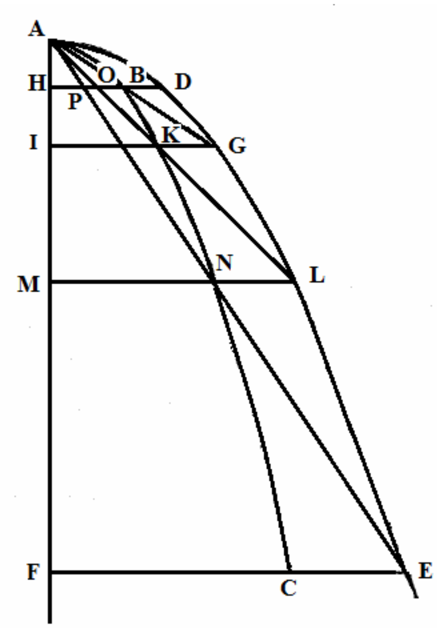
PARABOLA PART FOUR

We consider the properties of parabolas themselves, or of their intersection with circles.

PROPOSITION CLIII.

The parabolas ABC, ADE shall have the common axis AF, and the vertex A: from which the line AG sent crossing the parabolas at B and G, and the ordinate lines DH, GI shall be put through B and G to the axis: and indeed GI shall cross the parabola ABC at the point K, through which on detaching from A, AL shall be put in place; and from L, the ordinate line LM, crossing the parabola ABC at N : then AE shall be put in place through N, and from E the ordinate line ECF.

I say the lines DH, KI, likewise LM, CF to be equal to each other.



Demonstration.

The ratio AH to AI, that is HB to IG, is twice the ratio HB to IK, that is: HD to IG, therefore HB, IK, IG are proportionals, and as IK to IG, thus HB to IK, but HB is to HD, as IK is to IG; therefore HB is to HD, as HB to IK; therefore HD and IK are equal; by the same reasoning it may be shown the lines LM, CF to be equal.

PROPOSITION CLIV.

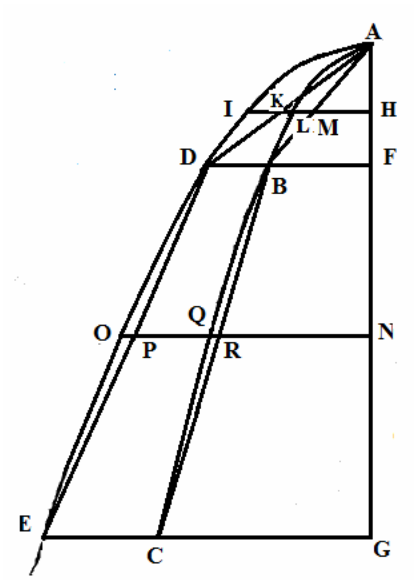
With the same in place, the right lines AL, AE will cut the line DH at O and P.

I say the right lines EF, LM, GI, DH, BH, OH, PH to be in continued proportion.

Demonstration.

Since it has been shown that NM and GI are equal, and EF, LM, NM are proportionals, and EF, LM, GI are lines in continued proportion: and because LM, GI, KI are proportionals, and the line KI is equal to the line DH, also LM, GI, DH will be continued in the same ratio with EF, LM, GI : besides moreover as GI is to DH, thus DH is to BH, therefore EF, LM, GI, DH, BH are proportionals; further since as GI shall be to KI, thus BH shall be to OH, moreover as GI shall be to KI, thus as GI to DH, as DH to BH, therefore the lines GI, DH, BO, OH are continued proportionals. Again since OH shall be to PH, as LM to NM, that is from the demonstration LM is to GI, as GI is to DH, i.e. as DH to BH, i.e. as BH to OH; therefore the lines GI, DH, BH, OH, PH, as well as all the lines EF, LM, GI, DH, BH, OH, PH shall be in continued proportion. Q.e.d.

PROPOSITION CLV.



The parabolas ABC, ADE shall have the common axis AF, and with the ordinate lines FBD, GCE put in place for that, and AB, AD, BC, DE shall be joined, HI, NO parallel to FD, and indeed HI shall cut the parabolas at I and L, the lines AD, AB at K and M; truly the right line NO will cross the lines DE, BC at P and R, and the parabolas at Q and O.

I say LM to be to RQ, thus as IK to OP.

Demonstration.

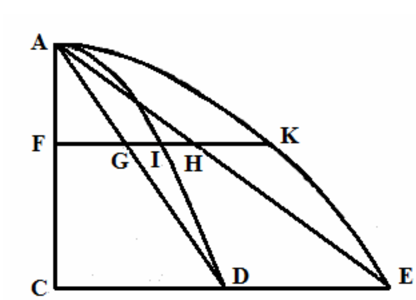
As IH to LH, thus DF to BF; but as DF to BF, thus KH to MH, therefore so that as IH to LH, thus KH to MH, and IK to LM, as IH to LH ; i.e. as DF to BF; but as DF to BF, thus ON also is to QN, and PN to RN. Therefore as ON to QN, thus PN to RN; and OP to QR, as ON to QN, that is as DF to BF, that is from the demonstration as IK to LM, and on interchanging, so that as IK to OP, thus LM to RQ. Q.e.d.

PROPOSITION CLVI.

Two parabolas set up on the same axis AC shall have the common apex A: and with the ordinate CDE established for the axis, AD, AE shall be joined with the right line FK drawn parallel to CE, cutting the parabolas at I and K , truly the right lines AD, AE at G and H.

I say the rectangle GFH to the rectangle IFK to have that ratio, which the line FG has to the line CD.

Demonstration.



The ratio of the rectangle GFH to IFK, is the rectangle composed from the ratio FG to FI, and the rectangle from the ratio HF to FK: moreover, as FG is to FI, thus FI to CD [§.42]; and as FH to FK, thus FK to CE ; therefore the ratio of the rectangle GFH to the rectangle IFK, is composed from the ratio FI to CD, and from FK to CE, but FK is to CE, as FI is to CD; therefore the rectangle GFH to twice the rectangle IFK has the ratio of that which the line the line FI has to CD : that is, the rectangle GFH to the rectangle IFK shall be as FG to CD, because FG, FI, CD are proportionals. Q.e.d.

PROPOSITION CLVII.

The two parabolas ABC, ADE shall exterior tangents to each other at the vertex, and with the secants AE, AC put in place from A, and with the tangent AH, some number of lines IK are drawn parallel to the common axis FAG.

I say the rectangle IHD, to be to the rectangle IHD, as the rectangle KHB to the rectangle KHB.

Demonstration

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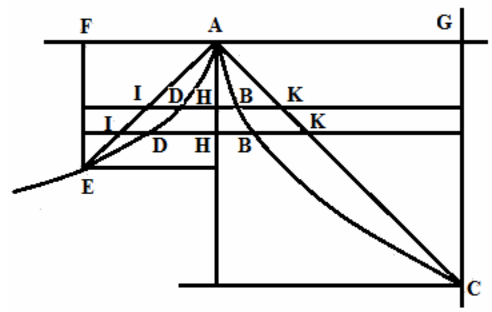


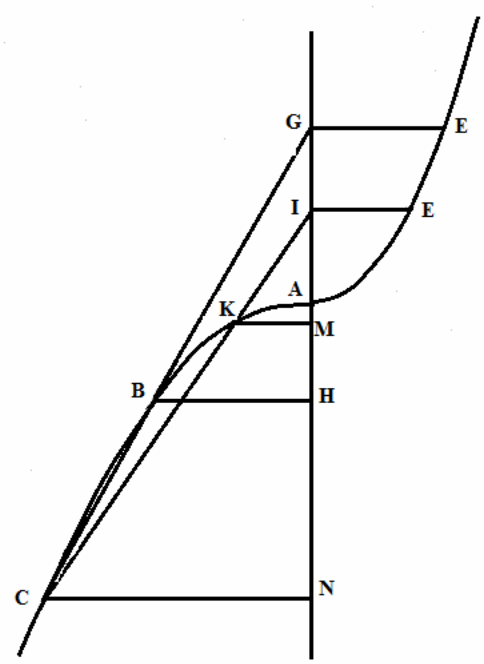
The rectangle IHD to the rectangle IHD, has triple the ratio [§.54] of that which IH has to IH, that is AH to AH; also the rectangle BHK to the rectangle BHK, has triple the ratio of that which HK has to HK, that is, AH to AH: therefore as the rectangle BHK is to the rectangle BHK, thus the rectangle IHD is to the rectangle IHD. Q.e.d.

PROPOSITION CLVIII.

Again the two parabolas ABC, AEF shall themselves be external tangents established to the same axes GH, and from some point C taken on the perimeter ABC the lines CG, CI may be drawn cutting the parabola ABC at B and K, and the axes GH at G and I : then the ordinates BH, KM, IE, GF shall be put in place.

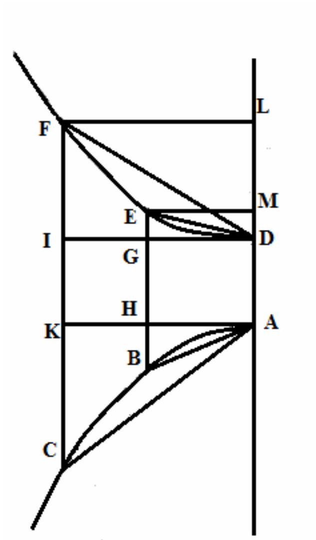
I say the line HB to the line MK, shall be double the ratio of that which the line GF has to the line IE.

Demonstration.



The ordinate line CN shall be drawn from C. Because both the lines NA, AI, AM as well as NA, AG, AH are in continued proportion, and NA the first common term of each series, the ratio AH to AM, of the third to the third, is the square of that which AG has to AI; but AH to AM, the second to the second: also will be in the square ratio BH to KM; therefore BH is to KM as AG is to AI : and HB to MK has the squares GF to IE. Q.e.d.

PROPOSITION CLIX.



Let the axis AD of the parabola ABC be equal to the latus rectum; and through the vertex D the inverted parabola DEF shall be described directed towards the apex of the parabola ABC and with a common axis; then some diameters EB, FC shall be drawn crossing the parabolas at the points B, C, E, F ; truly the tangents through A and D, will cut the diameters EB,FC at G, H, I, K.

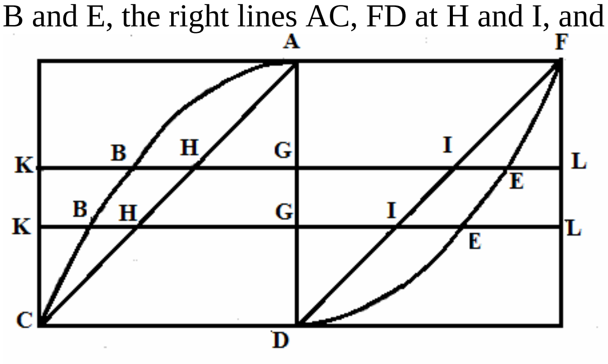
I say the rectangle EGB to be to the rectangle FIC, as the square ED is to the square FD.

Demonstration.

The ordinate lines FL, EM shall be put in place. Since the line AD is put equal to the latus rectum, and EM, FL are the applied ordinates, the square ED (that is the sum of the squares EM, MD) is equal to the rectangle DMA: and the square FD (that is the sum of the squares FL, LD) is equal to the rectangle DLA; therefore so that the square ED shall be to the square FD, thus as the rectangle DMA to the rectangle DLA: but DMA, that is the rectangle GEH, is equal to the rectangle EGB; and DLA is equal to FIC; therefore the rectangle EGB to the rectangle FIC, to be as the square ED to the square FD.

PROPOSITION CLX.

ABC, DEF shall be the parabolas placed inversely to each other for the same axis AD ; and with the ordinate lines AF, DC drawn from A and D; the points AC, FD shall be joined; in addition some lines BE shall be drawn parallel to AF crossing the parabolas at B and E, the right lines AC, FD at H and I, and the axis AD at G.



I say the square BG, the rectangle BGE and the square GE to be in continued proportion.

Demonstration.

As the line BG is to the line GE, thus the square BG is to the rectangle BGE: but also as BG is to GE, thus the rectangle BGE is to the square GE; therefore as the square BG to the rectangle BGE, thus the rectangle BGE is to the square GE. Q.e.d.

PROPOSITION CLXI.

With the same in place:
I say the rectangle AGD to the rectangle AGD, to be the square of the ratio of that which the rectangle BGE has to the rectangle BGE.

Demonstration.

The ratio of the rectangle AGD to the rectangle AGD is composed from the ratio AG to AG, and from GD to GD: truly the ratio of the rectangle BGE to the rectangle BGE is composed from the ratio BG to BG, and from the ratio GE to GE: but the ratio AG to AG is of the square of the ratio BG to BG; and from the ratio GD to GD, the square is of the ratio GE to GE, therefore the ratio of the rectangle AGD to the rectangle AGD, is the square of that which the rectangle BGE has to the rectangle BGE. Q.e.d.

PROPOSITION CLXII.

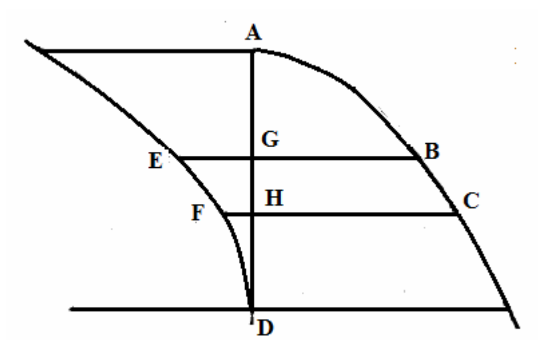
With the same in place: the diameters CK, FL shall be erected from C and F crossing the line BE in K and L.
I say the rectangles KGL, BGE, HGI to be in continued proportion.

Demonstration.

The rectangle KGL to the rectangle BGE, has a ratio composed from KG to BG, and from GL to GE : and the rectangle BGE to the rectangle HGI, has a ratio composed from BG to HG, that is KG to BG, and from GE to GI: that is, GL to GE; therefore the rectangle KGL is to the rectangle BQE, as the rectangle BGB to the rectangle HGI. Q.e.d.

PROPOSITION CLXIII.

Again the two parabolas shall be ABC, DEF : and indeed the apex of the parabola ABC shall be A; truly of the parabola DEF the apex shall be D, for which the axes AD shall be a tangent at D: and with the lines AG, DH made equal; the lines EB, FC normal to AD acting through G and H crossing the parabolas at E, B, F, C.
I say the rectangle EGB to the rectangle FHC to be in the quintuple ratio of that which the line GB has to the line HC.



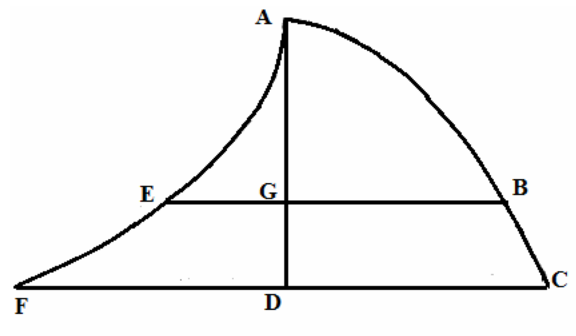
Demonstration.

The ratio of the rectangle EGB to the rectangle FHC, is composed from the ratio EG to FH, that is the square GD to HD; that is AH ad AG, (since the lines AG, BD shall be equal) that is to the quadruple ratio GB ad HC; and from the ratio GB to HC: therefore the rectangle EGB to FHC becomes the quintuple of the ratio GB to HC. Q.e.d.

PROPOSITION CLXIV.

AD shall be the axis of the parabola ABC, and A the vertex of the parabola AEF, at which AD shall be the tangent line, and with the ordinate EGB shall be drawn for AD and FDC for the other, so that the right lines GB, FD shall be equal.

I say the ratio EG to DC to be the quintuple of that which GB has to DC.



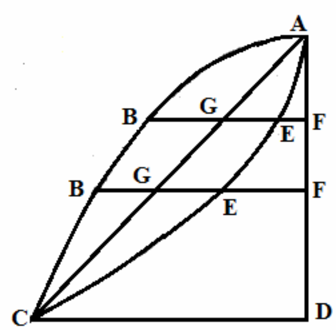
Demonstration.

The ratio EG to DC is composed from the ratio EG, that is FD, to DC; and therefore from the quadruple ratio GB ad DC. Q.e.d.

PROPOSITION CLXV.

CD shall be the ordinate put in place for the diameter AD of the parabola ABC, moreover the parabola AEC shall be described through A and C , having the vertex at A and connected to the tangent AD ; AC shall be put in place to AD, BG the ordinate of the line EF.

I say the rectangle GFE shall be to the rectangle GFE in the sextuple ratio of FB to FB.



Demonstration.

The rectangle GFE to the rectangle GFE has the triplicate ratio GF to GF, that is AF to AF; but AF to AF, has the duplicate ratio of that which BF has to BF; therefore the rectangle GFE to the rectangle GFE, has the sextuple ratio of FB to FB. Q.e.d.

PROPOSITION CLXVI.

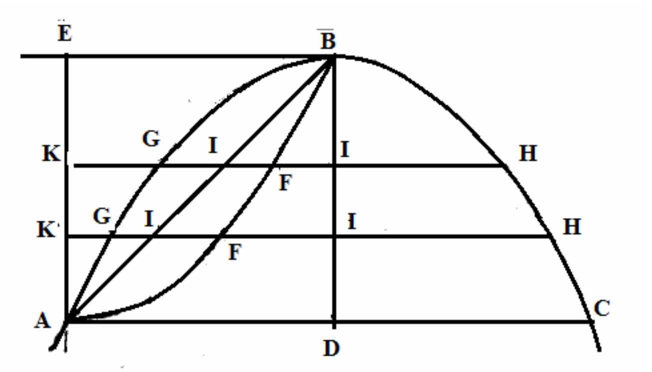
With the same in place:
I say the rectangle EFB to the rectangle EFB to have the quintuple ratio of that which FB has to FB.

Demonstration.

The rectangle EFB to the rectangle EFB, has the ratio composed from EF to EF, that is from the square of the ratio AF ad AF, that is from the quadruple of the ratio FB to FB, and from the ratio FB to FB; therefore the ratio of the rectangle EFB to the rectangle EFB is the quintuple of the ratio FB to FB.

PROPOSITION CLXVII.

The line BE shall be a tangent at B to the parabola ABC, of which the axis is BD, in which from some point E assumed the line EA shall be sent, crossing the parabola ABC at A; then the parabola AFB shall be described from the points A and B having the vertex at A, crossing the parabola ABC at B; and AE shall be its axis : moreover KG shall be drawn crossing the parabola AFB at F, and the axis BD at I.



I say GK, FK, HK to be proportional lines.

Demonstration.

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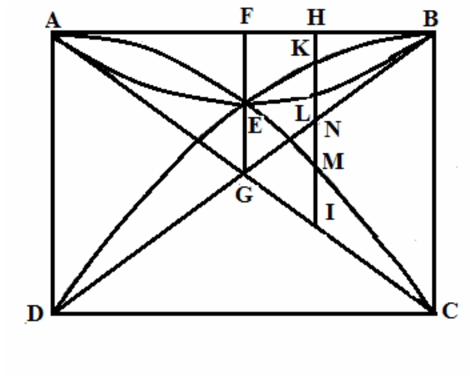
AB joined will cut the line HG at L. Because BD shall be parallel to the diameter AE, and IK shall be the ordinate applied for that, the rectangle LKI is equal to the square FK [§.42], moreover the rectangle LKI is equal to the rectangle GKH, therefore the square FK is equal to the rectangle GKH, and GK, FK, HK are lines in continued proportion. Q.e.d.

PROPOSITION CLXVIII.

ABCD shall be a parallelogram, and with a parabola described through A and C, of which the diameter shall be AD, and with the tangent AB, and another parabola shall be described through B and D, the diameter of which shall be BC, and moreover shall cross with the tangent AB of the other parabola AEC at E; then through the points A, E, C a third parabola is described having a common diameter with the other diameters.
I say the lines joined AC, BD to be tangents to the parabola AEC at A and C.



Demonstration

The diameter EF acts through E cutting the line DB at G, thus so that FE shall be to AD, as the square FB shall be to the square AB, and so that the line FE shall be to the line BC, thus as the square AF to the square AB: but by the hypothesis the lines AD, BC are equal, and therefore the square FB is to the square AB, as the square AF is to the square AB: therefore the squares AF, FB are equal, and the line AB is the double of AE, just as AD is twice FG: whereby FE also crossed the line AC at the point G, by which DB is bisected, by which the other diameter of the parallelogram is cut. Again since the square AB will be to the quadruple of the square FB, and the line BC will be to the quadruple of the line FE; but also AD, that is BC, shown to be twice FG, therefore FG is twice FE, and FE, EG are equal lines, from which the line BD is a tangent to the parabola AEB; similarly it is shown the line AC to be a tangent to the parabola AEB. Q.e.d.

Corollary.

Hence it is evident the lines AG, BD to intersect at that point, where EG is equal to the line FE: which it has been a pleasure to explain and to note with a word.

PROPOSITION CLXIX.

With the same in place: it will be required to show the intersection of the point E.

Construction & Demonstration.

With AB bisected at F, the diameter FE shall be dropped from F, equal to the fourth part of the line BC: I say E, the end of the line FE, to designate the point of the intersection, shall indeed be the point E of the intersection found, and through E the diameter EF shall be acting, as we have shown in the previous proposition, equal to the fourth part of the line BC; therefore through the composition since the diameter FE shall be given equal to the fourth part of the line BC, it is clear the point E to be the point of intersection, etc. Q.e.d.

PROPOSITION CLXX.

With the same in place HI cuts the parabolas at K, L, M.
I say the lines HK, HL, HM to be continued proportionals.

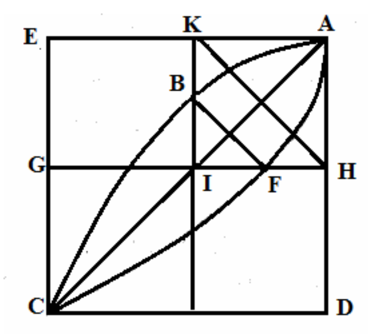
Demonstration.

The right line HI shall cut the line BD at N, and AC at I. So that the rectangle BHA thus shall be to the rectangle FA as the line HL is to the line FE, that is EG; and so that the rectangle BND thus shall be to the rectangle BGD, as KN to EG: but the rectangle BHA is to the rectangle BFA, as the rectangle BND to the rectangle BGD; so that therefore as HL to EG, thus KN to EG: and thus HL, KN are equal lines; and with the common line KL taken away, HK remains equal to LN; similarly it is shown the line LM to be equal to the line HL: whereby so that as HM to MI, thus HM is to HL: but as HM to MI, thus AI is to IC, that is AH to HB; Therefore as HM is to HL, thus AH is to HB, that is, HL to LN, that is HL to HK ; therefore the proportionals are HK, HL, HM. Q.e.d.

PROPOSITION CLXXI.

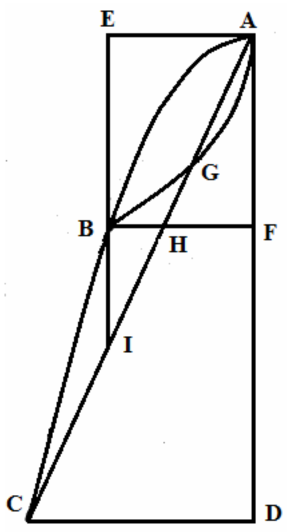
The line EA shall be a tangent to the parabola ABC at A of which the diameter is AD and with the diameter EC send from E, the parabola AFC shall be described through A and C parabola, the diameter of which shall be AE and the tangent AD: then the line GH shall be drawn parallel to AE, cutting the parabola AFC at F, and the line AG at the point I; through which the diameter IBK shall be put in place, and BF, HK shall be joined.
I say the lines BF, HK to be parallel.

Demonstration.



The ordinate line CD shall be put in place from C for the diameter AD: the ratio KB to EC is the square of the ratio KI to EC: likewise the ratio HF to CD squared, is the ratio of HI to CD, that is AK to AE, that is KI to EC; therefore as KB to EC, thus HF to CD, and as KB to KI, thus HF to Hl, therefore the lines FB, HK are assumed to be parallel. Q.e.d.

PROPOSITION CLXXII.



The line AE shall be a tangent to the parabola ABC at A, of which AD shall be the diameter, and with the diameter EB sent down, which will cross the parabola ABC at B, from B the ordinate line BF shall be put in place: and the parabola AGB will be described through A and B, of which the diameter shall be AE and the tangent AD shall be drawn from A and the line AC crossing the parabola AGB and lines BF, EB in G, H, & I.

I say the lines AG, AH, AI, AC to be continued proportionals.

Demonstration.

Because EB shall be parallel to the tangent AD, FB to the diameter AE, the right lines AG, AH, AI are continued proportionals, and moreover AH, AI, AC shall be continued in the same ratio. Q.e.d.

PROPOSITIO CLXXIII.

With the same in place CD shall be put for the ordinate.

I say AG to AC to be in the threefold ratio of that which BF has to CD..

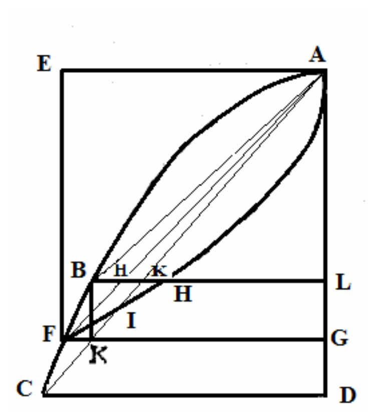
Demonstration.

Since by the preceding, the lines AG, AH, AI, AC are in continued proportion, the ratio AG to AC shall be the cube of that of which AH to AC is the square : but AH to AC, that is AF to AD, has the square ratio BF ad CD ; therefore AG to AC has the triplicate ratio of that which BF had to the line CD. Q.e.d.

PROPOSITION CLXXIV.

The line AE shall be a tangent at A to the parabola ABC, of which the diameter is AD; and with the ordinates CD, FG put in place, the parabola AHF shall be described through F and A, the tangent of which is AD, the diameter AE: and with AF drawn, AC shall be put in place, cutting the parabola AHF at I, the right line FG at the point K, from which with the diameter KB erected, the ordinate line BL shall be drawn, crossing the line AF at M, the right line AC at N, and the parabola AHF at H.

I say the lines CD, FG, BL, ML, NL, HL to be in continued proportion.



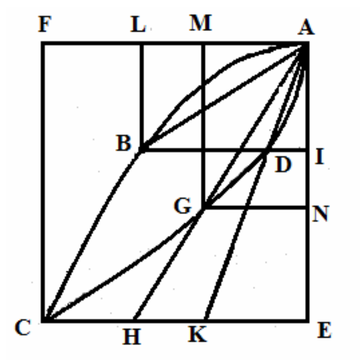
Demonstration.

The diameter FE shall be erected from F. Since the diameter AD shall be parallel to BK, the right lines CD, FG, BL are in proportion: and moreover FG, BL, ML also are proportionals ; therefore they shall be continued in the same ratio CD, FG, BL, ML. Again since the ratio NL to KG, that is, to BL; that is, the ratio AL to AG, shall be the square of the ratio BL to FG, that is LM to LB; also BL, ML, NL are proportionals; finally since the line FG to the line HL is as the square ratio of AL to AG, that is in the quadruple ratio of LB to GF, that is LB to LM; the lines LH, LN, LM; LB, LF are proportionals; therefore the lines CD, FG, BL, ML, NL, HL will continue in the same ratio. Q.e.d.

PROPOSITION CLXXV.

The lines AE, AF are tangents at A to the equal parabolas ABC; ADC, having the common vertex at A; moreover the parabolas cut each other at C, and the ordinates CE, CF are drawn from C, and with the equal lines AB, AG sent down from A; of which the one AB shall cut the parabola ABC at B, truly the other AG the parabola ADC at C and produced, the line CE at H; from B the ordinate line BI cutting the parabola ADC at D, and through D from A the line AK is drawn crossing EC at K.
I say AD to AK, to be as the square of that ratio which AG has to AH.

Demonstration.

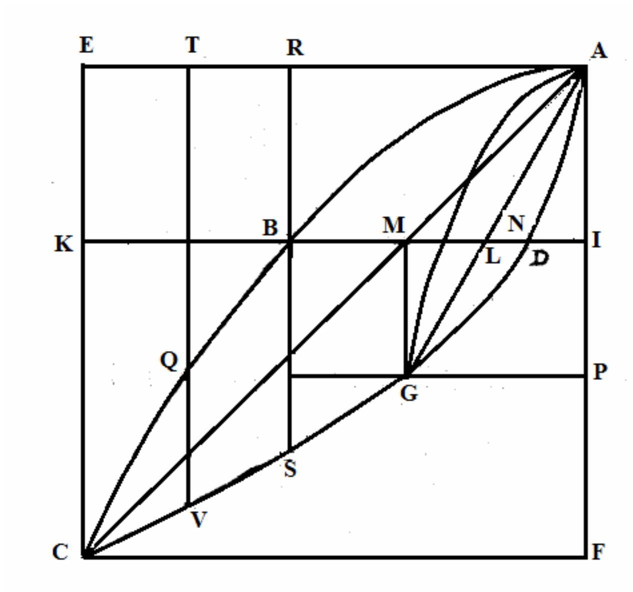


The diameters BL, GM, GN shall be drawn from B and G: and BL and GM indeed shall cross the line AF at L and M; GN truly the right line AE at N. Since both the parabolas ABC, ADC, as well as the lines AB, AG shall be equal, also the line LB shall be equal to the line GN, and the line IB shall be equal to MG. Again the ratio LB to FC, that is AI to AE; that is AD to AK, is the square of the ratio IB to EC, that is: MG to EC ; but as MG to EC, thus AN to AE (for AL that is IB is equal to the right line AN, and EC itself is equal to AE) that is AG to AH; therefore the ratio AD to AK, is the square of that ratio which AG to AH. Q.e.d.

PROPOSITION CLXXVI.

The lines AE, AF are tangents at A to the equal parabolas ABC, ADC having the common vertex at A, and the parabolas themselves in turn will intersect at C; from which with the ordinate lines drawn CE, CF, the parabola AHG shall be described through A having the vertex at A, and the tangent AE, moreover crossing the parabola ADC at G, and with the points AC, AG, joined the diameter GM shall be erected from G cutting the line AC at M, through which point the ordinate line IK shall be drawn cutting the parabolas at B,H,D, and the line AG at L.

I say the ratio IH to IB to be the cube of that, of which IL to IM, has the square ratio.



Demonstration.

The mean IN shall be found between ID and IL; therefore since ID, IL, IM [§.42]are proportionals, and IN the mean between ID, IL, both the lines ID, IN, IL, IH [§.74], as well as the lines ID, IL, IM, IB shall be continued proportionals: whereby the ratio IH to IB of the fourth to the fourth shall be the cube of that ratio IB ad IL, of the second to the second, of which IL to IM, the third to the third, has the square ratio. Q.e.d.

PROPOSITION CLXXVII.

With the same in place: the ordinate QP shall be acting through G.

I say the ratio ID to IH, to be in the cubic ratio of that which IH has to GP, and the ratio ID to IB, to be the cube of that ratio IB to CF.

Demonstration.

Indeed since IB shall be the mean between ID and IL, the right lines ID, IN, IL, IH, GP shall be proportionals, whereby ID to IH, first to fourth, has the cubic ratio IH to GP, of fourth to fifth. Which was the first part. Again since the mean IL shall be between ID, IM; the right lines ID, IL, IM, IB, IK that is CF, are continued proportionals, whereby ID to IB, first to fourth, to be in the cubic ratio of that which IB has to CF, the fourth to the fifth. Q.e.d.

PROPOSITIO CLXXVIII.

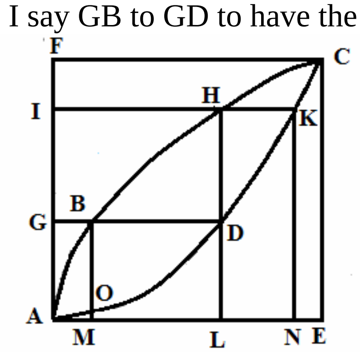
With the same in place acting through Q and B the diameters TV, RS meeting the parabola ADC at V & S, and the line AE at T and R.
I say the ratio ID to GP, to be as the eighth power of the ratio RS to TV.

Demonstration.

For the ratio ID to GP is the square of the ratio AI to AP, that is RB to TQ: and that is the fourth power of that which RA has to TA. But the ratio RA to TA, is the square of the ratio RS to TV, since the ordinates shall be put to AE; therefore the ratio ID to GP, is of the eighth power of the ratio RS to TV. Q.e.d.

PROPOSITION CLXXIX.

The parabolas ABC, ADC having a common vertex, the lines AE, AF shall be tangents at A, moreover the parabolas cut each other at the point C, from which CE, CF are put to be the ordinates of the lines: and with some point G assumed on AF, the ordinate ling GD shall be drawn from G, cutting the parabolas ABC, ADC at B and D, and with the diameter DH acting through D, which shall cross the parabola ABC at H, the ordinate line IK shall be drawn through H, cutting the line AF at I.
I say GB to GD to have the fourth power ratio of that which IH has to IK.



Demonstration.

The ratio GB to IH, that is : GD squared is of the ratio AG to AI; but AG to AI, squared has the ratio GD to IK, that is IH to IK ; therefore the ratio IH to IK, is of GB to GD, to the fourth power the ratio. Q.e.d.
[*i.e.* in modern terms, the *x* and *y* axes are interchanged.]

PROPOSITION CLXXX.

HD shall be produced, then it shall cross the line AE at L, the diameters BM, KN shall be sent from B and K crossing the line AE at M and N, & B and indeed the parabola ADC at O.
I say the ratio OM to DL, to be the quadruple of the ratio DL to KN.

Demonstration.

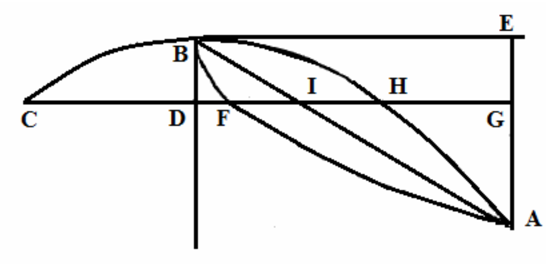
For the ratio OM to DL is the square of the ratio AM to AL, that is GB to IH: that is the fourth power of the ratio AG to AI, that is : LD to NK. Q.e.d.

PROPOSITION CLXXXI.

The line BE shall be a tangent at B to the parabola ABC of which the axis shall be BD, and with some point E assumed on which, a diameter EA shall be sent from E: then a parabola BFA will be described through the points A and B of which the axis shall be BE, and some ordinate line HC to the tangent BD, crossing the parabolas at C, F, H; and the lines BD, BA, EA at D, I, G.

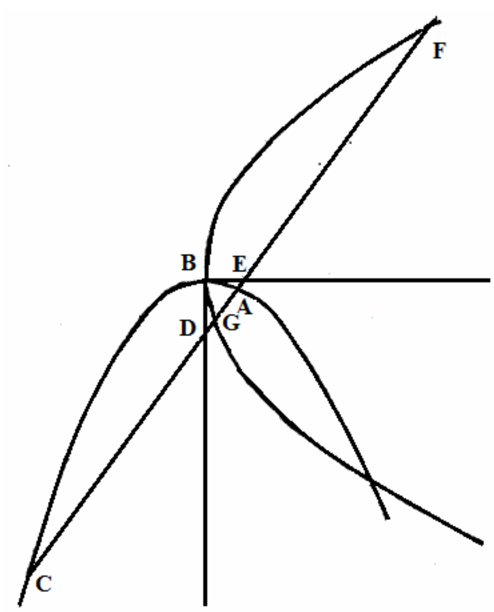
I say the rectangle HIC to be equal to the rectangle GDIF.

Demonstration.



The rectangle GDI, is equal to the square HD : but the square HD is equal to the square ID, that is, to the rectangle FDB, together with the rectangle HIC, therefore the rectangle GDI, is equal to the rectangles FDG, HIC. Similarly moreover the rectangle GDI, also is equal to the rectangles FDG, GDIF: therefore the rectangles FDG, HIC, are equal to the rectangles FDG, GDIF. Therefore with the common rectangle FDG taken away, the rectangle HIC remains equal to the rectangle GDIF. Q.e.d.

PROPOSITION CLXXXII.



The line BE shall be a tangent at B to the parabola ABC, of which the diameter is BD, and the parabola FBG described through B, of which the diameter is BE, and the tangent BD, some line FC shall be drawn, crossing, intersecting the parabolas at A, and G, F, C, and the diameters at E & D.

I say the rectangle AEC to be equal to the rectangle GDF.

Demonstration.

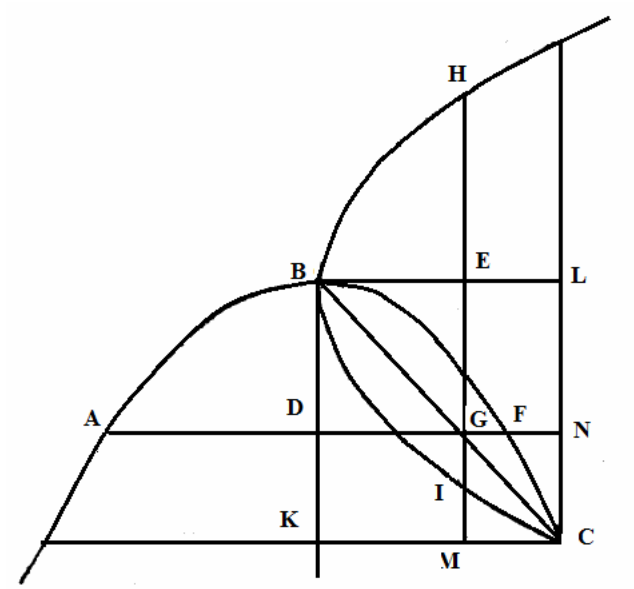
Because EB is a tangent line, the right lines AE, DE, CE [§.81] are proportionals, and thus the rectangle AEC is equal to the square ED*:* but the square ED is equal to the rectangle GDF, therefore the rectangle AEC is equal to the rectangle GDF. Q.e.d.

PROPOSITION CLXXXIII.

With the same in place: the two parabolas ABC, CBH shall intersect each other at C; and with BC joined, the ordinate ADF shall be put in place for BD in the parabola ABC, crossing the right line BC at G : and through G the right line HI shall be acting parallel to BD, crossing the diameter BL at E.

I say the rectangle IGH to be to the rectangle FGA, as the square IE to the square FD.

Demonstration.



The ordinate lines CK, CL shall be put in place from C for the diameters BD, BE ; because the parallelograms BC, BG have the common have the common diameter chord BGC, so that as CL is to GE, thus KC is to GD; but as CL to GE, thus the square IE is to the square GE, and as the line CK is to the line GD, thus the square FD is to the square GD; therefore as the square IE to the square FD, thus the square GE is to the square GD, and on interchanging so that as the square IE to the square FD, thus the square GE is to the square GD, but the square IE is equal to the square GE, together with the rectangle IGH; and the square FD is equal to the square GD, together with the rectangle FGA; and therefore the rectangle IGH is equal to the rectangle FGA as the square ID to the square FD. Q.e.d.

PROPOSITION CLXXXIV.

With the same in place, the line GH produced, shall cross the line CK at M, and AF produced the line LC at N. I say the rectangle IMH to be to the rectangle FNA, as the rectangle IGH is to the rectangle FGA.

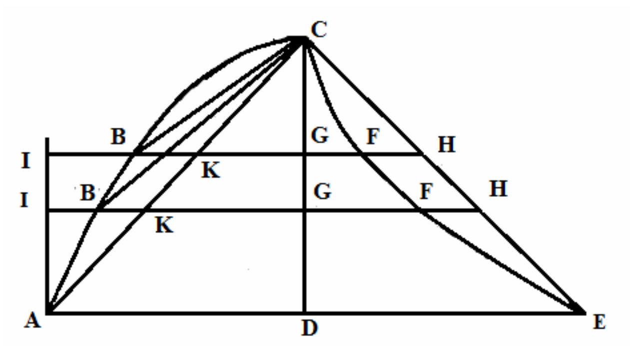
Demonstration.

It has been shown in the preceding proposition, the square KC to the square CL, to have the same ratio as the square DG to the square GE, and hence, as the square DF to the square EI : but the square KC, that is the square DN shall be equal to the square DF, together with the rectangle FNA; likewise the square CL that is EM to DN, just as the square EI to the square DF, also the rectangle IMH is to the rectangle FNA, as the square EI to the square DF, that is by the preceding, as the rectangle IGH to the rectangle FGA . Q.e.d.

PROPOSITION CLXXXV.

Let CD be the axis of the parabola ABC, equal to the latus rectum, and with the ordinate line DA acting through D, shall be produced to E, so that the lines AD, DE shall be equal and AC, EC shall be joined : then the parabola CFE shall be described through C and E, having the vertex at C and the tangent CD; and BG shall be drawn parallel to AD, crossing the parabola at B and F, and BC shall be joined.
I say as the square BC to be to the square BC, thus as the line KF to the line KF.

Demonstration.



The diameter AI shall be erected from A crossing the line BG at L and the square CB is equal to the squares BG, CG: but the square BG is equal to the rectangle KGI [§.42] , and the square CG or GH, (since CD, DE and thus CG, GH are equal lines) is equal to the rectangle FGI; therefore the square BC is equal to the rectangles FGI, KGI; that is to the rectangle IGDF: but the rectangle IGKF is to the rectangle IGKF, as the line FK to the line KF; therefore the square CB is to the square CB, as the line KF is to the line KF. Q.e.d.

PROPOSITION CLXXXVI.

With the same in place:
I say FG to GB, to have the cubic ratio of that which GB has to AD.

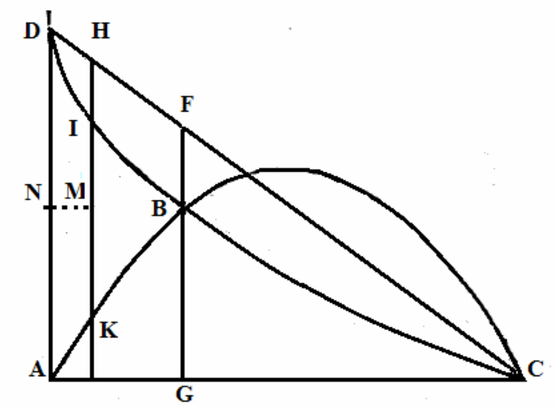
Demonstration.

Because both AD, BG, KG, as well as DE, GH, GF are proportionals, and the first AD is equal to the first DE, the ratio KG to GF, is the square of the ratio BG, GH that is, BG to KG, since indeed the lines AD, DC shall be equal, also they are equal to KG, GC; but the ratio FG to GB, is composed from the ratio FG ad GK, that is, from the square of the ratio KG to GB, that is BG to AD, (since AD, BG, KG shall be proportionals) and from the ratio KG to GB that is, therefore as FG to AD, therefore FG to GB, has the triplicate ratio of that which the line GB has to the line AD. Q.e.d. [§.42, §.26, §.9]

PROPOSITION CLXXXVII.

Again the two parabolas ABC, CBD will intersect at the points B, C, having the common diameters FG, HKL, DA which will intersect with the line CB at M and N.
I say IM to MK to have the same ratio as has DN to NA.

Demonstration.



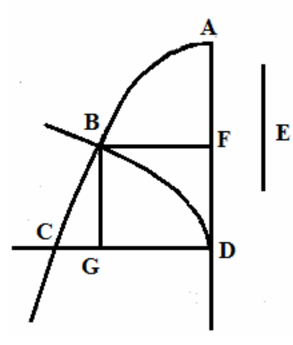
As the rectangle DHC is to the rectangle DIC, thus the rectangle ALC is to the rectangle AGC, but as DHC is to DFC, thus HI is to FB, and as ALC is to AGC, thus the line LK is to the line GB; therefore so that as HI to FB, thus KL to BG: and on converting by interchanging, so that as FB to BG, thus HI to KL. But as FB to BG, thus HM to ML; therefore as Hl to KL, thus HM to ML: and from which, IM is to MK, as HM to ML, that is DN to NA. Q.e.d.

PROPOSITION CLXXXVIII.

AD shall be the diameter of the parabola ABC which the ordinate line DC shall cut at D: then through D the parabola DB shall be described, having the tangent AD and the diameter DC and the common latus rectum E with the other section. But the parabola DB shall cross the parabola ABC in B, and the ordinate BF shall be drawn BF.
I say the line AF to the line FB, to have the square ratio of that which FB has to FD.

Demonstration.

BG shall be put parallel to AD. Therefore since the latus rectum E is common to both sections, and the ordinates FB, BG put in place, both the lines E, FB, AF as well as the lines E, BG, that is FD, and DC are proportionals. From which since the first E shall be common, the ratio AF to DG, that is to FB, is the square of the ratio FB to BG, that is, to ED. Q.e.d.



Corollary.

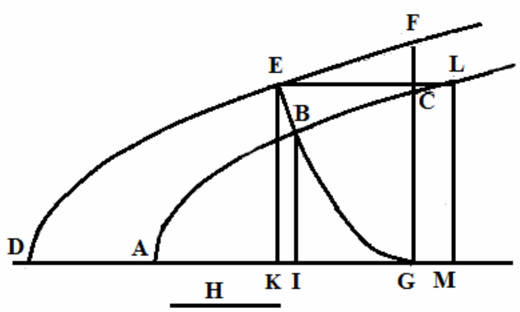
Hence it follows the ratio AF ad FD, to be the triplicate [i.e. cubic] of that ratio [recall that this work was executed before algebra was invented, so that the use of powers was not used in described ratios such as squares, cubes , etc. as we have done here and elsewhere for our convenience in understanding], of that ratio which is between AF and FD. Indeed the ratio AF to FD, is composed from the ratio AF ad FB. That is the square of BF to FD, and from the ratio BF that is FD: that is the cube of the ratio AF to FD.

PROPOSITION CLXXXIX.

The parabolas ABC, DEF shall be established for the same axes, with the common latus rectum H, and with some point G taken on the axis, from that the ordinate line GCF shall be drawn, and the parabola shall be described through G, having the axis GC, crossing in the parabolas ABC, DEF at the points B and E; from which the ordinate lines BI, EK shall be sent.

I say the ratio IA to KD, to be the fourth power of the ratio GI to GK.

Demonstration.

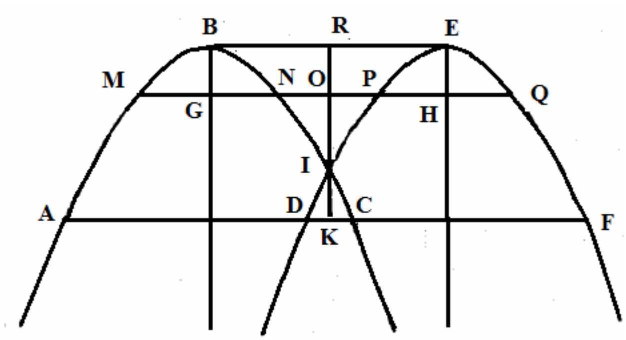


Indeed with the line EL drawn from E parallel to GD, with the parabola ABC being crossed at L, with the ordinate line LM dropped from L to the diameter GD : therefore the square LM shall be equal to the square EK, and from which the rectangle under H and MA, is equal to the rectangle under H and KD, and thus the line MA is equal to the line KD, therefore the ratio IA to KD, or to MA, shall be the square of the ratio IB to LM, or to EK; but the ratio IB to EK, is the square of the ratio GI to GK; therefore the ratio IA to KD, is the square of that ratio, which GI has to GK. Q.e.d.

PROPOSITIO CXC.

The two parabolas ABC, DEF, having the parallel axes BG, EH, shall intersect each other at I, and with the diameter acting through I, the right line ordinates MQ, AF are drawn.

I say the rectangle NOM to be to the rectangle POQ, as the rectangle CKA to the rectangle DKF.



Demonstration.

As IO is to IK, thus the rectangle NOM is to the rectangle CKA; but as IO to IK, thus the rectangle QOP is to the rectangle DKF; therefore so that as the rectangle NOM is to the rectangle CKA, thus the rectangle QOP is to the rectangle DKF, and on interchanging, the rect. NOM is to the rect. QOP as the rect. CKA to the rect. DKF. Q.e.d.

PROPOSITION CXCI.

With the same in place, if the parabolas ABC, DEF will have had the same height and the common tangent BE.
I say the rectangle NOM to be to the rectangle POQ, as the square BR to the square RE.

Demonstration.

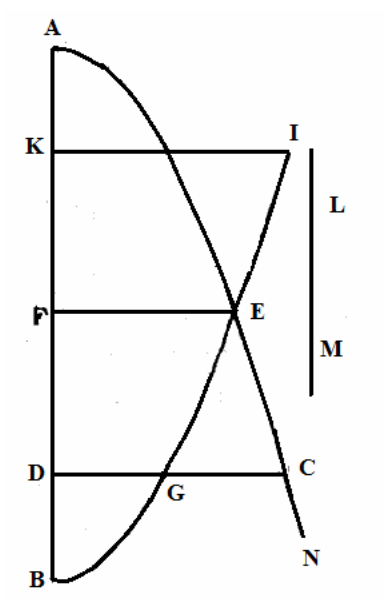
As the line RI to the line OI, thus the square BR to the rectangle NOM; and as RI to OI, thus the square RE to the rectangle QOP; therefore as the square BR to the rectangle NOM thus the square RE to the rectangle POQ, and on interchanging so that the square BR to the square RE, thus the rectangle NOM to the rectangle QOP. Q.e.d.

Corollary.

Hence also it is clear the rectangle AKC to be to the rectangle DKF, as the square BR to the square RE.

PROPOSITION CXCII.

AD shall be the axis of the parabola ABC and with the ordinate CD drawn, some right line DE may be taken on the axis and through E the parabola shall be described of which the axis shall be EA, cutting the parabola ABC at B and the line CD at G; and with AK made equal to DE, the ordinate KI shall be drawn for AE in the parabola EGB, and the ordinate BF for the parabola ABC.
I say the square CD to be to the square IK, as the line EF to the line FA.

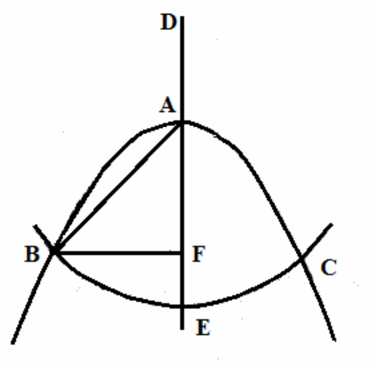


Demonstration.

L shall be the latus rectum of the parabola ABC, and M the latus rectum of the parabola EGB, the square CD is equal to the rectangle DAL; and the square IK is equal to the rectangle KEM, moreover the line AD is equal to the line KE, since AK, ED are placed equal; therefore the square CD is to the square IK, as I to M. Again since the square BF is equal both to the rectangle FAL as well as to the rectangle FEM; the rectangles FAL, FEM also are equal, therefore as AF to FE, thus as M to L; but as M to L, thus the square IK is to the square CD, therefore as AF to FE, thus the square IK is to the square CD. Q.e.d.

PROPOSITION CXCIII.

AD shall be the axis of the parabola ABC equal to the latus rectum, and with centre A some circular interval is described crossing the parabola at B,C; with the axis at E, and the ordinate BF put in place to the axis.



I say the lines DF, AE, AF to be in continued proportion.

Demonstration.

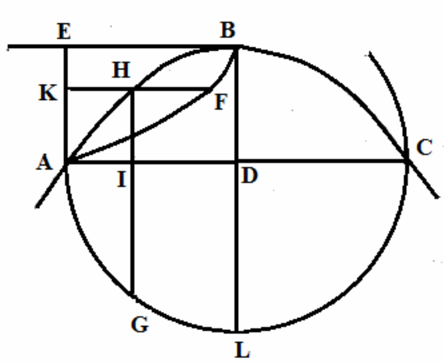
The right line AE is equal to AB : but AF, AB, DF, are proportionals, and therefore AF, AE, DF are continued in the same ratio. Q.e.d.

PROPOSITION CXCIV.

BD shall be the axis of the parabola ABC, and with the tangent BE acting through B, the ordinate of the applied line AC, which erected from the diameter A, will cross BE at E, the parabola AFG shall be described passing through AB, of which AE is the axis; and with centre D and with the radius AD, the circle AGC shall be described, which will cut the right line HG at some G, parallel to the axis BD, and crossing the parabola ABC at H, and the line AC at I: finally KF shall be put in place through H parallel to EB, crossing the parabola AFB at F and AE at K.

I say the lines GI, FK to be equal to each other.

Demonstration.

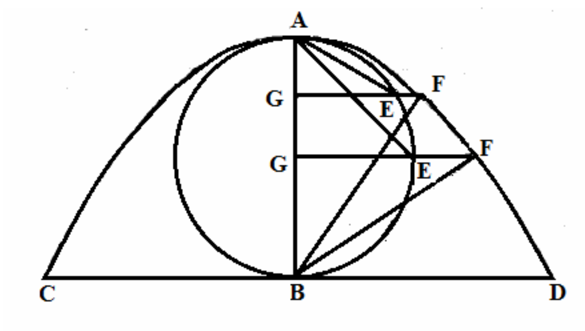


BD produced, shall cross the circle at L : so that AK to AE, that is HI to BD, thus as the square FK is to the square BE; but also as HI is to BD, thus the rectangle AIC is to the rectangle ADC, that is , as the square IG to the square DL; therefore as the square FK to the square BE, thus the square IG to the square DL, that is AD, that is EB, whereby the squares IG, FK, and thus the lines are equal to each other. Q.e.d.

PROPOSITION CXCV.

The circle AEB is described on the parabola ABC with the axis AB equal to the latus rectum, which at E will cut any ordinate GF put in place to the axis of the parabola, and AE shall be joined.
I say the lines AE, GF to be equal.

Demonstration.



Because the line AB is equal to the latus rectum, the square FG shall be equal to the rectangle GAB; and moreover the rectangle GAB is equal also to the square AE, since AGE is a right angle; therefore the squares AE, FG and thus the lines AE, GF between these shall be equal. Q.e.d.

PROPOSITION CXCVI.

With the same in place, FB shall be joined.
I say the square FB to be equal to the squares AG, GE, GB taken together.

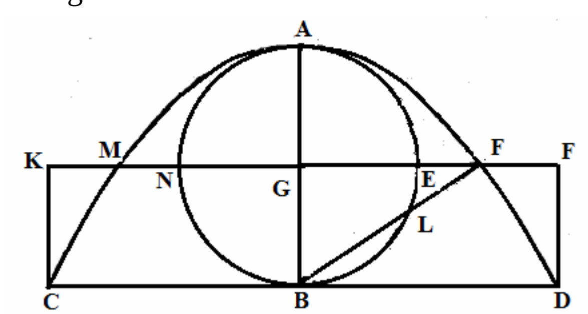
Demonstration.

The square FB, is equal to the squares BG, GF; but the square FG is equal to the square AE, that is, to the squares AG, GE, therefore the square FB is equal to the squares AG, GE, GB. Q.e.d.

PROPOSITION CXCVII.

With the same in place: the line FG shall cross the diameters DH, CK at H and K.
I say the rectangle HFK to be equal to the rectangle ABG.

Demonstration.



The square HG is equal to the square FG, together with the rectangle HFK; but the square of HG or of BD, is equal to the square AB, (since AB is put equal to the latus rectum) and therefore the square AB is equal to the square FG together with the rectangle HFK; but the square

AB also is equal to the rectangles GAB, GBA, that is to the square GF together with the rectangle GBA, therefore the square FG together with the rectangle GBA, is equal to the square FG with the rectangle HFK: therefore with the common square FG taken away, the remaining rectangles ABG, HFK are equal to each other. Q.e.d.

Corollary.

From the said above it is apparent the rectangle FEM to be equal to the square AG, for FE in the rectangle together with the square EG, shall be equal to the square FG, that is, to AE, that is to the squares AG, GE: therefore with the common square GE taken away, the rectangle FEM remains equal to the square AG.

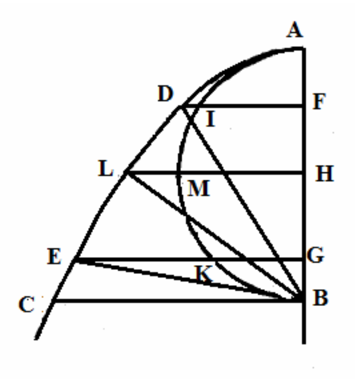
PROPOSITIO CXCVIII.

With the same in place: FG shall cross the circle at L.
I say the rectangle BFL, to be equal to the square AG.

Demonstration.

The rectangle BFL is equal to the rectangle NFE, that is equal to the rectangle MEF; but the rectangle MEF is equal to the square AG ; and therefore the rectangle BFI is equal to the square AG; and therefore the rectangle BFL is equal to the square AG. Q.e.d.

PROPOSITION CXCIX.



AB shall be the latus rectum of the parabola ABC, and with AF, BG taken equal, the right lines FD, GE shall be put the ordinates to the axis I say the lines BD, BE joined to be equal to each other.

Demonstration.

The circle shall be described on AB crossing the lines FD, GE at I and K; because the lines AF, BG are equal, and FI, GK normal to the axes AB, the right lines FI, GK, likewise AG, BF are equal to each other: but the square BD is equal to the squares AF, FI, FB; and the square BE is equal to the squares BG, GK, GA; therefore the square BD is equal to the square BE, and the line BD equal to the line BE. Q.e.d.

PROPOSITION CC.

With the same in place, the right line HML shall be drawn from the centre of the circle H to the circle AIB, parallel to FD; and BL are joined.
I say BL to be the shortest line of all which can be drawn from B to the periphery of the parabola.

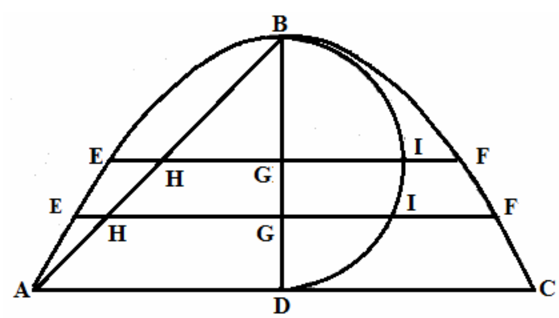
Demonstration.

Some other line BD may be put in place, and the ordinate DF; therefore the square BL shall be equal to the squares AH, HM, HB: and the square BD shall be equal to the squares AF, FI, FB ; truly since AH, HM, HB are semicircles, the squares of these are smaller than the squares AF, FI, FB (as is shown easily from the elements) and therefore the square BL is smaller than the square BD, and BD the shortest of all the lines, which can be drawn from B to the periphery of the parabola. Q.e.d.

PROPOSITION CCI.

The semicircle BID shall be described in the parabola ABC, with the axis BD equal to the latus rectum, which shall cut some ordinate FGE put in place to the axis at I, and with some ordinate AC acting through D, AB shall be drawn, crossing the line EF at H.
I say the rectangle FHE to be equal to the square GI.

Demonstration.



Because the axis BD is put equal to the latus rectum, and AD the applied ordinate, the right lines AD, BD and thus HG, GB are equal to each other, since truly the line EF is bisected at G and not bisected at H, therefore the rectangle EHF together with the square HG, that is BG, is equal to the square EG; but the square EG is equal to the rectangle GBD, that is, to the square BG together with the rectangle BGD; therefore the square BG together with the rectangle BGD shall be equal to the rectangle EHF together with the square HG, since that is the square BG; therefore with the common square BG removed, the rectangle EHF remains equal to the rectangle BGD, that is to the square GI. Q.e.d.

PROPOSITION CCII.

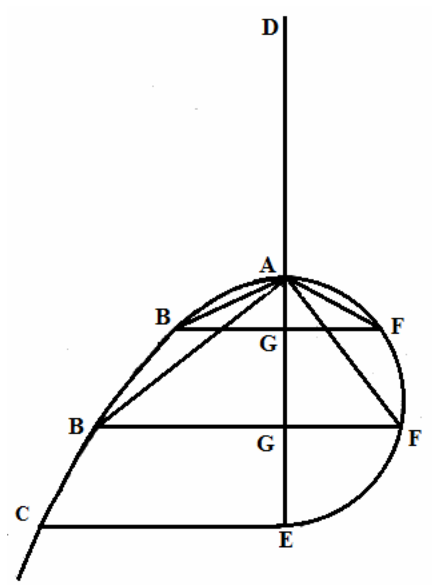
With the same in place:
I say the lines FI, GH, IE to be proportionals.

Demonstration.

Because the line EF is bisected at G and not bisected at I, the rectangle FIE together with the square IG, that is together with the rectangle BGD, is equal to the square GF, that is to the rectangle GBD, that is to the square BG together with the rectangle BGD, that is together with the square GI; therefore with the common square IG removed, the rectangle FIE remains equal to the square BG, that is to the square HG; and thus the proportionals are FI, GH, IE. Q.e.d.

PROPOSITION CCIII.

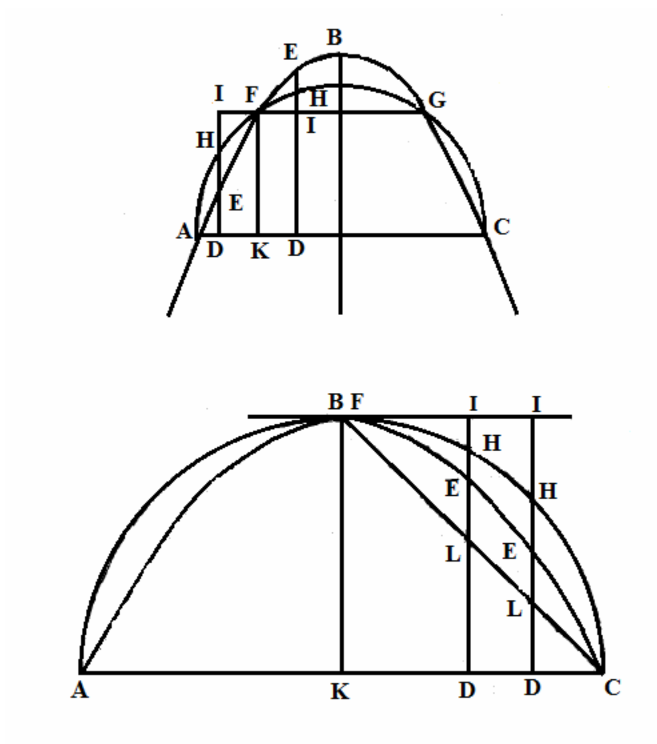
The semicircle AFE shall be described on the parabola ABC with the axis AG equal to the latus rectum, and with AD taken to be equal to AE, some ordinate BF may be drawn cutting the axis BF at G, and AB, AF shall be joined crossing the circle at F.
I say DG to DA, to be in the square ratio of that which AB has to AF.



Demonstration.

Because the square AF is equal to the squares FG, AG, that is to the rectangle DAG, the right lines AG, AF, DA are continued proportionals: but also AG, AB, GD are in a continued ratio [§.34] ; therefore since AG shall be the first of each common series, DG to DA, the third to the third, shall be as the square of the ratio AB to AF, of the second to the second. Q.e.d.

PROPOSITION CCIV.



The right line AC shall subtend the parabola ABC normal to the diameter ED: truly so that the semi-ellipse or semicircle AF, GC shall be described on AC crossing the parabola at F and G, or the same tangent at B may be drawn, and FI may be drawn parallel to AC itself, and the lines ID shall be placed parallel to ED, crossing the parabola at E, and the ellipse or semicircle at H, FI and the lines ID are put parallel to ED, crossing the parabola at F and G; or the same tangent at B, and the lines ID put parallel to ED, crossing the tangent line at I, and AC itself at D.

I say DE, DH, DI to be lines continued proportionals.

Demonstration.

The diameter FK shall be dropped from F, cutting the line AC at K, so that the rectangle AKC shall be to the rectangle ADC, thus as FK is to ED; but as the rectangle AKC to the rectangle ADC, thus the square FK also is to the square HD, therefore as the line FK is to the line ED, thus the square FK is to the square HD ; therefore the right line FK, that is ID, to ED, thus has the square ratio of that, which ID has to HD: whereby the lines DE, DH, DI are in continued proportion. Q.e.d.

PROPOSITION CCV.

With the second figure remaining, BC shall be drawn, crossing the lines ID at LL.
I say IE to ED, to have the square ratio of that which IL has to HD.

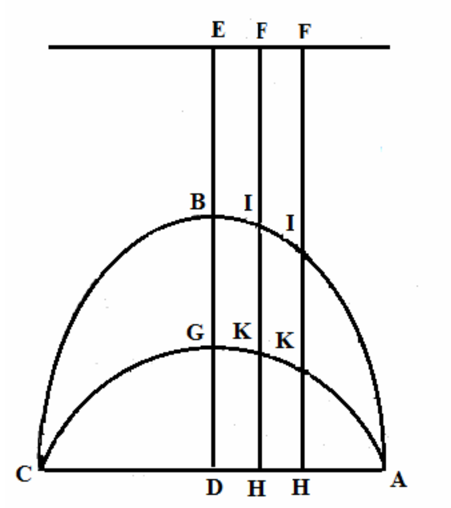
Demonstration.

Because both the lines ID, IL, IE, as well as the lines ID, HD, ED are in continued proportion, and ID is the first common term of each series, the ratio IE to ED, of the third to the third, is the square of the ratio IL to HD, of the second to the second. Q.e.d.

PROPOSITION CCVI.

AC, BD shall be the conjugate diameters of the ellipse ABC, or of the diameter of the circle; and some EF shall be drawn parallel to AC, with BD produced cut at E, ED, BD, GD shall become continued proportionals; and with the parabola described through the points AGC, of which the diameter is GD, some lines FH shall be put in place parallel to ED cutting the ellipse at I, the parabola at K, and the line AC at H.

I say the lines FH, IH, KH to be proportionals.



Demonstration.

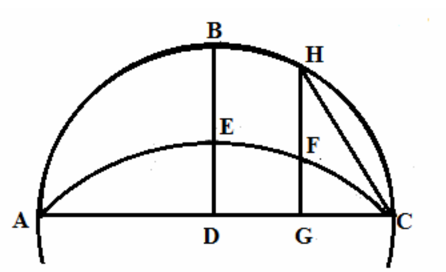
As the rectangle AHC to the rectangle ADC, thus the line HK to the line DG, and as the square IH to the square BD ; thus the line HK to the line DG has the square ratio of that, which the line IH has to the line BD. From which with the proportionals ED, BD. GD put in place, and the first terms ED, FH are equal to each other, and thus moreover the ratio DG to KH, of the third to the third term; have the square ratio of the second to the second, DB to HI, and therefore FH, IH, KH will be in continued proportion. Q.e.d.

PROPOSITION CCVII.

In the semicircle ABC the two diameters AC, BD are arranged crosswise to each other at right angles at D, of which the one BD is bisected at E, and the parabola shall be described by A, E, C, of which ED is the axis, and with the diameter FG drawn FG, which shall meet the semicircle at H; and HC shall be put in place.

I say FG to GC to have the square ratio of that, which HG has to HC.

Demonstration.



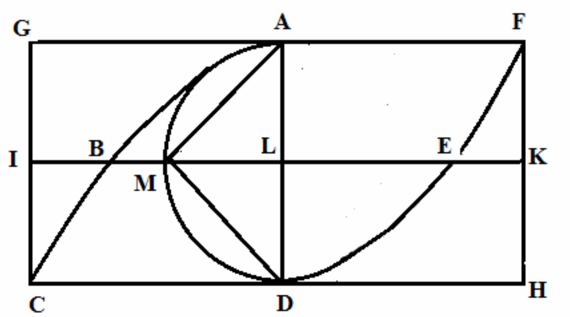
Because AC is twice BD, and with that twice ED, the right lines AC, BD, ED are in proportion ; moreover ED to FG, is in the square ratio of DB to GH, just as the rectangle ADC shall be to the rectangle AGC, as the square BD to the square HG; therefore the right lines GF, HG, AC are in continued proportion, moreover the lines AC, HC, GC are in proportion; therefore FG to GC, the third to the third, has the square ratio of that, which HG has to HC, the second to the second. Q.e.d.

PROPOSITION CCVIII.

Equal parabolas ABC, and DEF inverted, shall be established on the same axes AD, which shall be equal to the latus rectum put in place. The lines GF, HC shall be tangents to the parabolas at A and D; and indeed GF shall cross the parabola DEF at F; and truly the line HC shall cross the parabola ABC at C: and from C and F with the diameters which cross the lines FG, CH at G and H, some IK shall be drawn, parallel to FG, cutting the parabolas at B and E, the right lines CG, HF at I and K, the axes AD at L, then on AD as diameter, the semicircle AMD shall be described, crossing the line IK at M.

I say the rectangle ILM to be equal to the rectangle ELB.

Demonstration.



Because AD is equal to the latus rectum, the lines AD, CD, likewise AM, LB, likewise MD, LE are equal; therefore the rectangle AMD is equal to the rectangle ELB: but the rectangle AMD also is equal to the rectangle ADML, that is ILM; therefore the rectangles ILM, BLE are equal.
Q.e.d.

PROPOSITION CCIX.

With the same in place:
I say the rectangle ILBM to be equal to the rectangle BLEK.

Demonstration.

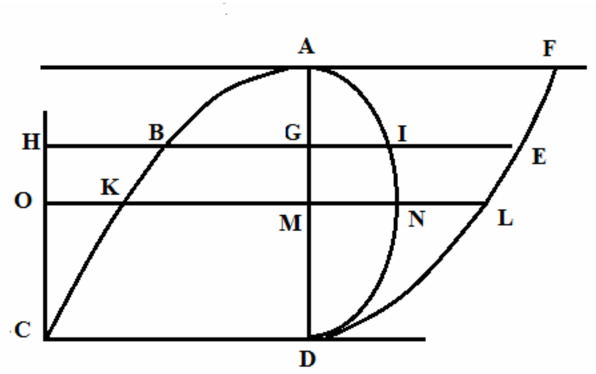
Because the rectangle ILM is equal to the rectangle ELB, so that as IL to LE, that is LK to LE, thus LB is to LM: and on interchanging terms, so that as LK to LB, thus EL is to LM: and EK remaining to MB, as LK to LB: that is IL to LB; therefore the rectangle ILBM is equal to the rectangle BLEK. Q.e.d.

PROPOSITION CCX.

With the same in place as before: the rectangle BGE shall become equal to the rectangle HGI: and an ellipse may be described through the points A, I, D, of which the axis shall be AD; and some line LK shall be drawn parallel to BE, crossing the parabolas at K and L, the axis AD at M, the ellipse at N, the diameter CH at O.

I say the rectangle KML to be equal to the rectangle OMN.

Demonstration.



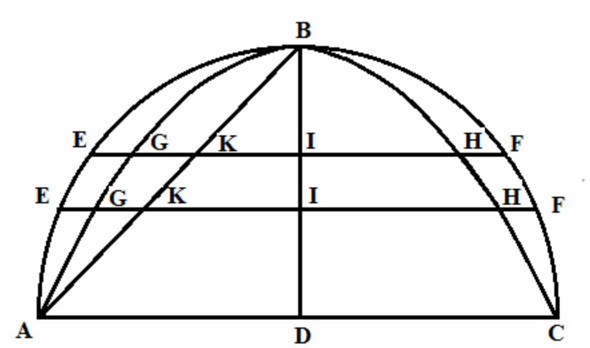
Because AD is the axis of the ellipse, and to that the ordinates IG, MN shall be put in place, so that as the square IG to the square NM, thus the rectangle AGD is to the rectangle AMD: therefore the rectangle AGD to the rectangle AMD, has the ratio IG to NM, that is the ratio of the rectangle HGI to the rectangle OMN: and because the ratio of the rectangle AGD, to the rectangle AMD, is composed from the ratio AG to AM, that is from the square ratio BG to KM, and from GD to MD, that is from the square ratio GE to ML, the ratio of the rectangle AGD to the rectangle AMD also is the square of that, which the rectangle BGE has to the rectangle KML, therefore as the rectangle HGI, to the rectangle OMN, thus the rectangle BGE to the rectangle KML: and on interchanging so that as the rectangle HGI to the rectangle BGE, thus the rectangle OMN is to the rectangle KML; but by the hypothesis, the rectangles HGI, BGE are equal ; and therefore the rectangles OMN, KML are equal to each other. Q.e.d.

PROPOSITION CCXI.

The two diameters AC, BD shall cut the semicircle ABC orthogonally: and with the parabola described through A,B,C, of which the axis is BD, some right lines EF shall be drawn, parallel to AC, crossing the circle at E and F, the parabola at G and H, truly the axis at I.

I say the rectangle EGF to be to the rectangle EGF, as the rectangle BID to the rectangle BID.

Demonstration.



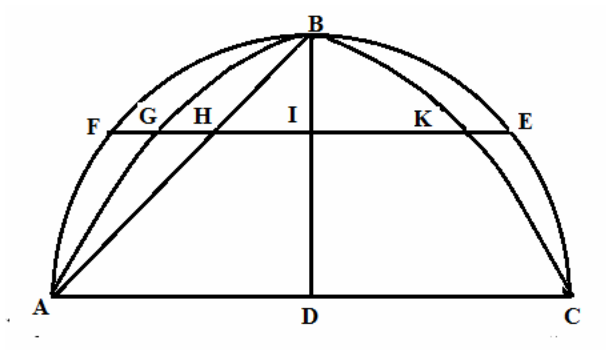
The line AB shall be put in place, it shall cut the line EF at K so that the rectangle BKA shall be to the rectangle BKA, thus as the rectangle EKF is to the rectangle EKF , but as the rectangle BKA to the rectangle BKA, thus the rectangle GKH is to the rectangle GKH; therefore as the rectangle EKF is to the rectangle EGF, thus the rectangle GKH is to the rectangle GKH, therefore the remaining rectangle EGF is to the rectangle EGF, as the rectangle EKF to the rectangle EKF, that is the rectangle BKA to the rectangle BKA, that is: the rectangle BID to the rectangle BID. Q.e.d.

PROPOSITION CCXII.

The two diameters AC, BD shall cut the semicircle ABC orthogonally: and the parabola described through A and B of which the axis is BD, and the other parabola described through B and C, having the axes DC and the vertex C, and the line EF shall be drawn parallel to AC, meeting the circle at E and F, the parabolas at G and H, the axis BD at I, and the joined line AB at K.

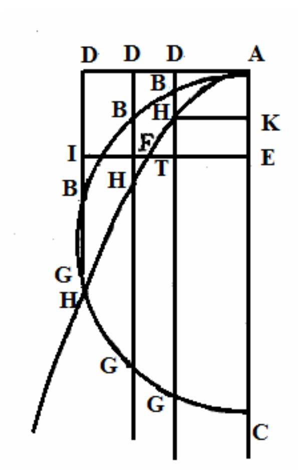
I say the square of the line EI shall be to the square GI, as the line IH is to the line IK.

Demonstration.



Because both the lines AD, GI, KI , as well as DC, FI, HI are proportionals, and the first of each series AD, CD are equal, the ratio HI to IK, of the third to the third, is the square of the ratio FI to IG, that is EI to IG of the second to the second; therefore so that the line HI to the line IK thus shall be as the square EI to the square GI. Q.e.d.

PROPOSITION CCXIII.



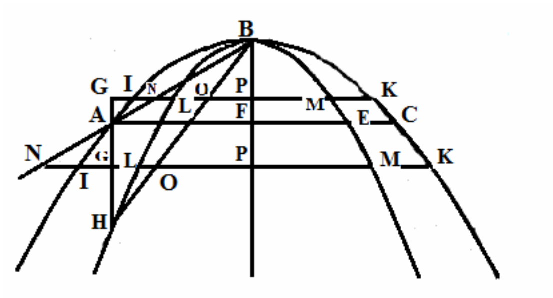
The line AD shall be a tangent at A to the semicircle ABC, of which the diameter shall be AC; and the parabola described through A, of which the axis AC, and the tangent AD, and the latus rectum shall be taken equal to AE; and with the ordinate EF acting through E, some diameter DG shall be drawn in the parabola crossing the circle at B and G, the parabola at H, and the ordinate FE put in place at I.

I say the rectangle BDG, to be equal to the rectangle HDI.

Demonstration.

The ordinate line HK shall be drawn through H: Because the line AD shall be a tangent to the circle at A, the rectangle BDG is equal to the square AD, that is to the square HK; moreover the square HK is equal to the rectangle KAE, that is to HDI (because by the hypothesis AE is equal to the latus rectum); therefore the rectangle BDG is equal to the rectangle HDI. Q.e.d.

PROPOSITION CCXIV.



The two parabolas ABC, DBE shall have the same axis BF, and DBE itself being an interior tangent to the common vertex at B; some diameter GH shall intersect the parabolas at H and A, and with AB, HB joined, the ordinate line IK shall cut the parabolas at I, K, L, & M; the right lines AB, HB, AH at N, O, G , and the axis BF at P.

I say the rectangle ILK to be equal to the rectangle GPNO.

Demonstration.

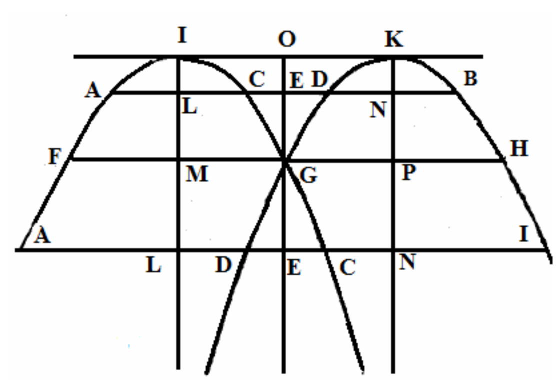
The rectangle GPN is equal to the square IP, but the square IP is equal to the square LP, that is, to the rectangle GPO, together with the rectangle ILK ; therefore the rectangle GPN is equal to the rectangle GPO ; but the rectangle ILK is equal to the rectangle GPN, equal to the rectangles GPO, GPNO, therefore with the common rectangle GPO taken away, the rectangle ILK will remain equal to the rectangle GPNO. Q.e.d.

PROPOSITION CCXV.

The given line AB to be divided anew at C & D to be divided again at E so that the rectangle AEC thus shall be to the rectangle BED as the square AC to the square DB.

Construction.

With the right lines AC, BD bisected at L & N, with the perpendiculars LI, NK erected equal to each other, and the parabolas NK are described through the points A, I, C and D, K, B, as if mutually crossing each other at G. Then from G the normals DE to AB are drawn.

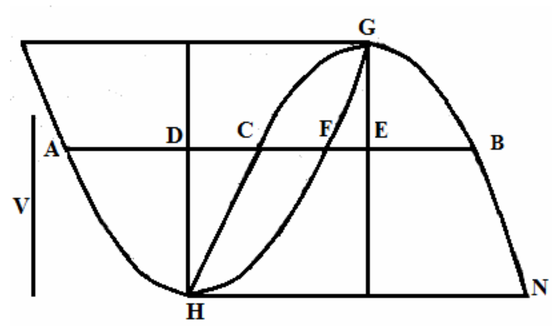


Demonstration.

With the points I, K joined and EG produced to O, through G draw FM, PH parallel to AB and IK. Because both IL & OE, KN, as well as IK, FH are parallel, the square IO will be to the square OK as the square MG to the square PG, that is, as the square FG ; then the square AC is to the square FG as LI to MI, that is as the square DB to the square GH. Therefore on interchanging, the square AC is to the square DB, as the square FG to the square GH, that is (as shown before) as the square IO to the square OK. And the rectangle AEC is to the rectangle BED as the square IO to the square OK. Therefore also the rectangle AEC is to the rectangle BED as the square AC to the square DB. Therefore what was desired has been done.

PROPOSITION CCXVI.

The given right line AB divided at C, to be cut again at D, so that the rectangle BDC shall be equal to the square DA.



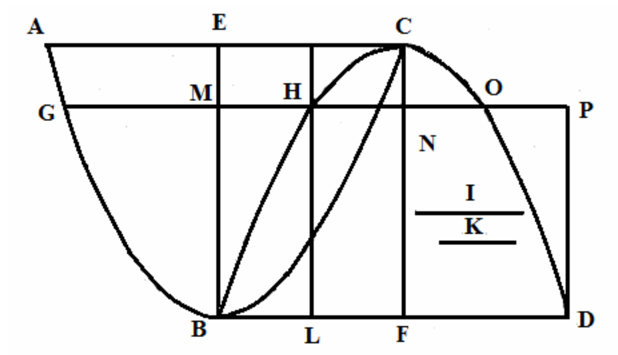
Construction and Demonstration.

CB shall be bisected at E, and made so that AE to CE, thus shall be as CE to FE; then AF shall be bisected again at D. I say the what is desired has been done.
From the points E and D with normals erected, one of which EG shall be the magnitude desired, the other DH infinite. Then through the points C, G, B a parabola shall be described having the axis EG and crossing DH at H. Again through the points AHF a parabola is understood to be described having the axis DH; moreover EG, DH will be able to be the axes of the parabolas, since both will be able to be bisected equally from the construction of the right lines CB, FA at right angles. Therefore since from §.167, the construction AE, CE, FE are considered to be in continued proportion, according to which the parabola AHF to be transformed through the vertex G of the other parabola. Therefore by that same proposition BD, FD, CD are continued in proportion; as from the construction FD and AD are equal, therefore BD, DA, CD are continued in proportion; therefore the rectangle BDC shall be equal to the square DA. Q.e.d.

PROPOSITION CCXVII.

The two equal parabolas ABC, BCD shall be put in place parallel to the axes EB, CF, passing through the common apices ; the ordinate BF shall be drawn from B.
It shall be required to put GH parallel to BF, which shall be divided by the right line BE, following the given ratio I to K.

Construction & Demonstration.



The line BD shall be divided according to the ratio of the square I, to the square K, at the point L: then IH shall be erected, which shall be parallel to the axis CF, and it shall cross the section at H and through H, GH shall be draw parallel to BD, cutting BE at the point M. I say the right line GH to be the cut at M, according to the ratio I to K; from D PD shall be erected parallel to BE crossing GO at P. Since the rectangle HMO shall be equal to the square MN; MH, MN, MO shall be in continued proportion. Therefore the square MH shall be to the square MN, as MH to MO; that is, as MH to HP, that is as the square K to the square I; but the square MN shall be equal to the square GM; therefore the square MH is to the square GM as the square K to the square I and by inverting, therefore GM is to the right line MH as the right line I to the right line K. Therefore we have established, etc. Q.e.d.

PARABOLAE

PARS QUARTA

Proprietates contemplatur parabolarum sese invicem vel circulos intersecantium.

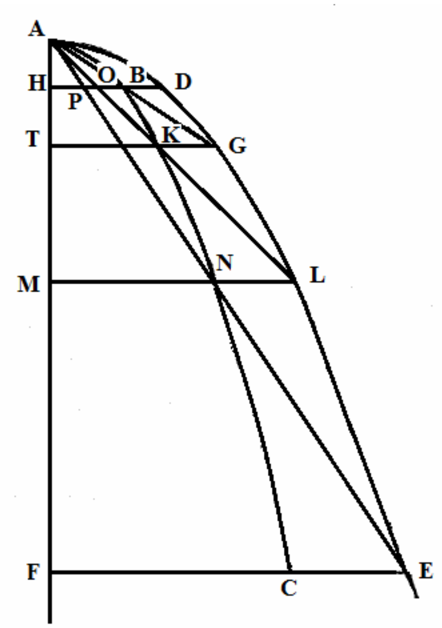
PROPOSITIO CLIII.

Habeant ABC, ADE parabolae communem axem AF, & verticem A: ex quo demissa linea AG quae parabolis occurrat in B & G ordinatim ad axem lineae DH, GI: & GI quidem parabolae ABC occurrat in K puncto, per quod ex A secans ponatur AL; & ex L, ordinatim recta LM, occurrens ABC parabolae in N : dein per N ponatur AE, & ex E ordinatim linea ECF.

Dico DH, KI lineas, item LM, CF inter se aequales esse.

Demonstratio.

Ratio AH ad AI, hoc est HB, ad IG, duplicata est rationis HB ad IK, hoc est:
HD ad IG, igitur HB, IK, IG proportionales sunt, & ut IK ad IG, sic HB ad IK, sed HB est ad HD, ut est IK ad IG; igitur HB est ad HD, ut HB ad IK; aequales igitur sunt HD & IK; eadem ratione ostendetur lineas LM, CF aequales esse.



PROPOSITIO CLIV.

Iisdem positis, rectae AL, AE secent DH lineam in O & P.

Dico rectas EF, LM, GI, DH, BH, OH, PH in continua esse proportione.

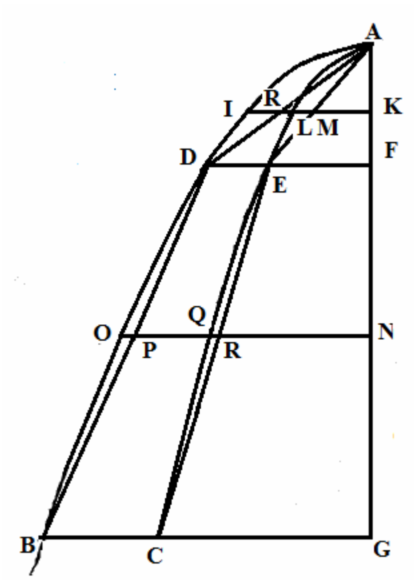
Demonstratio.

Ratio AH ad AI, hoc est HB ad IG, duplicata est rationis HB ad IK, hoc est HG ad IG; igitur HB, IK, IG proportionales sunt, & ut IK ad IG, sic HB ad IK, sed HB est ad HD, ut est IK ad IG; igitur HB est ad HD, ut HB ad ; aequales igitur sunt HD & IK; aequales igitur sunt HD & IK; eadem ratione ostendetur lineas LM, CF aequales esse.

PROPOSITIO CLIV.

Iisdem positis, rectai AL, AE secent DH lineam in O & P.
Dico rectas EF, LM, GI, DH, BH, OH, PH in continua esse proportione.

Demonstratio.



Quoniam NM aequalis est ostensa GI, & EF, LM, NM proportionales sunt, erunt & EF, LM, GI lineae in continua analogia: & quia LM, GI, KI proportionales sunt, & KI lineae aequalis linea DH, continuabunt quoque LM, GI, DH, rationem cum EF, LM, GI : est autem praeterea ut GI ad DH, sic DH ad BH, proportionales igitur sunt EF, LM, GI, DH, BH; ulterius cum sit ut GI ad KI, sic BH ad OH, sit autem ut GI ad KI, sic GI ad DH, & DH ad BH, rectae GI, DH, BO, OH continuae proportionales sunt. Iterum cum OH sit at PH, ut LM ad NM, id est ex demonstratis LM ad GI, id est GI ad DH, hoc est DH ad BH, hoc est BH ad OH; erunt GI, DH, BH, OH, PH lineae, adeoque omnes EF, LM, GI, DH, BH, OH, PH in continua analogia. Quod erat demonstrandum.

PROPOSITIO CLV.

Habeant ABC, ADE parabolae communem axem AF, positisque ad illum ordinatim lineis FBD, GCE, iungantur AB, AD, BC, DE ducanturque HI, NO parallelae FD, & HI quidem secet parabolas in I & L, lineas AD, AB in K & M, recta vero NO; occurrat DE, BC lineas AD, AB in K & M, recta vero NO; occurrat DE, BC, lineis in P & R, parabolis in Q & O.
Dico esse ut LM ad RQ, sic IK ad OP.

Demonstratio.

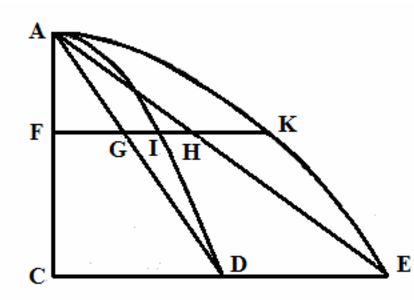
Ut IH ad LH, sic DF ad BF; sed ut DF ad BF, sic KH ad MH, igitur ut IH ad LH, sic KH ad MH, & IK ad LM, ut IH ad LH, ut IH ad LH; id est ut DF ad BF, sed ut DF ad BF, sic ON quoque est ad QN, sic PN ad RN; & OP ad QR, ut ON ad QN, id est DF ad BF, id est ex demonstratis IK ad LM, & permutando ut IK ad OP, sic LM ad RQ.
Quod erat demonstrandum.

PROPOSITIO CLVI.

Parabolae duae ad eundem axem AC constitute communem habeant apicem A: positaque ad axem ordinatim CDE, iungantur AD, AE & recta ducatur FK parallela CE, secans parabolas in I & K , rectas vero AD, AE, in G & H.

Dico GFH rectangulum ad rectangulum IFK eam habere rationem, quam habet FG ad lineam CD.

Demonstratio.



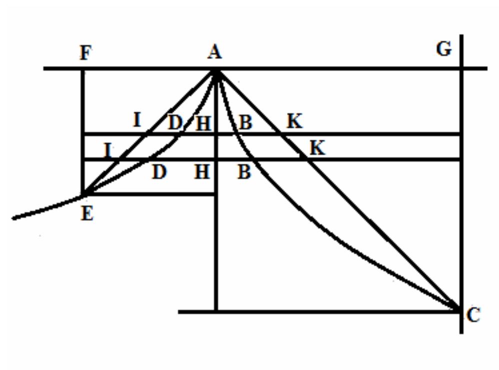
Ratio rectanguli GFH ad IFK, rectangulum composita est ex ratione FG ad FI, & ex HF ad FK: est autem ut FG ad FI, sic FI ad CD; & ut FH ad FK, sic FK ad CE ; ratio igitur rectanguli GFH ad rectangulum IFK, composita est ex ratione FI ad CD, ac ex FK ad CE, sed FK est ad CE, ut FI est ad CD; rectangulum igitur GFH ad IFK rectangulum duplicatam habet rationem eius quam habet linea FI ad CD : id est rectangulum GFH ad IFK rectangulum, est ut FG ad CD, quia FG, FI, CD proportionales sunt. Quod fuit demonstrandum.

PROPOSITIO CLVII.

Contingant se exterius in vertice, parabolae duae ABC, ADE, positisque ex A secantibus AE, AC, & contingente AH, ducantur quotcunque IK parallelae axi communi FAG.

Dico IHD rectangulum,esse ad rectangulum IHD, ut KHB rectangulum ad rectangulum KHB.

Demonstratio.



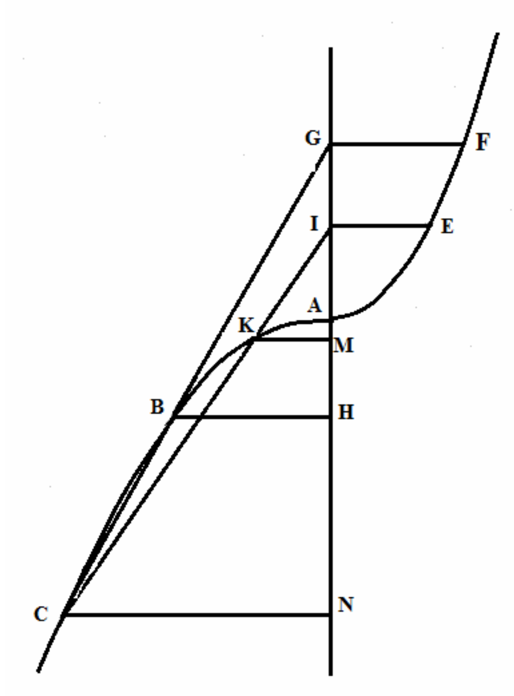
Rectangulum IHD ad rectangulum IHD, triplicatam habet rationem eius quam habet IH ad IH, id est AH ad AH; sed & BHK rectangulum ad retangulum BHK, triplicatam habet rationem eius quam habet HK ad HK, id est AH ad AH: igitur ut BHK rectangulum est ad rectangulum BHK, sic IHD rectangulum est ad rectangulum IHD. Quod erat demonstrandum.

PROPOSITIO CLVIII.

Contingant se rursum exterius in vertice parabolae duae ABC, AEF, ad eundem axem GH constitutae, sumptoque in ABC perimetro puncto quovis C ducantur ex C lineae CG, CI secantes ABC parabolam in B & K, & axem GH in G & I : dein ordinatim ponantur BH, KM, IE, GF.

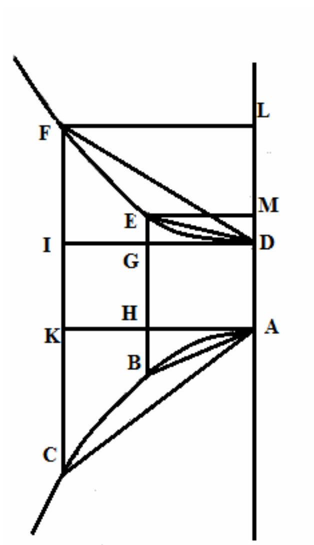
Dico HB lincam ad lineam MK, duplicatam habere rationem eius quam habet GF linea ad lineam IE.

Demonstratio.



Ducatur ex C ordinatim linea CN. Quoniam tam NA, AI, AM lineae quam NA, AG, AH continuae proportionales sunt & NA prima utrique seriei communis, ratio AH ad AM, tertiae ad tertiam, duplicata est eius quam habet AG ad AI, secunda ad secundam: sed AH ad AM. duplicata quoque rationem habet BH ad KM; igitur BH est ad KM ut AG est ad AI : & HB ad MK duplicatam habet GF ad IE. Quod erat demonstrandum.

PROPOSITIO CLIX.



Esto ABC parabolae axis AD aequalis lateri recto; & per D parabola descripta DEF aequalis parabola ABC apicem habeat D, obversum apici parabolae ABC & axem communem; dein quaevis ducantur diametri EB, FC occurrentes parabolis in B, C, E, F punctis; actae vero per A & D, contingentes, diametros EB,FC secent in G, H, I, K.

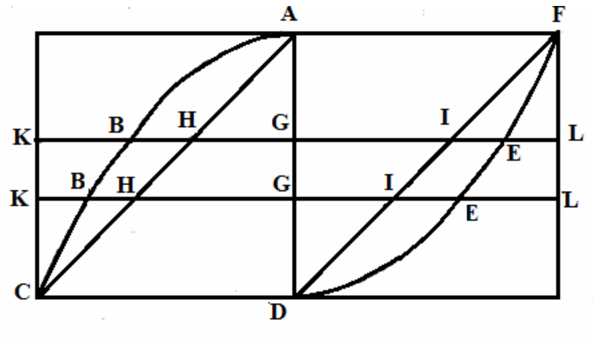
Dico EGB rectangulum esse ad rectangulum FIC, ut est quadratum ED ad quadratum FD.

Demonstratio.

Ponantur ordinatim lineae FL, EM. Quoniam AD linea, aequalis ponitur lateri recto,
and EM, FL ordinatim applicatae sunt, quadratum ED (hoc est quadrata EM, MD) aequale est rectangulo DMA:& quadratum FD (hoc est quadrata FL, LD) aequale rectangulo DLA; igitur ut ED quadratum ad quadratum FD, sic DMA rectangulum ad rectangulum DLA: sed DMA id est GEH, rectangulo aequale est rectangulum EGB; & DLA aequatur FIC; rectangulum igitur EGB ad FIC, rectangulum est ut quadratum ED ad rectangulum FD.

PROPOSITIO CLX.

Sint ABC, DEF parabolae, ad eundem axem AD inverse positae; ductisque ex A & D, ordinatim lineis AF, DC, iungantur puncta AC, FD; iungantur puncta AC, FD; ducantur praeterea quotvis BE parallelae AF occurrentes parabolis in B & E, rectis AC, FD in H & I, & axi AD in G.



Dico quadratum BG, rectangulum BGE & quadratum GE in continua esse analogia.

Demonstratio.

Ut BG linea ad lineam GE, sic quadratum BG est ad rectangulum BGE: sed etiam ut BG ad GE, sic BGE rectangulum est ad quadratum GE; igitur ut quadratum BG ad rectangulum BGE, sic BGE rectangulum est ad quadratum GE. Quod erat demonstrandum.

PROPOSITIO CLXI.

Iisdem positis:
Dico AGD rectangulum ad rectangulum AGD, duplicatam habere rationem eius quam habet BGE rectangulum ad rectangulum BGE.

Demonstratio.

Ratio AGD rectangulum ad rectangulum AGD composita est ex ratione AG ad AG, & ex GD ad GD: ratio vero rectangulum BGE ad BGE rectangulum, composita est ex ratione BG ad BG, & ex GE ad GE: sed ratio AG ad AG duplicata est: rationis BG ad BG, & ratio GD ad GD; duplicata est rationis GE ad GE, ratio igitur rectangulum AGD ad rectangulum AGD, duplicata est eius quam habet BGE rectangulum ad rectangulum BGE. Quod erat demonstrandum.

PROPOSITIO CLXII.

Iisdem positis: erigantur ex C & F diametri CK, FL occurrentes lineis BE in K & L.
Dico rectangula KGL, BGE, HGI in continua esse analogia.

Demonstratio.

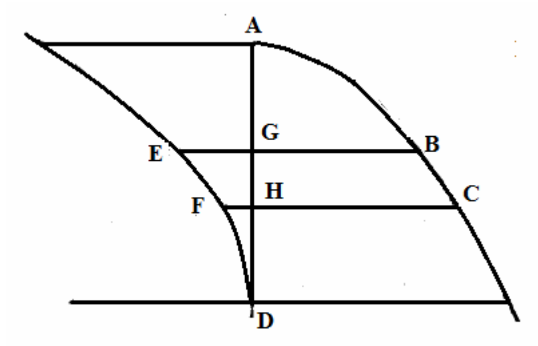
Rectangulum KGL ad rectangulum BGE, rationem habet compositam ex KG ad BG, & ex GL ad GE : & BGE rectangulum ad rectangulum HGI; rationem habet compositam ex BG ad HG, id est KG ad BG, & ex GE ad GI: id est GL ad GE; rectangulum igitur KGL est ad rectangulum BQE, ut BGB rectangulum ad rectangulum HGI. Quod erat demonstrandum.

PROPOSITIO CLXIII.

Sint rursum parabolae duae ABC, DEF : & ABC parabolae quidem apex sit A; parabolae vero DEF sit apex D, in quo contingat axem AD in D: factisque AG, DH lineis aequalibus; agantur per G & H, normales ad AD, lineae EB, FC occurrentes parabolis in E, B, F, C.

Dico EGB rectangulum ad rectangulum FHC quintuplicatam habere rationem eius quam habet GB linea ad lineam HC.

Demonstratio.

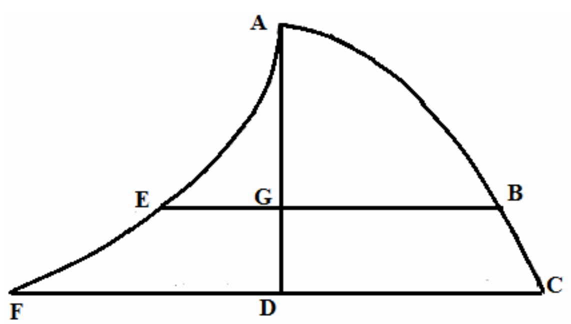


Ratio EGB rectanguli ad rectangulum FHC, composita est ex ratione EG ad FH, hoc est duplicata GD ad HD; id est AH ad AG, (cum AG, BD lineae aequales sint) hoc est ex quadruplicata ratione GB ad HC; & ex ratione GB, ad HC: rectangulum igitur EGB ad FHC rectangulum quintuplicatam habet rationem GB ad HC. Quod fuit demonstrandum.

PROPOSITIO CLXIV.

Sit ABC parabolae axis AD, & AEF parabolae vertex A, in quo illam contingat linea AD, positaque EGB ordinarim ad ducatur & altera FDC, ut GB, FD rectae aequales sint.

Dico rationem EG ad DC quintuplicatam esse eius quam habet GB ad DC.



Demonstratio.

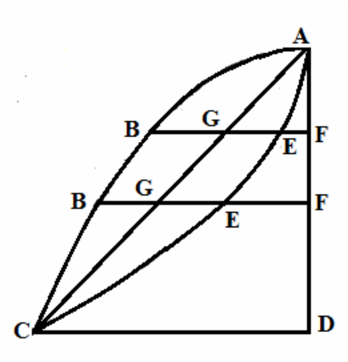
Ratio EG ad DC componitur ex ratione EG, hoc est FD, ad DC; igitur ex quadruplicata ratione GB ad DC. Quod fuit demonstrandum.

PROPOSITIO CLXV.

Sit ad ABC parabolae diametrum AD ordinarim posita CD, describatur autem per A & C , parabola AEC habens verticem in A & AD, contingentem, iunctisque; AC ponantur ad AD, ordinatim lineae BG, EF.

Dico GFE rectangulum esse ad rectangulum GFE in sextuplicata ratione FB ad FB.

Demonstratio.



Rectangulum GFE ad GFE rectangulum triplicatam habet rationem GF ad GF, id esse AF ad AF; sed AF ad AF, rationem habet duplicatam eius quam habet BF ad BF; rectangulum igitur GFE ad GFE, rectangulum sextuplicatam habet rationem FB ad FB. Quod erat demonstrandum.

PROPOSITIO CLXVI.

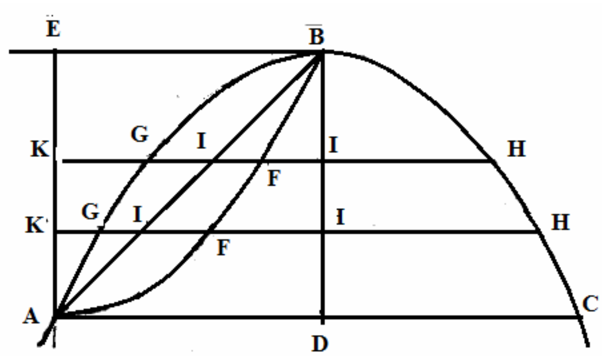
Iisdem positis:
Dico EFB rectangulum ad rectangulum EFB quintuplicatam habere rationem eius quam habet FB ad FB.

Demonstratio.

Rectangulum EFB ad EFB, rectangulum rationem habet compositam ex EF ad EF, id est ex duplicata rationis AF ad AF, id est ex quadruplicata rationis FB ad FB, & ex ratione FB ad FB; ratio igitur EFB rectangulum ad rectangulum:EFB quintuplicata est rationis FB ad FB.

PROPOSITIO CLXVII.

Parabolam ABC cuius axis BD, contingat in B linea BE, in qua assumpto quovis puncto E demittatur EA, occurrens ABC parabolae in A; tum per A & B parabola describatur AFB habens verticem in A, occurrens ABC parabolae in B; sitque eius axis AE: ducatur autem KH secans AFB parabolam in F, axem BD in I.



Dico GK, FK, HK lineas esse proportionales.

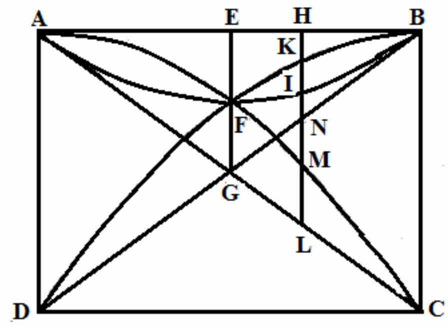
Demonstratio.

Iuncta AB secet HG lineam in L. Quoniam BD aequidistat diametro AE, & IK ordinatim ad illam applicatur, rectangulo LKI aequale est quadratum FK, sed & LKI rectangulo aequale est rectangulo GKH, quadratum igitur FK aequale est rectangulo GKH. & G K, FK, HK lineae in continua analogia. Quod erat demonstrandum.

PROPOSITIO CLXVIII.

Esto ABCD parallelogrammum, descriptaque per A & C, parabola cuius diameter sit AD, & contingens AB, describatur & altera per B & D, cuius diameter sit B C &

contingens AB: occurat autem parabolae AEC in E; deinde per A, E, C puncta tertia describatur parabola communes habens cum aliis diametros.
Dico iunctas AC, BD, parabolam AEC contingere in A & C.



Demonstratio .

Agatur per E diameter EF secans DB lineam in G, ut FE ad AD, sic FB quadratum ad quadratum AB, & ut FE linea , ad lineam BC, sic AF, quadratum ad quadratum AB: sunt autem AD, BC lineae per hypothesim aequales, igitur & FB quadratum est ad quadratum AB, ut AF quadratum ad quadratum AB: quadrata igitur AF, FB aequalia sunt, & linea AB dupla AE, uti AD dupla FG: quare FE etiam AC lineae occurrit in G puncto, quo bifariam a DB, altera parallelogrammi diametro secatur. Rursum quia AB quadratum quadruplum est quadrati FB, erit & BC linea, quadrupla lineae FE; est autem AD id est BC, ostensa dupla FG, igitur FG dupla est FE, & FE, EG lineae aequales, unde BD linea est contingens; similiterr ostenditur AC lineam, parabolam AEB contingere. Quod erat demonstrandum.

Corollarium.

Hinc patet AG, BD lineas, in illo puncto se intersecare, vbi EG est aequalis rectae FE: quod singulatim & explicite annotare verbo placuit.

PROPOSITIO CLXIX.

Iisdem positis: oportet E punctum intersectionis exhibere.

Constructio & Demonstratio.

Divisa AB bifariam in F, demittatur ex F diameter FE, aequalis quartae parti lineae BC: dico E, terminum lineae FE, designare punctum intersectionis, sit enim E punctum intersectionis inventum & per E diameter agatur EF. erit EF, vt in praecedenti propositione ostendimus, aequalis quartae parti lineae BC; igitur per compositionem cum FE diameter detur aequalis quartae parti lineae BC, patet E punctum esse intersectionis, &c. Quod erat exhibendum;

PROPOSITIO CLXX.

Iisdem positis ducatur quaevis diameter HI secans parabolas in K, L, M.
Dico HK, HL, HM lineas in continua esse analogia.

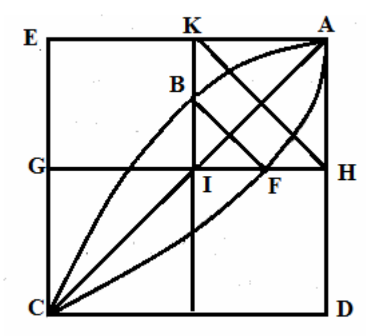
Demonstratio.

Recta HI secet BD lineam in N, & AC in I. Ut BHA rectangulum ad rectangulum BFA sic HL linea est ad lineam FE, id est EG; & ut BND rectangulum ad rectangulum BGD, sic KN ad EG: est autem BHA rectangulum ad rectangulum BFA, ut BND rectangulum ad rectangulum BGD; igitur ut HL ad EG, sic KN ad EG: adeoque HL, KN aequales sunt lineae; & dempta communi KL, manet HK aequalis LN; similiterr ostenditur LM aequalis lineae HL: quare ut HM ad MI, sic HM est ad HL: sed ut HM ad MI, sic AI est ad IC, id est AH ad HB; Igitur ut HM ad HL, sic AH ad HB, id est HL ad LN, id est HL ad HK proportionales igitur sunt HK, HL, HM. Quod fuit demonstrandum.

PROPOSITIO CLXXl.

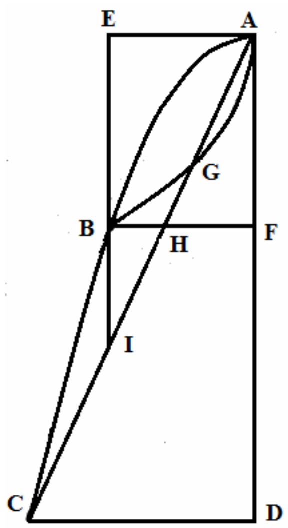
Parabolam ABC cuius diameter AD contingat in A linea AE, demissaque ex E, diametro EC, describatur per A & C parabola AFC cuius diameter sit AE & contingens AD: dein linea ducatur GH parallela AE, secans AFC parabolam in F, & AG lineam in I puncto; per quod diameter ponatur IBK, iunganturque BF, HK.
Dico BF, HK lineas aequidistare.

Demonstratio.



Ponatur ex C ordinatim ad diametrum AD linea CD: ratio KB ad EC , duplicata est rationis KI ad EC: similiterr ratio HF ad CD, duplicata est rationis HI ad CD, id est AK ad AE, id est KI ad EC; igitur ut KB ad EC, sic HF ad CD, & ut KB ad Kl, sic HF ad Hl, aequidistant igitur FB, HK lineae. Quod erat demonstrandum.

PROPOSITIO CLXXIl.



Parabolam ABC cuius diameter AD, contingat in A linea AE, demissaque diametro EB, quae ABC parabolae occurrat in B, ponatur ex B ordinatim linea BF: descriptaque per A & B parabola AGB cuius diameter AE & contingens AD, ducatur ex A linea AC occurrens AGB parabolae, & lineis BF, EB in G, H, & I.

Dico AG, AH, AI, AC lineas esse continue proportionales.

Demonstratio.

Quoniam EB aequidistat contingenti AD, & FB, Diametro AE, rectae AG, AH, AI continuae sunt proportionales, sunt autem & AH, AI, AC eandem continuant analogiam. Quod erat demonstrandum.

PROPOSITIO CLXXIII.

Iisdem positis ponatur ordinatim CD.
Dico AG ad AC triplicatam habere rationem eius quam habet BF ad CD.

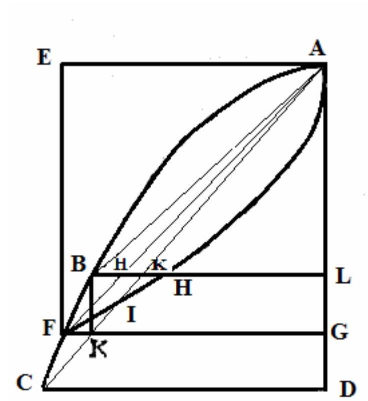
Demonstratio.

Quoniam AG, AH, AI, AC lineae per praecedentem continuae proportionales sunt, ratio AG ad AC triplicata est eius cuius AH ad AC est duplicata : sed AH ad AC, id est AF ad AD, rationem habet duplicatam BF ad CD ; igitur AG ad AC rationem habet triplicatam eius quam habet BF linea ad lineam CD. Quod erat demonstrandum.

PROPOSITIO CLXXlV.

Parabolam ABC cuius diameter AD contingat in A linea AE; positisque ordinatim CO, FG, per F & A parabola describatur AHF cuius tangens AD, diameter AE: ductaque AF, ponatur AC, secans AHF parabolam in I, rectam FG in K puncto, ex quo erecta diametro KB, ducatur ordinatim linea BL, occurrens AF lineae in M, rectae AC in N, & parabolae AHF in H.

Dico CD, FG, BL, ML, NL, HL lineas in continua esse analogia.



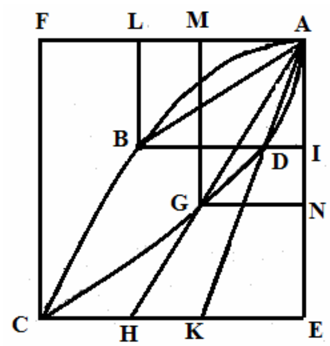
Demonstratio.

Erigatur ex F diameter FE. Quoniam BK aequidistat diametro AD, rectae CD, FG, BL proportionales sunt : sed & FG, BL, ML quoque sunt proportionales ; eandem igitur continuant rationem CD, FG, BL, ML. Rursum cum ratio NL ad KG, id est ad BL; id est: ratio AL ad AG, duplicata sit rationis BL ad FG, id est LM ad LB, rectae quoque BL. ML NL proportionales sunt; postremo quia FG linea ad lineam HL duplicatam habet rationem AL ad AG, id est quadruplicatam LB ad GF, id est LB ad LM; lineae LH, LN, LM; LB, LF proportionales sunt; igitur comtinuant eandem rationem lineae CD, FG, BL, ML, NL, HL. Quod erat demonstrandum.

PROPOSITIO CLXXV.

Parabolas aequales ABC; ADC, & communem habentes verticem contingant in A lineae AE, AF aequales lateribus rectis; secent autem sese parabolae in C, & ex C ordinatim ducantur CE, CF, demissisque ex A lineis aequalibus AB, AG; quarum altera AB, quidem secet ABC parabolam in B, AG vero parabolam ADC in C & producta, CE lineam in H ducatur ex B ordinatim linea BI secans ADC parabolam in D, & per D ex A linea ducatur AK occurrens EC in K.

Dico Ad AK, duplicatam habere rationem eius quamhabet AG ad AH.



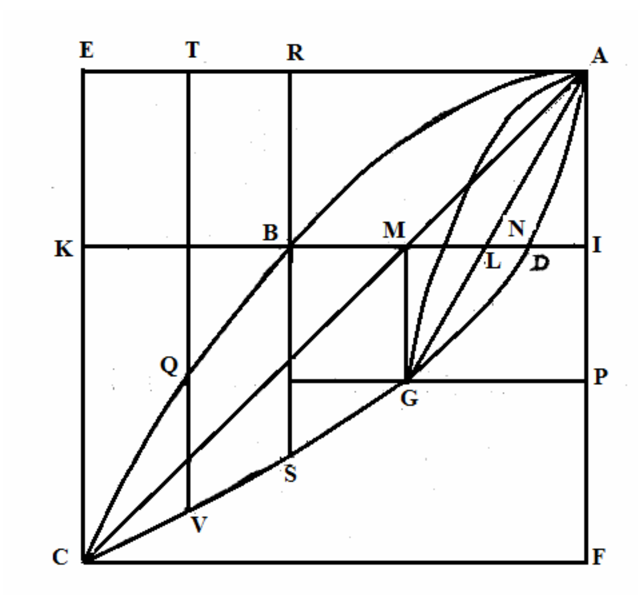
Demonstratio.

Ducantur ex B & G punctis, diametri BL, GM, GN : ac BL, & GM quidem occurrant AF linea in L & M; GN vero,rectae AE in N. Quoniam tam ABC, ADC parabolae, quam AB, AG lineae aequales sunt,erit LB linea quoque aequalis GN,& IB aequalis MG. Rursum ratio LB ad FC, id est: AI ad AE; id est Ad AK duplicata est rationis IB ad EC, id est: MG ad E C ; sed ut MG ad EC, sic AN ad AE (nam AL id est IB aequalis est rectae AN, & EC ipsi AE, est: aequalis) hoc est AG ad AH; igitur ratio Ad AK, duplicata est eius quam habet AG ad AH. Quod erat demonstrandum.

PROPOSITIO CLXXVI.

Parabolas aequales ABC, ADC communem habentes verticem contingant in A lineae AE, AF, secentque sese invicem parabolae in C; ex quo ductis ordinatim lineis CE; CF, describatur per A parabola AHG habens verticem in A, & contingentem AE, occurrens autem ADC parabolae in G, iunctisque punctis AC, AG, erigatur ex G diametcr GM secans AC lineam in M, puncto per quod ordinatim ducatur linea IK secans parabolas in B,H,D,lineam AG in L.

Dico rationem IH ad IB triplicatam esse eius, cuius IL ad IM, habet duplicatam.



Demonstratio.

Inveniatur inter ID & IL media IN; quoniam igitur ID, IL, IM, proportionales sunt, & IN media inter ID, IL,erunt tam ID,IN, IL, IH lineae, quam ID, IL, IM, IB continuae proportionales: quare ratio IH ad IB quartae ad quartam triplicata est rationis IB ad IL, secunda ad secundam, cuius lL ad IM, tertia ad tertiam, rationem habet duplicatam. Quod erat demonstrandum.

PROPOSITIO CLXXVII.

Iisdem positis: agatur per G ordinatim QP.
Dico ID ad IH, rationem habere triplicatam eius quam habet IH ad GP, & rationem ID ad IB, triplicam esse rationis IB ad CF.

Demonstratio.

Cum enim IB media sit inter ID & IL, rectae ID, IN, IL, IH, GP proportionales sunt, quare ID ad IH, prima ad quartam, triplicam habet rationem IH ad GP, quartae ad quintam. Quod erat primum. Rursum cum IL media sit inter ID,IM; rectae ID, IL, IM, IB, IK id est CF continuae proportionales sunt, quare ID ad IB, prima ad quartam, triplicatam habet rationem eius quam habet IB ad CF, quarta ad quintam. Quod erat demonstrandurn.

PROPOSITIO CLXXVIII.

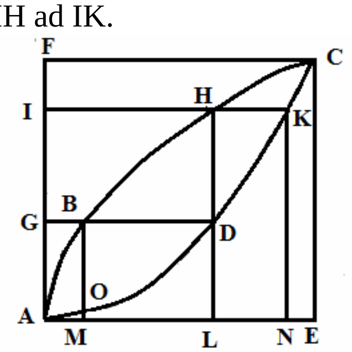
Iisdem positis agantur per Q & B diametri TV, RS occurrentes ADC parabolae in V & S, & AE lineae in T & R.
Dico rationem ID ad GP, octuplicatam esse rationis RS ad TV.

Demonstratio.

Est enim ratio ID ad CP, duplicata rationis AI ad AP, id est RB ad TQ: id est quadruplicata eius quam habet RA ad TA. Sed ratio RA ad TA, duplicata est: rationis RS ad TV, cum ordinatim sint positae ad AE; igitur ratio ID ad GP, octuplicata est rationis RS ad TV. Quod erat ostendendum.

PROPOSITIO CLXXIX.

Parabolas ABC, ADC communem habentes verticem, contingant in A lineae AE, AF, secent autem sese parabolae in C puncto, ex quo ordinatim ponantur lineae CE, CF: assumptoque in AF puncto quovis G, ducatur ex G ordinatim linea GD, secans ABC, ADC parabolas in B & D, actaque per D diametro DH, quae ABC parabolae occurrat in H ducatur per H ordinatim linea IK, secans AF lineam in I.
Dico GB ad GD quadruplicatam habere rationem eius quam habet IH ad IK.



Demonstratio.

Ratio GB ad IH, id est: GD duplicata est rationis AG ad AI; sed AG ad AI, duplicatam habet rationem GD ad IK, id est IH ad IK ; ratio igitur GB ad GD, quadruplicata est rationis IH ad IK, Quod erat demonstrandum.

PROPOSITIO CLXXX.

Producta HD donec AE lineae occurrat in L, demittantur ex B & K diametri BM, KN occurrentes AE lineae in M, & N, & B in quidem ADC parabolae in O.
Dico ationem OM ad DL, quadruplicatam esse rationis DL ad KN.

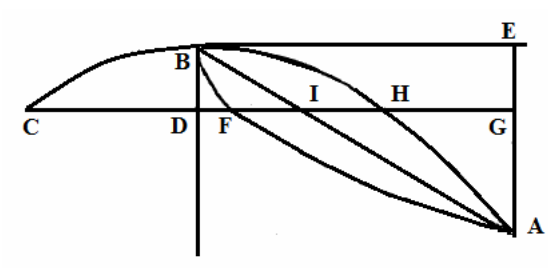
Demonstratio.

Est enim ratio OM ad DL duplicata rationis AM ad AL , id est GB ad IH: id est quadruplicata rationis AG ad AI, id est: LD ad NK. Quod erat demonstrandum.

PROPOSITIO CLXXXI.

Parabolae ABC cuius axis BD contingat in B linea BE,in qua assumpto quovis puncto E, demittatur ex E diameter EA: descripta deinde per A & B parabola BFA cuius axis BE, & contingens BD, ducatur ad BD quaevis ordinatim linea HC, occurrens parabolis in C, F, H; & BD, BA, EA lineis in D, I, G.
Dico HIC rectangulum, aequari rectangulo GDIF.

Demonstratio.



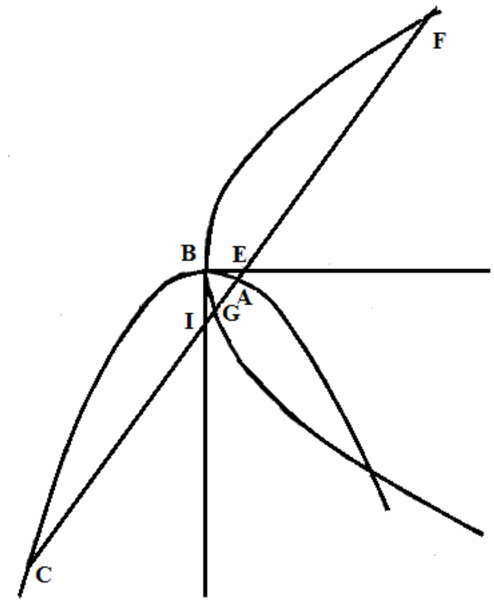
Rectangulum GDI, aequale est quadrato HD : sed HD quadratum aequale est quadrato ID, id est rectangulo FDB, una cum rectangulo HIC, rectangulum igitur GDI, aequale est rectangulis FDG, HIC. Est autem idem GDI rectangulum, aequale quoque rectangulis FDG, GDIF: rectangula igitur FDG, HlC, aequalia funt rectangulis FDG, GDIF. Dempto igitur communi rectangulo FDG, manet HIC rectangulum aequale rectangulo GDIF. Quod erat demonstrandum.

PROPOSITIO CLXXXII.

Parabolam ABC cuius diameter BD contingat in B, linea BE,descriptaque per B parabola FBG, cuius diameter BE,& contingens BD, ducatur linea quaecunque FC, occurrens parabolis in A, & *G*, F, C, diametris vero in E & D.

Dico AEC rectangulum, aequari rectangulo GDF.

Demonstratio.



Quoniam EB linea contingens est, rectae AE, DE, CE proportionales sunt, adcoque AEC rectangulum aequale quadrato ED*:* sed ED quadrato aequale est rectangulum GDF, rectangulum igitur AEC aequale est rectangulo GDF. Quod erat demonstrandum.

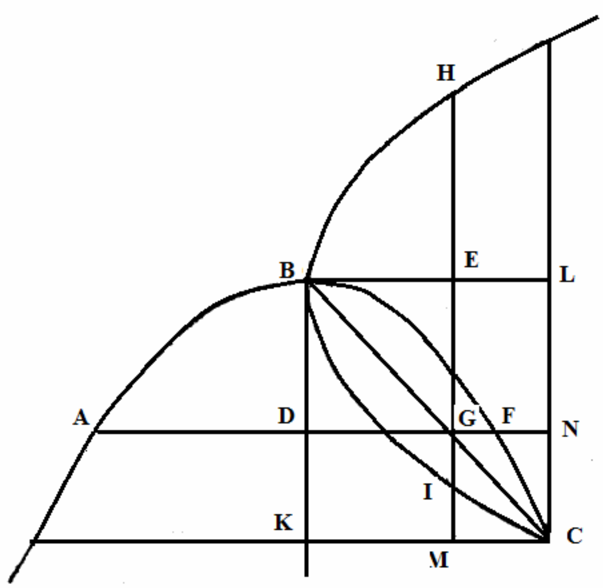
PROPOSITIO CLXXXIII.

Iisdem postis: occurrant sibi parabolae duae ABC, CBH in C; iunctaque BC, ponatur in ABC parabola ordinatim ad BD, linea ADF, occurrens recta; BC in G : & per G recta agatur HI parallela BD, occurrens diametro BL in E.

Dico IGH rectangulum esse ad rectangulum FGA, ut quadratum IE ad quadratum FD.

Demonstratio.

Ponantur ex C ordinatim ad diametros BD, BE , lineae CK, CL; quoniam BC, BG parallelogramma communem habent diametralem BGC, ut CL ad GE, sic KC est ad GD; sed ut CL ad GE, sic IE quadratum est ad quadratum GE, & ut CK linea ad lineam GD, sic FD quadratum est ad quadratum GD; igitur ut IE quadratum ad quadratum FD, sic quadratum GE est ad quadratum GD, & permutano ut IE quadratum ad quadratum FD, sic quadratum GE ad quadratum GD, est autem quadratum IE aequale quadrato GE, una cum rectangulo IGH; & FD quadratum aequale quadratum GD, una cum



rectangulo FGA; igitur & IGH rectangulum est ad rectatangulum FGA ut ID quadratum FD. Quod erat demonstrandum.

PROPOSITIO CLXXXIV.

Iisdem positis, producta linea GH, occurrat CK lineae in M, & AF producta lineae LC in N. Dico IMH rectangulum esse ad rectangulum FNA, ut IGH rectangulum est ad rectangulum FGA.

Demonstratio.

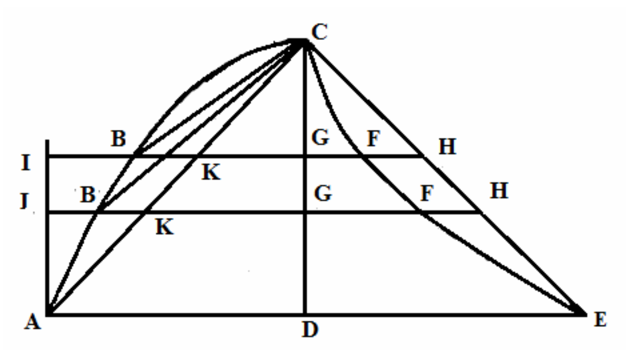
Ostensum est praecedenti propositione, quadratum KC ad CL, eandem habere rationem quam DG quadratum ad quadratum GE, ac proinde, quam quadratum DF ad EI, quadratum: sed quadratum KC hoc est DN aequatur DF quadrato, una cum rectangulo FNA; similiter quadratum CL hoc est EM ad DN, sicut quadratum EI ad DF, rectangulum quoque IMH est ad rectangulum FNA, ut quadratum EI ad DF quadratum, hoc est per praecedentem ut rectangulum IGH ad FGA rectangulum. Quod oportuit demonstrare.

PROPOSITIO CLXXXV.

Esto ABC parabolae axis CD, aequalis lateri recto, actaque per D ordinatim linea DA, producatur in E, ut AD, DE lineae aequales sint iunganturque AC, EC : dein per C & E parabola describatur CFE, habens verticem in C & contingentem CD; ducanturque BG parallelae AD, occurrentes parabolis in B & F, & iungantur BC.

Dico esse ut quadratum BC, ad quadratum BC, sic KF lineam ad lineam KF.

Demonstratio.



Erigatur ex A diameter AI occurrens BG lineis in L quadratum CB aequale est quadratis BG, CG: est autem quadratum BG aequale rectangulo KGI, & quadrato CG sive GH, (cum CD, DE adeoque & CG, GH lineae aequantur) aequale est rectangulum FGI; quadratum igitur BC aequale est rectangulis FGI, KGI; hoc est rectangulo IGDF: sed IGKF rectangulum est ad rectangulum IGKF, ut FK linea ad lineam KF; quadratum igitur CB est ad quadratum CB, ut KF linea est ad lineam KF. Quod erat demonstrandum.

PROPOSITIO CLXXXVI.

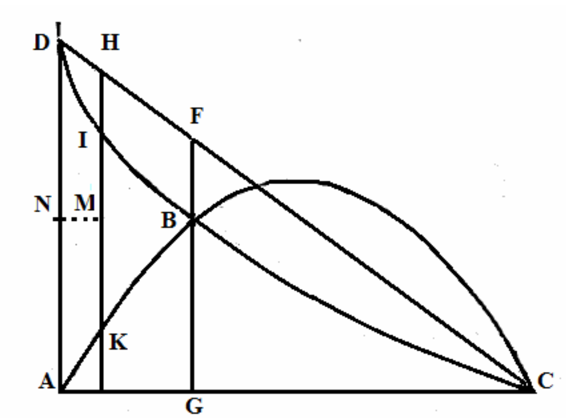
Iisdem positis:
Dico FG ad GB, triplicatam habere rationem eius quam habet GB ad AD.

Demonstratio.

Quoniam tam AD, BG, KG, quam DE, GH, GF proportionales sunt, & AD prima, aequales primar DE, ratio KG ad GF, duplicata est rationis BG, GH id est BG ad KG, cum enim AD, DC lineae aequales sint, aequantur etiam KG, GC; est autem ratio FG ad GB, composita ex ratione FG ad GK, id est ex duplicata ratione KG ad GB, hoc est BG ad AD, (cum AD, BG, KG, proportionales sint) & ex ratione KG ad GB id est, igitur FG ad AD, igitur FG ad GB, triplictam habet eius quam habet GB linea ad lineam AD. Quod fuit demonstrandum.

PROPOSITIO CLXXXVII.

Intersecent iterum parabolae duae ABC, CBD in punctis B, C, habentes communes diametros FG, HKL, DA quas in M & N, secet recta CB.
Dico IM ad MK eandem habere rationem quam habet: DN ad NA.



Demonstratio.

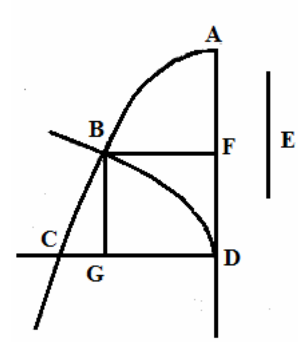
Ut DHC rectangulum est ad rectangulum DlC, sic ALC ad AGC , rectangulum, sed ut DHC ad DFC, sic HI ad FB, & ut ALC ad AGC, sic LK linea est ad GB lineam; igitur ut HI ad FB, sic KL ad BG: & permutando convertendo ut FB ad BG, sic HI ad KL. Est autem ut FB ad BG, sic HM ad ML; igitur ut Hl ad KL , sic HM ad ML: unde & IM est ad MK, ut HM ad ML, id est DN ad NA. Quod fuit demonstrandum.

PROPOSITIO CLXXXVIII.

Esto ABC parabolae diameter AD quam in D secet ordinatim linea DC: dein per D parabola describatur DB, habens AD contingentem & diametrum DC & commune cum altera sectione latus rectum E. Occurrat autem DB parabola, parabolae ABC in B,& ordinatim ducatur linea BF.
Dico AF lineam ad lineam FB, rationem habere duplicatam eius quam habet FB ad FD.

Demonstratio.

Ponatur BG aequidistans AD. Quoniam igitur E latus rectum utrique sectioni commune est, & FB, BG ordinatim positae, erunt tam E, FB, AF lineae, quam E, BG, id est FD, & DC proportionales. Unde cum E prima sit communis, ratio AF ad DG, id est ad FB, duplicata est rationis FB ad BG, id est ad ED. Quod fuit demonstrandum.



Corollarium.

Hinc sequitur AF ad FD, rationem triplicatam eius esse rationis, quae est inter AF & FD. Est enim ratio AF ad FD, composita ex ratione AF ad FB. Hoc est duplicata BF ad FD, & ex ratione BF ad FD: hoc est triplicata AF ad FD.

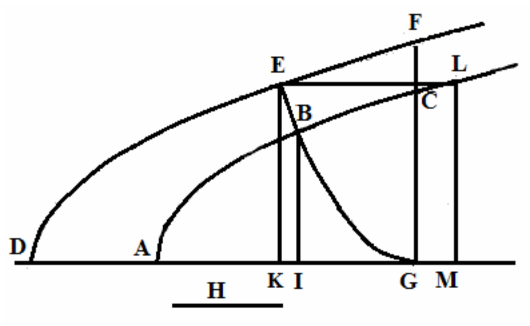
PROPOSITIO CLXXXIX.

Habeant ABC, DEF parabolae ad eundem axem constitutae, commune latus rectum H, assumptoque in axe puncto quovis G, ex eo ordinatim ducatur linea GCF, describaturque per G parabola, habens axem GC, occurrens parabolis ABC, DEF in B & E punctis ; ex quibus ordinadim demittantur lineae BI, EK.

Dico rationem IA ad KD, quadruplicatam esse rationis GI ad GK.

Demonstratio.

Ducta enim ex E linea EL parallela GD, occurrente ABC parabolae in L, demittatur ex L



ad diametrum GD, ordinatim linea LM: erit igitur quadratum LM aequale quadrato EK, unde &: rectangulum sub H & MA, aequale est rectangulo sub H & KD, adeoquc & MA linea aequalis KD, est igitur ratio IA ad KD, sive ad MA, duplicata rationis IB ad LM, sive ad EK; est autem ratio IB ad EK, duplicata rationis GI ad GK; ratio igitur IA ad KD, quadruplicata est rationis eius, quam habet GI ad GK. Quod fuit demonstrandum.

PROPOSITIO CXC.

Intersecent sese invicem in I parabolae duae ABC, DEF parallelos habentes axes BG, EH, actaque per I diametro, ducantur ordinatim rectae MQ, AF.

Dico NOM rectangulum esse ad rectangulum POQ, ut CKA rectangulum ad rectangulum DKF.

Demonstratio.

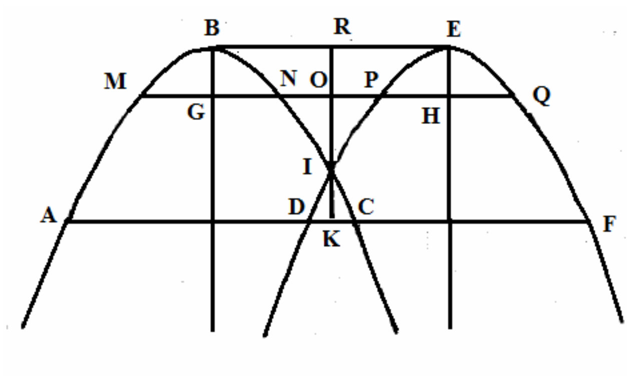
Ut IO est ad IK, sic NOM rectangulum est ad rectangulum CKA; sed ut IO ad IK, sic QOP rectangulum est ad rectangulum DKF; igitur ut NOM rectangulum ad rectangulum CKA, sic QOP rectangulum est ad rectangulum DKF, & permutando, NOM est ad rectangulum QOP ut rectangulum CKA ad rectangulum DKF. Quod fuit demonstrandum.

PROPOSITIO CXCI.

Iisdem politis: si ABC, DEF parabolae eandem habuerint altitudinem & communem contingentem BE.

Dico NOM rectangulum esse ad rectangulum POQ, ut BR quadratum ad quadratum RE.

Demonstratio.



Ut linea RI ad lineam OI, sic BR quadratum ad rectangulum NOM; & ut RI ad OI, sic RE quadratum ad rectangulum QOP; igitur ut BR quadratum ad NOM rectangulum sic quadratum RE ad rectangulum POQ, & permutando ut quadratum BR ad quadratum RE, sic NOM rectangulum ad rectangulum QOP. Quod fuit demonstrandum.

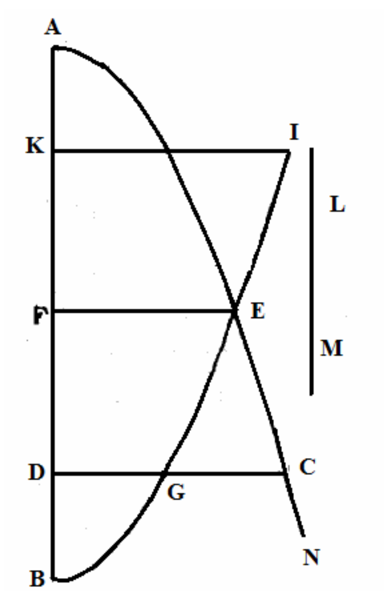
Corollarium.

Hinc patet etiam rectangulum AKC esse ad rectangulum DKF; ut BR quadratum ad quadratum RE.

PROPOSITIO CXCII.

Esto ABC parabolae axis AD ductaque ordinatim CD, sumatur in axe quaevis recta DE & per E describatur parabola cuius axis sit EA, secans parabolam ABC in B & CD lineam in G. factaque AK aequali DE, ducantur KI ordinatim ad AE in parabola EGB, & BF ordinatim ad ABC.

Dico CD quadratum esse ad quadratum IK. ut EF linea ad FA.



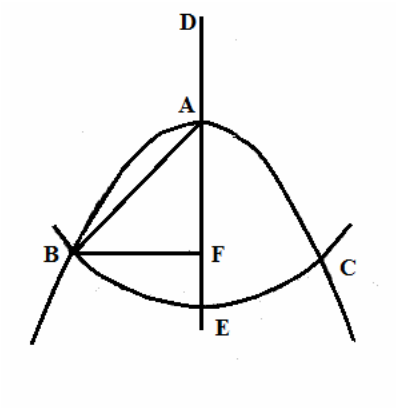
Demonstratio.

Sit ABC parabolae latus rectum L, & parabolae EGB latus rectum M, quadratum CD aequale est rectangulo DAL; & quadratum IK aequale rectangulo KEM, est autem AD linea aequalis KE, quia AK, ED aequales ponuntur; quadratum igitur CD est ad IK quadratum, ut I, ad M. Rursum quia quadrato BF tam est aequale

rectangulum FAL quam FEM; rectangula quoque FAL, FEM aequalia sunt, igitur ut AF ad FE, sic M ad L; sed ut
M ad L, sic IK quadratam est ad CD quadratum, igitur ut AF ad FE, sic IK quadratum est ad quadratam CD.

PROPOSITIO CXCIII.

Sit ABC parabolae axis AD aequalis lateri recto centroque A intervallo quovis, circulus describatur occurrens parabolae in B,C; axi in E, ponaturque ad axem ordinatim BF.



Dico DF, AE, AF lineas continuae esse proportionales.

Demonstratio.

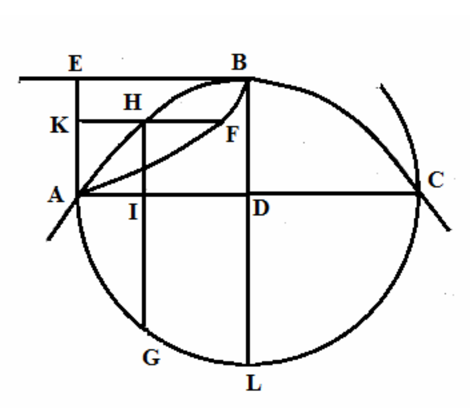
Recta AE aequalis est est.AB : sed AF, AB, DF, proportionales sunt, igitur & AF, AE, DF eandem continuant rationem. Quod fuit demonstrandum.

PROPOSITIO CXCIV.

Sit ad ABC parabolae axem BD, ordinatim applicata AC; actaque per B contingente BE, quae erectae ex A diametro, occurrat in E, describatur per AB parabola AFB, cuius axis AE ;ccntroque D intervallo AD, circulus describatur AGC, quem in G, secet recta quaedam HG, aequidistans axi BD, occurrensque ABC parabolae in H, & AC lineae in I: denique per H ponatur KF parallela EB, occurrens AFB parabolae in F & AE in K. ·

Dico GI, FK lineas esse inter se aequales.

Demonstratio.

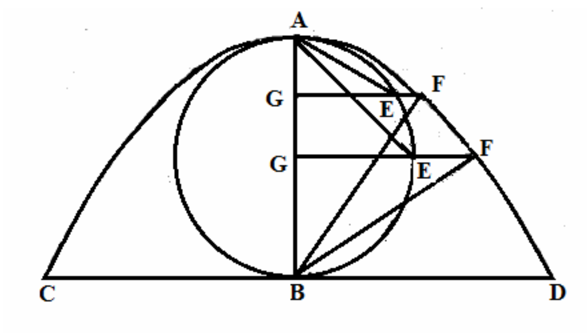


Producta BD, occurrat circulo in L : ut AK ad AE, id est HI ad BD; sic FK quadratum est ad quadratum BE: sed etiam ut HI ad BD ; sic AIC rectangulum est ad rectangulum ADC, id est IG quadratum ad quadratum DL; igitur ut quadratum FK ad quadratum BE, sic IG quadratum ad quadratum DL, id est AD, id est EB, quare IG, FK, quadratum adeoque & lineae inter se aequantur.

PROPOSITIO CXCV.

Super ABC parabolae axe AB aequali lateri recto, circulus describatur AEB, quem in E secent utcunque ordinatim positae GF, ad axem parabolae, iunganturque AE.
Dico AE, GF lineas aequari.

Demonstratio.



Quoniam AB, lateri recto aequalis est quadratum FG aequatur rectangulo GAB; sed & GAB rectangulo aequale quoque est quadratum AE, quia AGE angulus rectus est, quadrata igitur AE, FG adeoque & lineae inter se aequantur. Quod erat demonstrandum.

PROPOSITIO CXCVI.

Iisdem positis, iungantur FB.
Dico FB quadratum aequari quadratis AG, GE, GB simul sumptis.

Demonstratio.

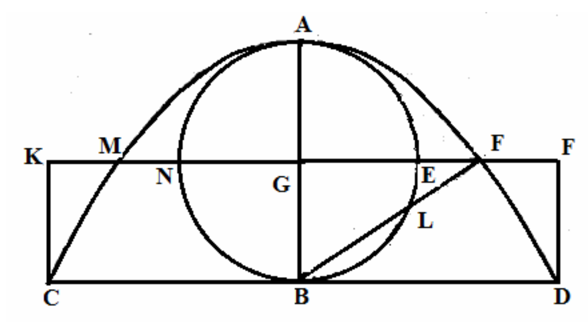
Quadratum FB, aequalc est quadratis BG, GF; sed FG quadratum aequale est quadrato AE, id est quadratis AG, GE, quadratum igitur FB aequale est quadratis AG, GE, G B. Quod erat demonstrandum.

PROPOSITIO CXCVII.

Iisdem posîtis: occurrat FG linea diametris DH, CK in H & K.
. Dico HFK rectangulum aequari rectangulo ABG.

Demonstratio.

Quadratum HG aequale est quadrato FG, una cum rectangugulo HFK; est autem quadrato HG sive BD, aequale quadratum AB, (cum AB aequalis ponatur lateri recto) igitur & quadratum AB, aequale est quadrato FG una cum rectangulo HFK; sed AB quadratum quoque est aequlae rectangulis GAB,



GBA, id est quadrato GF una cum rectangulo GBA, quadratum igitur FG una cum rectangulo GBA , aequale est quadrato FG cum rectangulo HFK: dempto igitur communi quadrato FG, residua rectangula ABG, HFK sunt inter se aequalia. Quod erat demonstrandum.

Corollarium.

Ex dictis patet FEM rectangulum aequari quadrato AG, nam FE in rectanguum una cum quadrato EG, aequatur quadrato FG, id est AE, id est quadratis AG, GE: dempto igitur communi quadrato GE, manet FEM rectangulum aequale quadrato AG.

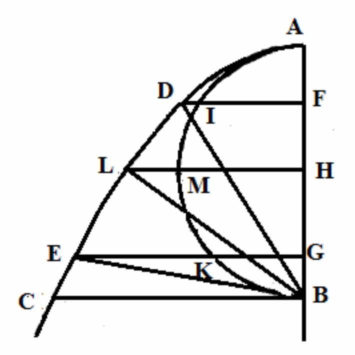
PROPOSITIO CXCVIII.

Iisdem postis: occurrat FB, circula in L.
Dico BFL rectangulo, aequari quadratum AG.

Demonstratio.

BFL rectangulum aequale est rectangulo NFE, id est MEF. sed MEF rectangulo aequale est quadratum AG ; igitur & rectangulo BFI aequale est quadratum AG; igitur & rectangulo BFL aequale est quadratum AG. Quod erat demonstrandum.

PROPOSITIO CXCIX.



Sit AB axis parabolae ABC lateri recto, sumptisque AF,BG aequalibus, ponantur ordinatim ad axem rectae FD,GE.
Dico iunctas BD,BE esse inter se aequales.

Demonstratio.

Super AB describatur circulus occurrens FD, GE lineis in I & K, quoniam AF , BG lineae aequales sunt, & FI, GK normales ad axem AB, rectae FI, GK, item AG, BF inter se aequales sunt : est autem quadratum BD aequale quadratis AF, FI,FB; & BE quadratum aequale quadratis BG,GK,GA; quadratum igitur BD aequale est quadrato BE, & BD linea aequalis BE. Quod erat demonstrandum.

PROPOSITIO CC.

Iisdem postis, ducatur ex H centro circuli AIB , recta HML, parallela FD; iunganturque B L.

Dico BL lineam brevissimam esse omnium quae ex B ad peripheriam parabolae duci possunt.

Demonstratio.

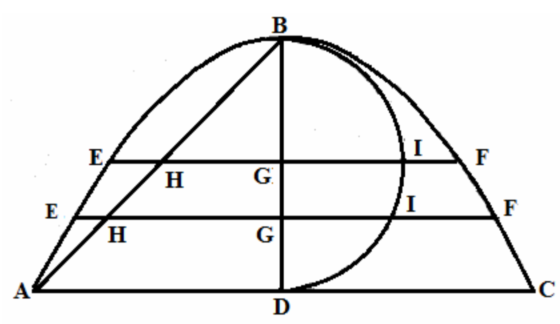
Ponatur quaevis alia BD, & ordinatim DF; erit igitur quadrarum BL, aequale quadratis AH, HM, HB: & BD quadratum aequale quadratis AF, FI, FB ; quia vero AH, HM, HB semidiametri sunt, quadrata illarum minora sunt quadratis AF, FI, FB (ut facile ex elementis ostenditur) igitur & quadratum BL minus est quadrato BD, & BD linea omnium brevissima, quae ex B ad peripheriam duci possunt. Quod erat demonstrandum.

PROPOSITIO CCI.

Esto super ABC parabolae axi BD aequali lateri recto descriptus semicirculus BID, quem in I secent quaecunque ordinatim ad axem positae FGE, actaque per D ordinatim AC, ducatur AB, occurrens EF lineis in H.

Dico rectangulo FHE aequari GI quadratum.

Demonstratio.



Quoniam axis BD aequalis ponitur lateri recto, & AD ordinatim applicata, rectae AD, BD adeoque & HG,GB inter se aequales sunt, quia vero EF linea in G diuisa est bifariam & non bifariam in H, rectangulum EHF una cum quadrato HG, id est BG, aequale est quadrato EG; sed EG quadratum est aequale rectangulo GBD, id est quadrato BG una cum rectangulo BGD; una cum rectangulo BGD; igitur quadratum BG una cum rectangulo BGD; igitur quadratum BG una cum rectangulo BGD aequatur EHF rectangulo una quadrato HG, cum id est BG; dempro igitur communi quadrato BG, manet EHF rectangulum aequale rectangulo BGD, id est quadrato GI. Quod erat demonstrandum

PROPOSITIO CCII.

Iisdem positis:
Dico FI, GH, IE lineas proportionales esse.

Demonstratio.

Quoniam EF linea divisa est bifariam in G & non bifariam in I, rectangulum FIE una cum quadrato IG, id est divida una cum rectangulo BGD, aequale est quadrato GF, id est rectangulo GBD, id est quadrato BG una cum rectangulo BGD, id est cum quadrato GI; dempto igitur communi quadrato IG, manet FIE rectangulum aequale quadrato BG, id est quadrato HG; proportionales itaque sunt FI, GH, IE. Quod erat demonstrandum.

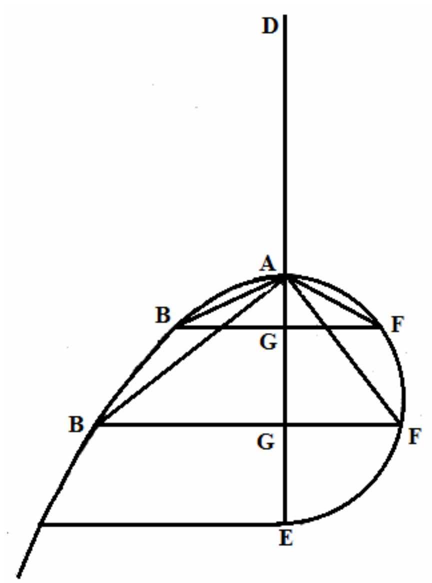
PROPOSITIO CCIII.

Esto super ABC parabolae axe AC aequali lateri recto descriptus semicirculus AFE, sumptaque AD aequali AE, ducatur ordinatim quaevis BF secans axem AE in G, occurrens circulo in F iunganturque AB, AF.

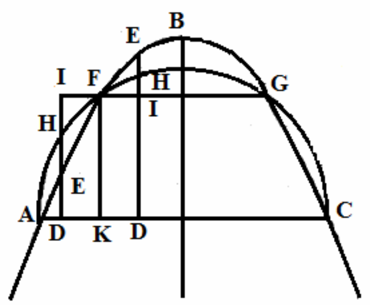
Dico DG ad DA, rationem habere duplicatam eius quam habet AB ad AF.

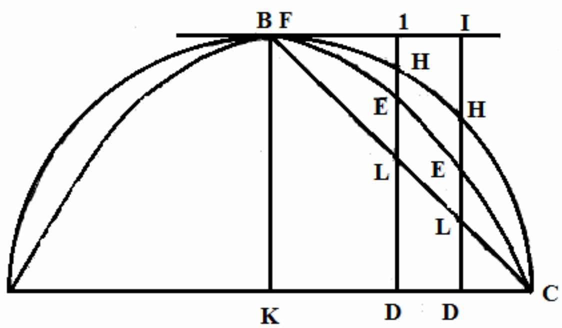
Demonstratio.

Quoniam AF quadratum aequale est quadratis FG, AG, id est rectangulo DAG recte
AG, AF, DA continuae sunt proportionales: sed & AG, AB, GD sunt in continua ratione; igitur cum AG prima utrique seriei communis sit, DG ad DA, tertia ad tertiam, in duplicata est ratione AB ad AF, secundae ad secundam. Quod erat demonstrandum.



PROPOSITIO CCIV.





Parabolam ABC subtendat recta AC normalis ad diametrum ED: super AC vero ut axe dcscribatur semiellipsis vel semicirculus AF, GC occurrens parabolae in F & G, vel eandem contingens in B, ducaturque FI parallela ipsi AC, & ID lineae ponantur aequidistantes ED, occurrentes parabolae in E, ellipsi vel semicirculo in H, FI lineae in I, & ipsi AC in D.

Dico DE, DH, DI lineas esse continuas proportionales.

Demonstratio.

Demittatur ex F diameter FK, secans AC lineam in K, ut AKC rectangulum ad rectangulum ADC, sic FK est ad ED; sed ut AKC rectangulum ad rectangulum ADC, sic FK quadratum quoque est ad quadratum HD, igitur ut FK linea est ad lineam ED, sic quadratum FK est ad quadratum HD ; recta igitur FK, id est ID. ad ED; duplicatam habet rationem eius, quam habet ID ad HD: quare DE, DH, DI lineae in continua sunt analogla. Quod erat demonstrandum.

PROPOSITIO CCV.

Manente secunda figura, ducatur BC, occurrens ID lineis in LL.
Dico IE ad ED, duplicatam habere rationem eius quam habet IL ad HD.

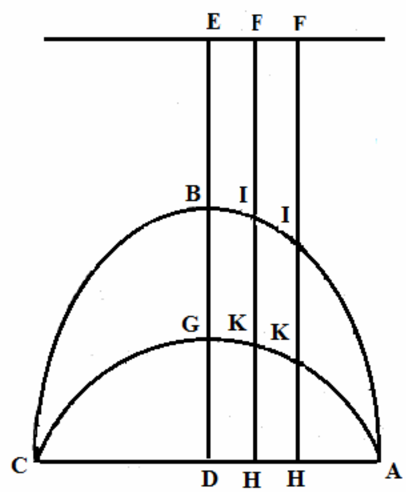
Demonstratio.

Quoniam tam ID, IL, IE lineae, quam ID, HD, ED in continua sunt analogia, & ID prima utrique seriei est communis, ratio IE ad ED, tertiae ad tertiam, duplicata est rationis IL ad HD, secundae ad secundam. Quod erat demonstrandum.

PROPOSITIO CCVI.

Sint ABC est ellipseos vel circuli diametri coniugatae AC, BD; ductaque quavis EF parallela AC, secante BD productam in E, fiant ED, BD, GD continuae proportionales; descriptaque per AGC puncta, parabola, cuius diameter GD, ponantur lineae quotcunque FH parallelae ED secantes ellipsim in I, parabolam in K, & AC lineam in H.

Dico FH, IH, KH lineas esse proportionales.



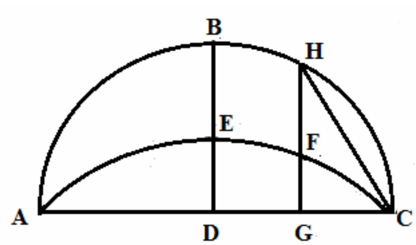
Demonstratio.

Ut AHC rectangulum ad rectangulum ADC , sic HK linea ad lineam DG, et IH quadratum ad quadratum BD ; igitur HK linea ad lineam DG duplicata habet rationem eius, quam habet IH linea ad lineam BD. Unde cum ED, BD. GD positione sunt proportionales, & ED, FH primae inter se aequales, sic autem & ratio DG ad KH, tertiae ad tcrtiam; duplicata rationis DB ad HI, secundae ad secundam , erunt FH, IH, KH in continua analogia. Quod erat demonstrandum.

PROPOSITIO CCVII.

In semicirculo ABC decussent sese orthogonaliter in D diametri duae AC, BD, quarum altera BD bifariam in E divisam, describatur per A, E, C parabola, cuius axis ED, ductque diametro FG, quae semicirculo occurrat in H; ponatur HC.

Dico FG ad GC ·duplicatam habère rationem eius, quam habet HG ad HC.



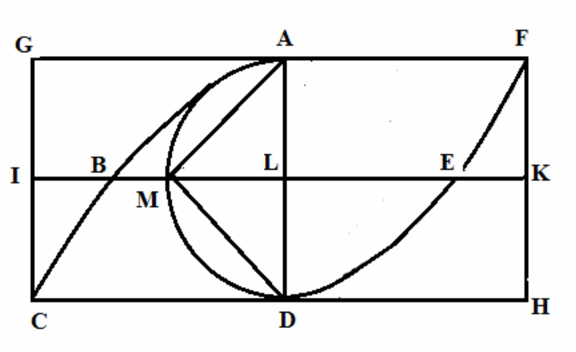
Demonstratio.

Quoniam AC dupla est BD, & illa dupla ED, rectae AC, BD, ED proportionales sunt ; est autem ED ad FG, in duplicata ratione DB ad GH, cum sit ut ADC rectangulum ad rectangulum AGC, id est BD quadratum ad quadratum HG; rectae igitur GF, HG, AC in continua sunt analogia, sed & AC, HC, GC lineae proportionales sunt ; igitur FG ad GC, tertia ad tertiam, duplicatam habet rationem eius, quam habet HG ad HC, secunda ad secundam. Quod erat demonstrandum.

PROPOSITIO CCVIII.

Parabolas aequales ABC, DEF inversem ad eundem axem AD, qui aequalis sit lateri recto constitutas, contingant in A & D, lineae GF, HC: & GF quidem parabolae DEF, occurrat in F, HC lineis occurrant in G, & H, ducatur quaevis IK, parallel a FG, secans parabolae in B & E, rectas CG, HF in I & K, axem AD in L, dein super AD ut diametro, describatur semicirculus AMD, occurrens IK lineae in M.
Dico ILM rectangulum aequari rectangulo ELB.

Demonstratio.



Quoniam AD aequalis est lateri recto, lineae AD, CD, item AM, LB, item MD, LE aequalis sunt, rectangulum igitur AMD aequale est rectangulo ELB: sed & AMD rectangulo ADML, id est ILM; rectangulae igitur ILM, BLE aequalia sunt.
Quod erat demonstrandum.

PROPOSITIO CCIX.

Iisdem positis:
Dico rectangulum ILBM aequari rectangulo BLEK.

Demonstratio.

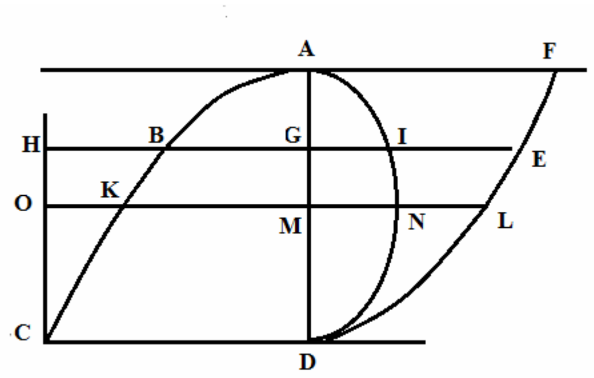
Quoniam ILM rectangulum aequale est rectangulo ELB, ut IL ad LE, id est LK ad LE, sic LB est ad LM: & permutando ut LK ad LB, sic EL est ad LM: & EK reliquum ad MB, ut LK ad LB. id est IL ad LB rectangulum igitur ILBM aequale est rectangulo BLEK.
Quod erat demonstrandum.

PROPOSITIO CCX.

Iisdem positis quae prius: fiat BGE rectangulo aequalo rectangulum HGI: & per A, I, D ·puncta ellipsis describatur , cuius axis sit AD; ducaturque recta quaevis LK parallela BE, occurrens parabolis in K & L, axi AD in M, ellipsi in N, diametro CH in O.

Dico KML rectangulum aequari rectangulo OMN.

Demonstratio.



Quoniam AD axis est ellipseos, & ad illam ordinatim ponuntur IG, MN, ut IG quadratum ad quadratum NM, sic AGD rectangulum est ad rectangulum AMD: rectangulum igitur AGD ad AMD rectangulum, rationem habet duplicatam, IG ad NM, id est rectanguli HGI ad rectangulum OMN: & quia ratio rectanguli AGD, ad AMD rectangulum, composita est ex ratione AG ad AM, id est ex duplicata ratione BG ad KM, & ex GD ad MD, id est ex duplicata ratione GE ad ML, ratio AGD rectangulum ad AMD rectangulum quoque duplicata est eius, quam habet BGE rectangulum ad rectangulum KML, igitur ut HGI rectangulum, ad rectangulum OMN, sic BGE rectangulum ad rectangulum KML: & permutando ut HGI rectangulum ad rectangulum BGE, sic OMN est ad rectangulum KML; sed HGI, BGE rectangula per hypothesim aequalia sunt; igitur & OMN, KML rectangula inter se aequantur. Quod erat demonstrandum.

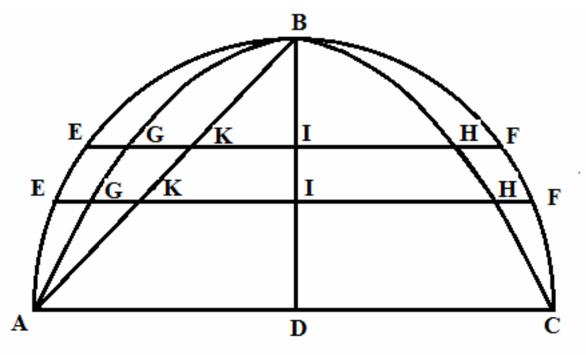
PROPOSITIO CCXI.

Secent ABC semicirculum orthogonaliter diametri duae: AC, BD: descriptaque per A,B,C, parabola, cuius axis BD, ducantur rectae quotcunque EF, parallelae AC, occurrentes circulo in E & F, parabolae in G & H, axi vero in l.

Dico EGF rectangulum esse ad rectangulum EGF, ut BID rectangulum ad rectangulum BID.

Demonstratio.

Ponatur AB, secat EF lineas in K ut BKA rectangulum ad rectangulum BKA, sic EKF rectangulum est ad rectangulum EKF, sed ut BKA rectangulum ad rectangulum BKA, sic GKH rectangulum est ad rectangulum GKH; igitur ut EKF rectangulum est ad rectangulum EGF, sic GKH rectangulum



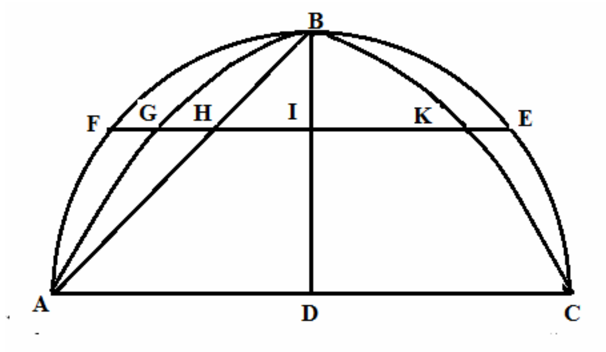
est ad rectangulum GKH, residiuum igitur EGF rectangulum, est ad rectangulum EGF, ut EKF rectangulum ad rectangulum EKF, id est rectangulum BKA ad rectangulum BKA, id est: rectangulum BID ad rectangulum BID. Quod erat demonstrandum.

PROPOSITIO CCXII.

Secent ABC semicirculum orthogonaliter diametri duae AC, BD: descriptaque per A & B parabola cuius axis BD, describatur & altera per B & C, habens axem DC & verticem C ducaturque linea EF parallela AC occurrens circulo in E & F, parabolis in G & H, axi BD in I, & AB iunctae in K.

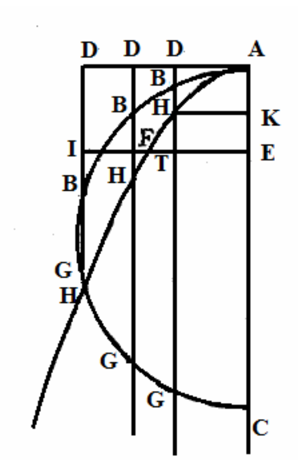
Dico EI quadratum esse ad quadrati GI, ut IH linea est ad lineam IK.

Demonstratio.



Quoniam tam AD, GI, KI lineae, quam DC, FI, HI proportionales sunt, & AD, CD utriusque seriei primae aequales, ratio HI ad IK, tertiae ad tertiam, duplicata est rationis FI ad IG, id est EI ad IG secundae ad secundam; igitur ut HI linea ad lineam IK; sic EI quadratum ad quadratum GI. Quod erat demonstrandum.

PROPOSITIO CCXIII.



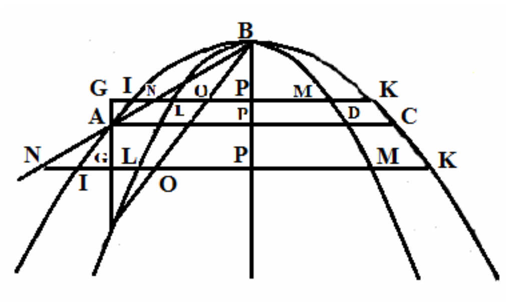
Semicirculum ABC, cuius diameter AC contingat in A linea AD; descriptaeque per A parabolam, cuius axis AC, & contingens AD, sumatur AE aequalis lateri recto; actaque per E ordinatum EF, ducatur in parabola diametcr quaecunque DG occurrens circula in B & G, parabolae in H, & FE ordinatim positae in I.

Dico BDG rectangulum , aequari rectangulo HDI.

Demonstratio.

Ducatur per H ordinatim linea HK: Quoniam AD linea in A circulii contingit, rectangulum BDG aequale est a quadrato AD, id est quodrato HK; sed & HK quadratum est aequale rectangulo KAE, id est HDI (quia AE per hypothesim aequalis est lateri recto) rectangulum igitur BDG aequale est rectangulo HDI. Quod erat demonstrandum.

PROPOSITIO CCXIV.



Parabolas duas ABC, DBE ad eundem axem BF positas, & contingentes sese f interius in B vertice secet in H & A diameter quaecunque GH, iunctisque AB,HB ducatur ordinatim linea IK , secans parabolas in, IK, L, & M, rectas AB, HB, AH in G, N, O, & axem BF in P.
Dico ILK rectangulum aequari rectangulo GPNO.

Demonstratio.

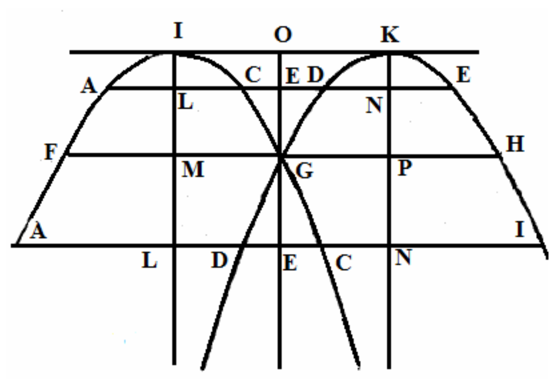
Rectangulum GPN aequale est quadrato IP, sed IP quadratum est aequale quadrato LP, id est rectangulo GPO, una cum rectangulo ILK rectangulum igitur GPN aequale est rectangulis GPO, ILK: est autem GPM rectangulum, aequale rectangulis GPO, GPNO, dempto igitur communi rectangulo GPO, manet ILK rectangulum aequale rectangulo GPNO. Quod erat demonstrandum.

PROPOSITIO CCXV.

Datam lineam AB divisam in C, E, I, D denuo ita partiri in E ut rectangulum AEC sit ad rectangulum BED sicut quadratum AEC quadratum DB.

Constructio.

Rectis AC, BD bisectis in LELN, erige perpendiculares LI, NK aequales inter se, & per puncta A, I, C, & D. K, B, describantur parabolae N,K,lib1tnfltllo:ovÇ4rf!
Cntes in G. Tum ex G duae G E normales ad A B. Dico factum.



Demonstratio.

Iunge puncta I, K, & EG producta in O, per G duc FMPH parallelam ad KB & IK. Quoniam tam ILOE, KN, quam IK, FH sunt parallele, erit quadratum IO ad quadratum OK ut quadratum MG ad PG quadratum, hoc est ut FG quadratum ; deinde AC quadratum est ad quadratum FG ut LI ad MI, hoc est ut quadratum DB ad quadratum GH. Ergo permutatim AC quadratum est ad quadratum DB, ut quadratum FG ad quadratum GH, hoc est (sicut ante ostendi) ut quadratum IO ad quadratum OK. Atqui rectangulum AEC est ad rectangulum BED ut quadratum IO ad

quadratum OK. Ergo etiam rectangulum AEC est ad rectangulum BED ut quadratum AC ad quadratum DB. Factum igitur est quod petebatur.
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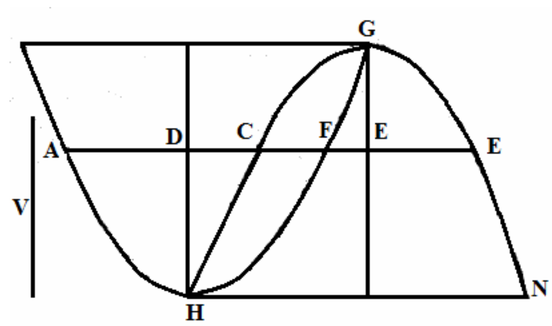
PROPOSITIO CCXVI.

Datam rectam AB divisam in C, iterum secare in D, ut rectangulum BDC aequale sit quadrato DA.

Constructio & Demonstratio.



Biseca CB in E, & fiat ut AE ad CE, ita CE ad FE; deinde AF etiam bisectaq in D. Dico factum quod petitur.

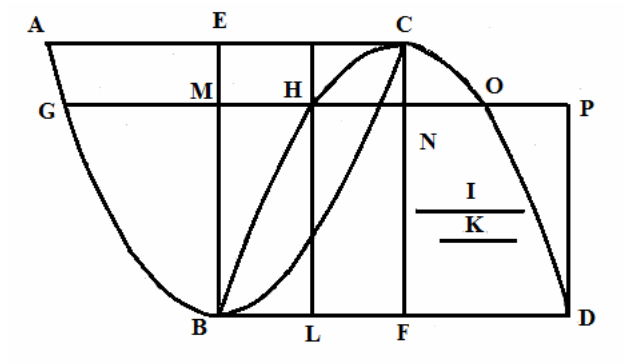
Ex punctis E ac D erige normales, quarum una EG sit magnitudinis placitae, altera DH infinita. Deinde per puncta C, G, B, describatur parabola axem habens EG & occurrens ipsi DH in H. Rursum per puncta AHF descripta intelligatur parabola axem habens DH; poterunt autem EG, DH axes esse parabolarum, cum ambae ex constructione rectas CB, FA ad angulos rectos bifariam secent. Quoniam igitur ex constructione AE, CE, FE sunt continuae colligitur ex 167, huius parabolam AHF transire per G verticem alterius parabolae. Ergo per eandem illam propositionem BD, FD, CD sunt continuae; sed FD,AD aequales sunt ex constructione, ergo BD, FD, CD sunt continuae; ergo rectangulum BDC aequatur quadrato DA. Quod erat faciendum.

PROPOSITIO CCXVII.

Sint duae parabolae aequales ABC, BCD ad axes parallelos constitutae EB, CF, per apices mutuos transeuntes; ex B sit ordinatim ducta BF.

Oporteat GH parallelam BF ponere, quae dividatur a recta BE, secundum datam rationem I ad K.

Constructio & Demonstratio.



Dividatur BD linea secundùm rationem quadrati I, ad K quadratum, in puncto L: erigatur deinde IH, quae aequidistet axi CF, occurratque sectioni in H & per H ponatur GH parallela BD, secans BE in puncto M. Dico rectam GH esse sectam in M, secundum rationem I ad K; erigatur ex D parallela ad FE occurrens ipsi GO in P. Quoniam rectangulum HMO aequatur quadrato MN, erunt MH, MN, MO continuae. Ergo quadratum MH ad quadratum MN, ut MH ad MO; hoc est ut MH ad HP, hoc est ut quadratum K ad quadratum I; sed quadratum MN aequatur quadrato GM; ergo quadratum MH est ad quadratum GM ut quadratum K ad I & invertendo ergo GM est ad rectam MH ut recta I ad K. posuimus igitur, &c. Quod erat faciendum.