CHAPTER III

CONCERNING THE OSCILLATORY MOTION OF AIR IN UNIFORMLY WIDE TUBES, APPLIED TO EXPLAINING THE SOUNDS OF FLUTES

PROBLEM 78

59. *If the tube AABB* (Fig. 86) *were open at each end and the air contained in that may be disturbed from equilibrium in some manner, to determine the motion of the oscillation, by which the air in the tube may be disturbed.*

SOLUTION

In the first place, for the calculation of the disturbance to be refer to at some place S in the tube, we may put the density to be $=Q$,with the normal density being $=B$, and the speed along the direction AB there to be Υ; hence above the line AB at the point S the two applied lines may be raised $SQ = l\frac{Q}{B}$ and $S\Upsilon = \Upsilon\sqrt{\frac{b}{2ga}}$, with a certain right line assumed for unity, with which made the point Q will be in accordance with the scale of the density, Υ truly with the scale of the speed. But since the tube is open at A and B , it is necessary,

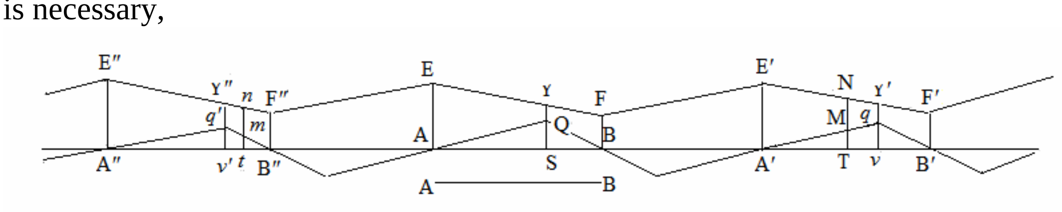

Fig. 86

that the scale of the density at each end A and B shall fall on the axis, as *AQB;* moreover, the whole scale of the speed shall be *ETF*. Now for the following motion requiring to be defined, each scale will be required to be continued indefinitely following the precepts given above. Hence finally, with the axis produced on both sides, and there with the abscissas intervals *BA′*, *A′B′*, *AB″*, *B″ A″* etc. equal to the length of the tube, in the first place the scale of the density *AQB* will be placed alternately below and above the axis, truly the scale of the speed shall be repeated at the same part of the axis *FE′*, *E′F′* etc., *EF″*, *F″ E″* etc., thus in order; as the inspection of the figure shows, with the individual parts being set out alternately to the right and to the left. With these prepared thus, for some elapsed time $=t$, if we may wish to know the state of the air, which initially was at S, from some point S on each side we may cut the intervals on the axis $ST = St = t\sqrt{\frac{2ga}{b}}$, and at the points T and t for each scale with the applied lines drawn we have shown above to become (13)

$$\text{density } q = Q(1 - SQ - \tfrac{1}{2}TN + \tfrac{1}{2}TM + \tfrac{1}{2}tn + \tfrac{1}{2}tm)$$

$$\text{and the speed } \mathfrak{T} = \tfrac{1}{2}(TN - TM + tn + tm)\sqrt{\frac{2ga}{b}},$$

by which motion the air at S will be moved towards B by the interval

$$=\tfrac{1}{2}TNtn-\tfrac{1}{2}SQTM+\tfrac{1}{2}SQtm,$$

which therefore, if the whole scale of the speed may be placed above the axis, however great it can emerge, that which arises, while air flows continually through the tube *AB* and that will be with new air entering, occupying the place of the air gone out ; the state of which is defined by the two formulas given for q and $\mathfrak{T}$.
Now I observe, if such a time t may be taken, so that there may become

$$t\sqrt{\tfrac{2ga}{b}}=2AB,$$

then the points T and t fall at s and s', which points are analogous to S itself, and thence to become

$$q=Q \text{ and } \mathfrak{T}=S\Upsilon\sqrt{\tfrac{2ga}{b}}=\Upsilon,$$

thus so that now the point S and likewise all the air contained in the tube shall be reduced to the same state, in which it was moving initially [*i.e.* a standing wave pattern]. Meanwhile therefore here the air is considered to be resolved into two oscillations, from which the times of the individual oscillations will become $=AB\sqrt{\tfrac{b}{2ga}}$; indeed it is evident all these times to be equal to each other, because, if there may be taken $t\sqrt{\tfrac{2ga}{b}}=2AB$, or $=4AB$, or $=6AB$, it will produce always $q=Q$ and $\mathfrak{T}=\Upsilon$; moreover it is required to be noted the formula $\sqrt{\tfrac{2ga}{b}}$ expresses the distance, through which sound is propagated in one second. For the sake of brevity we may put this distance $=k$, which is around 1000 feet, and since the time of each oscillation is $=\tfrac{AB}{k}$, in one second $\tfrac{k}{AB}$ oscillations will be resolved, to which the number of sounds thence produced may be put in place proportional.

COROLLARY 1

60. Therefore if the length of an open tube of equal width shall be $AB=d$ and the air contained in that may be disturbed from equilibrium, it will receive an oscillatory motion, in which $\tfrac{k}{d}$ oscillations will be performed in a time of one second, with k denoting the length through which the sound will be propagated in one second: and hence the hearing perceives the number represented by the number $\tfrac{k}{d}$.

COROLLARY 2

61. But since there is $k=\sqrt{\tfrac{2ga}{b}}$ where a can denote the height of mercury in a barometer, if the density of the mercury may be shown by one and b may be written for the density of air, then it is evident these sounds depend partially on the elasticity of the air and

partially on its density: thus so that the greater elasticity will render the sound sharper, truly the greater the density the deeper.

COROLLARY 3

62. If the length of the tube shall be of one foot, on account of $k = 1000$ ft., 1000 oscillations will be produced per second, and for a tube of length 8 feet, 125 oscillations, to which number the sound is accustomed to be indicated by the letter C in musical instruments. Hence for any musical sound the length of the open tube in harmony can be assigned.

SCHOLIUM 1

63. Therefore whatever disturbance were induced to the air contained in the tube *AB*, whether by an impressed motion or by a change in the density, an oscillatory motion will be generated in that, such as has been found. Yet meanwhile it can happen, that the number of oscillations may become greater either by two, three, or four, or by any multiple ratio ; that which will depend on the nature of the first disturbance. If indeed both scales determining the disturbances *AQB* and *ETF* may be formed thus, so that scales similar to the scale continued through twice the interval AA' or through the triple interval AB' , or the quadruple $A'A''$ etc., an oscillatory motion thus will be had, as if the tube *AB* were twice, or three or four times shorter, and thus the number of oscillations produced per second will become greater either by two, or three, or four; from which continually sharper sounds according to the numbers 1, 2, 3, 4 etc. will arise. But since for these sharper sounds, the property of the first individual disturbance shall be required, these for the following are required to be had, while the first sound is that, which is indicated by the formula $\frac{k}{AB}$, since here it demands no condition of this kind. Therefore from the presence of the principal sound $= \frac{k}{AB}$, the following sounds will become

$$\frac{2k}{AB},\ \ \frac{3k}{AB},\ \ \frac{4k}{AB}\ \text{ etc.}$$

SCHOLIUM 2

64. This property of sounds agrees so finely with these, which flutes blown into produce, so that there may be no doubt, why the sounds of flutes should not be explained from this principle. Indeed while the flute is blown into, the air contained in the tube not only by reason of the density being disturbed from the state of equilibrium, but also because a certain motion may be started up, which may be propelled through the tube *AB*; from which a certain scale of the density *AQB* and a certain scale of the speed $E\Upsilon F$ may arise , the applied lines of which will be positive, indicating motion along the direction *AB*. Therefore from a disturbance of this kind oscillatory motion and hence the sound is produced, and all the circumstance such as we have defined agree with our theory exceptionally well; while also all these secondary sounds as well as the primary ones produced are able to be grasped. But here it is especially noteworthy with the continued

blowing of the air contained in the tube also the air expelled from the tube to be imbued continually with an oscillatory motion and thus from that the propagation to be effected more easily into the external air; and is required to be attributed to this cause, because with the cessation of the inflation suddenly also the sound may be extinguished, since still the theory of oscillatory motion must endure in the tube itself for a long time.

SCHOLIUM 3

65. Thus indeed it is understood, why with the blowing ceasing, flutes will produce no further sound, but yet truly in general, when the same air remains in the tube, the reason is less evident, why the oscillatory motion may stop there so soon, and why in the cases of the preceding chapter the repetitions by no means as many tones may be heard, as the theory indicates, in which case the theory disagrees the most with experience. Indeed in the present case in the elapsed time, for however large a calculation, now the equally strong disturbance of the air shown in the tube and at once from the beginning, yet meanwhile we know from experience the whole initial disturbance produced soon disappears completely, unless it may be renewed again continually, from which the most serious argument against our theory is seen to be able to demand. Truly besides most other impediments of the motion arising from the nature of the tube must be seen chiefly to be required to be diminished ; indeed even if the tube shall be constructed from the hardest material, yet there is always a small decrease in the disturbances of the air from that, since still in the calculation the walls of the tube may be assumed to be endowed with the greatest stiffness, so that plainly they shall not be capable of any impression or disturbance; since in practice the opposite to this condition may apply especially, it is little wonder, when the agitations of the air of this kind vanish so quickly. But most materials, from which such tubes are accustomed to be made, evidently there may be lacking in the degree of stiffness, which may be required for the continuance of the oscillatory motion, on account of that single reason, even if the other impediments may not be present, this motion may not be able to endure for a long time. Yet meanwhile, where the material of the tube were stronger and harder, there the practice may be taken to depart less from the theory.

PROBLEM 79

66. *If* (Fig. 87) *a tube of equal cross-section were open at the one end AA, truly were closed at the other end BB, and the air contained in that may be disturbed from the state of equilibrium in some manner, to determine the motion of the oscillations, by which the air in the tube henceforth will be disturbed.*

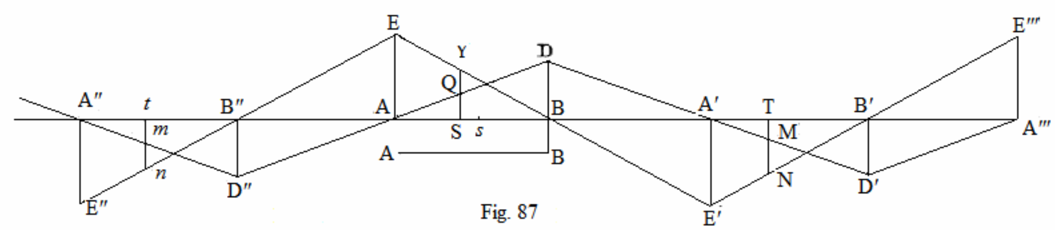

Fig. 87

SOLUTION

The most important features of the air moving around in the tube about *S* shall be the density induced $=Q$, with the natural density being $=B$, likewise the speed $=\Upsilon$ along the direction *AB*, thence the two scales will be formed, so that for the scale of the density *AD* the applied line shall be $SQ=l\frac{Q}{B}=\frac{Q-B}{B}$, for the scale of the speed *EB* truly with the applied line $S\Upsilon=\Upsilon\sqrt{\frac{b}{2ga}}$, of which for that scale, since the tube is open at *AA*, must pass through the point *A*, truly for this scale, since the tube is closed at *BB*, must pass through the point B. Now it will be required thus for these scales to be continued in both directions, so that with the axis *AB* taken produced with the intervals *BA'*, *A'B'*, *B'A'''* etc., likewise with *AB''*, *B''A''* etc. themselves equal to *AB* , *BD* shall be the diameter of the scale of the density, from which that will pass through *A'* thus, so that the branches around *A'* shall be alternately equal, thus so that the branch *A'D'* shall be equal and similar to the arc *A'D*, but placed on contrary sides of the axis, which likewise prevails for the point *A*, then truly the right lines *B'D'* and *B'' D''* again will be diameters, and the points *A''* and *A'''* the centres of equal alternate branches as the points *A* and *A'*, and so on thus again on each side indefinitely. But for the continuation of the scale of the speed *EB* the right line *AE* is its diameter, truly the points *B* and *B'* the centres of the equal branches, from which the right lines *A'E'*, *A''E''* in turn will be the diameters and thus both scales may be continued readily indefinitely. So that if now the time elapsed from the first disturbance shall be $=t$, the intervals $ST=St=t\sqrt{\frac{2ga}{b}}$ may be cut off on each side from the point *S* on the axis, and there with each scale of the applied lines drawn *TM*, *TN*, *tm*, *tn*, since these fall against each other here, as we have assumed above, the particles of the air, which initially were at *S*, now the density *q* will become

$$=Q(1-SQ+\tfrac{1}{2}TN-\tfrac{1}{2}TM-\tfrac{1}{2}tn-\tfrac{1}{2}tm)$$

and the speed

$$\mathfrak{T}=-\tfrac{1}{2}\left(TN-TM+tn+tm\right)\sqrt{\frac{2ga}{b}}$$

along *AB*, truly this particle will be moved to *s*, so that the small displacement shall become

$$Ss=\tfrac{1}{2}TNtn-\tfrac{1}{2}SQTM+\tfrac{1}{2}SQtm,$$

evidently in as much as these areas fall above the axis, indeed these parts which fall below the axis are to be considered negative.

Moreover we may put the time lapse so far to be *t*, so that there shall be $ST = St = 2AB$ and thus $t = 2AB\sqrt{\frac{b}{2ga}}$, and it is evident the points *T* and *t* likewise to be situated in the intervals *A'B'* and *A''B''* and the point *S* in the interval *AB*, and thus to become $TM = tm = SQ$ and $TN = tn = ST$; from which after this time the density at *S* evidently $q = Q(1-2SQ)$ and the speed

$$\mathfrak{T} = -S\Upsilon\sqrt{\frac{2ga}{b}} = -\Upsilon.$$

Hence if in the first place the density at *S* were $Q = B+O$ evidently a little greater than the natural density *B*, on account of $SQ = \frac{O}{B}$ there will be now $q = B-O$, just as much smaller than the natural density, then truly also the speed of the first is equal to $\mathfrak{T}$, but directed into the opposite region; from which in place, which is opposite to the first direction, it is understood now the air to have completed one oscillation, because only after twice the time greater will it revert to the initial state, thus so that the time of each oscillation shall be required to be considered $= 2AB\sqrt{\frac{b}{2ga}}$. But so that it may extend to the translation of the small interval *Ss*, on account of

$$TNtn = -AE\Upsilon S - ABE + BS\Upsilon + AE\Upsilon S + AEB - BS\Upsilon = 0,$$
$$SQTM = SQDB + ABD - ASQ$$

and

$$SQtm = ASQ - ABD - SQDB = SQTM,$$

from which it is evident this translation to vanish. Therefore we conclude, if the interval traversed in one second may be put $\sqrt{\frac{2ga}{b}} = k$, the time of one oscillation to become $= \frac{k}{2AB}$ and the number of oscillations performed per second to become $\frac{k}{2AB}$, which number likewise hence will show the nature of the sound.

COROLLARY 1

67. Therefore when the tube is closed from one end, the individual oscillations will endure for twice as long, than if the same tube were open at each end, and therefore in the same time only half as many oscillations will be resolved. Or the tube with one end closed will produce a sound deeper by one octave, than if it were open at both ends.

COROLLARY 2

68. Moreover the sounds here in tubes open at each end, equally maintain a ratio between themselves reciprocal to the length, thus so that, where the tube were longer, there its sound shall be going to become deeper; from which, if the length of the tube

producing the tone *C* shall be known, which in this case as if it were 4 feet, for all the remaining musical tones the length will be easily assigned.

SCHOLION l

69. But thus here the first disturbance may not be allowed to be applied, so that twofold oscillations may become more frequent, just as comes about in use in the case of tubes open at each end, if indeed here with the same scales remaining, we may attribute twice the length *AA'* to the tube, and that opening at *A'* we may put closed, then the figure of the speed scale, which must fall on the point *A'*, must be contrary to this hypothesis, and the same contradiction arises, if the length of the tube were four times, or six, eight or according to any even number of times by which we may wish to multiply the length, thus so that a tube of this kind in no way may be adapted to producing the sound, which it may hold according to the principle ratio, so that the ratios $2:1$, $4:1$, $6:1$ or $2i:1$. But this reduction exceeds surprisingly well according to the odd numbers : for we may consider a tube three times longer *AB'* open at *A*, truly closed at *B*, in which a disturbance of this kind may be induced in the air, so that the scale of the density shall be *ADA'D'* and the scale of the speed *EBE'B'*, the form of which generally agrees with the prescribed conditions ; and since the continuation of both scales themselves may be had as before, also the same oscillations thence may arise, from which it is understood the first disturbance in a tube of this kind thus can be prepared, in order that the oscillatory motion thence brought forth may agree completely with the tube three times shorter, so that also it prevails with the fifth, seventh etc. Therefore since the principal tone of the tube *AB* closed at the other end shall be $=\frac{k}{2AB}$, the same tube also under certain circumstances will be able to produce these tones $=\frac{3k}{2AB},\ \frac{5k}{2AB},\ \frac{7k}{2AB},$ etc.

SCHOLIUM 2

70. Since we shall have explained the nature of flutes open at the far end so very clearly from the preceding problem, here much less doubt will be allowed, why the oscillatory motion defined here may not contain an account of flutes closed at the far end. Moreover the experimental evidence we have taken all agree well with this theory, since if it may be agreed the same flute, if it may be closed at the far end, a tone lower by one octave to be produced: then also now it has been observed, if flutes of this kind closed at the far end may be blown into in a certain way, it can happen, that a tone may be produced higher by three or five times, but at no time twice as high, which clearly would be higher by a single octave for the main scale. Moreover with tubes of this kind now we see air alternately to enter and in turn to be expelled, and which expelled air now be the aforesaid oscillatory motion, since this motion thus will be communicated easily by the external air and will be propagated in that, yet meanwhile on account of this reason, some distinction will be present between the sounds of open and closed flutes, from which it may be allowed to discerned. Then also it is no wonder, that the tones of flutes may depend very little on the material from which the flutes have been constructed, and

thence evidently from that they bear a certain characteristic. Indeed now we may note the material of tubes can impede the motions of most oscillations, and now it may be allowed to add a certain rotational motion to the air to be shared with the tube itself, from which henceforth the sound of the flute itself in turn is affected. But if we may only consider the depth and sharpness of the sounds, an account of the length alone is required to be had and neither the material nor the form of any tube thus will be considered, provided that they shall be tubes of equal width. Yet flutes of unequal sizes shall be going to be produced sounds of this kind, the question is of a higher order of investigation, of which one would scarcely be allowed to hope for the solution.

PROBLEM 80

71. *If an equally wide tube* (Fig. 88) *were closed at each end AA and BB and the air contained in that were disturbed from the state of equilibrium, thence to determine the motion of the oscillations of the air then arising.*

SOLUTION

The initial density of the air induced in the tube at the location *S* shall be $=Q$, the density of the normal air being $=B$, truly the speed in the region $AB=\Upsilon$, and there may be taken

$$SQ = l\tfrac{Q}{B} = \tfrac{Q-B}{B} \quad \text{and} \quad S\Upsilon = \Upsilon\sqrt{\tfrac{b}{2ga}},$$

from which each scale of the density *CD* and of the speed $A\Upsilon B$ may be constructed, of which these by necessity must pass through the points *A* and *B*. Then following the precepts given each scale may be continued, produced above the axis by continued replication in each direction,

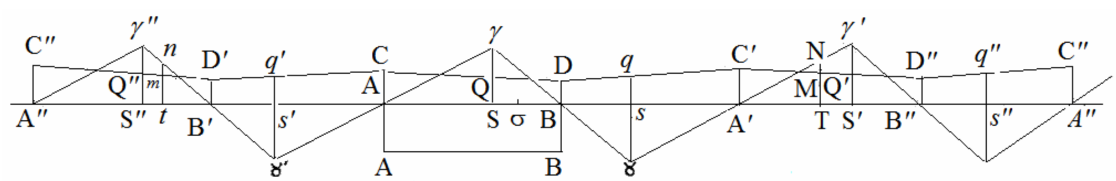

Fig. 88

as may be allowed to be seen from the figure. With which done, if after some lapse the time $=t$ on each side from the point *S* the intervals may be taken $ST = St = t\sqrt{\tfrac{2ga}{b}}$, now from each side the scales for the applied lines at the points *T* and *t* , now the properties of the air which initially was for *S*,

I. density $\quad q = Q(l - SQ - \tfrac{1}{2}TN + \tfrac{1}{2}TM + \tfrac{1}{2}tn + \tfrac{1}{2}tm)$

II. Speed $\quad T = \tfrac{1}{2}(TN - TM + tn + tm)\sqrt{\tfrac{2ga}{b}}$

and besides the interval, by which that will be advanced towards B , surely

$$\text{III.} \qquad S\sigma = \tfrac{1}{2}TNtn - \tfrac{1}{2}SQTM + \tfrac{1}{2}SQtm$$

and hence therefore the state of the air in the tube will be defined at some time t . Now we may put the elapsed time to become $t = 2AB\sqrt{\frac{b}{2ga}}$, so that the points T and t fall at S' and S'' agreeing with the point S itself, and then in place of S

$$\text{I. density} \qquad q = Q(1 - SQ - \tfrac{1}{2}S\Upsilon + \tfrac{1}{2}SQ + \tfrac{1}{2}S\Upsilon + \tfrac{1}{2}SQ) = Q$$

$$\text{II. speed} \qquad \mathfrak{T} = \tfrac{1}{2}(S\Upsilon - SQ + S\Upsilon + SQ)\sqrt{\frac{2ga}{b}} = S\Upsilon\sqrt{\frac{2ga}{b}} = \Upsilon$$

$$\text{III. interval} \quad S\sigma = 0,$$

thus so that the air shall be returned now to its original state, since with the lapse in the time only diminished by half $t = 2AB\sqrt{\frac{b}{2ga}}$ almost the opposite state may be had. Therefore since in that time it shall be agreed that two oscillations to be completed, the time for a single oscillation will be

$$= AB\sqrt{\frac{b}{2ga}} = \frac{AB}{k},$$

with the distance $k = \sqrt{\frac{2ga}{b}}$, through which the air is propagated in a time of one second; from which the number of complete oscillations produced per second will be $= \frac{k}{AB}$, which is had likewise for the measure of the sound produced.

COROLLARY 1

72. If the tube $= AA'$ were twice as long with the same scales remaining, certainly which for this tube both ends can be agreed closed, the same oscillatory motion may arise, from which it can happen that the tube closed at both ends may produce the same sound as an open tube half as long, which likewise prevails, if a tube longer by three or four times may be taken.

COROLLARY 2

73. Therefore since the fundamental tone shall be $= \frac{k}{AB}$, the same tube with both ends closed, if the first disturbance may be tempered in a certain way, it will be able to produce these tones also $\frac{2k}{AB}$, $\frac{3k}{AB}$, $\frac{4k}{AB}$ etc. : and generally there will be the same account of these tubes and of tubes open at both ends, while on the other hand those, which are open at one end, closed at the other end, produce tones one octave lower.

SCHOLIUM 1

74. Moreover just as it shall be able to induce a disturbance of this kind in an enclosed tube of air, from which also it is not at all clear, how any examples of tones generated by this method may be permitted to pass out : as well, in whatever way a certain disturbance may be set in motion there, since the tube is closed at both ends, neither may an oscillatory motion be communicated to the external air nor shall the sound be able to be heard clearly. Indeed if the walls of the tube may vibrate at the same time, the sound will be propagated by these rather stronger by the external air than by the enclosed air: and moreover the material of the tube itself at once adopts the vibratory motion, from which it follows that the motion of the enclosed air may be not be disturbed especially, nor further laws follow from the calculation of the air found: if indeed where it is compressed more, the sides of that tube may move only by a very small amount, thus the surrounding air may be moved more weakly, and it is seen to be for this particular reason, that the motion of such air is extinguished so quickly, since following the theorem it ought still to be continuous ; which is required not only to be concerned with these tubes closed at each end, but also to be understood for all the remaining tubes. But in whatever manner this motion may become weakened on account of the material of the tube, yet the duration of the individual oscillations is not affected, the times of which are taken to agree always with the calculation, even if for that reason the oscillatory motion will be continually weakened, and soon completely extinguished.

SCHOLIUM 2

75. Moreover although the individual oscillations in these tubes are absolved in certain times, and in the same tube are unable to disagree in the reckoning of the low and high tones, yet in these a great difference can be present in the accounting of the remaining qualities, arising from the diverse nature of the pulses excited in the air. Indeed since the air in tubes of this kind may be able to be disturbed from the state of equilibrium in an infinite number of ways, while either the density alone may be changed in the individual elements of the air, or from these only a certain motion may be induced, or each may happen at the same time, hence the maximum diversity is able to arise in the motion of the individual oscillations, from which it will seem astonishing, that the same flute blown into always produces a sound of the same nature; but I am not talking here about the depth of the tone, which allows no variation, but concerned with that quality, by which we may distinguish the tones of flutes from the tones of other corded instruments. But because inflated flutes are excited into producing tones, it is easily understood hence both the scales of density and of speed cannot be varied indefinitely, but always by the inflated air both a certain condensation as well as a certain motion must be produced, thus so that here boundless variations may not be found: the reason for this characteristic, by which the tones of flutes differ from the rest of the instruments, without doubt is required to be attributed to this cause. From which, if perhaps some mode of the air may be able to be induced in tubes closed at both ends by the disturbance, indeed the tone may be agreed to be calculated from the reckoning of the depth and sharpness, but may be able to differ especially from the sound of flutes.

CAPUT III

DE MOTU OSCILLATORIO AERIS IN TUBIS AEQUALITER AMPLIS AD SONOS TIBIARUM EXPLICANDOS

PROBLEMA 78

59. *Si* (Fig. 86) *tubus AABB utrinque fuerit apertus aërque in eo cententus quomodocunque de statu aequilibrii deturbetur, motum oscillatorium, quo aër in tubo deinceps agitabitur, determinare.*

SOLUTIO

Aequilibrii primam perturbationem ad calculum revocaturi ponamus in loco tubi quocunque *S* densitatem esse $=Q$, naturali existente $=B$, et celeritatem secundum directionem *AB* ibi esse Υ; hinc supra axem *AB* in puncto *S* erigantur duae applicatae $SQ = l\frac{Q}{B}$ et $S\Upsilon = \Upsilon\sqrt{\frac{b}{2ga}}$, certa quadam linea recta pro unitate assumta, quo facto punctum *Q* erit in scala densitatum, Υ vero in scala celeritatum. Quia autem tubus in *A* et *B* est apertus, necesse est,

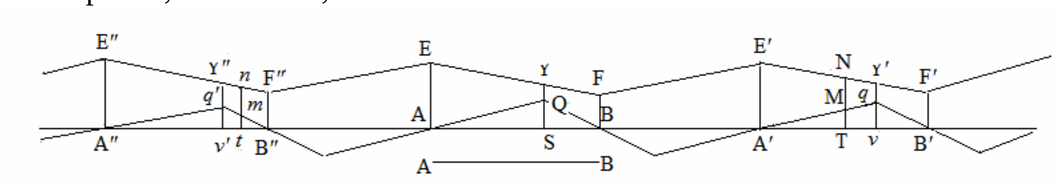

Fig. 86

ut scala densitatum in utroque termino *A* et *B* in axem incidat, uti *AQB;* tota autem scala celeritatum sit *ETF*. Nunc pro motu sequente definiendo utramque scalam secundum praecepta supra data utrinque in infinitum continuari oportet. Hunc in finem, axe utrinque producto in eoque abscissis intervallis *BA'*, *A'B'*, *AB''*, *B'' A''* etc. longitudini tubi aequalibus, scala primo densitatum *AQB* alternatim infra et supra axem applicetur, scala vero celeritatum ad eandem axis partem repetatur *FE'*, *E'F'* etc., *EF''*, *F'' E''* etc., eo ordine; quem figurae inspectio ostendit, singulas partes alternatim dextrorsum et sinistrorsum disponendo. His ita praeparatis si ad tempus quodcunque elapsum $=t$ statum aëris, qui initio erat in *S*, cognoscere velimus, a puncto *S* utrinque in axe abscindamus intervalla $ST = St = t\sqrt{\frac{2ga}{b}}$, atque in punctis *T* et *t* ad utramque scalam ductis applicatis supra ostendimus fore (13)

$$\text{densitatem } q = Q(1 - SQ - \tfrac{1}{2}TN + \tfrac{1}{2}TM + \tfrac{1}{2}tn + \tfrac{1}{2}tm)$$

$$\text{et celeritatem } \mathfrak{T} = \tfrac{1}{2}(TN - TM + tn + tm)\sqrt{\frac{2ga}{b}},$$

quo motu aër in *S* versus *B* promotus erit intervallo

$$=\tfrac{1}{2}TNtn-\tfrac{1}{2}SQTM+\tfrac{1}{2}SQtm,$$

quod ergo, si tota scala celeritatum supra axem sit sita, quantumvis magnum evadere potest, id quod evenit, dum aër continuo per tubum *AB* transfluit eoque continuo novus aër intrat, locum egressi occupans; cuius autem status binis prioribus formulis pro *q* et $\mathfrak{T}$ datis definitur. Iam observo, si tempus *t* tantum capiatur, ut fiat

$$t\sqrt{\tfrac{2ga}{b}}=2AB,$$

tum puncta *T* et *t* in *s* et *s′* cadere, quae puncta ipsi *S* sunt analoga, indeque fieri

$$q=Q \text{ et } \mathfrak{T}=S\Upsilon\sqrt{\tfrac{2ga}{b}}=\Upsilon,$$

ita ut nunc punctum *S* simulque omnis aër in tubo contentus in eundem statum sit reductus, in quo initio versabatur. Interea ergo hic aër duas oscillationes absolvisse est censendus, ex quo singularum oscillationum tempora erunt $=AB\sqrt{\tfrac{b}{2ga}}$; perspicuum enim est omnia haec tempora inter se esse aequalia, quia, sive capiatur $t\sqrt{\tfrac{2ga}{b}}=2AB$ sive $=4AB$ sive $=6AB$, semper prodit $q=Q$ et $\mathfrak{T}=\Upsilon$; notandum autem est formulam $\sqrt{\tfrac{2ga}{b}}$ exprimere distantiam, per quam sonus uno minuto propagatur. Ponamus hoc spatium brevitatis gratia $=k$, quod est circiter 1000 pedum, et quia tempus cuiusque oscillationis est $=\tfrac{AB}{k}$, uno minuto secundo absolventur $\tfrac{k}{AB}$ oscillationes, cui numero sonus inde editus proportionalis statuitur.

COROLLARIUM **1**

60. Si igitur tubi utrinque aperti et aequaliter ampli longitudo sit $AB=d$ aërque in eo contentus de statu aequilibrii deturbetur, motum oscillatorium recipiet, quo singulis minutis secundis $\tfrac{k}{d}$ oscillationes absolventur, denotante *k* spatium, per quod sonus uno minuto secundo propagatur: hincque auditus percipiet sonum numero $\tfrac{k}{d}$ repraesentatum.

COROLLARIUM 2

61. Quia autem est $k=\sqrt{\tfrac{2ga}{b}}$ ubi *a* denotare potest altitudinem mercurii in barometro, si densitas mercurii unitate exhibeatur et pro *b* densitas aëris scribatur, inde patet hos sonos partim ab elasticitate aëris pendere partim ab eius densitate: ita ut maior elasticitas sonum acutiorem reddat, maior vero densitas graviorem.

COROLLARIUM 3

62. Si tubi longitudo sit unius pedis, ob $k=1000$ ped. singulis minutis secundis edentur 1000 oscillationes et a tubo longitudinis 8 pedum 125 oscillationes, cui numero respondet

sonus in instrumentis musicis littera *C* indicari solitus. Hinc pro quovis sono musico longitudo tubi aperti consoni assignari potest.

SCHOLION 1

63. Quaecunque ergo agitatio aëri in tubo *AB* contento fuerit inducta, sive motus impressione sive densitatis immutatione, eiusmodi motus oscillatorius in eo generabitur, qualis est inventus. Interim tamen evenire potest, ut numerus oscillationum vel duplo vel triplo vel quadruplo vel in ratione quacunque multipla fiat maior; id quod a certa indole primae agitationis pendet. Si enim ambae scalae agitationem determinantes *AQB* et *ETF* ita formentur, ut similes fiant scalis per duplum spatium *AA'* continuatis vel per triplum *AB'* vel per quadruplum *A'A''* etc., motus oscillatorius perinde se habebit, ac si tubus *AB* esset vel duplo vel triplo vel quadruplo brevior, sicque numerus oscillationum uno minuto secundo editarum fiet vel duplo vel triplo vel quadrupla maior; unde soni continuo acutiores secundum numeros 1, 2, 3, 4 etc. orientur. Cum autem ad hos sonos acutiores singularis proprietas primae agitationis requiratur, ii pro secundariis sunt habendi, dum sonus primarius est is, qui formula $\frac{k}{AB}$ indicatur, quia hic nullam huiusmodi conditionem singularem postulat. Existente ergo sono principali $=\frac{k}{AB}$, soni secundarii erunt

$$\frac{2k}{AB},\quad \frac{3k}{AB},\quad \frac{4k}{AB}\ \text{etc.}$$

SCHOLION 2

64. Haec sonorum proprietas tam pulcre convenit cum iis, quos tibiae inflatae edunt, ut nullum sit dubium, quin soni tibiarum ex hoc principio explicari debeant. Dum enim tibia inflatur, aër in tubo contentus non solum ratione densitatis de statu aequilibrii deturbatur, sed etiam ad motum quendam concitatur, quo per tubum *AB* propellitur; unde nascetur scala quaedam densitatum *AQB* et celeritatum $E\Upsilon F$, cuius applicatae erunt positivae, motum secundum directionem *AB* indicantes. Ex huiusmodi igitur agitatione eiusmodi motus oscillatorius indeque sonus nascitur, qualem definivimus omnesque circumstantiae observatae cum nostra Theoria egregie conveniunt; dum etiam eadem tibia omnes illos sonos secundarios praeter primarium edere posse deprehenditur. Imprimis autem hic notandum est continua inflatione aërem in tubo contentum etiam motu oscillatorio imbutum continuo ex tubo expelli sicque eo facilius propagationem in aëre externo effici; huicque causae esse tribuendum, quod cessante inflatione sonus subito quoque extinguatur, cum tamen secundum Theoriam motus oscillatorius in ipso tubo diu durare deberet.

SCHOLION 3

65. Sic quidem intelligitur, cur inflatione cessante tibiae nullum amplius sonum edant, verumtamen in genere, quando idem aër in tubo manet, ratio minus est perspicua, cur motus oscillatorius in eo mox extinguatur et cur in casibus praecedentis capitis neutiquam tot vocis repetitiones percipiantur, quot Theoria indicat, in quo Theoria plurimum ab

experientia dissentit. In praesente enim casu elapso tempore quantumvis magno calculus etiam nunc aeque vehementem aëris agitationem in tubo ostendit ac statim ab initio, interim tamen experientia novimus totam agitationem initio productam mox penitus interire, nisi continuo de novo instauretur, unde certe gravissimum argumentum contra nostram Theoriam peti posse videtur. Verum praeter plurima alia motus impedimenta ipsa tubi indoles ad hanc obiectionem diluendum potissimum spectari debet; etiamsi enim tubus ex durissima materia sit confectus, tamen ea aëris agitationibus semper aliquantillum cedit, cum tamen in calculo latera tubi tanto rigore praedita assumantur, ut nullius plane impressionis et agitationis sint capacia; cui conditioni cum praxis maxime adversetur, minime mirum est, quando huiusmodi aëris agitationes tam cito pereunt. Pleraeque autem materiae, ex quibus tales tubi confici solent, manifesto eo rigoris gradu destituuntur, qui ad perennitatem motus oscillatorii requireretur, ob eamque solaro causam, etiamsi alia impedimenta non adessent, is motus diu durare non posset. Interim tamen, quo fortior et durior fuerit tubi materia, eo minus experientia a Theoria recedere deprehenditur.

PROBLEMA 79

66. *Si* (Fig. 87) *tubus aequaliter amplus in altero termino AA fuerit apertus, in altero vero BB clausus, aërque in eo contentus utcunque de statu aequilibrii deturbetur, motum oscillatorium, quo aër in tubo deinceps agitabitur, determinare.*

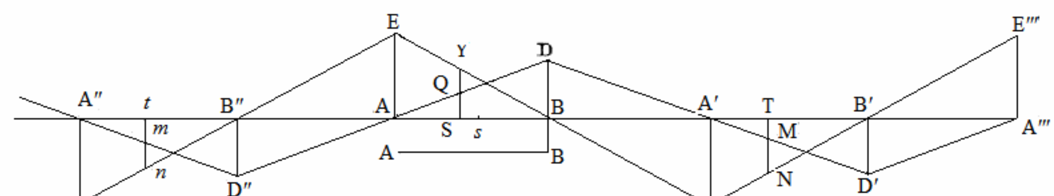

Fig. 87

SOLUTIO

Primo agitationis momento aëri in tubo circa *S* versanti sit inducta densitas $=Q$, naturali existente $=B$, simulque celeritas $=T$ secundum directionem *AB*, inde binae scalae ita formentur, ut pro scala densitatum *AD* sit applicata $SQ=l\frac{Q}{B}=\frac{Q-B}{B}$, pro scala vero celeritatum *EB* applicata $S\Upsilon=\Upsilon\sqrt{\frac{b}{2ga}}$, quarum illa scala, quia tubus ad *AA* est apertus, per punctum *A*, haec vero, quia tubus ad *BB* est clausus, per punctum *B* transire debet. Has iam scalas utrinque ita continuari oportet, ut sumtis in axe *AB* producto intervallis *BA′*, *A′B′*, *B′A‴* etc., item *AB″*, *B″A″* etc. ipsi *AB* aequalibus *BD* sit diameter scalae densitatum, unde ea per *A′* ita transibit, ut rami circa *A′* sint alternatim aequales, ita ut ramus *A′D′* aequalis sit et similis arcui *A′D*, sed ad axis partes contrarias positus, quod idem de puncto *A* valet, tum vero rectae *B′D′* et *B″ D″* iterum erunt diametri, punctaque *A″* et *A‴* centra ramorum alternatim aequalium ut puncta *A* et *A′*, sicque porro utrinque in infinitum. Pro scalae celeritatum continuatione autem recta *A E* est eius diameter, puncta vero *B* et *B′* centra ramorum alternatim aequalium, unde rectae *A′E′*, *A″E″* iterum erunt diametri sicque ambae scalae facile in infinitum continuantur. Quodsi iam a prima agitatione elapsum sit tempus $=t$, a puncto *S* utrinque super axe abscindantur spatia $ST=St=t\sqrt{\frac{2ga}{b}}$, ibique ductis ad utramque scalam applicatis *TM*, *TN*, *tm*, *tn*, quia eae hic contra cadunt, ac supra assumsimus, particulae aëris, quae initio erat in *S*, nunc erit

$$\text{densitas } q=Q(1-SQ+\tfrac{1}{2}TN-\tfrac{1}{2}TM-\tfrac{1}{2}tn-\tfrac{1}{2}tm)$$

et

$$\text{celeritas } \mathfrak{T}=-\tfrac{1}{2}\left(TN-TM+tn+tm\right)\sqrt{\tfrac{2ga}{b}}$$

secundum *AB*, ipsa vero haec particula translata erit in *s*, ut sit spatiolum

$$Ss=\tfrac{1}{2}TNtn-\tfrac{1}{2}SQTM+\tfrac{1}{2}SQtm,$$

quatenus scilicet hae areae supra axem cadunt, quae partes enim infra axem cadunt, negativae sunt censendae.

Ponamus autem tantum elapsum esse tempus *t*, ut sit $ST=St=2AB$ ideoque $t=2AB\sqrt{\frac{b}{2ga}}$, et manifestum est puncta *T* et *t* in intervallis *A'B'* et *A"B"* similiter esse sita ac punctum *S* in intervallo *AB*, ideoque fore $TM=tm=SQ$ et $TN=tn=ST$; unde post hoc tempus erit densitas in *S* nempe $q=Q(1-2SQ)$ et celeritas

$$\mathfrak{T}=-S\Upsilon\sqrt{\tfrac{2ga}{b}}=-\Upsilon.$$

Hinc si initio fuerit in *S* densitas $Q=B+O$ naturali scilicet *B* tantillo maior, ob $SQ=\frac{O}{B}$ erit nunc $q=B-O$, tantundem minor naturali, tum vero etiam celeritas $\mathfrak{T}$ primae est aequalis, sed in plagam oppositam directa; ex quo statu, qui primo directe est contrarius, intelligitur iam aërem unam oscillationem absolvisse, quia nonnisi post tempus duplo maius in statum initialem revertetur, ita ut tempus cuiusque oscillationis sit censendum $=2AB\sqrt{\frac{b}{2ga}}$. Quod autem ad translationis spatiolum *Ss* attinet, ob

$$TNtn=-AE\Upsilon S-ABE+BS\Upsilon+AE\Upsilon S+AEB-BS\Upsilon=0,$$
$$SQTM=SQDB+ABD-ASQ$$

et

$$SQtm=ASQ-ABD-SQDB=SQTM,$$

unde patet hanc translationem evanescere. Concludimus ergo, si spatium a sono uno minuto secundo percursum ponatur $\sqrt{\frac{2ga}{b}}=k$, tempus unius oscillationis fore $=\frac{k}{2AB}$ et singulis minutis secundis absolvi oscillationes $\frac{k}{2AB}$, qui numerus simul naturam soni hinc editi exhibet.

COROLLARIUM 1

67. Quando ergo tubus ex altera parte est clausus, singulae oscillationes duplo longius durant, quam si idem tubus utrinque esset apertus, eodemque propterea tempore dimidium tantum oscillationum numerum absolvit. Seu tubus ex altera parte clausus sonum una octava edit graviorem, quam si esset utrinque apertus.

COROLLARIUM 2

68. Soni autem hic aeque atque in tubis utrinque apertis inter se tenent rationem reciprocam longitudinum, ita ut, quo tubus fuerit longior, eius sonus eo futurus sit gravior; unde, si tubi longitudo sonum *C* edentis sit cognita, quae hoc casu quasi 4 erit pedum, pro omnibus reliquis sonis musicis longitudo facile assignabitur.

SCHOLION 1

69. Hic autem primam agitationem non ita accommodare licet, ut oscillationes duplo fiant frequentiores, quemadmodum in casu tuborum utrinque apertorum usu venit, si enim hic iisdem manentibus scalis tubo longitudinem duplam AA' tribuamus eumque in A' clausum ponamus, figura scalae celeritatum, quae in punctum A' incidere deberet, huic hypothesi adversatur, eademque repugnantia deprehenditur, si longitudinem tubi quater vel sexies vel octies vel secundum quemvis numerum parem multiplicare vellemus, ita ut huiusmodi tubus nullo modo ad sonum, qui ad principalem teneat rationem, ut $2:1,\ 4:1,\ 6:1$ seu $2i:1$ edendum sit aptus. Secundum numeros impares autem haec reductio egregie succedit: concipiamus enim tubum triplo longiorem AB' in A apertum, in B vero clausum, in quo aëri eiusmodi sit agitatio inducta, ut scala densitatum sit $ADA'D'$ et scala celeritatum $EBE'B'$, quarum forma conditionibus praescriptis utique convenit; et cum continuatio ambarum scalarum se quoque habeat ut ante, oscillationes etiam eaedem inde nascentur, ex quo intelligitur in huiusmodi tubo primam agitationem ita comparatam esse posse, ut motus oscillatorius inde genitus prorsus conveniat cum tubo triplo breviori, quod etiam de quintupla, septuplo etc. brevioribus valet. Cum igitur tubi AB in altera parte clausi sonus principalis sit $=\frac{k}{2AB}$, idem tubus quoque sub certis circumstantiis edere poterit hos sonos $=\frac{3k}{2AB},\ \frac{5k}{2AB},\ \frac{7k}{2AB},$ etc.

SCHOLION 2

70. Cum ex praecedente problemate naturam tibiarum superne apertarum tam dilucide explicaverimus, multo minus hic dubitare licet, quin motus oscillatorius hic definitus rationem contineat tibiarum superne clausarum. Experimenta autem consulentes omnia egregie cum hac theoria consentire deprehendimus, cum constet eandem tibiam, si superne claudatur, sonum una octava graviorem esse edituram: deinde etiam iam observatum est, si huiusmodi tibiae superne clausae certo modo inflentur, fieri posse, ut

sonus edatur triplo vel quintupla altior, nunquam autem duplo altior, qui scilicet principali foret una octava altior. In huiusmodi autem tibiis iam videmus aërem alternis vicibus in eas intrare iterumque expelli, qui aër expulsus et iam descripto motu oscillatorio praeditus, cum externo aëre hunc motum eo facilius communicabit in eoque propagabit, interim tamen ob hanc ipsam causam discrimen aliquod inerit inter sonos tibiarum apertarum et clausarum, unde eos dignoscere liceat. Deinde etiam mirum non est, quod tibiarum soni haud parum a materia, ex qua tibiae sunt confectae, pendeant indeque notam quandam manifestam secum gerant. Iam enim animadvertimus materiam tuborum motum oscillatorium plurimum impedire posse, et nunc addere licet ab aëre etiam quendam motum gyratorium cum ipso tubo communicari, a quo deinceps sonus tibiae vicissim afficitur. At si gravitatem et acumen sonorum tantum spectemus, solius longitudinis ratio est habenda neque materia vel forma tubi quicquam eo conferet, dummodo sint tubi aequaliter ampli. Cuiusmodi tamen sonos sint editurae tibiae inaequaliter amplae, quaestio est altioris indaginis, cuius solutionem vix adhuc sperare licet.

PROBLEMA 80

71. *Si* (Fig. 88) *tubus aequaliter amplus in utroque termina AA et BB fuerit clausus et aër in eo contentus utcunque de statu aequilibrii deturbetur, motum oscillatorium aëris inde oriundum determinare.*

SOLUTIO

Sit densitas aëris initio in tubi loco S inducta $=Q$, naturali existente $=B$, celeritas vero in plagam $AB=\Upsilon$, capiaturque

$$SQ=l\tfrac{Q}{B}=\tfrac{Q-B}{B} \quad \text{et} \quad S\Upsilon=\Upsilon\sqrt{\tfrac{b}{2ga}},$$

unde utraque scala densitatum CD et celeritatum $A\Upsilon B$ construatur, quarum haec necessario per puncta A et B transire debet. Tum secundum praecepta data utraque scala utrinque super axe producto per continuam replicationem,

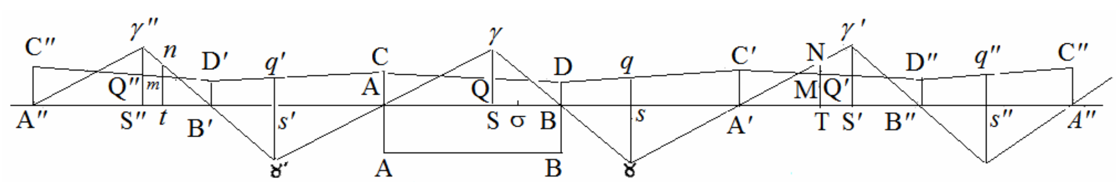

Fig. 88

uti ex figura videre licet, continuetur. Quo facto, si post tempus quodcunque elapsum $=t$ a puncto S utrinque capiantur intervalla $ST=St=t\sqrt{\tfrac{2ga}{b}}$, ex utriusque scalae applicatis in punctis T et t definitur nunc aëris, qui initio fuerat ad S,

I. densitas $q = Q(l - SQ - \frac{1}{2}TN + \frac{1}{2}TM + \frac{1}{2}tn + \frac{1}{2}tm)$

II. Celeritas $T = \frac{1}{2}(TN - TM + tn + tm)\sqrt{\frac{2ga}{b}}$

ac praeterea spatium, quo is versus *B* erit promotus, nempe

III. spatiolum $S\sigma = \frac{1}{2}TNtn - \frac{1}{2}SQTM + \frac{1}{2}SQtm$

hincque ergo status aëris in tubo ad quodvis tempus *t* definietur. Ponamus nunc tempus elapsum esse $t = 2AB\sqrt{\frac{b}{2ga}}$, ut puncta *T* et *t* cadant in *S′* et *S″* puncta ipsi *S* homologa, eritque tum in loco *S*

I. densitas $q = Q(1 - SQ - \frac{1}{2}S\Upsilon + \frac{1}{2}SQ + \frac{1}{2}S\Upsilon + \frac{1}{2}SQ) = Q$

II. celeritas $\mathfrak{T} = \frac{1}{2}(S\Upsilon - SQ + S\Upsilon + SQ)\sqrt{\frac{2ga}{b}} = S\Upsilon\sqrt{\frac{2ga}{b}} = \Upsilon$

III. spatiolum $S\sigma = 0$,

ita ut nunc aër in ipsum statum primitivum sit reductus, cum elapso tantum tempore dimidio $t = 2AB\sqrt{\frac{b}{2ga}}$ statum fere contrarium habuisset. Cum igitur illo tempore duas oscillationes absolvisse sit censendus, tempus singularum oscillationum erit

$$= AB\sqrt{\frac{b}{2ga}} = \frac{AB}{k},$$

denotante $k = \sqrt{\frac{2ga}{b}}$ spatium, per quod aër uno minuto secundo propagatur; unde numerus oscillationum singulis minutis secundis editarum erit $\frac{k}{AB}$, qui simul pro mensura soni hoc motu producti habetur.

COROLLARIUM 1

72. Si tubus esset duplo longior $= AA'$ iisdem manentibus scalis, quippe quae huic tubo utrinque clauso convenire possunt, motus oscillatorius idem orietur, unde fieri potest, ut tubus utrinque clausus eundem edat sonum ac tubus duplo brevior, quod idem valet, si tubus triplo vel quadrupla etc. longior acciperetur.

COROLLARIUM 2

73. Cum igitur sonus principalis sit $= \frac{k}{AB}$, idem tubus utrinque clausus, si prima agitatio certo quodam modo temperetur, hos quoque sonos $\frac{2k}{AB}$, $\frac{3k}{AB}$, $\frac{4k}{AB}$ etc. edere poterit : omninoque horum tuborum eadem erit ratio ac tuborum utrinque apertorum, dum contra ii, qui ex altera parte sunt aperti, ex altera vero clausi, sonos una octava graviores edunt.

SCHOLION 1

74. Quemadmodum autem aëri huiusmodi tubo incluso agitatio induci possit, haud liquet, unde etiam nulla exempla sonorum hac ratione genitorum proferre licet: quin etiam, quamvis ibi quaedam agitatio excitaretur, tamen, quia tubus undique est clausus, motus oscillatorius neque cum aëre externo communicari neque sonus exaudiri posset. Si enim tubi latera simul contremiscant, eorum sonus proprius potius quam aëris inclusi per aërem externum propagabitur: statim autem, atque ipsa tubi materia motum vibratorium concipit, aëris inclusi motus maxime turbatur neque amplius leges per calculum inventas sequitur: si enim aëri, ubi est magis compressus, ipsa tubi latera aliquantillum cedant, eo debilius inde aër vicinus concitabitur, haecque praecipua causa esse videtur, cur talis aëris motus tam cito extinguatur, cum tamen secundum Theoriam perennis esse deberet; quod non solum de his tubis utrinque clausis, sed etiam reliquis omnibus est intelligendum. Quomodocunque autem haec motus debilitatio ob tubi materiam eveniat, inde tamen duratio singularum oscillationum non afficitur, quarum tempora cum calculo semper consentire deprehenduntur, etsi ob illam causam motus oscillationum continuo debilitetur et mox prorsus extinguatur.

SCHOLION 2

75. Quanquam autem in his tubis singulae oscillationes certis temporibus absolvuntur atque in eodem tubo soni ratione gravitatis et acuminis discrepare non possunt, tamen in iis ratione reliquarum qualitatum maximum discrimen inesse potest, a diversa indole pulsuum in aëre excitatorum oriundum. Cum enim aër in huiusmodi tubis infinitis modis de statu aequilibrii deturbari possit, dum vel sola densitas in singulis aëris elementis alteratur, vel iis tantum motus quidam inducitur, vel utrumque simul evenit, hinc maxima diversitas in motu singularum oscillationum oriri poterit, unde mirum videbitur, quod eadem tibia inflata sonum semper eiusdem indolis edat; non loquor autem hic de gravitate, quae nullam variationem patitur, sed de ea qualitate, qua sonos tibiarum a sonis cordarum aliorumque instrumentorum distinguimus. Quia autem tibiae inflatione ad sonos edendos excitantur, facile intelligitur hinc ambas scalas densitatum et celeritatum non in infinitum variari posse, sed semper ab aëre inflato tam certam condensationem quam certum motum produci debere, ita ut hic non amplius infinita varietas locum invenire queat: huicque causae qualitas illa, qua soni tibiarum a reliquis instrumentis discrepant, sine dubio est tribuenda. Unde, si forte quocunque modo aëri in tubis utrinque clausis agitatio induci posset, sonus quidem ratione gravitatis et acuminis calcula foret consentaneus, sed a sono tibiarum maxime differre posset.