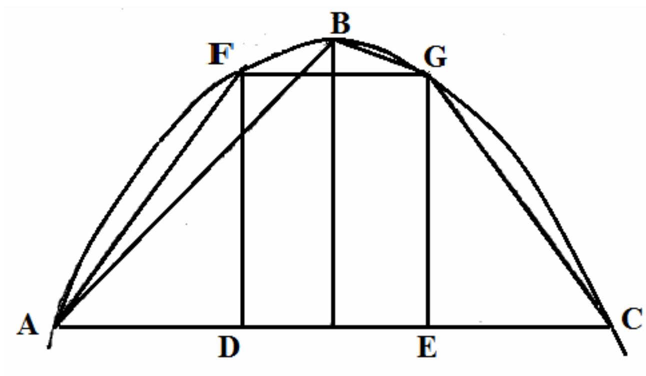


AC shall be the ordinate put in place for the diameter BD of the parabola ABC ; it is required to inscribe the maximum parallelogram of the parabola ABC, with DB divided at E, so that EB doubled shall be ED, the ordinate line FG shall be put through E, and the diameters FH, GI shall be dropped through F and G crossing the line AC at H and I. From the previous proposition it is clear, the parallelogram HGIF to be that which is required.

PROPOSITION CCLXXXVII.

To inscribe a regular polygon with a given number of sides in a terminated parabola.

I call a polygon regular, of which the individual side segments are equal, besides the subtending chords.



AC shall subtend the given parabola ABC: in order to inscribe a regular polygon in the parabola ABC, with four equal sides, AC put in place shall be trisected at D and E, and from D and E the diameters DF, EG shall be put in place and AF, FG, GC shall be joined; I say the polygon AFGC to be required to be satisfactory. Indeed since the lines AD, DE, EC shall be equal, also the segments AF, FG, GC are equal: therefore the regular polygon is the quadrilateral AFGC we have inscribed, etc. Q.e.d.

PROPOSITION CCLXXXVIII.

With the same in place:
I say the quadrilateral AFGC to be the maximum of these which are able to be inscribed in the terminated parabola ABC.

Demonstration.

For some other quadrilateral ABGC may be described : so that indeed in the first place it shall have the line CG common with that quadrilateral AFGC : therefore since AF, FG are equal segments, these are smaller than the segments AB, BG; therefore the remaining rectilinear figure AFGC is greater than the figure ABGC: similarly it is shown some other quadrilateral to be smaller than the quadrilateral AFGC: therefore the maximum of these is that one of these which can be inscribed in the parabola ABC.

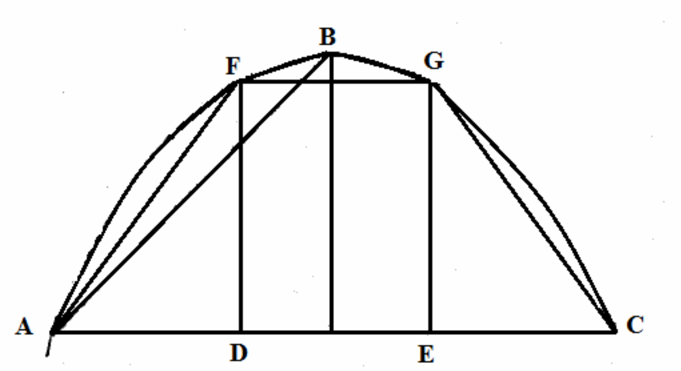
Corollary.

What I have said concerning regular quadrilaterals, the same can be said about any side of a regular polygon requiring to be understood: and all agreed from the same construction and demonstration.

PROPOSITION CCLXXXIX .

Given terminated parabolas, to inscribe the maximum of these polygons which can be inscribed with a given number of sides
.

Construction and demonstration.



The maximum quadrilateral shall be required to be inscribed in the terminated parabola : the regular quadrilateral AFGC to be inscribed in the terminated parabola ABC : I say that to be the maximum of these which are able to be inscribed with the same number of sides. The demonstration is evident from the preceding.

PARABOLAE

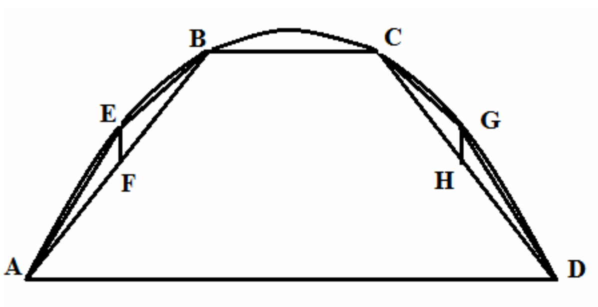
PARS SEXTA

Segmenta primum & parabolas inter se confert; dein figuras maximas sectioni inscribit.

PROPOSITIO CCLIII.

Secent ABC parabolam parallelae quaevis duae AD, BC, ducanturque lineae AB, CD. Dico illas segmenta auferre aequalia.

Demonstratio.



Segmentis AB, CD triangula inscribantur maxima AEB, CGD; quae cum BC, AD, aequidistent, inter se aequalia sunt :unde & segmenta aequantur. Quod erat demonstrandum.

PROPOSITIO CCLIV.

Eadem manetc figura oportet ex dato in peripheria puncto C, rectam ducere, quae segment auferat aequale dato AB.

Constructio & demonstratio.

Iuncta BC, ducatur AD parallela BC, iunganturque CD: manifestum est per praecedentem DC ex dato puncto eductam, segment auferre aequale dato AB. Quod erat demonstrandum.

PROPOSITIO CCLV.

Dato segmenta ABC & diametro GH oportet ad illam ordinatim applicare lineam, quae segment auferat dato aequale.

Constructio & demonstratio.

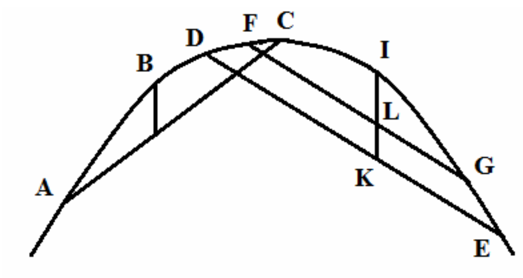
Divisae AB bifariam in F, erigitur diameter FE, cui fiat aequalis GH: & per H ordinatim ponatur CD, dico factum esse quod petitur: iungantur enim AEB, CGD; quoniam EF, GH diametri aequales sunt, triangulae quoque AEB, CGD aequalia sunt: quae cum maxima sint illorum quae segmentis AB, CD inscribi possunt, segmenta quoque AB, CD aequalia sunt; applicuimus igitur ad datam diametrum, &c. Quod erat faciendum.

PROPOSITIO CCLVI.

Data parabola ABC, & in illa segmento AC, oportet cuicunque ED equidistantem ducere, quae segment auferat dato aequale.

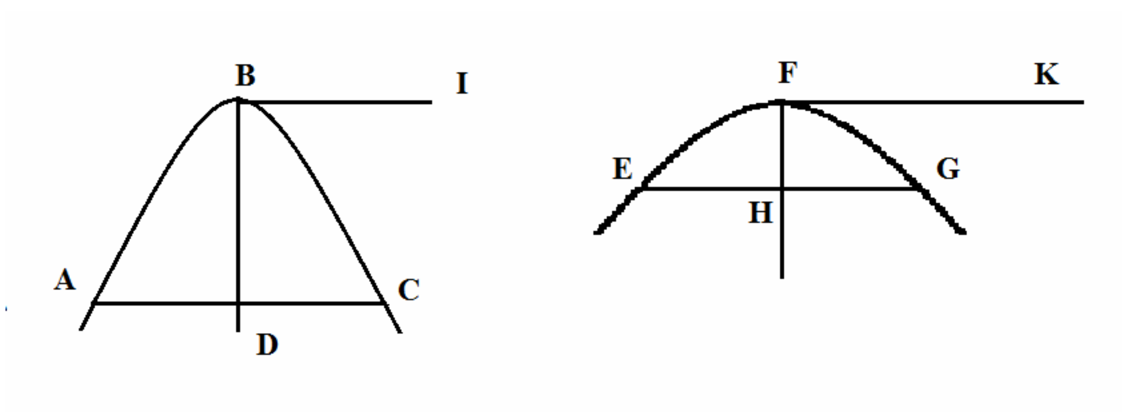
Constructio & demonstratio.

Divisis AC, DE bifariam in H & K; erigantur diametri HB, IK: factaque IL aequali HB ae ponatur per L, FG aequidistans DE, patet per praecedentem FIG segment dato AC, aequale esse; igitur lineam duximus aequidistance DE quae segment FIG auferat aequale dato. Quod erat postulatum.



PROPOSITIO CCLVII.

Sit ad ABC parabolae axe in BD cuius latus rectum BI, ordinatim posita AC: sit autem & EFG parabola, cuius axis FH; & latus rectum FK; oportet ex EFG parabola segment auferre; quod ad segment ABC, rationem habeat quam BI ad FK.



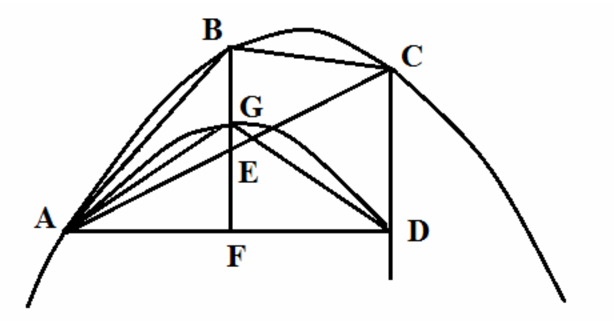
Constructio & demonstratio.

Fiat ut IB ad FK, sic FH ad BD, & per H ordinatim ponatur EG. Dico factum esse quod petitur: cum enim sit ut IB ad FK, sic FH ad BD, rectangulum super IBHD id est quadratum AD, aquale est rectangulo super FK, FH id est qua ratio EH unde AC, EG inter se aequales; sunt triangulum igitur maximum segmenti EFG, ad triangulum maximum segmenti ABC est ut FH ad BD , id est per constructionem BI ad FK; ergo & segment EFG ad segment ABC, ut IB ad FK; abstulimus igitur ex EFG parabola segment ABC, ut IB ad FK; abstulimus igitur ex EFG eam rationem continet quam latus rectum BI, ad latus rectum FK. Quod exhibendum erat.

PROPOSITIO CCLVIII.

Sit ad ABC parabolae diametrum BE ordinatim posita AC: ductaque AD normali, ad diametrum ex C demissam, fiant BE, FG lineae aequales; & per AGD, parabola describatur , cuius axis GF.
Dico AGD segment aequari segmenta ABC.

Demonstratio.

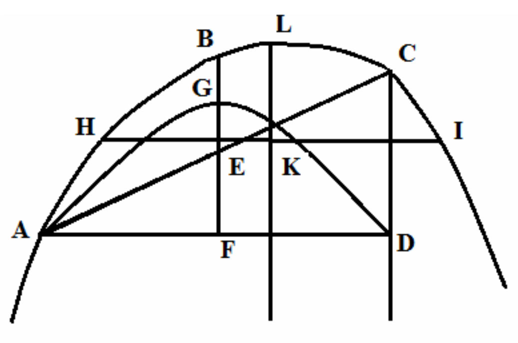


Quoniam BE, GF lineae aequales sunt, triangula BCE, GDF, item BAE, GAF, ac propterea ABC, AGD triangula inter se aequalia sunt, sed quoque maxima sunt illorum quae segmentis ABC, AGD, inscribi possunt ; segmenta igitur ABC, AGD aequalia funt. Quod erat demonstrandum.

PROPOSITIO CCLIX.

Ponatur LK axis parabolae ABC, aequalis diametro BE, & ordinatim per K linea HI.
Dico illam rectae AD aequalem existere.

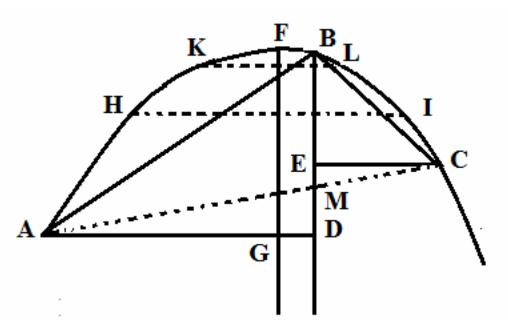
Demonstratio.



Est enim per praecedentem AGD parabola aequalis parabolae ABC id est HLI: igitur & triangula maxima AGD, HLI sunt aequalia : sunt autem ex hypothesi illorum altitudines LK, FG aequales, igitur & bases AD, HI inter se aequales sunt.
Quod fuit demonstrandum.

PROPOSITIO CCLX.

Secent ABC parabolam lineae quaevis AB, BC : demissaque ex B diametro BD, ponantur ad illam ex A & C normales AD, CE.
Dico segment AB esse ad segment BC in triplicata ratione lineae AD ad CE.



Demonstratio.

Invento axe FG applicentur ad illum ordinatim lineae HI, DL: & HI quidem sit aequalis AD; KL vero rectae CI: erit igitur segmenta AB aequale segment HFI, & segmento BC aequae segment KFL, sed HFI segment ad segment KFL, triplicatam habet rationem lineam HI ad lineam KL; igitur & segment AB ad segment BC, triplicatam habet rationem lineae HI ad KL, lineam, id est ex hypothesi AD ad EC. Quod erat demonstrandum.

Corollarium.

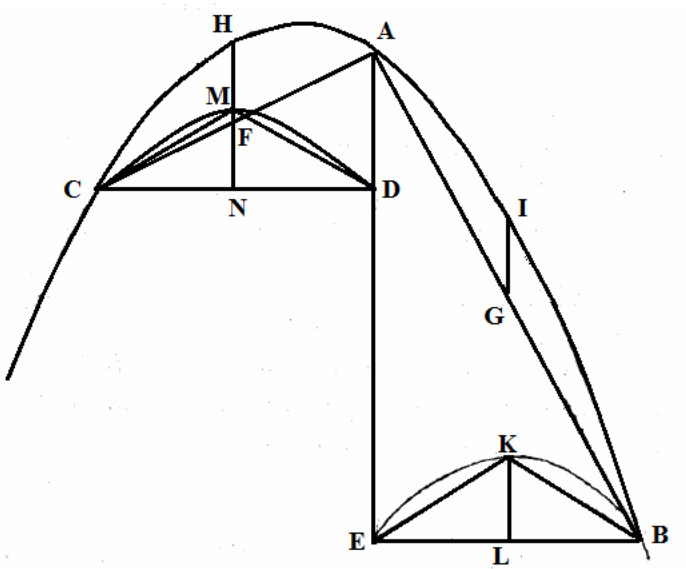
Hinc sequitur iuncta AC, quae occurrat BD in M, AB segment ad segment BC triplicatam habere rationem AM ad MC, patet, cum sit ut AD ad CE, sic AM ad MC, ergo, &c.

PROPOSITIO CCLXI.

Parabolam ABC, secent duae quaevis lineae AB, AC: demissaque ex A diametro AD, ponantur ad illam ex B & C normales BE, CD: dein AB, AC lineis bifariam divisis in F & G, erigantur diametri FH, GI.

Dico segment AHC ad segment AIB, rationem habere compositam, ex ratione FH ad IG, & CE ad BE.

Demonstratio.



Divisis EB, CD lineis bifariam in L & N, erigantur normales LK, NM: & LK quidem aequalis IG:NM vero aequalis HF: & per E, K, B, item C, M, D puncta, parabolae describantur, quarum axes sint LK, MN: iunganturque EKB, CMD. Quoniam LK aequalis est IG, segmenta EKB, AIB aequalia sunt: eadem de causa aequalia sunt segmenta AHC, DME ; segment igitur AHC est ad segment AIB ut DME segment; est ad segment EKB: sed DME segmentam est ad segment EKB ut DME triangulum ad triangulum EKB; igitur & AHC segment, ad segment AIB, est ut DME triangulum EKB, & invertendo ut triangulum DME ad triangulum EKB, sic
AHC segment est ad segment AIB : sed ratio trianguli DME ad triangulum

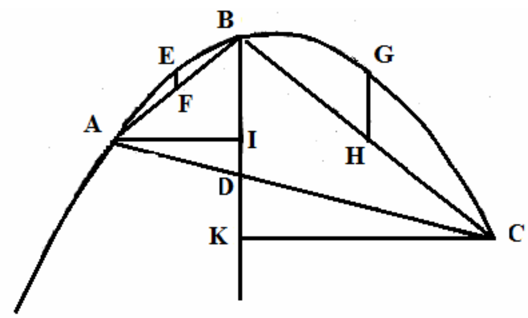
IKB est composita ex ratione NM, ad LK,id est FH ad IG, & ex DC ad EB; ratio igitur segmenti AHC ad segment AIB composita est :ex ratio HF ad IG, & DC ad EB. Quod erat demonstrandum.

PROPOSITIO CCLXII.

Auferant AB, BC lineae :segmenta quicunque, demissaque ex B diametro BD, ponatur AC occurrens BD lineae in D: dein AB, BC divisis bifariam in F & H, ponantur per F & H, diametri EF,GH.

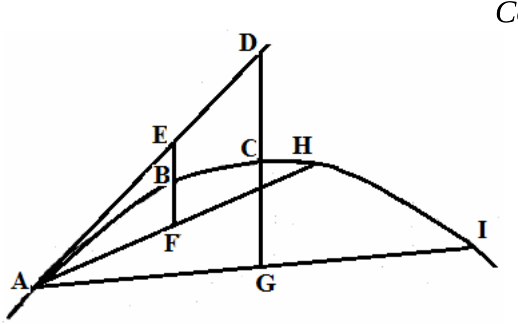
Dico EF ad GH, duplicatam habere rationem eius quam habet AD ad DC.

Demonstratio.



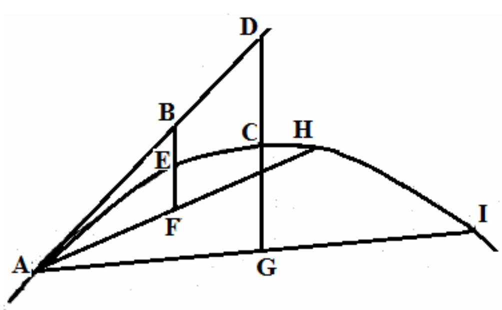
Segment AB ad segment BC in triplicata est ratione AD ad DC; sed ratio segmenti AB ad segment BC composita est ex ratione EF ad GH, & ex AD ad DC, ratio igitur EF ad GH duplicata est rationis eius, quam habet AD ad DC. Quod fuit demonstrandum.

Corollarium.



Hinc sequitur, ductis AI, CK normalibus ad diametrum BD ; rectam EF ad GH duplicatam quoque habere rationem eius quam obtinet AI linea, ad lineam CK, ut patet ex demonstratione.

PROPOSITIO CCLXIII.



Parabolam ABC contingat in A linea AD: ducanturque ex A linaeae quaevis AH, AI, quibus in F & G bifariam divisis, ponantur diametri FB, GC, occurrences AD contingenti in E & D.

Dico ABH segment, ad segment ACI, rationem habere compositam ex ratione AE ad AD, & EB ad DC.

Demonstratio.

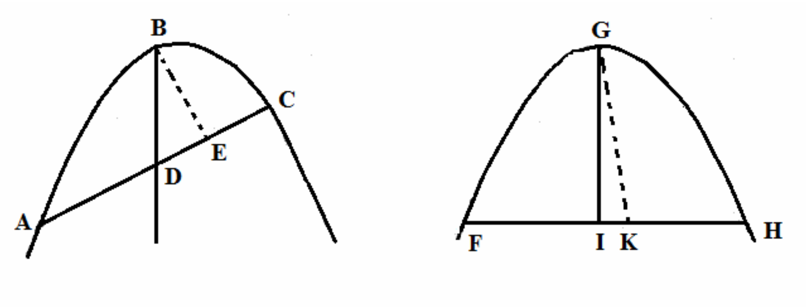
Quoniam AH, AI lineae ordinatim ponuntur ad diametros BF, CG, & AD linea contingens, lineae EB, BF, item DC, CG inter se aequales sunt; unde ABH triangulum aequale trianguli AEF ; igitur & segment ABH est ad segment ACI, ut triangulum ad triangulum, ACI , & ACI triangulum aequale triangulo ADC; est autem ut ABH triangulum ad triangulum; sic ABH segment ad segment ACI; igitur & segment ABH ad segment ACI, ut AEF triangulum ad triangulum ADG· sed ratio trianguli AEF ad triangulum ADG, composita est ex ratione AE ad AD, & ex ratione EF ad DG, id est EB ad DC: igitur & segment ABH ad segment ACI rationem habet compositam ex ratione AE ad AD, & EB ad DC. Quod erat demonstrandum.

PROPOSITIO CCLXIV.

Parabolas duas ABC, FGH subtendant rectae AC, FH segmenta auferentes aequalia; rectis autem AC, FH divisis in D & I bifariam, ponantur diametri BD, GI, & ex B & G, demittantur BE, GK normales ad AC, FH.

Dico esse ut BE ad GK, sic FH ad AC.

Demonstratio.



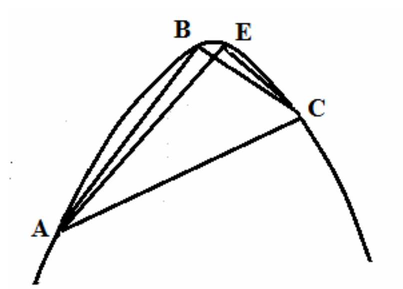
Cum enim segmenta ABC,FGH ponantur aequalia, triangula quoque illorum maxima inter se aequalia sunt, unde ut BE ad GK, sic FH ad AC. Quod erat demonstrandum.

PROPOSITIO CCLXV.

Parabolae ABC segmento, trianglula duo inscripta sint, & ABC quidem illorum maximum, quae segmento inscribi possunt, alterum vero AEC quodcunque.

Dico segmenta AE, EC simul sumpta, maiora esse segmentis AB, BC simul sumptis.

Demonstratio.

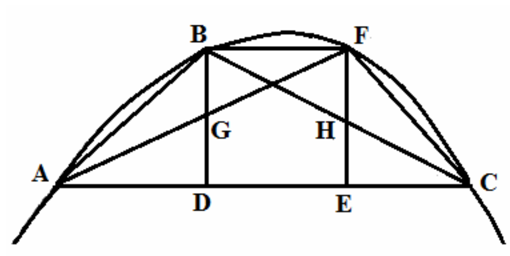


Cum triangulum AEC minus sit triangulo ABC, residua AE, EC segmenta, maiora sunt residuis segmentis AB, BC; eodem etenim excessu superat triangulum ABC triangulum AEC, quo segmenta super lineis AE, EC cxcedunt segmenta super AB, BC.

PROPOSITIO CCLXVI.

Parabolam ABC subtendat recta AC; qua divisa in quotvis partet aequales, in punctis D, E; igitur diametri DB, EF; iunganturque AB, BF, FC.
Dico segmenta AB, BF, FC aequalia esse.

Demonstratio.

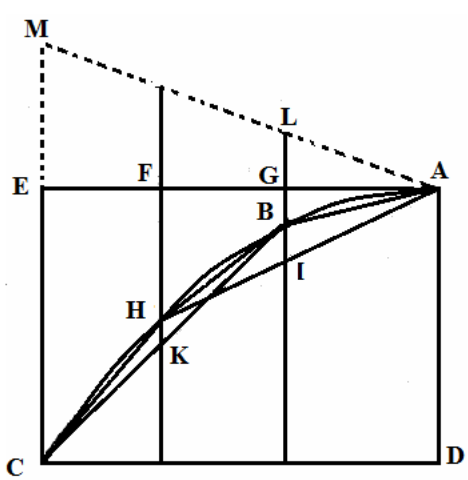


Ponantur AF, BC & AF quidem occurrat BD in G; BC vero rectae FE in H: ut AD ad DE, sic AG ad GF, sed AD, DE per hypothesin aequales sunt ; igitur & AG, GF quoque inter se aequantur, quare ABC triangulum maximum est eorumque ABFsegmento inscribi possunt, & AB, BF segmenta sunt aequalia; similiter aequalia ostenduntur segmenta BF, FC: segmenta igitur AB, BF, FC, aequalia sunt.

PROPOSITIO CCLXVII.

Parabolam ABC cuius diameter AD, contingat in A linea AE; qua divisa in partes aequales, punctis E, F, G, demittantur diametri EC, FH, GB, occurrences parabolae in B, H, C; iunganturquc AB, BH, HC:
Dico segmenta AB, BH, HC esse inter se aequalia.

Demonstratio.



Ducantur AH, BC; & AH quidem occurrat GB lineae productae in I; BC vero ipsi EH, in K. Quoniam IG, FH aequidistant & AG, GF ponuntur aequales, rectae AI, IH inter se aequales sunt: quare AH ordinatim posita est ad diametrum IB, & ABH triangulum maximum est eorumquae segmenta ABH inscribi possunt: adeoque & segmenta AB, BH aequalia sunt ; eodem modo ostenduntur segmenta BH, HC inter se aequalia sunt ; eodem modo ostenduntur segmenta BH, HC inter aequari l segmenta igitur AB, BH, HC aequalia sunt.
Quod erat demonstrandum.

Corollarium.

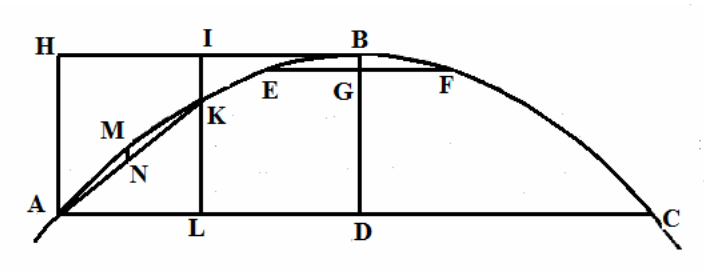
Propositio quoque vero est si ex A ducta secans AN dividatur in partes aequales punctis L, M: ex quibus in parabolam rectae emittantur LB, MH, NC parallelae diametro AD. demonstratio patet ex praecedenti.

PROPOSITIO CCLXVIII.

Sit ad ABC parabolae axem BD ordinatim posita recta EF; actaque per B contingente BH, sumantur in illa, portio HI aequalis EF: & ex H & I, diametri demittantur HA, IK, occurrentes parabolae in A & K, iunganturque AK.

Dico segment AK, aequari segmento EBF.

Demonstratio.

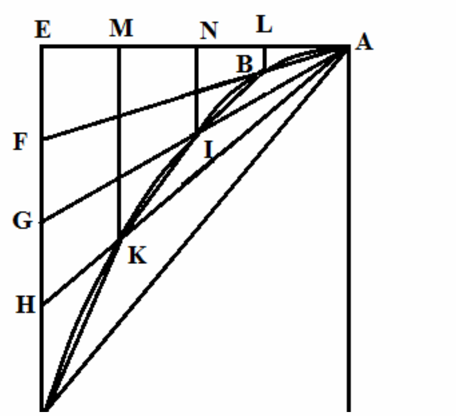


Divisa AK bifariam in N ducatur diameter NM. Quoniam AL, EF lineae ponuntur aequales & MN ad BG in duplicata est ratio AL ad EF, rectae MN, BG inter se aequales sunt ; sed ratio segmenti AK ad segment EBF composita est ex ratione MN ad BG, & AL ad EF; segment igitur AK aequale est segmento EBF. Quod erat demonstrandum.

PROPOSITIO CCLXIX.

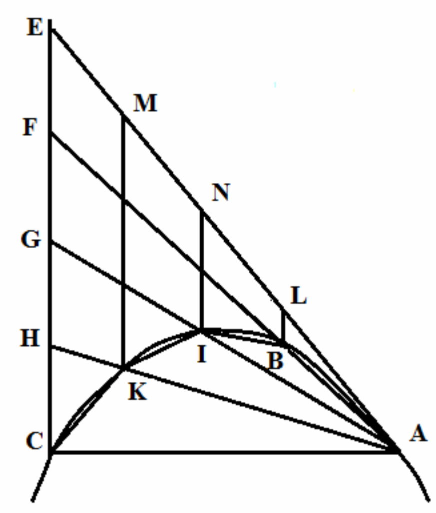
Parabolam ABC cuius diameter AD, contingat in A linea AE : in qua assumpto quovis puncto E; ponatur diameter EC, quae in F, G, H punctis secetur in partes aequales; ductisque AC, AH; AG ,AF lineis quae parabolae occurrant in B, I, K: iungantur AB, BI, IK, KC.

Dico segmenta AB,BI, IK, KC esse inter se aequalia.



Demonstratio.

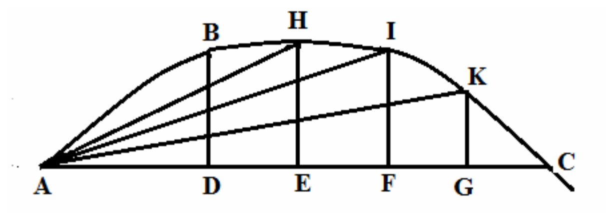
Erigantur ex B, I, K diametri BL, IN, KM: Quoniam BL, IN, KM, CE aequidistant axi, recta AE in L, N, M divisa est sicut EC divisa in F, G, H: igitur lineae AL, LN, NM, ME, aequales sunt ac proinde segmenta AB, BI, IK, KC inter se aequalia.
Quod erat demonstrandum.



PROPOSITIO CCLXX.

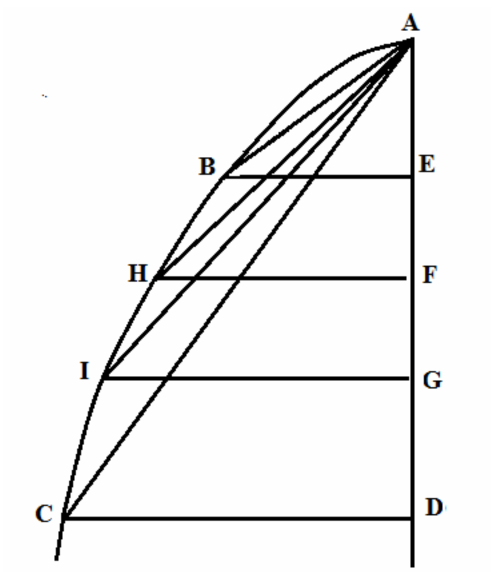
Parabolam ABC subtendat quaevis AC normalis ad axem parabolae, quam divisam in D, E, F, G: ut AD, AE, AF, AG, AC proportionales sint, ponantur diametri DB, EH, FI, GK: iunganturque AB, AH, Al, AK.
Dico segmenta AB, ABH, AHI, AIK, AKC in continua esse analogia.

Demonstratio.



Segment AB ad ABH segment, rationem habet triplicatam lineae AD ad AE: & AH segment ad segment AE triplicatam habet rationem AB ad AF, & sic de ceteris; igitur cum AD, SE, AF, AG, AC continues sint proportionales. Quod erat demonstrandum.

PROPOSITIO CCLXXI.



Sit ABC parabolae diameter AD divisa in E, F, G punctis ut A E, AF, AG, AD lineae sint continue proportionales: pofitisque ordinatim EB, FH, GI, CD; iungantur AB, AH, AI, AC.
Dico segmenta AB, ABH, ABI, ABC in continua esse analogia.

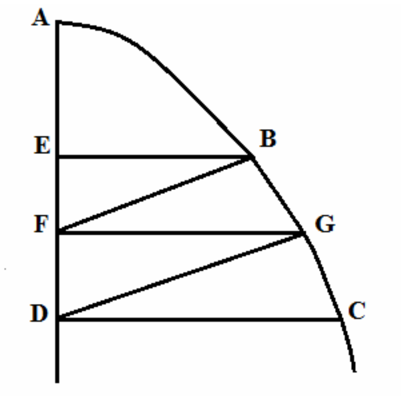
Demonstratio.

Ratio segmenti AB ad segment AH triplicata est eius quam habet BE ad HF; rursum ABH segment ad segment ABI triplicatam habet rationem HFad IG, & sic de caeteris; sed EB, FH, GI, CD lineae continue sunt proportionales, quoniam AE, AF,AG, AD in continua ponuntur analogia; igitur & segmenta AB, ABH, AHI, AIC sunt in ratione continuata.Quod erat demonstrandum.

PROPOSITIO CCLXXII.

Sit ABC parabolae diameter AD, divisa in E & F, ut AE, AF, AD sint proportionales, & ordinatim ponantur EB, FG; DC : iunganturque BG, GC.
Dico segment BG ad segment GC rationem habere triplicatam eius, quam habet EB linea ad lineam FG.

Demonstratio.

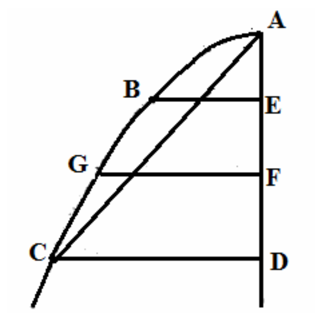


Ducantur FB, DG. Quoniam AE, AF, AD linea proportionales sunt, EF est ad FD, ut AE ad AF id est: ut quadratum EB ad quadraturo FG. Sed ratio trianguli FEB ad triangulum DFG composita ex ratione EF ad FD, & ex EB ad FG; triangulum igitur FEB ad DFG triangulum, triplicatam habet rationem EB ad FG. Eodem modo triangulum FBG ad DGC, triangulum triplicatam habet rationem FG ad DC, (cum rationem habeant compositam ex EF ad FD altitudine ad altitudineme & ex FG ad DC, id est EB ad FG; (cum EB, FG, DC proportonales sint) igitur totum rectilineum EBGF est ad totum rectilineum FGCD in triplicata ratione EB ad FG; sed & mixtilineum EBGF est ad mixtilineum DFGC in triplicata ratione EB ad FG, nam cum EB, GF est ad mixtilineum DFGC in triplicata ratione EB ad FG; nam cum EB, FG, DC proportionales sint parabolae quoque EAB, FAC, DAC in continua sunt analogia : adeoque ut ABE parabola est ad parabolam FAG, sic EBGF mixtilineum est ad mixtilincum FGCD, igitur & reliquum segment BG est ad reliquum GC, in triplicata ratione EB ad FG. Quod erat demonstrandum.

PROPOSITIO CCLXXIII.

Sint denuo proportionales AE, AF, AD, & AF aequalis lateri recto diametri AD, & iungantur AB, AC.
Dico segment AB esse ad segment AC ut quadratum AB ad quadratum AC.

Demonstratio.



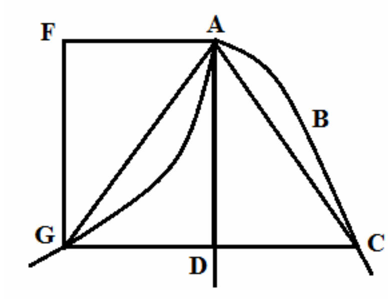
Segment AB est ad segment AC in triplicata ratione EB ad DC, id est sextuplicata EB ad FG, cum EB, FG, DC proportionales sint: sed AB; quadratum ad quadratum AC rationem habet sextuplicatam lineae EB ad FG, igitur ut quadratum AB ad quadratum AC, sic AB segment ad segment AC. Quod erat demonstrandum.

.

PROPOSITIO CCLXXIV.

Parabolam ABC cuius diameter AD, contingat in A linea AF; dein per A describatur parabola FAG cuius AF sit diameter & contingens AD, ducaturque in ABC parabola ordinatim linea GC , occurrens FAG parabolae in,G, iunganturque AC, AG.
Dico segment AG esse ad segment ABC, ut GD linea ad lineam DC.

Demonstratio.

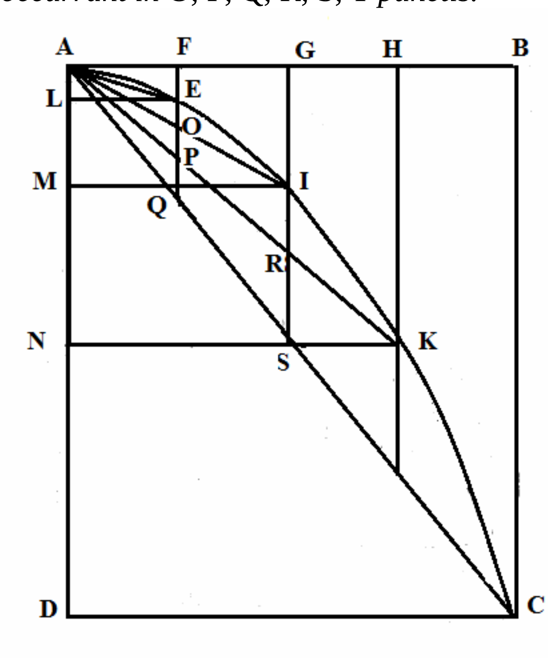


Erigitur ex G linea GF parallela contingenti AD, erit igitur segment GA, ad segment ABC, vt FAG triangulum id est triangulum GDA; ad triangulum DAC: sed GD est ad DC, ut GDA triangulum ad triangulum DAC: segment igitur AG est ad segment AC, ut GD linea ad lineam DC. Quod erat demonstrandum.

Scholion.

Lubet hoc loco propositionem ducentesimam sexagesimum septimam huius, secundum proportiones quasdam Arithmeticas contemplari, nimirum quae linearum, & segmentorum, tam convexorum quam concavorum Arithmetica sit progressio, sive incrementum.

Sit ABC *parabolae diamete*r AD, *& contingens* AE: *qua divsa in quotcunque partes aequales punctis* F, G, H *demittantur diametri* FB, GI, HK, EC *occurrentes parabolae in* B, I, K, C: ex quibus ordinatim ponatur BL, IM, KN, CDE: *ductisque* AB, AE, AK, AC *iungantur* BI, IK, KC, & FB, GI, HK *lineae producantur, donec* AI, AK, AC *lineis occurrant in* O, P, Q, R, S, T *punctis.*



Erunt igitur segmenta AB, BI, IK, KC *inter se aequalia: item lineae aequales* FB, BO, OP, PQ: *cum* FQ *linea sit divisa ut* AE.

Primo ut quadratum AF *ad quadratum* AG, *sic* FB *est ad lineam* GI: *sed* AG *quadratum quadratum est quadrati* AF *quia* AG *dupla est AF; igitur &* GI *quadruple est linea* EB, *id est* AM *quadruple ipsius* AL; *rursum quadratum* AH *est ad quadratum* AF, *ut* 9 *ad* 1 *cum AH tripla sit* AF, *igitur &* HK *linea est at lineam* AF, *id est* AN *ad* AL, *ut novem ad unum: iterum quadratum* AE *est ad quadratum* AF *est ut* 16. *ad* 1, *ergo &* EC *est ad* FB, *id est* AD *ad* AL, *ut* 16. *ad* 1 *& sic de caeteris: igitur* AL *dat unum; quorum* AM quatuor; AN *novem*; AD *se decim*.

Rursum cum IR, *sit ad* OP. *ut* AI *ad* AO, *id est* AG ad AF, *ponatur autem* AG *dupla* AF, *erit & IR dupla* OP: *eadem methodo ostenditur* KT *triplam esse* PQ *sive* OP, & *sesquialteram ipsius* IR; *unde incrementum innotescit linearum* BO, IR, KT.

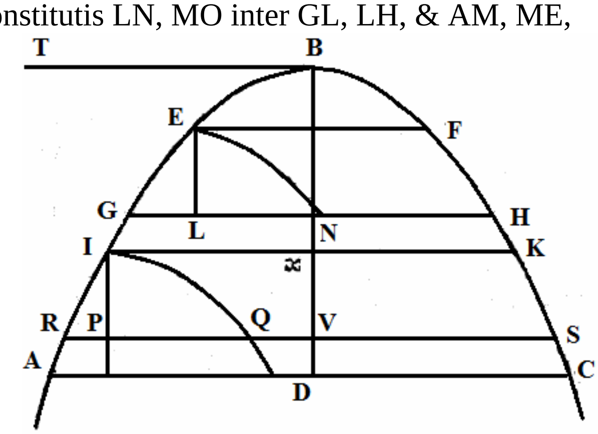
Secundo segmentorum convexorum AB, AI, AK, AC *arithmetica proportio sic instituitur . Triangulum* ABO *aequale est triangulo* AFB, (*cum* FB,BO *aequales ostensa sint*) *adeoque triplum segmentis AB: quare totum triangulum* ABI, *sextuplum est segmenti* AB; *additis igitur segmentis aequalibus* AB, BI, *erit totum segment* ABI *ad segment* AB *ut* 8 ad 1. *Rursum triangulum* AIR (*habens* IR *basim duplam baseos* OP, & IM *altitudinem duplam altitudinis* LB) *quadruplum est trianguli* ABO: *est autem triangulum* IKR *duplum trianguli* ABO, *quia eandem habent altitudinem, & basis* IR *dupla est baseos* OB, *igitur totum triangulum* AIK, *sextupli est trianguli* ABO, *unde est ad segment* AB, *ut* 18 *ad* 1 *addito igitur segmento* IK *aequalis segmenta AB*, & *segmento* AI, *quod octuplum est segmenti* AB, *erit totum segment* AK *ad segment* AB, *ut* 27 *ad* 1: *iterum, cum* AKT *triangulum, basim* TK *habeat triplam baseos* OB, & NK *altitudinem triplam altitudiis* LB, *erit* AKT *triangulum noncuplum trianguli* ABO: *est autem triangulum* KCT *triplum trianguli* ABO, *igitur totum triangulum* AKC, *duodecuplum erit trianguli* ABO: *quare & ad segment* AB *est ut* 36. *ad* 1. *addito ergo segmento* KC, *aquali ipsi* AB & AIK *segmento quod ad* AB *segment est ut* 27. *ad* 1.; *erit* AKC *segment ad segment* AB, *ut* 64. *ad unum; & sic de ceterus.*

Tertio parabola AIM *est ad* ABI, *parabolam, ut* AIM *triangulum ad triangulum* ABL, *est autem* AIM *triangulum octuplum trianguli* ABL, (*cum* AM *basis ostensa sit quadrupla baseos* AL, & MI *altitudo dupla ipsius* LB;) *igitur* AIM *parabola octupla est parabolae* ABL: *eodem modo cum* AN *noncupla sit* AL, & NK *tripla* LB, *erit triangulum* AKN *ad triangulum* ABL *est ut* 27. *ad* 1. *unde &* AKN *parabola toties continebit parabolam* ABL *eodem praxi procedendo, reliquorum proportio habibitur.*

Sed & concanorum, cum segmenta illa eandem servent rationem quam triangul AFB, AGI, AHK, *quorum nota est proportio.*

PROPOSITIO CCLXXV.

Sit parabolae ABC axis BD, ad quem ponantur ordinatim EF, GH, & IK, RS: ductis deinde diametris EL, IM; mediisque constitutis LN, MO inter GL, LH, & AM, ME, describantur circa axes EL, IM &puncta N & Q, parabolae ELN, lPQ. Dico parabolas has aequales esse.



Demonstratio.

Sit BT latus rectum axeos BV; erit ergo rectangulum VBT aequale quadrato VR, & rectangulum XBT aequale quadrato XI hoc est quadrato VP: itaque a rectangulo VBT dempto rectangulo XBT, remanet rectangulum VXBT, aequale rectagulo RPS, quod ex quadrato VR remanet per 5.2., dempto quadrato

VP; sed cum ex constructione RP, PQ, PS sint continuae, rectangulum RPS aequatur quadrato PQ; ergo BT latus rectum est parabolae IQP. Atqui eodem plano discursum BT latus rectum est parabolae ENL aequantur igitur parabolae. Quod erat demonstrandum.

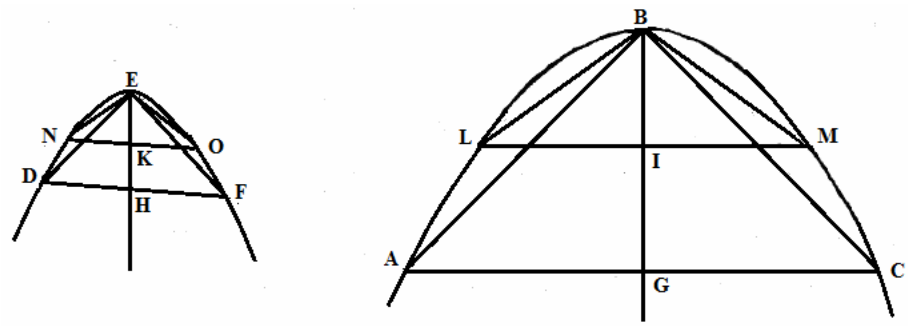
PROPOSITIO CCLXXVI.

Omnis parabola, parabolae similis est.

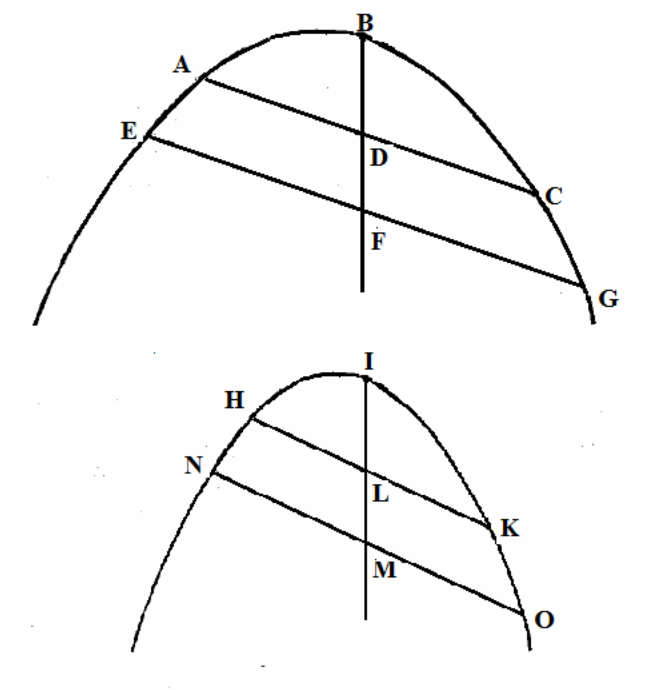
NOTA:

Duplici modo superfiies duas curvilineas dici similes: Primo, quando similes figurae in infinitum illis inscribi possunt: & hoc sensu Archimedes & Euclides, similia esse curvilinea quaedam ostendunt. Secundo similes dicuntur figurae curvilinea quarum essentiales proprietates eadem sunt. Nos duplicem hanc similitudinem parabolis inesse demonstrabimus.

Demonstratio.



Ponantur ABC, DEF parabolarum axes BG, EH aequales lateribus suis rectis; & per G & H ordinatim AC, DF: iunganturque ABC, DEF. Quoniam EH, BG lineae lateribus rectis aequales sunt & AC, DF ordinatim applicantur, anguli DHE, AGB recti sunt: & quia triangula DHE, AGB, ac proinde tota DEF, ABC similia sunt : Rursum divisis AG, EH lineis proportionaliter in I & K, ponantur per I & K ordinatim LM, NO: iungantur LBM, NEO. Quoniam igitur ut BI ad BG, sic EK est ad EH, ut LI quadratum ad quadratum AG, sic NI quadratum est ad quadratum DH, ut LI linea ad AG. sic NK linea ad DH; & permutando invertendo ut AG ad DH, sic LI ad NK: sed est ut AG ad DH, id est BG ad EH, (quia AG, GB item DH, HE aequales sunt) sic BI ad NK; per constructione igitur ut LI ad NK, sic BI est ad EK, & permutando ut LI ad BI, sic NK ad EK: quare cum LIB, NKE anguli lateribus proportionalibus conteri recti sint, triangula LIB, NKE, adeoque ; & tota LBM, NEO inter se similia sunt: similiter si iungantur ND, OF, LA, MC, ostendetur triangula DNE, EOF similia triangulis BLA, BMC : adeoque figura, totamDNEOF similem figurae ALBMC, quae operatio cum sine termino continuare possit: constat ABC, DEF similem figurae ALBMC, quae operatio cum sine termino continuari possit: constat ABC, DEF parabolas similes esse primo modo.



Secundo autem modo parabolas esse similes, sic ostento sit ABC parabolae diameter quaecunque AD divisa utcunque in D & E punctis, per quae ordinatim ponantur AC,FG ; sit autem & HIK parabolae diameter IL divisa proportionaler in L & M, & per L & M ordinatim positae HK, NO. Quoniam est ut BD ad BE, sic IL ad IM, erit ut quadratum AC ad quadratum NO, eodem modo si rursum diametri BD, IL proportionaliter dividantur, & per divisionum puncta ordinatim ponantur lineae, ostendentur quadrata ordinatim positarum in una parabola, proportionalia esse quadratis ordinatim positarum in altera. Quod cum in infinitum semper fieri possit, patet ABC, HIK parabolas esse similes secundo modo. Quod erat demonstrandum.

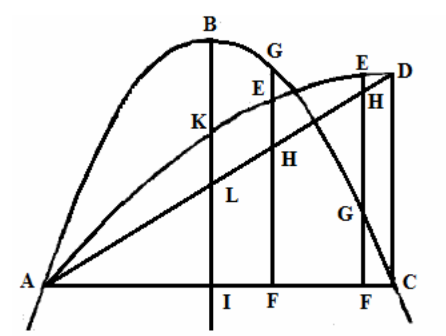
PROPOSITIO CCLXXVII.

Sit ad ABC parabolae axem BI ordinatim posita AC, erectaque diametro CD, sumatur in punctum quodvis D , & per A & D, parabola describatur cuius diameter DC, iunganturque AD: cum EF ponatur diameter,occurrens ABC parabolae in G, & AED in E, rectae vero AD in H.

Dico ABC parabolam esse ad segment AED, vt FG linea ad lineam. EH.

Demonstratio.

Occurrat axis BI, parabolae AEC in K, & AD in L·, cum igitur AC bifariam in I, sit divisam, & CD aequidistet BI, erit & AD quoque in L bifariam divisa & ordinarim ad LK diametrum posita: quia vero AC normalis ad CD, utrique parabolae est communis, erit ABC parabola ad segment AED; ut BI ad LK, (cum rationem habeant compositam ex ratione BI ad LK, sic GF ad EH, quia LK est ad EH, ut BI ad FG, id est ALD rectangulum ad rectangulum AHD, ut AIC ad AFC. rectangulum ; igitur ut FG ad EH, sic ABC parabola est ad segment AED. Quod erat demonstrandum.

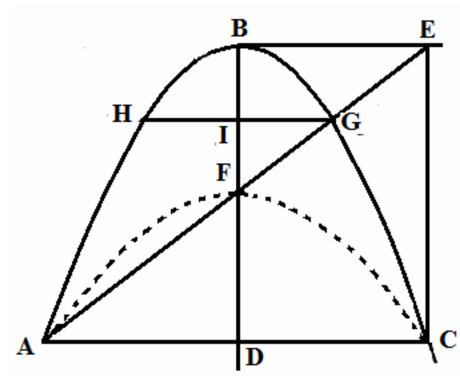


PROPOSITIO CCLXXVIII.

Sit ad ABC parabolae axem BD, ordinarim posita AC, contingens vero BE, quae erectae ex C diametro occurrat in E posita, autem AE quae axem secet in F: & parabolam in G, per A, F, C parabola describatur, habens apicem in F, ponaturque ordinatim GH, occurrens axi BD in I.

Dico HBG parabolam ad parabolam AFC duplicatum habere rationem HG ad AC.

Demonstratio.



Quoniam EB, AC aequidistant, ut AF ad FE, sic FD est ad FB, & EB ad AD: sed AF, FE aequales sunt, aequantur igitur EB, AD, & BF, FD: quia vero EF dupla est GF, id est BI dupla IF, erit & EB dupla GI ; unde tota HG aequalis est EB, id est AD dimidio rectae AC; quare ut BI ad BF, id est FD, sic HG est ad AC, est autem ratio parabolae HBG ad AFC parabola in composita ex ratione BI ad FD, & HG ad AC : igitur ratio parabolae HBG ad parabolam AFC duplicata HG ad AC. Quod erat demonstrandum.

PROPOSITIO CCLXXIX.

Iisdem positis:

Dico ABC parabolam, octuplam esse parabolae HBG.

Demonstratio.

Quoniam AD, HG lineae aequales sunt ostensae , & BD quadrupla ipsius BI, triangulum ABC octuplum est trianguli HBG, sed ABC parabola est ad parabolam HBG, ut ABC triangulum ad triangulum HBG; octupla igitur est parabola ABC, parabolae HBG. Quod erat demonstrandum.

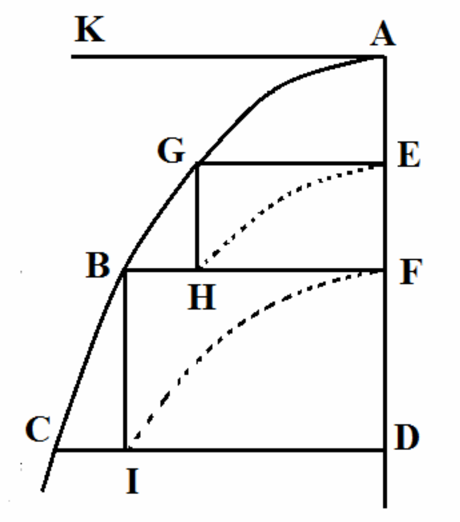
PROPOSITIO CCLXXX.

Sint ABC, DEF parabolarum axes BC, EF, divisoqae BC utcunque in H dividatur & EF proportionaliter in K, ponanturque ordinatim HG, IK.

Dico GBH parabolam IEK, ut ABC parabola est ad parabolam DEF.

Demonstratio.

Ponantur ordinatim CA, FD: ut BH ad BC, sic EK est ad EF, igitur ut quadratum GH ad quadratum AC, sic IK quadratum ad quadratum DF, & invertendo permutando ut AC



quadratum ad quadratum DF, sic GH quadratum ad quadratum IK, & ut AC ad DF, sic GH ad IK,sed ABC parabola est ad parabolam GBH in triplicata ratione AC ad GH ; & DEF parabola ad parabola in IEK, in triplicata ratione DF ad IK, erit igitur ut parabola ABC ad parabola in GBH, sic DEF parabola ad parabolam IEK: & permutando ut ABC parabola ad parabolam DEF, sic GBH parabola ad parabolam IEK; quod erat demonstrandum.

Si vero BC, EF lateribus rectis aequentur, erit GBH parabola ad parabolam IEK in duplicata ratione GH ad IK: quia ABC parabola ad parabolam DEF in duplicata est ratione AC ad DF, cum AC, CB lineae, item DF, FE ex quibus rationem habent compositam, aequales ponantur.

PROPOSITIO CCLXXXI.

Esto ABC parabolae axis AD divisus in E & F, vt AE, AF, AD proportionales sint, positisque ordinatim EG, FB, DC ex G & B, diametri demittantur GH, BI occurrentes FB, DC lineis in H & I: & per E, H & F, I parabolae describantur habentes apices in E & F.

Dico FEH parabolam esse ad parabolam DFI, in triplicata ratione FH ad DI.

Demonstratio.

Quoniam AE, AF, AD continuae proportionales sunt,vt AE ad AP, sic EF est ad FD: sed ut AE ad AF, sic EG quadratum est ad quadratum FB, id est quadratum FH ad quadratum DI; igitur ut EF ad FD, sic FH quadratum est ad quadratum DI: quia vero ratio parabolae FEH ad parabolam DFI composita est ex ratione EF ad FD, id est ex duplicata ratione FH ad DI, & iterum ex ratione FH ad DI, parabola FEH est ad parabolam DFI, in triplicata ratione FH ad DI. Quod erat demonstrandum.

Corollarium.

Hinc sequitur, figuram mixtilineam HGB, esse ad figuram IBC in triplicata ratione HB ad IC, cum enim EG, FB, DC proportionales sint, parabolae EAG, FAB, DAC in continua quoque sunt analogia: quare & FEGB mixtilineam ad mixtilineum DFBC est ut EAG parabola ad parabola in FAB, id est in triplicata ratione EG ad FB, id est FH ad DI: sed & rectangulum FG, ad rectangulum DB est in triplicata ratione lineae FH ad lineam DI, quia rationem habent compositam ex ratione EF ad FD, & FH ad DI. Igitur & residuum HGB ad residuum IBC in triplicata est ratione FH ad DI; quia vero est GE ad FB, id est FH ad DI, ut FB ad DC, recta HB est ad IC; reliquum ad reliquum ut FH ad DI : figura igitur HGB ad IBC figuram, triplicatam habet rationem HB ad IC. Quod erat demonstrandum.

PROPOSITIO CCLXXXII.

Esto ABC parabolae diameter BD, utcunque divisa in D & E, sic ut nec BE nec BD sit aequales lateri recto diameteri BD; & per E & D, ordinatim positis AC, FG, describantur per A,B,C, & F, B, G, puncta ellipses quarum coniugatae sint diametri AC, BD, FG, BE. Dico ABC parabolam esse ad parabolam FBG, vt ABC ellipsis ad ellipsim FBG.

Demonstratio.

Iugantur ABC, FBG; ut ABC triangulum ad triangulum FBG, sic ABC parabola est ad parabolam FBG: sed & ABC ellipsis est ad ellipsim FBG, ut ABC triangulum ad triangulum FGB, ut ABC triangulum ad triangulum FBG; igitur ut parabola ABC est ad parabolam FBG: igitur ut parabola ABC est ad parabolam FBG, sic ABC. Quod erat demonstrandum.

PROPOSITIO CCLXXXIII.

Parabolam ABC subtendat recta quaevas BC, oportet super illam describere parabolam quae ad ABC parabolam datam habeat rationem F ad G.

Constructio & demonstratio.

Divisa BC bifariam in D, erigitur diameter DA, quae dividatur in E, ut AD sit ad DE sicut G est ad F; tum per B, E, C puncta parabola describatur cuius diameter sit DE, & ordinatim ad illam applicata BC, dico factum esse quod petitur . Quoniam ABC, BEC parabolae communem subtensam habent BC, parabola BEC ad ABC, parabolam est, ut ED lineae ad lineam AD, id est ut F ad G.

PROPOSITIO CCLXXXIV.

Esto ABC parabolae diameter AD, divisa vtcunque in E & D, & ordinatim positae BE CD. Sit autem & FGH parabolae diameter FI, utcunque divisa in I & ordinatim posita HI; oportet FHG parabolam iterum dividere sicut ABC parabola divisa est.
.

Constructio & demonstratio.

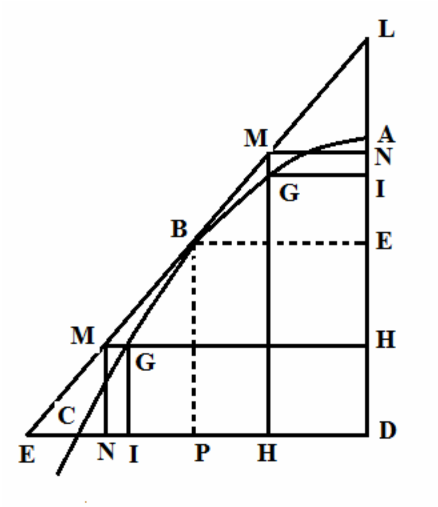
Fiat ut AD ad AE, sic FI ad FK, & ex K ordinatim ponatur KG: dico factum esse petitur. Quoniam est ut AE ad AD, sic FK ad FI, erit & GK ad HI, ut BE ad CD: sed BAE parabola ad parabolam CAD in triplicata est ratione lineae BE ad CD, & GFK parabola ad parabolam HFl: in triplicata ratione GK ad HI, igitur vt parabola BAE ad CAD, parabolam, sic GFK est ad parabolam HFI. Perfecimus igitur quod fuit postulatum.

PROPOSITIO CCLXXXV.

Sit ad ABC parabolae diametrum AD ordinatim positae DC; divisaque ; AD in E, ut ED dupla sit AE, ponatur ex E ordinatim EB, & ex B, demittatur diameter BF, occurrens rectae DC in F.

Dico parallelogrammum DEBF maximum esse illorum, quae in angulo EBF, parabolae ABCD terminatae inscribi possunt.

Demonstratio.

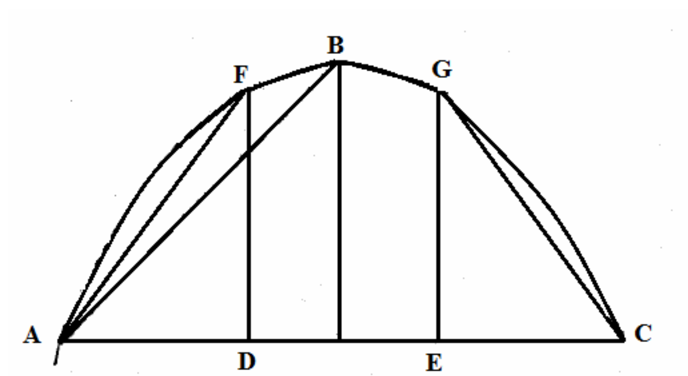


Inscribatur enim quodcunque parallelogrammum IGH habens angulum IGH aequalem angulo EBF : agaturque per B contingens LK, occurrens AD diametro in L, & DC lineae in K rectae vero HG in M ; ex quo recta ponatur MN equidistans GI. Quoniam BL est contingens, & EB ordinatim posita, rectae LA, AE aequales sunt ; adeoque tota LE, aequalis est ED quae duplae ponitur AE: unde LK in B quoque bifariam est divisae : parallelogrammum igitur DB maius est parallelogrammum DM: sed parallelogrammum DM maius est parallelogrammam DG: quia punctum M cadit extra parabolam; parallelogrammum migitur DB, multo maius est parallelogrammo DG: idem demonstratur de quovis alio; parallelogrammum igitur DB maximum est eorumque ABC parabolae terminatae in angulo EBF, inscribi possunt. Quod erat demonstrandum.

PROPOSITIO CCLXXXVI.

Datae parabolae terminatae maximum inscribere parallelogrammum.

Constructio & demonstratio.



Sit ad ABC parabolae diametrum BD ordinatim posita AC; oportet parabolae ABC maximum inscribere parallelogrammum, divisa DB in E, ut ED dupla sit EB, ponatur per E ordinatim linea FG, & ex F & G diametri demittantur FH, GI occurrences AC lineae in H & I. Manifestum est ex praecedenti propositione, parallelogrammum HGIF esse id quod quaeritur.

PROPOSITIO CCLXXXVII.

Datae parabolae terminae polygonum regulare inscribere, quod dato laterum numero.

Polygonum regulare voco, cuius singula latera segmenta auferunt aequalia, praeter subtensam.

Parabolam datam ABC subtendat AC, oporteat ABC parabolae, polygonum regulare , inscribere, quatuor constans lateribus, secetur AC in D & E, diametri ponantur, trifariam, & ex D & E diametri ponantur DF, EG, iunganturque AF, FG, GC; dico AFGC polygon satisfacere petitioni. Cum enim AD, DE, EC lineae aequales sint, segmenta quoque AF, FG, GC aequalia sunt: polygonum igitur regulare est quadrilaterum AFGC inscripsimus igitur, &c. quod erat faciendum.

PROPOSITIO CCLXXXVIII.

Iisdem positis:
Dico AFGC. quadrilaterum esse maximum illorum quae ABC parabolae terminatae inscribi possunt.

Demonstratio.

Inscribatur enim aliud quodvis quadrilaterum ABGC: quod primo quidem latus CG commune habeat cum quadrilatero AFGC : quoniam igitur AF, FG aequalia sunt segmenta, minora illa sunt segmentis AB, BG; residua igitur figura rectilina AFGC maior est figurea ABGC: similiter ostenditur quadrilaterum quodvis aliud minus esse quadrilatero AFGC: maximum igitur illud est eorum quae ABC parabolae inscribi possunt.

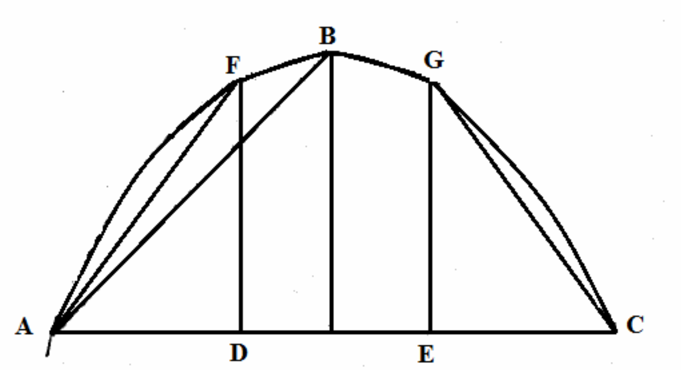
Corollarium.

Qvae de quadrilataero regulaeri dixi, eadem de quotiusss laterum polygono regulari; intelligenda sunt: eademque omnis constructio & demonstratio convenit.

PROPOSITIO CCLXXXIX .

Datae parabolae terminatae, polygonum inscribere maximum illorum quae dato numero laterum inscribi possunt.

Constructio & demonstratio.



Inscribendum sit parabolae, maximum quadrilaterum: inscribatur ABC parabolae quadrilaterum regulare AFGC : dico illud esse maximum eorum quae. pari numero laterum, parabolae inscribi possunt. Demonstratio ex praecedenti manifesta est.