PARABOLA

PART TWO

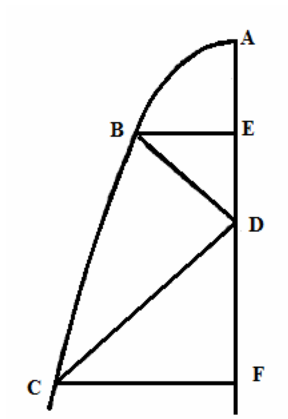
For the parabola, both continued as well as discrete proportions of lines, are to be considered.

PROPOSITION XXXII.

AD shall be the axis of the parabola ABC, in which for some point D assumed the lines DB, CD shall be drawn to the periphery making the equal angles ADB, CDF ; then the ordinate lines BE, CF may be put in place at right angles to the axis AF.

I say the right ordinate lines AE, AD, AF to be in continued proportion, and vice versa.

Demonstration.



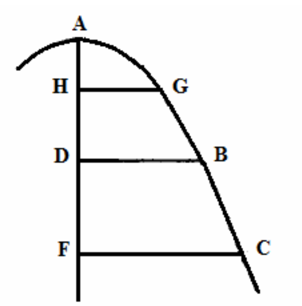
Since the ordinates EB, CF have been put in place to the axis, the angles BED, CFD are right : moreover the angles BDE, CDF put in place are equal; therefore the triangles BED, CFD are equal : and so that EB is to FC, thus as ED to FD: but EB to FC is the square of the ratio AE to AF, and therefore is the square of the ratio ED ad DF*:* thus AE, AD, AF are continued proportionals, as is readily deduced from the principles of geometric progressions.

Now AE, AD, AF shall be proportionals, and with the ordinates BE, CF put in place: BD, CD shall be joined: I say the angles BDE, CDF to be equal, since AE, AD, AF shall be proportionals, the ratio AE to AF is the square of the ratio AD to AF, that is : ED to DF, but also the ratio AE to AF, is the square of the ratio BE to CF, therefore as Ed to DF, thus BE ad CF, and as DE to BE, thus DF to FC: from which since the angles contained by sides in proportion shall be right, the triangles BED, CFD shall be similar, and the angles BDE, CDF equal. Q.e.d.

PROPOSITION XXXIII.

The diameter AD of the parabola ABC shall be divided at H, D, F so that AH, AD, AF shall be continued proportionals: moreover the ordinate lines HG, DB, FC shall be put in place.

I say these lines to be in continued proportion.

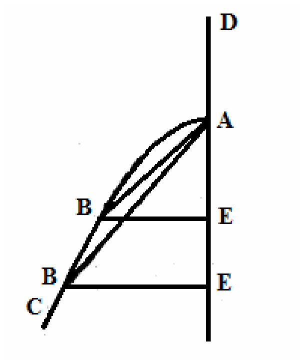


Demonstration.

It is evident, since AH, AD, AF shall be continued proportions, and GH, BD, CF shall have the ratio of their squares.

PROPOSITION XXXIV.

AD will be equal to the latus rectum of the parabola ABC, and with some ordinate line EB drawn, AB may be joined.
I say AE, AB, ED to be proportional lines.



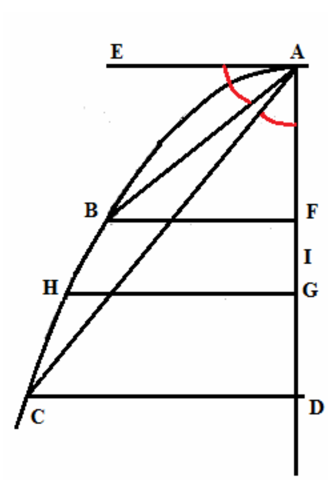
Demonstration.

The square AB is equal to the squares AE, EB: but the square EB is equal to the rectangle EAD; therefore the square AB is equal to the rectangle EAD together with the square AE, that is to the rectangle AED. Whereby AE, AB, ED are proportional lines . Q.e.d.

PROPOSITION XXXV.

The line AE shall be a tangent to the parabola ABC at A, the diameter of which shall be AD, and with the lines AB, AC sent from A, which cross the parabola at B and C so that the angles EAB, DAC shall be equal, and with the ordinates BF, CD drawn, the mean AG may be found between AF and AD.
I say, AG to be equal to the latus rectum ; and if the angles EAB, DAC were equal, and AG the line equal to the latus rectum, I say AF, AG, AD to be proportionals; and if AF, AG, AD were continued proportionals, and the mean AG were equal to the latus rectum : I say the angles EAB, DAC to be equal to each other.

Demonstration.



The ordinate line GH is drawn from G. Since, from the hypothesis, the angle DAC is equal to the angle EAB, either from the addition or subtraction of the common angle BAC, the angle BAD is equal to the angle EAC, moreover, they are both equal to the angle ACD ; also, the angles BFA, CDA are equal (on account of the parallel lines AE, CD), therefore, the triangles ABF, ACD are similar to each other: so that as BF to FA, thus AD to DC, from which the rectangle BF, CD is equal to the rectangle FAD ; but, from the hypothesis, the rectangle FAD is equal to the square AG, and the square HG is equal to the rectangle BF, CD, since BF, AG, CD shall be lines in proportion from the above ; therefore the square AG, is equal to the square HG, and the line AG is equal to the line HG; and thus equal to the latus rectum [§9 part 1]. Which is the first part.

Now the line AG shall be equal to the latus rectum, and the angles EAB, DAC to be equal, I say AF, AG, AD to be in continued proportion. Indeed if they shall not be proportionals, the mean AI shall be found between AF and AD: therefore from the first part of this, the line AI will be equal to the latus rectum, and thus the line AG, whereby AF, AG, AD are in continued proportion, nor may some other mean be found between AF and AD, besides AG. Which was the second part.

Again AG shall be equal to the latus rectum, and AF, AG, AD shall be proportionals, I say the EAB, DAC to be equal to each other, indeed since AF, AG, AD are continued proportionals, the rectangle FAD is equal to the square AG, that is to the square HG: but the square HG is equal to the rectangle BF, CD ; and therefore the rectangle FAD is equal to the rectangle BFCD: so that as AF to HF, thus CD to AD ; moreover the angles AFB, ADC are equal: with proportional sides in place, therefore the triangles AFB, ADC are similar to each other, and the angle BAF is equal to the angle ACD, that is to the angle EAC: therefore with the common angle BAC removed, the angle EAB remains equal to the angle CAD. Q.e.d.

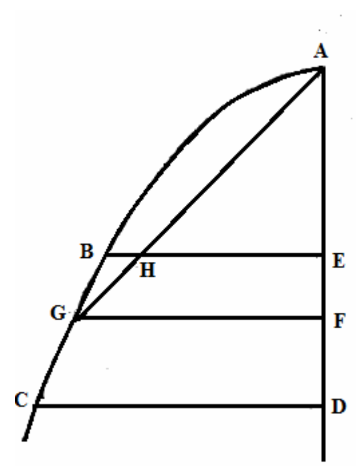
Corollary.

With the same in place, it follows the triangles BAF, CAD to be similar to each other; this itself to be apparent from the first part of the preceding proposition.

PROPOSITION XXXVI.

The diameter AD of the parabola ABC shall be divided at E and F so that AE, AF, AD shall be proportionals, and the mean AF equal to the latus rectum : moreover with the ordinate lines put in place EB, FG, DC, the right line AG may be drawn from A to G, crossing the line EB at H.

I say AD, DC, FG, EB, EH to be lines in continued proportion.



Demonstration.

Since AF is equal to the latus rectum, the lines GF, FA are equal: therefore the ratio AF to AD, that is GF to AD, is double the ratio of GF ad CD: therefore AD, CD, GF are proportionals; truly since AD, AF, AE are in continued proportion, also CD, GF, BF are proportionals; therefore they are continued in the same ratio AD, CD, GF, BE; then since the ratio AF to AE, that is GF ad HE, shall be the squared ratio of GF to BE, also GF, BE, HE are proportionals; therefore AD, CD, GF, BE, HE are continued in the same ratio. Q.e.d.

Corollary.

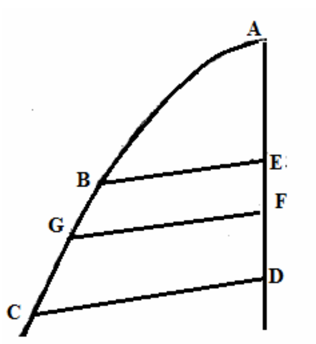
Hence it follows AE, FB, FG, CD also to be continued in the same ratio; for the ratio AF to AE, that is GF to BE, is double the ratio GF to BE, therefore AE, BE, GF are proportionals: but as BE to GF, thus GF is to CD, since AE, AF, AD shall be continued proportionals ; therefore AE, EB, GF, CD are proportionals.

PROPOSITION XXXVII.

Let the diameter AD of the parabola ABC be divided at E and F, so that the lines AE, AF, AD shall be proportionals, and with the end AD equal to the latus rectum ; the ordinate lines EB, FG, DC shall be drawn.

I say AE to EB, to be in the ratio that AF has to FG squared.

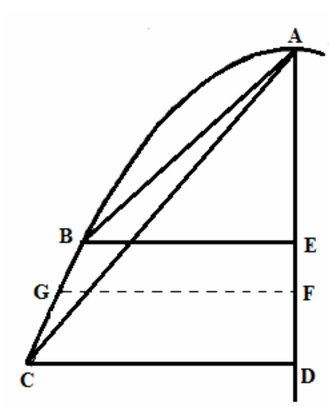
Demonstration.



Since the lines AE, AF, AD are placed in continued proportion, EB, FG, DC also will be in continued proportional: moreover AD, the first of the series AE, AF, AD is put equal [in length] to the latus rectum, and thus also to DC itself of the first series EB, FG, DC ; therefore the line AE has the ratio to the line EB squared, as AF has to FG, the second to the second. Q.e.d.

Corollary.

Hence it follows the rectangle EAF has the triplicate ratio to the rectangle EBFG of that which the line AF has to the line FG. Indeed the ratio of the rectangle EAF to the rectangle EBFG is composed from AE to EB, and AF to FG, but the ratio AE to EB, is in the square ratio AF to FG, therefore the rectangle EAF to the rectangle EBFG has the ratio of the cube of that ratio, which the line AF has to the line FG.



PROPOSITION XXXVIII.

AD shall be the diameter of the parabola ABC divided at E and F, so that AE, AF, AD shall be proportional lines, and the mean length AF shall be equal to the latus rectum, moreover the ordininate lines EB, DC may be put in place: and AB, AC shall be joined.

I say AB to AC to be in the ratio of the cube of that ratio, of which EB to DC is the square ratio.

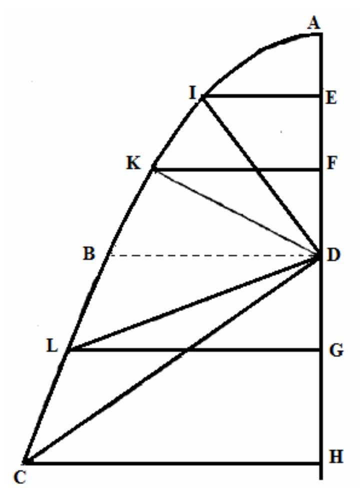
Demonstration.

The ordinate line FG shall be drawn. Because AE, AF,AD are proportional lines, and the mean length AF is equal to the latus rectum, the triangles AEB, ADC are similar, and thus as AB to AE, thus AC to CD: and by inverting and interchanging, so that as AE to CD, thus AB to AC; now truly since AF shall be made equal to the latus rectum, and FG made equal to the same: AE, EB, FG, DC will be in continued proportion [Cor.§35 above]; therefore the ratio AE to CD , of the first to the fourth is the triplicate [*i.e.* cube] of the ratio BE ad FG; and therefore of AB to AC of the second to the third, of which EB to DC [§33 above] has the square ratio. Q.e.d.

PROPOSITION XXXIX.

The axis of the parabola ABC shall be divided into continued proportionals; and indeed the midpoint AD shall be present between AE, AF, AG, AH; and with the ordinates drawn to the axis by the right lines EI, FK, DB, GI, HC, also there may be put in place DI, DK, DL, DC.

I say the ratio DI to DC to be the square of that ratio which KD has to DL.



Demonstration.

Since AE, AF, AD , AG, AH, are continued in the same ratio, if also EI, FK, DB, GL, HC, shall be proportionals, since truly the ratio AE to AH, is both the square of the ratio of EI to CH, as well as of the ratio AD to AH, that is, ED to DH, since AE, AD, AH, shall be proportionals, the ratio ED to DH, is the same as for the ratio EI to CH: therefore the triangles IED, CHD are similar: whereby as ID to DC, as ED to DH, that is AE to AD, that is in the ratio of the squares IE to BD. Similarly it may be shown the triangles FDK, GLD to be similar, and KD to be to LD, as FD to GD, that is as AF to AD, that is in the ratio of the squares KF to BD: but the ratio ID to BD, is the square of the ratio EK to BD, since LE, KE, BD shall be proportionals, therefore the ratio ID to CD, is the square of that ratio which DK has to LD. Q.e.d.

Corollary.

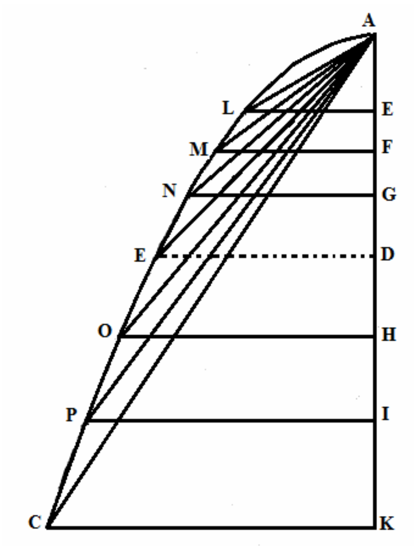
Now if AD were the midpoint between seven continued proportionals of which the end lengths were AE, AH, ID to DC will be shown similarly, then without doubt the two ends of these also to have KD to DL as a cubic ratio: and thus as far as the rest are concerned, always to be increasing in proportion.

PROPOSITION XL.

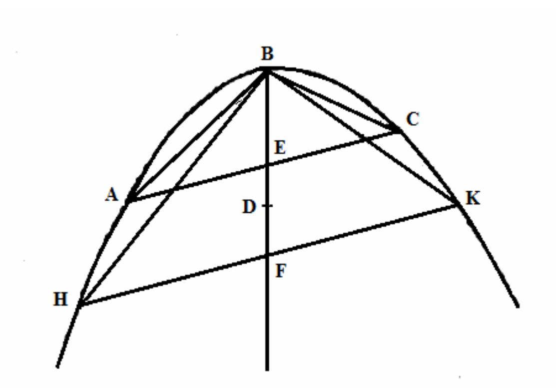
Let AK be the axis of the parabola ABC divided so that AE, AF, AG, AD, AH, AI, AK shall be continued proportionals, and with AD equal to the right line drawn to be the mean of the whole series ; the ordinates of the lines EL, FM, GN, DB, HO, IP, KC, shall be joined : AL, AM, AN, AB, AO, AP, AC.

I say the ratio AM to AP to be the square [or duplicate] of the ratio AN to AO, and AL to AC the cube [or triplicate] of this ratio which AN has to AO, and thus henceforth by proceeding indefinitely the increase of each ratio will be found.

Demonstration.



Since AE, AF, AG, AD , &c. are continued proportionals and AD is put to be the mean of the whole series; also AF, AD, AI will be continued proportionals, without doubt the second, fourth, and sixth, so that since AD shall be the mean of the right side, the ratio AM to AP, will be the cube of that, the square of which will have the ratio MF to PI: but the ratio MP to PI, is the square of the ratio MF to BD, since MF, BD, PI shall be proportionals; therefore the ratio AM to AP, is the cube of the ratio MF to BD, that is the sextuplet of that ratio which NG has to BD, since MF, NG, BD are continued, in the same manner as AG, AD, AH shall be proportionals, the ratio AN to AO is shown to be the triplicate if that, of which the duplicate has the ratio NG to OH, that is the triplicate of that which has NG to BD, from which since AM to AP has been shown to have the sextuple ratio of NG to BD, it is apparent AM to AP, to be the duplicate of the ratio AN to AO; evidently it is shown by the same method, the ratio AL to AC, to be the triple of the ratio AN to AO, and thus for the rest. Q.f.d.



PROPOSITION XLI.

BD shall be the diameter of the parabola ABC, divided at E and F, so that BE, BD, BF shall be proportionals, and the mean line BD equal to the latus rectum, and the ordinates AC, HK may be put in place through E and F:

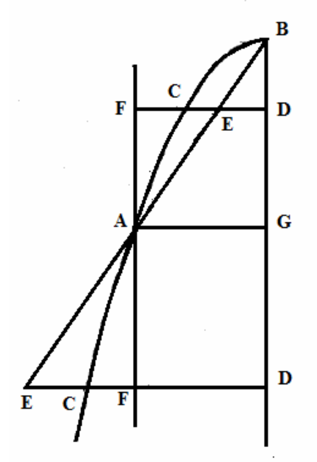
I say the joined lines AB, HB themselves to be proportional to CB, KB

Demonstration.

Since BE, BD, BF are continued proportionals, and the mean line BD is equal to the latus rectum, the ratio AB to HB is the triplicate of that, of which the duplicate has the ratio AE to HF: but for the same reason also the ratio BC to BK, is the triplicate of which the duplicate has EC to FK, that is AE to HF; therefore so that as AB to HB, thus CB to KB: q.e.d.

PROPOSITION XLII.

Any two diameters AF, BD shall cut the parabola ABC, and with the ends A, B of the diameters joined, the ordinate line CD may be put in place for the diameter BD, crossing the lines AB, AF at E and F.
I say FD, CD, ED to be in continued proportion.



Demonstration.

The ordinate line AG may be put in place from A ; so that as BD to BG, thus DE is to AG, that is to DF: but the BD to BG has DC to AG in the duplicate ratio, that is as DC to DF, therefore the ratio DE to DF, also is in the duplicate [*i.e.* square] ratio DC to DF: whereby DE, DC, DF are lines in continued progression. Q.e.d.

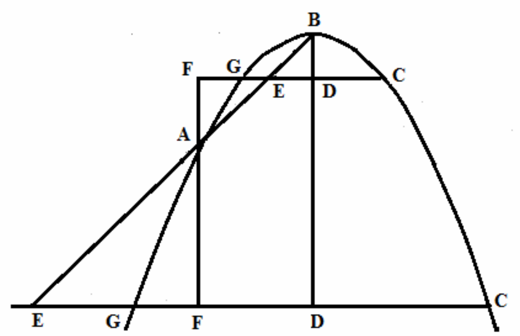
Corollary.

With the same in place : it follows the rectangle FCD to be equal to the rectangle FD, CE, since indeed FD, CD shall be proportionals, so that as FD to CD, thus FC is to CE : therefore the rectangle FDCE is equal to the rectangle FCD. Q.e.d.

PROPOSITION XLIII.

With the same in place as above:
I say the rectangle CFG to be equal to the rectangle DFE.

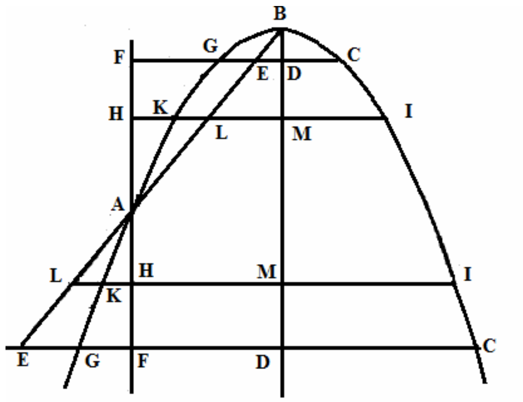
Demonstration.



At first the point F shall fall outside the parabola. Therefore since the right line GC is bisected at D and to that is added directly a certain line GF; the rectangle GFC together with the square GD, is equal to the square FD: but the square FD is equal to the rectangle EFD, together with the rectangle EFD, that is together with the square GD, by the preceding proposition. Therefore the rectangle GFC together with the square GD, is equal to the rectangle EFD together with the square GD: therefore with the common square GD removed, the rectangle GFC remains, equal to the rectangle EFD.

In the second case the point the point F shall fall within the parabola. Since the line GC is bisected at D and it is not bisected at F, the rectangle GFC together with the square FD, is equal to the square GD: but the square GD also is equal to the rectangle FDE, together with the square FD; therefore the rectangle GFC together with the square FD, is equal to the rectangle EFD, together with the square FD: whereby with the common square FD taken away, the rectangle GFC equal to the rectangle EFD. Q.e.d.

PROPOSITION XLIV.



Again there shall be some two diameters AF, BD in the parabola ABC : and to BD certain ordinate lines GC, KI may be put in place crossing the diameter FA at F & H .
I say that as AH shall be to AF, thus the HK to the rectangle CFG.

Demonstration.

With the line AB drawn, it will cut the right lines FC, HI at E and L, therefore rect. LHM is equal to rect. KHI, and rect. EFD is equal to rect. GFC; so that rect. KHI is to rect. GFC, as rect. LHM is to rect. EFD : but rect. LHM is to rect. EFD, as the line LH is to the line EF, that is as AH is to AF, and therefore rect. KHI to rect. GFC is as the line AH to the line AF. Q.e.d.

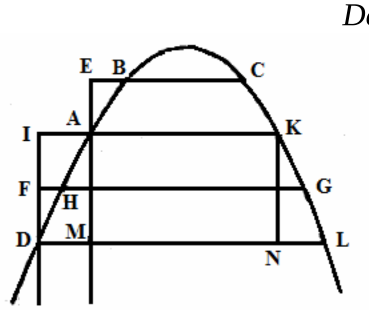
Corollary.

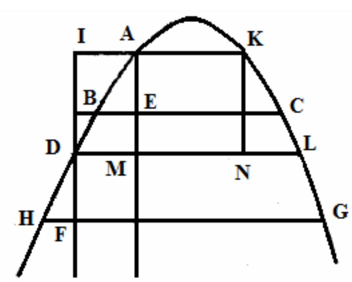
Hence it follows, whatever the ratio AH to AH; thus the rectangle IHK to the rectangle IHK, is indeed as AH to AH, thus as HL to HL, that is rect. LHM to rect. LHM; but the rectangles LHM have been shown have been shown to be equal to the rectangles IHK : therefore as AH to AH, thus the rect. IHK is to the rect. IHK.

PROPOSITION XLV.

The two equal diameters AE, DF will cut the parabola ABC at A and D : and some parallels EC, FG are put from E and F crossing the parabola at the points B, C, H, G.
I say the rectangles BEC, HFG to be equal to each other.

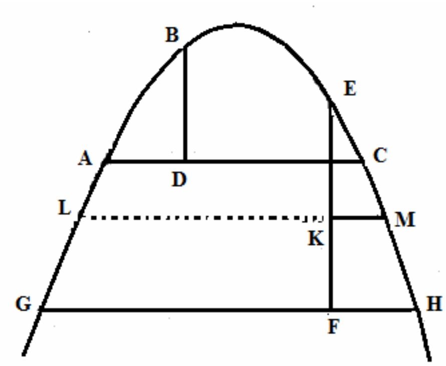
Demonstration.





From D and A the lines DL and AK may be put in place parallel to FG; and indeed AK shall cross the diameter DF at I, and DL with the diameter at N sent from K : truly the right line AE produced shall cut DL at M, so that as DF to DI, thus the rectangle HFG is to the rectangle AIK, and as AE to AM, thus BEC is to the rectangle DML: therefore since EA, DF and DI, MA shall be equal lines, the rectangle HFG to the rectangle IAK, as the rectangle BEC is to the rectangle DML; and on interchanging the rectangle HFG is to the rectangle BEC, as the rectangle DML is to the rectangle IAK; but the rectangle DML, that is MDN, on account of the equal lines DM, NL, is equal to the rectangle IAK; therefore the rectangle HFG is equal to the rectangle BEC. Q.e.d.

PROPOSITION XLVI.

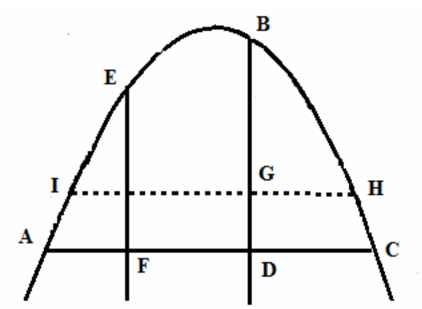


Any two diameters BD, EF shall cut the parabola ABC, which shall cut any two parallel lines AC, GH at D and F.
I say that as BD shall be to EF, thus the rectangle ADC to the rectangle GFH.

Demonstration.

Either the line EF, or the line EF produced, shall be cut at K, so that EK shall be equal to BD, and through K the line LM may be put parallel to GH, therefore so that EK to EF will become thus, the rectangle LKM to the rectangle GFH; truly since EK shall be equal to BD, thus the rectangle LKM shall be equal to the rectangle AD, therefore so that as BD to EF, thus the rectangle ADC to the rectangle GFH. Q.e.d.

PROPOSITIO XLVII.



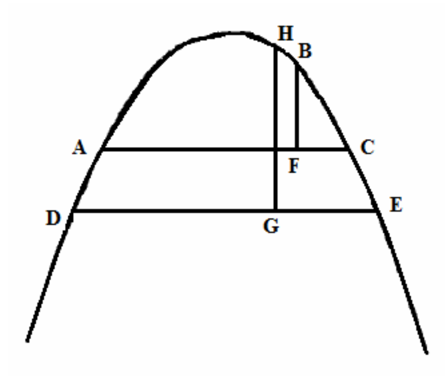
Again, BD, EF shall be some two diameters of the parabola ABC, which some right line AC shall cut at F and D.
I say rect. ADC to be to the rect. AFC, as BD to EF.

Demonstration.

EF shall be made equal to BG; and through G, IH is placed parallel to AC, therefore so that BG to BD thus will become as rect. IGH to rect. ADC : but rect. IGH is equal to rect. AFC, since BG, EF are equal lines, therefore so that as EF to BD, rect. AFC to rect. ADC. Q.e.d.

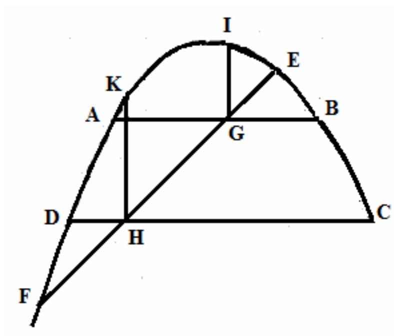
PROPOSITION XLVIII.

Some two parallel lines AC, DE subtend the parabola ABC, by which the diameters BF, HG are divided proportionally at F and G.
I say BF to HG to have the square ratio of that which AC to DE has.



Demonstration.

Since the lines AC, DE are divided proportionally at F and G, rect. AFC shall be to rect. DGE in the duplicate ratio AF to DG, that is AC to DE, but BF is to HG, as rect. AFC to rect. DGE; therefore BF to HG, has the duplicate ratio of that which AC has to DG, that is AC to DE, but BF is to HG as rect. AFC to rect. DGE; therefore BF to HG has the duplicate ratio of that which AD has to DE. Q.e.d.



PROPOSITION XLIX.

Some two parallel lines AB, CD cut the parabola ABC, which the line EF will divide in some manner at G and H.
I say that as rect. AGB shall be to rect.DHC, thus rect.FGE shall be to rect. FHE.

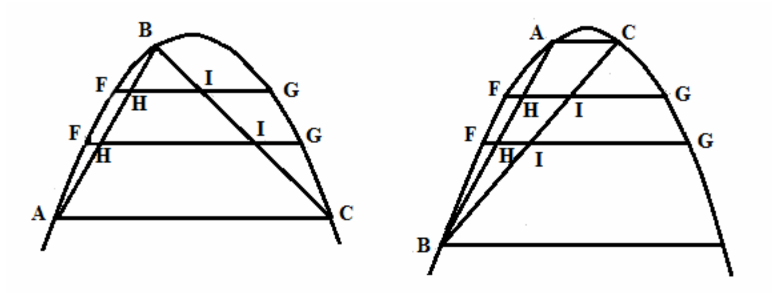
Demonstration.

The diameters HK, GI shall be erected from H and G. Therefore as GI to HK, thus rect. AGB to rect. DHC: but as GI to HK; thus rect.FGE also to rect. FHE; therefore as rect. AGB to rect. DHC, thus rect. FGE is to rect. FHE. Q.e.d.

PROPOSITION L.

Let the triangle ABC be inscribed in the parabola ABC, of which the two sides AB, CB, will cut some parallel lines FG and AC at H and I.
I say that rect. FHG to rect. FHG to be thus as rect. FIG to rect. FIG.

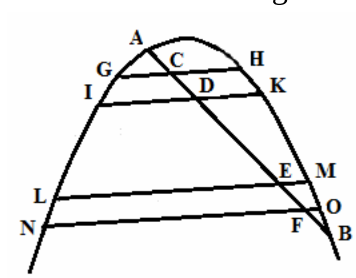
Demonstration.



Rect. FHG is to rect. FHG, as rect. AHB is to rect. AHB: and rect. FIG is to rect. FIG, as rect. CIB is to rect. CIB : moreover as rect. AHB is to rect. AHB, thus rect. CIB is to rect. CIB, since they are composed from the same ratio, therefore so that
as rect. FHG to rect. FHG, thus rect. FIG is to rect. FIG. Q.e.d.

PROPOSITION LI.

Some right line AB shall cut the parabola at A and B, with which divided in some manner at C and D, shall be divided at E and F, so that the individual lines AC, AD shall be equal lines to the individual lines BF, BE ; then through the points C D, E F the parallel lines GH, IK, LM, NO may be drawn.
I say that GCH to be to IDK so that thus, the rectangle OFN shall be to the rectangle MEL.



Demonstration.

Indeed by the 49th Proposition of this section, rect. GCH to be to rect. IDK, as rect.ACB to rect. ADB,
that is as rect. BFA shall be to rect.BEA, since AC, AD are equal to FB, EB, but so that as rect.BFA to rect.BEA, thus rect. NFO is to rect. LEM: therefore the rectangles GCN to IDK, provide the same ratio as rect. NFO to rect. LEM*:* which was required to be demonstrated, nor to be wondered, since rect.GCH to rect. NFO, and rect. IDK is equal to rect. LEM.
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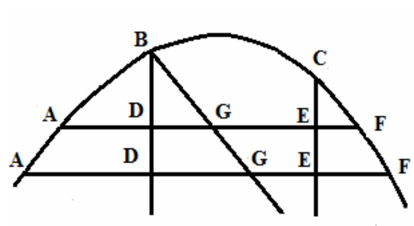
PROPOSITION LII.

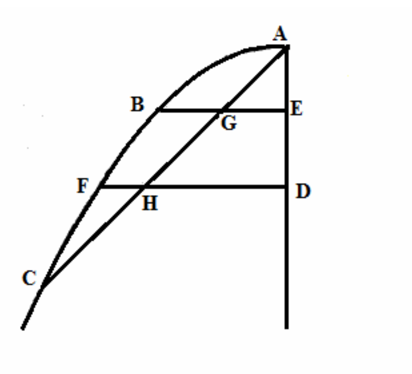
Any two diameters BD, CE shall cut the parabola ABC, and shall divide both the parallel lines AF at D and E, then from B, some line BG shall be drawn cutting the lines AF at GG.
I say the rectangle GDE to be to the rectangle GDE, as the rectangle ADF is to the rectangle ADF.

Demonstration.



Since by hypothesis the lines DE shall be parallel, and BD, CE are the diameters, the right lines DE are parallel to each other on account of the parallelogram ED, whereby the rectangle GDE is to the rectangle GDE as GD to GD , that is BD to BD, thus BD to BD, thus as the rectangle ADF is to the rectangle ADF: and therefore the rectangle GDE is to the rectangle GDE, as the rectangle ADF is to the rectangle ADF. Q.e.d.

PROPOSITION LIII.

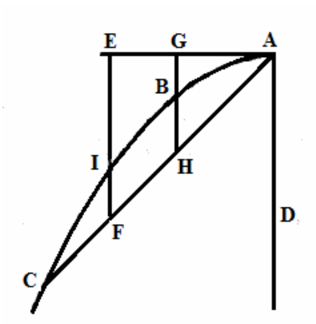


AD shall be the diameter of the parabola ABC, which shall cut the ordinate lines BE, FD at E & D, and from A the line AC shall be drawn, cutting the ordinates put in place at BE, FD, at G and H.
I say the rectangle BEG to the rectangle FDH to have the ratio of the cube of that which BE has to FD.

Demonstration.

The rectangle BEG to the rectangle FDH has the ratio composed from BE to FD, and GE to HD, that is the rectangle AE to the rectangle AD: but the square of the ratio AE to AD is the ratio of BE to FD: therefore the rectangle BEG to the rectangle FDH has the ratio of the cube of that which BE has to FD. Q.e.d.

PROPOSITION LIV.



The line AE touches the parabola ABC at the point A, of which the diameter is AD ; and with some line AC drawn which shall meet the parabola again at C, some points F, H, may be taken on AC, from which the diameters FE, HG, may be raised crossing AE at E and G, crossing the parabola at B, and I.

I say the rectangle BGH to have the triple [*i.e.* cubic] ratio to the rectangle IEF of that which GH has to EF.

Demonstration.

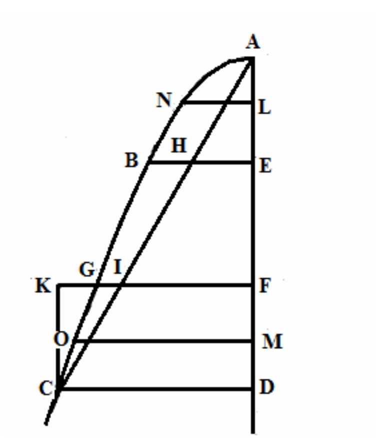
The rectangle BGH to the rectangle IEF, has the ratio composed from BG ad IE, and from GH to EF, that is AG to AE: but the ratio BG to IE is the square of the ratio AG to AE, therefore the rectangle BGH to the rectangle IEF has the cubic ratio of that which GH has to EF. Q.e.d.

PROPOSITION LV.

AD shall be the diameter of parabola ABC, which shall be cut by the ordinate line CD at D: and AD shall be divided at E and F, so that AE, DF shall be equal, EB, FG are put to be the ordinates.

I say the squares EB, FG taken together, to be equal to the square CD.

Demonstration.



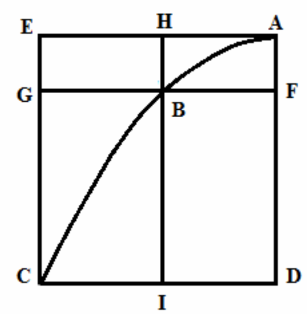
With AC drawn it shall cross the line EB at H, and FG at I, and the diameter CK will be erected from C diameter CK , cutting FG at K. Since by hypothesis AE is equal to FD, it is equal to CK, the angle AHE is equal to the angle AIF, that is to the angle KIC (because HE, GF are parallel) and the angle AEH is equal to the angle CKI, the triangle AHE is equal to the triangle CKI; and the lines HE and KI are equal. Again, since both CD, GF, IF as well as CD, BE, HE shall be proportional lines, the mean squares FG, BE are equal to the rectangles CDIF, CDHE; that is, to the rectangles IFK, IKF, since HE, KI shall be equal lines, but the square FK is equal to the rectangles IFK, IKF, and therefore the squares FG, BE taken together are equal to the square FK, that is to the square CD.
Q.e.d.

Corollary.

Hence it follows, if again AD may be divided at L and M, so that AL, DM shall be equal lines and the ordinates LN , MO may be put in place, the squares LN, MO taken together to be equal to the squares EB, FG likewise taken together: it is apparent from the demonstration, since both the squares EB,FG as well as LN, MO taken together are equal to the square CD.

PROPOSITION LVI.

The line AE is a tangent at A to the parabola ABC, the diameter of which is AD and the ordinate to that CD put in place, which will cut the diameter CE at E: and with some point B taken in the section, the ordinate line FG may be put through B crossing the line AD at F, and EC at G: then through B the diameter HI crossing the ordinate CD placed at I.



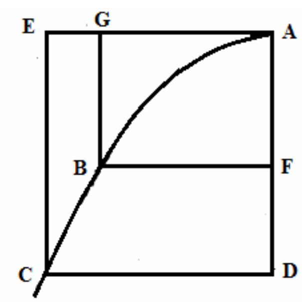
I say the parallelograms AB, AG, AI, AC to be in continued proportion.

Demonstration.

Because the lines AE, FG shall be parallel, the parallelogram AB to the parallelogram AG, is as the line FB to the line FG, that is equal to CD: but also the parallelogram AB to the parallelogram AI, shall be as the line AF to the line AD, that is in the square ratio FB ad DC: therefore the parallelogram AB, to the parallelogram AI, has the square ratio of that, which the parallelogram AB has to the parallelogram AG : therefore the parallelograms AB, AG, AI are in continued proportion: Again the parallelogram AI is to the parallelogram AC, as the line DI to the line DC, that is, as the parallelogram AB to the parallelogram AG. Therefore the parallelograms AB, A G, AI, AC are in continued proportion. Q.e.d.

PROPOSITION LVII.

The line AE is a tangent to the parabola ABC at A, of which the diameter is AD; and with the ordinates FB, DC, erected from C and B, the diameters EC, BG crossing the tangent at G & E.



I say the parallelogram AGB to the parallelogram AEC to have the threefold ratio [*i.e.* cubic] of that which the line AG has to the line AE.

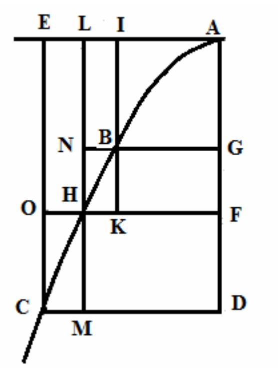
Demonstration.

The ratio of the parallelogram AGB to the parallelogram AEC is composed from the ratio EC to AE, and from the ratio GB to EC; but the ratio GB to EC, that is AF to AD, is the twofold ratio [*i.e.* square] of AG ad AE; that is, FB to DC, therefore the parallelogram AGB to the parallelogram AEC has the threefold [*i.e.* cubic] ratio of that which the line AG has to the line AE. Q.e.d.

PROPOSITION LVIII.

The line AE shall be a tangent to the parabola ABC at A, of which the diameter shall be AD, and the ordinate to that CD, and which shall cut the diameter CE at E, and with AD, AF, AG made to be in continued proportions, and with the ordinates of the lines FH, GB drawn through B and H, the diameters IK, LH shall be put in place crossing the lines FH, DC at K and M: moreover the line GB shall cut the diameter LH in N, and the line FH the diameter EC in O.

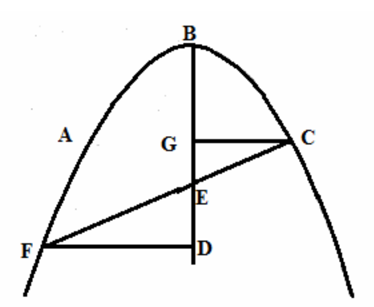
I say the parallelogram HD to be to the parallelogram FB, as the parallelogram HE is to the parallelogram BL.



Demonstration.

The ratio of the parallelogram HD to the parallelogram FB, is composed from the ratios FH to GB, and FD to FG: but as FH is to GB, thus DC to FH; that is EA to LA, that is in turn as EL to LI (since AG, AF, AD, and thus GB, FH, DC, that is EA, LA, IA shall be continued proportionals) and as FD to FG, thus DA to FA; that is FA ad GA; that is HL to BI: therefore the ratio of the parallelogram HD to the parallelogram FB, is composed from the ratio EL to LI, and from the ratio LH to IB; but also from the same the ratio is composed of the parallelogram HE to the parallelogram BL; therefore so that as the parallelogram HD to the parallelogram FB, thus the parallelogram HE is to the parallelogram LB. Q.e.d.

PROPOSITION LIX.



Let BD be the diameter of the parabola ABC, which shall cut some right line FC at E and both sides of the parabola at C & F, moreover the ordinate lines EG, FD shall be drawn from C and F.

I say the lines BG, BE, BD to be proportionals.

Demonstration.

Because the ordinates CG, FD are put in place for the diameter BD, the ratio BG to BD, is the square of that which GC has to FD, that is GE to

ED; therefore BG, BE, BD are proportional lines, indeed with BE put to be the mean between BG, BD thus there will be BG to BE, as GE to ED, and the ratio BG to BD, to be the duplicate [*i.e.* square] of the ratio GE to ED. Therefore, etc.
What was required to be shown.

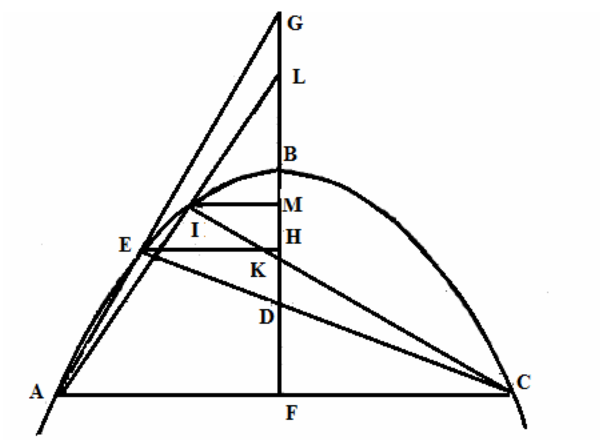
Corollary.

Hence it follows that CE to EF, thus to be as BE to BD, for as CE to EF, thus GE is to ED, that is BE to BD.

PROPOSITION LX.

Let BF be the diameter of the parabola ABC ; in which with some point H taken, the continued proportionals BH, BG, BF may be put in place : and with the ordinate HE drawn, the line acts through F parallel to HE, crossing the line GE at A and the parabola at C : and EC shall be drawn crossing the diameter at D.
I say DB, BG to be lines equal to each other.

Demonstration.



Since BH, BG, BF are placed in continued proportion, and the ordinate CF applied to the diameter GB, A will be an applied point for the parabola ABC by Corr. 25 of this section, moreover by the preceding also BH, BD, BF are proportionals ; therefore the mean DH is equal to the mean DF.

PROPOSITION LXI.

With the same in place some other CI may be drawn from C crossing the diameter BD at K; truly the parabola at I; then from A, a line drawn through I shall meet with the diameter at L, and the ordinate IM may be put in place from I.
I say GB to be to LB, as EH to IM.

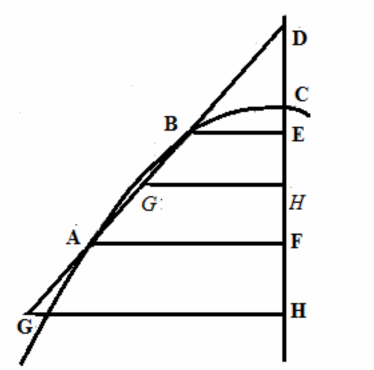
Demonstration.

If the mean between BM and BF may become BL, it will be shown that in the first place LB to be equal to BK itself, and the join LI of the parabola and the right line AC to occur at the point A, from which since the lines BF, BD, BH, as well as BF, BK, BM having the same first term BF are continued proportionals, the ratio BH to BM, of the third to the third is the square ratio BD to BK, that is BG to BL of the second to the second: and also the ratio BH to BM, is also the square of the ratio EH to IM: therefore as EH to IM, thus GB to LB. Q.e.d.

PROPOSITION LXII.

CD shall be the diameter of the parabola ABC, moreover AB may be put to be the line crossing the parabola at the two points A B, truly crossing the diameter beyond the section at D, and the ordinate BE may be put in place, and the ordinates BE, AF to be put in place.
I say the lines CE, CD, CF to be continued proportionals.

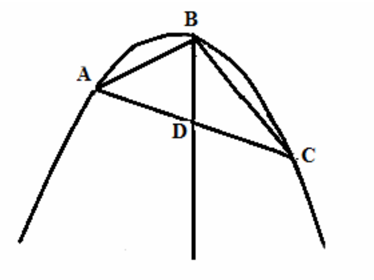
Demonstration.



Truly if the continued proportionals CE, CD, CH may be put in place and from H the ordinate HG may be put in place, crossing the line AB at G: therefore the point G belongs to the parabola ABC*:* and thus the line AB crosses the parabola at three points. Which cannot happen; from which CE, CD, CF are continued proportionals. Q.e.d.

PROPOSITION LXIII.

Let BD be some diameter of the parabola ABC equal to the latus rectum, and with the ordinate AC acting through D, which will meet the parabola at A and C; AB, BC may be joined. I say the angle ABC to be right.

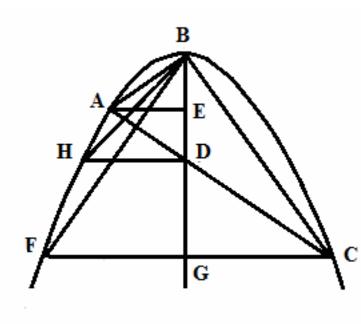


Demonstration.

Because the diameter BD is equal to the latus rectum, and the ordinate AC put in place of the line DB; DA, DC, are equal and thus the points A B C belong to the circle of which AC is the diameter: therefore the angle ABC is right. Q.e.d.

PROPOSITION LXIV.

Let the axis BD of the parabola ABC be equal to the latus rectum, and with some line AC passing through D, which shall cross the parabola at A and C; AB, BC shall be joined:
I say the angle ABC to be right.



Demonstration.

The ordinate lines AE, CGF shall be put in place from A and C ; therefore BE, BD, BG will be proportional lines [*Prop.* 59]; truly since the mean BD is equal to the latus rectum, ABE, FBG will be similar triangles [*Coroll. Prop*. 35] ; and the angle BAE equal to the angle FBG, that is, CBG : but the angle BAE together with the angle ABE is equal to a right angle, because the angle AEB is right to the axis : therefore the angle CBD together with the angle ABE are equal to one right angle; therefore the angle ABC is right. Q.e.d.

PROPOSITION LXV.

With the same in place:
I say AB to BC, to be in the triplicate [*i.e.* cubic] ratio of that, of which AD to DC is the duplicate [*i.e.* square].

Demonstration.

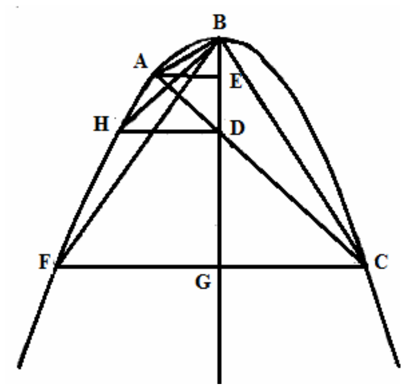
Since in the triangles GBF, CBG the angles at G are right, and by the hypothesis the lines FG, GC are equal, the triangles FBG, CBG are equal to each other, and also the lines FB, CB are equal: from which since the ratio AB to FB, shall be the triple of that, of which the duplicate has the ratio AE to FG , and the ratio AB to BC, shall be the triplicate [*i.e.* cube] of the ratio AE to FG, that is AE to CG, which is AD to DC. Q.e.d.

PROPOSITION LXVI.

With the same in place the ordinate line DH may be drawn from D, and with HB joined: I say the angle ABF, to be bisected by the line HB.

Demonstration.

Because the angles AEB, HDB, are right, the remaining two angles ABE, BAE are equal to the remaining two HBD, DHB. But the angle BAE is equal to the angle FBG [*Coroll. Prop*. 35], therefore the two angles FBG, ABE are equal to the two angles HBD, BHD: that is, to the angle HBD taken twice, on account of the equal lines HD, DB : therefore with the common angle FBG removed, the two angles remain HBD, HBF equal to the angle ABE: from which again if the common angle HBD be taken away, the angles left ABH, HBF remain equal. Q.e.d.



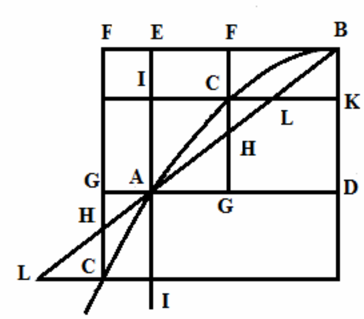
PROPOSITION LXVII.

The line BE shall be a tangent to the parabola ABC, the diameter of which is BD ; moreover from some point A assumed on the perimeter with the ordinate AD drawn and AB joined so that some point F may be taken on the tangent, from which the line FG may be dropped, parallel to the diameter BD, crossing the parabola at C, connected to AB at H, and with the ordinate put in place at G.

I say the lines FC, FH, FG to be in continued proportion.

Demonstration.

The diameter AE erected from A diameter AE shall meet the tangent at E, and the ordinate IK through C may be put in place cutting the lines AB, AE at L and I, and the diameter BD at K, so that as KL is to KC, thus BL is to BH, that is FC to FH: and so that CK to IK, that is FC to EH, thus as FH is to EA, as FG: but the lines LK, CK, IK, are in continued proportion; and therefore FC, FH , FG are lines in continued proportion. Q.e.d.



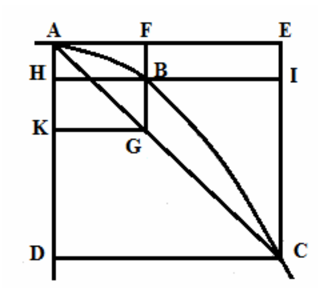
PROPOSITION LXVIII.

With the same given some point F may be taken on the tangent from which FH may be drawn parallel to BD, crossing the line AB at H.
I say HC to be to CF as AH to HB.

Demonstration.

Because GF, HF, CF are proportionals, HF is to CF, as GF is to HF, that is, as AE to HF, that is as AB to HB : therefore on dividing, HC is to CF, as AH to HB. Q.e.d.

PROPOSITION LXIX.

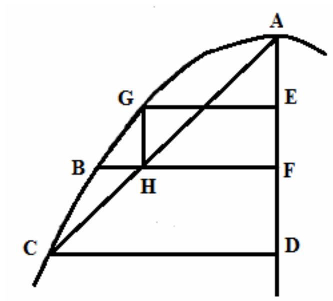


The line AE shall be a tangent to the parabola ABC, of which the diameter is AD, and the ordinate for that CD put in place, which shall cut the diameter CE at E , and with the points AC joined, some diameter FG may be drawn, crossing the parabola at B; then with the ordinate HI acting through B, which shall cross the right line EC at I, and the diameter AD at H, the line GK may be drawn through G parallel to DC.
I say the parallelogram EB to be equal to the parallelogram KB.

Demonstration.

As AG to GC, thus AF to FE, that is HB to BI: but as AG to GC, thus FB to BG: therefore as HB to BI, thus as FB to BG; but the opposite angles at B are equal B ; therefore the [area] parallelogram EB is equal to the [area] parallelogram KB. Q.e.d.

PROPOSITION LXX.



AD shall be the diameter of the parabola ABC, divided at E and F; so that AE, AF, AD shall be proportionals, and the ordinates EG, FB, DC put in place, and AC will cross FB at H :

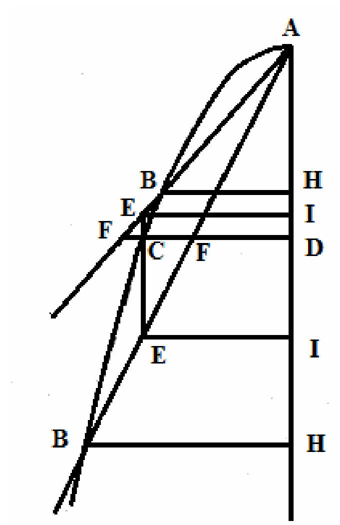
I say HG to be parallel to the diameter AD, and on the other hand if HG shall be parallel to the diameter AD, and the ordinate FB may be put through H, I say AE, AF, AD to be proportionals.

Demonstration.

Since AE, AF, AD shall be proportionals, the right lines CD, BF, GE also shall be in continued proportion ; but also DC, FB, FH shall be proportionals ; therefore the lines HF, GE shall be equal, whereby HG shall be parallel to the diameter AD, which was the first part.

Now HG shall be parallel to AD, and the ordinate BF will be put in place through H ; I say AE, AF, AD also to be continued in proportion, since indeed the lines CD, BF, HF, shall be in proportion, and GE to be equal to HF, and CD, BF, GE shall be continued proportionals; from which AD, AF, AE also are in continued proportion. Q.e.d.

PROPOSITION LXXI.



AD shall be the diameter of the parabola ABC, which the ordinate line DC shall cut at D, and with the diameter CE acting through C, the line AF shall be drawn from A cutting the parabola at B, and the diameter EC at E, truly meeting the line DC at F.

I say AB, AE, AF to be lines in continued proportion.

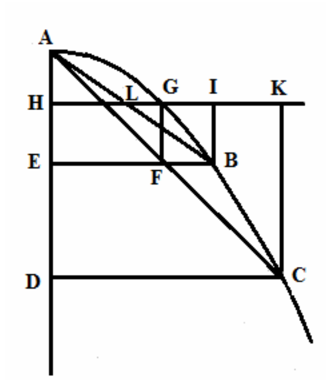
Demonstration.

The ordinate lines BH, EI shall be put in place from B and E. The ratio AH to AD is the square of the ratio HB to DC : that is HB to IE, but as HB is to IE, thus AH is to AI; therefore the ratio AH to AD, is the square of the ratio AH to AI: whereby the lines AH, AI, AD, that is, AB, AE, AF are proportionals. Q.e.d.

PROPOSITION LXXII.

Let AD be the diameter of the parabola ABC: and with some lines AB, AC drawn from A which meet the parabola at B and C, the ordinate lines CD, BE may be put in place; and indeed BE will meet the line AC at F; and FG shall be erected parallel to the diameter AD; the ordinate HG shall be put in place through G, meeting the diameters IB, KC at I and K, moreover the right line AB at L:

I say HL, HG, HI, HK to be lines in continued proportion.



Demonstration.

Since HG, EB, DC are the ordinates put in place, and FG parallel to the diameter AD; the right lines HG, HI, HK [§70] are in continued proportion; but HL, HG, HI are proportionals [§42]; therefore HL, HG, HI, HK are continued proportionals. Q.e.d.

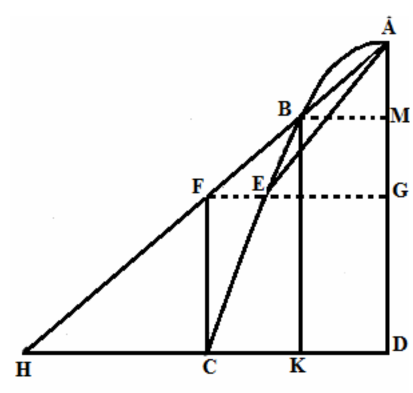
PROPOSITION LXXIII.

Let AD be the diameter of the parabola ABC, and with CD put to be the ordinate for that; moreover the line AE shall be drawn from A, meeting the section at E: and with the ordinate FG acting through E, the diameter CF shall be erected from C, meeting FG in F; and AF shall be joined, which shall meet the line CD at H, truly at the point B of the section B from which the diameter BX may be dropped, cutting the line FG at L, and CD at K.

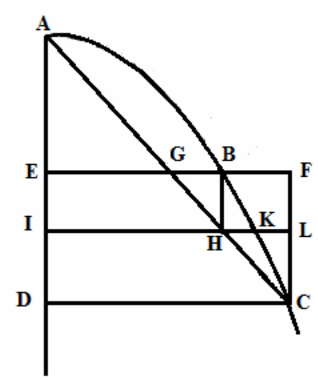
I say that HC to CK to be as the square of the ratio of that which FE has to EL.

Demonstration.

The ordinate line BM shall be put in place from B. Since FC is the diameter and HCD the ordinate put in place for the diameter AD, the right lines AB, AI, AH shall be proportionals and therefore AM, AG, AD, and likewise the line BM, EG, CD, that is KD, CE, HD are in continued proportion. Whereby since each of the first of each series BM, KD shall be equal, the ratio HD to CE, of the third to the third, is the square of the ratio CD to EG, that is FG to EG: moreover as HD to CD, thus HC to CK, and as FG to EH, thus FE to EL, therefore the ratio HC to CK, is the square of the ratio FE to EL. Q.e.d.



PROPOSITION LXXIV.



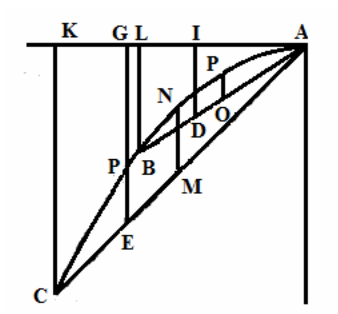
AD shall be the diameter of the parabola ABC, and with the ordinate DC put in place from C with the diameter CF, and CA shall be joined, and some ordinate FE may be put in place meeting the parabola at B, and AC joined at G; then with a diameter dropped from B, which shall meet the line AC at H, and the ordinate line IL may be put in place through H crossing the diameter FC at L, and the section at K:

I say the line GB to BF, to be in the square ratio of that which HK has to KL.

Demonstration.

Because both the lines FE, BE, [§.42] GE, as well as LI, KI , HI are proportionals having equal first terms FE, LI, the ratio GE to HI, third to third, that is the ratio GE to BE [§.1 *De prog.*], that is GB to BF, is the square [§.27 *De prog.*] of the ratio EB to IK, of the second to the second, that is of the ratio IH to IK [§.1 *De prog.*], that is, HK ad KL. Q.e.d.

PROPOSITION LXXV.



With the same figure present:

I say the rectangle EGB to the rectangle EFB, to have the cubic ratio of that which the line GB has to the line BF.

Demonstration.

The rectangle EGB to the rectangle EFB, has the ratio composed from the ratio EG to EF, that is from the square ratio EG to EB, that is GB to BF, (since EG, EB, EF are proportionals) and from GB to BF; therefore the rectangle EGB to the rectangle EFB has the cubic ratio of that which the line GB has to the line BF. Q.e.d.

PROPOSITION LXXVI.

Let there be the parabola ABC, and with some point A assumed on the periphery, the right lines AB, AC may be drawn crossing the parabola again: but with the lines AB, AC crossing the parabola again at D and E, the diameters EG, DI may be erected, meeting the parabola at F and H, and the tangents at G and I acting through the point A.

I say the lines GE, DI to be divided proportionally at F and H.

Demonstration.

The diameters CK, BL shall be erected from C and B. Because AC, AB to put to be divided proportionally at E and D, so that BA to DA, that is BL to DI; thus as CA is to EA, that is KC to GE; but as BL to DI, thus D1 is to HI, and as KC to GE, thus as GE is to GF; therefore so that DI to HI, thus GE to GF. Q.e.d.

PROPOSITIO LXXVII.

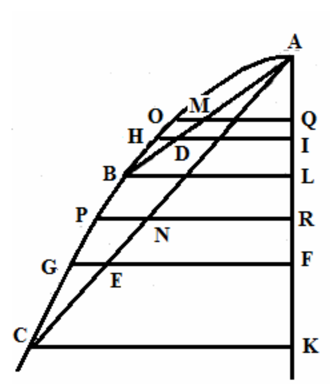
With the same in place, again the lines AB, AC may be divided proportionally at M and O, and the diameters MN, OP shall be erected.
I say FE to be to MN, as DH is to OP.

Demonstration.

Indeed as the rectangle CEA is to the rectangle CMA, thus FE is to NM and as the rectangle BDA to the rectangle BOA, thus DH is to PO; but BDA to BOA, the rectangle is as CEA to CMA, the rectangle, since CA, BA are put in place divided proportionally ; therefore so that as FE to NM, thus DH to PO. Q.e.d.

PROPOSITION LXXVIII.

With the same in place the ordinate lines HDI, FEG to the diameter sent from A acting through D and E.
I say the lines HI, FG to be divided proportionally at D and E.



Demonstration.

The ordinate lines BL, CK shall be put in place. Therefore since the lines AB, AC are divided proportionally at D and E, so that as CK is to EG, thus BL is to DI; but CK to EG, has the square ratio FG to EG, and BL to DI has the square ratio HI to DI, therefore so that as FG to EG, thus HI to DI, and on being divided, so that as FE to EG, thus HD to DI. Q.e.d.

PROPOSITION LXXIX.

With the same in place, again the lines AB, CA shall be divided proportionally at M and N, and the lines MOQ, NPR shall be drawn parallel to HI, FG themselves.
I say that as HD to OM, thus FE to be to PN.

Demonstration.

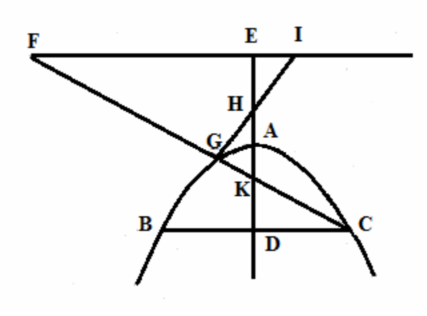
Indeed since the lines AB, AC shall be divided proportionally by the points at in M, D, N, E, MQ will be to DI, as NR to EG, and on interchanging , so that as MQ to NR, thus DI to EG; but by the preceding, so that as MQ to NR, thus OM is to PN; and as DI to EG, thus HD to FE; therefore as OM to PN, thus HD to FE, and on interchanging so that as OM to HD, thus PN to FE. Q.e.d.

PROPOSITION LXXX.

The ordinate line CB shall be put in place for the diameter AD of the parabola ABC : and with AE equal to AD itself the line EF may be put in place through E parallel to the right line BC, then from C the line CF shall be drawn, crossing the parabola at the point G, through which from B a line may be put in place crossing the axis AD at H; and the line EF at I.

I say that FE, BD, EI to be lines in continued proportion.

Demonstration.



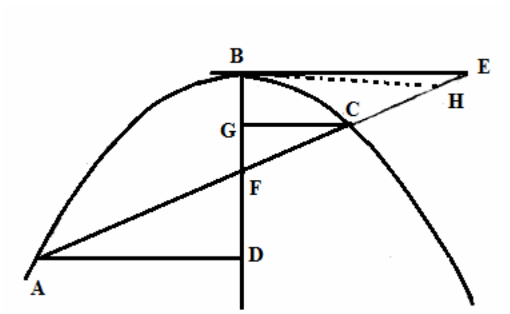
The line CF shall cross the diameter at K. Because the line AE is put equal to the line AD; and HA, AK also shall be equal to each other, the remaining line EH shall be equal to the remaining line KD. Again the triangle FKE to the triangle DKC has the square ratio of the line EK to the line KD, that is as HD to EH; moreover the triangle BHD to the triangle EHI, is in the square ratio HD to HE; therefore so that as triangle FKE to triangle DKC, and thus triangle BHD to triangle EHI are composed from these same ratios, but the ratio of FKE to triangle DKC, is composed from the ratio FE to DC, that is BD; and from EK to KD, that is DH to HE: and the ratio of triangle BHD to triangle EHI, is composed from HD to HE, and from BD to EI, with the common ratio HD to HE taken away, the ratio FE to BD remains, with the same ratio as BD to EI; whereby FE, BD, EI are lines in continued proportion. Q.e.d.

PROPOSITION LXXXI.

The line BE shall be a tangent to the parabola ABC at B, of which the diameter is BD ; and the line EA dropped from E shall cross the parabola at C and A, and truly the diameter BD at F.

I say the lines EC, EF, EA to be in continued proportion, and on the other hand if EC, EF, EA were proportionals, I say EB to be a tangent to the [conic] section.

Demonstration.



The ordinate lines AD, CG from A and C may be put in place to the diameter BD: therefore since BE; GC shall be parallel, so that GB will be to FB, thus as CE to FE; and as FB to DB; thus FE to AE; but BG, BF, BD are proportional lines; and therefore EC, EF, EA are lines in continued proportion.
Which was the first part.

Now if EC, EF, EA shall be proportionals, and EB may be joined, I say the line EB to be a tangent to the section at B. Truly if the tangent shall not act through B, it shall meet the line AE at H, and therefore HC, HF, HA will be continued proportionals, and CF to FA, shall be as HC to HF: but CF is to FA, as EC to EF (since EC, EF, EA shall be proportionals) ; therefore so that as HC to HF, thus EC to EF: and by dividing so that as HC to CF, thus EC to CF, which is absurd, since from the hypothesis, HC shall be greater or the right line EC smaller, whereby HB is not a tangent, nor any other line besides EB. Q.e.d.

PROPOSITION LXXXII,

With the same figure remaining proposed, to draw the tangent from an external point of the section.

Construction and demonstration.

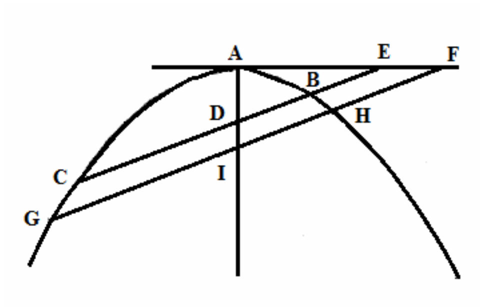
The point E shall be given, from which some section cutting the parabola at C and A may be drawn : EC shall be the first of the continued parts, and AC the excess of the remainder; therefore the mean EF between CE, EA may be found through F, the diameter BD shall be acting and EB joined EB, it is evident from the preceding, the line EB to the section to be a tangent at B, therefore from a given point beyond the parabola, etc. Q.e.d..

PROPOSITION LXXXIII.

The line AE shall be a tangent to the parabola ABC at A, the diameter of which shall be AD, on which line with the two points E and F taken two parallel lines EC, FG shall be drawn from E and F, the two parallel lines crossing the parabola at B, C, H, G, and the diameter AD at D & and I.

I say the rectangle BEC to the rectangle HFG to have the square ratio of that which the line EA has to the line FA.

Demonstration.



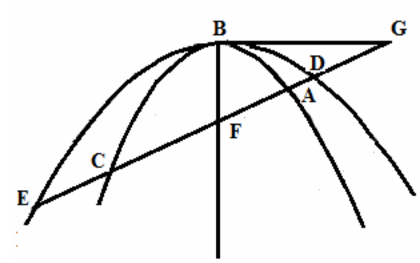
Since both the lines EB, ED, EC, as well as FH, FI, FG are proportionals, the rectangle BEC is equal to the square ED, and the rectangle HFG is equal to the square FI, therefore so that as the square ED is to the square FI, thus the rectangle BEC is to the rectangle HFG; but the square ED is to the square FI, as the square AE is to the square AF; and therefore the rectangle BEC is to the rectangle HFG as the square AE is to the square AF. That is the squares have the ratio EA to FA. Q.e.d.

PROPOSITION LXXXIV.

The two parabolas ABC, DBE having the common diameter BF shall touch at the same point B, the line BG may be drawn from some point G, crossing the parabolas at D, A, C, E; truly with the diameter BF at F.

I say the rectangle DGE to be equal to the rectangle AGC.

Demonstration.

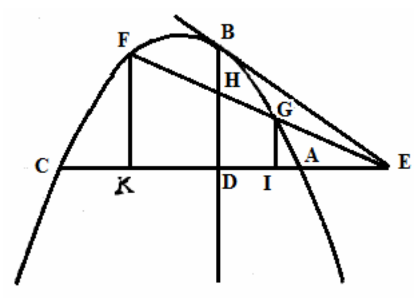


Indeed by Prop. LXXXI, in the first place both the rectangle DGE, as well as the rectangle ACG, is equal to the square FG: and therefore the rectangles AGC, DGE are equal to each other. Q.e.d.

PROPOSITION LXXXV.

The line EB shall be a tangent to the parabola ABC of which BD is a diameter: from the point E some two lines EF, EC shall be drawn, cutting the parabola at the points F, G, A, C and the diameter BD at D and H, and the diameters FK, GI dropped from F and G, cross the line AC at K and I.

I say: GI to FK, to be in the square ratio of that which GH has to HF.



Demonstration.

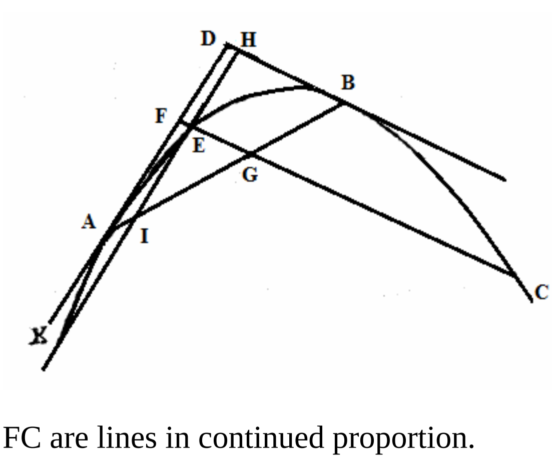
As GI to FK, thus EG is to EF; but EG to EF has the ratio to the squares EG to EH [Prop. 82], that is GH to HF [§.1 *De prog.*]; and therefore GI to FK is in the square ratio of that which the line GH has to the line HF.
Q.e.d.

PROPOSITION LXXXVI.

The lines AD, BD shall be tangents to the parabola ABC at A and B meeting at D, and with the point E assumed from the point E on the periphery, the line FC shall act through E, parallel to the right line BD meeting the line AD at F.

I say that the square BD to the square AD, thus shall be as the rectangle EFC to the square AF.

This is from Apollonius bk.3.prop.16. clearly proposed in the same manner: but from this we will infer further, if the points of contact A, B may be joined together, the right lines FE, FG, FC to be in continued proportion, just as the lines HE, HI, HK.

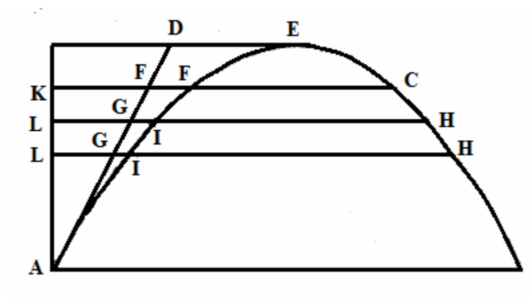


Demonstration.

Indeed from the supposition that the square AD to the square DB, thus the square AF shall be to the rectangle EFC: and on interchanging, so that the square AD to the square AF, thus the square DB to the square FG; therefore so that the square DB to the rectangle EFC, thus likewise the square DB to the square FG, from which the square FG is equal to the rectangle EFC; and FE, FG, FC are lines in continued proportion.

PROPOSITION LXXXVII.

With the same in place and with the line GH drawn parallel to the line FC as well, cutting the parabola at I and H.
I say the rectangle EFC to the rectangle IGH, to be as the square FA to the square GA.



Demonstration.

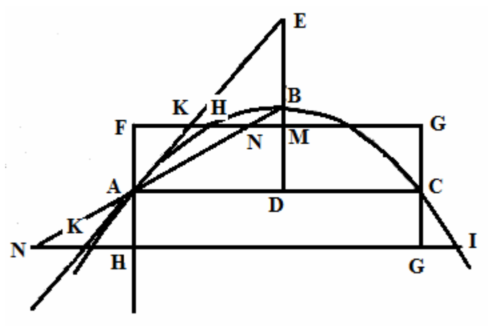
Indeed since both the rectangle EFC to the square FA, as well as the rectangle IGH to the square IGH to the square GA shall be in the ratio of the square DB to the square DA, thus it is agreed the rectangle EFC to be to the rectangle IGH as the square FA to the square GA. Q.e.d.

Corollary.

If the diameter AK shall be drawn from A, crossing the ordinates of the applied lines at K and L, the rectangle CFE to the rectangle HGI, will be in the square ratio of that which the rectangle CKE has to the rectangle HLI; indeed the ratio of the rectangle CKE to HLI, is the ratio of the line KA to the line LA, that is FA to GA; but the ratio of the rectangle CFE to HGI, is the ratio of the squire FA to the square GA: therefore the ratio of the rectangle CFE to the rectangle HGI, is the square of that which the rectangle CKE has to the rectangle HLI.

PROPOSITION LXXXVIII.

The line AE shall be a tangent at A to the parabola ABC, of which the diameter is BD, and that ordinate ADC put in place, agreeing with the diameter at E, and with the diameters AF, CG erected from A and C, some line HI may be drawn, parallel to AC, meeting the tangent line AE at K, the line AF at F, and the diameter BD at M, the line joined AB in N, and the right line CG at G .
I say the rectangle KFG to be equal to the rectangle HFI.



Demonstration.

Since both [§18] FN is to be bisected at K, as well as FG at M, FK will be to FN, as FM to FG; so that the rectangle FKFG, shall be equal to the rectangle FNFM: but the rectangle NFM has been shown to be equal to the rectangle HFI [§43]. Therefore also the rectangle HFI is equal to the rectangle KFG. Q.e.d.

PROPOSITION LXXXIX.

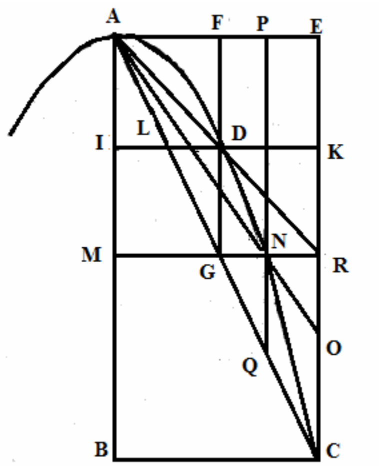
With the same in place:
I say HK, HF, FG to be lines in continued proportions.

Demonstration.

Because the rectangle KFG is equal to the rectangle HFI, so that as FH is to FK, thus FG is to FH; and on interchanging so that as FH to HK, thus FG is to GI, that is to FH; therefore the square FH is equal to the rectangle HKFG; and HK, HF, FG are proportionals. Q.e.d.

PROPOSITION XC.

AB shall be the diameter of the parabola ADC and with the ordinate BC put in place, and the tangent AE to the diameter, and in addition EC parallel to AB, and with some AR crossing the parabola at D; AC shall be put in place, and FDG shall be put parallel to AB, and meeting AC at G.
I say FG to be equal to the right line ER.



Demonstration.

The right lines IK, MR may be put in place through the points D and G, parallel to AE ; and thus there will be the three lines IL, ID, IK and MG, MN, MR in continued proportion [§42]. Therefore the square ID to the square MN , has the same ratio as IL to MG [$IL.IK : MG.MR = IL : MG$, as $IK = MR$]; that is IA to MA ; that is DA to RA; that is DF to RE, but FG is equal to MA: therefore RE is equal to the same FG. Q.e.d.

[There are several typographical errors in the original text of this proposition and the next, and in the diagram, which have been corrected here. I.B.]

PROPOSITION XCI.

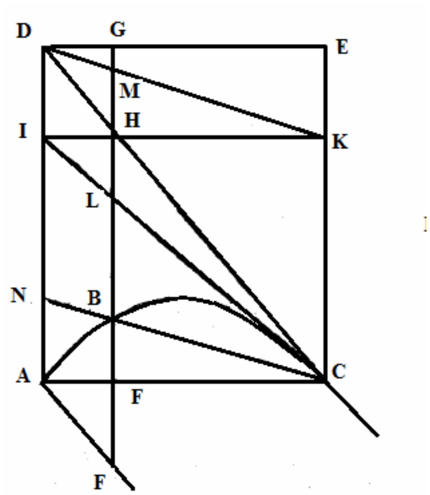
With the same in place; ANO shall be drawn through N. I say ID to MN to have the same ratio as ER to EO.

Demonstration.

PNQ shall be put parallel to AB, by the preceding, there will become PQ equal to EO. Similarly by the same there will become FG equal to ER: but FG is to PQ, as AF to AP, that is ID to MN, therefore ER to EO, maintains the same ratio as ID to MN. Q.e.d.

PROPOSITION XCII.

The line AC sustains the parabola ABC, and with the tangent acting through C, which meets the diameter through A at D, the parallelogram ADEC shall be completed: then with some point B assumed in the section, the diameter FG shall act through B cutting the line AC at F, the tangent CD at the point H, through which with IK drawn parallel to AC, IC may be put which meets the line FG at L.
I say GF, HF, LF to be lines in continued proportion.



Demonstration.

As GF is to HF, thus DA is to IA: but as DA is to IA, thus HF is to LF; therefore as GF to HF thus HF to LF, therefore GF, HF, LF are the proportionals. Q.e.d.

PROPOSITION XCIII.

With the same in place the line DK is drawn crossing GF at M.
I say GM, GH, GF to be lines in continued proportion.

Demonstration.

Because EC is parallel to AD (on account of the parallelogram DC) and IKD, ICD triangles having the same base ID, the lines MH, HL are equal, but GF, HF, LF are lines in continued proportion; and therefore GM, GH, GF are lines in continued proportion. Q.e.d.

PROPOSITION XCIV.

With the same in place:
I say as CF shall be to FA, thus HB to BF.

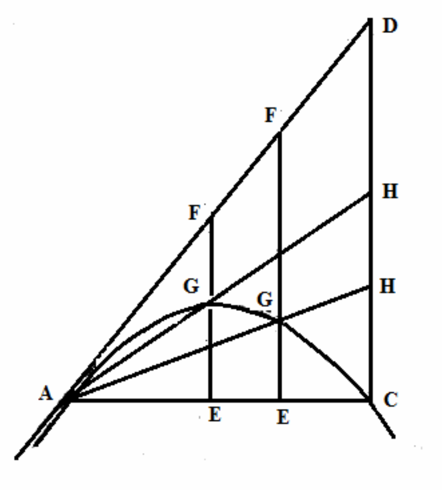
Demonstration.

The line CN is drawn from C through B crossing the line AD at N : and from A the right line AO is drawn parallel to DC, cutting EC produced at O, and the line FB at P. Because DC is a tangent and AO parallel to the same, the right lines HB, HF, HP are proportionals. But also LF, HF,GF are continued, and the line GF is equal to the line HP (on account of the parallelograms AE, AC upon the same base AD, and put in place between the same parallel lines), and therefore the line HB is equal to the right line LF: from which with the common line LB removed, HL remains of the right line BF, and NA equal to ID itself; and thus NC parallel to DK. Whereby as DN if to NA, that is CK to DI, thus HB is to BF; but as CK to DI, thus CH is to HD, that is CF to FA; therefore so that as CF is to FA, thus HB is to BF. Q.e.d.

This Archimedes has shown otherwise.

PROPOSITION XCV.

The line AC shall subtend the parabola ABC and with the tangent AD acting through A which will cross the diameter CD drawn through C at D, some diameters FE may be put in place cutting the parabola at G, and through G from A, the lines AH shall be drawn crossing the diameter CD at H.
I say the lines AC, CD to be divided proportionally at E and H.



Demonstration.

The demonstration is evident from the preceding ; for always, as AE shall be to EC, thus FG to GE, that is DH to HC.

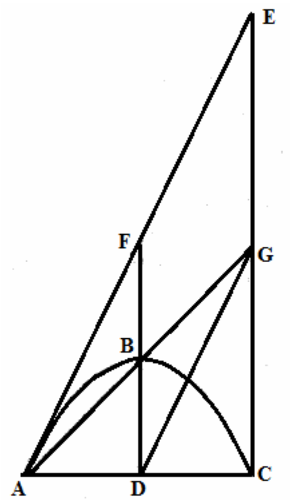
PROPOSITION XCVI.

The line AD shall be a tangent to the parabola ABC at A, on which with some points D and E taken ; the diameters DB, EC shall be dropped: and from A the right line AF shall be drawn through B crossing the diameter BC at F. I say EC to be to CF, as AF to FB.

Demonstration.

The right line AC is drawn crossing the line DB at G; therefore there will be AG to GC, as DB to BG [§.94], that is EF to FC. But as AG to GC, thus AB to BF; therefore so that as AB to BF, thus EF to FC: and on taking together so that as AF is to FB, thus EC is to CF. Q.e.d.

PROPOSITION XCVII.



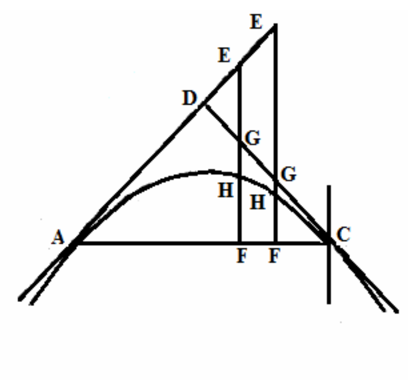
The right line AC shall subtend the parabola ABC, and with the diameter GE erected from C, some line AG meeting CE at G, the point B of the parabola, through which the diameter BD may be put in place.

I say the line joined DG to be parallel to the tangent drawn through A.

Demonstration.

For the tangent drawn through A shall meet the diameters BD, CE at F and E. Therefore [§.94] FB will be to BD, as AD to DC, that is AB to BG: whereby AE, DG are parallel lines. Q.e.d.

PROPOSITION XCVIII.



The line AC shall subtend the parabola ABC, and with the tangents through A and C, which shall meet at D, some diameter FE may be drawn, cutting the line AD at E, DC at G; and the parabola at H.

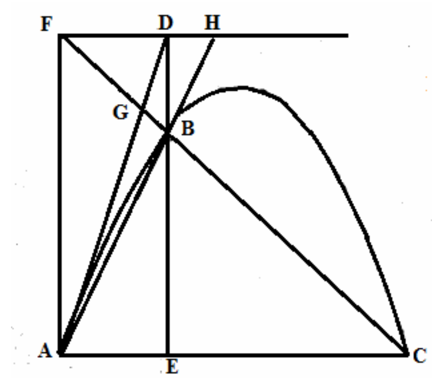
I say GH, HF, HE, to be lines in continued proportion

Demonstration.

Indeed as AF is to FC, thus EH is to HF [§.94], but also as AF to FC, thus FH to HG, therefore as HE to HF, thus HF to HG : therefore GH, HF, HE are proportionals. Q.e.d.

PROPOSITION XCIX.

The line AD shall be a tangent at A to the parabola ABC, subtended by the line AC, on which with some point taken a diameter DE shall be dropped crossing the diameter DE of the parabola at B, moreover through B the right line CF may be put in place from C, crossing the diameter erected from A at F.
I say the lines FA, DE are equal to each other.



Demonstration.

As AE to EC, thus DB is to BE; and on adding together so that as AC to CE, thus DE ad BE. But as AC to EC, thus FA is to BE; therefore so that as DE to BE, thus FA to BE: therefore the lines FA, DE are equal to each other. Q.e.d.

PROPOSITION XCX.

With the same in place:
I say BG to be to GF, as FB to FC.

Demonstration.

Since by the preceding FA, DE are equal, DE is to BE, as FA as BE; that is as AC to EC; and by inverting AE to AC, that is FB ad FC, as DB to DE; that is to FA. But as BD to FA, thus BG to GF, therefore BG to GF, as FB is to FC. Q.e.d.

PROPOSITION CI.

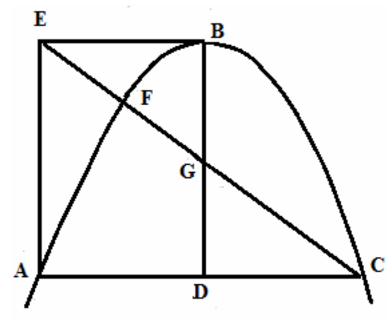
With the same in place AB crossing FD joined at H.
I say DH, DF, EC to be lines in continued proportion.

Demonstration.

For DH is to AE, that is FD, as DB to BE, but as DB to BE, thus FD to EC, therefore DH is to FD as FD to EC. Q.e.d.

PROPOSITION CII.

The line BE shall be a tangent to the parabola ABC at B, the diameter of which is BD, and with the diameter EA dropped, the ordinates ADC are drawn for BD, and EC may be put in place crossing the parabola at F and the diameter BD at G.
I say the line EF to be equal to the line FG.



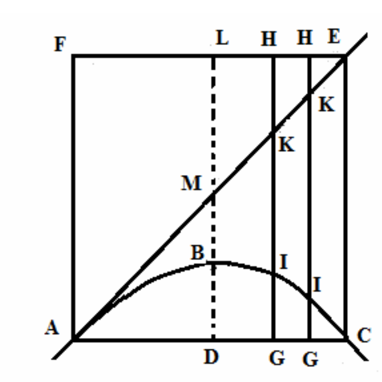
Demonstration.

Because EB is a tangent to the section, EF will be to EG, as EG to EC: and on dividing, EF will be to FG, as EG to GC, but EG is equal to GC, since AD, DC are equal, and therefore the right lines EF, FG are equal to each other. Q.e.d.

PROPOSITION CIII.

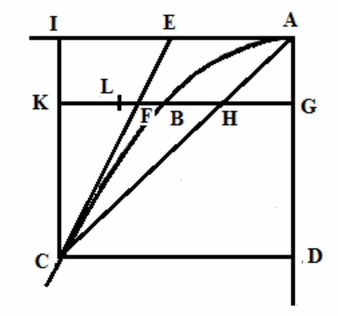
The line AE will be a tangent at A to the parabola ABC, of which the diameter is BD, and the ordinate for that is put to be AC, and with the diameter CE from C for that put in place, which shall meet the tangent AE at E, the parallelogram ACE may be completed ; some diameter HG may be drawn meeting the parabola at I, and the right line AE at K.
I say the rectangle HGI to be equal to the rectangle HKG.

Demonstration.



The diameter BD produced, will cut the lines EF, AE at L and M. Because the line AC is parallel to EF, and is equal to the same AD, which is half of the right line AC, is equal to LE, half of EF; and whereby LM is equal to MD, and thus LD is bisected at M : from which since LD shall not be bisected at B, the rectangle LMD, that is the square MD, is equal to the rectangle LBD, together with the square MB, that is, equal to the square BD, and this shall be equal to the rectangle LDB. Again since as BD shall be to IG, thus the rectangle LDB shall be to the rectangle HGI, likewise the rectangle ADC to the rectangle AGC, that is the rectangle AME to the rectangle AKE, therefore so that the rectangle LDB to the rectangle HGI, thus the rectangle AME is to the rectangle AKE; but so that as the rectangle AME to the rectangle AKE, thus the rectangle LMD to the rectangle HGI, and thus the rectangle LMD is to the rectangle HKG, and on interchanging so that the rectangle LDB is to the rectangle LMD, thus the rectangle HGI to the rectangle HKG; but the rectangles LDB, LMD have been shown to be equal, and therefore the rectangle HGI is equal to the rectangle HKG. Q.e.d.

PROPOSITION CIV.



The lines AE, CE meeting at E are tangents to the parabola ABC at A and C, of which the diameter is AD and the ordinate put in place for that CD ; and with AC drawn, the ordinate BG shall be put in place which will meet the lines EC, AC at F and H.

I say that half the square HB to be equal to the rectangle FB, HG.

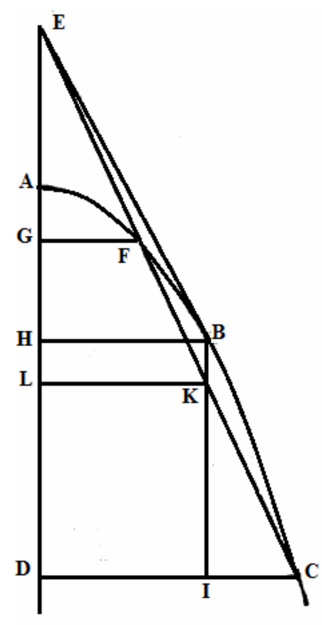
Demonstration.

The diameter CI shall be erected from C, crossing the line BG at K, and HB shall be made equal to KL. Therefore since KG, BG, HG shall be proportionals, and the difference BH may be put equal to LK, the right lines GH, HB, BL also are proportionals, and thus the square BH to be equal to the rectangle LB, HG. Again since AI at E and therefore HK shall be bisected at F, and the line HB shall be equal to KL, and the remaining LB will be bisected at F ; whereby the rectangle FB, HG is the half of the rectangle LB, HG, and thus equal to the half of the square HB. Q.e.d.

PROPOSITION CV.

The line BE shall be a tangent to the parabola ABC at B , of which the diameter is AD, meeting with the diameter at E; and with the line EC dropped from E, which cuts the parabola at F and C, the ordinate lines FG, BH, CD shall be drawn.

I say that FG, BH, CD to be lines in continued proportion.



Demonstration.

The diameter BI shall be dropped from B cutting the line EC at K: and KL shall be put parallel to HB.

Because the line BE is a tangent to the section, the lines AE, EK, EC and thus the lines FG, KL,CD will be continued proportions : but the line HB is equal to KL, therefore FG, BH, CD are in continued proportion. Q.e.d.

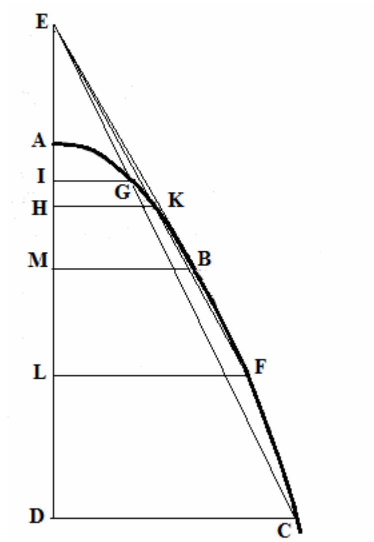
Corollary.

With the same figure remaining, some point E is taken on the diameter AD, outside the parabola; from which EFC shall be dropped cutting the parabola, and with the ordinates FG, CD drawn, the line AE shall become equal to AH, and from H, the ordinate HB shall be put in place: I say the lines FG, BH, CD to be proportionals; indeed BE joined will be a tangent; from which the true is agreed from the preceding.

PROPOSITION CVI.

AD shall be the diameter of ABC, on which with some point E assumed outside the section, from E some two lines EC, EF may be dropped cutting the parabola at G, K, C, F: and the ordinates GI, KH, FL, CD shall be put in place.

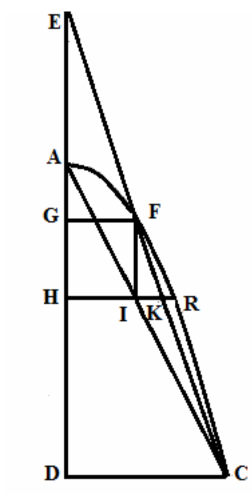
I say as GI shall be to KH, thus FL shall be to CD.



Demonstration.

The tangent EB shall be dropped from E, and from B the ordinate BM shall be drawn BM, therefore both the lines GI, BM, CD, as well as the lines HK, BM, FL shall be proportionals; whereby since the line BM shall be common to each series and the mean proportion between these same lines, the rectangle GICD is equal to the rectangle HKEI ; therefore so that as GI to KH, thus FL to CD. Q.e.d.

PROPOSITION CVII.



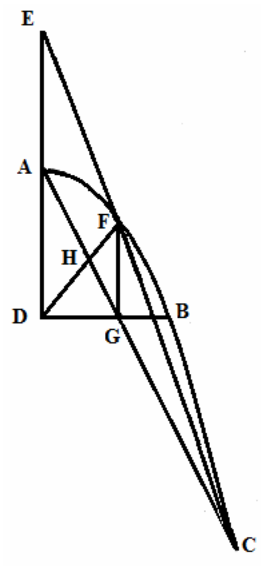
AD shall be the diameter of the parabola ABC, on which some point E assumed outside the section, from E the line EFC shall be dropped cutting the parabola, and with the ordinates FG, CD put in place, the line AE shall be made equal to AH, and from H the ordinate HB may be put in place crossing the lines AC, FC at I and K.

I say the line EC to be divided at F and K in the extreme and mean ratio of proportionality, that is CE to be to EF as CK to KF.

Demonstration.

FI shall be joined: Since the line AH is put equal to AE, the lines FG, BH, CD are proportionals, and the lines FG, IH are equal, and thus FI is parallel to parallel to AE; whereby so that as EC to EF, thus AC to AI, that is AD to AH, that is AH to AG, (Since AD, AH, AG shall be proportionals.) that is DH to HG, that is CK to KF. Q.e.d.

PROPOSITION CVIII.



The line BE shall be a tangent at B to the parabola ABC, of which the diameter is AD, meeting with the diameter at E, from which the right line EC shall be drawn, meeting the parabola at F and C, and with the ordinate BD, AC shall be drawn meeting the line BD at G, and the right line FD cutting AC at H.

I say the right line AC to be divided at the extreme and mean ratio of the proportional, H and G : that is AC, CG: and AH, HG to be proportionals.

Demonstration.

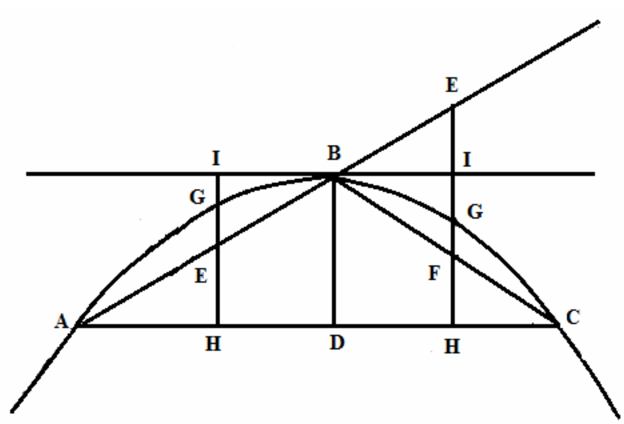
FG shall be joined : Since it has been shown in the preceding proposition, the line FG to be parallel to AE, so that as AC to CG, thus AE is to FG: but AE is equal to AD, therefore so that as AC to CG, thus AD is to FG, that is as AH is to HG. Q.e.d.

PROPOSITION CIX.

BD shall be the diameter for the parabola ABC, AC the ordinate put in place: and with AB, CB joined some diameter EF shall be drawn.

I say that as AD is to DH, thus HE is to EG.

Demonstration.



In the first place the line AB crossed the diameter EF at E outside the section ; the tangent IB shall be put in place through B: so that as AD shall be to AH, thus DB, that is IH, is to HE ; truly since IG, IF (that is IE) and IH are proportionals [§42]; so that HI to IE, or IF, first to second, thus HI with IF or IE, that is HE, first with second, to IE together with IG, that is EG, second to third, therefore so that AD to DH, thus HE to EG. Q.e.d.

Now AB will cross the diameter HG at E within the parabola, therefore so that as AH to AD, thus HE to BD, that is, to HI; and as AD to HE, thus HI to IE, but so that HI to IE shall be the first to the second, (since HI, IE, IG are proportionals) thus HE is to EG, therefore as AD to DH, thus HE is to EG.

This is Prop. 4 of Archimedes, *de quadrature Parabolam* demonstrated otherwise.

Corollary.

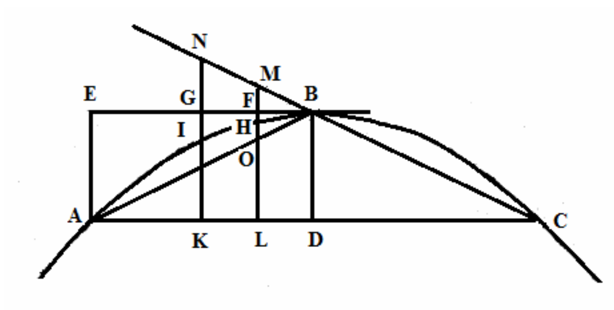
Hence it follows the line HI to IG, to be in the square ratio of that which HF has to IG: since indeed HI, FI, GI shall be in continued proportionals, the ratio HI to IG, will be the square of the ratio HI to FI, that is, HF to FG.

PROPOSITION CX.

The line BE is a tangent to the parabola ABC at B, of which the diameter is BD ; with which divided at F and G, so that BF, BG, BE shall be proportionals, the diameters FH, GI, EA shall be dropped, the ordinate line AC shall be drawn from A to BD, which cuts the right lines FH, GI at K and L, the line CM is acting through B from C, meeting the diameters FH, GI at M and N.

I say the ratio LM to MH to be the square of the ratio KN to NI.

Demonstration.



Because the lines BF, BG, BE are continued proportionals; also the diameters FH, GI, EA are in continued proportions. Whereby the ratio AE to FH, that is LF to FH, is the square of the ratio AE to GI, that is KG to GI; but LF to FH, has the square ratio LO to OH, (since LF, FO, FH shall be proportionals), that is by the preceding, LM to MH, and likewise KG to GI, has the square ratio of that which KN has to NI, and therefore the ratio LM to MH, is the square of the ratio KN ad NI, and therefore the ratio LM to MH, is the square of the ratio KN to NI. Q.e.d.

PROPOSITION CXI.

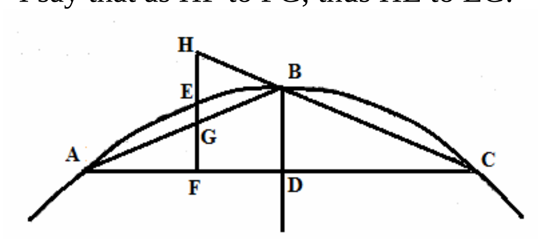
With the same in place the line AB shall be drawn cutting the line FL at O.
I say the rectangle MHLF to be equal to the rectangle MLFO.

Demonstration.

Because both the ratio LF to FH is the square both of the ratio LM to MH, as well as FL to FO, so that there shall be LM to MH, thus FL to FO: whereby the rectangle MHLF is equal to the rectangle MLFO.
Q.e.d.

PROPOSITION CXII.

BD shall be the diameter of the parabola ABC for the ordinate line AC put in place, and with the points AB, CB joined, some diameter EF shall be drawn crossing the lines AB, CB at G and H.
I say that as HF to FG, thus HE to EG.



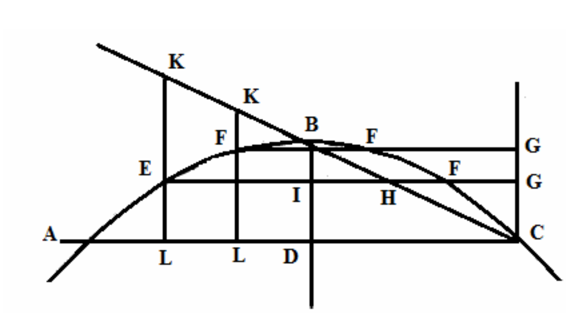
Demonstration.

As AD to DF, that is DC to DF, thus FH is to HE, but also as AD to DF, thus FG is to GE, therefore as FH to HE, thus FG is to GE, and on interchanging, so that as HF to FG, thus HE to EG. Q.e.d.

PROPOSITION CXIII.

BD shall be the diameter of the parabola ABC with the ordinate AC put in place, and with the points BC joined the ordinate EF crossing the diameters CG, BD at G and I, and the right line CB at H.
I say the rectangle GIEH to be equal to the rectangle GEIF.

Demonstration.



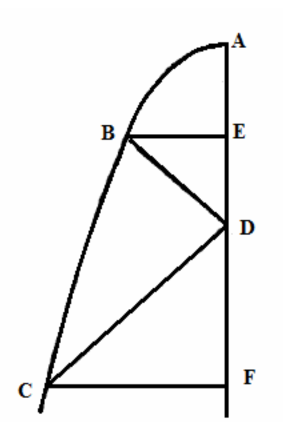
The diameters KL are acting through E so that as AD to DL, thus LK is to KE, but as AD to DL, thus CD is to DL, that is, GI to IE; therefore GI is to IE, as LK to KE; moreover so that LK as to KE, thus LC is to EH, that is GE to EH ; therefore so that as GI to IE, thus GE to EH, from which the rectangle GI, EH is equal to the rectangle GE,IE, that is, GE,IF. Q.e.d.

PARABOLA

PARS SECUNDA

Linearum in parabola tam continuam, quam discretam contemplatur proportionem.

PROPOSITIO XXXII.



Sit ABC parabolam axis AD, in quo assumpto quovis puncto D, educantur ex D ad peripheriam lineae DB, CD constituentes angulos ADB, CDF aequales ; tum rectae ponantur BE, CF ordinatim ad axem AF.
Dico AE, AD, AF, in continua esse analogia & contra.

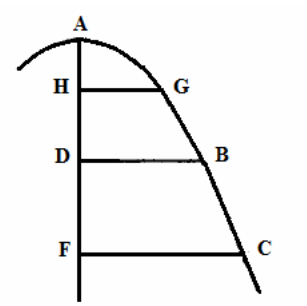
Demonstratio.

Quoniam EB, CF ordinatim positae sunt ad axem, anguli BED, CFD recti sunt : aequales autem ponuntur anguli BDE, CDF; similia igitur sunt triangula BED, CFD: & ut EB ad FC, sic ED ad DF: rationis autem EB ad CF, duplicata est ratio AE ad AF, ergo & duplicata est rationis ED ad DF*:* unde AE, AD, AF, continuae sunt proportionales. Ut facile deducitur ex prima de progress. Geometricis.
Sint iam proportionales AE, AD, AF, positisque ordinatim BE, CF: iungantur BD, CD : dico angulos BDE, CDF aequati, cum proportionales sint AE, AD, AF, ratio AE ad A F duplicata est rationis AD ad AF id est: ED ad DF , sed etiam ratio AE ad AF, duplicata est rationis BE ad CF, igitur Ut Ed ad DF, sic BE ad CF, & ut DE ad BE, sic DF ad FC: unde cum anguli proportionalibus lateribus contenti recti sint, similia sunt triangula BED, CFD, & anguli BDE, CDF aequales. Q.e.d.

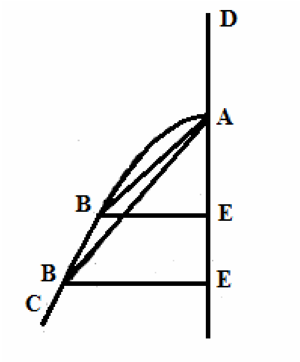
PROPOSITIO XXXIII.

Sit ABC parabolam diameter AD divisa in H, DF ut AH, AD, AF continuae sint proportionales: ponantur autem ordinatim lineae HG, DB, FC.
Dico illas in continua esse analogia.

Demonstratio.



Patet; cum AH, AD, A F sint continuae proportionales & duplicatam rationem habeant linearum GH, BD, CF.



PROPOSITIO XXXIV.

Esto ABC parabolam axis AD aequalis lateri recto, ductaque ordinatim linea quacunque EB, iungantur AB.

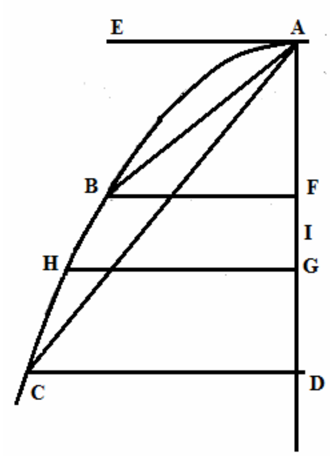
Dico AE, AB, ED lineas esse proportionales.

Demonstratio.

Quadratum AB aequales est quadratis AE, EB: sed EB quadratum aequatur rectangulo EAD; igitur quadratum aequale est rectangulo EAD una cum quadrato AE, id est rectangulo AED. Quare AE, AB, ED lineae sunt proportionales . Q.e.d.

PROPOSITIO XXXV.

Parabolam ABC cuius diameter AD contingat in A linea AE, demissisque ;ex A lineis AB, AC quae parabolam occurrant in B & C ut anguli EAB, DAC aequales sint ductisque ordinatim BF, CD, inveniatur AG, media inter AF & AD.



Dico AG aequari lateri recto; & si anguli EAB, DAC aequales fuerint, & AG linea aequalis lateri recto, dico AF, AG, AD esse proportionales & si AF, AG, AD fuerint continuae, & AG media aequalis lateri recto: dico angulos EAB, DAC esse inter se aequales.

Demonstratio.

Ducatur ex G ordinatim linea GH. Quoniam angulus DAC ex hypothesi aequalis est angulo EAB, addito vel dempto communi angulo BAC, angulus BAD aequalis est angulo EAC, id est angulo ACD; (ob AE, CD parallelas) aequales autem sunt & anguli BFA, CDA, triangula igitur ABF, ACD inter se similia sunt & ut BF ad FA, sic AD ad DC, unde FAD rectangulo, aequale rectangulum BF, CD; est autem FAD rectangulo ex hypothesi aequale quadratum AG, & quadratum HG aequale rectangulo BF, CD, cum BF, AG, CD linae proportionales sint per pennultimam; igitur quadratum AG, aequale est quadrato HG, & AG linea aequalis lineae HG; adeoque & lateri recto. Quod erit primum.

Sit iam AG linea aequalis lateri recto, & anguli EAB, DAC aequales, dico AF, AG, AD in continua esse analogia. Si enim non sint proportionalles, inveniatur inter AF & AD media AI: erit igitur per primam partem huius linea AI, aequalis lateri recto, adeoque & AG linea, quare AF, AG, AD in continua sunt analogia, nec quaevis alia media inter AF & AD, praeter AG. Quod erat secundum.

Rursum sit AG aequalis lateri recto, & AF, AG, AD proportionales, dico angulos EAB, DAC esse inter se aequales, quoniam enim AF, AG, AD continuae sunt proportionales, FAD rectangulum aequale est quadrato AG, id est quadrato HG: sed & HG quadrato aequale est rectangulum BF, CD; igitur & rectangulo FAD aequatur rectangulo BFCD: unde ut AF ad HF, sic CD ad AD ; aequales autem sunt anguli AFB, ADC: lateribus proportionalis contenti, igitur triangula AFB, ADC inter se similia sunt, & angulus BAF aequalis angulo ACD, id est angulo EAC: dempto ergo communi angulo BAC, manet angulus EAB aequalis angulo CAD. Q.e.d.

Corollarium.

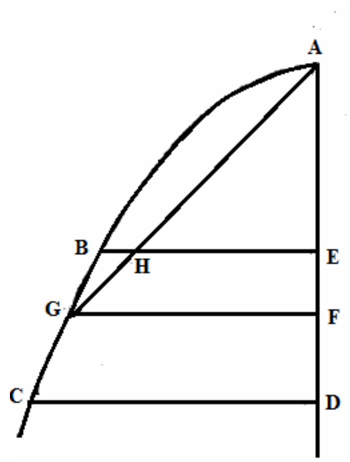
Iisdem positis sequitur triangula BAF, CAD esse inter se similia patet per primam partem praecedentis propositionis.

PROPOSITIO XXXVI.

Sit ABC parabolam diameter AD divisa in E & F ut AE, AF, AD sint proportionales, & AF media aequalis lateri recto: positis autem ordinatim lineis EB, FG, DC, ex A ad G, ducatur recta AG occurrens EB lineae in H.
Dico AD, DC, FG, EB, EH lineas continue esse proportionales.

Demonstratio.

Quoniam AF aequalis est lateri recto, GF, FA lineae aequales sunt: ratio igitur AF ad AD, id est GF ad AD, duplicata est rationis GF ad CD: proportionales igitur sunt AD, CD, G F. quia vero AD, AF, AE sunt continuae, etiam CD, GF, BF proportionales sunt; continuant igitur eandem rationem AD, CD, GF, BE; deinde cum ratio AF ad AE, id est GF ad HE, duplicata sit rationis GF ad BE, proportionales quoque sunt GF, BE, HE; continuae igitur sunt in eadem ratione AD, CD, GF, BE, HE: Q.e.d.



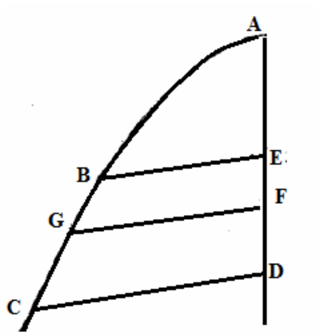
Corollarium.

Hinc sequitur AE, FB, FG, CD quoque in continua esse analogia: ratio enim AF ad AE, id est GF ad BE, duplicata est rationis GF ad BE, proportionales igitur sunt AE, BE. GF: sed ut BE ad GF, sic GF est ad CD, cum AE, A F, AD sint continuae; proportionales igitur sunt AE, EB, GF, CD.

PROPOSITIO XXXVII.

Esto ABC parabolam diameter AD divisa in E & F, ut AE, AF, AD lineae sint proportionales, & AD extrema aequalis lateri recto; ducantur ordinatum linea EB, FG, DC.
Dico AE ad EB, duplicatam habere rationem, quam habet AF ad FG.

Demonstratio.



Quoniam AE, AF, AD lineae ponuntur continuae, proportionales quoque erunt EB, FG, DC: ponitur autem AD prima seriei AE, AF, AD aequalis lateri recto, adeoque ipsi DC primae seriei EB, FG, DC igitur AE linea ad lineam EB tertia ad tertiam duplicatam habet rationem eius quam habet AF ad FG, secunda ad secundam. Q.e.d.

Corollarium.

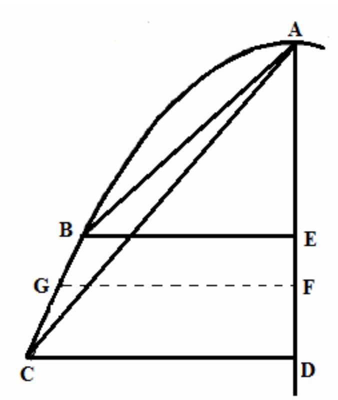
Hinc sequitur rectangulum EAF ad rectangulum EBFG rationem habere triplicaram eius quam habet AF linea ad lineam FG. Est enim ratio rectanguli EAF ad rectangulum EBFG composita ex AE ad EB, & AF ad FG, sed ratio AE ad EB, duplicata est rationis AF ad FG, igitur rectangulum EAF ad rectangulum EBFG triplicatam habet rationem eius quam habet AF linea ad lineam FG.

PROPOSITIO XXXVIII.

Sit ABC parabolam diamcter AD divisa in E & F, ut AE, AF, AD lineae sint proportionales, & AF media aequalis lateri recto, ponantur autem ordinatim linea EB, DC: iunganturque AB, AC.

Dico AB ad AC, rationem habere triplicatam eius, cuius EB ad DC est duplicata.

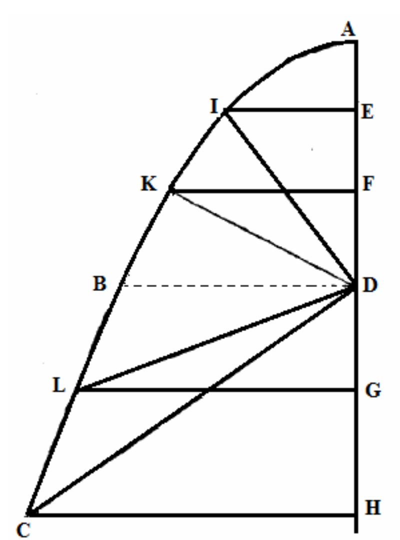
Demonstratio.



Ducatur ordinatim linea FG. Quoniam AE, AF,AD lineae proportionales sunt, & AF media aequalis lateri recto, triangula AEB, ADC similia sunt, adeoque ut AB ad AE, sic AC ad CD: & invertendo permutando ut AE ad CD, sic AB ad AC; iam vero cum AF aequalis facta sit lateri recto, adeoque & FG eidem aequalis AE, EB, FG, DC erunt continuae; ratio igitur AE ad CD , primae ad quartam triplicata est rationis BE ad FG, secundae ad tertiam, igitur & AB ad AC, triplicatam habet rationis EB ad FG, cuius EB ad DC, habet duplicatam. Q.e.d.

PROPOSITIO XXXIX.

Axis parabolam ABC divisus sit in continue proportionales; & AD quidem media existat inter AE, AF, AG, AH. ductisque ordinatim ad axem rectis EI, FK, DB, GI, HC, ponantur quoque DI, DK, DL, DC.



Dico rationem DI ad DC duplicatam eius esse quam habet KD ad DL.

Demonstratio.

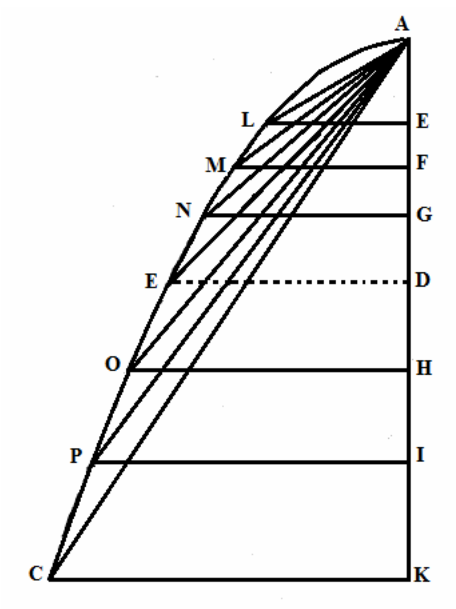
Quoniam eandem continuant rationem AE, AF, AD , AG, AH, proportionales quoque EI, FK, DB, GL, HC, quia vero ratio AE ad AH, duplicata est tam rationis EI ad CH, quam rationis AD ad AH, id est ED ad DH, cum AE, AD, AH, proportionales sint, ratio ED ad DH, eadem est cum ratione EI ad CH: similia igitur sunt triangula IED, CHD : quare ID ad DC, ut ED ad DH, id est AE ad AD, hoc est in duplicata rationis IE ad BD. Similiter ostendentur triangula FDK, GLD similia, & KD esse ad LD, ut FD ad GD, id est ut AF ad AD, id est in duplicata rationis KF ad BD: sed ratio ID ad BD, duplicata est rationis EK ad BD, cum LE, KE, BD proportionales sint, ratio igitur ID ad CD, duplicata est eius quam habet DK ad LD. Q.e.d.

Corollarium.

Si iam AD fuerit media inter septem continue proportionales quarum extremae AE, AH, similiter ostendetur ID ad DC, nimirum duas extremas triplicatam habere rationem eius quam habent KD ad DL, duae quoque extremae: & sic de reliquis & accrescit semper proportio.

PROPOSITIO XXXL.

Esto ABC parabolam axis AK divisus ut AE, AF, AG, AD, AH, AI, AK sint continuae proportionales & AD media totius seriei aequalis lateri recto ductisque; ordinatim lineis EL, FM, GN, DB, HO, IP, KC, ungantur AL, AM, AN, AB, AO, AP, AC.
Dico rationem AM ad AP duplicatam esse rationis AN ad AO & AL ad AC rationem triplicatam eius quam habet AN ad AO, atque ita deinceps in infinitum procedendo invenietur augmentum unius rationis.

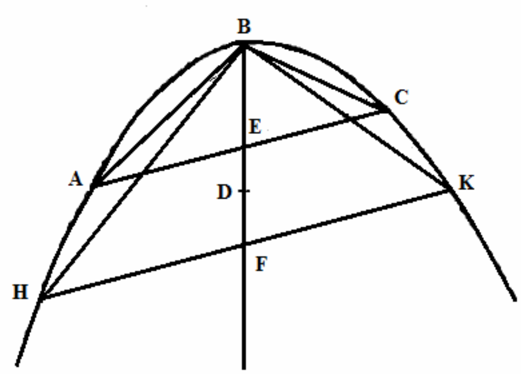


Demonstratio.

Quoniam AE, AF, AG, AD , &c. sunt continuae proportionales & totius seriei media ponitur AD; erunt quoque AF, AD, AI continuae proportionales, nimirum secunda, quarta, & sexta, unde cum AD media aequalis ponatur lateri recto, erit ratio AM ad AP, triplicata eius cuius duplicaram habet MF ad PI: est autem ratio MP ad PI, duplica rationis MF ad BD, cum MF, BD, PI sint proportionales; igitur

ratio AM ad AP, triplicata est rationis MF ad BD, id est sextuplicata eius quam habet NG ad BD, quia MF, NG, BD sunt continuae, eodem modo cum AG, AD, AH sint proportionales,ostenditur rationem AN ad AO, triplicatam esse eius cuius duplicatam habet NG ad OH, id est triplicatam eius
quam habet NG ad BD, unde cum AM ad AP, ostensa ut rationem habere sextuplicatam ipius NG ad BD, patet rationem AM ad AP, duplicatam esse rationis AN ad AO; eadem prorsus methodo ostenditur , rationem AL ad AC, triplicatam esse rationis AN ad AO, & sic de ceteris. Quod fuit demonstrandum.

PROPOSITIO XLI.



Sit ABC parabolam diameter BD, divisa in E & F, ut BE, BD, BF sint proportionales, & BD media aequalis lateri recto, ponanturque per E & F ordinatim lineae AC, HK:
Dico iunctas AB, HB ipsis CB, KB esse proportionales.

Demonstratio.

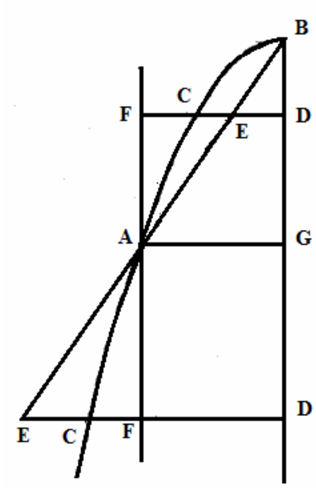
Quoniam BE, BD, BF continuae sunt proportionales, & BD media aequalis lateri recto, ratio AB ad HB triplicata est eius, cuius duplicatam habet AE ad HF: sed eadem de causa quoque ratio BC ad BK, triplicata est eius cuius duplicatam habet EC ad FK, id est AE ad HF; igitur ut AB ad HB, sic CB ad KB: quod erat demonstrandum .

PROPOSITIO XLII.

Secent ABC parabolam diametri duae quaevis AF, BD, iunctisque; A, B diametrorum terminis, ponatur ad diametrum BD ordinatim linea CD, occurrens AB, AF lineis in E & F.
Dico FD, CD, ED in continua esse analogia.

Demonstratio.

Ponatur ex A ordinatim AG ; ut BD ad BG, sic DE est ad AG, id est ad DF: sed BD ad BG, duplicatam habet rationem eius quam habet DC ad AG, id est DF, ratio igitur DE ad DF, quoque duplicata est rationis : DC ad DF: quare DE, DC, DF lineae sunt in continua analogia. Q.e.d.



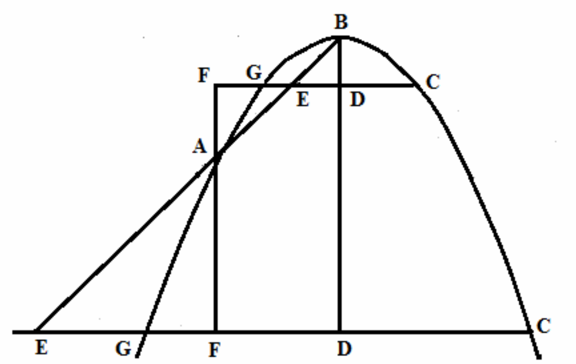
Corollarium.

Iisdem positis : sequitur rectangulum FCD aequale esse rectangulo FD, CE, cum enim FD, CD proportionales sint, ut FD ad CD , sic FC est ad CE : rectangulum igitur FDCE aequale est rectangulo FCD. Quod erat propositum.

PROPOSITIO XLIII.

Iisdem politis quae suprae:
Dico CFG rectangulum aequari rectangulo DFE.

Demonstratio.



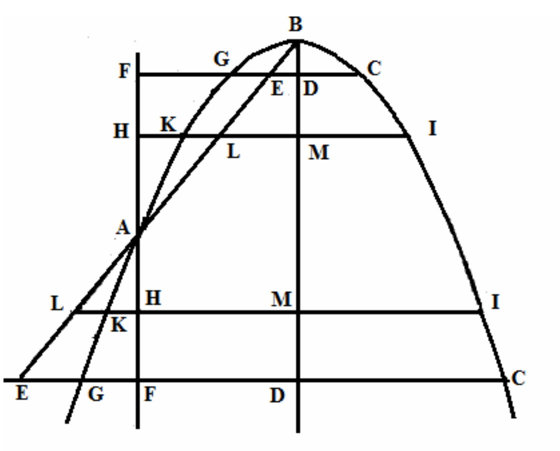
Primo cadat F punctum extra parabolam. Quoniam igitur recta GC in D bisecta est & ei in directum adiecta quaedam GF; rectangulum GFC una cum quadradrato GD, aequale est quadrato FD: sed FD quadrato aequale rectangulum EFD, una cum rectangulum EFD, id est una cum quadrato GD per praecedente propos. Igitur & rectangulo GFC una cum quadrato GD, aequale est rectangulum EFD una cum quadrato GD: dempto igitur communi quadrato GD, manet GFC rectangulo, aequale rectangulum EFD.

Secundo cadat F punctum intra parabolam. Quoniam GC linea in D secta est bifariam & non bifariam in F, rectangulum GFC una cum quadrati FD, aequale est quadrato GD: sed GD quadrato aequale quoque est rectangulum FDE, una cum quadrato FD; rectangulum igitur GFC una cum quadrato FD, manet GFC rectangulum aequale est rectangulo EFD, una cum quadrato FD: quare dempro communi quadrato FD, manet GFC rectangulum aequale rectangulo EFD. Q.e.d.

PROPOSITIO XLIV.

Sint iterum in parabola ABC diametri dua quaevis AF, BD: & ad BD quidem ordinatim ponantur lineae GC, KI occurrentes diametro FA in F & H .

Dico esse ut AH ad AF, sic HK rectangulum ad rectangulum CFG.



Demonstratio.

Ducta AB linea secet FC, HI rectas in E & L, erit igitur LHM rectangulum aequale rectangulo KHI, & EFD rectangulum aequale rectangulo GFC; unde KHI rectangulum est ad rectangulum GFC, ut LHM rectangulum ad rectangulum EFD : sed LHM rectangulum est ad rectangulum EFD, ut LH linea ad lineam EF, id est ut AH ad AF, igitur & KHI rectangulum, ad rectangulum GFC est ut AH linea ad lineam AF. Q.e.d.

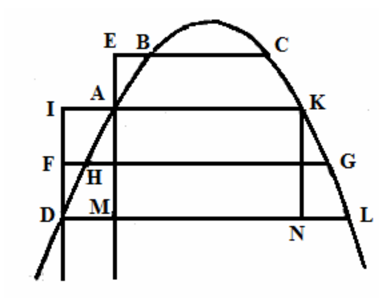
Corollarium.

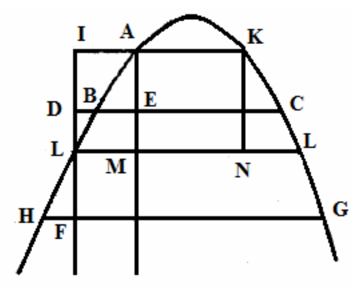
Hinc sequitur, quoque esse ut AH ad AH; sic IHK rectangulum ad rectangulum IHK, est enim ut AH ad AH, sic HL ad HL, id est LHM rectangulum ad rectangulum LHM; sed rectangulis LHM aequalia ostensa sunt rectangula IHK : igitur ut AH ad AH, sic IHK rectangulum est ad rectangulum IHK.

PROPOSITIO XLV.

Parabolam ABC secent in A & D, diametri duae aequales AE, DF: & ex E & F quaevis ponantur parallelae EC, FG occurrentes parabolam in B, C, H, G punctis.
Dico rectangula BEC, HFG esse inter se aequalia.

Demonstratio.





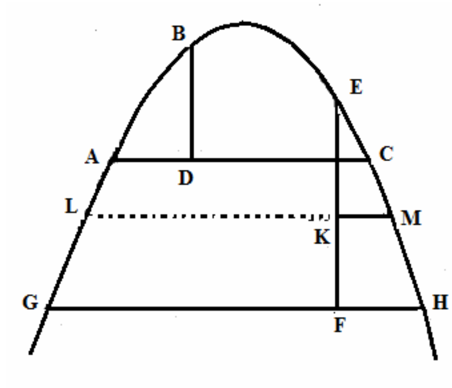
Ponantur ex D & A lineae DL, AK parallelae FG; & AK quidem occurrat DF diametro in I, & DL demissae ex K diametro in N : recta vero AE product secet DL in M, ut DF ad DI, sic HFG rectangulum est ad rectangulum AIK, & ut AE ad AM, sic BEC est ad rectangulum DML: igitur cum EA, DF & DI, MA lineae aequales sint, rectangulum HFG ad IAK, rectangulum est ut BEC rectangulum ad rectangulum DML; & permutando HFG rectangulum BEC, ut DML rectangulum est ad rectangulum IAK; sed DML rectangulum, id est MDN, ob DM, NL aequales lineas, aequatur rectangulo IAK; rectangulum igitur HFG, rectangulo BEC aequale est. Q.e.d.

PROPOSITIO XLVI.

Parabolam ABC secent duae quaevis diametri BD, EF, quas in D & F secent quaecunque parallelae AC, GH.

Dico esse ut BD ad EF, sic ADC rectangulum ad rectangulum GFH.

Demonstratio.



Secetur EF linea vel producatur in K, ut EK sit aequalis BD, & per K ponatur LM parallela GH, erit igitur ut EK ad EF, sic LKM rectangulum ad rectangulum GFH; quia vero EK aequatur ipsi BD, erit LKM rectangulum aequale rectangulo AD, igitur ut BD ad EF, sic ADC rectangulum ad rectangulum GFH. Q.e.d.

PROPOSITIO XLVII.

Sint iterum in ABC parabola duae quaevis diametri BD, EF, quas in F & D secet recta quaevis AC.

Dico ADC rectangulum esse ad rectangulum AFC, ut BD ad EF.

Demonstratio.

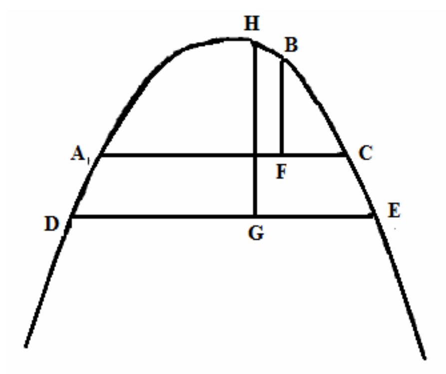
Fiat EF aequalis BG; & per G ponatur IH aequidistans AC, erit igitur ut BG ad BD , sic IGH rectangulum ad rectangulum ADC : sed IGH rectangulum aequale est rectangulo AFC, cum BG, EF lineae sint aequales, igitur ut EF ad BD, sic AFC rectangulum ad rectangulum ADC. Q.e.d.

PROPOSITIO XLVIII.

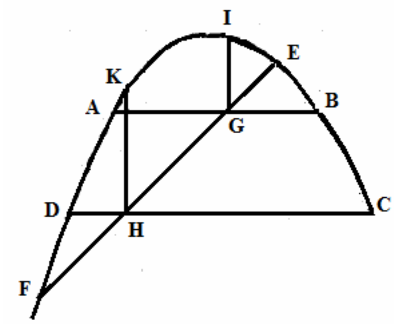
Parabolam ABC subtendant duae quaevis parallelei AC, DE, quibus proportionaliter in F & G, divisis ponantur diametri BF, HG.

Dico BF ad HG duplicatam habere rationem eius quam habet AC ad DE.

Demonstratio.



Quoniam AC, DE lineae in F , & G proportionaliter sunt divisae, erit AFC rectangulum ad DGE rectangulum in duplicata ratione AF ad DG ut AFC, id est AC ad DE, sed BF est ad HG, sed BF est ad HG ut AFC rectangulum DGE; igitur BF ad HG, duplicatam habet rationem eius quam habet AC ad DG, id est AC ad DE, sed BF est ad HG ut AFC rectangulum ad rectangulum DGE; igitur BF ad HG, duplicatam habet rationem eius quam habet AD ad DE. Q.e.d.



PROPOSITIO XLIX.

Secent ABC parabolam duae quaevis parallelae AB, CD, quas utcunque in G & H, dividat linea E F.
Dico esse ut AGB rectangulum ad rectangulum DHC sic FGE rectangulum ad rectangulum FHE.

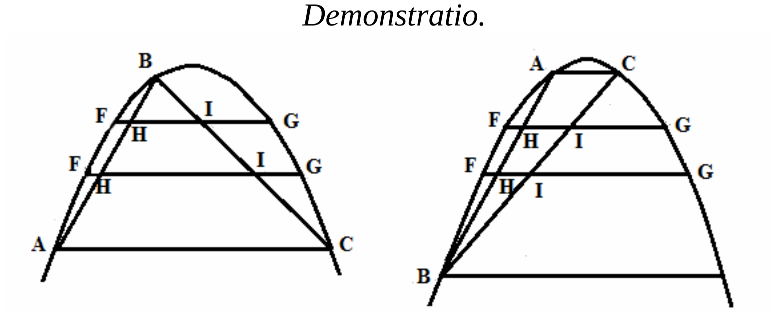
Demonstratio.

Erigantur ex H & G diametri HK;GI. Erit igitur ut GI ad HK; sic AGB rectangulum ad rectangulum DHC: sed ut GI ad HK; sic FGE rectangulum quoque ad rectangulum FHE; igitur ut AGB rectangulum, ad rectingulum DHC, sic FGE rectangulum est ad rectangulum FHE. Q.e.d.

PROPOSITIO L.

Esto ABC parabolam inscriptum triangulum ABC cuius duo latera AB, CB, duae quaevis secent FG aequidistantes AC in H & I.
Dico esse ut FHG rectangulum ad rectangulum FHG sic FIG rectangulum ad rectangulum FIG.

Demonstratio.



Rectangulum FHG est ad rectangulum FHG, ut AHB rectangulum est ad rectangulum AHB: & FIG rectangulum est ad rectangulum FIG, ut CIB rectangulum est ad rectangulum CIB : est autem ut AHB rectangulum ad rectangulum AHB, sic CIB rectangulum ad rectangulum CIB quia ex iisdem rationem habeant compositam, igitur ut FHG rectangulum ad rectangulum FHG, sic FIG rectangulum est ad rectangulum FIG. Q.e.d.

PROPOSITIO LI.

Secet AB parabolam recta quaevis AB in A & B, quam divisam utcunque in C & D dividatur in E & F, ut AC, AD lineis sint aequales BF, BE singulae fingulis; dein per CD, EF puncta parallelae ducantur GH, IK, LM, NO.
Dico esse ut GCH ad IDK, rectangulum sic OFN ad MEL rectangulum.

Demonstratio.

Est enim per quadragesimam nonam huius rectangulum GCH ad IDK, ut ACB ad ADB, hoc est BFA ad BEA, quia AC, AD aequantur FB, EB, sed ut BFA ad BEA, sic est NFO ad LEM: igitur GCN ad IDK, rectangulum, eandem obtinet rationem quam NFO rectangulum ad LEM*:* quod fuit demonstrandum, nec mirum, cum GCH rectangulum ipsi NFO, & rectangulo IDK aequetur LEM rectangulum.
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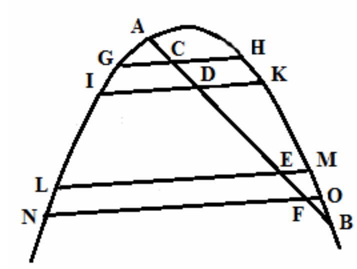
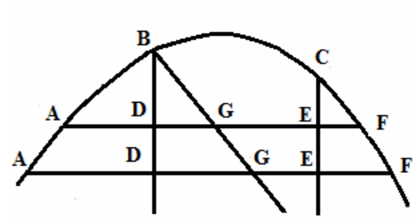


PROPOSITIO LII.

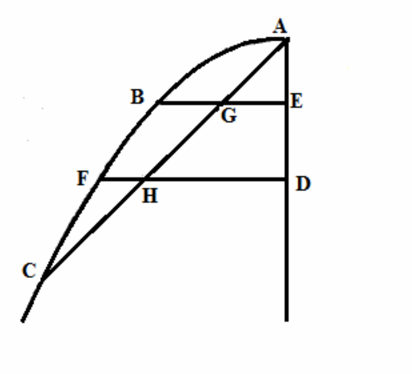
Secent ABC parabolam duae quaevis diametri BD, CE, quas in D & E, dividant utcunque parallelae duae AF: dein ex B, linea ducatur quaevis BG secans AF lineas in GG.
Dico esse GDE rectangulum ad rectangulum GDE, ut ADF rectangulum est ad rectangulum ADF.

Demonstratio.

Quoniam DE lineae per hypothesim aequidistant, & BD, CE sunt diametri, rectae DE Inter se aequales sunt ob ED parallelogrammum, quare GDE rectangulum ad rectangulum GDE est ut GD ad GD , id est BD ad BD. sic BD ad BD, sic ADF rectangulum est ad rectangulum ADF: igitur & GDE rectangulum, est ad rectangulum GDE, ut ADF rectangulum est ad rectangulum ADF. Q.e.d.



PROPOSITIO LIII.



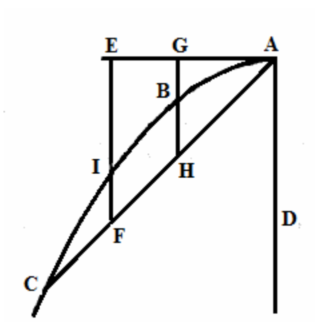
Esto ABC parabola diameter AD, quam in E & D, secent ordinatim lineae BE, FD, ducaturque ex A linea AC, secans BE, FD ordinatim positas ut cunque in G & H.

Dico BEG rectangulum ad rectangulum FDH triplicatam habere rationem eius, quam habet BE ad F D.

Demonstratio.

Rectangulum BEG ad rectangulum FDH rationem habet compositam ex BE ad FD, &; GE ad HD, id est AE ad AD: sed ratio AE ad AD duplicata est rationis BE ad FD: rectangulum igitur BEG ad rectangulum FDH triplicatam habet rationem eius quam habet BE ad FD. Q.e.d.

PROPOSITIO LIV.



Parabolam ABC cuius diameter AD contingat in A linea AE; ductaque quavis AC quae parabolam iterum occurrat in C, sumantur in AC linea puncta quaecunque F, H, ex quibus erigantur diametri FE, HG, occurrentes AE, contingenti in E & G, parabolam vero in B, & I.

Dico BGH rectangulum ad rectangulum IEF rationem habere triplicatam eius quam habet GH ad EF.

Demonstratio.

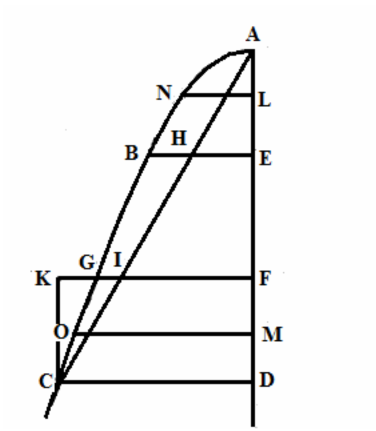
Rectangulum BGH ad rectangulum IEF, rationem habet compositam ex BG ad IE, & ex GH ad EF, id est AG ad AE: sed ratio BG ad IE duplicata est rationis AG ad AE, igitur rectangulum BGH ad rectangulum IEF rationem habet triplicatam eius quam habet GH ad EF. Q.e.d.

PROPOSITIO LV.

Esto ABC parabolam diameter AD, quam in D secet ordinatim linea CD : divisque AD in E & F, ut AE, DF sint aequales , ponantur A ordinatim EB, FG.

Dico EB, FG quadrata simul sumpta, aequari quadrato CD.

Demonstratio.



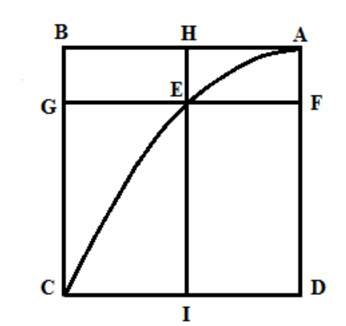
Ducta AC occurrat EB lineae in H, & FG in I, erigaturque ex C diameter CK , secans FG in K. Quoniam AE per hypothesim aequalis est FD, est CK, angulus AHE aequalis angulo AIF id est angulo KIC (ob HE, GF aequidistantes) & angulus AEH angulo aequalis CKI, erit AHE triangulum aequale triangulo CKI; & HE lineae aequalis KI. Rursum cum tam CD, GF, IF; quam CD, BE, HE lineae proportionales sint, quadrata FG, BE mediarum, aequalia sunt rectangulis CDIF, CDHE; hoc est rectangulis IFK, IKF, quia HE, KI lineae aequales sint, sed FK quadratum aequales est rectangulis IFK, IKF, igitur & quadrata FG, BE simul sumpta aequalia sunt quadrato FK, id est quadrato CD.

Q.e.d.

Corollarium.

Hinc sequitur: si rursum AD dividatur in L & M, ut AL, DM lineae aequales & ordinatim ponantur LN , MO , quadrata LN, MO simul sumpta aequari quadratis EB, FG simul sumptis: patet ex demonstratis, quia tam EB,FG quadrata, quam LN, MO simul sumpta aequalia sunt quadrato CD.

PROPOSITIO LVI.



Parabolam ABC cuius diameter AD & ordinatim ad illam posita CD contingat in A linea AE, quam in E secet diameter CE: assumptoque in sectione puncto quovis B, ponatur per B ordinatim linea FG occurrens AD lineae in F, & EC in G: dein per B ducatur diameter HI secans ACD ordinatim positam in I.

Dico parallogramma AB, AG, AI, AC in continua esse analogia.

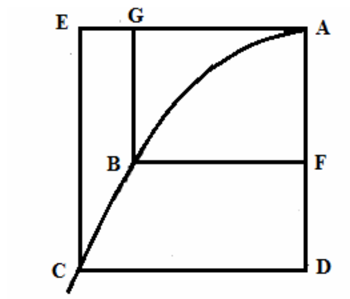
Demonstratio.

Quoniam AE, FG lineae aequidistant, parallelogrammum AB ad parallelogrammum

AG, est ut FB linea ad lineam FG, id est CD: est autem parallelogrammum AB ad parallelogrammum AI, ut AF linea ad lineam AD, hoc est in duplicata ratione FB ad DC: igitur parallelogrammum AB, ad parallelogrammum AI, duplicatam habet rationem eius, quam habet AB parallelogrammum ad parallelogrammum AG : parallelogramma igitur AB, AG, AI in continua sunt analogia: Rursum parallelogrammum AI est ad parallelogramum AC, ut DI linea ad lineam DC, hoc est ut AB parallelogrammum ad parallelogrammum AG. Parallelogramma igitur AB, A G, AI, AC sunt in continua proportione. Q.e.d.

PROPOSITIO LVII.

Parabolam ABC, cuius diameter AD, contingat in A linea AE; ductisque ordinatim FB, DC, erigantur ex C & B, diametri EC, BG occurrentes contingenti in G & E.
Dico AGB parallelogrammum ad parallelogrammum AEC triplicatam habere rationem eius quam habet AG linea ad lineam AE.

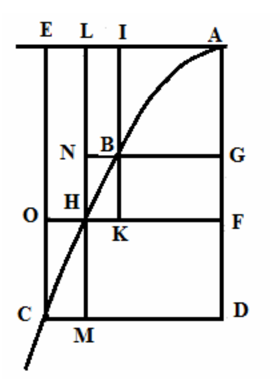


Demonstratio.

Ratio parallelogrammi AGB ad AEC parallalogrammum composita est, ex ratione EC ad AE, & GB ad EC ; sed ratio GB ad EC, id est AF ad AD, duplicata est rationis.
AG ad AE; id est FB ad DC, parallelogrammum igitur AGB ad parallelogrammum AEC triplicatam habet rationem eius quam habet AG linea ad lineam AE. Q.e.d.

PROPOSITIO LVIII.

Parabolam ABC cuius diameter AD, & ordinatim ad illam posita CD, contingat in A linea AE, quam in E secet diameter CE, factisque AD, AF, AG continue proportionalibus, ducatur ordinatim lineae FH, GB: & per B & H, diametri agantur IK, LH occurrentes FH, DC lineis in K & M: secet autem GB linea diametrum LH in N, & FH linea diametrum EC in O.
Dico HD parallelogrammum esse ad parallelogrammum FB, ut HE parallelogrammum est ad parallelogrammum BL.



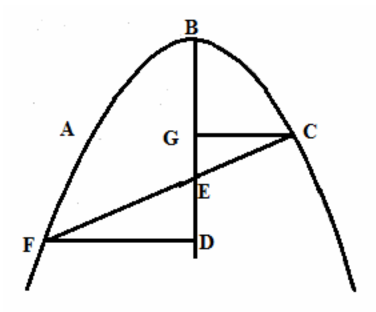
Demonstratio.

Ratio parallelogrammi HD ad parallelogrammum FB, composita est ex ratione FH ad GB, & FD ad FG: est autem ut FH ad GB, sic DC ad FH id est EA ad LA, id est EL ad LI (cum AG, AF, AD, adeoque GB, FH, DC, id est EA, LA, IA sint continuae proportionales) & ut FD ad FG, sic DA ad FA; id est FA ad GA; id est HL ad BI: ratio igitur parallelogrammi HD ad parallelogrammum FB, composita est ex ratione EL ad LI, & ex ratione LH ad IB; sed ex iisdem quoque composita est ratio parallelogrammi HE, ad parallelogrammum BL; igitur ut HD parallelogrammum, ad parallelogrammum FB, sic HE parallelogrammum est ad parallelogrammum LB. Q.e.d.

PROPOSITIO LIX.

Esto ABC parabolam diameter BD, quam in E secet utcunque rectae FC occurrens utrimque parabolam in C & F, ducantur autem ex C & F, ordinatim lineae EG, FD.
Dico BG, BE, BD lineas esse proportionales.

Demonstratio.



Quoniam CG, FD ordinatim positae sunt ad diametrum BD, ratio BG ad BD, duplicata est eius quam habet GC ad FD, id est GE ad ED; igitur BG, BE, BD lineae sunt proportionales, posita enim media BE inter BG, BD erit BG ad BE, ita GE ad ED, & ratio BG ad BD, duplicata rationis GE ad ED.Igitur, &c.
Quod fuit demonstrandum.

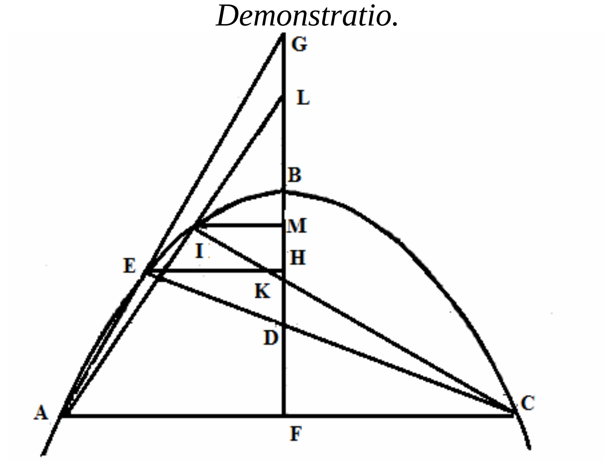
Corollarium.

Hinc sequitur esse ut CE ad EF, sic BE ad BD, nam ut CE ad EF, sic GE est ad ED, id est BE ad BD.

PROPOSITIO LX.

Esto ABC parabolam diameter BF; in qua sumpto quovis puncto H, ponantur continuae proportionales BH, BG, BF : ductaque ordinatim HE, agatur per F linea ipsi HE aequidistans, occurrens GE lineae in A & parabolam in C : ducaturque EC occurrens diametro in D.
Dico DB, BG lineas esse inter se aequales.

Demonstratio.



Quoniam BH, BG, BF ponuntur continuae proportionales, & CF ordinatim ad diametrum GB applicata erit A pundum ad parabolam ABC per Corr. 25 huius sunt autem per praecetentem proportionales quoque BH, BD, BF; media igitur DH aequalis est mediae DF.

PROPOSITIO LXI.

Iisdem positis ducatur ex C alia quaevis CI occurrens diametro BD in K; parabolam vero in I; tum ex A per I ducta linea conveniat cum diametro in L, ponaturque ex I ordinatim linea IM.
Dico esse GB ad LB, ut EH ad IM.

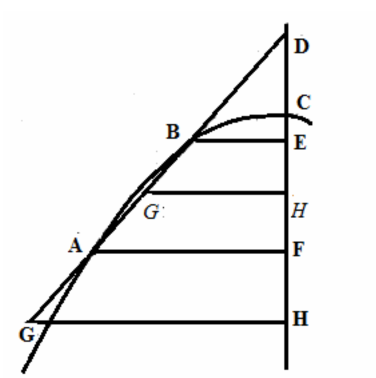
Demonstratio.

Si inter BM & BF, media fiat BL ostendetur ut prius LB aequalc esse ipsi BK, & iunctam LI occurrere parabolam & AC rectae in A puncto, unde cum tam BF, BD, BH lineae, quam BF, BK, BM habentes communem primam BF continuae sunt proportionales, ratio BH ad BM, tertiae ad tertiam duplica est rationis BD ad BK, id est BG ad BL secundae ad secundam: sed & ratio BH ad BM, quoque duplicata est rationis EH ad IM: igitur ut EH ad IM, sic GB ad LB.Q.e.d.

PROPOSITIO LXII.

Sit ABC parabolam diameter CD, ponatur autem AB linea occurrens parabolam in duobus punctis AB, diametro vero extra sectionem,in D, ponanturque ordinatim BE, AF.
Dico CE, CD, CF lineas continues esse proportionales.

Demonstratio.

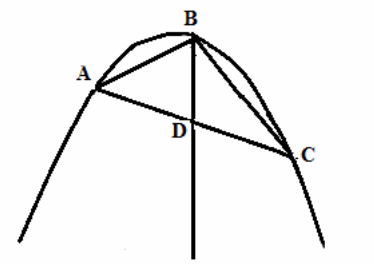


Sin vero: ponantur continuae proportionales CE, CD, CH & ex H ordinatim ducatur HG, occurrens AB lineae in G: erit igitur punctum G ad parabolam ABC*:* adeoque linea AB parabolam in tribus punctis occurrit. Quod fieri non potest; unde CE, CD, CF continuae sunt proportionales.

PROPOSITIO LXIII.

Esto parabolam ABC diameter quaecumque BD aequalis lateri recto, actaque per D ordinatim AC, quae parabolam occurrat in A & C, iungantur AB, BC.
Dico angulum ABC esse rectum.

Demonstratio.

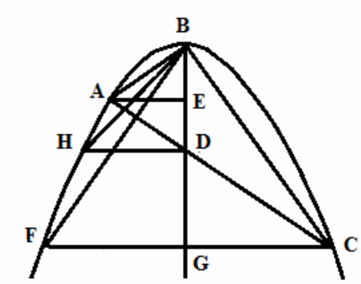


Quoniain BD diameter lateri recto aequalis est, & AC ordinatim posita lineae DB, DA, DC, aequales sunt adeoque & puncta ABC, ad circulum cuius AC diameter est: angulus igitur ABC rectus est. Q.e.d.

PROPOSITIO LXIV.

Esto ABC parabolam axis BD aequalis lateri recto, actaque per D quivis AC, quae parabolam occurrat in A & C, iungantur AB, BC:
Dico angulum ABC esse rectum.

Demonstratio.



Ponantur ex A & C ordinatim lineae AE, CGF; erunt igitur BE, BD, BG lineae proportionales; quia vero BD media aequalis lateri recto est, erunt ABE, FBG triangula similia ; & angulus BAE aequalis angulo FBG, id est CBG : sed angulus BAE una cum angulo ABE, recto est aequalis, quia angulus AEB ad axem rectus est: igitur & angulus CBD una cum angulo ABE, uni recto sunt aequales; rectus igitur angulus ABC. Q.e.d.

PROPOSITIO LXV.

Iisdem positis:
Dico AB ad BC, triplicatam habere rationem eius, cuius AD ad DC duplicata est.

Demonstratio.

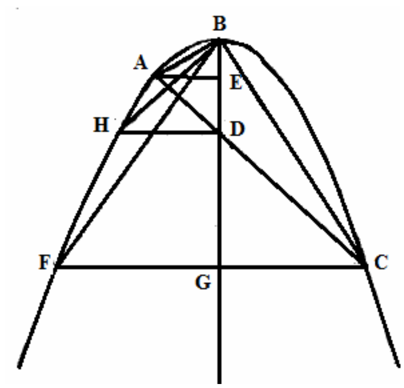
Quoniam in triangulis GBF, CBG anguli ad G recti sunt, lineaeque FG, GC aequales ex hypothesi, triangula FBG, CBG inter se aequalia sunt, & FB, CB lineae quoque aequales: unde cum ratio AB ad FB, triplicata sit eius, cuius duplicatam habet AE ad FG , erit & AB ad BC, ratio triplicata rationis AE ad FG, id est AE ad CG, id est AD ad DC. Q.e.d.

PROPOSITIO LXVI.

Iisdem positis ducatur ex D ordinatim linea DH, iungaturque; HB:
Dico angulum ABF, linea HB divisum esse bifariam.

Demonstratio.

Quoniam anguli AEB, HDB, recti sunt, reliqui duo anguli ABE, BAE reliquis HBD, DHB aequales sunt. Sed BAE angulo aequatur angulus FBG, duo igitur anguli FBG, ABE aequales sunt duobus HBD, BHD: id est angulo HBD bis sumpto, ob HD, DB lineas aequales : dempto igitur communi angulo FBG, manent angult duo HBD, HBF aequales angulo ABE: a quibus rursum si communem demas angulum HBD; rcliqui ABH, HBF aequales manent. Q.e.d.

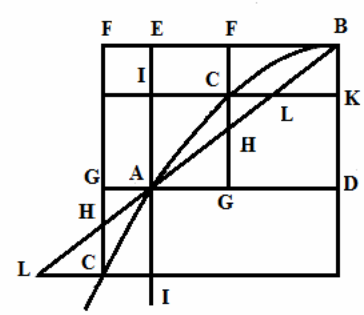


PROPOSITIO LXVII.

Parabolam ABC, cuius diameter BD, contingat in B linea BE; ex quovis autem puncto A in perimetro assumpto ducta ordinatim AD iunctisque; AB punctis sumatur in contingente punctum quod cumque F, ex quo linea demittatur FG, parallela diametro BD, occurrens parabolam in C, AB iunctae in H, & ordinatim positae in G.
Dico FC, FH, FG lineas in continua esse analogia.

Demonstratio.

Erecta ex A diameter AE occurrat contingenti in E , & per C ordinatim ponatur IK secans AB, AE lineas in L & I, & BD diametrum in K, ut KL ad KC, sic BL est ad BH, id est FC ad FH: & ut CK ad IK , id est FC ad EH, sic FH est ad EA, id est ad FG: sed LK, CK, IK, lineae sunt continuae proportionales. igitur & FC, FH , FG lineae in continua sunt analogia. Q.e.d.



PROPOSITIO LXVIII.

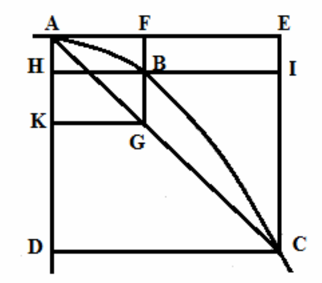
Iisdem datis sumatur in contingentc quodvis punctum F ex quo ducatur FH parallela BD, occurrans lineae AB in H.

Dico HC esse ad CF ut AH ad HB.

Demonstratio.

Quoniain GF, HF, CF proportionales sunt, HF est ad CF, ut GFad HF, hoc est ut AE ad HF, hoc est ut AB ad HB : igitur dividendo HC est ad CF, ut AH ad HB. Q.e.d.

PROPOSITIO LXIX.



Parabolam ABC cuius diameter AD, & ordinatin ad illam posita CD, contingat in A linea AE, quam in E secet diameter CE, iunctisque; punctis AC, ducatur diameter quaecumque FG, occurrens parabolam in B; deinde acta per B ordinatim linea HI, quae EC rectae occurrat in I, & AD diametro in H, ducatur ex G linea GK parallela DC.

Dico parallelogrammum EB aequari parallelogrammo KB.

Demonstratio.

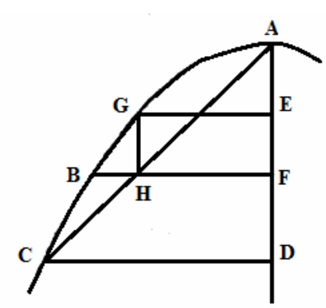
Ut AG ad GC, sic AF ad FE, id est HB ad BI: sed ut AG ad GC, sic FB ad BG: igitur ut HB ad BI, sic FB ad BG; sunt autem anguli ad B oppositi aequales; igitur parallelogrammum EB aequale est parallelogrammo KB. Q.e.d.

PROPOSITIO LXX.

Sit ABC, parabolam diameter AD divisa in E & F; ut AE, AF, AD sint proportionales, positisque ordinatim EG, FB, DC, iuncta AC occurrat FB in H :

Dico HG aequidistare diametro AD, & contra HG aequidistet diamtro AD, & per H ordinatim ponatur FB, dico AE, AF, AD esse proportionales.

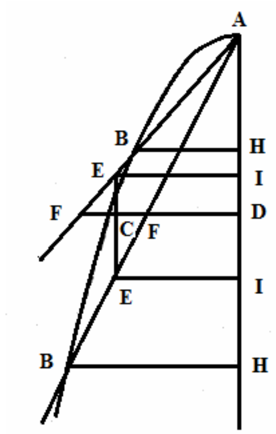
Demonstratio.



Cum AE, AF, AD proportonalaes sint, rectae quoque CD, BF, GE in continua sunt analogia ; sed & DC, FB, FH quoque sunt proportionales ; aequales igitur sunt HF, GE linea quare HG aequidistat e diametro AD, quod fuit primum.

Sit iam HG parallela AD, & per H ordinatim applicetur BF; dico AE, AF, AD quoque in in continua esse analogia, cum enim CD, BF, HF, linea proportionales sint, & GE aequetur HF, erunt & CD, BF, GE continuae proportionales; unde AD, AF, AE in continua quoque sunt analogia. Q.e.d.

PROPOSITIO LXXI.



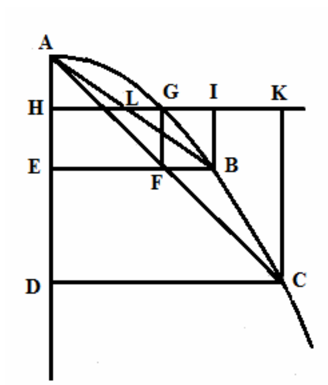
Esto ABC parabolam diametcr AD, quam in D secet ordinatim linea DC, actaque per C diametro CE, ducatur ex A linea AF secans parabolam in B, & EC diametrum in E, occurens vero DC lineae in F.

Dico AB, AE, AF lineas esse in continua analogia

Demonstratio.

Ponantur ex B & E ordinatim lineae BH, EI. Ratio AH ad AD, duplicata est rationis HB ad AD, duplicata est rationis HB ad DC: id est HB ad IE, sic AH est ad AI; igitur ratio AH ad AD, duplicata est rationis AH ad AI: quare AH, AI, AD lineae, id est AB, AE, AF sunt proportionales. Q.e.d.

PROPOSITIO LXXII.



Esto ABC parabolam diameter AD: ductisque ex A lineis quibusvis AB, AC, quae parabolam occurrant in B & C, ponantur ordinatim lineae CD, BE; & BE quidem AC lineae occurrat in F; erectaque FG parallela diametro AD; ponatur per G ordinatim HG, occurrens diametris IB, KC in I & K, rectae autem AB in L:

Dico HL, HG, HI, HK lineas in continua esse analogia.

Demonstratio.

Quoniam HG, EB, DC, ordinatim positas sunt, & FG parallela diametro AD; rectae HG, HI, HK in continua sunt analogia; sed & HL, HG, HI proportionales sunt igitur HL, HG, HI, HK in continua sunt analogia. Q.e.d.

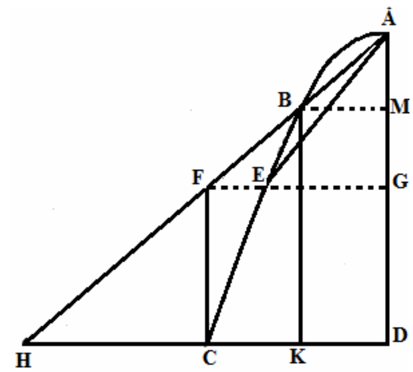
PROPOSITIO LXXIII.

Esto ABC parabolam diameter AD, & ordinatim ad illam posita CD; ducatur autem ex A linea AE, sectioni occurrens in E: actaque per E ordinatim FG, erigatur ex C diameter CF, occurrens FG in F; iunganturque AF, quae CD lineae occurrat in H, sectioni vero in B puncto ex quo diameter demittatur BK, secans FG liineam in L, & CD in K.

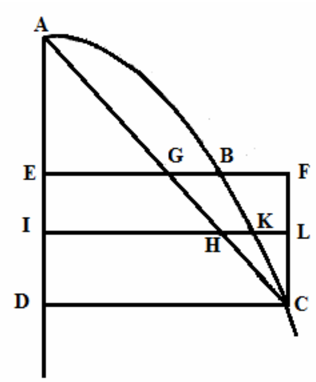
Dico HC ad CK duplicatam habere rationem eius quam habet FE ad EL.

Demonstratio.

Ponatur ex B ordinatim linea BM. Quoniam FC diameter est & HCD ordinatim posita ad diametrum AD, rectae AB, AI, AH proportionales sunt igitur & AM, AG, AD , item BM, EG, CD lineae, id est KD, CE, HD in continua sunt analogia. Quare cum utriusque seriei primae BM, KD aequales sint, ratio HD ad CE, tertiae ad tertiam, duplicata est rationis CD ad EG, id est FG, ad EG: est autem ut HD ad CD, sic HC ad CK, & ut FG ad EH, sic FE ad EL, ratio igitur HC ad CK, duplicata est rationis FE ad EL. Q.e.d.



PROPOSITIO LXXIV.

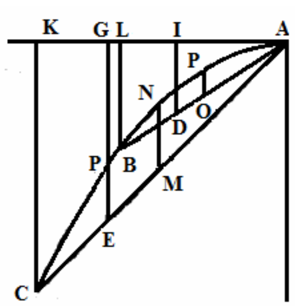


Sit ad ABC parabolam diametrum AD, ordinatim posita DC erectaque; ex C diametro CF, iungatur CA, ponaturque ordinatim quaevis FE occurrens parabolam in B, & AC iunctae in G; tum ex B demissa diametro, quae AC lineae occurrat in H, ponatur per H ordinatim linea IL occurrens diametro FC in L, & sectioni in K:

Dico lineam GB ad BF, duplicatam habere rationem eius quam habet HK ad KL.

Demonstratio.

Quoniam tam FE, BE, GE lineae, quam LI, Kl , HI sunt proponionales habentcs aequales primos terminos FE, LI, ratio GE, ad HI, tertiem ad tcrtiam, id est ratio GE ad BE, id est GB ad BF, duplicata est rationis EB ad IK, secundae ad secundam, id est rationis IH ad IK, id est: HK ad KL. Q.e.d.



PROPOSITIO LXXV.

Eadem figura manente:

Dico rectangulum EGB ad rectangulum EFB, triplicatam habere rationem eius quam habet linea GB ad lineam BF.

Demonstratio.

Rectangulum EGB ad rectangulum EFB, rationem habet compositam ex ratione EG ad EF, id est ex duplicata ratione EG ad EB, id est GB ad BF, (quia EG, EB, EF sunt proportionales) & ex GB ad BF;

igitur rectangulum EGB ad rectangulum EFB triplicatam habet rationem eius quam habet linea GB ad BF. Q.e.d.

PROPOSITIO LXXVI.

Esto ABC parabola, assumptoque in peripheria puncto quovis A, ducantur rectae AB, AC occurrentes iterum parabolam: divisis autem AB, AC lineis proportionalitcr in D & E, erigantur diametri EG, DI, occurrentcs parabolei in F & H, & actae per A contingenti in G & I.
Dico GE, DI lineas in F & H, proportionaliter esse divisas.

Demonstratio.

Erigantur ex C & B, diametri CK, BL. Quoniam AC, AB proportionaliter in E & D ponuntur divisae, ut BA ad DA, id est BL ad DI; sic CA est ad EA, id est KC ad GE; sed ut BL ad DI, sic D1 est ad HI, & ut KC ad GE, sic ad GE est ad GF; igitur ut DI ad HI, sic GE ad GF. Q.e.d.

PROPOSITIO LXXVII.

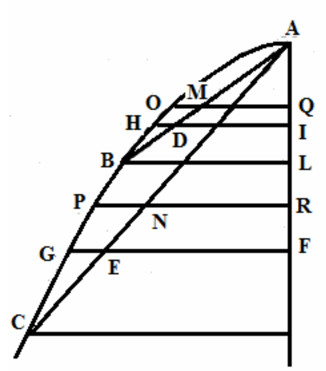
Iisdem positis, dividantur rursum portionaliter lineae AB, AC in M & O, eriganturque diametri MN, OP.
Dico FE esse ad MN, ut DH est ad OP.

Demonstratio.

Est enim ut CEA rectangulum, ad rectangulum CMA sic FE ad NM & ut BDA rectangulum ad rectangulum BOA, sic DH ad PO; sed BDA ad BOA, rectangulum est ut CEA ad CMA, rectangulum, quia CA, BA proportionaliter ponuntur divisae; igitur ut FE ad NM , sic DH ad PO. Q.e.d.

PROPOSITIO LXXVIII.

Iisdem positis agantur per D & E ordinatim ad diametrum ex A demissam lineae HDI, FEG.
Dico HI, FG lineas in D & E proportionaliter esse divisas.



Demonstratio.

Ponantur ordinatim lineae BL, CK. Quoniam igitur AB, AC lineae in D & E proportionaliter sunt divisae, ut CK ad EG, sic BL est ad DI; sed CK ad EG, duplicatam habet rationem FG ad EG, & BL ad DI; duplicatam habet rationem HI ad DI, igitur ut FG ad EG, sic HI ad DI, & dividendo ut FE ad EG, sic HD ad DI. Quod fuit demonstrandum.

PROPOSITIO LXXIX.

Iisdem positis, dividantur rursum AB, CA lineae proportionaliter in M & N, & rectae ducantur MOQ, NPR aequidistates ipsis HI, FG.
Dico ut HD ad OM, sic FE esse ad PN.

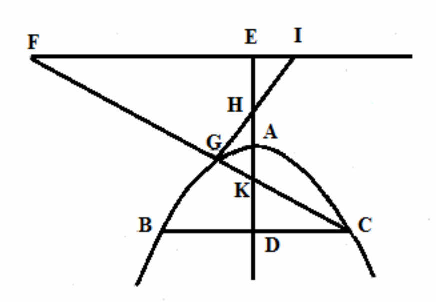
Demonstratio.

Cum enim lineae AB, AC proportionaliter in M, D, N, E punctis sint divisae, erit MQ ad DI, ut NR ad EG, & permutando MQ ad NR, ut DI ad EG; sed per praecedentem est ut MQ ad NR, sic OM ad PN; & ut DI ad EG, sic ad HD, sic HD ad FE; igitur ut OM ad PN, sic HD ad FE, & permutando ut OM ad HD, sic PN ad FE. Quod fuit demonstrandum.

PROPOSITIO LXXX.

Sit ad ABC parabolam diametrum AD ordinatim posita linea CB: factaque AE aequali ipsi AD ponatur per E linea EF parallela rectae BC, dein ex C: ducatur linea CF, occurrens parabolam in G puncto, per quod ex B agatur linea occurrens axi AD in H; & EF lineae in I;
Dico FE, BD, EI lineas in continua esse analogia.

Demonstratio.

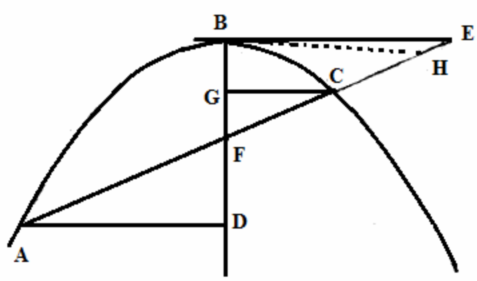


Linea CF occurrat diametro in K. Quoniam AE linea aequalis ponitur lineae AD; & HA, AK lineae quoque inter se aequantur, erit EH reliqua aequalis reliquae KD. Rursum triangulum FKE ad triangulum DKC duplicatam habet rationem EK lineae ad lineam KD; id est HD ad EH, est autem & triangulum BHD, ad triangulum EHI in duplicata ratione HD ad HE; igitur ut triangulum FKE ad triangulum DKC, sic BHD triangulum, ad triangulum EHI adeoque rationes ex iisdem habent compositas, sed ratio trianguli FKE ad triangulum DKC, est composita ex ratione FE ad DC, id est BD; & ex EK ad KD, id est DH ad HE: & ratio trianguli BHD, ad triangulum EHI est composita ex HD ad HE, & ex BD ad EI, ablata igitur communi ratione HD ad HE, manet ratio FE ad BD, eadem cum ratione BD ad EI; quare FE, BD, EI lineae sunt continuae proportionales. Q.e.d.

PROPOSITIO LXXXI.

Parabolam ABC cuius diameter BD contingat in B in linea BE; demissaque ex E linea EA occurrat in C & A, diametro vero BD in F.
Dico EC, EF, EA lineas in continua esse analogia, & contra si EC, EF, EA fuerint proportionales, dico EB, sectionem contingere.

Demonstratio.



Ponantur ex A & C ordinatim lineae AD,CG ad BD, diametrum : quoniam igitur BE; GC lineae aequidistant, erit ut GB ad FB, sic CE ad FE; & ut FB ad DB; sic FE ad AE sed BG, BF, BD lineae sunt proportionales; igitur & EC, EF, EA lineae in continua sunt analogia.
Quod erat primum.
Sint iam EC, EF, EA proportionales, iunganturque EB, dico EB lineam, sectionem in B contingere. Sin vero, agatur per B tangens, quae AE lineae occurrat in H, erunt igitur HC, HF, HA continuae proportionales, & CF ad FA, ut HC ad HF: sed & CF est ad FA, ut EC ad EF (cum EC, EF, EA sint proportionales) igitur ut HC ad HF, sic EC ad EF: & dividendo ut HC ad CF, sic EC ad CF, quod absurdum, cum HC ex hypothesi, maior sit vel minor recta EC, quare HB non est tangens, nec quaevis alia praeter EB. Q.e.d.

PROPOSITIO LXXXII,

Eadem manente figura propositum sit a dato puncto extra sectionem contingentem ducere.

Constructio & demonstratio.

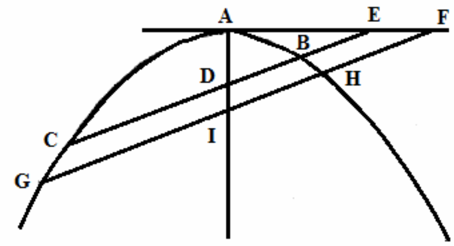
Sit datum punctum E, ex quo ducatur quaevis secans parabolam in C & A: erit EC prima trium continuarum & AC excessus reliquarum: inveniantur igitur inter CE, EA media EF per F, diameter agatur BD iunganturque EB, patet per praecedentem, lineam EB sectionem contingere in B, a dato igitur extra parabolam puncto tangentem duximus, &c. Quod erat quaesitum.

PROPOSITIO LXXXIII.

Parabolam ABC cuius diameter AD, contingat in A linea AE, in qua sumptis duobus punctis E, F, ducantur ex E & F, duae parallelae EC, FG occurrentes parabolam in B, C, H, G, & AD diametro in D & I.

Dico BEC rectangulum ad rectangulum HFG duplicatam habere rationem lineae eius quam habet EA linea ad lineam FA.

Demonstratio.



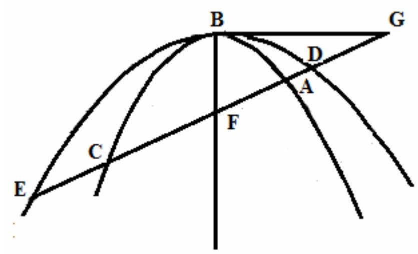
Quoniam tam EB, ED, EC lineae, quam FH, FI, FG sunt proportionales, erit BEC rectangulum aequale quadrato ED , & HFG rectangulum aequale quadrate FI, igitur ut quadratum ED ad quadratum FI, sic BEC rectangulum est ad rectagulum HFG; sed ED quadratum est ad quadratum FI, ut AE quadratum est ad quadratum AF; igitur & BEC rectangulum est ad rectangulum HFG ut AE quadratum ad quadratum AF. hoc est duplicatam habent rationem, EA ad FA. Q.e.d.

PROPOSITIO LXXXIV.

Parabolas duas ABC, DBE habentes communem diametrum BF contingat in eadem puncto, linea BG, & ex G quaevis ducatur linea, occurrens parabolis in D, A, C, E; diametro vero BF in F.

Dico DGE rectangulum aequari rectangulo AGC.

Demonstratio.

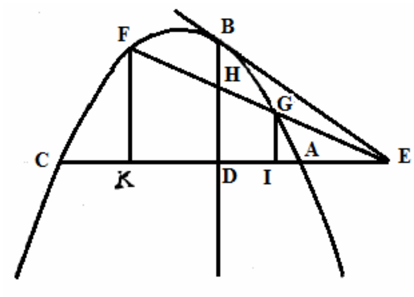


Est enim per octuages, primam huius tam DGE rectangulum, quam rectangulum DGF aequale quadrato FG: igitur & AGC, DGE rectangula inter se aequalia sunt. Quod fuit demonstrandum.

PROPOSITIO LXXXV.

Parabolam ABC cuius diameter BD contingat in B linea EB: ex puncto E duae quaevis ducantur lineae EF, EC, secantes parabolam in F, G, A, C punctis & BD diametrum in D & H, demissaeque ex F & G diametri FK, GI, occurrant AC lineae in K & I.

Dico: GI ad FK, duplicatam habere rationem eius quam habet GH



ad HF.

Demonstratio.

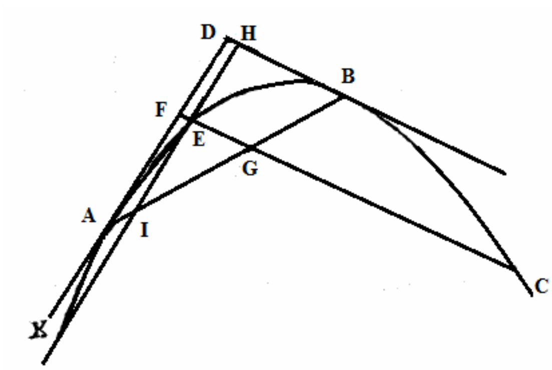
Ut GI ad FK, sic EG est ad EF; sed EG ad EF, duplicatam habet ratione EG ad EH, id est GH ad HF; igitur & GI ad FK, duplicatam habet rationem eius quam habet GH linea ad lineam HF.
Q.e.d.

PROPOSITIO LXXXVI.

Parabolam ABC contingant in A & B lineae AD, BD convenientes in D, assumptoque puncto E in sectionis peripheria, agatur per E linea FC, parallela rectae BD occurrens AD lineae in F.

Dico ut quadratum BD ad quadratum AD, sic EFC rectangulum esse ad quadratum AF.

Est haec ab Apollonio lib.3.prop.16.eodem plane modo proposita: nos autem hanc supponendo ulterius inferimus, si A, B puncta contactuum coniungantur, rectas FE, FG, FC continua esse analogia, uti & HE, HI, HK lineas.

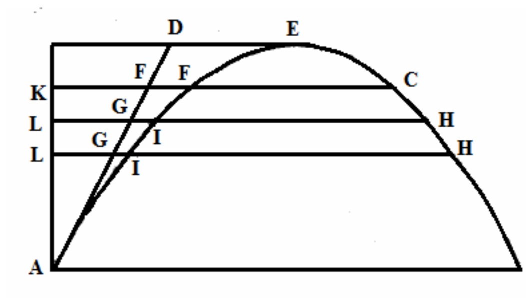


Demonstratio.

Est enim ex supposito ut AD quadratum ad quadratum DB, sic AF quadratum ad rectangulum EFC: & permutando ut AD quadratum ad quadratum AF, sic DB quadratum ad quadratum FG; igitur ut DB quadratum ad rectangulum EFC, sic idem quadratum DB ad quadratum FG, unde FG quadratum aequale est rectanglo EFC; & FE, FG, FC lineae sunt in continua proportione.

PROPOSITIO LXXXVII.

Iisdem positis ducatur & altera GH parallela FC, secans parabolam in I & H.
Dico rectangulum EFC ad rectangulum IGH, esse ut quadratum FA ad GA, quadratum.



Demonstratio.

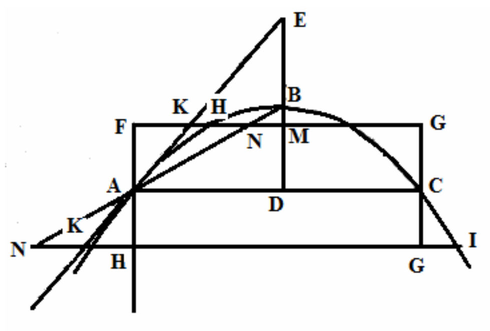
Cum enim tam rectangulum EFC ad quadratum FA, quam rectangulum IGH ad quadratum IGH ad quadratum GA habeant rationem quadrati DB ad quadratum DA, constat ita esse EFC rectangulum ad IGH ut quadratum FA ad GA. Quod demonstrandum fuit.

Corollarium.

SI ex A ducatur diameter AK, occurrens ordinatim applicatis in K & L, rectangulum CFE ad HGI, duplicatam habebit rationem eius quam CKE habet ad rectangulum HLI; ratio enim rectanguli CKE ad HLI, est ratio lineae KA ad LA, hoc est FA ad G A; sed ratio rectanguli CFE ad HGI, est ratio quadrati FA ad GA: igitur ratio rectanguli CFE ad HGI, duplicata est eius quam habet CKE ad HLI, rectangulum.

PROPOSITIO LXXXVIII.

Parabolam ABC cuius diameter BD, & ordinatim ad illam posita ADC, contingat in A linea AE, conueniens cum diametro in E, erectisque ex A & C diametris AF, CG ducatur quaecunque HI, parallela AC, occurrens AE contingenti in K, lineae AF in F, diametro BD in M, iunctae AB in N, & rectae CG in G .
Dico KFG rectangulum aequare rectangulo HFI.



Demonstratio.

Quoniam tam FN in K, quam FG in M bifariam divisa est, erit FK ad FN, ut FM ad FG. unde rectangulo FKFG, aequatur FNFM rectangulum: sed rectangulo NFM ostensum est aequari rectangulum HFI. Igitur etiam rectangulo HFI aequale est KFG rectangulum. Q.e.d.

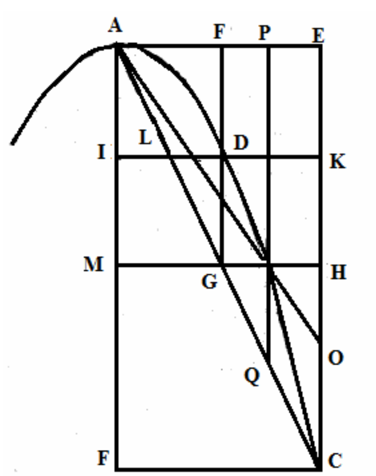
PROPOSITIO LXXXIX.

Iisdempositis:
Dico HK, HF, FG lineas esse continue proportionales.

Demonstratio.

Quoniam rectangulum KFG aequale est rectangulo HFI, ut FH, ad FK, sic FG est ad FH; & convertendo ut FH ad HK, sic FG est ad GI, id est ad FH; quadratum igitur FH aequale est rectangulo HKFG; & HK, HF, FG proportionales fiunt. Quod fuit demonstrandum.

PROPOSITIO XC.



Sit AB diameter parabolam ADC & ordinatim posita BC & contingens AE diametro insuper AB aequidistet EC ductaque AH quacunque occurrente parabolam in D ponatur AC, & FDG aequidistans AB, occurrensque; AC in G.
Dico FG aequalem esse rectae EH.

Demonstratio.

Ponantur per D & G puncta rectae IK, MR parallelae BE; erunt itaque tres in continua analogia IL, ID, I K & MG, MN, NR. Igitur quadratum ID ad MN, quadratum eam habet rationem quam IL ad MG; hoc est IA ad M A ; hoc est DA ad HA; hoc est DF ad HE, est autem FG aequalis MA. igitur HE eidem FG, aequalis est. Quod fuit demonstrandum.

PROPOSITIO XCI.

Iisdem positis; per N ducatur ANO. Dico ID ad MN eandem habere rationem quam habet EH ad EO.

Demonstratio.

Ponatur PNQ aequidistans AB, erit haec PQ, aequalis EO per pracedentem. Similiter per eandem erit FG aequalis EH: est autem FG ad PQ, ut AF ad AP, hoc est ID ad MN, igitur EH ad EO, eandem obtinet rationem quam ID ad MN. Quod fuit demonstrandam.

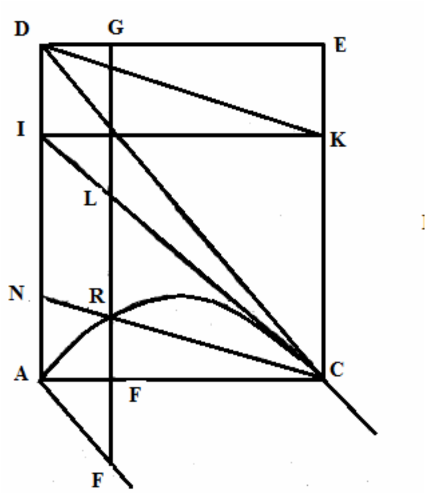
PROPOSITIO XCII.

Parabolam ABC sustendat linea AC, actaqae per C contingente, quae diametro per A positae occurrat in D, perficiatur parallelogrammum ADEC: dein sumpto in sectione puncto quovis B, agatur per B diameter FG secans AC lineam in F, tangentem CD in H puncto, per quod ducta IK parallela AC, ponatur IC quae FG lineae occurrat in L.

Dico GF, HF, LF lineas in continua esse analogia.

Demonstrato.

Ut GF ad HF, sic DA est ad IA: sed ut DA est ad lAE sed ut DA ad IA, sic HF est ad LF; igitur ut GF ad H F sic HF ad LF, proportionales igitur sunt GF, HF, LF. Q.e.d.



PROPOSITIO XCII.

Iisdem positis ducatur linea D K occurrens· GF in M. Dico GM, GH, GF lineas in continua esse analogia.

Demonstratio.

Quoniam EC aequidistat AD (ob DC, parallelogrammum) & IKD, ICD triangula eandem habent basim ID, lineae MH, HL aequales sunt, sunt autem continuae proportionales GF, HF, LF; igitur & GM, GH, GF lineae in continua sunt analogia. Q.e.d.

PROPOSITIO XCIII.

Iisdem positis:
Dico esse ut CF ad FA, sic HB ad B F.

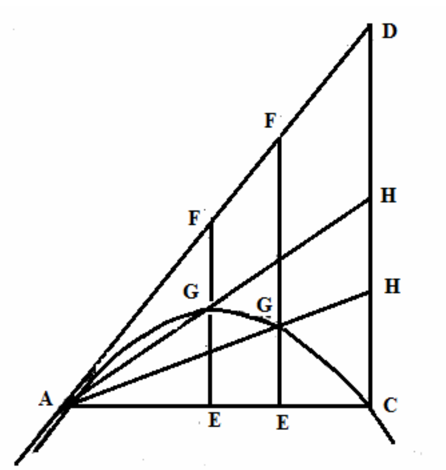
Demonstratio.

Ducatur ex C per B linea CN occurrens AD lineae in N : & ex A recta AO parallela DC, secans EC productam in O, & FB lineam in P. Quoniam DC est contingens & AO eidem parallela, rectae HB, HF, HP proportionales sunt. Sunt autem etiam continuae LF, HF,GF, & GF linea aequalis lineae HP(ob AE, AC parallelogramma super eadem basi AD, & inter easdem parallelas constituta.) igitur & HB ,aequalis est rectae LF: unde dempta communi LB, manet HL rectae BF, & NA ipsi ID aequalis; adeoque & NC parallela DK. Quare ut DN est ad NA, id est CK ad DI, sic HB est ad BF; sed ut CK ad DI, sic CH est ad HD, id est CF ad FA; igitur ut CF ad FA , sic HB est ad BF. Q.e.d.

Est haec Archimedis aliter demonstrata.

PROPOSITIO XCIV.

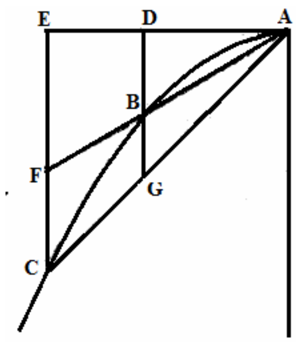
Parabolam ABC subtendat linea AC actaque per A contingente AD quae diametro CD ductae per C,occurrat in D ponatur quotvis diametri FE secantes parabolam in G, & per G ex A, ductae lineae AH, occurrant diametro CD in H.
Dico AC, CD lineas proportonaliter in E & H esse divisas.



Demonstratio.

Demonstratio manifesta est per praecedentem; nam semper est ut AE ad EC, sic FG ad GE, id est DH ad HC.

PROPOSITIO XCVI.

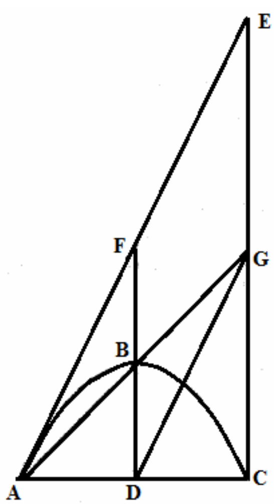


Parabolam ABC contingat in A linea AD, in qua sumptis quibusvis punctis D & E; demittantur diametri DB, EC: & ex A per B recta ducatur AF occurrens BC diametro in F. Dico esse EC ad CF, ut AF ad FB.

Demonstratio.

Ducatur recta AC occurrens DB lineae in G; erit igitur AG ad GC, ut DB ad BG, id est EF ad : FC. Sed est ut AG ad GC, sic AB ad BF; igitur ut AB ad B F, sic EF ad F C: & componendo ut AF ad FB, sic EC ad CF. Quod fuit demonstrandum.

PROPOSITIO XCVII.

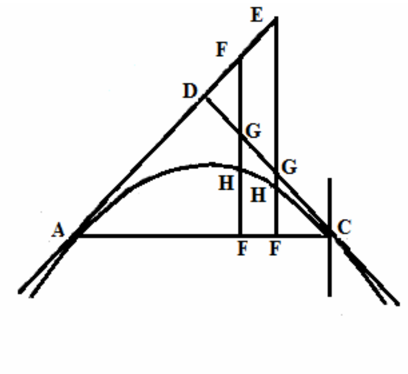


Parabolam ABC subtendat recta AC, erectaque ex C diametro GE, ponatur quaecunque
AG occurrens CE lineae in G, parabolam in B puncto, per quod diameter ponatur BD.
Dico iunctam DG aequidistare contingenti per A ductae.

Demonstrando.

Ducta enim per A contingens occurrat BD, CE diametris in F & E. Erit igitur FB ad BD, ut AD ad DC, id est AB ad BG: quare AE, DG lineae sunt parallelae. Q.e.d.

PROPOSITIO XCVIII.



Parabolam ABC subtendat lineat AC, actisque per A & C, contingentibus quae conveniant in D, ducatur diameter quaecunque FE, secans AD lineam in E, DC in G; & parabolam in H.

Dico GH, HF, HE, lineas in continua esse analogia

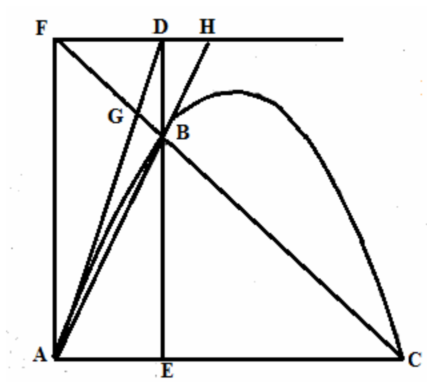
Demonstratio.

Est enim ut AF ad FC, sic EH ad HF, sed etiam ut AF ad FC, sic FH ad HG, igitur ut HE ad HF, sic HF ad HG : proportionales igitur sunt GH, HF, HE. Q.e.d.

PROPOSITIO XCIX.

Parabolam ABC cuius subtensa AC; contingat in A linea AD, in quasumpto quovis puncto D demittatur diameter DE occurrens parabolam in B, per B autem ex C recta ponatur CF, occurrens erectae ex A diametro in F.

Dico FA, DE lineas esse inter se aequales.



Demonstratio.

Ut AE ad EC, sic DB est ad BE; & componendo ut AC ad CE, sic DE ad BE. Sed ut AC ad EC, sic FA est ad BE; igitur ut DE ad BE, sic FA ad BE: igitur FA, DE lineae sunt inter se equales. Q.e.d.

PROPOSITIO XCX.

Iisdem positis:

Dico esse BG ad GF, ut FB ad FC.

Demonstratio.

Quonium FA, DE lineae per precedentem aequales sunt, DE est ad BE, ut FA as BE; id est ut AC ad EC; & invertendo AD ad AC, id est FB ad FC, ut DB ad DE; id est ad FA. Sed ut BD ad FA, sic BG ad GF, igitur BG ad GF, ut FB est ad FC. Q.e.d.

PROPOSITIO CI.

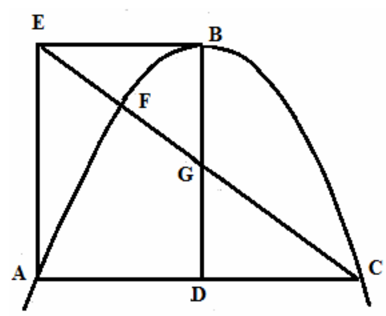
Iisdem positis ducatur AB occurrens FD iuncte in H.
Dico DH, DF, EC lineas in continua esse analogia.

Demonstratio.

Est enim DH ad AE, id est ad FD, ut DB ad BE, sed ut DB ad BE, sic FD ad EC, igitur DH est ad FD ut FD ad EC. Q.e.d.

PROPOSITIO CII.

Parabolam ABC cuius diameter BD, contingat in B linea BE, demissaque diametro EA ducatur ad BD, ordinatim ADC, ponaturque EC occurrens parabolam in F & BD diametro in G.
Dico EF lineam aequari lineae FG.

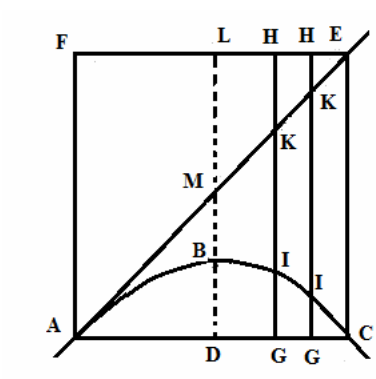


Demonstratio.

Quoniam EB linea sectionem contingit, erit EF ad EG, ut EG ad EC: & dividendo EF ad FG, ut EG ad GC, sed EG est aeqalis GC, quia AD, DC aequales sunt, igitur & EF, FG rectae inter se aequantur. Q.e.d.

PROPOSITIO CIII.

Parabolam ABC cuius diameter BD, & ordinatim ad illam posita AC, contingat in A linea AE, erectaque ex C diametro CE quae contingenti AE occurrat in E, perficiatur parallelogrammum ACE, ducaturque; diameter quaevis HG occurrens parabolam in I, & AE rectae in K.

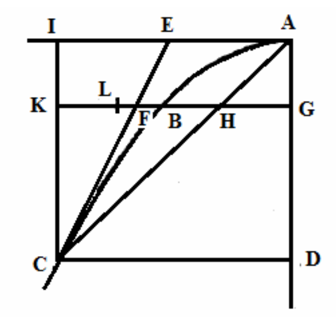


Dico HGI rectangulum aequari rectangulo HKG.

Demonstratio.

Producta diameter BD, secet EF, AE lineas in L & M. Quoniam AC linea aequidistat EF, eidemque est aequalis , erit AD, dimidia rectae AC, aequalis LE, dimidiae ipsius EF; quare & LM aequalis est MD, adeoque LD in M bifariam divisa: unde cum & LD in B divisa sit non bifariam, rectangulum LMD, id est quadratum MD aequale est rectangulo LBD, una cum quadrato MB, id est quadrato BD, ideae aequalc erit rectangulo LDB. Rursum cum ut BD ad IG, sic LDB rectangulum sit ad rectangulum HGI, item ADC rectangulum ad rectangulum AGC, Id est rectangulum AME ad rectangulum AKE, igitur ut LDB rectangulum ad rectangulum HGI, sic AME rectangulum est ad rectangulum AKE; est autem ut AME rectangulum ad rectangulum AKE, sic LMD rectangulum ad rectangulum HGI, sic LMD rectangulum est ad rectangulum HKG, & permutando ut LDB rectangulum est ad rectangulum LMD, sic HGI rectangulum ad rectangulum HKG; sed LDB, LMD rectangula ostensa sunt aequalia, igitur & rectangulum HGI aequale est rectangulo HKG. Q.e.d.

PROPOSITIO CIV.



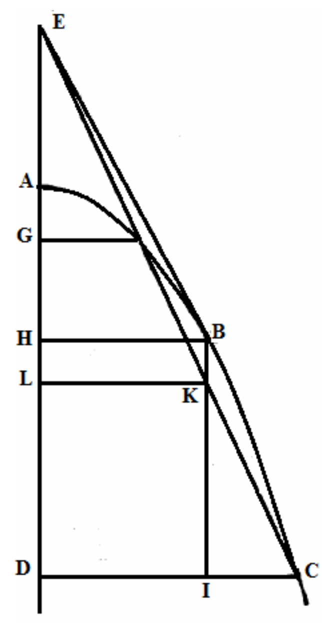
Parabolam ABC cuius diameter AD & ordinatim ad illam posita CD contingant in A & C lineae AE, CE convenientes in E; ductaque AC, ponatur ordinatim BG quae EC, AC lineis occurrat in F, &H.

Dico HB quadrati dimidio aequale esse rectangulum FB, HG.

Demonstratio.

Erigatur ex C diameter CI, occurrens BG linaea in K, fiatque HB aequalis KL. Quoniam igitur KG, BG, HG proportionales sunt, & BH differentia ponitur aequalis LK, rectae GH, HB, BL quoque proportionales sunt, adeoque BH quadrato aequale rectangulum LB, HG. Rursum cum AI in E ac propterea HK in F divisa sit bifariam, & HB lineae aequalis KL, erit & LB reliquae in F divisa bifariam; quare FB, HG rectangulum dimidium est rectanguli LB, HG, adeoque & aequale dimidio quadrati HB. Q.e.d.

PROPOSITIO CV.



Parabolam ABC cuius diameter AD contingat in B linea BE,conveniens cum diametro in E; demissaque ex E linea EC , quae parabolam in F & C, ducantur ordinatim lineae FG, BH, CD.

Dico FG, BH, CD lineas in continuae esse analogia.

Demonstratio.

Demittatur ex B diameter BI secans EC lineam in K: ponaturque KL parallela HB.

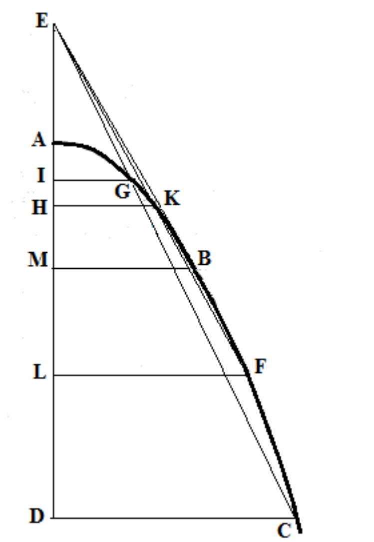
Quoniam BE linea sectionem contingit, erunt AE, EK, EC lineae, adeoque & FG, KL,CD continuae proportionals : sed HB linea aequalis est KL, igitur FG, BH, CD lineae in continua sunt analogia. Quod erat demonstrandum.

Corollarium.

Eadem manente figura sumatur in AD diametro quodcunque punctum E, extra parabolam; ex quo in parabolam secans demittatur EFC ,ductisque ordinatim FG, CD, fiat AE lineae aequalis AH, & ex H, ordinatim ponatur HB: dico FG, BH, CD lineas esse proportionales; iuncta enim BE erit contingens; unde per praecedentem constat veritas assertionis.

OPOSITIO CVI.

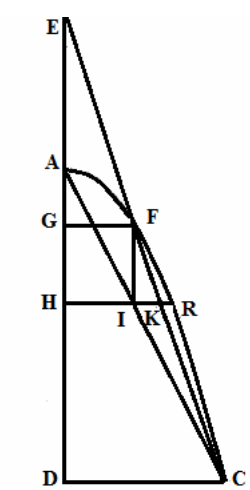
Esto ABC diameter AD, in qua assumpto extra sectionem puncto quovis E, demittantur ex E duae quaevis EC, EF secantes parabolam in G, K, C, F: ponanturque
ordinatim GI, KH, FL, CD.
Dico esse ut GI ad KH, sic FL ad CD.



Demonstratio.

Demittatur ex E cotingens EB, & ex B ordinatim ducatur linea BM, erunt igitur tam GI, BM, CD lineae, quam HK, BM, FL proportionales; quare cum BM linea sit utrique seriei communis & media proportionalis inter easdem lineas, GICD rectangulum aequale est rectangle aequale est rectanglo HKEI ; igitur ut GI ad KH, sic FL ad CD. Quod erat demonstrandum.

PROPOSITIO CVII.

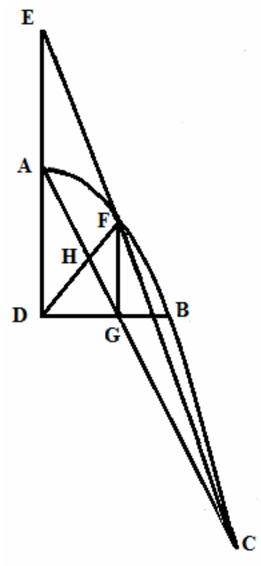


Esto ABC parabolam diameter AD, in qua assumpto extra sectionem puncto quovis E, demittatur ex E secans EFC , positisque ordinatim FG, CD, fiat AE lineae aequalis AH, & ex H ordinarim ponatur HB occurrens AC, FC, lineis in I & K.
Dico EC lineam extrema & media ratione proportionali in F & K esse divisam id est CE esse ad EF ut CK ad KF.

Demonstratio.

Iungantur FI : Quoniam AH linea aequalis ponitur AE, rectae FG, BH, CD proportionales sunt, & FG, IH lineae aequales, adeoque & FI parallela AE. quare ut EC ad EF, sic AC ad AI, id est AD ad AH, id est AH ad AG, (cum AD, AH, AG proportionales sint.) id est DH ad HG, id est CK ad KF. Quod erat demonstrandum.

PROPOSITIO CVIII.



Parabolam ABC cuius diameter AD, contingat in B linea BE, conveniens cum diametro in E, ex quo recta ducatur EC, occurrens parabolam in F & C, positaque ordinatim BD, ducatur AC occurrens BD lineae in G, & recta FD secans AC in H.

Dico rectam AC divisam esse in H, & G, extrema & media ratione proportionali : id est AC, CG: & AH, HG esse proportionales.

Demonstratio.

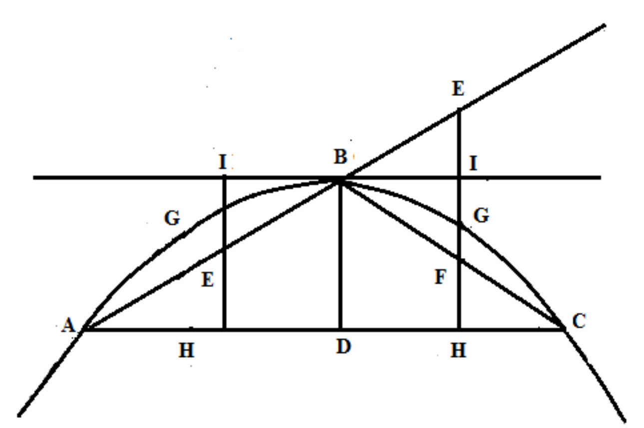
Iungantur FG : Quoniam ostensum est in pracedenti propositione FG lineam aequidistare AE, ut AC ad CG, sic AE est ad FG: sed AE est aequalis AD, igitur ut AC ad CG, sic AD est ad FG, id est AH ad HG.Q.e.d.

PROPOSITIO CIX.

Sit ad ABC parabolam diametrum BD, ordinatim posita AC: iunctisque AB, CB ducatur quaeuis diameter EF.

Dico esse ut AD ad DH, sic HE ad EG.

Demonstratio.



Diameter EF, primo AB lineae occurrat in E extra sectionem; ponatur per B contingens IB: ut AD ad AH, sic DB id est IH ad HE ; quia vero IG, IF (id est IE) & IH proportionales sunt; ut HI ad IE sive IF,prima ad secundam, sic HI cum IF vel IE, id est HE, prima cum secunda, ad IE una cum IG id est EG, secundam cum tertia, igitur ut AD ad DH, sic HE ad EG. Q.e.d.

Occurret iam diameter HG rectae AB in E intra parabolam, erit igitur ut AH ad AD, sic HE ad BD, id est ad HI; & ut AD ad HE, sic HI ad IE, sed ut HI ad IE prima ad secundam, (quia HI, IE, IG proportionales sunt) sic HE est ad EG, igitur ut AD ad DH, sic HE est ad EG.

Est haec Archimedis prop. 4.de quadrature Parabolam aliter demonstrata.

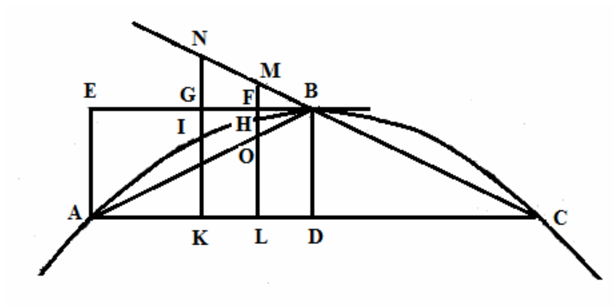
Corollarium.

Hinc sequitur lineam HI ad IG, rationem habere duplicatam eius quam habet HF ad IG: cum enim sint continuae proportionales HI, FI, GI, erit ratio HI ad IG, duplicata rationis HI ad FI, id est, HF ad FG.

PROPOSITIO CX.

Parabolam ABC cuius diameter BD, contingat in B linea BE; qua divisa in F & G, ut BF, BG, BE sint proportionales, demittantur diametri FH, GI, EA ductaque ; ex A ad BD ordinatim linea AC, quae FH, GI rectas secet in K & L, agatur per B ex C, linea CM occurrens FH, GI diametris in M & N.
Dico rationem LM ad MH duplicatam esse rationis KN ad NI.

Demonstratio.



Quoniam BF, BG, BE lineae continuae proportionales sunt; diametri quoque FH, GI, EA in continua sunt analogia. Quare ratio AE ad FH, id est LF ad FH, duplicata est rationis AE ad GI, id est : KG ad GI; sed LF ad FH, duplicatam habet rationem, LO ad OH, (cum LF, FO, FH proportionales sint) id est per praecedentem LM ad MH, similiter & KG ad GI, duplicatam habet eius quam habet KN ad NI, igitur & ratio LM ad MH, duplicata est rationis KN ad NI, igitur & ratio LM ad MH, duplicata est rationis KN ad NI
Q.e.d.

PROPOSITIO CXI.

Iisdem positis ducatur linea AB secans FL lineam in O.
Dico MHLF rectangle esse aequale rectanglo MLFO.

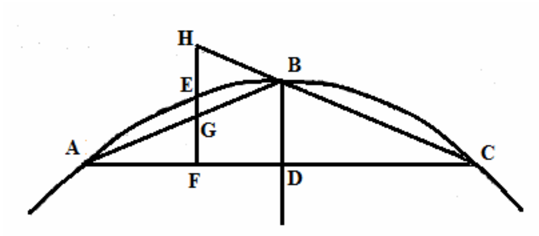
Demonstratio.

Quoniam ratio LF ad FH tam duplicata est rationis LM ad MH, quam FL ad FO, erit ut LM ad MH, sic FL ad FO: quare rectangulum MHLF aequale est rectangulo MLFO.
Q.e.d.

PROPOSITIO CXII.

Sit ad ABC parabolam diametrum BD, ordinatim posita linea AC, iunctisque punctis AB, CB, ducatur diameter quaecunque EF occurrens AB, CB lineis in G & H.

Dico esse ut HF ad FG, sic HE ad EG.



Demonstratio.

Ut AD ad DF, id est DC ad DF, sic FH est at HE , sed etiam ut AD ad DF, sic FG est ad GE, igitur ut FH ad HE, sic FG est ad GE, & permutando ut HF ad FG, sic HE ad EG. Q.e.d.

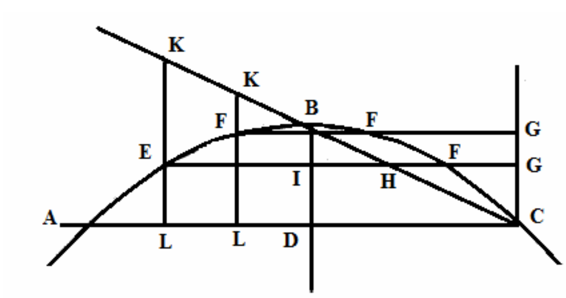
PROPOSITIO CXIII.

Sit ad ABC parabolam diametrum BD ordinatim posita AC, iunctisque punctis BC ducantur ordinatim lineae EF occurrentes CG.

BD diametris in G & I, & CB rectae in H.

Dico rectangulum GIEH aequari rectangulo GEIF.

Demonstratio.



Agantur per E diametri KL ut AD ad DL, sic LK est ad KE, sed ut AD ad DL, sic CD est ad DL, id est: GI ad IE; igitur GI est ad IE, ut LK ad KE; est autem ut LK ad KE, sic LC ad EH, id est GE ad EH ; igitur ut GI ad IE, sic GE ad EH, unde GI, EH rectangulum aequale est rectangulo GEIE, id est GE, IF. Q.e.d.