CHAPTER SEVEN

CONCERNING THE MOTION OF BODIES FLOATING ON WATER

PROPOSITION 74

PROBLEM

762. *If some body, secured in the plane of its vertical diameter, may be moved straight through water at rest* (Fig. 101), *to determine the motion with which it shall begin to move, to be diminished by the resistance arising from the water, and the speed at the individual points of the path, which it will describe.*

SOLUTION

Since the body may be put with the diametrical plane secured vertical, the submerged part, certainly which is divided into two similar and equal parts by that plane, the centre of magnitude of which will be situated in that plane itself; from which the centre of gravity of the whole body also will be required to be located in this plane. Truly again since this body is put in place to maintain a straight course, thus so that it shall move following a horizontal straight line in the plane of the diameter itself, the mean direction of the resistance itself will lie in this plane. Therefore the force of the resistance will be directly opposite to the direction of the motion, and on this account will only retard the motion ; truly it will not affect the direction of the motion. Truly the force of the vertical resistance, if which may be present, will disturb neither the motion of the body nor its direction, but will be used up only in raising the body. Then, unless the mean direction of the resistance shall pass through the centre of gravity of the body, the body will be inclined about the latitudinal axis [*i.e. AB* in the diagram], by which inclination neither the direction of the motion, nor the position of the keel or axis of the ship leading from the prow to the stern of the ship, will be changed. On account of which otherwise the motion of the ship will not be disturbed by the resistance of the water, except for the diminution of the speed ; and both the direction of the motion, as well as the straight course of the ship, will be maintained. With these observed, a horizontal section of the body *AEBF* shall be made through its centre of gravity *G*, with the horizontal right line *AB* in the diametric plane extended from the prow *A* to the stern *B* which likewise will represent the direction of the motion, and the right line *CGL* will represent the manner in which the centre of gravity may be introduced, in which likewise both the prow *A* as well as the stern *B* will remain in place permanently. Now we may put the body to be starting from the point *C,* where the initial speed will be had

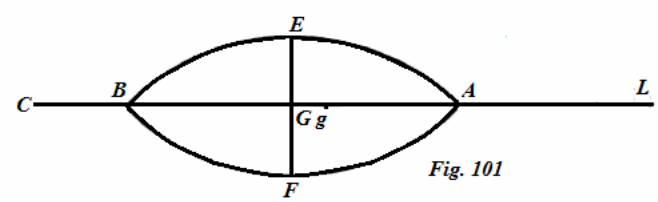


Fig. 101

corresponding to the altitude k; truly while the centre of gravity G is moving , its speed [at some instant], with which it will continue to progress in the direction GL shall correspond to an altitude υ. Again the mass or weight of the whole body $= M$, the volume of its submerged part $= V$; truly this body may experience a resistance of just as great a magnitude as may be experienced by a plane figure of area ff moving directly against the water with the same speed; from which the resistance, which the body may experience while its centre of gravity G is moving forwards, will be equal to the weight of the volume of water $ff\upsilon$, so that the weight of the submerged part itself will be had to the weight of the whole body M as ffu to V, thus so that the force of the resistance retarding the motion shall become equal to the weight $\frac{Mff\upsilon}{V}$.

[This is essentially Newton's law for resistance: the drag force on the plane area ff to be proportional to the square of the velocity, here represented by the equivalent altitude υ, where the resistance of the water is taken as 1. The resistance acting on the body shall be to the wt. of the body as the volume swept out $ff\upsilon$ to the volume of the submerged part V, $= ff\upsilon : V$; hence the resistive force becomes $\frac{Mff\upsilon}{V}$. This result can be regarded as no more than a rule of thumb; useful as it leads at once to a d.e., which may be integrated. The theory of ship bow waves lay far in the future at this stage: an interesting talk on the subject of Ship Waves was given by Sir William Thompson in Edinburgh at the Institute of Mechanical Engineers meeting on the evening of 3rd Aug. 1887, published in the proceedings of the R.S.E. for that year, which can be accessed on the web these days.]

Now the distance shall be $CG = s$, which the body has completed from the beginning, and while the element $Gg = ds$ will be transverse, the decrement of the speed shall be so that

$$-d\upsilon = \frac{ff\upsilon ds}{V};$$

which equation integrated gives

$$l\frac{k}{\upsilon} = \frac{ffs}{V}$$

with the integration put in place thus so that there may become $\upsilon = k$, on putting $s = 0$, as the condition of the question requires. Therefore there will become

$$\frac{k}{\upsilon} = e^{ffs:V},$$

with e denoting the number, of which the logarithm is $=1$, and hence again $u = ke^{-ffs:V}$, from which formula the speed of the body at the individual points of the path which the body describes is known. Finally since the speed itself shall be $= e^{-ffs:2V}\sqrt{k}$, the element of the time, in which the element $Gg = ds$ will be traversed, will be

$$= \frac{e^{ffs:2V}ds}{\sqrt{k}}$$

and thus the total time which has been spent, for the distance $CG = s$ being resolved, will become

$$= \frac{2V\left(e^{ffs:2V} - 1\right)}{ff\sqrt{k}}.$$

Q.E.I.

COROLLARY 1

763. Since the height corresponding to the speed of the body at G shall be $= ke^{-ffs:V}$, it is understood at no time is the body going to lose all its motion: indeed the speed does not vanish, unless there may be put $s = \infty$, that is the body actually must resolve an infinite distance before it shall lose all motion.

COROLLARY 2

764. The expression of the speed also can be transformed conveniently into a series, through which there will become

$$\upsilon = k - \frac{kffs}{V} + \frac{kf^4s^2}{2V^2} - \frac{kf^6s^3}{6V^3} + \frac{kf^8s^3}{24V^2} - \text{etc.}$$

which converges quickly, unless the distance s may be taken very large.

COROLLARY 3

765. Then also the decrease of the speed also thus is seen to be greater, when the body resolves the given distance s, where the greater will have been the area ff, to which we have reduced the resistance, and where the smaller will have been the part submerged in the water, that is where the lighter were the body.

COROLLARY 4

766. Therefore if several similar bodies may begin to move with the same speed, the resistance or area ff will be maintained under the three on two ratio of the weights, the submerged parts truly will hold the same ratio of the weights, from which it is understood the greater bodies to be slowed down less than the smaller ones. [$\frac{Mffu}{V}$.]

COROLLARY 5

767. Also the time, in which a given body may complete the given distance $CG = s$, may be expressed by this series, indeed will become

$$=\frac{s}{Vk}+\frac{f^2s^2}{4V\sqrt{k}}+\frac{f^4s^2}{24V^2\sqrt{k}}+\frac{f^6s^4}{192V^3\sqrt{k}}+\text{etc.}$$

But if the body may be progressing with an initial uniform motion, experiencing no resistance, then the time for the same distance s will become $\frac{s}{\sqrt{k}}$; from which quantity it is understood there is need for a greater time on account of the resistance.

COROLLARY 6

768. If the time may be desired, in which a body will traverse a given distance, in given units of time measurements, then in the expression of the time,

$$\frac{2V\left(e^{ffs:2V}-1\right)}{ff\sqrt{k}}$$

the quantities k, s, ff and V may be expressed in thousandth parts of Rhenish feet; with which done the time expressed divided by 250 will give the time in minutes and seconds.

COROLLARY 7

769. In a similar manner, if that speed may be desired to be expressed by the length, which in a given time may be traversed with that speed uniformly, the distance may be put which is resolved in a second if time to be n thousandth parts of a Rhenish foot, and there will become

$$\frac{n}{250\sqrt{\upsilon}}=1,$$

with υ given equally in thousandth parts of Rhenish feet, from which there will become $n=250\sqrt{\upsilon}$.

COROLLARY 8

770. But if moreover the speed n may be given by the distance traversed per second, and shall be given in the thousandth parts of a Rhenish foot, the height will be found corresponding to that speed

$$\upsilon=\frac{n^2}{62500},$$

equally in thousandth parts of the same feet: from which these two methods of measuring the speed to be compared with each other, and the one to be formed from the other.

[Note: 1 Rhenish foot = 12.356 inches..]

SCHOLIUM 1

771. Indeed it is not agreed on by experiment, that bodies floating on water may at no time be at rest, but must go on moving always: since it is agreed well enough the motion finally to cease completely. Truly it is required here to note that water, as well as having that resistance which is proportional to the square of the speed [of the moving body], has in addition another resistance to be put in place, not depending on the speed, but proportional to the interval of time, as NEWTON discusses, and the height of which corresponding to the speed must be diminished in the ratio of the elements of the distance traversed. But this resistance of the water is so great, that unless the motion shall be the slowest, that resistance will vanish besides the other resistance; and hence on that account in the solution of this problem we will ignore that resistance, since our principles shall be not to pursue the slowest motion from the professed investigation. Yet meanwhile this same resistance does not render the calculation more difficult; for this same constant resistance for the case offered shall be $=g$, or for the equivalent weight g, will produce in place of the equation:

$$-d\upsilon=\frac{ff\upsilon ds}{V}$$

that same equation

$$-d\upsilon=\frac{ff\upsilon ds}{V}+\frac{gds}{M};$$

which integrated gives

$$\upsilon=\left(k+\frac{gV}{ffM}\right)e^{-ffs:V}-\frac{gV}{ffM}.$$

Therefore from this equation it is certainly understood, the body is not going to be progressing beyond a given limit, since its speed shall vanish on traversing the distance s, the magnitude of this quantity will be given from this equation

$$e^{ffs:V}=\left(\frac{kffM+gV}{gV}\right)\quad\text{or}\quad s=\frac{V}{ff}l\frac{kffM+gV}{gV}.$$

Why not also from that same equation may that resistance g itself become known, from the distance traversed, then the whole motion will be lost, if indeed this distance may then be found from experiment to become $=s$,there will become

$$g=\frac{kffM}{V\left(e^{ffs:V}-1\right)},$$

defined from a single experiment, which will prevail for all cases, for which the same body will be moving through water in a straight line.

SCHOLIUM 2

772. We have introduced this chapter by a motion or direction of the course of the ship, and in addition to be rectilinear, and we have defined the diminution of this motion arising from the resistance. Moreover from these circumstances, of which we make mention in the solution, for the direction and linearity of the course requiring to be conserved, likewise it will be required to gather how the matters themselves may be allowed to be disturbed. Evidently in the first place the rectilinear motion may be disturbed, if the direction of the mean resistance will not lie in the diametric plane, or if the horizontal force from that may arise in a direction not directly opposite to the direction of the motion; indeed with everything arising satisfactorily, it is apparent if the direction of the resistance does not agree with the direction of the motion, then not only will the motion be retarded, but also to be deflected from rectilinear path; which indeed pertain only to the progressive motion of the centre of gravity, which we consider here especially. But even if the motion may not be made in a straight line, yet the direction of the course can remain, evidently if the axis of the longitude drawn from the prow to the stern may remain drawn parallel to the direction of the motion; indeed through the direction of the course we understand a motion of the ship of this kind, the direction of which proceeds directly from the stern to the prow, and in which the same anterior part of the ship is placed opposite the water resistance. Therefore when forces of this kind may be present, which may rotate around the vertical axis, even if these may not affect the progressive motion, yet they may disturb the direction of the course, and they may produce an oblique course. Whereby since in the case proposed, no forces of this kind shall be present, also not only is the motion found to be performed along a right line, but also the direction of the course remains constant. Therefore at first we set up the direction of the course and likewise subject the right lines to be examined, both in water at rest as well as flowing, and that it will be required to propose cases concerning this kind, for which both the direct course as well as the rectilinear motion may be preserved; with which cases set out more easily to proceed to the oblique courses and the curvilinear motion requiring to be examined. But these floating bodies themselves, whether they shall be either free or abandoned, or not free or restricted as if bound to some anchor will supply the requirements of the first division of this first chapter. Then truly subdivisions will be assumed for the forces on which the bodies are acted on, concerning which if they may be present, indeed so that for each action, here, initially we may consider no forces to be acting, and it is required to see clearly, not only how many forces shall be present, but also whatever direction they may maintain; also how for a variety of bodies how they may be changed both in speed and direction. If indeed ships may be propelled by the wind, the force of the wind there shall become smaller where the ships are progressing faster, indeed when they may be moving in that region in which the wind is present; but in the remaining cases an account of the wind acting sideways is required to be had. Thence also the direction of the wind, on which the force of the wind on the ship

depends, is required to be considered especially; indeed since once the same position maintained with respect to the ship is established, it will remain unchanged, whatever the course of the ship. But otherwise an account of oars is required to be prepared, since they shall exercise the same force in the same direction with respect to the ship, however both the speed and direction of the wind may change. Therefore this distinction of the forces will be required to be attended to properly, when we inquire into the effects of these; that which also even now will not yet be treated generally, since the effect both of the wind as well as that arising from the oars; for they will be set out more accurately in the following book. On account of which thus it will suffice here for the discussion to be treated generally, as its use shall be come apparent well enough in the following book.

PROPOSITION 75

PROBLEM

773. *If a body provided with the diametric plane thus shall be situated in a river, so that the axis of the body led from the bow to the stern may lie in the direction of the river* (Fig. 101), *to define the motion which the force of the river will impress on the body.*

SOLUTION

The body may be represented by the horizontal cross-section *AEBF* made through the centre of gravity *G*, and the body now may be put into the river now to be propelled in this position, since its centre of gravity shall be moving from *C* initially, where the body will have no speed at this stage. It is evident therefore from the conditions prescribed the body to be going to take a straight course and to be rectilinear since no force shall be present, which either may deflect the linear motion, or rotate the body about the vertical axis, so that thence an oblique course would be able to arise. And thus since the speed of the body at *C* were put to be zero, its acquired speed when it will arrive at *G* must be put equal to the altitude υ, truly the distance traversed by the centre of gravity *CG* shall be $= s$. Again the speed of the river must correspond to the altitude *b*. Therefore while the body shall be moving at *G* where its speed is $\sqrt{\upsilon}$, the river will act on the body with its excess speed, which is $\sqrt{b}$ above the speed of the body $\sqrt{u}$, that is with a speed $\sqrt{b}-\sqrt{\upsilon}$ and it will exert a force on the body with this same speed, as if the body were to be moving along the direction *AB* with this same speed in water at rest. Moreover we may put the plane figure $= ff$, which in this case will experience the same resistance, if indeed it were to strike the water directly with the same speed. Therefore from these it follows to become the force propelling the body along the direction *GL* to become equal to the weight of the water, of which the volume shall be

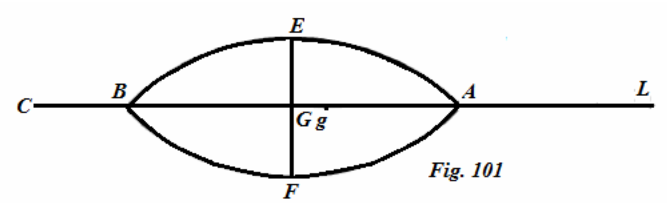


Fig. 101

$=\left(\sqrt{b}-\sqrt{\upsilon}\right)^2 ff$. Therefore with the mass or weight of the body $=M$ and with the volume of the submerged part $=V$ the force propelling the body at G will be

$$=\frac{\left(\sqrt{b}-\sqrt{\upsilon}\right)^2 ffM}{V};$$

and by this force thus the motion of the body thus will be accelerated, so that while it is progressing through the element of distance $Gg=ds$, there shall be

$$d\upsilon=\frac{\left(\sqrt{b}-\sqrt{\upsilon}\right)^2 ffds}{V} \text{ or } \frac{d\upsilon}{\left(\sqrt{b}-\sqrt{\upsilon}\right)^2}=\frac{ffds}{V},$$

of which the integral is

$$\frac{2\sqrt{\upsilon}}{\sqrt{b}-\sqrt{\upsilon}}-2l\frac{\sqrt{b}}{\sqrt{b}-\sqrt{\upsilon}}=\frac{ffs}{V},$$

with the integration put in place thus so that there shall become $\upsilon=0$ on putting $s=0$. But the time in which the body resolves the distance $CG=s$ is

$$=\int\frac{ds}{\sqrt{\upsilon}}=\frac{V}{ff}\int\frac{d\upsilon}{\left(\sqrt{b}-\sqrt{\upsilon}\right)^2\sqrt{\upsilon}}=\frac{2V\sqrt{\upsilon}}{ff\left(\sqrt{b}-\sqrt{\upsilon}\right)\sqrt{b}}.$$

Q.E.I.

COROLLARY 1

774. Therefore the distance s is assigned much easier from the speed $\sqrt{\upsilon}$ given, with which traversed the body will have acquired that speed, than in turn from the given distance s to assign the speed $\sqrt{\upsilon}$. And hence for that reason it shall not be preferred to determine the time to pass through that region, but to determine the speed itself.

COROLLARY 2

775. But from the formulae found it is understood at no time is the body able to acquire such a speed, however great the flow of the river becomes ; for if there may be put $\upsilon=b$, both the distance s and likewise the time become infinite when there becomes $\upsilon=b$.

COROLLARY 3

776. Moreover, if once there were $\upsilon=b$, since that can happen, if the body may be presented with a force with such a speed , then on account of $d\upsilon=0$, the body on progressing cannot accept either an increase or decrease of the speed, and thus will be moved forwards with a uniform speed.

COROLLARY 4

777. If the logarithm which in the equation, by which the relation between the distance *s* and the speed $\surd\upsilon$ is contained, may be converted into a series, there will be had

$$\frac{ffs}{2V}=\frac{\upsilon}{2b}+\frac{2\upsilon\surd\upsilon}{3b\surd b}+\frac{3\upsilon^2}{4b^2}+\frac{4\upsilon^2\surd\upsilon}{5b^2\surd b}+\text{etc.}$$

from which expression it is evident the speed acquired in the given distance *s* thus to be greater where the round figure *ff* were greater.

COROLLARY 5

778. Therefore since the body may be washed away by the speed of the river, of that part, which the force of the water may drive out, which is the latter part of the body, thus so that it will be required to be prepared, so that the maximum resistance may be experienced, if it travels directly into the water. Therefore the acceleration will be a maximum if the plane posterior part were normal to the course of the river.

COROLLARY 6

779. As it is difficult to assign the speed of the body for a given distance traversed, thus the speed can be defined more easily after a given interval of time. Indeed for the time put in place from the beginning of the motion $=t$, there will become

$$t=\frac{2V\surd\upsilon}{ff\left(\surd b-\surd\upsilon\right)\surd b},$$

from which in turn there becomes

$$\surd\upsilon=\frac{ffbt}{2V+fft\surd b}.$$

COROLLARY 7

780. If the magnitudes b, υ, ff, and V are expressed in the thousandth parts of Rhenish feet, the time *t,* in which the given speed $\surd\upsilon$ is acquired, will become known in seconds by this equation

$$t=\frac{V\surd\upsilon}{125ff\left(\surd b-\surd\upsilon\right)\surd b}.$$

COROLLARY 8

781. Therefore in turn if the time *t* may be given in seconds, and the magnitudes *b*, *ff* and *V* may be expressed in the thousandth parts of Rhenish feet, that same equation

$$\surd\upsilon = \frac{125 ffbt}{V + 125 fft\surd b}$$

will produce the altitude corresponding to the speed υ in thousandth parts of the same feet.

SCHOLIUM 1

782. Here at the start *C* we have placed the body initially to be at rest, and all the motion for that to be acquired, from the impressed motion of the water: but the problem can be treated in a like manner, if a given motion may be attributed to the body from the start, the direction of which may fall on the same right line *CL*, on which both the direction of the river, as well as the position of the axis of the body *AB* are situated. Moreover if the question may be extended in this manner, several cases completely different from each other are required to be distinguished ; of which the first is, if the body while at *C* may be released into the water, then it shall have a motion in the direction of the river *CL* but smaller than that the river itself has; which case brought from the solution itself will be resolved easily; for since the speed of the river is greater, the body will be accelerating, and if the initial speed must correspond to the altitude *k*, the integration of the differential equation

$$\frac{d\upsilon}{\left(\surd b - \surd\upsilon\right)^2} = \frac{ffds}{V}$$

will become

$$\frac{ffs}{V} = \frac{2\surd\upsilon}{\surd b - \surd\upsilon} - \frac{2\surd k}{\surd b - \surd k} - 2l\frac{\surd b - \surd k}{\surd b - \surd\upsilon}$$

and the time, in which the distance *s* is resolved, or the speed $\surd\upsilon$ is acquired, will be found

$$= \frac{2V\surd\upsilon}{ff\left(\surd b - \surd\upsilon\right)\surd b} - \frac{2V\surd k}{ff\left(\surd b - \surd k\right)\surd b};$$

which here will relate to everything, so that this motion may be able to be considered as a part of the motion departing from rest. For in the beginning the motion is considered to have been beyond the point *C*, with the interval

$$s=\frac{2V\surd k}{ff(\surd b-\surd k)}-\frac{2V}{ff}l\frac{\surd b}{\surd b-\surd k}.$$

From this it is apparent for a point, at the individual points of the distance *CL,* to go from the resting speed to having the same speed as the body, where the body departs from the point *C* itself with a given speed $\surd k$ in the oblate case. Then if the speed of the body initially at *C* will have been contrary to the direction of the river, then the body at first will rise against the course of the river, until its motion shall be completely reduced to zero, and then it will be taken along as if from rest by the river. Moreover the establishment of this case follows from the preceding on putting $\surd k$ negative, if indeed at *C* the speed $\surd k$ may be had, from which there will become

$$\frac{ffs}{V}=\frac{2\surd \upsilon}{\surd b-\surd \upsilon}+\frac{2\surd k}{\surd b+\surd k}-2l\frac{\surd b+\surd k}{\surd b-\surd \upsilon}.$$

From this equation the distance will be obtained, through which the body will progress beyond *C* against the flow of the river, if there may be put $\upsilon=0$; then truly there will become

$$\frac{ffs}{V}=\frac{2\surd k}{\surd b+\surd k}-2l\frac{\surd b+\surd k}{\surd b}=\frac{-2\surd k}{2\left(\surd b+\surd k\right)^2}-\frac{2k\surd k}{3\left(\surd b+\surd k\right)^3}-\frac{2k^3}{4\left(\surd b+\surd k\right)^4}\text{ etc.}$$

from which the required value of the distance $-s$ will be produced. Finally the third case disagrees entirely with these, where the body initially at *C* will have a motion along the direction of the river, but swifter. Then indeed not only will the body be slowed down in the river, but also the surface of the body will experience another force against *A* due to the action of the water, truly towards the posterior part at *B,* which so far will be the only point allowed to be free from the force of the water. Therefore in this case the body will encounter a resistance which shall be equivalent to the resistance, which the plane surface *ff* will experience on being struck by water with the same speed. Whereby if the initial speed at *C* may be put $=\surd k$ and at $G=\surd \upsilon$, the resistance will become

$$=\frac{Mff\left(\surd b-\surd \upsilon\right)^2}{V}$$

from which there becomes

$$d\upsilon=-\frac{ffds\left(\surd b-\surd \upsilon\right)^2}{V}$$

and on integrating :

$$\frac{ffs}{V}=\frac{2\sqrt{\upsilon}}{\sqrt{\upsilon}-\sqrt{b}}-\frac{2\sqrt{k}}{\sqrt{k}-\sqrt{b}}+2l\frac{\sqrt{k}-\sqrt{b}}{\sqrt{\upsilon}-\sqrt{b}}:$$

from which it is understood the body finally to have traversed an infinite distance to reach the speed of the river.

SCHOLIUM 2

783. Though everything shall follow to be put in place correctly by this calculation, yet if we may look further into that matter itself, there is need for an amendment. Indeed that circumstance had been disregarded, of which before we have made mention, apart from the water, for the air above the water also there is another resistance which exercises a resistance proportional to the square of the speed, it will be required to consider also the resistance of the air above the water, which although it shall be almost eight hundred times smaller than the resistance from the water on the rest of the parts, yet it disturbs the motion to some extent. Thus just as the air resistance is the reason bodies acted on by the river, at no time approach properly to the speed of the river, as the above calculation indicates, nor also for this same reason will the motion of a body be conserved, if it were equal to the motion of the river, but will be retarded. Then if the body may descend deeper into the river with the speed which the river itself has, then on account of the air resistance, it will it acquire only so much of the speed of the river itself, but also less, since the air resistance may become equal to the impulse of the water. Since it is required to estimate this effect in a certain way, we may put the part of the body present in the air to be acted on by the air resistance, as the plane surface hh with the same speed may endure against the motion of the air. Therefore if the speed of the body, which must be acted on by the air, shall correspond to the altitude υ, the resistance will be equal to the weight of a mass of air of which the volume is $=hh\upsilon$, or to the weight of a mass of water, of which the volume is $=\frac{hh\upsilon}{800}$. Therefore if an account of this force may be had in the solution of the problem, there will be produced:

$$du=\frac{\left(\sqrt{b}-\sqrt{\upsilon}\right)^{2}ffds}{V}-\frac{hh\upsilon ds}{800V},$$

from which the final speed which the body will acquire is understood not to be $\sqrt{b}$, but smaller, evidently there will become

$$f\sqrt{b}-f\sqrt{\upsilon}=\frac{h\sqrt{\upsilon}}{28}$$

approximately, or

$$\sqrt{\upsilon}=\frac{28f\sqrt{b}}{28f+h}.$$

On account of which if the portion of the surface of the body emerging above the water shall become greater by n times, than that, which is situated under the water, there will be approximately $hk = nff$, and thence the final speed

$$\surd\upsilon = \frac{28\surd b}{28+\surd n}.$$

Hence also changes in the remaining cases arising from the air will be able to be deduced. But for all these cases we may put the air to be at rest, for another matter will arise if the air may be disturbed by a wind, which motion equally will be led into the calculation without difficulty.

PROPOSITION 76

PROBLEM

784. *If the body AB* (Fig. 101) *may be moving in still water, not only may it be moved directly in the direction BAL, but also shall be propelled along that direction by some constant force, that is of such a size, which body equally will be accelerated or become steady; to define the motion of this body.*

SOLUTION

Here at first we may put the force acting for the body moved to be absolute or so great, which in a given increment of time will produce the same acceleration and shall be moved with some speed ; clearly a force of this kind will exercise the forces of the oars, with which accordingly the oarsmen always shall produce the same force, by which ships are accustomed to be propelled equally. And thus the force $= p$ shall be propelling this body in the direction *AL,* with p denoting the weight equal to that force: and the resistance, which the forwards portion *EAF* experiences in the water, shall be just as great a quantity ff as the surface will allow if indeed it will be driven directly against the water. Now we may prepare the centre of gravity of the body at the distance $CG = s$ and have began the motion at the point *C*, truly at *G* it will have a speed due to the altitude υ, from which the resistance, which it will experience at *G* will become $= ff\upsilon$; or if the mass or weight may be called M and the volume of the submerged part $= V$, the force of the resistance $=$ to the weight $\frac{Mffu}{V}$. Therefore from these while the body has traversed the element $Gg = ds$, there will become

$$d\upsilon = \frac{pds}{M} - \frac{ff\upsilon ds}{V} \text{ or } d\upsilon + \frac{ff\upsilon ds}{V} = \frac{pds}{M}$$

with which taken with $e^{ffs:V}$ becomes integrable, and the integrated equation will be :

$$e^{ffs:V}\upsilon=\frac{\int e^{ffs:V}pds}{M}=\frac{pV}{Mff}\left(e^{ffs:V}-1\right),$$

with the integration thus put in place so that υ may vanish on putting $s=0$. Whereby this equation is obtained :

$$\upsilon=\frac{pV}{Mff}\left(1-e^{-ffs:V}\right)$$

from which the speed of the body being describe at the individual points of the distance *CGL* will become known. Truly the time when the distance $CG=s$ from the centre of gravity *G* is traversed will become known from the integral of $\frac{ds}{\surd\upsilon}$, which is found

$$=\frac{2\surd MV}{f\surd p}l\left(e^{ffs:V}+\surd\left(e^{ffs:2V}-1\right)\right).$$

Q.E.I.

COROLLARY 1

785. Therefore the body will be accelerated continually for with υ increasing, *s* will increase ; and now with an infinite distance passed through, the body will acquire a speed, of which the corresponding altitude will be $=\frac{pV}{Mff}$; or the maximum speed , which it can acquire will become $=\frac{\surd pV}{f\surd M}$.

COROLLARY 2

786. Moreover it is understood from the formula found,

$$\upsilon=\frac{pV}{Mff}\left(1-e^{-ffs:V}\right),$$

soon the body will reach such a speed, which will be indistinguishable from the final speed. For if the distance *s* were moderately great, the quantity $e^{-ffs:V}$ now will become so small a fraction, which will vanish besides 1 indeed provided there becomes

$\frac{ffs}{V}=10$ or $s=\frac{10V}{ff}$, as now the magnitude $e^{-ffs:V}$ becomes less than $\frac{1}{10000}$.

COROLLARY 3

787. Therefore with the first part of the motion ignored the body can be considered without risk as it were progressing with a uniform motion: and the speed with which it will be moving uniformly will be $=\frac{\surd pV}{f\surd M}$; which expression if f and V may be expressed in thousandths of Rhenish feet, then $\frac{250\surd pV}{f\surd M}$ will give the distance in the same measure, which the body will resolve in one second.

COROLLARY 4

788. Therefore the speed, with which the ship propelled by the oars will be moved forwards in still water, is in the square root ratio of the strength of oars: from which, if the number of oarsmen may be quadrupled, the ship will progress at twice the speed.

COROLLARY 5

789. Hence if two ships may be driven forwards between themselves with similar oars, and the length of the greater AB shall be $=A$, of the lesser $=a$, the greater will be driven forwards by a force $=P$, the smaller truly by a force p, the speeds with which they may be advancing between themselves, will be as $\frac{\surd P}{A}$ to $\frac{\surd p}{a}$. Therefore so that if the ships will be advancing with the same speed, it is necessary that the forces of the rowers shall maintain the square ratio of the lengths.

COROLLARY 6

790. Then also it is understood, where the resistance of the ship shall be smaller, there a greater speed is generated by the same force of the oars. Indeed since the absolute resistance shall be as ff, the speed produced will be in the inverse square root ratio of the resistance, that is if the resistance were four times smaller, the same force of the oars will impress twice as great a speed on the ship.

COROLLARY 7

791. Finally since V [*i.e.* the volume of the submerged part of the ship] holds a constant ratio to M [the mass of the ship]; for indeed V multiplied by the specific gravity of water, is equal to M; it is evident the speeds of propulsion of ships from the motion of oars to be in the ratio composed directly from the square root of the forces of the oars, and inversely as the square root of the absolute resistance.

SCHOLIUM

792. Since these determinations are applicable only to water at rest tae, yet they are easy to be transferred to the motion of ships in rivers from the propulsion of oars ; if indeed the motion may be made along the direction of the river. For if the speed of the flow shall correspond to the altitude b or that speed itself $=\sqrt{b}$, then if the shall be going down in the river, its speed acquired from the force of the oars will be augmented by the speed of the river, thus so that such a body, as we have considered by descending in a river may acquire a velocity

$$=\frac{\sqrt{pV}}{f\sqrt{M}}+\sqrt{b}.$$

But if the same body may be propelled upwards against the flow, then the speed will be acquired

$$=\frac{\sqrt{pV}}{f\sqrt{M}}-\sqrt{b}.$$

from which expression it is understood, unless $\frac{\sqrt{pV}}{f\sqrt{M}}$ is greater than $\sqrt{b}$, the body cannot overcome the flow, nor to ascend. But since this is with regard to the force of the oars, it is required to be observed the forces of the oars must be applied equal and similar on both sides, from which the direction of the force resulting from these jointly may pass through the middle of the ship, or may lie on the right line BA; indeed unless this shall be observed, the body or ship cannot hold a direct course, for indeed by calling to mind here the action of the rudder, by which certainly it may be able to come to the aid of this inconvenience.

PROPOSITION 77

PROBLEM

793. *If the plane surface put in place situated vertically may be moved parallel to its own situation in a straight line along CGL* (Fig. 102), *and on that may strike fluid in the direction VG with a given speed, to determine the force, which the fluid by its own motion, will exert on the surface.*

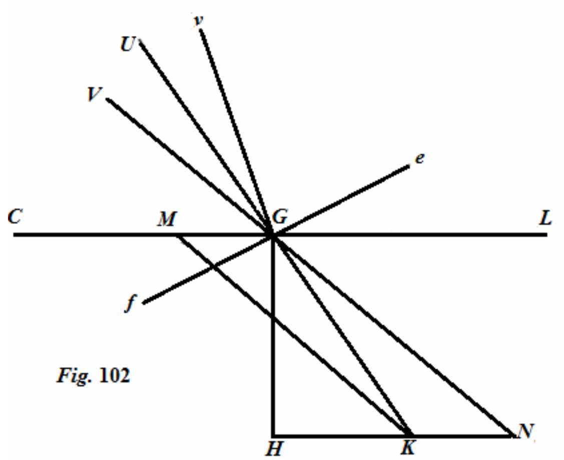


Fig. 102

SOLUTION

The speed with which the plane surface *ef* advances corresponds to the altitude υ, or $= \sqrt{\upsilon}$, and the speed, with which the fluid moves, $= \sqrt{c}$, moreover the sine of the angle *CGV,* which the direction of motion of the fluid *VG* makes with the direction of motion of the surface *CGL* may be put $= \mu$, and the cosine $= \nu$. But the sine of the angle *VGf,* which the direction of the motion of the fluid *VG* makes with the plane surface shall be $= m$ and the cosine $= n$, with the whole sine $= 1$; and finally $gg =$ to the area of the surface, the centre of gravity of which shall be at the point *G.* Now if the surface may be at rest, from the previous demonstration the force which the fluid will exert on the surface $= m^2 g^2 c$, or shall be equal to the force of the mass corresponding to the same fluid material, of which the volume $= m^2 g^2 c$. But since the surface may not be at rest but shall be considered to be progressing in the direction *GL* with the speed $= \sqrt{v}$, the whole system may be taken with the fluid and the surface to be moving backwards in the direction *GC* with the speed $= \sqrt{u}$, so that the surface *ef* shall be brought back to rest; but the force of the fluid on the surface will be the same in each case. Moreover by the known composition, the motion thus will become known, both of the resulting speed of the fluid as well as the direction. Indeed since now the fluid may be carried by two motions, the one along the direction *GN* with the speed $= \sqrt{c}$, truly with the other in the direction *GM* with the speed $= \sqrt{v}$. If there may be taken $GN = \sqrt{c}$ and $GM = \sqrt{\upsilon}$, and the parallelogram *GMKN* may be formed, both the resultant speed of the diagonal flow *GK*, it will suggest as well as the direction, thus so that the fluid shall strike against the resulting surface *ef* with the speed *GK* in the direction *UG*. But with the perpendicular *GH* dropped from *G* onto *NK* produced, there will be on account of the sine of the angle *GNH*= μ, and the cosine $= \nu$, the perpendicular $GH = \mu\sqrt{c}$, and $NH = \nu\sqrt{c}$, from which there will become

$KH = \nu\sqrt{c} - \sqrt{\upsilon}$ and $GK = \sqrt{\left(c - 2\nu\sqrt{c\nu} + \upsilon\right)}$. From these there is found the sine of the angle *NGK* or sine $UGV = \frac{\mu\sqrt{\upsilon}}{\sqrt{\left(c-2\nu\sqrt{c\upsilon}+\upsilon\right)}}$ and the cosine $= \frac{\sqrt{c}-\nu\sqrt{\upsilon}}{\sqrt{\left(c-2\upsilon\sqrt{c\nu}+\upsilon\right)}}$, and hence the sine of the angle *UGf*

$$= \frac{m\sqrt{c} - (m\nu - n\mu)\sqrt{\upsilon}}{\sqrt{\left(c - 2\nu\sqrt{c\upsilon} + \upsilon\right)}},$$

therefore which is the sine of the angle of incidence under which the fluid is forced onto the surface, whereby since the speed of the fluid shall be $= \sqrt{\left(c - 2\nu\sqrt{c\upsilon} + \upsilon\right)}$, the force will be produce, which the fluid, truly moving in the direction *VG* with the speed $\sqrt{c}$, will produce a force, which will be exercised on the surface *ef* moving with the speed $\sqrt{\upsilon}$ in the direction *GL*,

$$= \left(m\sqrt{c} - (m\nu - n\mu)\sqrt{\upsilon}\right)^2 g^2,$$

and the direction of this force will be passed through the centre of gravity *G* of the surface, and will be normal to the surface itself. Q. E. I.

COROLLARY I

794. If the sine of the angle *CGf* shall be put $= q$, since there shall be $q = m\nu - n\mu$, the force of the fluid will become, which it exerts on the surface $= \left(m\sqrt{c} - q\sqrt{\upsilon}\right)^2 gg$, or as great as to weigh the same as the volume of the fluid.

COROLLARY 2

795. Since the sine of the angle *UGf* found

$$= \frac{m\sqrt{c} - q\sqrt{\upsilon}}{\sqrt{\left(c - 2\nu\sqrt{c\upsilon} + \upsilon\right)}},$$

it is evident there must become $m\sqrt{c} > q\sqrt{\upsilon}$, if indeed the plane surface must be urged towards the region *GK*. For if there shall become $m\sqrt{c} < q\sqrt{\upsilon}$ then the surface thus will be forced towards the region *UV*.

COROLLARY 3

796. If the plane surface *ef* may be put in place normally to the course of the river *VG*, thus so that there shall become $m = 1$ and $n = 0$, the force which the surface experiences will become

$$= \left(\surd c - v \surd \upsilon\right)^2 gg :$$

which force therefore will thus be smaller, when the cosine *v* were of the greater angle *VGC*.

COROLLARY 4

797. Moreover with the position of the surface remaining *ef* the same with respect of the direction of motion of *GL* itself, the force of the river thus will be greater where the sine *m* will have been greater. Whereby the maximum force will be experienced by the surface, if the angle *VGf* were right.

COROLLARY 5

798. But if the surface *ef* were placed nearly along the direction *GL* of the motion itself, the angle *CGf* will be vanishing and consequently $q = 0$; therefore in this case the surface will experience that same force as if it were at rest.

COROLLARY 6

799. If the river may have come from the region υG, thus so that the direction υG shall be inclined so much to *Ge,* the direction *VG* will be inclined by just as much to *Gf,* the sine of the angle υGf will remain the same $= m$; and thus on account of the changed angle CGf, the sine of which is *q,* the force acting on the surface will remain the same, which in the other case evidently $= \left(m \surd c - q \surd \upsilon\right)^2 gg$.

COROLLARY 7

800. Now we may consider the angle *VGC* to remain unchanged; it will be possible to define the angle *VGf,* or the position of the surface *ef,* so that it shall experience the maximum force from the fluid. Moreover the tangent of the angle *VGf* will be found

$$= \frac{m}{n} = \frac{\surd c - v \surd \upsilon}{\mu \surd \upsilon},$$

and the force will become $= (c - 2v\sqrt{c\upsilon} + \upsilon)g^2$.

SCHOLIUM

801. This proposition will become especially necessary for us in the following, where we are going to investigate both the force of the wind on the sail, as well as the force of the river on the ship moving forwards. But it is readily apparent unless the speed of the wind shall be the greatest or almost as large as possible, the motion of the sails cannot generally be ignored; if indeed the sails may be progressing in the same direction as the wind acts, it is evident the force of the wind on the sails there will become smaller, when the sails are moving forwards more quickly, and thus to vanish, if the sails may have the same speed as the wind. On account of which it will be allowed for us to set out the following problems into which we will enquire, in whatever manner ships may be propelled by the wind, both in a direct course, or oblique in some manner.

PROPOSITION 78

PROBLEM

802. *If a body or ship, with the aforementioned diametric plane AB, may be acted on by the wind thus, so that it shall be moved in still water directly along the direction GL* (Fig. 103), *to determine the motion of this ship, and the maximum speed, which it will be able to acquire.*

SOLUTION

Because the ship is put to be moving in a straight course in the direction *BAL*, in which likewise the direction of the resistance happens, it will be required that the mean direction of the wind shall fall in the same direction. Whereby since the force of the wind always shall be normal to the plane of the sails, and its mean direction shall pass through the centre of gravity of the sails, it is required that the plane of the sails shall be normal to the diametric plane *AB,* and so that the centre of gravity of the sails shall lie in this same plane. And thus *EF* will represent the plane surface of the sails, of which the area shall be $= gg$, and *G* shall be the centre of gravity of the sails in place on the axis ; and in

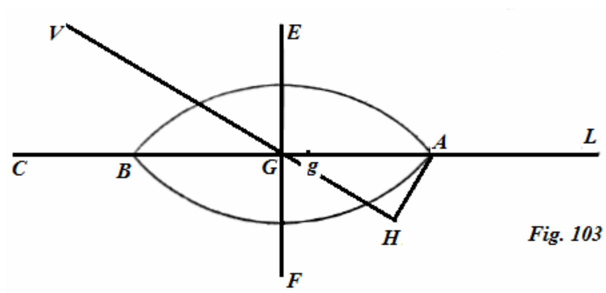


Fig. 103

this manner it will become the case that the mean direction of the wind shall lie on the right line *GL*, and thus so that both the direction of the course, as well as the progressive motion may be kept along the line *GL*. Now the wind may strike the sails in some oblique direction *VG,* and the speed of the wind shall correspond to the altitude *c,* and the sine of the angle *VGC,* which the direction of the wind makes with the direction of the motion, shall be $=\mu$, and the cosine $=\nu$; and the sine of the angle *VGF*, which the direction of the wind *VG* makes with the plane area of the sails, will be $=\nu$, which before was put to be *m,* and the cosine, which before was *n,* in this case will become $=-\mu$, since the angle *VGF* is obtuse. Again we may put the ship to be starting to move from the point *C*, and now the distance $CG=x$ may be resolved, and here the speed of the ship to have the speed corresponding to the altitude υ. With these in place from the preceding proposition the force of the wind will become, by which the ship is acted on in the direction *GL,*

$$=\left(m\surd c-(m\nu-n\mu)\surd\upsilon\right)^2 g^2=\left(\nu\surd c-\surd\upsilon\right)^2 g^2,$$

evidently the volume of the air will have a weight equal to the propelling force of the wind; or since the specific gravity of the air itself may be had approximately to that of water as 1 to 800 or as 1 to 784, since we will make use of the latter number since 784 is a square number, and we need to extract the square root, the weight of the force of water equal to that volume will be had

$$=\frac{\left(\nu\surd c-\surd\upsilon\right)^2 gg}{784}.$$

Now the plane surface *ff* shall express the resistance, which the ship experiences in the direct course from the water, or the plane surface *ff* may express the same resistance as the ship, if it may impinge directly against the water with the same speed. Hence therefore the force of the resistance will be equal to the weight of a mass of water, of which the volume is $=ff\upsilon$, since we assume the water to be at rest. Therefore from these, if the mass or weight of the ship may be said $=M$, and the volume of the submerged part $=V$, then the force propelling the ship in the direction *GL*

$$=\frac{M\left(\nu\surd c-\surd\upsilon\right)^2 gg}{784V};$$

but the repelling force $=\frac{Mff\upsilon}{V}$; fro which the acceleration is produced, while the ship progresses through the element $Gg=dx$,

$$d\upsilon=\frac{M\left(\nu\surd c-\surd\upsilon\right)^2 g^2dx}{784V}-\frac{ff\upsilon dx}{V}$$

Therefore meanwhile the ship will be accelerating, as long as $\frac{\left(v\sqrt{c}-\sqrt{\upsilon}\right)^2 g^2}{784}$ were greater than $ff\upsilon$; but when at last it will have acquired so great a speed $\sqrt{\upsilon}$ so that there shall become

$$\frac{\left(v\sqrt{c}-\sqrt{\upsilon}\right)^2 g^2}{784}=ff\upsilon,$$

which certainly finally occurs after an infinite distance will have been completed, but as soon as proper, that speed will have been attained so that the difference will be insignificant. On account of which, in accordance with the initial motion of the ship in the direction *GL,* which will be moved with a speed due to the altitude υ, the value of which is found from the above equation

$$\sqrt{\upsilon}=\frac{vg\sqrt{c}}{28f+g};$$

thus, so that the speed of the ship itself will be had to the speed of the wind as *vg* to $28f+g$. Q.E.I.

COROLLARY 1

803. Since the speed, which the ship acquires propelled by this wind shall be $=\frac{vg\sqrt{c}}{28f+g}$, it is understood the speed of the ship, with the ratio of the other parts to maintain a simple ratio to the speed of the wind, thus so that in which ratio the speed of the wind may be increased, the speed of the same ship may be increased in the same ratio.

COROLLARY 2

804. But as far as it concerns the surface of the sails *gg*, it is seen the speed of the body indeed to increase, if the sails may be multiplied, but not to maintain a fixed ratio. For if the sails my be increased to infinity, the speed of the boat will not be increased beyond a given limit, clearly on making $gg=\infty$, the speed acquired $=v\sqrt{c}$.

COROLLARY 3

805. Therefore when the sails now were increased to that point so that $28f$ shall almost vanish with respect to *g*, then the size of the force may be multiplied further by the sails, yet a greater speed of the ship will not be impressed. From which it is able to be deduced to be useless to multiply the sails beyond a given limit; which limit will be determined easily in practice from the resistance.

COROLLARY 4

806. It is concluded from these also, thus no considerable gain to be obtained, even if the resistance were diminished considerably. For if so great sails may be taken so that a multiple of g may exceed $28f$, then it will be of little concern, if the resistance also may be removed further. Nevertheless where the resistance is smaller, thus there will be a need for fewer sails.

COROLLARY 5

807. Furthermore it will be apparent at once with the arrangement of the ship remaining the same, the speed of the wind therefore to become greater where the angle VCG were made smaller; from which the wind acting directly along CG or from the stern of the ship will impress the maximum velocity.

SCHOLIUM 1

808. And from these it is understood well enough how great a distinction lies between ships which are propelled by the wind, and these propelled by oars. Indeed the greatest interest for those which are advanced by oars so that the resistance may be diminished as much as possible, since the speeds impressed may be held in the inverse square root ratio of the resistances : truly on the other hand in these ships which are propelled by the wind not so much is gained from a diminution of the resistance ; from which the greatest distinction arises in the construction of ships, as they are resolved either for sails or oars. But this difference itself, if we may consider the ships received in use, in the first place it will be required to be observed. Indeed three oared or ships of this kind, which are moved by oars, have the sharpest foremost part, from which the resistance arising to be extremely small. Truly we see the forward part of the other kind of ships, destined for the wind, to be endowed suitably obtuse, which it appears will be suitable for the resistance requiring to be reduced. Moreover it follows at once, ships of this kind, which are accustomed to be moved both by oars as well as by the wind, ought to be prepared, so that they shall be especially suitable for each; it is apparent evidently some means to be required between each kind being treated. Even now so much is demanded from this distinction for the direction of the course, but which may be considered when we have subjected oblique courses to examination, for which ships equipped with sails must be adapted especially, since on the other hand for ships being propelled by oars, for an oblique course, generally there will be no need for anything at all to be considered. Otherwise the calculation will be resolved easily from the manner of the solution, if besides the wind, oars also may be used, and jointly may be propelled by oars and sails. Any calculation may be put in place in a similar manner without difficulty, if the motion shall not be made in still water, but in a river, provided that the directions of the river and the motion of the body shall agree, and the course shall be straight ; on account of which we will not linger over investigations of this kind for a long time.

SCHOLIUM 2

809. Because here we have put the surface of the sails to be perfectly plane, that solution will be minimally disturbed even if the sails may be extended to a concave figure by the wind: indeed, in the following book where the instructions regarding sails will be put in place, it will be established a plane sail can always be taking the same force, thus so that what are established here with regard to plane sails, will prevail equally for sails, in whatever manner they may be taken. Then also the solution from trials concerning this is seen to disagree because that wind may be put in place especially gainful, which comes straight from the stern, since still it will be agreed from observation that ships prefer to be propelled by not too oblique winds. But the reason for this discrepancy will be placed in the accustomed location of the sails to be expanded together in the stern, the prow, and also in the middle of the ship: from which it is easily deduced, if the wind shall blow straight from the stern to the prow, then the posterior sails remove the wind from the anterior sails, and to be an impediment, so that less wind may be able to blow on the anterior sails. But since this may not arise, if the direction of the wind is oblique, it is no wonder an oblique wind to be accustomed to produce a greater speed than a direct wind. But these are to be understood only, if the ship shall be equipped with several masts, but in this case only will this inconvenience be found, for if a single mast shall be present, and the wind to be free in all the sails it shall be able to meet, then the disagreement of theory with practice mentioned is not be observed, but rather there the ship may be taken to be progressing faster, where the direction of the wind may differ less from the direction of the ship. But apparent differences will occur more often especially in this instruction of the motion of ships, but always they will be refuted easily if all the circumstances are considered properly.

SCHOLIUM 3

810. Therefore these are almost all the cases, for which ships are able to advance in a direct course and moving in a straight line in water both at rest as well as flowing, for which course it is required, so that both the direction of the motion itself, as well as both the mean direction of the resistance and the direction of the force acting, and also the direction of the river, must agree amongst themselves, and lie on the axis or right line drawn from the stern to the prow. On account of which if a single condition may have failed, or the direct course or rectilinear motion will have been disturbed, and it may arise that either the direction of the motion may depart from the axis of the ship, or from the longitudinal diameter stretching from the stern to the prow, or also the centre of gravity may be found to lie on a curved line, which all to be distinguished properly between each other, and to the cause of each all attention will be rendered. Meanwhile from the satisfactory handling, if the force shall be had acting along the length of the ship, then even if the course may become exceedingly oblique, nevertheless the course direction to be changed shortly. For when the force vanishes, acting continually in the same direction, then the motion, will soon disappear if it were oblique. And hence it occurs that ships

which are propelled by oars shall be progressing by always following along their own length, since the direction of the force of the oars always shall act there, although the course may be changed repeatedly with the aid of the rudder. Then indeed the curvilinear motion endures, as if for a moment only, and changes in direction at once, the account of which matter resides mainly in the side resistance, which is large in these ships especially, and the oblique motion will cease at once. And for this reason the direct course is appropriate for that kind of ships, which are propelled by oars, for since the force of the oars can be exercised equally in any region, and the motion along the length of the ship is made most easily on account of the minimum resistance, it would be absurd to construct ships of this kind for oblique motion. But otherwise there is an account of ships prepared with length, which are moved by the wind, since the direction of the wind shall not be allowed to be formed as you wish, but for that wind, which chance suggests, for again it will be required for that construction used to be done thoroughly. Therefore as often it happens so that the intended course may differ from the direction of the wind to such an extent, that the direct course generally cannot be used, then it is required to have recourse to an oblique course, which will be able to be used there quite happily, from which it will be able to navigate closer towards the direction from which the wind blows. Therefore with these ships, which are propelled by the wind, the oblique courses must be attended to especially, and thence the rules aim at the construction and sailing of such ships. On account of that same oblique course, by which ships are not progressing along their length, initially we will subject these ships to be examined, which are not moved by oars but by the wind alone.

PROPOSITION 79

PROBLEM

811. *If the body or ship AEBF* (Fig. 104) *in quiet water may have taken an oblique course along the direction GL with a given speed, to determine both path itself, which the centre of gravity G will describe, as well as the obliqueness of the course everywhere, or the position of the longitudinal axis AB.*

SOLUTION

AGL shall be the angle of inclination of the course, which the direction of motion *GL* makes with the position of the longitudinal axis or the spine *BA*, and s shall be put for the sine of this angle, truly the cosine $= r$; moreover the speed impressed on the body along the direction *GL* shall correspond to the altitude v. Then *RM* shall be the mean direction of the resistance, which the body will experience

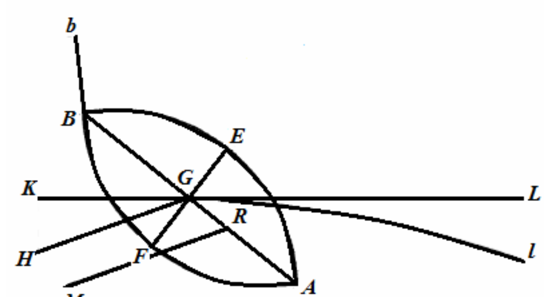


Fig. 104

from the water in this oblique motion, which shall constitute the angle *MRB* with the direction of the spine or axis of the ship *AB*, of which the sine shall be $=\sigma$, truly the cosine $=\rho$; and the resistance which the ship experiences moving in the water shall be as great as the plane *uu* surface will experience moving directly opposite to the water in the direction *MR* ; from which the resistive force, by which the body will be urged along the direction *RM*, will be equal to the weight of the mass of water, the volume of which is $=uuv$. Moreover the quantities σ, ρ and u will depend on the angle of the obliquity *AGL* or to the sine of that *s* and on the structure of the whole body. Whereby if the mass or weight of the whole body may be put $=M$ and the volume of the part submerged in the water $=V$, the force of the resistance acting in the direction *RM* , $=\frac{Muuv}{V}$; which force will exert a twofold effect, the first of which consists in the progressive motion of the centre of gravity *G* requiring to be changed, truly the other acting by turning the body about the vertical axis through the centre of gravity *G*. But before effecting the required investigation it will be required to consider the first force *RM* as if applied to the centre of gravity *G* itself in the direction parallel to *GH*. Therefore the sine of the angle *HGK,* which establishes the direction of the resistance *GH* with the direction of the motion *GK* , will be $=r\sigma-s\rho$, and the cosine $=s\sigma+r\rho$. Hence the force of the resistance, which is $=\frac{Muu\upsilon}{V}$; will be resolved into the two sides *GK, KH,* of which the one *GK* lies in the sane direction as the direct motion itself *GL*, truly the other *KH* will be normal to this, from which the tangential force *GK* will be

$$=\left(s\sigma+r\rho\right)\frac{Muu\upsilon}{V},$$

and the normal force *KH*

$$=\left(r\sigma-s\rho\right)\frac{Muu\upsilon}{V}.$$

Therefore the tangential force will retard the motion of the body, and it will bring about, that while the body traverses the space $Gg=dx$, it shall become

$$=-\frac{\left(s\sigma+r\rho\right)Muuvdx}{V}.$$

But the normal force will deflect the body from the rectilinear path and force it to describe the arc of a circle concave towards the region M , the radius of which will be

$$=\frac{2V}{\left(r\sigma-s\rho\right)u^{2}}.$$

So that in fact it shall pertain to the other effect, where the body will be rotated by the resistive force about the vertical axis passing through the centre of centre of gravity *G*, it is apparent the first rotation to be done in the region *AF,* thus so that the angle may be increased more by that declination *AGL*, if indeed the centre of the resistance *R* may fall between the centre of gravity *G* and the prow *A*. Moreover with the said interval $GR = z$ the moment of the resistive force for that rotation requiring to be produced $= \frac{\sigma Muu\upsilon z}{V}$, which moment divided by the moment of inertia with respect of the same vertical axis passing through the centre of gravity *G*, will give the rotational force to which the momentary angular motion is proportional. But if the angular motion of the body now may be had, then the increment of this may be known from the rotational force. Q. E. I.

COROLLARY 1

812. Therefore the retardation of the motion will be greater there, when the angle *HGK,* were smaller, that is, where the difference between the angles *GRM* and *AGL* were smaller. From which it follows the more the angle *GRM* may exceed the angle *AGL*, thus the diminution of the motion to become smaller.

COROLLARY 2

813. Again since generally the resistance shall be greater there, where the course obliqueness differs more from the direct course, or where the angle *AGL* were greater, while it may not exceed a right angle, the value of *uu* thus will become greater, where the angle *AGL* were greater, and thus a greater retardation of the motion arises there.

COROLLARY 3

814. Therefore the maximum retardation of the motion will be agreed on, if the angle *AGL* shall become right, for then not only the value of all the terms *uu* will become a maximum, if indeed the resistance of the sides may greatly exceed the resistance of the prow. But then also the direction of the resistance *RM* lies in the direction of the motion, so that as $s\sigma + r\rho$ becomes the maximum value it may obtain, and will become $= 1$.

COROLLARY 4

815. Since the radius of curvature of the path, in which the centre of gravity *G* advances, shall be $= \frac{2V}{(r\sigma - s\rho)u^2}$, the body will be deflected from its own impressed direction *GL,* and while it traverses the element of distance $Gg = dx$, it will be deflected by the angle

$$= \frac{(r\sigma - s\rho)u^2 dx}{2V}.$$

COROLLARY 5

816. Therefore the deflection from a rectilinear course will not depend on the speed of the body, but only on the obliquity of the course. Indeed both the value of *uu*, as well as also $r\sigma - s\rho$ will depend on the obliquity of the course, or the sine of the difference of the angles *MRG* and *AGL.*

COROLLARY 6

817. Therefore, if the angle *MRG* shall become equal to the angle *AGL,* then the deflection from a rectilinear course will be zero everywhere, and the body will not progress in a right line. But if the angle *MRG* were greater than the angle *AGL*; then it will be deflected towards *A,* and it will describe the curved path *Gl*, situated between *GA* and *GL*. But if the angle *AGL* were greater than the angle *MRG,* then it will be deflected into the opposite part.

COROLLARY 7

818. If the centre of resistance *R* may coincide with the centre of gravity *G* itself, then the rotational force vanishes, therefore in this case the position of the axis therefore in this case the position of the axis *AB* always will remain parallel to itself. From which if the angle *MRG* or *HGB* shall be equal to the angle *KGB,* then not only does the body proceed to move along the line *GL,* but also the same oblique course will be preserved.

COROLLARY 8

819. But with *R* and *G* meeting, or which likewise is the case, provided that the points *G* and *R* may lie on the same vertical right line, if the angle *MRG* were greater than the angle *AGL,* then the direct line course *Ggl* approaches to the position of the axis *AB* and finally will become the motion along the direct course, to the point of being conserved, since all motion shall cease on account of the resistance. But if the angle *MRG* shall be less than the angle *AGL,* then the course will continually be deflected more from being a straight line, thus so that finally its direction may become normal to *AB.*

COROLLARY 9

820. But if the centre of resistance *R* may not fall on *G* but rather towards the prow *A*, then the body will be changed from rotating about the vertical axis drawn through the centre of gravity, and the axis *BGA* will rotate within the region *AF;* with which done so that the obliquity of the course or the angle *AGL* may increase perpetually.

COROLLARY 10

821. But if the centre of resistance *R* may fall beyond *G* towards the stern *B*, then conversely it will occur in the opposite region; from which the obliquity of the course will soon be removed, and the axis of the ship *AB* will be turned into that motion of the ship in the direction *GL;* so that if that happens, the direction of the course will be conserved.

SCHOLIUM 1

822. From these it is understood also, if the ship *AB,* which before shall be advancing in the direction of the course, thus may be turned around by an external force, so that its keel *AB* shall make an oblique angle *AGL* with the direction of motion, changes of this kind thence shall arise. Indeed especially that will be with regard to the centre of resistance *R,* corresponding to the angle of obliqueness present *AGL*: so that if the centre of gravity *G* were put in place further towards the prow, then the ship itself will be restored to the former situation, and the direction of the course will be recovered. Truly on the other hand if the centre of resistance *R* may be fall towards the prow, then not only will the ship not be perceived to be moving in the direct course, but also the obliquity of the course will be increased until the axis of the width *EF* may lie on the direct course, so that if this situation should arise, the ship will progress in this direction. On account of which if the centre of resistance *R* shall lie in the stern, the direct course is largely agreed to be firmly established, since the ship, if it may be dislodged from that, at once to be spontaneously restored; but on the other hand, if the centre of resistance *R* may fall towards the prow, then the course of the direction will be unstable, thus so that if the ship may deviate minimally from the direct course, the obliquity will continue to become greater. On account of which the direct course will be maintained with difficulty, unless it may be firmly managed, that is unless the centre of resistance *R* may fall towards the stern; for the smallest force will suffice to destroy the direct course completely. Meanwhile yet even if the centre of resistance *R* shall be placed in the prow, yet the direct course will be able to be maintained with the aid of the rudder *Bb* ; but with that, the force required for restoration will be greater in the direct course, on account of which the point *R* may lie towards the prow *A*, and likewise where a greater angle *GRM* may be apparent. But the oblique course in the direction under the angle *AGL* in general be maintained unless the centre of resistance *R* shall fall at *G*, and the angle *HGK* shall vanish. For if *R* shall fall at the prow not even with the aid of the rudder can the course be maintained with the same obliquity: indeed since the force of the rudder placed at *Bb* may be able to impair the motion by rotation of the ship about the vertical axis, yet by the same force of the rudder the motion will be turned away more from the right line *GL*. If indeed the angle *MRG* were greater than the angle *AGL,* just as indeed must happen in ships. But when the centre of resistance *R* falls in the stern, then the same oblique course can happen with the aid of the rudder, and the rectilinear motion will be maintained; as that will happen, if the force of the rudder shall not only impede the rotational motion,

but also likewise may cancel out the normal force. From all these effects it is evident for the rectilinear motion requiring to be conserved within an oblique direction there is a need for significant external forces to be applied with caution, which indeed we will soon see regarding this matter. Moreover so that any case may be able to be considered, we will advance several examples, how the values σ, ρ and *uu* together with the interval $GR = z$ may depend on a given oblique course, in which these values themselves will be able to be shown.

EXAMPLE 1

823. At first the figure *AEBF* shall be composed from two equal circular segments put in place with a common chord *AB*, or all the horizontal sections of the ship shall be equal horizontal sections of this figure; and the radius of the circle of which the arcs *AEB* and *AFB* are parts $= c$; and since the centre of gravity *G* will be situated at the centre of the chord *AB*, there shall become

$$AG = BG = a,\ EG = FG = b,$$

thus so that there shall be $2bc = a^2 + b^2$; $\left[i.e.\ a^2 + b^2 + a^2 + \left(2c - b\right)^2 = 4c^2, \text{etc.} \right]$; and there may be put, for the sake of brevity $c - b$ or $\surd\left(c^2 - a^2\right) = d$.
From these compared with proposition 58 the interval will be produced :

$$GR = z = \frac{ra^3 d}{c^3 - rd^3}$$

and the tangent of the angle *GRM*

$$= \frac{\sigma}{\rho} = \frac{2sc^3 - 2rsd^3}{2rc^3 - 3rrc^2 d + \left(rr - ss\right) d^3},$$

and finally the force of the resistance

$$= \frac{2\upsilon}{3cc} \surd\left(4c^6 - 12r^3 c^5 d + 9r^4 c^4 d^2 \left(r^2 - 3ss\right) c^3 d^3 - 6rr\left(r^2 - s^2\right) c^2 d^4 + d^6\right),$$

which force of the resistance, if the course shall be direct, gives

$$= \frac{2(c-d)^2 \left(2c + d\right)\upsilon}{3cc}.$$

Therefore if *ff* shall be the plane surface providing the same resistance, as the body experiences in the direct course, there will be

$$\frac{uu}{ff}=\frac{\surd\left(4c^6-12r^3c^5d+9r^4c^4d^2+4r\left(r^2-3s^2\right)c^3d^3-6rr\left(r^2-s^2\right)c^2d^4+d^6\right)}{\left(c-d\right)^2\left(2c+d\right)}.$$

If the obliquity were very small so that s may vanish before r, there will become

$$\frac{\sigma}{\rho}=\frac{2s\left(c^3-d^3\right)}{2c^3-3c^2d+d^3}=\frac{2s\left(c^3+cd+d^2\right)}{\left(c-d\right)\left(2c+d\right)},$$

from which there will be

$$\rho=1 \text{ and } \sigma=\frac{2s\left(c^3+cd+d^2\right)}{\left(c-d\right)\left(2c+d\right)}=\frac{\left(3a^4+b^4\right)s}{\left(3a^2+b^2\right)b^2}\ *,$$

and

$$z=\frac{a^3d}{c^3-d^3}=\frac{2a^3\left(a^2-b^2\right)}{3a^4+b^4}*.$$

From which on putting the angle *AGL* infinitely small there will be produced

$$s\sigma+r\rho=1 \text{ and } r\sigma-s\rho=\sigma-s=\frac{3a^2\left(a^2-b^2\right)s}{\left(3a^2+b^2\right)b^2}*$$

and $u^2=f^2$. With which substituted there will be had $d\upsilon=-ff\upsilon dx$, and the radius of the curvature of the curve described

$$=\frac{2bb\left(3a^2+b^2\right)V}{3aa\left(a^2-b^2\right)s}*$$

and the moment of the force rotating the body about the vertical axis drawn through the centre of gravity

$$=\frac{2a^3\left(a^2-b^2\right)Mffsv}{b^2\left(3a^2+b^2\right)V}\ *$$

[*values corrected by C. Truesdell in the *O.O.* edition.]

COROLLARY 1

824. Therefore the rotational force or the increasing obliquity of the course will become greater there, where the length of the ship exceeds the width. And likewise thus the curvature of the force will be greater, in which the centre of gravity of the body advances.

COROLLARY 2

825. Also it is evident where the length of the ship *AB* shall exceed the width *EF,* so that the excess of the angle *GRM* over the angle *AGL* will become greater. For indeed the angle *GRM* itself is had to the angle *AGL* in the square ratio of the length to the width of the ship.

COROLLARY 3

826. Therefore if the width *EF* shall be made equal to the length *AB,* or $b = a$, in which case the figure will be changed into a whole circle, then there will become $c = a$ and $d = 0$; from which the centre of the resistance will fall at *G* and there will become $\frac{\sigma}{\rho} = \frac{s}{r}$, on account of which the beginning course will be continued without any change ; it will also be apparent there, that with the circle, no oblique course may be given.

EXAMPLE 2

827. The figure of the ship, besides the diametric plane *AB* which we always put in place, shall be prepared thus, so that the centre of resistance *R* shall always fall on the right vertical line passing through the centre of gravity *G* . Then if the resistance, which the body experiences in moving in the direct course in the direction *GA*, may be put to be as great as the resistance the plane figure *ff* may experience moving with the same speed directly against the water; and the resistance of the side, which it will experience, if it were moving in the direction *GE*, to be as great as the amount the plane figure *hh* driven against the water with the same speed will experience ; the oblique motion thus shall have this property, that the tangent of the oblique angle *HGB*

$$= \frac{\sigma}{\rho} = \frac{shh}{rff};$$

or, the tangent of the angle *MRB,* or of the angle *HGB,* to the tangent of the oblique angle *AGL* of the course, may hold the ratio so that *hh* to *ff* shall be as the resistance of the side to the resistance of the prow. Finally the force of the resistance shall be $= \upsilon\sqrt{\left(s^2h^4 + r^2f^4\right)}$, acting on the body in the direction *GH* ; thus, so that for the obliquity of the course offered, there shall become $uu = \sqrt{\left(s^2h^4 + r^2f^4\right)}$. Therefore with these in place there will be no force at all striving to rotate the body about the vertical axis passing through the centre of gravity, and on that account the position of the axis of the ship *AB* will remain the same always, or to be parallel to itself. But afterwards, since there shall be

$$\frac{\sigma}{\rho}=\frac{shh}{rff},\text{ there will become }\frac{1}{\rho}=\frac{\surd\left(r^2f^4+s^2h^4\right)}{rff}$$

and thus

$$\sigma=\frac{shh}{\surd\left(r^2f^4+s^2h^4\right)}\text{ and }\rho=\frac{rff}{\surd\left(r^2f^4+s^2h^4\right)};$$

from which there will become

$$s\sigma+r\rho=\frac{s^2h^2+r^2f^2}{\surd\left(r^2f^4+s^2h^4\right)}\text{ and }s\sigma+r\rho=\frac{rff}{\surd\left(r^2f^4+s^2h^4\right)};$$

from which, in the first place, the retardation of the motion

$$d\upsilon=-\frac{(s^2h^2+r^2f^2)\upsilon dx}{V},$$

and the declination will be from so great a rectilinear path that the body will describe a circular arc *Gg,* of which the radius will be

$$=\frac{2V}{rs\left(hh-ff\right)}.$$

COROLLARY I

828. Since the sine of the angle *HGK* shall be

$$=\frac{rs\left(hh-ff\right)}{\surd\left(s^2h^4+r^2f^4\right)}$$

it is evident if there were $h>f$ then always the angle *MRG* or *HGB* to be greater than the angle *AGL,* as well as the two cases in which there is either *s* or $r=0$, that is if the inclination of the course *AGL* were either zero or 90 degrees.

COROLLARY 2

829. Therefore it is understood from this formula the difference to be somewhere between the angles *HGR* and the maximum *AGL*, which position will be there, if the tangent of the angle *AGL*

$$=\frac{f}{h}, \text{ or } \frac{s}{r}=\frac{f}{h};$$

moreover then the tangent of the angle *HGB* or *MRG* will become $=\frac{h}{f}$.

COROLLARY 3

830. Moreover since here no force is present rotating the body, and thus the axis *AB* will always retain the same orientation, but the direction of the course *GL* will continually be changed towards *AB* by a normal force, thus so that finally the course will be changed in direction.

COROLLARY 4

831. But there the motion of the centre of gravity will be deflected more from being a right line, where the difference between the resistance of the prow and the resistance of the side were greater, that is, where a smaller resistance shall be experienced along the direction of the ship *BA*, and likewise where it will be greater than the resistance it will have experienced when moved in the direction *FE*.

SCHOLIUM 2

832. Not without serious reasoning, we have arrive at this case, for this property is observed which we have presented here of a body to be moved forwards obliquely in water, to be agreed especially in ships, which are accustomed to be propelled by wind. For in the first place in ships of this kind for this to be attended to especially, so that the centre of resistance from the prow towards the stern may be removed, and as if it may lie on that same vertical right line through the centre of gravity. Then the resistance of the sides is accustomed to greatly exceed the resistance of the direct course, from which it follows at once, as we have now noted above, in an oblique course the direction of the resistance will be turned away much more from the axis. Likewise moreover it may be understood well enough, the formula assumed by means of which the angle of the obliquity of the course of which the tangent is $=\frac{s}{r}$ will correspond to the angle, which the mean direction of the resistance makes with the spine of the ship, the tangent of which $=\frac{s}{r}\cdot\frac{hh}{ff}$; which angle therefore vanishes, if the obliquity vanishes, and it will become a right angle, if the direction will be changed into a right angle, if the direction of the ship shall become normal to the spine, which in the first place agree with the known shape of the ship. In the third place because we have put the force of the resistance to be $=\sqrt{\left(s^2h^4+r^2f^4\right)}$, indeed that squared agrees wonderfully well with the known structure

of the ship; indeed by making $s = 0$ and $r = 1$, which is the case of the direct course, the resistance will become $= ff$, as we have assumed, and in a like manner if the obliquity of the course may be turned to a right angle, the outstanding resistance produced $= hh$. Truly besides it is clear if the resistance of the side *hh* may be put equal to the resistance of the prow *ff*, then the resistance of all the courses likewise to become the same ; and finally this expression of the resistance is to be prepared thus, so that in the first place it will agree with the angle *MRB,* of which the tangent is $\frac{shh}{rff}$, if indeed it may be compared with the cases treated above. But the truth of these distinctive properties, if not able to be demonstrated directly yet shall be of this kind, for which it may be allowed to agree, that a place may be found, by which these can be shown for ships, both for the direction of the resistance as well as an account of the magnitude. Evidently the motion made following the oblique direction *GL* may be resolved into two sides ; of which one may become in the direction *GA* of which the speed will be $= r\sqrt{u}$, truly the other in the direction *GE* for that same normal, of which the speed will be $= s\sqrt{u}$. Now since in the calculation of the resistances it may not be allowed to decompose the motion, yet for our purposes it may depart a little from the truth, if we may put the body to be carried by a twofold motion: the one in the direction *GA* with the speed $= r\sqrt{u}$, the other in the direction *GE* with the speed $= s\sqrt{u}$. Moreover on account of that motion, the resistance which the prow itself can be considered to have, to the resistance which the side bears as *rff* to *shh,* from which the direction of the mean resistance *GH* will constitute the angle *HGB,* of which the tangent will be $= \frac{shh}{rff}$, and the magnitude of the resistance itself will become $= \upsilon\sqrt{(s^2h^4 + r^2f^4)}$, or the plane expressing the resistance will be $uu = \sqrt{(s^2h^4 + r^2f^4)}$. From this consideration a new hypothesis will be able to be formed by putting the resistance of the prow not as we have made here $= rff$, but rather, how great it may become if it may be moving forwards with so great a speed, evidently $= r^2f^2$, and in a similar manner the resistance of the side $= s^2h^2$, from which the tangent of the angle $HGB = \frac{s^2h^2}{r^2f^2}$, and the force of the resistance itself $= \upsilon\sqrt{\left(s^4h^4 + r^4f^4\right)}$; but this other hypothesis departs further from the truth than the first, if the figure may be put to be a circle. For in this case the tangent of the angle *HGB* always becomes $= \frac{s}{r}$, and the resistance is constant or $uu = f = hh$; that which justifies the first hypothesis, but not the second. On account of which it will be agreed deservedly to prefer the first hypothesis to the second, and thus we will consider that hypothesis before the others in the following. There that first hypothesis will differ less from the truth, if ships thus were actually prepared thus, so that they may maintain some oblique course, in which the centre of resistance always may fall in the centre of gravity of the ship itself; indeed as this hypothesis itself postulates, there is no doubt, why the remaining thus may not be going to be less in error, if ships were to agree about this.

PROPOSITION 80

PROBLEM

833. *To determine the force of the wind and the arrangement of the sails by which it shall be brought about so that for the rectilinear motion of the ship under a given course of obliquity it shall be progressing uniformly; and likewise to define the velocity of the motion.*

SOLUTION

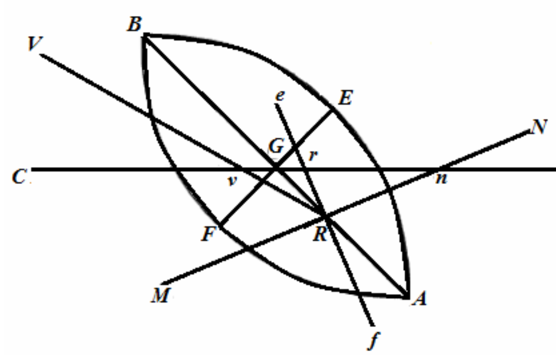


Fig. 105

AEBF (Fig. 105) shall represent the figure of the ship, in which *G* shall be the centre of gravity of the ship, and the position of the spine, or of the longitudinal axis from the prow *A* to the stern *B*, by the right line *AB* drawn. Moreover the centre of gravity *G* shall be moved forwards with a uniform motion in the direction through *GL* and the height must correspond to the speed itself $= \upsilon$. Therefore the angle *AGL* will show the obliquity of the course, of which the sine shall be $= s$ and the cosine $= r$; whereby since the obliquity of the ship may be put to be retained constantly, the resistance likewise will remain the same and its direction will be *RM,* which shall constitute the angle *MRB* with *AB*, the sine of which shall be $= \sigma$ and the cosine $= \rho$, truly the resistance shall be so great, how much will depend on the plane figure with the flat surface *uu* striking straight against the water with the same speed $\sqrt{\upsilon}$; which will all be given from the structure of the ship and from the given obliquity, thus so that with σ, ρ and u^2 arising from r, s and from the quantities which may depend on the ship given. Hence the force of the resistance $uu\upsilon$ acting along the direction *RM*, or on putting the weight of the ship $= M$ and with the volume of the submerged part $= V$, the force of the resistance will be equal to the weight $\frac{Mu^2\upsilon}{V}$. Therefore since this force not only shall retard the motion, but also will change the direction and course of the obliquity, that resistance will be required to be destroyed by the force of the wind. For if this were maintained, it is seen with the cause of the motion disturbed, the ship to retain its own speed $\sqrt{\upsilon}$, and that to proceed uniformly in direction and in addition to have to maintain its own obliquity. Otherwise moreover this force of the resistance in short cannot be destroyed, unless the direction of the force of the wind shall be incident in the direction of the resistance *RM,* and likewise the force of the wind shall be equal and opposite to the force of the resistance. Therefore since the force of the wind shall be normal to the sail, it will be required so that the surfaces of the sails shall be normal to the right line *RM*, and the common centre of gravity of the sails shall fall on the same vertical line with the centre of the resistance *R*, if indeed the sails always have their centre of gravity placed on the axis *AB*, and thus the direction *ef* of the sails, which shall be normal to *MR,* and the force of the wind must be so great so that it shall equal the force of the resistance $\frac{Mu^2\upsilon}{V}$. Moreover the plane surface of the sails may be put $= gg$, and the wind will become in the direction *VR* with the speed $\surd c$, and the sine of the angle $VvC = \mu$ and the cosine $= \nu$. Truly for the force of the wind requiring to become known, the sines of the angles *VRf* and *Crf* are required to become known, of which if that were $= m$ and this truly $= q$ the force of the wind acting in the direction *RN* becomes $= \left(m\surd c - q\surd \upsilon\right)^2 gg\sqrt{a^2+b^2}$ (§ 794) , or it will be equal to the weight

$$\frac{M\left(m\surd c - q\surd \upsilon\right)^2 gg}{784V}.$$

But there is :

$$\sin Crf = \cos Rnr = \cos\left(MRG - AGL\right)$$

and

$$\cos Crl = -\sin Rnr = -\sin(MRG - AGL);$$

from which the sine of the angle *Crf,* $q = s\sigma + r\rho$ and the cosine $= s\rho - \sigma r$. Then since there shall be $\sin VRf = \sin(Crf + C\upsilon V)$ there will become $m = \nu\left(s\sigma + r\rho\right) - \mu\left(r\sigma - s\rho\right)$; and thus the force of the wind

$$= \frac{M\left(\nu(s\sigma + r\rho)\surd c - \mu(r\sigma - s\rho)\surd c - (s\sigma + r\rho)\surd \upsilon\right)^2 \mathrm{g}^2}{784V},$$

the direction of which now is contrary to the direction of the resistance *RM,* therefore it remains so great, that the force itself may become equal to the force of the resistance, $\frac{Mu^2\upsilon}{V}$, from which this same equation will be obtained:

$$28u\surd \upsilon = \nu(s\sigma + r\rho)g\surd c - \mu\left(r\sigma - s\rho\right)g\surd c - (s\sigma + r\rho)g\surd \upsilon$$

from which equation either the speed if the wind $\surd c$, or the speed of the ship $\surd \upsilon$ will be able to be determined. Therefore if we may put the speed of the wind to be given, the speed will be found, with which the ship will be moving along the course *AGL* with the given obliquity on the given right line *GL,*

$$= \surd \upsilon = \frac{\left(v\left(s\sigma + r\rho\right) - \mu\left(r\sigma - s\rho\right)g\surd c\right)}{28u + \left(s\sigma + r\rho\right)g}.$$

Moreover it will be required that *m* or its value $v(s\sigma + r\rho) - \mu\left(r\sigma - s\rho\right)$ shall be positive, for if it were negative, the wind would not stretch out the sails in the direction *RN* but in the opposite direction *RM*; on account of which these cases properly are required to be exceptions. [The Bernoulli Effect was discovered some 30 years previously in 1738, which can be applied to sails as aerofoils to provide horizontal 'lift' , but which would appear to have been an unknown application at that time.] Indeed the same expression found may be indicated more clearly, since if *m* may obtain a negative value, the speed $\surd \upsilon$ also will become negative, which then has shown the ship not to be advancing in the direction *GL* but to be moving forwards in the opposite direction *GC*. Q. E. I.

COROLLARY1

834. If the sine of the angle *RnG,* which is the excess of the angle *MRB* over the angle *AGL,* may be put $= p$ and the cosine $= q$, there will become
$p = r\sigma - s\rho$ and $q = s\sigma + r\rho$, from which the speed of the ship is found

$$\surd \upsilon = \frac{\left(vq - \mu p\right)g\surd c}{28u + gg}.$$

COROLLARY 2

835. But $vq - \mu p$ expresses the cosine of the sum of the angles, of which the sines are p and μ. Whereby lest the same cosine to be used may not become negative, it will be required that the sum of the angles $V\upsilon C + RnG$ shall be less than a right angle.

COROLLARY 3

836. Therefore if the direction of the speed *VR* may be given, the path *GL* being described and the obliquity of the curve *AGL* on which the position of the sails will depend, there the speed of the ship will be greater, where the speed of the wind were greater, and thus to be in the same ratio.

COROLLARY 4

837. If the centre of resistance *R* may have a variable location, for courses with various obliquities, then the centre of gravity of the sails also will have to be changed, since the force of the resistance of the wind can be can be removed in a single way.

SCHOLIUM 1

838. Therefore unless the centre of resistance *R* shall hold a fixed position for all the obliquities of the curve with a single mast no outstanding use is required to be conserved in the various obliquities courses. But if a ship with several masts were constructed, then certainly sails can be fitted thus, so that the common centre of gravity of these may arise from some point *R*; but for this case for any obliquity it will be necessary to know the position of the centre of resistance *R* most precisely, that which in practice can scarcely be hoped for. But by testing the difficulty with the sails of the masts thus will be moderated, so that they may preserve the desired effect, since it must be perfectly satisfied by two conditions. In the first place that force of the resistance must be cancelled, which is known to turn the ship away from a rectilinear course, and this force is normal to the path *GL,* arising from the resolution of the resistive force; therefore this force can cancel the force of the wind in an indefinite number of ways, provided the force of the normal to the path *GL* arising from the resolution of the force of the wind shall be equal and opposite to that, nor to refer to that case, in which it shall be applied to the position of the axis *AB*. Then truly the force of the resistance is required to be cancelled out, which also tends to rotate the ship around the vertical axis passing through the centre of gravity, whereby the moment of the force of the wind with respect of this axis must be equal and opposite to the moment of the force of the resistance with respect of the same axis, of which condition again it is possible to be satisfied in innumerable ways. But so that each resistive force may be cancelled in a single given manner, which hence by seizing upon some case requiring to be tested is scarcely to be hoped for. Hence it will be on account of the most useful case so that ships will be constructed thus, so that the centre of the resistance in these may hold a fixed position, and that shall lie precisely on the centre of gravity, for then, since the force of the resistance turning the ship shall be zero, it will be easy enough to cancel the other force.

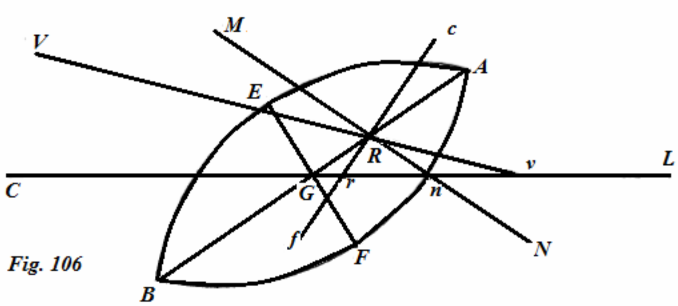


Fig. 106

SCHOLIUM 2

839. In the solution of this problem we have put the prow *A* in the part of the way *CL,* to fall contrary to that, from which the wind *V* comes, but a similar solution will emerge, if the prow may be moved aside from the right line *GL,* a case of this kind alleged here (Fig. 106) is represented in the figure. Indeed if, as before, the sine of the angle of declination *AGL* were $= s$ and the cosine $= r$, truly for the angle *MRB,* which the mean direction of the resistance makes with the axis of the ship, the sine will be $= \sigma$ and the cosine $= \rho$; thus the sails *ef* must be spread out so that they shall be normal to *RM.* But as before *gg* shall be the surfaces of the sails, and the speed of the wind $\surd c$, and the sine of the angle $V\upsilon C = \mu$, which the direction of the wind makes with the path requiring to describe the way *CL* , and the cosine of the same angle $= \nu$; and finally the plane surface *uu* shall express the force of the absolute resistance. It is evident the first case to be reduced to this, if *s* and σ may become negative, because the angles *AGL* and *MRB* fall into opposite parts. Hence therefore the speed will be produced, by which the ship will be able to progress uniformly in the direction *GL*,

$$= \frac{\left(\nu(s\sigma + r\rho) + \mu\left(r\sigma - s\rho\right)\right) g \surd c}{28u + (s\sigma + r\rho)g},$$

which same expression would be produced if only μ were put negative. If therefore the sine of the angle *GnR*, which is the excess of the angle *MRB* over *AGL,* may be put $= p$ and the cosine $= q$, there will be $p = r\sigma - s\rho$ and $q = s\sigma + r\rho$, and the speed of the ship for uniform motion in a straight lie requiring to be conserved will be produced

$$= \frac{\left(\nu q + \mu p\right) g \surd c}{28u + qg},$$

which will only differ from the above form, because the sine of the angle $V\upsilon C$, which is $= \mu$, shall be taken negative.

COROLLARY 1

840. Because the sine of the angle *VR f,* which is $= \nu q + \mu p$ must always be positive, there will have to become $\nu q + \mu p > 0$ > C; but $\nu q + \mu p$ is the cosine of the difference of the angle $V\upsilon C$ and *RnC,* whereby the difference of the angles must be smaller than a right angle.

COROLLARY 2

841. Therefore the angle $V\upsilon C$ will be greater than a right angle, provided the smaller angle, which is the angle *RnC,* may not exceed a right angle. On account of which if the angle *MRB* were greater than the angle *AGL,* thus the course indeed will be able to be established so that the angle $V\upsilon L$ from which the wind comes may be put to be acute.

COROLLARY 3

842. Therefore an adverse wind will be allowed to put in place the course of the ship, if there the obliquity of the course may be put in place, in which the angle *MRB* especially exceeds the angle *AGL.*

COROLLARY 4

843. Because moreover, where the wind establishes a greater adverse course, the expression $vq + \mu p$ thus will become smaller, it is evident where the ship is propelled by the greater adverse force of the wind, thus its motion to become slower; and thus no advantage to be gained. Therefore an arrangement of this kind will be given, by which it will be able especially to run into the direction of the wind.

EXAMPLE 1

844. The ship may have that property (Fig. 105), which we have discussed above, so that the centre of resistance *R* may fall on the centre of gravity *G* of the ship, and if the direction of the resistance in the direct course shall be as great as the resistance of the plane figure *ff* striking the water with the same speed, truly the resistance in the maximum oblique course, of which the obliquity shall be 90 degrees, shall be reduced in the same manner to the plane *hh;* then truly the tangent of the angle *MRB* shall be

$$\frac{\sigma}{\rho} = \frac{shh}{rff}$$

or

$$\sigma = \frac{shh}{\sqrt{\left(s^2h^4 + r^2f^4\right)}} \text{ and } \rho = \frac{rff}{\sqrt{\left(s^2h^4 + r^2f^4\right)}} ;$$

and the resistance in this oblique course, or *uu,* shall be

$$= \sqrt{\left(s^2h^4 + r^2f^4\right)}, \text{ from which there becomes } u = \sqrt[4]{\left(s^2h^4 + r^2f^4\right)}.$$

With these in place there will become:

$$p = r\sigma - s\rho = \frac{rs\left(h^2 - f^2\right)}{\sqrt{\left(s^2h^4 + r^2f^4\right)}} \text{ and } q = s\sigma + r\rho = \frac{s^2h^2 + r^2f^2}{\sqrt{\left(s^2h^4 + r^2f^4\right)}},$$

from which the speed of this ship is found, with which it will be able to proceed uniformly in the direction *GL,* or

$$\sqrt{\upsilon} = \frac{\left(v\left(s^2h^2 + r^2f^2\right) - \mu rs\left(h^2 - f^2\right)\right)g\sqrt{c}}{28\sqrt[4]{\left(s^2h^4 + r^2f^4\right)^3} + \left(s^2h^4 + r^2f^4\right)g}.$$

But if the prow *A* may be directed into that region of the right line *GL* from which the wind comes (Fig.106), then with everything remaining as before, only μ will become negative, with the speed of the ship progressing

$$= \frac{\left(v\left(s^2h^2 + r^2f^2\right) + \mu rs\left(h^2 - f^2\right)\right)g\sqrt{c}}{28\sqrt[4]{\left(s^2h^4 + r^2f^4\right)^3} + \left(s^2h^4 + r^2f^4\right)g}.$$

COROLLARY 1

845. Therefore the ship will obtain a greater speed, if in the latter case the axis *AB* of the ship may be inclined thus so that the prow *A* may be pointing into that region, from which the wind comes, with all else evidently maintaining the same course obliquity.

COROLLARY 2

846. Therefore these courses are to be preferred in which the prow *A* lies on the path *GL*, with these for which the prow *A* lies below *GL* : because not only are these obliquities associated with the greater speed, but also many more obliquities can be located.

COROLLARY 3

847. But with the obliquity of the course observed downwards as in the first case, the limits of all the courses will be $s = 0$ and this equation

$$vs^2h^2 + vr^2f^2 = \mu rs\left(hh - ff\right),$$

from which the tangent of the angle of the obliquity becomes

$$=\frac{\mu\left(h^2-f^2\right)\pm\sqrt{\left(\mu^2\left(h^2-f^2\right)^2-4v^2f^2h^2\right)}}{2vh^2},$$

from which twice the angle *VRf* vanishes.

COROLLARY 4

848. But in the other case, where the prow *A* is turned above *CL* all the levels of the curves of the obliquity will be contained between these limits, of which the one constitutes the tangent of the obliquity $=0$, the other so that it may be equal to the tangent

$$-\frac{\mu\left(h^2-f^2\right)\pm\sqrt{\left(\mu^2\left(h^2-f^2\right)^2-4v^2f^2h^2\right)}}{2vh^2};$$

where the sign is limited to the first case, thus so the above sign + may prevail only for the first case.

COROLLARY 5

849. But so that for the others remaining cases concerned with the direction of the wind, it is evident generally that the fastest wind to propel the ship to be in the direction which shall be normal to the plane surface of the sails. For the speed, with all else remaining the same, is directly as the sine of the angle, which the direction of the wind makes with the sails.

COROLLARY 6

850. If the sails may be increased indefinitely then the speed of the ship produced

$$=v\sqrt{c}\mp\frac{\mu rs\left(h^2-f^2\right)}{s^2h^2+r^2f^2}\sqrt{c},$$

from which the maximum speed of the ship will be obtained if the tangent of the angle of the obliquity may become $=\frac{f}{h}$, unless the angle $V\upsilon C$ were exceedingly small.

COROLLARY 7

851. But if the obliquity of the course may be sought, by which the ship may be advancing most quickly in a given direction propelled by a given wind, the equation will

be found to be exceedingly complex, so that thence nothing may be able to be concluded; but which with the *g* made infinite there will be provided $\frac{s}{r}=\frac{f}{h}$. But the following is for the first case, with the equation generally determining the angle *AGL*, with the prow placed above *CL* :

$$\frac{1}{14}\mu g\left(r^2+s^2\right)\left(r^2f^2-s^2h^2\right)\sqrt[4]{s^2h^4+r^2f^4}=\left(2\mu s^4+\mu r^2s^3-vrs^2\right)h^4$$
$$-3vrs\left(r^2+s^2\right)f^2h^2+\left(-2\mu r^4-2\mu r^2s^2+vr^3s\right)f^4.$$

[Corrected from the original.]

COROLLARY 8

The analysis of section 852 therefore is false.

EXAMPLE 2

853. Another property of the ship mentioned above may be had (§ 832) so that the centre of the resistance *R* may fall on the centre of gravity *G* (Fig. 105), and if as before the resistance of the prow and of the side may be expressed by the plane figures *ff* and *hh,* so that the tangent of the angle *MRB*

$$\frac{\sigma}{\rho}=\frac{s^2h^2}{r^2f^2}$$

or

$$\sigma=\frac{s^2h^2}{\sqrt{\left(s^4h^4+r^4f^4\right)}}\quad\text{and}\quad\rho=\frac{r^2f^2}{\sqrt{\left(s^4h^4+r^4f^4\right)}},$$

and the resistance in this course or *uu* shall be $=\sqrt{(s^4h^4+r^4f^4)}$, from which there will become :

$$u=\sqrt[4]{s^4h^4+r^4f^4}.$$

With these in place there will be had:

$$p=r\sigma-s\rho=\frac{sr\left(sh^2-rf^2\right)}{\sqrt{\left(s^4h^4+r^4f^4\right)}},$$

and

$$q=s\sigma+r\rho\sigma=\frac{s^3h^2+r^3f^2}{\sqrt{\left(s^4h^4+r^4f^4\right)}}:$$

from which the speed of the ship is found, with which it will be progressing uniformly in the direction *GL*

$$=\frac{\left(v\left(s^3h^2+r^3f^2\right)-\mu\left(s^2rh^2-sr^2f^2\right)\right)g\sqrt{c}}{28\sqrt[4]{\left(s^2h^4+r^4f^4\right)^3}+\left(s^3h^2+r^3f^2\right)g}.$$

But if the prow *A* shall be directed in the other direction of the right line *CL,* this same expression will prevail except that in place of μ, there must be written $-\mu$.

SCHOLIUM 3

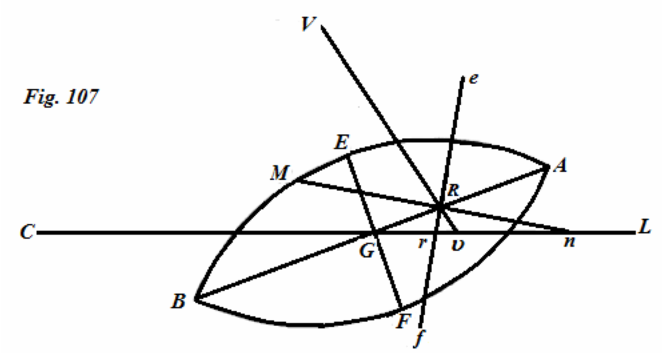
854. A conspicuous and especially useful problem occurs here, where from the given direction of the wind *VR*, and the path to be described by the ship *CGL*, either the obliquity of the course is define from the given angle $C\upsilon V$ or the angle AGL, which may be done so that the ship may progress the fastest. Indeed we have now resolved this problem according to the first example (§ 851); truly extending to an equation of this kind, from which the obliquity desired can be elicited with difficult, not even with approximations will it be allowed to be used, since the equation reduced to rationality may become of sixteen dimensions. Moreover a major difficulty will arise, if we may wish the same problem to be examined for the second example (Fig. 106). Yet meanwhile the matter can be treated generally by considering how the most advantageous course can be estimated. For since the speed found shall be $=\frac{(vq+\mu p)g\sqrt{c}}{28u+qg}$, in which expression μ is the sine and *v* the cosine of the given angle $V\upsilon C$, truly *p* and *q* the sine and cosine of the angle *RnG,* which depends on the angle sought AGL; moreover *uu* expresses the resistance, which accompanies the ship along the oblique course, which thus also depends on *p* and *q*. On account of which since the connection between *u* and *p* may not be agreed on, the value of *p* or *q* cannot be defined by the method of maxima and minima, so that the speed of the ship may become a maximum. But since the numerator of this expression evidently, $vq+\mu p$ will bear the sine of the angle *VRf,* which established the direction of the wind *VR* established with the plane of the sails *ef*, it is clear this numeration not to be changed, if the position of the sails *ef* thus may be changed so that the angle *VRf* may become obtuse, then with the position of the first angle *VRf,* that is, if the obliquity of the course may be taken thus, so that the mean direction of the resistance *RM* shall lie within the angle $V\upsilon C$, which case we have expressed in the figure presented (Fig.107) in which as before so that *VR* is the direction of the wind, *RM* the mean direction put in place of the resistance between *Vv* and *Cn*. Therefore since in this case the sine of the angle *VRf* will agree with the preceding, and thus the numerator of the fraction expressing the velocity of the ship shall be the same, the denominator must be considered, which is $28u+qg$, from which first is apparent *u* to represent a smaller magnitude than in the preceding case, since the course here is less oblique, and the resistance will be greater, so

that the obliquity will increase more. Truly the other part of the denominator qg, in which q is the cosine of the angle GnR, either can increase or decrease, or to remain the same. For if the obliquity is very small then indeed the angle GnR becomes exceedingly small, and thus q, may be almost equal to the whole sine, but truly also, if the obliquity becomes exceedingly great, then equally the angle GnR decreases, indeed with the obliquity increased to 90, the mean direction of the resistance again will lie in the direction of the course. Wherefore with this change of the course, since q will be able equally to be increased or diminished, from the other part $28u$, which certainly has been made smaller it is required to be concluded that same course put in place before to be preferred and thus the ship to be moving forwards faster. Therefore we shall see with these predicted, how by an approximation the course may be able to be defined with the greatest swiftness, if indeed the direction of the wind VR were given and the path requiring to be resolved GL or the angle $V\upsilon C$ of which the sine is μ and the cosine v.

PROPOSITION 81

PROBLEM

855. *If VR were the direction of the wind and GL were the direction of the path to be followed, to determine the obliquity of the course AGL, with which the ship may move along with the greatest speed*
(Fig. 107**).**



SOLUTION

$V\upsilon C$ shall be the angle, of which the sine $=\mu$ is given and the cosine $=v$, and the sine of the obliquity of the course AGL is given $=s$, and the cosine $=r$; which it will be required to define, so that the maximum speed of the ship shall be produced. But the speed of the ship, which it will have within this course, is

$$=\frac{(vq+\mu p)g\sqrt{c}}{28u+qg};$$

in which expression p and q are the sine and cosine of the angle RnG, which is the excess of the angle MRB over the angle AGL. Moreover in the first place it is required to be observed this angle RnG, to increase obliquely from the beginning, but to be increasing only as far as to a given limit, which when it will be reached, if the obliquity of the course may be increased more, to be diminished again, truly the resistance or the magnitude u to be increased continually, as long as the obliquity may increase. Therefore we may put that course AGL assumed to be the oblique to which the maximum angle

RnG shall correspond ; it is evident, if the obliquity may be increased or diminished a little, the angle *RnG* to conserve its magnitude, and hence p and q not to be changed. Hence therefore it is understood if the angle *AGL* may be increased more, then on account of the increase u the speed of the ship to be diminished, on the other hand to be increased, if the obliquity of the course of the ship to be diminished, otherwise to be increased, if the obliquity of the course *AGL* may be diminished. With this satisfied very well it follows for the fastest motion being obtained, the angle *AGL* thus must be taken smaller, to which the maximum angle *RnG* will correspond. Therefore here the angle *AGL* must be taken smaller, or such that it shall increase with the angle *RnG* increasing ; and we may put the angle *AGL* to be increased a little amount, thus so that its sine s may increase by small amount ds; therefore also the sine p of the angle *RnG* will increase by a small amount, which shall be $= mds$; truly the cosine q will decrease by the small amount $\frac{mpds}{q}$; truly the resistance thus shall increase so that u shall be increased by the small amount nds, and thus there will become :

$$dp = mds,\ dq = -\frac{mpds}{q} \text{ and } du = nds.$$

Moreover from these the speed will be produced, which is $= \frac{(vq + \mu p) g \sqrt{c}}{28u + qg}$; the increase

$$= \frac{\left(\mu gm + 28mu\left(\mu q - vp\right) - 28ng\left(\mu p + vq\right)\right) gds \sqrt{c}}{q\left(28u + qg\right)^2}.$$

The numerator of which fraction, if it were positive, then the ship with the obliquity increased will support an actual increase, but if the numerator were negative then the speed will decrease. But $\mu q - vp$ is the sine of the angle *VRM* and $pp + vq$ the cosine of the same angle. Whereby if the sine of the angle *VRM* is $= x$ and the cosine $= y$, with the increase of the obliquity of the course , the ship will be progressing faster if there were

$$\mu gm + 28mux - 28nqy > 0;$$

but slower if there were

$$\mu gm + 28mux - 28nqy < 0.$$

On account of which if the ship were progressing the fastest, the course would have to be put in place thus, so that there shall become

$$\frac{\mu gm}{28} = nqy - mux.$$

From which the difference of the angle *VRM* must be of the two angles, of which the sine of the greater shall be

$$=\frac{nq}{\surd\left(n^2q^2+m^2u^2\right)};$$

the sine of the smaller truly

$$=\frac{\mu gm}{28\surd\left(n^2q^2+m^2u^2\right)}.$$

Moreover all the magnitudes which occur here are positive, from which it will not be difficult to judge the obliquity of the course in whatever case offered. But the most convenient arrangement will be put in place, if for whatever case, some angle to be taken between the assigned limits, and it will be sought, whether with that increased a little, the speed will be increased or diminished, and from that the state will be gathered, whether the obliquity desired shall exceed that assumed, or it shall be less than that. Q.E.I.

COROLLARY 1

856. If the first obliquity may be assumed to be zero, or the course to be direct, there will be put $p=0$ and $q=1$ and $u=f$, accordingly ff shall express the resistance of the direct course ; and the speed of the body will be $=\frac{vg\surd c}{28f+g}$. Now if the obliquity may be made infinitely small, there will be $du=nds=0$, and with there remaining $dp=mds$, the speed of the increment will become

$$=\frac{\left(\mu gm+28\mu mf\right)gds\surd c}{\left(28f+g\right)^2}.$$

COROLLARY 2

857. Therefore it will be apparent unless there shall be $\mu=0$ or unless the wind may come from the stern, the direct course not to produce a swifter motion, but some oblique course is to be preferred. Yet with the case excepted where $\frac{dp}{ds}$ vanishes in the direction of the course, certainly where the direct course always has a maximum or minimum, but not always with such a kind as may be desired here.

COROLLARY 3

858. $\frac{dp}{ds}$ may vanish in the direction of the course, so that *m* and *n* shall become infinitely small quantities ; and the direct course will produce the fastest speed, if there were

$$\mu gm + 28\mu mf < 28vn,$$

for in this case, the speed will be reduced with the minimum obliquity.

COROLLARY 4

859. Therefore with *s* vanishing, likewise $\frac{dp}{ds}$ will vanish the ship will be progressing fastest in the direct course as long as the tangent $\frac{\mu}{v}$ of the angle $V\upsilon C$ does not exceed this limit $\frac{28n}{m(28f+g)}$. But if $\frac{dp}{ds}$ may not vanish with *s* vanishing, an oblique course is always going to be preferred, unless the wind blows from the stern.

[According to C.Truesdell, who edited the O.O. edition of this work, the analysis of the following sections 860, 861, and 862 were incorrect.]

****

COROLLARY 5

860. Therefore if perhaps for the minimum obliquity there were

$$\frac{\sigma}{\rho} = \frac{s^2h^2}{r^2f^2} \text{ and } u^2 = \sqrt{(s^4h^4 + r^4f^4)}$$

s will be a minimum or infinitely small:

$$\frac{dp}{ds} = -1 = m \text{ and } \frac{du}{ds} = 0,$$

from which the speed also will be produced greater, if the prow *A* were reclined on the opposite side of the right line *GL*. For in this case with *s* vanishing there will become $\frac{\sigma}{\rho}<\frac{s}{r}$.

COROLLARY 6

861. But the case where there becomes

$$\frac{\sigma}{\rho}=\frac{sh^2}{rf^2},$$

which agrees more with ships will become

$$\frac{dp}{ds}=\frac{h^2-f^2}{f^2}$$

with *s* vanishing. Whereby since $h>f$ with these cases the oblique course will be gong to be used always, unless the wind may be incident directly in the direction of the course.

SCHOLIUM

862. Nevertheless in the case, where there is put

$$\frac{\sigma}{\rho}=\frac{s^2h^2}{r^2f^2}$$

in that itself initially, or may adopt a negative value with the minimum obliquities *p*, yet when at first there may be put $\frac{s}{r}>\frac{f}{h}$ its value shall become positive. On account of which in this case also, unless the angle $V\upsilon G$ shall be a minimum, the oblique course will be directed to the part above the right line *GL*, if indeed the ship must be progressing quickly. Therefore with these same anomalies ignored, which are offered only in this single case with the minimum obliquities, for the most rapid motion the obliquity will have to be directed into the part above the line *GL* thus so that the angle *AGL,* thus in the manner in which we may consider the matter, shall become positive. But then from the circumstances brought forward it is easy to deduce, so that the angle $V\upsilon G$ shall be greater, thus the obliquity must be taken to be greater. But the obliquity at no time is agreed to be taken greater than that for which the maximum angle *RnG* will correspond, therefore if this same angle *AGL* of the obliquity for which the difference between the angle *MRG* and *AGL* is the maximum may be put to be α degrees, and the angle *RnG,* β degrees, the limits between which angles the obliquity of the course *AGL* must be contained will be 0° and $\alpha°$, of which that limit 0° has a place if the angle $V\upsilon C$ shall vanish, but the other only can be called into use when the angle $V\upsilon C$ will be approximately $90+\beta$ degrees. If indeed the angle $V\upsilon C$ were greater than $90+\beta$ degrees then the ship at no time could move forwards in the given direction *GL*, from which since the angle $V\upsilon C=0$ degrees, the angle $AGL=0$ degrees may correspond and for the angle $V\upsilon C=90+\beta$ degrees the angle $AGL=\alpha$ degrees may correspond, close enough for the intermediate angle $V\upsilon C$ to be allowed to assign the angle *AGL*, and that therefore would be established more easily where for one or another with the intermediate angle $V\upsilon C$ may be defined more suitably by the given method the given angle *AGL* may be defined more suitably. But only with an estimate of the oblique course *AGL* will be accepted close enough to the true value, if for the angle $V\upsilon C$ containing *x* degrees the oblique course *AGL* may be taken

$$=\frac{\alpha x}{90+\beta}\text{ degrees.}$$

or perhaps $\frac{\alpha x^2}{(90+\beta)^2}$ degrees, or more generally $\frac{\alpha x^n}{(90+\beta)^n}$; of which the formula will be able to be called into use, if for a certain given angle $V\upsilon C$ the angle *AGL* were agreeing especially with what actually will have been determined, from that indeed the exponent *n* will be defined. But the angle α and β will be determined easily from a given property of the ship; indeed if there were $\frac{\sigma}{\rho}=\frac{shh}{rff}$, the tangent of the angle *RnG* will be $=\frac{sr(hh-ff)}{s^2h^2+r^2f^2}$; which thus will be a maximum if there were $\frac{s}{r}=\frac{f}{h}$; or the tangent of the angle $AGL=\frac{f}{h}$. But then the tangent of the angle $MRB=\frac{h}{f}$; and the tangent of the difference $RnG=\frac{hh-ff}{2fh}$; or if $\beta=90°-2\alpha$ from which for a varied relation between the quantities *ff* and *hh,* which have that ratio between themselves, which the resistance of the ship moved in the direction *GA* to the resistance of the ship moved in the direction *GE*, will be known for the angles α and β, so that which may be more apparent the following table is seen to be adjoined

hh= *ff*	$\alpha=45°$	$\beta=0°$, 0'
hh=2 *ff*	$\alpha=35°$, 16'	$\beta=19°$, 28'
hh=3 *ff*	$\alpha=30°$, 0'	$\beta=30°$, 0'
hh= 4*ff*	$\alpha=26°$, 34'	$\beta=36°$, 52'
hh=5 *ff*	$\alpha=24°$, 6'	$\beta=41°$, 48'
hh=6 *ff*	$\alpha=22°$, 12'	$\beta=45°$, 36'
hh=7 *ff*	$\alpha=20°$, 42'	$\beta=48°$, 36'
hh= 8*ff*	$\alpha=19°$, 28'	$\beta=51°$, 4'
hh=9 *ff*	$\alpha=18°$, 26'	$\beta=53°$, 8'
hh= 10*ff*	$\alpha=17°$, 33'	$\beta=$ 54°, 54'

But since where the angle β is greater, the course of the wind there may be put to be more adverse, it is clear where the length of the ship were greater with respect to the width there the more adverse wind can be more difficult to navigate; indeed if *ff* to *hh* maintains a ratio close to the ratio of the width of the ship to its length. But with ships the customary ratio in use is approximately $kk = 4ff$, from which these are more suitable to navigate adverse winds, thus so that the angle $V\upsilon L$ may become almost 53°, 8' or the angle $V\upsilon C$, 126°, 52' that which agrees very well with experience where ships are compared to 11 rhombohedra, or able to be directed at $123\frac{3}{4}$ degrees.

****

PROPOSITION 82

PROBLEM

863. *To define the course to be established by a ship, so that it may sail the quickest into the region, from which the wind comes.*

SOLUTION

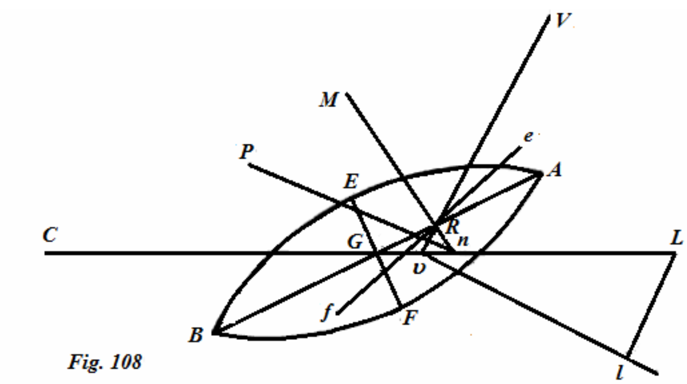


Fig. 108

First the obliquity *AGL* of the ship may be taken as it pleases (Fig. 108) of which the sine of the angle shall be *s,* the cosine $= r$; and the wind may come in the direction $V\upsilon$ equally given, and the position of the line *GL* is sought with respect to $V\upsilon$ so that the course will be going into the region from which the wind comes, to be progressing as quickly as possible. Therefore the angle $V\upsilon L$ is sought, of which the sine shall be $= \mu$, and the cosine $= v$. Moreover from the angle of the obliquity *AGL* given, the mean direction of the resistance *RM* will be given which shall make the angle *RnG* with the direction of the course *GL,* the sine of which shall be $= p$ and the cosine $= q$. Now as before with the speed of the wind put $= \sqrt{c}$, the plane area of the sails *gg,* and with the plane resistance expressed $= uu$, the speed with which the ship will be moving forwards in the direction *GL*

$$= \frac{(\mu p - vq) g \sqrt{c}}{28u + qg};$$

were we make *v* negative, because we put the angle $V\upsilon L$ acute. Therefore the ship shall advance in the direction *GL* with an acute angle put in place with the direction of the wind $V\upsilon$, while it traverses the distance υL, into the region from which the wind comes, the distance *Ll* to be added, with υl drawn perpendicular to the direction of the wind $V\upsilon$ and *Ll* parallel to that. Hence the ship will proceed on this course with the speed

$$=\frac{v(\mu p-vq)g\sqrt{c}}{28u+qg};$$

which must be a maximum. Therefore since the given obliquity of the course *AGL* may be put in place, that may be differentiated with the variables μ and v put in place, and there the differential may be put $=0$; but on account of $dv=-\frac{\mu d\mu}{v}$ this equation will be produced $(\mu^2-v^2)p=2\mu vq$, from which there becomes $\frac{\mu}{v}=\frac{q+1}{p}$. Whereby since the tangent of the angle $V\upsilon L$ shall be $=\frac{q+1}{p}$, the angle itself $L\upsilon V=90°-\frac{1}{2}Rng$ or the angle *Lvl* will be equal to half the angle *Rng.* And thus with the ship provided with a course of given obliquity *AGL*, the angle *RnG* will be bisected by the right line *nP,* and thus the ship shall be turned towards the wind so that the direction of the wind *Vv* shall be normal to that line *nP*. Truly since there is

$$\frac{\mu}{v}=\frac{q+1}{p} \text{ there will be } \mu=\frac{q+1}{\sqrt{(2+2q)}}=\sqrt{\frac{q+1}{2}} \text{ and } v=\sqrt{\frac{1-q}{2}};$$

from which the speed with which heads into the wind will be

$$=\frac{(1-q)g\sqrt{c}}{2(28u+qg)}.$$

Now if the angle of the obliquity of the course *AGL* may be sought, which shall return this expression $=\frac{1-q}{28u+qg}$, then that course of the ship will be had, where with the greatest speed of all, the ship it will be moved into the direction of the wind. But before everything it is understood from the solution of the preceding problem the obliquity of the course must be taken smaller than that, to which the maximum angle *RnG* will correspond. And if we may put while p increases by the element dp, meanwhile u to increase by the element ndp, the tangent of the angle *RnG* will be produced

$$\frac{p}{q}=\frac{28n(1-g)}{28u+g} \text{ [Corrected expression.]}$$

which expression if it were greater than the tangent of the angle *RnG,* if it is a maximum, then the maximum angle itself or a little smaller will be required to be used. But in any particular case offered that same part of the question, which may be considered for the determination of the obliquity of this course, will be easily resolved. Q. E. I.

COROLLARY 1

864. Since the angle *VRe,* within which the wind rushes into the sail shall be the complement of the angle $nR\upsilon$ for the right angle, also the angle *VRe* will be equal to half of the angle *GnR,* and thus the sine of that will be

$$\sqrt{\frac{1-q}{q}}.$$

COROLLARY 2

865. Then also it is required to note the angle $V\upsilon L$ which the direction of the wind makes with the path requiring to be described *GL,* with the angle *VRe* to be made right.

COROLLARY 3

866. Since the speed with which the ship shall approach towards the wind shall be

$$=\frac{(1-q)g\sqrt{c}}{2(28u+qg)},$$

it is evident the speed hence to become $=0,$ if $q=1$ and to become a maximum if $q=0$ or the angle *RnG* to become right. But since the angle *RnG* shall not be able to increase beyond a certain limit, it is understood the maximum approach to the wind if the maximum angle *RnG* may be taken. On account of which the obliquity of the course is required to be so great, so that the corresponding angle *RnG* may not differ sensibly from its own maximum value.

COROLLARY 4

867. Therefore if the angle of the obliquity of the course, the maximum of which will correspond to the angle *RnG,* may be put $=\alpha$, and the maximum $RnG=\beta$, the angle AGL must be taken a little smaller than α; thus so that *RnG* may remain $=\beta$.

COROLLARY 5

868. We may put the angle AGL itself equal to the angle α or to be taken a very small amount smaller, the angle $RnG=\beta$; from which on account of the isosceles triangle $Rn\upsilon$ the angle $V\upsilon L$ will become $=90°-\frac{1}{2}\beta$, and the angle *VRA,* which constitutes the angle between the wind blowing with the direction of the spine of the ship *AB* will be

$= 90^\circ - \alpha + \frac{1}{2}\beta$ degrees or a little more. Moreover the angle which the wind makes striking the sails or *VRe* will be $= \frac{1}{2}\beta$.

COROLLARY 6

869. Hence therefore with the aid of the table given above, which contains the relation between α and β, the course of any given ship thus will be able to be directed, so that an especially adverse journey against the wind may be put in place.

SCHOLIUM

870. We have assumed in these propositions, the ship now to have that speed, with which it will be able to progress along a given direction with the wind acting; but not yet have we been able to investigate, how that speed will have been acquired. On account of which these properties as well, which we have found not to be understood fully, unless that same speed of the ship which it now has, we have attributed to have been obtained from elsewhere. Evidently these propositions are concerned with uniform motion, where the ship can be carried along propelled by the wind nor produced from these and the acceleration of the motion, if the ship were placed in a state of rest, or would have a given speed in a given direction, it can be known, but these propositions advanced are in agreement whereby it is understood, how the ship, if now a certain speed shall follow, that with the aid of the wind may be able to be conserved and by that may be able to progress along a straight line. Whereby since these shall required to be explained satisfactorily, we will investigate how a ship shall be able to accept that motion from the wind and that to be able to be increased: in which, initially, it will be required to inquire if the ship now may have some speed in a given direction and what obliquity of the course it shall maintain; how a given wind rushing into each sail set out may affect that motion, either by increasing or diminishing that motion, or by changing the direction itself, or finally with the obliquity of the course unchanged. Then if this were defined, questions of this kind will be allowed, for which a given disposition of the ship and of the sails is required, the whole motion which the ship may receive from the wind acting to be treated ; and finally from these, it may be possible to obtain the motion of each kind of ship will be able to judged, which we have now put in place in these preceding propositions, or otherwise? and if it may be able to be done, so that the motion of such may itself be acquired, and will be agreed on in a like manner, so that motion of this kind shall be going to be produced. Indeed in navigation uniform motion in a straight line is required especially, which if now were of the form, we have established how it may be conserved ; but this may be of no use, unless it may be agreed, what will be the direction of the ship and what will be the arrangement of the sails, if the ship at first shall be at rest, and that motion of this kind shall be able to be continued. Then truly it will be required to know, how from one constant motion some other given shall be required to be formed, questions of this kind are of the greatest concern in navigation.

PROPOSITION 83

PROBLEM

871. *If the ship AEBF may hold some oblique course* (Fig. 109) *AGL, thus so that its centre of gravity G shall be endowed in a progressive motion in the direction GL; moreover this ship will be acted on by a wind blowing in the direction VL and striking the sails ef; to determine both the change in the motion as well as in the obliquity of the course thus arising.*

SOLUTION

The speed of the ship shall be progressive, which has its centre of gravity G following the direction GL due to the height υ, or $=\surd\upsilon$. And the sine of the angle of the obliquity of the course $AGL = s$ and the cosine $= r$, with the whole sine always put $=1$. Then the centre of gravity of the sails at which the force of the wind has been gathered is required to be considered from the point r from the axis AB with there being $Gr = y$; and the position of the sails ef shall constitute the angle Are with the axis AB or Brf of which the sine shall be $= m$ and the cosine $= n$; truly the plane area of the sails shall be gg. Finally the speed of the wind shall be $=\surd c$, and the sine of the angle VrB, which the direction of the wind makes with the axis of the ship AB shall be $=\mu$ and the cosine $= v$. From these presented, the direction of the force of the wind will be had in the direction rs normal to the surface of the sails ef, and the sine of the angle $Ars = n$ and the cosine $= m$; truly the sine of the angle rsG will be $= nr - ms$, and the cosine $= ns + mr$; and the sine of the angle Vrf will be $= n\mu + mv$; finally of the angle Gtf, which constitutes the position of the sails ef with the direction of motion GL, the sine will be $= ns + mr$; from which the magnitude of the force of the wind will be $\left((n\mu + mv)\surd c - (ns + mr)\surd v\right)^2 gg$, or equal to the weight of so great a volume of air.

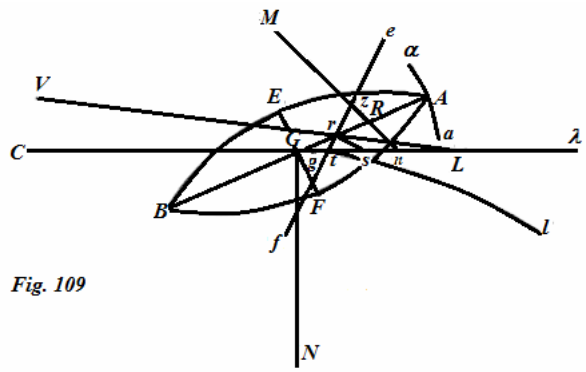


Fig. 109

Whereby if the weight of the ship may be put $= M$, and the volume of the submerged part $= V$ that force of the wind will be

$$\frac{M\left((n\mu + mv)\surd c - (ns + mr)\surd v\right)^2 g^2}{784V}.$$

Again for this oblique course the centre of resistance R with there being $GR = z$; and the mean direction of the resistance RM, which since it shall make the angle MRG with AB, the sine of which shall be $=\sigma$ and the cosine $=\rho$, the sine of the angle RnG will be

$= r\sigma - s\rho$, and the cosine $= s\sigma + r\rho$, truly the whole resistance shall be equal to that, which the plane surface *uu* experiences with the speed directly opposite to the water motion ; from whatever the resistance, it will be $= uu\upsilon$, or equal to the weight $\frac{Muu\upsilon}{V}$.
On account of which the ship will be acted on by the two forces, the first of which arises from the wind is equal to

$$\frac{M\left((n\mu + mv)\surd c - (ns + mr)\surd v\right)^2 gg}{784V}$$

and the ship is driven in the direction *rs;* truly the latter is the force of the resistance $= \frac{Muu\upsilon}{V}$, which pushes the ship in the direction *RM.* So that now the motion may be able to progress unchanged, each force is considered to be applied to the centre of gravity, and each to be resolved into two sides, clearly placed tangential in the direction *GL* and maintaining the normal direction *GN* ; moreover the force will be produced acting on the centre of gravity *G* in the direction *GL,* or the tangential force

$$= \frac{M(ns + mr)\left((n\mu + mv)\surd c - (ns + mr)\surd \upsilon\right)^2 g^2}{784V} - \frac{M(s\sigma + r\rho)uu\upsilon}{V}.$$

But the force acting in the direction *GN* normal to the direction of the motion *GL*, or the normal force will be

$$= \frac{M(nr - ms)\left((n\mu + mv)\surd c - (ns + mr)\surd \upsilon\right)^2 g^2}{784V} - \frac{M(r\sigma - s\rho)uu\upsilon}{V}.$$

For the sake of brevity the tangential force shall be put $= T$ and the normal force $= N$; and the centre of gravity *G* may be considered to be moving with its speed $\surd \upsilon$ through the increment $Gg = dx$, meanwhile it will be accelerated thus so that there shall become $d\upsilon = \frac{Tdx}{M}$. Likewise truly the normal force *N* will try to deviate the right line from the rectilinear path, and to describe the line *Ggl* convex towards *GL*, of which the radius of curvature *GN* at *G* will be $= \frac{2M\upsilon}{N}$; or, while the ship traverses the element $Gg = dx$, it will be deflected from the course *GL* towards *l* by the angle $= \frac{Ndx}{M\upsilon}$. Besides each force, if indeed the direction of neither passes through the centre of gravity *G*, will try to rotate the ship about the vertical axis passing through the centre of gravity *G*, and the force of the wind which indeed will try to rotate the prow *A* towards *a*, and its moment for the effect will be

$$\frac{Mn\left((n\mu+mv)\sqrt{c}-(ns+mr)\sqrt{\upsilon}\right)^2 g^2 y}{784V}.$$

The force of the resistance will produce a rotation in the opposite direction, and the prow *A* will be rotated towards *α* and the moment of this effect will be

$$=\frac{M\sigma uu\upsilon z}{V};$$

from which it will be agreed the moment of the forces for rotating the prow *A* towards *a* will become

$$=\frac{Mn\left((n\mu+mv)\sqrt{c}-(ns+mr)\sqrt{\upsilon}\right)^2 g^2 y}{784V}-\frac{M\sigma u^2\upsilon z}{V}$$

which quantity if it were negative, will act on the prow in the direction *Aα* about *G*. Q. E. I.

COROLLARY 1

872. If therefore with the aid of the rudder the required hindrance to the motion shall be rotating the ship about the vertical axis, so great a force must be exerted by the rudder, of which the moment with respect to the same vertical axis shall be equal and opposite to that moment arising from the wind and the water resistance.

COROLLARY 2

873. If both the centre of the forces followed by the wind *r*, as well as the centre of the resistance *R,* may pass through the centre of gravity *G*, then the rotational force will vanish completely, and there will be no need for the force of the rudder requiring to maintain the direction *AB*.

COROLLARY 3

874. But unless both *y* as well as *z* may vanish, it is evident the direction *AB* cannot be maintained without the action of the rudder. For even if the moment of this may be put to become $=0$, yet with a change in the speed, that will cease to be $=0$.

COROLLARY 4

875. If the speed of the ship at *G* may be put $=0$, then the resistance will vanish, and the wind will express the motion of the ship in the direction *rs:* which since never shall it be able to make an acute angle with the direction of the wind *Vr*, the initial motion will not be able to be made against the wind.

COROLLARY 5

876. But if the ship were to have a motion in the direction *GL* with the speed $V\upsilon$, the direction of the motion will be conserved, if the normal force *N* vanishes, that is if there were :

$$\left((n\mu+mv)\sqrt{c}-(ns+mr)\sqrt{\upsilon}\right)^2 g^2=\frac{784(r\sigma-sp)uu\upsilon}{nr-ms}.$$

But if that magnitude were greater than this, then the angle *VLλ* would become greater, truly on the other hand if that quantity were smaller than this, the angle *VLλ* would emerge smaller.

COROLLARY 6

877. Therefore unless such a motion of the ship shall be going to be produced, the direction of which *GLλ* shall constitute an angle *VLl* with the direction of the wind *VL* to be either right or acute, motion of this kind will be easy to be produced, by bringing about so that the first motion in the direction *rs* may be made in the given direction, which can be done, and then so that the normal force shall vanish; with that done a direct course can be put in place.

COROLLARY 7

878. But if a motion may be desired against the wind, thus so that the angle *VLA* shall be made acute, that initial motion cannot be obtained. Whereby after the start of the course thus it must be directed so that there shall be always

$$784(r\sigma-s\rho)u^2\upsilon>(nr-ms)\left((n\mu+mv)\sqrt{c}-(ns+mv)\sqrt{\upsilon}\right)^2 g^2.$$

But this will be obtained most conveniently, if at least the initial motion concerning the sails *ef* may be maintained normal to the present course *GL,* or also may be inclined so that the angle *rtL* shall become obtuse. Truly when the ship will have acquired some speed, then the sails will be able to be set out gradually into that state, which is required for equitable motion.

COROLLARY 8

879. But in this course, which must be put in place against the wind, it is required to be observed properly, the direction of the prow *GA* always to be required to be kept within the directions *VL* and *Lλ*, and hence an oblique course to be maintained, so that the angle *RnG* may become a maximum.

COROLLARY 9

880. If the sine of the angle $Vrf = \zeta$, or $n\mu + mv = \zeta$, and the sine of the angle $rtL = \eta$ and the cosine $= \vartheta$, thus so that there shall be $\eta = ns + mr$ and $\vartheta = nr - ms$, and likewise the sine of the angle $RnG = p = r\sigma - s\rho$ and the cosine $= q = s\sigma + r\rho$, the accelerating tangential force will be

$$T = \frac{M\eta(\zeta\sqrt{c} - \eta\sqrt{\upsilon})^2 g^2}{784V} - \frac{Mqu^2\upsilon}{V},$$

and the normal force deflecting the course GL towards l

$$N = \frac{M\vartheta(\zeta\sqrt{c} - \eta\sqrt{\upsilon})^2 g^2}{784V} - \frac{Mpu^2\upsilon}{V}.$$

COROLLARY 10

881. Therefore so that the course of the ship may be able to become directed against the wind , it is required that there shall be, just as we have now shown:

$$784pu^2\upsilon > \vartheta(\zeta\sqrt{c} - \eta\sqrt{\upsilon})^2 g^2;$$

and likewise so that the motion may be accelerated there must become

$$\eta(\zeta\sqrt{c} - \eta\sqrt{\upsilon})^2 p^2 > 784qu^2\upsilon :$$

from which there must become much more $p\eta > q\vartheta$ or the angle $RnG > rsG$; or the angle Rzr must be acute.

COROLLARY 11

882. Therefore besides the acute angle Rzr, also to be

$$\zeta\sqrt{c} - \eta\sqrt{\upsilon} > \frac{28u}{g}\sqrt{\frac{p\upsilon}{\vartheta}};$$

and likewise

$$\zeta\sqrt{c} - \eta\sqrt{\upsilon} > \frac{28u}{g}\sqrt{\frac{q\upsilon}{\eta}}.$$

But each of the conditions to be especially satisfied, if that obliquity of the course may be taken, to which the maximum angle *RnG* corresponds*,* for then *p* will obtain the maximum value, *q* truly the minimum.

SCHOLIUM

883. Therefore the known motion of a given ship not only depends on the speed of the wind acting in a given direction, but also on the magnitude of the sails and especially on the resistance from the two angles, evidently firstly from the angle *Vrf* which the direction of the wind makes with the position of the sails *ef*, secondly from the angle *rtL,* which the direction of the sails *ef* makes with the direction of the motion *GL*, of which the difference of the angles is the obliquity of the course *AGL,* from which the direction of the resistance *nRM,* or the angle *RnG* is determined. Therefore just as these angle remain constant in the course, or one or the other proves to be variable, the ship will describe another course and will have another speed at the individual points. But it will be with the greatest difficulty to define this motion if the angle of the obliquity *AGL* were variable, since from its variation not only may the angle *RnG* be changed, but also the absolute magnitude of the resistance which is expressed by *uu*, each of which has been assigned with difficulty, in whatever manner the obliquity of the course either has been increased or decreased. Otherwise it is understood how much it may expedite ships to be constructed thus, so that the centre of resistance for any oblique course shall pass through the vertical line of the centre of gravity, thus indeed if this were put in place, and in addition the sails were set out on masts, so that the mean direction of the wind which they receive may fall on the same vertical line, not only will the action of the rudder be desired so much, but also the ship be directed by a light force of the rudder, since at no time will a force of this kind occur which will try to turn the ship. Truly besides even if it may occur so that the centre, both of the resistance as well as of the force of the wind may not fall exactly at the same place, yet provided the distinction were small enough, with the aid of the rudder another clearly stronger force would not be needed, more than that required for the rotation of the ship about a vertical axis passing through the centre of gravity, yet with the aid of the rudder the direction of the motion is changed strongly. Indeed while the obliquity of the course may be changed with the aid of the rudder, likewise the mean direction of the resistance is changed into another region, so that it will become as the direct motion itself soon to be turned aside. Thus by the action of the rudder, not only the position of the ship or the length of its axis drawn from the stern to the prow is affected, but also the direction of the motion in the same region, in which the prow is directed, indeed to be deflected if indeed ships to be progressing straight in the resistance they experience will be extremely small compared to that which they endure if they are carried along in an oblique course. On this account the direction of the wind requiring to be put in place, is to be performed with the aid of the rudder, in order that the oblique course may be maintained, to which the maximum angle *RnG* may correspond. Then likewise so that the motion of ships will have the maximum acceleration, and in the opposing direction from which the wind comes, may be changed, the sails thus are continually required to be set out, so that an angle as great as *rts* may be maintained, as

great as the remaining circumstances permit ; so that in fact the greater the angle *rts* thus the greater will be its sine η and in the opposite sense therefore the smaller will be its cosine ϑ, from which both the maximum acceleration will be obtained, as well also the maximum declination of the course against the wind. But if the course now the course may make an acute angle with the direction of the wind, then certainly the angle *rts* will be required to be a greater acute angle, since otherwise the wind will not be impelled by the sails. So that this may always be effected, so that both the angle *Vre* shall be acute, as well as the magnitude of the wind will permit. But now since the speed will be made constant, then this direction of the course is required to be put in place, which above has been shown for equitable motion.

PROPOSITIO 84

PROBLEM

884. *If the ship AEBF* (Fig. 110) *may be moving obliquely in the river, the course of which shall be CGD, thus so that the axis AB of the ship shall make an oblique angle AGC with the direction of the river CD; to determine the change of the motion arising from the force of the river.*

SOLUTION

The speed of the river shall correspond to the height *k,* and for the angle *AGC,* which the axis or spine of the ship *AB* makes with the course of the river *CD*, the sine $=\mu$, and the cosine $=v$; moreover the motion of the ship may now be had, with which its centre of gravity *G* shall be progressing in the direction

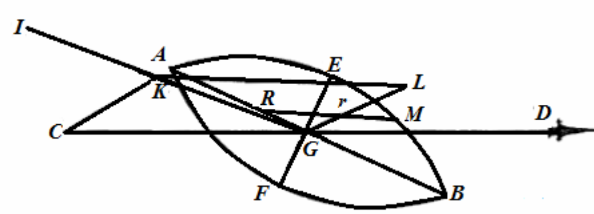


Fig. 110

GL with a speed corresponding to the height υ, which may make the angle *LGD* with the course of the river *GD,* of which the sine shall be $=m$ and the cosine $=n$. Now so that the force of the water on the ship may be found, the whole system composed from the river and the ship may be considered to be moving with the speed $\sqrt{\upsilon}$ in the direction opposite to *GL* itself, in order that the river may strike the ship at rest. Therefore with $GC=\sqrt{k}$, and $GL=\sqrt{\upsilon}$, the parallelogram *GCKL* may be completed, and the diagonal *GK* then will represent the direction which the speed, with which the river is considered to strike the ship at rest. Moreover, in triangle *GCK* the angle *C* is given, of which the sine $=m$ and the cosine $=n$, and both the sides $CK=\sqrt{\upsilon}$ and $CG=\sqrt{k}$, from which the speed of the river arises running in the direction $IG=\sqrt{\left(k-2n\sqrt{k\upsilon}+\upsilon\right)}$, and the sine of the angle *CGI*

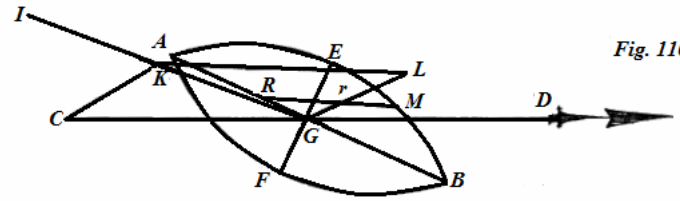


Fig. 110

$$=\frac{m\sqrt{\upsilon}}{\sqrt{\left(k-2n\sqrt{k\upsilon}+\upsilon\right)}},$$

and its cosine

$$=\frac{\sqrt{k}-n\sqrt{\upsilon}}{\sqrt{\left(k-2n\sqrt{k\upsilon}+\upsilon\right)}},$$

from which the angle AGI becomes known; moreover the sine of the angle AGI

$$=\frac{\upsilon\sqrt{k}-\left(\mu n+\nu m\right)\sqrt{\upsilon}}{\sqrt{\left(k-2n\sqrt{k\upsilon}+\upsilon\right)}},$$

and its cosine

$$=\frac{\upsilon\sqrt{k}+\left(\mu m-\nu n\right)\sqrt{\upsilon}}{\sqrt{\left(k-2n\sqrt{k\upsilon}+\upsilon\right)}}.$$

Now we may consider the ship to be moving in the direction *GI* in water at rest, *R* will be the centre of resistance, and *Rm* the mean direction of the resistance, and the resistance itself shall be so great, of such a size the plane figure *uu* will appear moving against the water with the same speed. With these in place the ship will be acted on by the flow of the water in the direction *RM,* by a force, which is equal to the weight of the volume of water equal to $\left(k-2n\sqrt{k\upsilon}+\upsilon\right)u^2$, or with the weight of the ship $=M$ and with the volume of the submerged part $=V$, the force of the water striking the ship

$$=\frac{\left(k-2n\sqrt{k\upsilon}+\upsilon\right)u^2}{V},$$

by which force the ship will be propelled along the direction RM. Therefore this force resolved into two parts, of which the one lies along *GL*, the other normal to *GL*, since it will give the tangential force, by which the motion will be accelerated in the direction *GL*, then the normal force, by which the direction of the motion is deflected from *GL* towards *GD.* Finally unless the point *R,* which is the centre of the resistance corresponding to the obliquity of the course *AGI* may coincide with *G* likewise the centre of the ship will be rotating about the vertical axis drawn through the centre of gravity *G* and indeed in the region *AEL* if *R* were situated between *A* and *G*, truly in the other region if the point *R* may fall between *B* and *G*; but indeed if the angle *CGI* were smaller than the angle *AGC,* for if the angle *CGI* were greater than the angle *AGC* all these will be become opposite. Q.E.I.

COROLLARY 1

885. If the ship either were at rest or may be moving along the direction of the river *GD* then the angle *CGI* will vanish, and *R* will be the centre of resistance corresponding to the obliquity of the curve *AGC*. Therefore the turning force will try to be increasing the obliquity *AGC*, if *R* may lie between the points *A* and *G* , truly on the other hand the axis *AB* will be disposed along the course of the river *CD*, if *R* may fall between *G* and *B*.

COROLLARY 2

886. Therefore if the ship were put in place initially following the course of the river *CD* directly, and it may be turned aside a little from that, if the centre of resistance *R* may fall between the points *G* and *B*. But on the other hand, if *R* were situated between *G* and *A*, the direction *AB* will be declined more from the course of the river *CD*.

COROLLARY 3

887. Therefore if the force of the river on the exposed part *EAF* were of a great enough weight, and a great force may be experienced from the river, then the ship or whatever body will be able to descend straight down the river, since in this case the point *G* approaches towards *A*, and truly recedes from *R*.

COROLLARY 4

888. But if the part *EAF* were extremely sharp and light, thus so that the centre of the resistance *R* will be located close to *A*, then the ship will descend with difficulty straight down the river, but will be turned aside by the smallest force from the direct course *CD*, and it will be moved aside from the direct position.

COROLLARY 5

889. Truly in the same case, where the motion of the ship *GL* happens on the course of the river *CD*, since the angle *GRM* will be greater than the angle *AGC*, the ship will be dislodged from its own direction, and forced to the bank of the river, to which *A* alas will be forced to be turned.

COROLLARY 6

890. Therefore we may put the ship now to be accepting a motion in the direction *GL* towards the bank, and the angle *AGO* to remain the same ; this motion will be accelerated, if indeed the angle *LrM* were acute, truly likewise it will be deflected from this direction, towards *GD*, if indeed the angle *LGD* were greater than *MRG*.

SCHOLIUM

891. Therefore it is evident the motion of bodies to be especially irregular on being dragged away by a river, even if these may be provided with a vertical diametric plane and the two parts may be had equal and similar on both sides. Indeed in the first place unless the axis *AB* may be incident in the direction of the river itself, the body is forced towards that bank to which it is inclined with the prow *A* (for it will be agreed that part of the ship which contends with the water to be called the prow). In the second place the body may be turned around the vertical axis through the centre of gravity by the water in various ways, according to the diverse situation of the centre of resistance *R,* which with the rotation endured continually changes position, unless it shall be fixed for every obliqueness. Thirdly, on account of a force arising from the river normal to the direction of the motion *GL*, the direction of the motion itself is affected and that either shall be inclined more to the course of the river or for that to be reduced, just as the angle *MRG* either shall have been either greater or less than *LGD.* But truly even if motion may be changed towards the course of the river, yet it is unable to recede from that, since with the angle *LGD* diminished to that point so that it may become smaller than *MRG,* a deflection may arise. Finally, even if the one direction *GL* may become parallel to the direction *RM,* in which case the normal force will vanish, yet on account of the rotation of the body either by the force of the water, or arising from some other of the lightest causes, at once another mean direction of the force of the water will be present, which will disturb the motion. Yet meanwhile ships of this kind, in which the centre of the resistance falls directly towards the stern *B* may be washed away by the river, if indeed the direction of the motion from the course may be began. But irregular bodies, which do not even have the advantage from the two equal and similar parts, will be required to be carried by an especially irregular motion, from which soon they will be set down at one or other bank. But because it restrains ships, with the aid of the rudder applied at *B* regular motion will be obtained easily, and a course either following the river directly downwards or it will be able to be put in place at the bank; but always the descending motion will prevail, in the direction of the river, so that it cannot be transferred entirely from this course to the normal course; but the ship on being transferred between will be sent of more downwards where the speed of the river will have been faster. Since the force of the river now may require another force to be acting, whether it be of the oars or of the wind, or it may be able to be jointly, and to be determined, how any intended course may be able to be put in place most conveniently, but since these may be better suited to another book, in which the navigation of ships is required to be constituted from a professed treatment, here many things have not been advanced with regard to motions of this kind, especially since it shall be easy from the method treated questions of this kind, which may be able to proposed, to be recalled and resolved according to the calculation. Therefore it remains that we may expound on the motion of ships which are not free, but are bound in some manner. Therefore it remains that we may expound on the motion of a few ships which are not free, but occasionally are bound, which knowledge

for the passage of ships without outside force is especially useful, [as with ferry boats on fast flowing rivers, such as the Rhine at Basel, Euler's home town.]

PROPOSITION 85

PROBLEM

892. *If the ship AD* (Fig. 111) *in the river flowing in the direction ZV, with the aid of the rope IZ thus will have been firmly attached to the point Z, so that it shall hold the same position with respect to the rope IZ, that which may be considered to be made by the rope BC, the point B of the ship connected with C; to determine the motion, which the course of the river will impress on this ship.*

SOLUTIO

As before we may put every part of the ship submerged in the water to be a parallelepiped, because otherwise the forces which the river exerts on different positions, shall only be able to be compared amongst themselves with difficulty. And thus *ABDC* shall be the rectangular water section, of which the side *AB* shall be $= a$, the side $AC = b$, and the depth of the ship in the water $= c$, the force of the river acting on *AB* $= ac$, and the surface under *AC* moving within the water $= bc$. Again the speed of the river must correspond to the height *k* and the length of the rope $ZI = f$, which shall be much greater than the magnitudes *a, b,* and *c* ; the sine of the angle $ZIA = m$ and the cosine $= n$. Now so that what follows is concerned with the motion of this ship, it is understood the ship cannot have another motion besides that of rotation about the point *Z,* so that the point *I* will be carried on the arc of the circle *PVQ,* of which the centre is *Z*. Moreover we may put the ship at *P* to have began moving, and by running through the arc *PVQ* to have come to the position *ABDC,* at which point *I* it shall have the speed corresponding to the height υ, with which it will try to progress through the arc *IVQ*. In addition corresponding to the angle *IZV,* which the position of the rope *IZ* makes with the direction of the river *ZV*, the sine $= x$ and the cosine $= y$. With these in place we will consider the force of the river, which it will exercise on the ship. And indeed in the first place it will rush against the side *AB,* of which the area is $= ac$ in the direction *ME* within the angle $MEA = ZIA + IZV$, of which the sine of the angle will be $= my + nx$, truly the

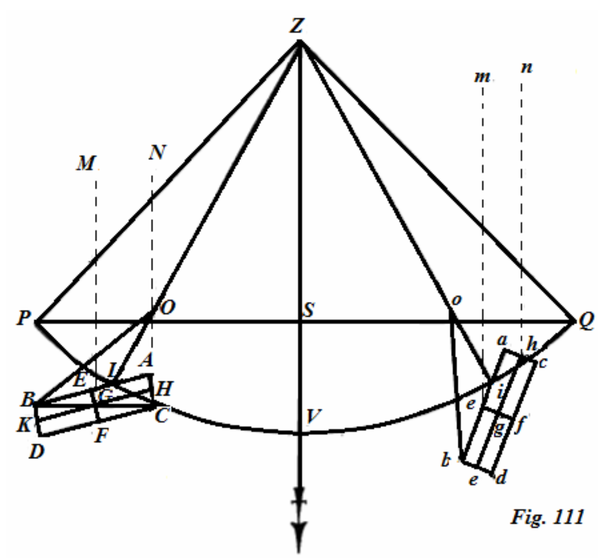


Fig. 111

cosine $= ny - mx$, which cosine likewise is the sine of the angle *NHA* within which the river impinges on the side *AC*. Now if the ship shall be at rest, the force of the river may be had at once, but since now the ship shall be put to have a motion, the force will be found, which the river will exert on the side $AB = ac$, in the direction of the normal *EF* to *AB* at its mid-point *E*

$$= ac\left((my+nx)\sqrt{k} - n\sqrt{\upsilon}\right)^2 ;$$

truly the force which will impress on the side *AC* in the direction *HG* in a similar manner will be

$$= bc\left((ny-mx)\sqrt{k} + m\sqrt{\upsilon}\right)^2 .$$

Moreover with the weight of the ship put $= M$ and with the volume of the submerged part $= V$, the expressions will be found, which if it may be multiplied by $\frac{M}{V}$, will give the weights equivalent to these forces. Hence therefore the moment of these two forces jointly for turning the ship about the pole *Z* in the region *IVQ*

$$= \frac{Mac \cdot nf}{V}\left((my+nx)\sqrt{k} - n\sqrt{\upsilon}\right)^2 - \frac{Mbc \cdot nf}{V}\left((ny-mx)\sqrt{k} + m\sqrt{\upsilon}\right)^2 ,$$

which divided by the moment [of inertia] of the ship with respect to the pole *Z,* which on account of the maximum length f is $= Mff$ will give the rotational force [i.e. rotational acceleration]. Therefore we may put the ship to progress through the element of arc *ds* towards *V*, there will be $ds = -fdx$; and thus meanwhile it will be accelerated so that there shall become:

$$Vd\upsilon = \frac{-n \cdot ac \cdot dx}{y}\left((my+nx)\sqrt{k} - n\sqrt{\upsilon}\right)^2 + \frac{m \cdot bc \cdot dx}{y}\left((ny-mx)\sqrt{k} + m\sqrt{\upsilon}\right)^2 .$$

But for the motion itself it will suffice to be known to be turning the rotation into the region *VQ* and to continue as long as there were $ank(my+nx)^2 > bmk(ny-mx)^2$, or the tangent of the angle $MEA > \sqrt{\frac{bm}{an}}$; which limit arises if there may be put $\upsilon = 0.$ For if has a motion in the direction *IV,* thus it will progress either with an acceleration or retardation ; but if it may be at rest somewhere, then if it will be driven forwards by the river in the same direction *aP* to *Q.* Therefore if somewhere between *P* and *Q* there became tang. $MEA > \sqrt{\frac{bm}{an}}$, then the ship will go on from *P* to *Q* as far as it will reach, which all suffice in abundance for the motion to become known. Q. E. I.

COROLLARY 1

893. If the ship may arrive beyond V in the position *abdc* all will remain as before, except that the sine of the angle VZi must become $= -x$. Thus so that the force on the ship at i will be acting towards Q on the other side, hence there will become [in the above formulas]

$$an\left((my-nx)\surd k - n\surd \upsilon\right)^2 - bm\left((ny+mx)\surd k + m\surd \upsilon\right)^2.$$

COROLLARY 2

894. Therefore the motion hence will continue beyond V towards Q , as long as there were

$$na\left(my-nx\right)^2 > mb\left(ny+mx\right)^2$$

or tang. $mea > \sqrt{\frac{mb}{na}}$. The angle of which the tangent is $\sqrt{\frac{mb}{na}}, = \alpha,$ there will become $mea > \alpha$ or $ZIA - VZi > \alpha$. Therefore so that the ship may be able to arrive at Q as far as it shall be able to come, it will be required that the angle VZQ shall not be greater than $ZIA - \alpha$.

COROLLARY 3

895. Again if we may put the motion of the ship to have begun around the pole Z at P , where motion of this kind through V to Q to be continued as far as it is necessary, so that there shall be $PZV + ZIA > \alpha$, with α denoting the angle of which the tangent is $\sqrt{\frac{mb}{na}}$: for unless this were the case, the motion will not even be able to begin; moreover here all the angles we put to be acute, otherwise it would be required to accept other tangents.

COROLLARY 4

896. If the right line ZV may cross over the middle of the river, and it may be required so that the ship itself thus may be driven across from the bank P of the river to the bank Q, it will be required that there shall be $\text{ang.}VZQ = \text{ang.}ZIA - \text{ang.}\alpha$. For if ang. VZQ were smaller, then the ship would be driven to Q by a very large force, which is to be avoided.

COROLLARY 5

897. If therefore the width of the river *PQ* may be given, and both that kind of ship, as well as the angle *AIZ,* from that equality the length of the rope *ZV* found will be discovered for the required transfer, and then the point *Z* is required to be accepted in the middle of the river, to which the ship is attached.

COROLLARY 6

898. If half of the width of the rive $PS = QS = h$, and the interval $Zs = z$, and the sine of the angle QZV shall $= x$, the cosine $= y$, there will be $\frac{h}{z} = \frac{x}{y}$. Therefore since there will be

$$\sqrt{\frac{mb}{na}} = \frac{my - nx}{ny + mx},$$

there will become $mz\sqrt{na} - nh\sqrt{na} = nz\sqrt{mb} + mh\sqrt{mb}$, and hence

$$ZS = z = \frac{h\left(n\sqrt{na} + m\sqrt{mb}\right)}{m\sqrt{na} - n\sqrt{mb}};$$

and the length of the rope

$$ZV = f = \frac{h\sqrt{na + mb}}{m\sqrt{na} - n\sqrt{mb}}.$$

COROLLARY 7

899. Moreover it is observed as before, generally, there must become

$$m\sqrt{na} > n\sqrt{mb}, \text{ either } \frac{a}{b} > \frac{n}{m} \text{ just as } \frac{b}{a} < \frac{m}{n}.$$

Therefore with the diagonal *BC* drawn in the rectangle *ABDC*, the angle *ZIA* ought to be greater than the angle *ABC.* Finally, the rope must be tied at some point of the right line *AB*.

COROLLARY 8

900. So that, with all else being equal, the ship may reach bank *Q* from bank *P* most quickly, it is required to bring into effect that the acceleration shall be a maximum at the mid point *V* . But if the ship may be at rest at *V*, that force pushing towards *Q* shall

become as $m^2na - mn^2b$, which quantity shall be a maximum if on putting $\frac{m}{n} = t$, there were

$$at^3 - 2bt^2 - 2at + b = 0.$$

COROLLARY 9

901. Moreover since the length of the ship *a* may exceed the width *b* many times, there will become approximately

$$t = \frac{m}{n} = \sqrt{2} + \frac{3b}{4a}.$$

On account of which it will be expedient to take the angle *AIZ* to be around 60°, if indeed the width *b* were around a third or a quarter of the length *a*. And from this, the angle *AIZ* will be several times greater than the angle *ABC*.

COROLLARY 10

902. If the figure of the ship were a square so that there shall be $b = a$, there will become $t = \frac{m}{n} = \frac{3+\sqrt{5}}{2}$, or the angle *AIZ* will be $69°\ 6'$; from which there becomes

$$m = \frac{1+\sqrt{5}}{2\sqrt{3}} \text{ and } n = \frac{\sqrt{5}-1}{2\sqrt{3}}.$$

And hence again the length of the rope produced $= f = h\sqrt{\left(\frac{15}{2} + 3\sqrt{5}\right)} = 3{,}8h$, [The final three values of *f* have been corrected by C.T.] and the angle *VZQ* shall become $10°\ 48'$.

COROLLARY 11

903. If the angle *AIZ* or its tangent *t* were given, the most convenient ratio of the length of the ship *a* to the width *b* will be found, from this equation:

$$\frac{b}{a} = \frac{t^3 - 2t}{2tt - 1},$$

from which it is required to understand by necessity there must become $tt > 2$ or the angle *AIZ* to be greater than $54^{\circ},\ 45'$, but from which at once there becomes $t > \frac{b}{a}$ as required. Therefore there will become

$$f=\frac{h\sqrt{(1+tt)}\sqrt{(tt-1)}}{t\sqrt{(2tt-1)}-t\sqrt{(tt-2)}}$$

COROLLARY 12

904. If the angle sixty degree angle *AIZ* may be put in place or $t=\sqrt{3}$, it will produce a convenient enough figure of ship, for there will become $\frac{b}{a}=\frac{\sqrt{3}}{5}$ or *b* to *a* shall become as 53 to 153 approx. Then the length of the rope *ZV* will be found

$$=f=\frac{4h\sqrt{2}}{\sqrt{15}-\sqrt{3}}=\frac{h\left(\sqrt{5}+1\right)\sqrt{2}}{\sqrt{3}}$$

or approx. $f=2,64222h$ or $\frac{f}{h}=\frac{37}{14}$. Truly the angle *PZQ* will become $44°,\ 28'$.

SCHOLIUM

905. This manner of crossing rivers without oars or sails is exceedingly suitable and adapted for use, since these conditions shall be easy to fulfill, by which the crossing not only shall be possible, but also may be rendered most quickly. But the case established in the final corollary is especially commendable, for which indeed a long rope is not required and the proportion between the length of the ship $AB=a$ and the width $AC=b$, has been found most conveniently; evidently the length to be almost three times the width. Moreover it is easily understood when the crossing from the bank *P* as far as to the bank *Q* will have been performed, how in turn the course from *Q* to *P* shall be put in place ; clearly the ropes *ZI* and *OB* are required to be attached to the other side *CD*. Truly so that this transformation may be able to be done more easily, the rope may be transferred from the point *I* to the point *H* , to be assumed from which it will be obtained, so that on the return there shall be no need for this rope to be tied firmly in some other location ; indeed likewise this is where the rope may be taken in place of *I*. Then it is required to be noted the rope *BC* must be taken so long, so that the angle which the direction of the rope *ZI* will make with *KH* produced shall be 60 degrees. But neither can the point *O* be taken exceedingly close to the point *I* nor exceedingly distant from that, lest the minimum elongation of the rope *CB* shall notably diminish the angle *AIZ* ; moreover the smallest angle *AIZ* will not be changed if there may be taken $OI=IB$. Finally even if with the elongation of the rope *BO* may allowed to assign a magnitude to the angle *AIZ*, yet immediately a remedy will be able to be brought forwards from which there is no need to be warned further. And thus I progress to the motion of ships to be determined in the river, which indeed as before are moveable around a fixed point, but their position with respect to the ropes is not constant; clearly I may put these to be free around both the points *Z* and *I* .

PROPOSITION 86

PROBLEM

906. *If the ship ABDC* (Fig. 112) *in the river shall be attached to the fixed point Z with the aid of the rope IZ, so that not only shall it be mobile about the point Z, but also about the point I to which it is attached by a rope to the ship; to determine the motion of this ship arising from the interaction with the river.*

SOLUTION

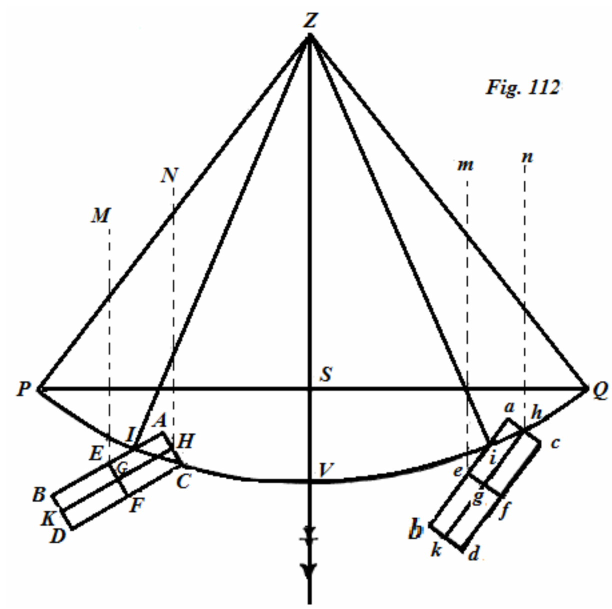


Fig. 112

ZV will represent the direction of the river, and the speed of the river must correspond to the height *k;* truly again as before we will attribute the figure of this ship, of which any horizontal section shall be a rectangular parallelogram *ABDC.* And the length shall be $AB = a$, the width $AC = b$, and the depth under water shall be $= c$. Truly the point I, at which a rope has been fixed to the ship shall be on the side *AB,* and there may be put $EI = i$. Now the ship shall be *in situ* at *ABCD,* where the sine of the angle *AIZ* shall be $= m$; the cosine $= n$, and here the motion may be had progressing through the arc *IVQ* described with centre *Z*, of which the radius or length of the rope *IZ* shall be $= f$; and the sine of the angle *IZV,* which the direction of the rope makes with the course of the river, shall be $= x$, and the cosine $= y$, and the speed of the point I must correspond to the height υ. With these in place from the solution of the previous problem, the force which the river expresses on the side *AB* in the direction *EF* $= ac\left((my+nx)\surd k - n\surd \upsilon\right)^2$, truly the force, which the side *AC* will exert in the direction *HK*, will be $= bc\left((ny-mx)\surd k + m\surd \upsilon\right)^2$. Now since the ship shall be mobile about the point *I* of the ship it is necessary to accept a situation of this kind, where the moments of all the forces with respect of the point *I* will cancel each other out. Truly the moment of the first force with respect of the point $I = ac\cdot i\left((my+nx)\surd k - n\surd \upsilon\right)^2$, and the contrary moment of the other acting

$$= \frac{bbc}{2}\left((ny-mx)\surd k + m\surd \upsilon\right)^2.$$

On account of which this equation will be had

$$\frac{(my+nx)\surd k-n\surd \upsilon}{(ny-mx)\surd k+m\surd \upsilon}=\frac{b}{\surd 2ai},$$

from which equation the prevailing value of m and n and thence the angle *AIZ,* from which the ship at once will be composed. Again the moment of these forces for the ship around the pole Z requiring to be rotated through the arc *IVQ* will be as before:

$$=\frac{M\cdot ac\cdot f\cdot n}{V}\Big((my+nx)\surd k-n\surd \upsilon\Big)^2-\frac{M\cdot bc\cdot f\cdot m}{V}\Big((ny-mx)\surd k+m\surd \upsilon\Big)^2$$

with the weight of the ship $=M$ and the volume of the submerged part $=V$. Truly by the first condition this simpler formula will emerge, and on account of $V=abc$ the moment of the rotation about Z

$$=\frac{MF}{bb}(bn-2im)\ \Big(\big(my+nx\big)\surd k-n\surd \upsilon\Big)^2;$$

therefore as long as $bn>2im$ the rotation along the arc *IVQ* will be accelerated. But the first equation set out gives

$$m=\frac{by\surd k-x\surd 2aik+\surd 2ai\upsilon}{\surd (b^2+2ai)(k-2x\surd k\upsilon+\upsilon)}$$

and

$$n=\frac{\surd y\surd 2aik+bx\surd k-b\surd \upsilon}{\surd\Big(b^2+2ai)(k-2x\surd k\upsilon+\upsilon\Big)}.$$

From these there becomes:

$$\Big(\big(my+nx\big)\surd k-n\surd \upsilon\Big)^2=\frac{b\surd\Big(k-2x\surd k\upsilon+\upsilon\Big)}{b^2+ai};$$

and

$$bn-2im=\frac{\Big(b^2+2i\surd 2ai\Big)\Big(x\surd k-\surd \upsilon\Big)+by\Big(\surd 2ai-2i\Big)\surd k}{\surd (b^2+2ai)(k-2x\surd kv+\upsilon)}.$$

Therefore as long as the ship will be progressing in the arc *IVQ* there will become

$$b^2x+2ix\surd 2ai+by\surd 2ai-2biy>0,$$

that which evidently thus will be had, before the ship reaches V. But since it will progress beyond the middle of the river, and it will have arrived at the situation *abdc,* the sine of the angle *VZi* will be negative. Whereby if the sine of the angle *VZ i* also may be put $=x$ and the cosine $=y$, the ship will advance in the arc *VQ* to that point, so that there will become

$$by\sqrt{2ai}-2biy>b^2x+2ix\sqrt{2ai}.$$

Therefore the final end will be the point *Q*, with the tangent of the angle *QzV*

$$=\frac{b\sqrt{2ai}-2bi}{b^2+2i\sqrt{2ai}},$$

or if the angle *QZV* were the excess of the two angles, of which the tangent of the greater is $\frac{b}{2i}$, truly of the lesser the tangent $=\frac{b}{\sqrt{2ai}}$; which suffices for the motion requiring to be known.
Q. E. I.

COROLLARY 1

907. If the point *I* may be taken at the same point *A* thus so that there shall be $i=\frac{a}{2}$, the ship progressing from the point *P* will not reach beyond the point ; indeed in this case the tangent of the angle $QZV=0$.

COROLLARY 2

908. In a similar manner the ship will not advance beyond the point V, if there were $i=0$, or if the point I at which the rope may be tied may be taken to be the midpoint of the side to *AB* : from which it is evident the point *I* to be given between *A* and *E* so that the ship attached to the rope may progress the furthest beyond *V*.

COROLLARY 3

909. Moreover if the position of the point *I* may be sought, so that the angle *VZQ* may become a maximum, this equation is found: $(bb+4ii)\sqrt{a}=((2bb+4ai)\sqrt{2i})\sqrt{2i}$ or this $l6ai^4-32a^2i^3-24ab^2i^2-8b^4i+ab^4=0$ which gives

$$b^2=\frac{12ai^2\ \pm\ 4i(2i-a)\sqrt{2ai}}{a-8i};$$

therefore there must become $a>8i$ or $EI<\frac{1}{8}AB$.

COROLLARY 4

910. If there may be taken $a = 18i$ there will become $b^2 = \frac{648}{10}i^2$ or $b = \frac{18i}{\surd 5} = 8,05i,$ [corrected: $b^2 = 60i^2$ so that the following is incorrect;] from which there becomes $a : b = \surd 5 : 1,$ and $EI = \frac{1}{9}AE,$ and hence the tangent of the angle VIQ produced $= \frac{3\surd 5}{16}$, or the angle VZQ will be $22°, 45'$: so that from the given width of the river the length of the rope will be determined.

COROLLARY 5

911. Where the smaller part *EI* of *AB* is taken there a smaller width of the ship *AC* arises ;and if the point *I* may lie towards *E*, then the width *Aa* will vanish completely. Therefore all the points *I* from which the ship will be able to progress beyond *V*, will be contained between *E* and a certain point situated between *E* and *A*, the distance of which from *E* is the eighth part of the length *AB*.

SCHOLIUM

912. Therefore in this manner without oars or sails it will be able to establish a crossing through the river; but a huge caution will be required to be observed both in the shape of a suitable ship to be selected, as well as in the points *Z* and *I* to be found, to which the rope *ZI* is the strongest and firmly attached. Indeed we may put a ship of kind to be selected, the length of which *AB* shall be had to the width as $\surd 5$ ad 1, the point *I* at which the rope must be tied, thus will be required to be taken so that it may stand apart from the middle point *E* of the length *AB* by the interval $AI = \frac{1}{18}AB.$ Then if the width of the river shall be put to be $PQ = 2h$ this width must be bisected at *S* so that there shall become $PS = QS = h$; and with the strongest anchor at *Z*, so that *ZS* shall be perpendicular to *PQ,* and the interval *ZS* will be required to be taken so great so that the angle *SZP* or *BZQ* shall become $22°,\ 45'$. Therefore there will become $ZS = h \cdot 2,3847 = \frac{31}{13}h$ approx. ; and the length of the rope $ZP = h \cdot 2,5859 = \frac{75}{29}h$ approx. Moreover so that a complete passage from *P* to *Q* and from *Q* in *P* across the river may come upon, with the rope being required to be bound to the side *CD* at a point situated in the position so that *C* will be separated from *F* by the distance $CF = \frac{1}{18}CD$. And this arrangement will be seen to be the most convenient in practice.

CAPUT SEPTIMUM

DE MOTU PROGRESSIVO CORPORUM AQUAE INNATANTIUM

PROPOSITIO 74

PROBLEMA

762. *Si corpus quodcunque plano diametrali verticali praeditum in aqua quiescente moveatur cursu directo* (Fig. 101), *determinare eius motus quo moveri coepit, diminutionem a resistentia aquae ortam, atque celeritatem in singulis locis viae, quam describet.*

SOLUTIO

Quoniam corpus plano diametrali verticali praeditum ponitur, eius partis submersae, quippe quae per illud planum diametrale in duas partes similes at aequales dividitur, centrum magnitudinis in ipso hoc plano situm erit ex quo etiam centrum gravitatis totius corporis in hoc plano collocatum esse oportet.Quia vero porro hoc corpus cursum directum tenere ponitur, ita ut moveatur secundum directionem horizontalem in ipso plano diametrali positam, media directio resistentiae in hoc ipsum planum cadet. Vis resistentiae horizontalis igitur directioni motus erit directe contraria, et hancobrem solum motum retardabit, directionem motus vero non afficiet. Vis resistentiae vero verticalis si quae adest neque motum corporis neque eius directionem turbabit, sed in corpore allevando tantum consumetur. Deinde nisi resistentiae media directio per ipsum corporis centrum gravitatis transeat, corpus circa axem latitudinalem inclinabit, qua inclinatione neque motus directio neque positio spinae seu axis navis a prora ad puppim ductus mutabitur. Quamobrem a resistentia aquae motus aliter non turbabitur, nisi diminutione celeritatis; ac tam motus directio, quam cursus directus conservabitur. His notatis sit *AEBF* sectio corporis horizontalis per eius centrum gravitatis *G* facta, *AB* recta horizontalis in plano diametrali a prora *A* ad puppem *B* extensa quae simul directionem motus repraesentabit, atque recta *CGL* repraesentabit viam in qua centrum gravitatis ingredietur, in qua simul tum prora *A* tum puppis *B* perpetuo sitae manebunt. Ponamus nunc corpus egressum esse ex puncto *C,* ubi celeritatem initialem habuerit altitudini k debitam; dum vero corporis centrum gravitatis in *G* versatur, sit eius celeritas, qua in directione sua *GL* moveri perget debita altitudini u. Sit porro massa seu pondus totius corporis $= M$, eius partis submersae volumen $= V$; resistentiam vero hoc corpus motu directo in aqua progrediens tantam patiatur, quantam figura plana ff eadem celeritate directe contra aquam mota pateretur; ex quo resistentia, quam corpus, dum eius centrum gravitatis in *G* versatur, patietur aequabitur ponderi voluminis aquae ffu, quod pondus se habebit ad pondus totius corporis *M* ut ffu ad V ita ut vis resistentiae motum retardans futura sit aequalis ponderi $\frac{Mffu}{V}$. Sit nunc spatium $CG = s$, quod corpus ab initio motus iam confecit, atque dum elementum $Gg = ds$ percurret tantum celeritatis decrementum ut sit

$$-d\upsilon = \frac{Mff\upsilon}{V};$$

quae aequatio integrata dat

$$l\frac{k}{\upsilon} = \frac{ffs}{V}$$

integratione ita instituta ut fiat $\upsilon = k$, posito $s = 0$, uti conditio quaestionis requirit. Erit ergo

$$\frac{k}{\upsilon} = e^{ffs:V}$$

denotante e numerum, cuius logarithmus est $=1$, hincque porro $\upsilon = ke^{-ffs:V}$, ex qua formula celeritas corporis in singulis punctis viae, quam describit cognoscitur. Denique cum ipsa celeritas sit $= e^{-ffs:2V}\sqrt{k}$ erit tempusculum, quo elementum $Gg = ds$ percurretur

$$= \frac{e^{ffs:2V}ds}{\sqrt{k}}$$

indeque tempus totum, quod insumsit ad spatium $CG = s$ absolvendum, erit

$$= \frac{2V\left(e^{ffs:2V} - 1\right)}{ff\sqrt{k}}.$$

Q.E.I.

COROLLARIUM 1

763. Cum altitudo celeritati corporis in G debita sit $= ke^{-ffs:V}$, intelligitur corpus omnem motum nunquam esse amissurum: celeritas enim non evanescit, nisi ponatur $s = \infty$, hoc est corpus actu spatium infinitum absolvet antequam omnem perdat motum.

COROLLARIUM 2

764. Expressio celeritatis commodo etiam in seriem potest transformari, per quam fiet

$$\upsilon = k - \frac{kffs}{V} + \frac{kf^4s^2}{2V^2} - \frac{kf^6s^3}{6V^3} + \frac{kf^8s^3}{24V^2} - \text{etc.}$$

quae satis cito convergit, nisi spatium s capiatur valde magnum.

COROLLARIUM 3

765. Deinde etiam perspicitur decrementum celeritatis eo fore maius, dum corpus datum spatium s absolvit, quo maior fuerit area ff, ad quam resistentiam reduximus, et quo minor portio aquae fuerit submersa, hoc est quo levius fuerit corpus.

COROLLARIUM 4

766. Si igitur plura corpora similla eadem celeritate moveri incipiant, tenebit resistentia seu area ff rationem subsesquiplicatam ponderum, partes submersae vero ipsam rationem ponderum, unde intelligitur corpora maiora minus retardari quam minora.

COROLLARIUM 5

767. Tempus etiam, quo corpus datum spatium $CG = s$ absolvit, commode per seriem exprimitur, erit enim

$$= \frac{s}{Vk} + \frac{f^2 s^2}{4V\sqrt{k}} + \frac{f^4 s^2}{24V^2\sqrt{k}} + \frac{f^6 s^4}{192V^3\sqrt{k}} + \text{etc.}$$

At si motu uniformi initiali progrederetur, nullam patiens resistentiam, tum tempus per idem spatium s foret $\frac{s}{\sqrt{k}}$; ex quo quanto maiori tempore propter resistentiam opus sit intelligitur.

COROLLARIUM 6

768. Si tempus, quo corpus datum spatium percurrit desideretur in data temporis mensura, tum in expressione temporis

$$\frac{2V\left(e^{ffs:2V} - 1\right)}{ff\sqrt{k}}$$

quantitates k, s, ff et V in partibus millesimis pedis rhenani exprimantur; quo facto expressio per 250 divisa dabit tempus in minutis secundis.

COROLLARIUM 7

769. Simili modo si celeritas ipsa desideretur expressa per spatium, quod dato tempore percurritur ea celeritate uniformiter, ponatur spatium quod uno minuto secundo, absolvitur essen partium millesimarum pedis rhenani, eritque

$$\frac{n}{250\sqrt{\upsilon}} = 1,$$

dato v pariter in particulis millesimis pedis rhenani, unde fiet $n = 250\sqrt{\upsilon}$.

COROLLARIUM 8

770. Sin autem celeritas detur per spatium n uno minuto secundo percursum, atque n datum sit in partibus millesimus pedis rhenani, invenietur altitudo celeritati illi debita

$$u = \frac{n^2}{62500},$$

pariter in partibus millesimis eiusdem pedis: ex quibus facile erit hos duos celeritates mensurandi modos inter se comparare, alterumque ex altero formare.

SCHOLION 1

771. Quod corpora aquae innatantia nunquam omnem motum omittant, sed perpetuo moveri pergant, id quidem experientiae non est consentaneum, qua satis constat, motum, tandem penitus cessare. Verum hic notari oportet, aquam praeter eam resistentiam, quae quadrato celeritatis est proportionalis aliam insuper resistentiam opponere, a celeritate non pendentem, sed ipsis momentis temporum proportionalem, prout NEUTONUS loquitur, seu quae sit constans, atque altitudinem celeritati debitam diminuat in ratione ipsius elementi spatii percursi. Haec autem resistentia aquae tam est exigua, ut nisi motus sit lentissimus, ea prae altera resistentia evanescat; hancque ob causam in solutione huius problematis istam resistentiam neglexímus, cum institutum nostrum non sit motus tardissimos ex professo prosequi. Interim tamen ista resistentia calculum non reddit difficiliorem; sit enim ista resistentia constans pro casu oblato $= g$, seu ponderi g, aequivalens, prodibit loco aequationis

$$-dv = \frac{ffuds}{V}$$

ista

$$-du = \frac{ffuds}{V} + \frac{gds}{M};$$

quae integrata dat

$$\upsilon = \left(k + \frac{gV}{ffM}\right)e^{-ffs:V} - \frac{gV}{ffM}.$$

Ex hac igitur aequatione utique intelligitur, corpus non ultra datum terminum esse progressurum, cum eius celeritas evanescat percurso spatio s, cuius quantitas ex hac aequatione dabitur

$$e^{ffs:V} = \left(\frac{kffM + gV}{gV}\right) \quad \text{seu} \quad s = \frac{V}{ff} l \frac{kffM + gV}{gV}.$$

Quin etiam ex ista aequatione cognoscetur ipsa haec resistentia g, ex spatio percurso, donec totus motus fuerit amissus, si enim hoc spatium per experientiam definitum sit $= s$, erit

$$g = \frac{kffM}{V\left(e^{ffs:V} - 1\right)},$$

quae unico experimento definita, pro omnibus casibus, quibus idem corpus cursu directo in aqua movetur, valebit.

SCHOLION 2

772. Initium fecimus huius capitis a motu seu cursu directo, atque insuper rectilineo, motusque huius diminutionem a resistentia ortam definivimus. Ex iis autem circumstantiis, quarum mentionem fecimus in solutione, ad cursum directum et rectilineum conservandum requisitis, simul colligere licet quibus rebus iste cursus turbetur. Primo scilicet motus rectilineus turbaretur,si directio media resistentiae non in planum diametrale incideret, vel si vis horizontalis ex ea orta directioni motus non esset directe contraria; ex supra enim allatis satis patet si resistentiae directio non congruat cum directione motus, tum motum non solum retardari sed etiam a semita rectilinea deflecti; quae quidem pertinent ad solum centri gravitatis motum progressivum, quem hic imprimis consideramus. Etiamsi autem motus non fieret in linea recta, tamen cursus manere potest directus, si scilicet perpetuo axis longitudinalis a prora ad puppem ductus maneat directioni motus parallelus; per cursum enim directum intelligimus eiusmodi navium motum, cuius directio directe a puppi ad proram tendit, et in quo eadem navis pars anterior resistentiae aquae opponitur. Quando igitur eiusmodi vires adessent, quae navem circa axem verticalem converterent, etiamsi illae motum progressivum non afficerent, tamen cursum directum turbarent, et cursum obliquum producerent. Quare cum in casu proposito, nullae istius modi vires adsint, etiam motus non solum in linea recta fieri inventus est, sed etiam cursus mansit directus. Primum igitur constituimus cursus directos simulque rectilineos examini subiicere, tam in aqua quiescenti quam fluvio, et id circo eiusmodi casus proponere oportet, quibus tam cursus directus quam motus rectilineus conservetur; quibus casibus evolutis facilius erit ad cursus obliquos motusque curvilineos examinandos progredi. Corpora autem ipsa aquae innatantia, prout sunt vel libera seu sibi relicta, vel non libera seu termino cuipiam veluti anchorae alligata primariam huius capitis divisionem suppeditabunt. Deinde vero subdivisiones sumentur a potentiis quibus corpora sollicitantur, de quibus si affuerint, primo enim in quaque tractione ut hic fecimus nullas potentias sollicitantes consideramus, dispiciendum est, non solum quantae sint et quamnam directionem teneant, sed etiam quomodo pro varia corporum celeritate et directione immutentur. Si enim naves a vento propelluntur, vis venti fit eo minor quo celerius naves progrediuntur, quando quidem in eam plagam in quam ventus tendit, moventur; in reliquis autem casibus obliquitatis venti ratio est habenda. Deinde etiam velorum directio, a qua directio vis venti pendet, imprimis est

contemplanda, quippe quae semel fixa eandem respectu navis tenent positionem, utcunque eius cursus immutetur. Remorum autem ratio aliter est comparata, cum eandem vim exerceant atque in eadem directione respectu navis, quantumvis tam celeritas quam motus directio mutetur. Ad hanc igitur potentiarum distinctionem probe attendi oportebit, quando in earum effectus inquiremus; id quod etiam nunc non nisi generatim facere licet, cum ipsi effectus tam a vento quam remis oriundi nondum sint penitus perspecti; sed in sequenti demum libro accurate evolventur. Quamobrem sufficiet hoc argumentum ita generaliter pertractasse, ut eius usus ad sequentem librum satis pateat.

PROPOSITIO 75

PROBLEMA

773. *Si corpus plano diametrali praeditum in fluvio ita sit collocatum, ut axis corporis a prora ad puppem ductus in fluvii directionem incidat* (Fig. 101**),** *definire motum quem fluvii vis corpori imprimet.*

SOLUTIO

Repraesentetur corpus per sectionem horizontalem *AEBF* per centrum gravitatis *G* factam, et ponatur corpus a fluvio iam propulsum esse in hunc situm, cum initio versaretur eius centrum gravitatis in *C,* ubi corpus nullam adhuc habuit celeritatem. Manifestum igitur est ex conditionibus praescriptis corpus cursum directum atque rectilineum

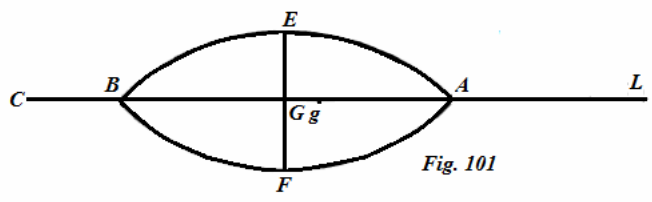


Fig. 101

esse accepturum cum nulla adsit vis, quae vel motum rectilineum deflectat, vel corpus circa axem verticalem convertat, ut inde cursus obliquus oriri posset. Cum itaque corporis in *C* celeritas nulla fuisset, ponatur eius celeritas acquisita cum in *G* pervenerit debita altitudini υ, spatium vero a centro gravitatis percursum *CG* sit $= s$. Porro fluvii celeritas debita sit altitudini *b.* Dum ergo corpus versatur in *G* ubi eius celeritas est $\surd u$ fluvius in corpus aget excessu suae celeritatis, qua est $\surd b$ supra celeritatem corporis $\surd u$, hoc est celeritate $\surd b - \surd u$ hacque celeritate eandem vim in corpus exeret, ac si corpus eadem celeritate in aqua quiescente secundum directionem *AB* moveretur. Ponatur autem figura plana $= ff$, quae hoc casu eandem resistentiam pateretur si eadem celeritate directe contra aquam impingeret. Ex his ergo sequitur fore vim corpus secundum directionem *GL* propellentem aequalem ponderi aquae, cuius volumen sit $\left(\surd b-\surd u\right)^2 ff$. Positis igitur massa seu pondere corporis $=M$ et volumine partis submersae $=V$ erit vis corpus in G propellens

$$=\frac{\left(\surd b-\surd u\right)^2 ffM}{V};$$

ab hacque vi ita motus corporis accelerabitur, ut dum per spatii elementum $Gg=ds$ progreditur, sit

$$dv=\frac{\left(\surd b-\surd u\right)^2 ffds}{V} \quad \text{seu} \quad \frac{dv}{\left(\surd b-\surd u\right)^2}=\frac{ffds}{V},$$

cuius integrale est

$$\frac{2\surd u}{\surd b-\surd u}-2l\frac{\surd b}{\surd b-\surd u}=\frac{ffs}{V},$$

integratione ita instituta ut fiat $u=0$ posito $s=0$ Tempus autem quo corpus spatium $CG=s$ absolvit est

$$=\int\frac{ds}{\surd u}=\frac{V}{ff}\int\frac{dv}{\left(\surd b-\surd u\right)^2\surd u}=\frac{2V\surd u}{ff\left(\surd b-\surd u\right)\surd b}.$$

Q.E.I.

COROLLARIUM 1

774. Ex data igitur celeritate $\surd u$ multo facilius spatium s assignatur, quo percurso corpus illam celeritatem acquisivit, quam vicissim ex dato spatio s celeritas $\surd u$. Hancque ob rem tempus non per spatium sed per ipsam celeritatem determinare licuit.

COROLLARIUM 2

775. Intelligitur autem ex formulis inventis corpus nunquam tantam celeritatem acquirere posse, quanta est celeritas fluvii; nam si ponatur $u=b$, fit spatium s itemque tempus quo fit $u=b$, infinitum.

COROLLARIUM 3

776. Sin autem semel fuerit $u=b$, id quod accidere potest, si corpori a vi externa tanta celeritas tribuatur, tum ob $du=0$, corpus progrediendo neque augmentum celeritatis capiet, neque decrementum, ideoque tum motu uniformi promovebitur.

COROLLARIUM 4

777. Si logarithmus qui in aequatione, qua relatio inter spatium s et celeritatem $\surd \upsilon$ continetur in seriem convertatur habebitur

$$\frac{ffs}{2V}=\frac{u}{2b}+\frac{2\upsilon\sqrt{u}}{3b\sqrt{b}}+\frac{3u^2}{4b^2}+\frac{4\upsilon^2\sqrt{u}}{5b^2\sqrt{b}}+\text{etc.}$$

ex qua expressione patet celeritatem per datum spatium s acquisitam eo fore maiorem quo maius fuerit circa ff.

COROLLARIUM 5

778. Quo igitur corpus quam celerrime a fluvio abripiatur, eam eius partem, quae impulsum aquae excipit, quae est corporis pars postica, ita oportet esse comparatam, ut ea si directe in aquam occurreret maximam pateretur resistentiam. Maxima igitur erit acceleratio si pars postica fuerit plana ad cursum fluvii normalis.

COROLLARIUM 6

779. Uti difficile est ad datum spatium percursum celeritatem corporis assignare, ita facilius post datum quodvis temporis intervallum celeritas corporis definiri potest. Posito enim tempore ab initio motus praeterlapso

$$t=\frac{2V\sqrt{u}}{ff\left(\sqrt{b}-\sqrt{u}\right)\sqrt{b}},$$

unde vicissim fit

$$\sqrt{u}=\frac{ffbt}{2V+fft\sqrt{b}}.$$

COROLLARIUM 7

780. Si quantitates $b,\ u,\ f\,f$, et V exprimuntur in partibus millesimis pedis Rhenani, tempus t, quo data celeritas $\sqrt{\upsilon}$ acquiritur, innotescet in minutia secundis per hanc aequationem

$$t=\frac{V\sqrt{u}}{125ff\left(\sqrt{b}-\sqrt{u}\right)\sqrt{b}}.$$

COROLLARIUM 8

781. Vicissim vero si tempus t detur in minutis secundis, atque quantitates b, ff et V partibus millesimis pedis Rhenani exprimantur, ista aequatio

$$\sqrt{u}=\frac{125ffbt}{V+125fft\sqrt{b}}$$

praebebit celeritati acquisitae altitudinem debitam v in particulis millesimis eiusdem pedis.

SCHOLION 1

782. Posuimus hic in initio C corpus nullam habuisse celeritatem, eique omnem motum quem acquirit, a motu aquae imprimi: sed pari modo problema tractari potest, si corpori ab initio datus motus tribuatur, cuius directio cadat in eandem rectam CL, in qua tum directio fluminis, tum positio axis corporis AB sunt sitae. Si autem quaestio hoc modo extendatur, casus nonnulli inter se prorsus diversi a se invicem probe sunt discernendi; quorum primus est, si corpus dum in C aquae immittitur, iam habeat motum in directione fluvii CL sed minorem quam ipse habet fluvius; qui casus ex ipsa solutione allata facile resolvetur; nam quoniam celeritas fluvii maior est, corpus accelerabitur, atque si celeritas initialis debita sit altitudini k, aequationis differentialis

$$\frac{dv}{\left(\sqrt{b}-\sqrt{u}\right)^2}=\frac{ffds}{V}$$

integratio fiet

$$\frac{ffs}{V}=\frac{2\sqrt{u}}{\sqrt{b}-\sqrt{u}}-\frac{2\sqrt{k}}{\sqrt{b}-\sqrt{k}}-2l\frac{\sqrt{b}-\sqrt{k}}{\sqrt{b}-\sqrt{u}}$$

atque tempus, quo spatium s absolvit, seu celeritatem $\sqrt{v}$ acquirit, reperietur

$$=\frac{2V\sqrt{u}}{ff\left(\sqrt{b}-\sqrt{u}\right)\sqrt{b}}-\frac{2V\sqrt{k}}{ff\left(\sqrt{b}-\sqrt{k}\right)\sqrt{b}};$$

quae omnia huc redeunt ut hic motus tanquam pars motus a quiete profecti considerari queat. Initium enim motus censendum est fuisse supra punctum C intervallo

$$s=\frac{2V\sqrt{k}}{ff(\sqrt{b}-\sqrt{k})}-\frac{2V}{ff}l\frac{\sqrt{b}}{\sqrt{b}-\sqrt{k}}.$$

Ex hoc scilicet puncto corpus ex quiete motum in singulis spatii CL punctis easdem habebit celeritates, quas in ipso casu oblato, quo corpus data celeritate $\sqrt{k}$ ex puncto ipso C egreditur. Deinde si celeritas corporis initialis in C directioni fluminis directe fuerit contraria, tum corpus primum contra cursum fluminis ascendet, donec eius motus penitus sit extinctus, indeque quasi ex quiete a fluvio deorsum abripietur. Evolutio autem huius casus sequitur ex praecedente ponendo $\sqrt{k}$ negativum, si quidem in C celeritatem habeat $\sqrt{k}$ unde fiet

$$\frac{ffs}{V}=\frac{2\sqrt{u}}{\sqrt{b}-\sqrt{u}}+\frac{2\sqrt{k}}{\sqrt{b}+\sqrt{k}}-2l\frac{\sqrt{b}+\sqrt{k}}{\sqrt{b}-\sqrt{u}}.$$

Ex hac aequatione obtinebitur intervallum, per quod corpus ultra *C* contra fluvii cursum progredietur si ponatur $\upsilon = 0$; tum vero fit

$$\frac{ffs}{V} = \frac{2\sqrt{k}}{\sqrt{b}+\sqrt{k}} - 2l\frac{\sqrt{b}+\sqrt{k}}{\sqrt{b}} = \frac{-2\sqrt{k}}{2\left(\sqrt{b}+\sqrt{k}\right)^2} - \frac{2k\sqrt{k}}{3\left(\sqrt{b}+\sqrt{k}\right)^3} - \frac{2k^3}{4\left(\sqrt{b}+\sqrt{k}\right)^4} \text{ etc.}$$

ex qua valor ipsius $-s$ desideratum spatium praebebit. Tertius denique casus ab his maxime discrepat, quo corpus initio in *C* motum habet velociorem secundum fluvii directionem sed maiorem. Tum enim motus corporis non solum in flumine retardabitur, sed etiam altera corporis superficies versus *A* in aqua constituta actionem aquae sentiet, posterior vero pars in *B,* quae hactenus sola vim ab aqua est passa, erit libera. Offendet igitur corpus hoc in casu resistentiam quae sit aequivalens resistentiae, quam superficies plana *ff* eadem celeritate in aquam impingens sentiret. Quare si celeritas initialis in *C* ponatur $= \sqrt{k}$ et in $G = \sqrt{u}$, erit resistentia

$$= \frac{Mff\left(\sqrt{b}-\sqrt{u}\right)^2}{V}$$

unde fit

$$dv = -\frac{ffds\left(\sqrt{b}-\sqrt{u}\right)^2}{V}$$

atque integrando

$$\frac{ffs}{V} = \frac{2\sqrt{u}}{\sqrt{u}-\sqrt{b}} - \frac{2\sqrt{k}}{\sqrt{k}-\sqrt{b}} + 2l\frac{\sqrt{k}-\sqrt{b}}{\sqrt{\upsilon}-\sqrt{b}}:$$

ex qua intelligitur corpus demum infinito spatio percurso ipsam fluvii celeritatem adipisci.

SCHOLION 2

783. Quanquam haec omnia ex calculo recte instituto consequantur, tamen si ad rem ipsam spectemus, correctione indigent. Missa enim ea circumstantia, cuius ante mentionem fecimus, qua aqua aliam exercet resistentiam praeter eam quae quadratis celeritatum est proportionalis, in hoc motu super fluviis ad aerem quoque respici oportet, qui parti corporum ex aqua eminenti nonnullam resistentiam opponit, quae quamvis fere octingenties minor sit quam resistentia aquae ceteris paribus, tamen eventus a sola aqua oriundos nonnihil turbat. Ita resistentia aeris in causa est, cur corpora a fluvio abrepta nunquam tam prope ad celeritatem fluvii accedant, quam calculus superior indicat, neque etiam ob hanc ipsam causam corporis motus si fuerit aequalis motui fluminis, conservabitur, sed retardabitur. Deinde si corpus in fluvio maiore descendat celeritate, quam ipse fluvius habet, tum ob aeris resistentiam non solum tandem ipsum fluvii

celeritatem acquiret, sed etiam minorem, quoad resistentia aeris aequalis fiat impulsui aquae. Ad hunc effectum quodammodo aestimandum ponamus partem corporis in aere versantem eandem ab aere pati resistentiam, quam perpeteretur superficies plana *hh* eadem celeritate contra aerem mota. Si ergo celeritas corporis, qua in aerem impingit debita sit altitudini *v,* erit resistentia aequalis ponderi molis aereae cuius volumen est $= hhv$, seu ponderi molis aquae, cuius volumen est $= \frac{hhv}{800}$. Huius vis igitur si ratio habeatur in solutione problematis, prodibit

$$du = \frac{\left(\surd b - \surd \upsilon\right)^2 ffds}{V} - \frac{hh\upsilon ds}{800V},$$

ex qua intelligitur ultimam celeritatem quam corpus acquiret non fore $\surd b$ sed minorem, fiet scilicet

$$f\surd b - f\surd u = \frac{h\surd u}{28}$$

circiter, seu

$$\surd u = \frac{28f\surd b}{28f + h}.$$

Quamobrem si portio superficiei corporis extra aquam eminentis sit *n* vicibus maior, quam ea, quae sub aqua versatur, erit proxime $hk = nff$, indeque celeritas ultima

$$\surd u = \frac{28\surd b}{28 + \surd n}.$$

Hincque etiam immutationes in reliquis casibus ab aere oriundae colligi poterunt. Sed in his omnibus aerem quietum posuimus, aliter enim res se habebit, si aer vento agitetur, qui motus pariter non difficulter in calculum inducetur.

PROPOSITIO 76

PROBLEMA

784. *Si corpus AB* (Fig. 101) *in aqua quiescente non solum moveatur cursu directo in directione BAL sed etiam secundum hanc directionem propellatur a vi quacunque constante, hoc est tali, quae corpus motum aeque acceleret ac quiescens; definire motum huius corporis.*

SOLUTIO

Potentiam corpus ad motum sollicitantem hic primum ponimus absolutam seu talem, quae dato tempusculo eandem producit accelerationem quacunque celeritate moveatur; eiusmodi scilicet potentiam exercent vires remorum, quibus siquidem remiges perpetuo eandem vim adhibeant, naves semper aequaliter propelli solent. Sit itaque potentia ista corpus in directione *AL* propellens $= p$, denotante *p* pondus illi vi aequale: atque resistentia, quam portio antica *EAF* in aqua patitur, tanta sit quantam pateretur superficies ff si eadem celeritate directe contra aquam impingeret. Ponamus nunc corporis centrum gravitatis iam spatium $CG = s$ confecisse atque in puncto *C* motum inchoasse, in *G* vero celeritatem habere debitam altitudini *v,* unde resistentia, quam in *G* sentiet erit $= ffu$; seu si corporis massa seu pondus dicatur *M* et volumen partis submersae $= V$, erit vis resistentiae = ponderi $\frac{Mffu}{V}$. Ex his igitur dum corpus elementum $Gg = ds$ percurrit fiet

$$du = \frac{pds}{M} - \frac{ffvds}{V} \text{ seu } du + \frac{ffvds}{V} = \frac{pds}{M}$$

quae ducta in $e^{ffs:V}$ fit integrabilis, atque aequatio integrata erit

$$e^{ffs:V}u = \frac{\int e^{ffs:V}pds}{M} = \frac{pV}{Mff}\left(e^{ffs:V} - 1\right),$$

integratione ita instituta ut evanescat *v* posito $s = 0$. Quocirca habebitur ista aequatio

$$\upsilon = \frac{pV}{Mff}\left(1 - e^{-ffs:V}\right)$$

ex qua celeritas corporis in singulis spatii describendi *CGL* punctis innotescit. Tempus vero quo spatium $CG = s$ a centro gravitatis *G* percurritur innotescet ex integrali ipsius $\frac{ds}{\sqrt{u}}$ quod reperitur

$$= \frac{2\sqrt{MV}}{f\sqrt{p}} l\left(e^{ffs:V} + \sqrt{\left(e^{ffs:2V} - 1\right)}\right).$$

Q.E.I.

COROLLARIUM 1

785. Corpus ergo continuo accelerabitur crescente enim *s* crescit *v;* atque spatio iam infinito emenso acquiret celeritatem, cuius altitudo debita erit $= \frac{pV}{Mff}$; seu celeritas maxima, quam acquirere potest erit $= \frac{\sqrt{pV}}{f\sqrt{M}}$.

COROLLARIUM 2

786. Intelligitur autem ex formula inventa
$$\upsilon = \frac{pV}{Mff}\left(1 - e^{-ffs:V}\right)$$
corpus mox tantam adipisci celeritatem, quae insensibiliter differat a celeritate ultima. Nam si fuerit spatium s modice magnum, quantitas $e^{-ffs:V}$ iam abibit in tam exiguam fractionem, quae prae 1 evanescat dummodo enim fit

$$\frac{ffs}{V} = 10 \text{ seu } s = \frac{10V}{ff},$$
quantitas $e^{-ffs:V}$ iam minor fit quam $\frac{1}{10000}$.

COROLLARIUM 3

787. Neglecto ergo ipso motus initio corpus satis tuto concipi potest quasi motu uniformi progrederetur: atque celeritas, qua uniformiter pro movebitur erit $= \frac{\surd pV}{f\surd M}$; quae expressio si f et V exprimantur in particulis millesimis pedis Rhenani; tum $\frac{250\surd pV}{f\surd M}$ dabit spatium in eadem mensura, quod corpus uno minuto secundo absolvet.

COROLLARIUM 4

788. Celeritas ergo, qua navis remis propulsa in aqua quiescente promovebitur, est in subduplicata ratione virium remorum: unde si remigum numerus quadruplicetur, navis duplo celerius progredietur.

COROLLARIUM 5

789. Hinc si duae naves inter se prorsus similes remis propellantur, atque maioris longitudo AB sit $= A$, minoris $= a$, maior vero propellatur vi $= P$, minor vero vi p, erunt celeritates, quibus incedent inter se ut $\frac{\surd P}{A}$ ad $\frac{\surd p}{a}$. Quo igitur ambae naves aequali celeritate progrediantur, necesse est ut vires remorum teneant rationem duplicatam longitudinum.

COROLLARIUM 6

790. Deinde etiam intelligitur, quo minor sit resistentia navis, eo maiorem fore celeritatem quam eadem vis remorum generat. Cum enim sit resistentia absoluta ut ff, erit celeritas producta in reciproca subduplicata ratione resistentiae, id est si resistentia quadruplo fit minor, eadem vis remorum duplo maiorem celeritatem navi imprimet.

COROLLARIUM 7

791. Quoniam denique V ad M rationem tenet constantem; namque V ductum in gravitatem specificam aquae, aequatur ipsi M; manifestum est celeritates navium remis propulsarum esse in ratione composita ex directa subduplicata virium remorum et reciproca subduplicata resistentiarum absolutarum.

SCHOLION

792. Quanquam hae determinationes tantum ad aquam quiescentem sunt accommodatae, tamen facile ad motum navium in fluviis propulsarum a remis transferri possunt; siquidem motus fiat secundum ipsius fluvii directionem. Nam si celeritas fluvii sit debita altitudini b seu ipsa celeritas $=\sqrt{b}$, tum si navis in fluvio descendat, eius celeritas a vi remorum acquisita augenda est celeritate fluvii, ita ut tale corpus, quale contemplati sumus in fluvio descendendo acquirat velocitatem

$$=\frac{\sqrt{pV}}{f\sqrt{M}}+\sqrt{b}.$$

At si idem corpus contra fluvii cursum sursum propellatur, tum celeritatem acquiret

$$=\frac{\sqrt{pV}}{f\sqrt{M}}-\sqrt{b},$$

ex qua expressione intelligitur, nisi $\frac{\sqrt{pV}}{f\sqrt{M}}$ maior sit quam $\sqrt{b}$, corpus cursum fluminis superare non posse, neque ascendere. Quoniam autem haec ad vim remorum respiciunt, notandum est vires remorum utrinque debere esse aequales et similiter applicatas, quovis ex iis coniunctim resultantis directio per medium navis transeat, seu in rectam BA incidat; nisi enim hoc observetur, corpus seu navis cursum directum tenere non poterit,animum namque hic abstrahimus ab actione gubernaculi, qua utique huic incommodo subveniri posset.

PROPOSITIO 77

PROBLEMA

793. *Si superficies plana in situ verticali posita e motu sibi parallelo moveatur uniformiter in directum secundum directionem CGL* (Fig. 102), *atque in eam impingat fluidum in directione VG data cum celeritate, determinare vim, quam fluidum allapsu suo in superficiem exercebit.*

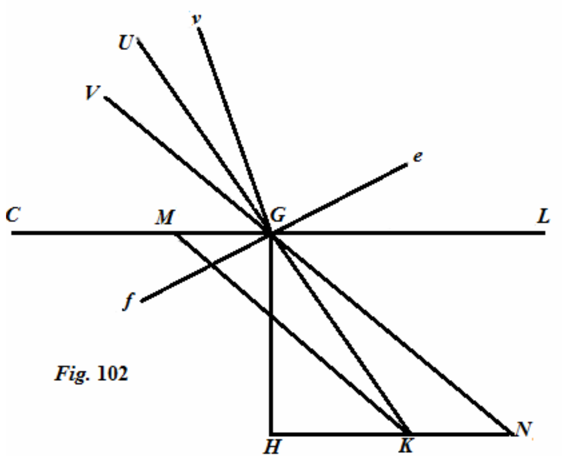


Fig. 102

SOLUTIO

Sit celeritas qua superficies plana ef progreditur debita altitudini v, seu $=\sqrt{u}$, atque celeritas, qua fluidum movetur $=\sqrt{c}$, anguli autem CGV, quem directio motus fluidi VG cum directione motus superficiei CGL constituit sinus ponatur $=\mu$, et cosinus $=u$. Anguli autem VGf, quem directio motus fluidi VG constituit cum planitie superficiei sinus sit $=m$ et cosinus n, posito sinu toto $=1$; denique sit $gg=$ ipsi superficiei, cuius centrum gravitatis sit in puncto G. Iam si superficies quiesceret, ex ante demonstratis foret vis, quam fluidum in superficiem exereret $=m^2g^2c$, seu aequaretur ponderi molis ex eadem materia fluida constantis, cuius volumen est $=m^2g^2c$. At cum superficies non quiescat sed celeritate $=\sqrt{v}$ progrediatur in directione GL concipiatur totum systema ex fluido et superficie constans retro in directione GC celeritate $=\sqrt{u}$ promoveri, quo fiet ut superficies ef in quietem redigatur; vis autem fluidi in superficiem exerta utroque casu erit eadem. Per compositionem motus autem innotescet, tam celeritas fluidi resultans quam directio. Cum enim nunc fluidum duplici feratur motu, altero secundum directionem GN celeritate $=\sqrt{c}$ altero vero in directione GM celeritate $=\sqrt{v}$. Si capiatur $GN=\sqrt{c}$ et $GM=\sqrt{v}$, atque formetur parallelogrammum $GMKN$, diagonalis GK tam celeritatem fluidi resultantem, quam eius directionem suggeret, ita ut fluidum censendum sit celeritate GK in directione UG in superficiem ef quiescentem impingere. Demisso autem ex G in NK productam perpendiculo GH, erit ob anguli GNH sinum $=\mu$, et cosinum $=u$, perpendiculum

$$GH=\mu\sqrt{c},\quad \text{et}\quad NH=u\sqrt{c},$$

unde fiet $KH = u\surd c - \surd u$ atque $GK = \surd\left(c - 2u\sqrt{cv} + u\right)$. Ex his reperietur anguli *NGK* seu *UGV* sinus $= \frac{\mu\surd u}{\surd\left(c-2u\sqrt{cv}+u\right)}$ et cosinus $= \frac{\surd c - u\surd u}{\surd\left(c-2u\sqrt{cv}+u\right)}$, atque hinc prodit anguli *UGf* sinus

$$= \frac{m\surd c - (mu - n\mu)\surd u}{\surd\left(c-2u\sqrt{cv}+u\right)}$$

qui ergo est sinus anguli incidentiae sub quo fluidum impinget in superficiem, quare cum fluidi celeritas sit $= \surd\left(c - 2v\sqrt{cu} + u\right)$, prodibit vis, quam fluidum in directione vera *VG* celeritate $V c$ motum, in superficiem *ef* motam celeritate *Vv* in directione *GL* exercebit $= \left(m\surd c - (mv - n\mu)\surd u\right)^2 g^2$, huiusque vis directio transibit per superficiei centrum gravitatis *G* atque ad ipsam superficiem erit normalis. Q. E. I.

COROLLARIUM I

794. Si anguli *CGf* sinus ponatur $= q$, cum sit $q = mu - n\mu$, erit vis fluidi, quam in superficiem exerit $= \left(m\surd c - q\surd u\right)^2 gg$, seu tantum fluidi volumen pondere adaequabit.

COROLLARIUM 2

795. Quoniam anguli *UG f* sinus inventus est

$$= \frac{m\surd c - q\surd u}{\surd\left(c-2u\sqrt{cv}+u\right)},$$

manifestum est esse debere $m\surd c > q\surd u$, siquidem superficies plana versus plagam *GK* debeat urgeri, Nam si esset $m\surd c < q\surd u$ tum superficies adeo urgeretur versus plagam *UV*.

COROLLARIUM 3

796. Si superficies plana *e f* normaliter ad cursum fluidi *VG* constituatur, ita ut sit $m = 1$ et $n = 0$, erit vis quam superficies patietur

$$= \left(\surd c - u\surd u\right)^2 gg :$$

quae vis ideo eo minor erit, quo maior fuerit anguli *VGC* cosinus *v*.

COROLLARIUM 4

797. Manente autem positione superficiei ef eadem respectu directionis motus ipsius *GL*, vis fluidi eo maior erit quo maior fuerit sinus *m.* Quare maximam patietur vim superficies, si angulus VGf fuerit rectus.

COROLLARIUM 5

798. Sin autem superficies ef iuxta motus sui directionem *GL* collocata fuerit, erit angulus CGf evanescens et consequenter $q = 0$; hoc igitur casu superficies eadem patietur vim ac si quiesceret.

COROLLARIUM 6

799. Si fluidum veniret ex regione uG, ita ut directio *uG* tantum inclinet ad *Ge,* quantum directio *VG* inclinat ad *Gf,* manebit anguli *uGf* idem sinus = *m;* ideoque ob angulum *CGf* in variatum, cuius sinus est *q,* erit vis quam superficies sufferet eadem, quae in altero casu scilicet $= \left(m\sqrt{c} - q\sqrt{u}\right)^2 gg$.

COROLLARIUM 7

800. Ponamus angulum *VGC* manere invariatum; definiri poterit angulus *VGf,* seu positio superficiei *ef,* ut maximam vim a fluido sufferat. Reperietur autem anguli *VGf* tangens

$$= \frac{m}{n} = \frac{\sqrt{c} - v\sqrt{u}}{\mu\sqrt{u}},$$

atque vis erit $(c - 2v\sqrt{cu} + u)g^2$.

SCHOLION

801. Haec propositio in sequentibus maxime nobis erit necessaria, ubi tum vim venti in vela mota tum vim fluvii in navem promotam sumus investigaturi. Facile autem patet nisi venti celeritas sit maxima seu prope infinita, ipsum velorum motum negligi omnino non posse; si enim vela in eandem plagam progrediantur in quam ventus tendit, perspicuum est vim venti in vela eo fore minorem, quo celerius vela promoventur, atque adeo evanescere, si vela eandem, quam ipse ventus, habeant celeritatem. Quamobrem hac propositione praemissa licebit nobis sequentia problemata aggredi in quibus inquiremus, quomodo naves a vento propellantur, tam cursu directo, quam utcunque obliquo.

PROPOSITIO 78

PROBLEMA

802. *Si corpus seu navigium plano diametrali AB praeditum a vento ita sollicitetur, ut cursu directo secundum directionem GL in aqua quiescente promoveatur* (Fig. 103), *determinare motum huius navigii, et celeritatem maximam, quam recipere poterit.*

SOLUTIO

Quoniam navis cursu directo in directione *BAL* moveri ponitur, in quam simul directio resistentiae incidit, oportet ut media directio venti in eandem directionem incidat. Quare cum vis venti semper normalis sit in planum velorum, atque eius media directio per centrum gravitatis velorum transeat, requiritur ut planum velorum normale sit ad planum diametrale *AB,* atque ut velorum centrum gravitatis in idem hoc planum incidat. Repraesentet itaque *EF* velorum planitiem, cuius area sit $= gg$, sitque *G* centrum gravitatis velorum in axe *AB* positum; hocque modo fiet ut media directio venti in rectam *GL* incidat, eaque tam cursus directus, quam motus progressivus in recta *GL* conservetur. Impingat nunc ventus in vela in directione quacunque obliqua *VG,* sitque celeritas venti debita altitudini *c,* atque anguli *VGC,* quem directio venti cum directione motus constituit, sinus sit $= \mu$ et cosinus $= \nu$; eritque anguli *VGF*, quem directio venti *VG* cum planitie velorum constituit sinus $= \nu$, qui ante positus erat *m,* et cosinus, qui ante erat *n,* hoc casu erit $= -\mu$, quoniam angulus *VGF* est obtusus. Ponamus porro navem in puncto *C* motum incepisse, atque iam absolvisse spatium $CG = x$, hicque habere celeritatem debitam altitudini υ. His positis ex praecedente propositione erit vis venti, qua navem urgebit in directione *GL,*

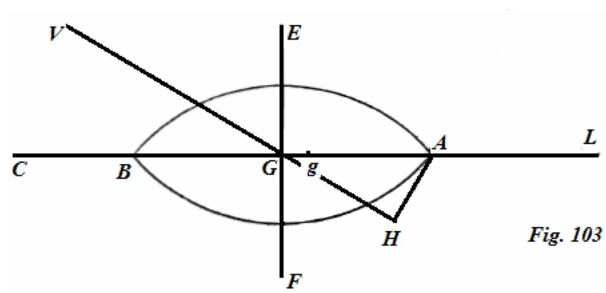


Fig. 103

$$= \left(m\sqrt{c} - (mu - n\mu)\sqrt{u}\right)^2 g^2 = \left(\nu\sqrt{c} - \sqrt{u}\right)^2 g^2$$

tantum scilicet aeris volumen pondus habebit vi isti propellenti aequalem; sive cum aeris gravitas specifica se habeat proxime ad aquam ut 1 ad 800 seu 1 ad 784, quam posteriorem rationem usurpabimus quia 784 est numerus quadratus, nobisque radicis extractione est opus, pondus aquae vi illi aequale volumen habebit

$$= \frac{\left(\nu\sqrt{c} - \sqrt{u}\right)^2 gg}{784}.$$

Exprimat nunc superficies plana *ff* resistentiam, quam navis in cursu directo ab aqua patitur, seu superficies plana *ff* eandem patiatur quam navis, si directe contra aquam eadem celeritate impingeret. Hinc igitur erit vis resistentiae aequalis ponderi molis aquae, cuius volumen est $= ffv$, quoniam aquam quiescentem assumimus. Ex his ergo si navis massa seu pondus dicatur $= M$ et volumen partis submersae $= V$ erit vis navem propellens in directione GL

$$= \frac{M\left(v\sqrt{c}-\sqrt{u}\right)^2 gg}{784V}$$

vis autem repellens $= \frac{Mffv}{V}$; ex quibus conficitur acceleratio, dum navis per elementum $Gg = dx$ progreditur,

$$du = \frac{M\left(v\sqrt{c}-\sqrt{u}\right)^2 g^2 dx}{784V} - \frac{ffudx}{V}$$

Tamdiu igitur navis accelerabitur, quam diu fuerit $\frac{\left(v\sqrt{c}-\sqrt{u}\right)^2 g^2}{784}$ maius quam ffv; at quamprimum tantam acquisiverit celeritatem $\sqrt{v}$ ut sit

$$\frac{\left(v\sqrt{c}-\sqrt{u}\right)^2 g^2}{784} = ffu$$

quod quidem demum post spatium infinitum confectum eveniet, sed mox tam prope istam celeritatem assequetur ut discrimen sit insensibile. Hancobrem excepto motus initio navis motu uniformi in directione GL cursu que directo promovebitur celeritate altitudini u debita, cuius valor ex superiore aequatione reperitur

$$\sqrt{u} = \frac{vg\sqrt{c}}{28f+g}$$

ita ut ipsa navis celeritas se habitura sit ad celeritatem venti ut vg ad $28f+g$.
Q.E.I.

COROLLARIUM 1

803. Cum celeritas, quam navis hoc vento propulsa acquirit sit $= \frac{vg\sqrt{c}}{28f+g}$, intelligitur celeritatem navis ceteris paribus rationem tenere simplicem celeritatum venti, ita ut in qua ratione venti celeritas augeatur, in eadem navis celeritas crescat.

COROLLARIUM 2

804. Quod autem ad superficiem velorum *gg* attinet, perspicitur celeritatem corporis quidem crescere, si vela multiplicentur, sed nullam tenere rationem fixam. Si enim vela in infinitum augeantur, celeritas navis tamen ultra datum terminum non augebitur, acquiret scilicet facto $gg = \infty$ celeritatem $= v\sqrt{c}$.

COROLLARIUM 3

805. Quando ergo vela iam eousque fuerint aucta ut 28*f* respectu ipsius *g* fere evanescat, tum quantum vis amplius vela multiplicentur, navi tamen maior celeritas non imprimetur. Ex quo colligi licet inutile esse vela ultra datum terminum multiplicare; qui terminus ex resistentia in praxi facile determinabitur.

COROLLARIUM 4

806. Ex his etiam concluditur lucrum non adeo considerabile obtineri, etiamsi resistentia vehementer diminuatur. Si enim vela tanta accipiantur ut *g* multum excedat 28*f,* tum parum intererit, si resistentia etiam penitus tolleretur Attamen quo minor est resistentia, eo paucioribus velis erit opus.

COROLLARIUM 5

807. Ceterum sponte patet manente navis dispositione eadem, celeritatem venti eo fore maiorem quo minor fuerit angulus *VCG;* unde ventus directe secundum *CG* seu a puppi urgens navi maximam imprimet velocitatem.

SCHOLION 1

808. Ex his satis superque intelligitur quantum intersit discrimen inter naves quae vento, easque quae remis propelluntur. In iis enim quae remis promoventur plurimum interest ut resistentia quantum fieri potest diminuatur, cum celeritates impressae teneant rationem reciprocam subduplicatam resistentiarum: contra vero in iis navibus quae a vi venti propelluntur diminutio resistentiae non tantum lucrum affert; ex quo in constructione navium maximum oritur discrimen, prouti vel velis vel remis destinantur. Haec autem ipsa differentia in praxi, si naves usu receptas intueamur, apprime observata deprehendetur. Triremes enim seu eiusmodi naves, quae remis moventur, partem anticam habent acntissimam, unde resistentia oritur perquam exigua. Alteram vero navium speciem vento destinatam videmus parte antica satis obtusa praeditam, quae parum sit idonea ad resistentiam diminuendam. Ex his autem sponte sequitur, quommodo eiusmodi naves, quae tam remis quam vento coniunctim promoveri solent, comparatas esse oporteat, ut sint maxime aptae; scilicet perspicuum est medium quoddam esse eligendum inter utramque speciem tractatam. At hoc discrimen tantum etiamnum est petitum ex cursu directo, maius deprehendatur cum cursus obliquos examini subiecerimus, ad quos naves velis instructae praecipue debent adaptari, cum contra in navibus remis propellendis ad cursum obliquum omnino non opus sit respicere. Ceterum

ex modo solutionis facile erit calculum absolvere, si praeter ventum etiam remi urgeant, atque navis coninnctim a remis et velis propellatur. Simili modo quilibet non difficulter calculum instituet, si motus non fiat in aqua quiescente, sed in fluvio, dummodo directiones fluvii et motus corporis congruant, atque cursus sit directus; quamobrem huiusmodi investigationibus diutius non adhaerebimus.

SCHOLION 2

809. Quod hic superficiem velorum perfecte planam posuimus, id solutionem datam minime turbat etiamsi vela a vento in figuram concavam extendantur: in sequenti enim libro quo velorum doctrina imprimis excutietur, demonstrabitur semper velum planum assignari posse eandem vim excipiens, ita ut quae hic de velis planis afferuntur, aeque valeant pro velis, quemadmodum in praxi usurpantur. Deinde etiam solutio ab experientia in hoc dissentire videtur quod ventum maxime lucrosum statuat eum, qui directe a puppi venit, cum tamen observatione constet naves felicius a vento non nimis obliquo propelli. Ratio autem huius discrepantiae sita est in consueta collocatione velorum qua vela tum in puppi tum in prora tum etiam in medio navis expandi solent: unde facile colligitur, si ventus recta a puppi ad proram tendat, tum posteriora vela anterioribus ventum adimere, atque impedire, quominus ventus in vela anteriora impingere queat. Cum autem hoc non eveniat, si directio venti est obliqua, mirum non est ventum obliquum maiorem celeritatem producere solere quam directum. Sed haec tantum sunt intelligenda, si navis pluribus malis sit instructa, hoc enim casu tantum illud uncommodum locum habet, at si unicus adsit malus, ventusque adeo libere in omnia vela incurrere possit, tum memoratus dissensus theoriae cum praxi non observatur, sed potius navis eo celerius progredi deprehenditur, quo minus directio venti a directione cursus navis aberrat. Eiusmodi autem dissensus apparentes saepius occurrunt praecipue in hac doctrina de motu navium, sed semper si omnes circumstantiae probe perpendentur facile diluentur.

SCHOLION 3

810. Hi igitur fere sunt casus, quibus naves cursu directo motuque rectilineo in aqua tam quiescente quam fluente incedere possunt, ad quem cursum requiritur, ut tum ipsius motus directio, tum media directio resistentiae tum directio vis sollicitantis tum etiam fluvii directio inter se congruant, atque in axem seu rectam a puppi ad proram ductam incidant. Quarum conditionum unica si defecerit, vel cursus directus vel motus rectilineus turbabitur, evenietque ut vel motus directio ab axe navis seu diametro longitudinali a puppi ad proram porrecta declinet, vel etiam centrum gravitatis cogatur in linea curva incedere, quae omnia probe inter se discernere, et quodque ex suis causis derivare omni attentione erit opus. Interim ex traditis satis liquet si vis sollicitans directionem habeat secundum navis longitudinem, tum etiamsi cursus vehementer esset obliquus, tamen brevi in cursum directum mutatum iri. Quando enim vis sollicitans perpetuo in eandem

plagam tendit, tum motus si quis affuerit obliquus mox tam destruetur, ut eius directio in directionem vis sollicitantis incidat. Atque hinc fit, ut naves quae remis propulsantur, perpetuo secundum suam longitudinem progrediantur, cum directio vis remorum semper eo tendat, quamvis subinde ope gubernaculi directio cursus immutetur. Tum enim quasi ad momentum tantum durat motus curvilineus, statimque in directum transmutatur, cuius rei ratio potissimum in resistentia laterali est sita, quae in his navibus vehementer est magna, motumque obliquum statim destruit. Atque ob hanc rationem cursus directus proprius est illi navium speciei, quae remis propelluntur, nam quoniam vis remorum in quamvis plagam aeque exerceri potest, atque motus secundum longitudinem ob minimam resistentiam est facillimus, absurdum foret huiusmodi naves ad motum obliquum instruere. Longe aliter autem comparata est ratio navium, quae vento ad motum cientur, cum directionem venti non ad arbitrium formare liceat, sed eo vento, quem fortuna suggerit, ad iter institutum conficiendum uti oporteat. Quoties igitur evenit ut cursus intentus a directione venti tantopere discrepet, ut cursus directus omnino institui nequeat, tum ad cursum obliquum est confugiendum, qui eo felicius usurpabitur, quo propius versus regionem unde ventus flat navigari poterit. In his igitur navibus, quae vento propelluntur, praecipue cursus obliquus attendi debet, indeque potissimum regulae pro constructione et velificatione navium sunt petendae. Quamobrem istum cursum obliquum, quo navis non secundum longitudinem suam progreditur, imprimis in iis navibus examini subiiciemus, quae non remis sed solo vento ad motum cientur.

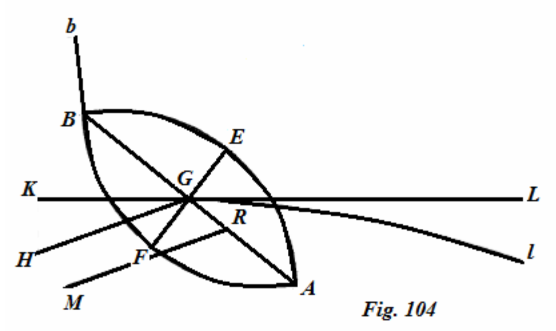


Fig. 104

PROPOSITIO 79

PROBLEMA

811. *Si corpus seu navis AEBF* (Fig. 104) *in aqua quiescente acceperit cursum obliquum secundum directionem GL data cum celeritate, determinare tam ipsam viam, quam eius centrum gravitatis G describet, quam ubique cursus obliquitatem, seu positionem axis longitudinalis AB.*

SOLUTIO

Sit *AGL* angulus declinationis cursus, quem directio motus *GL* cum positione axis longitudinalis *BA* seu spinae constituit, huiusque anguli sinus ponatur s, cosinus vero $= r$; celeritas autem corpori impressa secundum directionem *GL* debita sit altitudini v. Deinde sit *R M* media directio resistentiae, quam corpus hoc motu obliquo ab aqua patietur, quae cum directione spinae seu axis navis *AB* angulum *M RB* constituat, cuius sinus sit $= \sigma$, cosinus vero $= \rho$; atque resistentia quam navis hac obliquitate in aqua mota patitur, tanta sit, quantam pateretur superficies plana *uu* eadem celeritate directe contra aquam in directione *MR* mota; unde vis resistentiae, qua corpus secundum directionem *RM* urgebitur, aequalis erit ponderi molis aqueae, cuius volumen est $=uuv$. Pendebunt autem quantitates σ, ρ et u ab angulo obliquitatis *AGL* eiusve sinu s atque structura totius corporis. Quare si massa seu pondus totius corporis ponatur $=M$ atque volumen partis aquae submersae $=V$ erit vis resistentiae in directione *RM* urgentis $=\frac{Muuv}{V}$; quae vis duplicem exeret effectum quorum alter consistit in motu progressivo centri gravitatis *G* alterando, alter vero in corpore circa axem verticalem per centrum gravitatis *G* ductum convertendo. Ad priorem autem effectum investigandum oportet vim *RM* tanquam in ipso centro gravitatis *G* in directione sibi parallela *GH* applicatam concipere. Anguli igitur *H G K,* quem directio resistentiae *GH* cum directione motus *GK* constituit, sinus erit $=r\sigma-s\rho$, atque cosinus $=s\sigma+r\rho$. Hinc vis resistentiae quae est $=\frac{Muuv}{V}$; resolvetur in binas laterales *GK, KH,* quarum alterius *GK* directio in ipsam motus directionem *GL* incidit, altera *KH* vero ad hanc erit normalis, ex quibus vis tangentialis *GK* erit

$$=(s\sigma+r\rho)\frac{Muuv}{V},$$

et vis normalis *KH*

$$=(r\sigma-s\rho)\frac{Muuv}{V}.$$

Vis igitur tangentialis retardabit corporis motum, efficietque, ut dum corpus elementum spatii $Gg=dx$ percurrit, futurum sit

$$=-\frac{(s\sigma+r\rho)Muuvdx}{V}.$$

Vis normalis autem corpus a semita rectilinea deflectet cogetque arcum circularem concavum versus regione *M* describere, cuius radius erit $=\frac{2V}{(r\sigma-s\rho)u^2}$.

Quod denique ad alterum effectum attinet, quo corpus a vi resistentiae convertetur circa axem verticalem per centrum gravitatis *G* transeuntem, patet primo conversionem fieri in regionem *AF,* ita ut per eam declinatio *AGL* magis augeator, si quidem centrum resistentiae *R* intra centrum gravitatis G et proram *A* cadat. Dicto autem intervallo $GR=z$ fiet momentum vis resistentiae ad hanc conversionem producendam $=\frac{\sigma Muuvz}{V}$, quod momentum divisum per ipsius corporis momentum inertiae respectu eiusdem axis verticalis per centrum gravitatis *G* transeuntis, dabit vim gyratoriam cui motus angularis momentaneus est proportionalis. Sin autem corpus motum angularem iam habuerit, tum ex vi gyratoria eius incrementum cognoscetur. Q. E. I.

COROLLARIUM 1

812. Retardatio ergo motus eo erit maior, quo minor fuerit angulus *HGK,* hoc est quo minor fuerit differentia inter angulos *GRM* et *AGL.* Ex quo sequitur quo magis angulus *GRM* excedat angulum *AGL* eo fore diminutionem motus minorem.

COROLLARIUM 2

813. Quia porro plerumque resistentia eo fit maior, quo magis cursus obliquus a directo differt, seu quo maior fuerit angulus *AGL,* dummodo rectum non excedat, valor ipsius *uu* eo maior erit, quo maior fuerit angulus *AGL,* indeque eo maior motus retardatio orietur.

COROLLARIUM 3

814. Maxima igitur accedet motus retardatio, si angulus *AGL* fiat rectus, tum enim non solum valor ipsius *uu* omnium fiet maximus, siquidem resistentia lateralis multum superet resistentiam prorae. Sed etiam tum directio resistentiae *RM* in motus directionem incidet, quo fit ut $s\sigma + r\rho$ maximum valorem obtineat, fiatque $=1$.

COROLLARIUM 4

815. Cum radius curvedinis viae, in qua centrum gravitatis *G* incedet, sit $=\frac{2V}{(r\sigma - s\rho)u^2}$, corpus a directione sua impressa *GL* deflectet, atque dum elementum spatii $Gg = dx$ percurrit, deflectet angulo

$$=\frac{(r\sigma - s\rho)u^2dx}{2V}.$$

COROLLARIUM 5

816. Deflexio ergo a cursu rectilineo non pendet a celeritate corporis, sed tantum a cursus obliquitate. A cursus enim obliquitate pendet tum valor ipsius *uu,* tum etiam $r\sigma - s\rho$, seu sinus differentiae angulorum *MRG* et *AGL.*

COROLLARIUM 6

817. Si igitur angulus *MRG* aequalis fiat angulo *AGL,* tum deflectio a cursu rectilineo omnino erit nulla, corpusque in linea recta progredi perget. At si angulus *MRG* maior fuerit angulo *AGL;* tum deflectet versus *A,* atque viam curvilineam *Gl* describet, inter *GA*

et *GL* sitam. Sin autem fuerit angulus *AGL* maior angulo *MRG,* tum in partem oppositam deflectet.

COROLLARIUM 7

818. Si centrum resistentiae *R* incidat in ipsum gravitatis centrum *G,* tum vis gyratoria evanescit, hoc igitur casu positio axis *AB* perpetuo manebit sibi parallela. Unde si angulus *MRG* seu *HGB* aequalis sit angulo *KGB,* tum corpus non solum perget moveri in recta *GL,* sed etiam eadem cursus obliquitas conservabitur.

COROLLARIUM 8

819. Incidente autem *R* in *G,* seu quod perinde est, dummodo puncta *G* et *R* in eandem rectam verticalem cadant, si angulus *MRG* maior fuerit angulo *AGL,* tum cursus directio *Ggl* accedet ad positionem axis *AB* tandemque abibit motus in cursum directum, eousque conservandum, quoad motus per resistentiam omnis extinguatur. Sin autem angulus *M RG* minor sit angulo *AGL* tum cursus continuo magis deflectet a directo, ita ut tandem eius directio fiat normalis ad *AB.*

COROLLARIUM 9

820. Si autem centrum resistentiae *R* non in *G* sed versus proram *A* cadat, tum corpus inter movendum convertetur circa axem verticalem per centrum gravitatis ductum, atque axis *BGA* gyrabitur secundum plagam *AF;* quo fiet ut obliquitas cursus seu angulus *AGL* perpetuo crescat.

COROLLARIUM 10

821. Sin autem centrum resistentiae *R* ultra *G* versus puppim *B* cadat, tum conversio fiet in regionem oppositam; unde cursus obliquitas mox tolletur, atque axis navis *AB* convertetur in ipsam motus directionem *GL;* quod si evenerit cursus directus conservabitur.

SCHOLION 1

822. Ex his etiam intelligitur, si navis *AB,* quae ante cursu directo promovebatur, a vi externa ita convertatur, ut eius spina *AB* angulum obliquum *AGL* cum motus directione constituat, cuiusmodi mutationes inde sint oriturae. Praecipue enim respiciendum erit ad centrum resistentiae *R,* angulo obliquitatis cursus praesenti *AGL* respondens: quod si ultra centrum gravitatis *G* versus proram fuerit collocatum, tum navis sese in pristinum situm restituet, cursumque directum recuperabit. Contra vero si centrum resistentiae *R* versus proram cadat, tum navis non solum in cursum directum se non recipiet, sed etiam obliquitas cursus augebitur donec axis latitudinalis *EF* in cursus directionem incidat, quod si evenerit hoc situ in directum progredietur. Quocirca si centrum resistentiae *R* in

puppim cadat, cursus directus aliquam censendus est habere firmitatem, cum navis si ex eo depellatur, eo sponte se restituat; e contrario autem si centrum resistentiae *R* versus proram cadat, tum cursus directus quasi erit infirmus, eo quod si navis quam minime de cursu directo declinetur, obliquitas continuo maior evadit. Quamobrem cursus directus difficulter conservabitur, nisi habeat firmitatem, hoc est nisi centrum resistentiae *R* versus puppim cadat; minima enim vis sufficeret ad cursum directum penitus destruendum. Interim tamen etiamsi centrum resistentiae *R* in prora situm sit, tamen cursus directus ope gubernaculi *Bb* conservari poterit; eo maiore autem opus erit vi ad restitutionem in cursum directum, quo propius punctum *R* ad proram *A* ceciderit, simulque quo maior angulus *GRM* extiterit. Cursus autem obliquus sub angulo *AGL* in directum conservari omnino nequit nisi centrum resistentiae *R* in *G* cadat, atque angulus *HGK* evanescat. Nam si *R* cadat in proram nequidem ope gubernaculi motus rectilineus sub eadem obliquitate conservari potest: quam vis enim gubernaculum in situm *Bb* deflexum motum conversionis navis circa axem verticalem impedire queat, tamen per ipsam gubernaculi vim motus magis a via rectilinea *GL* declinabitur. Si quidem angulus *MRG* maior fuerit angulo *AGL,* prout id quidem in navibus accidere debet. Quando autem centrum resistentiae *R* in puppim cadit, tum fieri potest ut ope gubernaculi eadem obliquitas cursus, motusque rectilineus conservetur; id quod eveniet, si vis gubernaculi non solum motum gyratorium impediat, sed etiam simul vim normalem destruat. Ex his omnibus perspicuum est ad motum rectilineum sub directione obliqua conservandum opus esse viribus externis insigni cautione applicandis, qua quidem de re mox videbimus. Quo autem quovis casu aestimari liceat, quomodo valores σ, ρ et *uu* una cum intervallo $GR = z$ a data cursus obliquitate pendeant, exempla quaedam afferamus, in quibus isti valores exhiberi poterunt.

EXEMPLUM 1

823. Sit primo figura *AEBF* composita ex duobus segmentis circularibus aequalibus ad communem chordam *AB* dispositis, seu sint navis omnes sectiones horizontales huic figurae aequales; et ponatur radius circuli cuius arcus *AEB* et *AFB* sunt portiones $= c$; atque cum centrum gravitatis *G* in medio chordae *AB* erit situm, sit

$$AG = BG = a,\ EG = FG = b,$$

ita ut sit $2bc = a^2 + b^2$; et ponatur brevitatis gratia $c - b$ seu $\sqrt{\left(c^2 - a^2\right)} = d$.
Ex his cum propositione 58 comparatis prodibit intervallum

$$GR = z = \frac{ra^3 d}{c^3 - rd^3}$$

atque anguli *GRM* tangens

$$= \frac{\sigma}{\rho} = \frac{2sc^3 - 2rsd^3}{2rc^3 - 3rrc^3 d + \left(rr - ss\right) d^3},$$

tandemque vis resistentiae

$$=\frac{2v}{3cc}\surd\left(4c^6-12r^3c^5d+9r^4c^4d^2\left(r^2-3ss\right)c^3d^3-6rr\left(r^2-s^2\right)c^2d^4+d^6\right),$$

quae vis resistentiae, si cursus sit directus, prodit

$$=\frac{2(c-d)^2\left(2c+d\right)v}{3cc}.$$

Si ergo sit *ff* superficies plana eandem patiens resistentiam, quam patitur corpus in cursu directo erit

$$\frac{uu}{ff}=\frac{\surd\left(4c^6-12r^3c^5d+9r^4c^4d^2+4r\left(r^2-3s^2\right)c^3d^3-6rr\left(r^2-s^2\right)c^2d^4+d^6\right)}{\left(c-d\right)^2\left(2c+d\right)}.$$

Si fuerit obliquitas valde parva ut *s* prae *r* evanescat fiet

$$\frac{\sigma}{\rho}=\frac{2s\left(c^3-d^3\right)}{2c^3-3c^2d+d^3}=\frac{2s\left(c^3+cd+d^2\right)}{\left(c-d\right)\left(2c+d\right)}$$

unde erit

$$\rho=1 \text{ et } \sigma=\frac{2s\left(c^3+cd+d^2\right)}{\left(c-d\right)\left(2c+d\right)}=\frac{\left(3a^4+b^4\right)s}{\left(3a^2+b^2\right)b^2}\ *,$$

et

$$z=\frac{a^3d}{c^3-d^3}=\frac{2a^3\left(a^2-b^2\right)}{3a^4+b^4}*.$$

Ex quibus ponendo angulo *AGL* infinite parvo prodibit

$$s\sigma+r\rho=1 \text{ et } r\sigma-s\rho=\sigma-s=\frac{3a^2\left(a^2-b^2\right)s}{\left(3a^2+b^2\right)b^2}*$$

atque $u^2=f^2$. Quibus substitutis habebitur $dv=-ffvdx$, et radius curvedinis curvae descriptae

$$=\frac{2bb\left(3a^2+b^2\right)V}{3aa\left(a^2-b^2\right)s}*$$

atque momentum vis corpus circa axem verticalem per centrum gravitatis ductum convertens $= \frac{2a^3\left(a^2-b^2\right)Mffsv}{b^2\left(3a^2+b^2\right)V}$ *

[* corrected values.]

COROLLARIUM 1

824. Vis igitur gyratoria seu obliquitatem cursus adaugens eo erit maior, quo magis longitudo navis excedit latitudinem. Atque simul eo maior erit curvatura viae, in qua corporis centrum gravitatis incedet.

COROLLARIUM 2

825. Patet etiam quo magis longitudo *AB* superet latitudinem *EF,* eo magis fore excessum anguli *GRM* supra angulum *AGL.* Namque angulus *GRM* se habet ad angulum *AGL* in duplicata ratione longitudinis ad latitudinem navis.

COROLLARIUM 3

826. Si ergo latitudo *EF* aequalis fiat longitudini *AB,* seu $b=a$, quo casu figura abibit in integrum circulum, tum erit $c=a$ et $d=0$; unde centrum resistentiae in *G* cadet atque fiet $\frac{\sigma}{\rho}=\frac{s}{r}$, quamobrem cursus inceptus sine ulla mutatione continuabitur; id quod etiam eo patet, quod in circulo non detur cursus obliquitas.

EXEMPLUM 2

827. Sit figura navis, praeterquam quod habeat planum diametrale *AB* id quod semper ponimus, ita comparata ut centrum resistentiae *R* perpetuo cadat in rectam verticalem per centrum gravitatis *G* transeuntem. Deinde si ponatur resistentia, quam patitur corpus cursu directo in directione *GA* motum, tanta, quantam pateretur figura plana *ff* eadem celeritate directe contra aquam mota atque resistentia lateralis, quam sufferet, si in directione *GE* moveretur, tanta, quantam pateretur figura plana *hh* eadem celeritate in aquam impingens; habeat motus obliquus hanc proprietatem ut sit tangens anguli *HGB* $=\frac{\sigma}{\rho}=\frac{shh}{rff}$;
seu anguli *MRB* sive *HGB* tangens teneat ad tangentem anguli obliquitatis cursus *AGL* rationem ut *hh* ad *ff* hoc est ut resistentia lateralis ad resistentiam prorae. Vis denique resistentiae sit $=v\sqrt{\left(s^2h^4+r^2f^4\right)}$, in directione *GH* corpus urgens; ita ut sit pro oblata obliquitate cursus $uu=\sqrt{\left(s^2h^4+r^2f^4\right)}$. His igitur positis nulla omnino erit vis tendens ad corpus circa axem verticalem per centrum gravitatis transeuntem convertendum et

hancobrem positio axis navis *AB* perpetuo in motu manebit eadem seu sibi parallela. Postmodum autem cum sit

$\frac{\sigma}{\rho}=\frac{shh}{rff}$, erit $\frac{1}{\rho}=\frac{\surd\left(r^2f^4+s^2h^4\right)}{rff}$

ideoque

$\sigma=\frac{shh}{\surd\left(r^2f^4+s^2h^4\right)}$ et $\rho=\frac{rff}{\surd\left(r^2f^4+s^2h^4\right)}$;

unde fiet

$s\sigma+r\rho=\frac{s^2h^2+r^2f^2}{\surd\left(r^2f^4+s^2h^4\right)}$ et $s\sigma+r\rho=\frac{rff}{\surd\left(r^2f^4+s^2h^4\right)}$;

ex quibus elicitur primo retardatio motus

$$du=-\frac{(s^2h^2+r^2f^2)udx}{V}$$

atque declinatio a semita rectilinea tanta erit ut arculum circularem *Gg 2V* describat corpus cuius radius erit

$$=\frac{2V}{rs\left(hh-ff\right)}.$$

COROLLARIUM I

828. Cum anguli *HGK* sinus sit

$$=\frac{rs\left(hh-ff\right)}{\surd\left(s^2h^4+r^2f^4\right)}$$

patet si fuerit $h>f$ tum semper angulum *MRG* seu *HGB* fore maiorem angulo *AGL* praeter duos casus quibus est vel s vel $r=0$, hoc est si declinatio cursus *AGL* fuerit vel nulla vel 90 graduum.

COROLLARIUM 2

829. Ex hac igitur formula intelligitur fore alicubi differentiam inter angulos *HGR* et *AGL* maximam, qui locus ibi erit, si fuerit tangens anguli *AGL*

$=\frac{f}{h}$, seu $\frac{s}{r}=\frac{f}{h}$;

tum autem anguli *HGB* seu *MRG* tangens erit $=\frac{h}{f}$.

COROLLARIUM 3

830. Quoniam autem hic nulla adest vis corpus convertens, ideoque axis *AB* eandem positionem perpetuo retinet, cursus directio *GL* a vi normali continuo versus *AB* inflectetur ita ut tandem cursus in directum mutetur.

COROLLARIUM 4

831. Eo magis autem motus centri gravitatis a linea recta deflectetur, quo maior fuerit differentia inter resistentiam prorae et resistentiam lateris, hoc est quo minorem navis in cursu directo secundum directionem *BA* patiatur resistentiam, simulque quo maior fuerit resistentia quam pateretur in directione *FE* mota.

SCHOLION 2

832. Non sine gravi ratione casum hunc attulimus, videtur enim haec proprietas, quam corpori in aqua oblique promoto hic tribuimus, maxime competere in naves, quae vento propelli solent. Primo enim in huius generis navibus ad id imprimis attenditur, ut centrum resistentiae ex prora versus puppim removeatur, et quasi in ipsam rectam verticalem per centrum gravitatis ductam incidat. Deinde resistentia lateralis vehementer excedere solet resistentiam cursus directi, ex quo sponte sequitur, quod supra iam annotavimus, in cursu obliquo directionem resistentiae multo magis ab axe navis declinare. Idem autem satis commode formula assumta declarat, per quam angulo obliquitatis cursus cuius tangens est $=\frac{s}{r}$ respondet angulus, quem media directio resistentiae cum spina navis constituit, cuius tangens est $=\frac{s}{r}\cdot\frac{hh}{ff}$ qui angulus ergo evanescit, si obliquitas evanescit, atque in rectum abit, si navis directio ad spinam fit normalis, quae apprime conveniunt cum figura navium recepta. Tertio quod posuimus vim resistentiae esse $=\sqrt{\left(s^2h^4+r^2f^4\right)}$ id quidem mirifice in structuram navium receptam quadrat; facto enim $s=0$ et $r=1$, qui est casus cursus directi, resistentia fit $=ff$ uti assumsimus, similique modo si obliquitas cursus ad angulum rectum declinet egregie prodit resistentia $=hh$. Praeterea vero patet si resistentia lateralis *hh* resistentiae prorae *ff* aequalis ponatur, tum omnium cursuum resistentiam fore quoque eandem; ac tandem ista expressio resistentiae ita est comparata, ut cum angulo *MRB* cuius tangens est $\frac{shh}{rff}$ apprime conspiret, siquidem cum casibus supra tractatis conferatur. At harum proprietatum probatio si non apodictica tamen eiusmodi, in qua acquiescere liceat, afferri potest, qua evincetur hanc tum directionis resistentiae tum quantitatis rationem in navibus locum invenire. Resolvatur scilicet motus secundum directionem obliquam *GL* factus in duos laterales; quorum alter fiat in directione *GA* cuius celeritas erit $=r\sqrt{u}$, alter vero in directione *GE* ad istam normali,

cuius celeritas erit $= s\sqrt{u}$. Iam quamvis in calculo resistentiarum non liceat motum decomponere tamen pro nostro instituto parum a veritate aberrabitur, si corpus duplici motu altero in directione *GA* cum cleritate $= r\sqrt{u}$, altero in directione *GE* cum celeritate $= s\sqrt{u}$ ferri ponamus. Propter illum autem motum resistentia quam patietur prora censeri potest se habere ad resistentiam quam latus perferet ut *rff* ad *shh,* unde directio media resistentiae *GH* angulum *HGB* constituet cuius tangens erit $= \frac{shh}{rff}$, atque ipsius resistentiae quantitas fiet $= v\sqrt{(s^2h^4 + r^2f^4)}$, seu planum resistentiam exprimens erit $uu = \sqrt{(s^2h^4 + r^2f^4)}$. Ex hac consideratione nova hypothesis formari poterit ponendo resistentiam prorae non ut hic fecimus $= rff$, sed tantam, quanta foret si actu tanta celeritate promoveretur, scilicet $= r^2f^2$, similique modo resistentiam lateris $= s^2h^2$, unde fiet anguli *HGB* tangens $= \frac{s^2h^2}{r^2f^2}$, atque ipsa vis resistentiae $= v\sqrt{\left(s^4h^4 + r^4f^4\right)}$ sed haec altera hypothesis quam prior plus a veritate recedit, si figura ponatur circulus. Nam hoc casu semper fit anguli *HGB* tangens $= \frac{s}{r}$, atque resistentia est constans seu $uu = f = hh$; id quod indicat prior hypothesis, posterior autem secus. Quamobrem priorem hypothesin posteriori merito praeferre convenit, ideoque eam in sequentibus prae aliis considerabimus. Eo minus autem prior illa hypothesis a veritate aberrabit, si revera naves ita fuerint comparatae, ut semper quemcunque cursum obliquum teneant, centrum resistentiae in ipsum navis centrum gravitatis cadat; quoniam enim hoc ipsum hypothesis postulat, dubium non est, quin si naves in hoc convenerint, reliqua eo minus sint erratura.

PROPOSITIO 80

PROBLEMA

833. *Determinare vim venti et dispositionem velorum quibus efficiatur ut navis motu rectilineo sub data cursus obliquitate uniformiter progrediatur; simulque velocitatem motus definire.*

SOLUTIO

Repraesentet *AEBF* (Fig. 105) figuram navis, in qua sit *G* centrum gravitatis navis, atque recta *AB* positio spinae seu axis longitudinalis a prora *A* ad puppim *B* ducti. Promoveatur autem

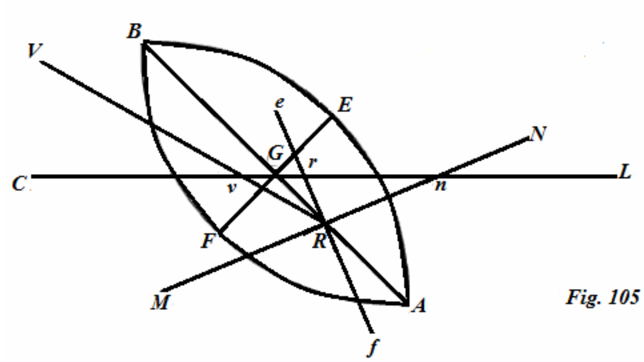


Fig. 105

centrum gravitatis *G* motu uniformi in directum per *GL* sitque altitudo ipsius celeritati debita $= v$. Exhibet igitur angulus *AGL* cursus obliquitatem cuius sinus sit $= s$ cosinusque $= r$; quare cum navis hanc obliquitatem constanter retinere ponatur, resistentia quoque perpetuo manebit eadem eiusque directio erit *RM,* quae cum *AB* angulum *MRB* constituat, cuius sinus sit $= \sigma$ et cosinus $= \rho$, resistentia vero ipsa tanta sit, quantam pateretur figura plana *uu* planitie sua directe contra aquam eadem celeritate $\sqrt{v}$ impingens; quae omnia dabuntur ex data navis structura et data obliquitate, ita ut σ, ρ et u^2 ab r, s et quantitatibus per navem datis pendeant. Hinc erit vis resistentiae in directione *RM* urgens *uuv,* seu posito navis pondere $= M$ et volumine partis submersae $= V$, aequabitur vis resistentiae ponderi $\frac{Mu^2v}{V}$. Cum igitur haec vis non solum motum retardet, sed etiam directionem et cursus obliquitatem mutet, eam per vim venti destrui oportebit. Hoc enim si fuerit praestitum, perspicuum est cessante causa motum perturbante, navem celeritatem suam $\sqrt{v}$ retinere, eaque uniformiter in directum incedere atque insuper obliquitatem suam immutatam conservare debere. Aliter autem haec vis resistentiae prorsus destrui non potest, nisi directio vis venti incidat in directionem resistentiae *RM,* atque simul vis venti aequalis sit et contraria vi resistentiae. Cum igitur vis venti ad vela sit normalis, oportet ut superficies velorum ad rectam *RM* sit normalis, atque centrum gravitatis commune velorum in eandem verticalem cum centro resistentiae *R* cadat, siquidem vela semper centrum suum gravitatis in axe *AB* habent positum, sit itaque *ef velorum* directio, quae sit normalis ad *MR,* atque venti vis tanta esse debebit ut aequet vim resistentiae $\frac{Mu^2v}{V}$. Ponatur autem superficies velorum plana $= gg$, atque ventus fiet in directione *VR* celeritate $\sqrt{c}$, sitque anguli *VvC* sinus $= \mu$ et cosinus $= \nu$. Ad vim venti vero cognoscendam, definiendi sunt sinus angulorum *VRf* et *Crf,* quorum ille si fuerit $= m$ hic vero *q* erit vis venti in directione *RN* urgens $= \left(m\sqrt{c} - q\sqrt{v}\right)^2 gg$ (§ 794) seu aequalis erit ponderi $= \frac{\left(m\sqrt{c} - q\sqrt{v}\right)^2 gg}{784V}$. At est

$$\sin Crf = \cos Rnr = \cos\left(MRG - AGL\right)$$

et

$$\cos Crl = -\sin Rnr = -\sin(MRG - AGL);$$

unde prodit anguli *Crf* sinus $q = s\sigma + r\rho$ et cosinus $= s\rho - \sigma r$. Deinde cum sit $\sin VRf = \sin(Crf + C\upsilon V)$ erit $m = \nu\left(s\sigma + r\rho\right) - \mu\left(r\sigma - s\rho\right)$; ideoque vis venti

$$=\frac{M\left(v(s\sigma+r\rho)\sqrt{c}-\mu(r\sigma-s\rho)\sqrt{c}-(s\sigma+r\rho)\sqrt{\upsilon}\right)^{2}\mathrm{g}^{2}}{784V}$$

cuius directio iam est contraria directioni resistentiae *RM,* superest ergo tantum ut ipsa vis fiat aequalis vi resistentiae $\frac{Mu^{2}v}{V}$ unde obtinebitur ista aequatio

$$28u\sqrt{\upsilon}=v(s\sigma+r\rho)g\sqrt{c}-\mu\left(r\sigma-s\rho\right)g\sqrt{c}-(s\sigma+r\rho)g\sqrt{v}$$

ex qua aequatione vel celeritas venti $\sqrt{c}$ vel celeritas navis $\sqrt{v}$ determinari poterit. Si igitur ponamus velocitatem venti datam, reperietur celeritas, qua navis sub data obliquitate cursus *AGL* in data linea recta *GL* movebitur

$$=\sqrt{\upsilon}=\frac{\left(v\left(s\sigma+r\rho\right)-\mu\left(r\sigma-s\rho\right)g\sqrt{c}\right)}{28u+\left(s\sigma+r\rho\right)g}$$

Oportet autem ut sit *m* seu eius valor $v(s\sigma+r\rho)-\mu\left(r\sigma-s\rho\right)$ sit affirmativus, nam si fieret negativus, ventus vela non in directionem *RN* sed oppositam *RM* intenderet; quamobrem hi casus probe sunt excipiendi. Idem quidem ipsa expressio inventa luculenter declarat, cum si *m* obtineat valorem negativum, quoque celeritas $\sqrt{v}$ fiat negativa, quod indicio est tum navem non in directione *GL* sed contraria *GC* esse incessuram. Q. E. I.

COROLLARIUM 1

834. Si anguli *RnG,* qui est excessus anguli *MRB* supra angulum *AGL,* sinus ponatur $=p$ et cosinus $=q$ erit $p=r\sigma-s\rho$ et $q=s\sigma+r\rho,$ unde reperietur celeritas navis

$$\sqrt{\upsilon}=\frac{\left(vq-\mu p\right)g\sqrt{c}}{28u+gg}.$$

COROLLARIUM 2

835. At $vq-\mu p$ exprimit cosinum summae angulorum, quorum sinus sunt p et μ. *Q*uare ne iste cosinus uti requiritur fiat negativus, oportet ut summa angulorum $V\upsilon C+RnG$ minor sit angulo recto.

COROLLARIUM 3

836. Si ergo dentur venti directio *VR,* via describenda *GL* et obliquitas cursus *AGL* a qua velorum positio pendet, celeritas navis eo maior erit, quo maior fuerit celeritas venti idque in eadem ratione.

COROLLARIUM 4

837. Si centrum resistentiae *R* locum habeat variabilem, pro variis cursus obliquitatibus, tum centrum gravitatis velorum debebit quoque mutari, quoniam vis venti unico modo vim resistentiae destruere potest.

SCHOLION 1

838. Nisi igitur centrum resistentiae *R* pro omnibus obliquitatibus cursus fixum teneat locum unicus malus nullum praestabit usum in variis cursibus obliquis conservandus. Sin autem navis pluribus malis fuerit instructa, tum utique vela ita attemperari poterunt, ut eorum commune centrum gravitatis verticaliter puncto *R* quovis casu immineat; sed hoc casu pro quolibet obliquitatis cursu necesse foret locum centri resistentiae *R* exactissime nosse, id quod in praxi vix sperari potest. Tentando autem difficillimum foret vela malorum ita moderari, ut desideratum effectum praestarent, cum duabus conditionibus perfecte satisfieri debeat. Primo enim ea resistentiae vis destrui debet, quae navem de cursu rectilineo declinare conatur, haecque est vis normalis ad semitam *GL* ex resolutione vis resistentiae orta; haec igitur vis infinitis modis per ventum destrui potest, dummodo vis normalis ad semitam *GL* ex resolutione vis venti orta illi sit aequalis et contraria, neque ad hoc refert, in quo loco axis *AB* ea sit applicata. Deinde vero vis resistentiae etiam quae tendit ad navem circa axem verticalem per centrum gravitatis transeuntem convertendam est destruenda, quare momentum vis venti respectu huius axis aequale et contrarium esse debebit momento vis resistentiae respectu eiusdem axis cui conditioni iterum innumeris modis satisfieri potest. At quo utraque vis resistentiae destruatur unicus datur modus, quem proin tentando deprehendere quovis casu vix est sperandum. Hanc ob causam utilissimum erit naves ita construere, ut centrum resistentiae in iis fixum teneat locum, atque adeo in centrum gravitatis incidat, tum enim, cum vis resistentiae navem convertens sit nulla, satis facile erit alteram vim destruere.

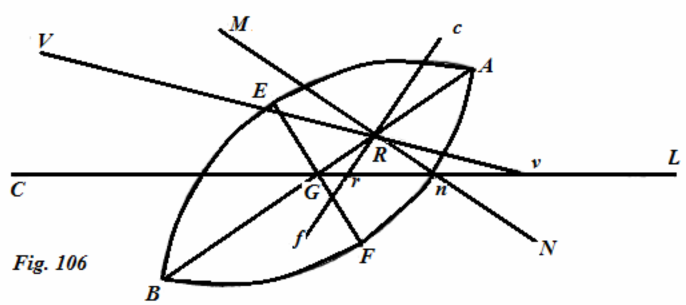


Fig. 106

SCHOLION 2

839. Posuimus in solutione huius problematis proram *A* in partem viae *CL* contrariam cadere ei, ex qua ventus *V* venit, similis autem evadet solutio, si prora in eandem plagam declinet a recta *GL,* cuiusmodi casus in figura hic allegata (Fig. 106) est repraesentatus. Si enim ut ante fuerit anguli declinationis cursus *AGL* sinus $= s$ [et] cosinus $= r$, anguli vero *MRB,* quem media directio resistentiae cum axe navis constituit, sinus $= \sigma$ et cosinus $= \rho$; vela *ef* ita expandi debebunt ut sint normales ad *RM.* Sit porro ut ante planities velorum *gg,* et venti celeritas $\sqrt{c}$, atque anguli $V\upsilon C$ quem directio venti cum via describenda *CL* constituit sinus $= \mu$, cosinusque $= v$; ac tandem superficies plana *uu* exprimat vim resistentiae absolutam. Manifestum est priorem casum ad hunc reduci, si fiant *s* et *u* negativa, quoniam anguli *AGL* et *MRB* in contrarias partes cadunt. Hinc igitur prodibit celeritas, qua navis uniformiter in directione *GL* progredi poterit,

$$= \frac{\left(v(s\sigma + r\rho) + \mu\left(r\sigma - s\rho\right)\right) g\sqrt{c}}{28u + (s\sigma + r\rho)g}$$

quae eadem expressio prodiisset si tantum μ positum fuisset negativum. Si ergo anguli *GnR* qui est excessus anguli *MRB* supra *AGL,* sinus ponatur *p* et cosinus $= q$, erit $p = r\sigma - s\rho$ et $q = s\sigma + r\rho$, atque celeritas navis ad motum uniformem in directum conservandum prodibit

$$= \frac{\left(vq + \mu p\right) g\sqrt{c}}{28u + qg},$$

quae a superiore forma hoc tantum discrepat, quod sinus anguli $V\upsilon C$, qui est $= \mu$ negative sit sumtus.

COROLLARIUM 1

840. Quoniam sinus anguli *VR f,* qui est $= vq + \mu p$ semper debet esse affirmativus, debebit esse $vq + \mu p > 0$ > C; at $vq + \mu p$ est cosinus differentiae angulorum $V\upsilon C$ et *RnC,* quare horum angulorum differentia debet esse recto minor.

COROLLARIUM 2

841. Angulus ergo $V\upsilon C$ poterit esse recto maior, dummodo rectum minore angulo excedat, quam est angulus *RnC.* Quamobrem si angulus *MRB* maior fuerit quam angulus *AGL,* cursus adeo ita institui poterit ut angulum $V\upsilon L$ cum plaga e qua ventus venit acutum constituat.

COROLLARIUM 3

842. Maxime igitur adversus ventum licebit cursum navis instituere, si navi ea obliquitas cursus tribuatur, in qua angulus *MRB* maxime superat angulum *AGL.*

COROLLARIUM 4

843. Quoniam autem, quo magis cursus adversus ventum dirigitur, expressio $vq+\mu p$ eo fit minor, manifestum est quo magis navis adversus ventum propellatur, eo tardiorem fore eius motum; indeque lucrum non augeri. Dabitur ergo eiusmodi dispositio, qua maxime in venti regionem curri poterit.

EXEMPLUM 1

844. Habeat navis eam proprietatem (Fig. 105), quam supra recepimus, ut centrum resistentiae *R* in ipsum navis centrum gravitatis *G* incidat, atque si resistentiae in cursu directo tanta sit quanta figurae planae *ff* resistentia eadem celeritate contra aquam impingentis, resistentia vero in cursu obliquo maximo, cuius obliquitas sit 90 grad. simili modo reducta sit ad planum *hh;* tum vero sit anguli *MRB* tangens

$$\frac{\sigma}{\rho}=\frac{shh}{rff}$$

seu

$$\sigma=\frac{shh}{\sqrt{\left(s^2h^4+r^2f^4\right)}} \text{ et } \rho=\frac{rff}{\sqrt{\left(s^2h^4+r^2f^4\right)}};$$

atque resistentia in hoc cursu obliquo, seu *uu* sit

$=\sqrt{\left(s^2h^4+r^2f^4\right)}$, unde fit $u=\sqrt[4]{\left(s^2h^4+r^2f^4\right)}$.

His positis erit

$$p=r\sigma-s\rho=\frac{rs\left(h^2-f^2\right)}{\sqrt{\left(s^2h^4+r^2f^4\right)}} \text{ et } q=s\sigma+r\rho=\frac{s^2h^2+r^2f^2}{\sqrt{\left(s^2h^4+r^2f^4\right)}},$$

ex quibus reperitur celeritas huius navis, qua uniformiter in directione *GL* incedere poterit seu

$$\sqrt{v}=\frac{\left(v\left(s^2h^2+r^2f^2\right)-\mu rs\left(h^2-f^2\right)\right)g\sqrt{c}}{28\sqrt[4]{\left(s^2h^4+r^2f^4\right)^3}+\left(s^2h^4+r^2f^4\right)g}.$$

At si prora *A* dirigatur in eam regionem rectae *GL* ex qua ventus venit (Fig.106), tum manentibus omnibus ut ante, erit tantum μ in negativum transmutato celeritas navis progressiva

$$\frac{\left(v\left(s^2h^2+r^2f^2\right)+\mu rs\left(h^2-f^2\right)\right)g\sqrt{c}}{28\sqrt[4]{\left(s^2h^4+r^2f^4\right)^3}+\left(s^2h^4+r^2f^4\right)g}.$$

COROLLARIUM 1

845. Maiorem igitur navis obtinebit celeritatem, si ut in casu posteriore axis navis *AB* ita inclinetur ut prora *A* in eam plagam collocetur, ex qua ventus venit, ceteris paribus scilicet manente eadem cursus obliquitate.

COROLLARIUM 2

846. Anteferendi igitur sunt illi cursus in quibus prora *A* supra viam *GL* cadit, iis quibus prora *A* infra *GL* cadit: quia non solum istae obliquitates cum maiore celeritate sunt connexae sed etiam multo plures obliquitates locum inveniunt.

COROLLARIUM 3

847. Posita autem obliquitate cursus deorsum spectante ut in priore casu, limites omnium cursuum erunt $s=0$ et haec aequatio

$$vs^2h^2+vr^2f^2=\mu rs\left(hh-ff\right)$$

ex qua anguli obliquitatis cursus tangens fit

$$=\frac{\mu\left(h^2-f^2\right)\pm\sqrt{\left(\mu^2\left(h^2-f^2\right)^2-4v^2f^2h^2\right)}}{2vh^2}$$

unde bis angulus *VRf* evanescit.

COROLLARIUM 4

848. Casu autem altero, quo prora *A* supra *CL* convertitur omnes obliquitatis cursus gradus continentur inter hos limites, quorum alterum constituit obliquitatis tangens $=0$, alterum quo ista tangens aequatur

$$-\frac{\mu\left(h^2-f^2\right)\pm\sqrt{\left(\mu^2\left(h^2-f^2\right)^2-4v^2f^2h^2\right)}}{2vh^2}$$

ubi signum limitem prioris casus praebet, ita ut superius signum + tantum pro hoc casu valeat.

COROLLARIUM 5

849. Quod autem manentibus ceteris ad directionem venti attinet,manifestum est generaliter eum ventum celerrime navem propellere cuius directio ad planitiem velorum sit normalis. Celeritas enim ceteris paribus est directe ut sinus anguli, quem directio venti cum velis constituit.

COROLLARIUM 6

850. Si vela in infinitum augerentur tum prodiret celeritas navis

$$= v\sqrt{c} \mp \frac{\mu rs\left(h^2 - f^2\right)}{s^2h^2 + r^2f^2}\sqrt{c}$$

ex quo navis maximam obtinebit celeritatem si anguli obliquitatis cursus [tangens] fiat $= \frac{f}{h}$, nisi angulus $V\upsilon C$ fuerit vehementer exiguus.

COROLLARIUM 7

851. Sin autem quaereretur cursus obliquitas, qua navis a dato vento propulsa in data directione celerrime promoveatur, aequatio reperitur vehementer perplexa, ut nil inde concludi queat; quae autem facto g infinito praebet $\frac{s}{r} = \frac{f}{h}$. Ipsa autem aequatio generaliter determinans angulum AGL est sequens pro casu priore, prora supra CL sita:

$$\frac{1}{14}\mu\mathrm{g}\left(r^2+s^2\right)\left(r^2f^2-s^2h^2\right)\sqrt[4]{s^2h^4+r^2f^4} = \left(2\mu s^4+\mu r^2s^2-vrs^2\right)h^4$$
$$-3vrs\left(r^2+s^2\right)f^2h^2+\left(-2\mu r^4-2\mu r^2s^2+vr^3s\right)f^4.\text{[Recte.]}$$

COROLLARIUM 8

Analysis sectionis 852 igitur falsa est.

EXEMPLUM 2

853. Habeat navis alteram proprietatem supra memoratam (§ 832) ut centrum resistentiae R in navis centrum gravitatis G incidat (Fig. 105), atque si ut ante resistentia prorae et lateris exprimatur figuris planis *ff* et *hh,* ut sit anguli *MRB* tangens

$$\frac{\sigma}{\rho}=\frac{s^2h^2}{r^2f^2}$$

seu

$$\sigma=\frac{s^2h^2}{\sqrt{\left(s^4h^4+r^4f^4\right)}} \text{ et } \rho=\frac{r^2f^2}{\sqrt{\left(s^4h^4+r^4f^4\right)}},$$

atque resistentia in hoc cursu seu *uu* sit $=\sqrt{(s^4h^4+\ r^4f^4)}$, unde erit

$$u=\sqrt[4]{s^4h^4+\ r^4f^4}.$$

His positis habebitur

$$p=r\sigma-s\rho=\frac{sr\left(sh^2-rf^2\right)}{\sqrt{\left(s^4h^4+r^4f^4\right)}},$$

et

$$q=s\sigma+r\rho\sigma=\frac{s^3h^2+r^3f^2}{\sqrt{\left(s^4h^4+r^4f^4\right)}}:$$

ex quibus reperitur celeritas navis, qua in directione *GL* aequabiliter progredietur

$$\frac{\left(v\left(s^3h^2+r^3f^2\right)-\mu\left(s^2rh^2-sr^2f^2\right)\right)g\sqrt{c}}{28\sqrt[4]{\left(s^2h^4+r^4f^4\right)^3}+\left(s^3h^2+r^3f^2\right)g}.$$

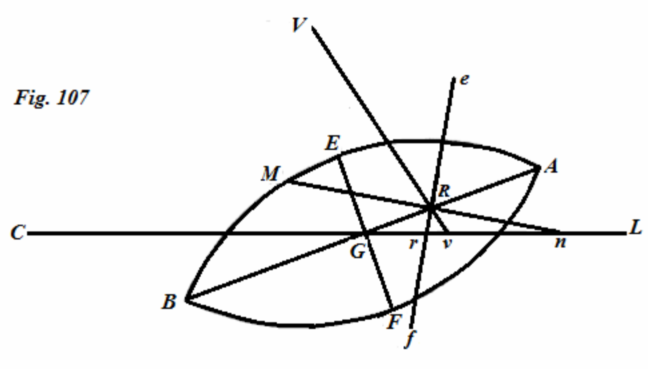
At si prora *A* dirigatur in alteram partem rectae *CL,* haec eadem expressio valebit praeterquam quod loco μ, scribi debeat $-\mu$.

SCHOLION 3

854. Insigne atque maxime utile problema hic occurrit, quo ex datis directione venti *VR* et via a navi describenda *CGL* seu ex dato angulo $C\upsilon V$ definienda est cursus obliquitas seu angulus *AGL,* quo fiat ut navis celerrime promoveatur. Problema quidem hoc iam resolvimus pro exemplo primo (§ 851); verum ad eiusmodi aequationem pertingimus, ex qua obliquitas desiderata difficillime erui potest, neque etiam approximationibus uti licet cum aequatio ad rationalitatem reducta fiat sedecim dimensionum. Maior autem difficultas oriretur, si idem problema pro exemplo secundo tentare vellemus (Fig. 106). Interim tamen rem generaliter considerando quodammodo cursus maxime lucrosus aestimari poterit. Cum enim celeritas inventa sit $=\dfrac{(vq+\mu p)g\sqrt{c}}{28u+qg}$ in qua expressione μ est sinus et v cosinus anguli dati $V\upsilon C$, p vero et q sinus et cosinus anguli *RnG* qui ab

angulo quaesito *AGL* pendet; *uu* autem exprimit resistentiam, quam navis cursu obliquo perfert, quo adeo etiam a p et q pendet. Quamobrem cum nexus inter u et p non constat, per methodum maximorum et minimorum valor ipsius p vel q definiri non poterit, quo celeritas navis fiat maxima. At quoniam numerator expressionis istius scilicet $vq+\mu p$ praebet sinum anguli *VRf,* quem directio venti *VR* cum planitie velorum *ef* constituit, manifestum est hunc numeratorem non mutari, si positio velorum *ef* ita immutetur ut angulus *VRf* fiat obtusus, deinceps posito angulo priori *VRf* hoc est, si obliquitas cursus ita sumatur, ut media directio resistentiae *RM* intra angulum *VvC* cadat quem casum in figura citata (Fig.107) expressimus in qua ut ante *VR* est directio venti, *M* media directio resistentiae intra *Vv* et *Cn* posita. Cum igitur hoc casu sinus anguli *VRf* cum praecedente congruat, ideoque numerator fractionis velocitatem navis exprimentis sit idem, denominator spectari debet, qui est $28u+gg$, de quo primum patet u minorem habere quantitatem quam a casu praecedente, quoniam cursus hic minus est obliquus, atque resistentia augetur, quo magis obliquitas crescit. Altera vero denominatoris pars *qg,* in qua q est cosinus anguli *GnR,* vel crescere potest vel decrescere, vel etiam eadem manere. Si enim obliquitas est valde parva tum quidem angulus *GnR* fit perquam exiguus, ideoque *q,* fere sinui toti aequatur, at vero quoque, si obliquitas vehementer fit magna, tum pariter angulus *GnR* decrescit, obliquitate enim ad 90 gradus aucta, media directio resistentiae iterum in directionem cursus incidit. Quocirca ista cursus mutatione, cum q aeque augeri ac dimininui potuerit, ex altera parte 28*u,* quae certe minor est facta concludendum est istam cursus dispositionem antecedenti esse praeferendam eaque navem celerius promoveri. His igitur praenotatis videamus, quomodo per approximationem cursus maxime velox definiri queat, si quidem datae fuerint venti directio *VR* et via absolvenda *GL* seu angulus $V\upsilon C$ cuius sinus est pet cosinus *v.*

Fig. 107



PROPOSITIO 81

PROBLEMA

855. *Si data fuerit venti directio VR atque itineris conficiendi via GL, determinare cursus obliquitatem AGL, qua navis maxima celeritate promoveatur*
(Fig. 107**).**
SOLUTIO

Sit anguli $V\upsilon C$, qui est datus, sinus $=\mu$ et cosinus $=\nu$, cursus autem obliquitatis *AGL* sinus $=s$ et cosinus $=r$; quos definire oportet, ut prodeat celeritas navis maxima. At celeritas navis, quam sub hac cursus obliquitate habebit est

$$=\frac{(\nu q+\mu p)g\sqrt{c}}{28u+qg};$$

in qua expressione p et q sunt sinus et cosinus anguli *RnG,* qui est excessus anguli *MRB* super angulum *AGL.* Primo autem notandum est hunc angulum *RnG,* ab initio crescente obliquitate crescere, at ad certum tantum terminum augeri, quem cum attigerit, si obliquitas cursus magis augeatur, iterum diminui, resistentiam vero seu quantitatem u continuo augeri, quamdiu obliquitas crescat. Ponamus igitur eum assumtum esse cursum obliquum *AGL* cui maximus angulus *RnG* respondeat; manifestum est, si obliquitas aliquantillum vel augeatur vel diminuatur angulum *RnG* quantitatem suam conservare, et proinde p et q non mutari. Hinc ergo perspicuum est si angulus *AGL* magis augeatur, tum ob crescentem u velocitatem navis diminui, contra vero augeri, si obliquitas cursus *AGL* diminuatur. Ex quo satis luculenter sequitur ad motum celerrimum obtinendum angulum *AGL* minorem accipi debere eo, cui angulus *RnG* maximus respondet. Sit igitur hic angulus *AGL* minor seu talis, quo crescente angulus *RnG* crescat; ponamusque angulum *AGL* aliquantillum augeri, ita ut eius sinus s crescat particula ds; augebitur igitur etiam anguli *RnG* sinus p aliqua particula, quae sit $=mds$; cosinus vero q decrescet parti~ cula $\frac{mpds}{q}$; resistentia vero ita crescat ut u augeatur particula nds eritque ideo

$$dp=mds,\ dq=-\frac{mpds}{q} \text{ et } du=nds.$$

Ex his autem prodibit celeritatis quae est $=\frac{(\nu q+\mu p)g\sqrt{c}}{28u+qg}$ augmentum

$$=\frac{\left(\mu gm+28mu(\mu q-\nu p)-28ng(\mu p+\nu q)\right)gds\sqrt{c}}{q(28u+qg)^2}.$$

Cuius fractionis numerator si fuerit affirmativus, tum navis aucta obliquitate cursus uti posuimus reale accipiet augmentum, at si numerator fuerit negativus tum celeritas decrescet. At est $\mu q-\nu p$ sinus anguli *VRM* et $pp+\nu q$ eiusdem anguli cosinus. Quare si ponatur anguli *VRM* sinus $=x$ et cosinus $=y$, aucta cursus obliquitate navis celerius progredietur si fuerit

$$\mu gm+28mux-28nqy>0$$

tardius autem si fuerit

$\mu gm + 28mux - 28nqy < 0$.

Hancobrem quo navis celerrime progrediatur, oportet cursum ita institui, ut sit

$$\frac{\mu gm}{28} = nqy - mux.$$

Ex qua reperitur angulum *VRM* differentiam esse debere duorum angulorum, quorum maioris sinus sit

$$= \frac{nq}{\sqrt{\left(n^2q^2 + m^2u^2\right)}}$$

minoris vero sinus

$$= \frac{\mu gm}{28\sqrt{\left(n^2q^2 + m^2u^2\right)}}.$$

Sunt autem omnes quantitates quae hic occurrunt, affirmativae, unde non difficile erit obliquitatem cursus aestimare quovis casu oblato. At commodissime negotium conficietur, si pro quovis casu angulus quicunque inter limites assignatos accipiatur, atque inquiratur, utrum eo aliquantillum aucto celeritas augeatur, an diminuatur, ex quo statim colligetur, utrum obliquitas desiderata excedat assumtam, an ea sit minor. Q.E.I.

COROLLARIUM 1

856. Si assumatur primum obliquitas nulla seu cursus ponatur directus, erit $p = 0$ et $q = 1$ atque $u = f$, siquidem planum *ff* exprimat resistentiam cursus directi; celeritasque corporis erit $= \frac{vg\sqrt{c}}{28f + g}$. Si nunc obliquitas infinite parva constituatur erit $du = nds = 0$, atque manente $dp = mds$, erit celeritatis incrementum

$$= \frac{\left(\mu gm + 28\mu mf\right)gds\sqrt{c}}{\left(28f + g\right)^2}.$$

COROLLARIUM 2

857. Apparet igitur nisi sit $\mu = 0$ seu nisi ventus a puppi veniat, cursum directum non celerrimum producere motum, sed cursum quendam obliquum esse praeferendum. Excepto tamen eo casu quo $\frac{dp}{ds}$ in cursu directo evanescit, quippe quo cursus directus semper habet maximum minimumve sed non semper eius modi quale hic desideratur.

COROLLARIUM 3

858. Evanescat in cursu directo $\frac{dp}{ds}$, ut m et n sint quantitates infinite parvae; atque cursus directus celerrimum motum producet, si fuerit

$$\mu gm + 28\mu mf < 28vn,$$

hoc enim casu, minima obliquitate celeritas diminueretur.

COROLLARIUM 4

859. Si igitur evanescente s simul evanescet $\frac{dp}{ds}$ cursu directo navis celerrime progredietur quamdiu anguli $V\upsilon C$ tangens $\frac{\mu}{v}$ non excedit hunc limitem $\frac{28n}{m(28f+g)}$.
At si $\frac{dp}{ds}$ non evanescat evanescete s cursus obliquus semper est praeferendus, nisi ventus a puppi flet.

COROLLARIUM 5

860. Si ergo fuerit saltem pro obliquitatibus minimis
$\frac{\sigma}{\rho} = \frac{s^2h^2}{r^2f^2}$ et $u^2 = \surd(s^4h^4 + r^4f^4)$
erit si s minimum seu infinite parvum

$$\frac{dp}{ds} = -1 = m \text{ et } \frac{du}{ds} = 0,$$

unde celeritas etiam maior prodibit si prora A in partem oppositam rectae GL declinetur.
Hoc enim casu evanescente s fit $\frac{\sigma}{\rho} < \frac{s}{r}$.

COROLLARIUM 6

861. At casu quo fit

$$\frac{\sigma}{\rho}=\frac{sh^2}{rf^2},$$

qui magis navibus congruit erit

$$\frac{dp}{ds}=\frac{h^2-f^2}{f^2}$$

evanescente *s*. Quare cum $h>f$ his casibus cursus obliquus semper erit usurpandus, nisi ventus directe in cursus directionem incidat.

SCHOLION

862. Quanquam in casu, quo ponitur

$$\frac{\sigma}{\rho}=\frac{s^2h^2}{r^2f^2}$$

in ipso initio seu obliquitatibus minimis *p* negativum induit valorem, tamen quam primum fit $\frac{s}{r}>\frac{f}{h}$ eius valor fit affirmativus. Quamobrem etiam in hoc casu, nisi angulus $V\upsilon G$ sit minimus, obliquitas cursus ad superiorem partem rectae *GL* erit dirigenda, siquidem navis celerrime debeat progredi. Neglectis igitur anomaliis istis, quae tantum in minimis obliquitatibus hoc solo casu se offerunt, ad motum velocissimum obtinendum obliquitas debebit dirigi in superiorem partem lineae *GL* ita ut angulus *AGL,* eo modo quo rem sumus contemplati fiat affirmativus. Deinde autem ex circumstantiis allatis facile colligitur, quo maior sit angulus $V\upsilon G$ eo maiorem capi debere obliquitatem. At obliquitatem nunquam maiorem accipi convenit quam est ea cui respondet angulus *RnG* maximus, si igitur iste angulus obliquitatis *AGL* pro quo maxima est differentia inter angulos *MRG* et *AGL* ponatur α graduum, et angulus *RnG* β graduum, limites intra quos angulus obliquitatis cursus *AGL* contineri debet erunt 0° et α°, quorum limitum ille 0° locum habet si angulus $V\upsilon C$ evanescat, alter autem solus in usum vocari potest quando angulus $V\upsilon C$ proxime erit $90+\beta$ graduum. Si enim angulus $V\upsilon C$ maior fuerit quam $90+\beta$ graduum tum navis nequidem in directione data *GL* promoveri potest, ex quo cum angulo $V\upsilon C=0$ grad. respondeat angulus $AGL=0$ grad. atque angulo $V\upsilon C=90+\beta$ graduum respondeat angulus *AGL* ex graduum, satis prope pro angulis $V\upsilon C$ intermediis convenientes angulos *AGL* assignare licebit, idque eo facilius praestabitur si pro uno alterove angulo $V\upsilon C$ intermedio per methodum datam angulus *AGL* aptissimus definiatur. Sola autem aestimatione ad veritatem satis prope accedetur, si pro angulo $V\upsilon C$ continente *x* gradus capiatur cursus obliquitas *AGL*

$= \frac{\alpha x}{90+\beta}$

graduum vel forte $\frac{\alpha x^2}{(90+\beta)^2}$ graduum, vel generalius $\frac{\alpha x^n}{(90+\beta)^n}$ graduum;
quae formula in usum vocari poterit, si pro dato quodam angulo $V\upsilon C$ angulus AGL maxime congruus actu fuerit determinatus, eo enim exponens n definietur. Anguli autem α et β ex data navis proprietate facile determinabuntur; si enim fuerit $\frac{\sigma}{\rho} = \frac{shh}{rff}$ erit anguli RnG tangens

$= \frac{sr(hh-ff)}{s^2h^2+r^2f^2}$;

qui ideo erit maximus si fuerit $\frac{s}{r} = \frac{f}{h}$; seu anguli AGL tangens $= \frac{f}{h}$. Tum autem erit anguli MRB tangens $= \frac{h}{f}$; atque differentiae RnG tangens $= \frac{hh-ff}{2fh}$; seu erit $\beta = 90° - 2\alpha$ unde pro varia relatione inter quantitates ff et hh, quae eam inter se rationem habent, quam habet resistentia navis in directione GA mota ad resistentiam navis in directione GE mota, anguli α et β cognoscentur, quod quo facilius pateat sequentem tabellam adiungere visum est

$hh = ff$	$\alpha = 45°$	$\beta = 0°, 0'$
$hh = 2ff$	$\alpha = 35°, 16'$	$\beta = 19°, 28'$
$hh = 3ff$	$\alpha = 30°, 0'$	$\beta = 30°, 0'$
$hh = 4ff$	$\alpha = 26°, 34'$	$\beta = 36°, 52'$
$hh = 5ff$	$\alpha = 24°, 6'$	$\beta = 41°, 48'$
$hh = 6ff$	$\alpha = 22°, 12'$	$\beta = 45°, 36'$
$hh = 7ff$	$\alpha = 20°, 42'$	$\beta = 48°, 36'$
$hh = 8ff$	$\alpha = 19°, 28'$	$\beta = 51°, 4'$
$hh = 9ff$	$\alpha = 18°, 26'$	$\beta = 53°, 8'$
$hh = 10ff$	$\alpha = 17°, 33'$	$\beta = 54°, 54'$

Cum autem quo maior est angulus β, eo magis adversus ventum cursus institui queat, liquet quo maior fuerit longitudo navis respectu latitudinis eo magis adversus ventum navigari posse; tenet enim ff ad hh proxime rationem latitudinis navis maximae ad ipsius longitudinem. In navibus autem usu receptis proxime est $kk = 4ff$, ex quo eae aptae sunt adversus ventum navigare, ita ut angulus $V\upsilon L$ fiat fere 53°, 8' seu angulus $V\upsilon C$, 126°, 52' id quod cum experientia egregie convenit qua naves observantur ad 11 rhombos seu $123\frac{3}{4}$ gradus dirigi posse.

PROPOSITIO 82

PROBLEMA

863. *Definire cursum a navi instituendum, quo celerrime in regionem, ex qua ventus venit, provehatur.*

SOLUTIO

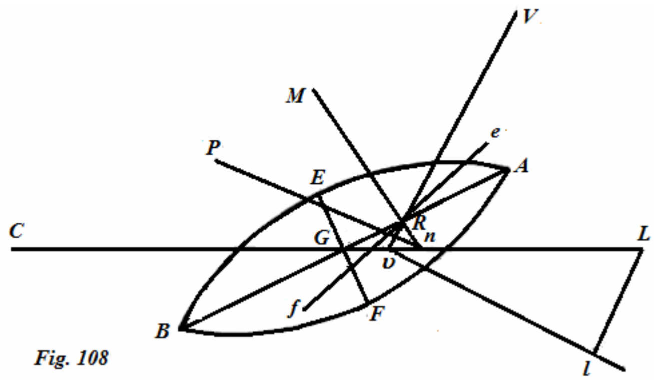


Fig. 108

Sumatur primo ad lubitum obliquitas navis *AGL* (Fig. 108) cuius anguli sinus sit *s*, cosinus $= r$; veniatque ventus in directione $V\upsilon$ pariter data, et quaeratur positio lineae *GL* respectu $V\upsilon$ ut cursus in regionem ex qua ventus venit, maxime properetur. Quaeritur ergo angulus $V\upsilon L$ cuius sinus sit $=\mu$ cosinus $=\nu$.

Ex angulo autem obliquitatis cursus dato *AGL,* dabitur media directio resistentiae *RM* quae cum directione cursus *GL* angulum faciat *RnG* cuius sinus sit $=p$ cosinus $=q$. Iam positis ut ante celeritate venti $=\sqrt{c}$, planitie velorum *gg,* et plano resistentiam exprimente $=uu$, erit celeritas qua navis in directione *GL* ingredietur

$$=\frac{(\mu p-\nu q)g\sqrt{c}}{28u+qg};$$

ubi ν fecimus negativum, quia angulum $V\upsilon L$ acutum ponimus. Cum igitur navis incedat in directione *GL* angulum acutum cum directione venti $V\upsilon$ constituente, dum spatium υL percurrit, in regionem, ex qua ventus venit, accessit spatio *L l,* ducta υl perpendiculari ad directionem venti $V\upsilon$ et *Ll* ipsi parallela. Hinc navis hoc cursu ad ventum accedit celeritate

$$=\frac{\nu(\mu p-\nu q)g\sqrt{c}}{28u+qg};$$

quae maxima esse debet. Cum ergo obliquitas cursus *AGL* data ponatur, differentietur ea ponendis μ et ν variabilibus, atque differentiale ponatur = 0; prodibit autem ob $d\nu=-\frac{\mu d\mu}{\nu}$ ista aequatio $(\mu^2-\nu^2)p=2\mu\nu q$, ex qua oritur $\frac{\mu}{\nu}=\frac{q+1}{p}$. Quare cum anguli $V\upsilon L$ tangens sit $=\frac{q+1}{p}$ erit angulus ipse $L\upsilon V = 90° - \frac{1}{2}Rng$ seu angulus *Lvl* aequabitur semissi anguli *Rng.* Nave itaque ad cursus obliquitatem datam *AGL* instructa, angulus *RnG* bisecetur recta *nP,* navisque vento ita obvertatur ut directio venti *Vv* ad illam rectam *nP* sit normalis. Quoniam vero est

$$\frac{\mu}{\nu}=\frac{q+1}{p} \text{ erit } \mu=\frac{q+1}{\sqrt{(2+2q)}}=\sqrt{\frac{q+1}{2}} \text{ et } \nu=\sqrt{\frac{1-q}{2}};$$

unde celeritas qua navis ad ventum accedit erit

$$=\frac{(1-q)g\sqrt{c}}{2(28u+qg)}.$$

Si nunc quaeratur angulus obliquitatis cursus *AGL,* qui reddat hanc expressionem $=\frac{1-q}{28u+qg}$ maximam, tum habebitur ille navis cursus, quo omnium celerrime navis in regionem venti promovetur. Ante omnia autem intelligitur ex praecedentis problematis solutione cursus obliquitatem minorem accipi debere, quam est ea, cui angulus *RnG* maximus respondet. Ac si ponamus dum *p* crescit elemento *dp,* interea crescere *u* elemento *ndp,* prodibit anguli *RnG* tangens

$$\frac{p}{q}=\frac{28n(1-g)}{28u+g}$$

quae expressio cum maior fuerit, quam tangens anguli *RnG,* si est maximus, tum ipse angulus maximus seu proxime minor erit adhibendus. Quovis autem casu particulari oblato ista quaestionis pars, quae ad ipsius obliquitatis cursus determinationem spectat, facile resolvetur. Q. E. I.

COROLLARIUM 1

864. Cum angulus *VRe,* sub quo ventus in vela irruit sit complementum anguli $nR\upsilon$ ad rectum, aequabitur quoque angulus *VRe* semissi anguli *GnR* eiusque ideo sinus erit

$$\sqrt{\frac{1-q}{q}}.$$

COROLLARIUM 2

865. Deinde etiam notandum est angulum $V\upsilon L$ quem directio venti cum via describenda *GL* constituit cum angulo *VR e* angulum rectum conficere.

COROLLARIUM 3

866. Cum celeritas qua navis versus ventum appropinquat sit

$$=\frac{(1-q)g\sqrt{c}}{2(28u+qg)},$$

manifestum est celeritatem hanc fore $=0$, si $q=1$ maximamque si $q=0$ seu angulus *RnG* rectus. At cum angulus *RnG* ultra datum limitem crescere nequeat, intelligitur maximam fore accessionem ad ventum si angulus *RnG* capiatur maximus. Quamobrem obliquitas cursus tanta est sumenda, ut angulus respondens *RnG* a valore suo maximo sensibiliter non discrepet.

COROLLARIUM 4

867. Si ergo angulus obliquitatis cursus, cui maximus respondet angulus *RnG,* ponatur $=\alpha$, et maximus angulus $RnG=\beta$, debebit angulus *AGL* aliquantulum minor accipi quam α; ita ut *RnG* maneat $=\beta$.

COROLLARIUM 5

868. Ponamus angulum *AGL* ipsi angulo α aequalem vel aliquantillum minorem capi, erit angulus $RnG=\beta$; unde ob triangulum $Rn\upsilon$ isosceles fiet angulus $V\upsilon L=90°-\frac{1}{2}\beta$, et angulus *V RA,* quem plaga venti cum directione spinae navis *AB* constituti erit $=90°-\alpha+\frac{1}{2}\beta$ grad. vel aliquanto maior. Angulus autem quo ventus in vela incidit seu *VR e* erit $=\frac{1}{2}\beta$.

COROLLARIUM 6

869. Hinc igitur ope tabellae supra datae, qua relatio inter α et β continetur, cuiusvis navis datae cursus ita dirigi poterit, ut iter maxime adversus ventum instituatur.

SCHOLION

870. In his propositionibus assumsimus navem iam habere eam celeritatem, qua a vento sollicitata secundum datam directionem progredi queat; neque solliciti fuimus, unde eam celeritatem acquisiverit. Quamobrem etiam istae proprietates, quas invenimus locum non habent, nisi navis iam eam ipsam celeritatem, quam ipsi tribuimus aliunde sit nacta. Hae scilicet propositiones respiciunt motum uniformem, quo navis a vento propulsa provehi potest neque ex iis productis et acceleratio motus, si navis vel in quiete fuerit posita, vel datam celeritatem in data directione habuerit, cognosci potest, sed istas propositiones praemittere conveniens visum est quo intelligitur, quomodo navis, si iam quandam

celeritatem sit consecuta, eam ope venti conservare eaque in directum progredi queat. Quare cum haec satis sint explicata, investigabimus quomodo navis a vento motum accipere eumque augere possit: in quo primum erit inquirendum si navis quamcumque iam habeat celeritatem in data directione atque quamvis teneat cursus obliquitatem, quomodo datus ventus in vela utrunque disposita irruens motum illum afficiat, eum vel augendo vel diminuendo vel directionem ipsam alterando vel denique cursus obliquitatem immutando. Deinde si hoc fuerit definitum, licebit eiusmodi quaestiones tractare, quibus pro data dispositione navis et velorum totus motus requiritur, quem navis a vento sollicitata accipiet; ex iisque demum iudicari poterit utrum navis eiusmodi motum, quem ipsi in his praecedentibus propositionibus iam insitum posuimus, nancisci queat, an secus? et si fieri poterit ut ipsi talis motus concilietur simul modus constabit, quo eiusmodi motus sit producendus. In navigatione quidem praecipue motus uniformis in directum requiritur, qui si iam fuerit formatus, quomodo conservetur exposuimus; sed hoc nullius foret usus, nisi constaret, quanam navis directione et qua velorum dispositione, si navis primum quieverit, ea ad eiusmodi motum perennem redigi queat. Deinde vero etiam nosse oportet, quomodo ex uno motu constante alius quicunque datus sit formandus, cuiusmodi quaestiones in navigatione maximi sunt momenti.

PROPOSITIO 83

PROBLEMA

871. *Si navis quaecunque AEBF* (Fig. 109) *cursum teneat obliquum AGL, ita ut eius centrum gravitatis G motu progressivo sit praeditum in directione GL; sollicitetur autem haec navis a vento in directione VL flante et in vela e f impingente; determinare immutationem tam motus quam obliquitatis cursus inde ortam.*

SOLUTIO

Sit navis celeritas progressiva, quam eius centrum gravitatis G habet secundum directionem GL debita altitudini v seu $=\sqrt{v}$. Sitque anguli obliquitatis cursus AGL sinus $=s$ et cosinus $=r$ posito semper sinu toto $=1$. Deinde centrum gravitatis velorum in quovis venti collecta est concipienda sit in axis AB puncto r existente $Gr = y$; atque velorum positio ef cum axe AB constituat angulum Are seu Brf cuius sinus sit $= m$ et cosinus $= n$; planities vero velorum sit gg. Denique venti celeritas sit $=\sqrt{c}$, atque anguli VrB, quem directio venti VrL cum axe navis AB constituit, sinus sit $=\mu$ et

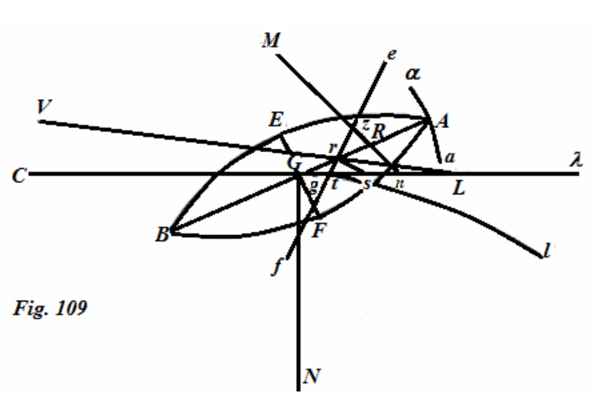


Fig. 109

cosinus $= v$. His praemissis vis venti habebit directionem *rs* normalem ad superficiem velorum *ef,* eritque
anguli *Ars* sinus $= n$ et cosinus $= m$; anguli vero *rsG* sinus erit $= nr - ms$, et cosinus $= ns + mr$; et anguli *Vrf* sinus erit $= n\mu + mv$; anguli tandem *Gtf,* quem positio velorum *ef* cum directione motus *GL* constituit, sinus erit $= ns + mr$; unde quantitas vis venti erit $\left((n\mu + mv)\surd c - (ns+mr)\surd v\right)^2 gg$, $= r\sigma - s\rho$ seu tanti aeris voluminis ponderi erit aequalis. Quare si pondus navis ponatur $= M$, et volumen partis aquae submersae $= V$ erit ipsa vis venti

$$\frac{M\left((n\mu + mv)\surd c - (ns+mr)\surd v\right)^2 g^2}{784V}.$$

Sit porro pro hoc cursu obliquo centrum resistentiae *R* existente $GR = z$; et *RM* media directio resistentiae, quae cum *AB* angulum *MRG* constituat, cuius sinus sit $= \sigma$ et cosinus $= \rho$, erit anguli *RnG* sinus $= r\sigma - s\rho$, et cosinus $= s\sigma + r\rho$, resistentia vero aequalis sit illi, quam pateretur superficies plana *uu* eadem celeritate directe contra aquam mota; ex quovis resistentiae erit $= uuv$, seu ponderi $\frac{Muu\upsilon}{V}$. Quamobrem navis a duabus potentiis sollicitabitur quarum prior ex vento orta est

$$= \frac{M\left((n\mu + mv)\surd c - (ns+mr)\surd \upsilon\right)^2 gg}{784V}$$

ac navem pellit in directione *rs;* posterior vero est vis resistentiae $= \frac{Muu\upsilon}{V}$, quae navem urget in directione *RM.* Quo nunc immutatio motus progressivi eruatur, utraque vis in ipso gravitatis centro applicata est censenda, utraque resolvenda in binas laterales, tangentiales scilicet in directione *GL* sitas et normales directionem *GN* tenentes; prodibit autem vis urgens centrum gravitatis *G* in directione *GL,* seu vis tangentialis

$$= \frac{M(ns+mr)\left((n\mu + mv)\surd c - (ns+mr)\surd \upsilon\right)^2 g^2}{784V} - \frac{M(s\sigma + r\rho)uu\upsilon}{V}.$$

At vis urgens in directione *GN* ad directionem motus *GL* normali, seu vis normalis erit

$$= \frac{M(nr-ms)\left((n\mu + mv)\surd c - (ns+mr)\surd v\right)^2 g^2}{784V} - \frac{M(r\sigma - s\rho)uu\upsilon}{V}.$$

Ponatur brevitatis gratia vis tangentialis $= T$ et vis normalis $= N$; et concipiatur centrum gravitatis *G* sua celeritate $\surd \upsilon$ progredi per spatiolum $Gg = dx$, accelerabitur interea ita ut sit $d\upsilon = \frac{Tdx}{M}$. Simul vero a vi normali N cogetur viam rectilineam deserere, atque lineam Ggl convexam versus GL describere, cuius in G radius curvaturae GN erit $= \frac{2M\upsilon}{N}$; seu dum elementum $Gg = dx$ percurrit deflectet a cursu GL versus l angulo $= \frac{Ndx}{M\upsilon}$. Utraque praeterea vis, siquidem neutrius directio per centrum gravitatis G transit, convertere conabitur navem circa axem verticalem per centrum gravitatis G transeuntem, et vis venti quidem proram A versus a rotari coget, eiusque momentum ad hunc effectum erit

$$\frac{Mn\left((n\mu + m\upsilon)\surd c - (ns + mr)\surd \upsilon\right)^2 g^2 y}{784V}.$$

Vis resistentiae contra conversionem in plagam oppositam producet, proramque A versus α rotabit eiusque momentum ad hunc effectum erit

$$= \frac{M\sigma uu\upsilon z}{V};$$

ex quo conficietur momentum virium ad proram A versus a convertendam

$$= \frac{Mn\left((n\mu + m\upsilon)\surd c - (ns + mr)\surd \upsilon\right)^2 g^2 y}{784V} - = \frac{M\sigma u^2\upsilon z}{V};$$

quae quantitas si fuerit negativa, prora in plagam $A\alpha$ circa G agetur. Q. E. I.

COROLLARIUM 1

872. Si ergo ope gubernaculi impedienda sit conversio navis circa axem verticalem, tanta vis a gubernaculo exerceri debet, cuius momentum respectu eiusdem axis verticalis aequale fit et contrarium illi momento ex vi venti et resistentia orto.

COROLLARIUM 2

873. Si tam centrum virium a vento exceptarum r quam centrum resistentiae R in rectam verticalem per centrum gravitatis navis G transeuntem incidat, tum vis gyratoria prorsus evanescit, nullaque vi opus est gubernaculi ad directionem AB conservandam.

COROLLARIUM 3

874. Nisi autem tam *y* quam *z* evanescat, perspicuum est directionem *AB* sine actione gubernaculi conservari non posse. Nam etiamsi illius momentum fieri possit $=0$, tamen mutata celeritate, id cessabit esse $=0$.

COROLLARIUM 4

875. Si celeritas navis in *G* ponatur $=0$, tum resistentia evanescet, atque ventus motum navi imprimet in directione *rs:* quae cum nunquam cum directione venti *Vr* angulum acutum conficere possit, initium motus non adversus ventum institui poterit.

COROLLARIUM 5

876. Si autem navis habeat motum in directione *GL* celeritate $V\upsilon$, directio motus conservabitur, si vis normalis *N* evanescit, hoc est si fuerit

$$\left((n\mu+m\nu)\sqrt{c}-(ns+mr)\sqrt{\upsilon}\right)^2 g^2=\frac{784(r\sigma-sp)uu\upsilon}{nr-ms}.$$

Si autem illa quantitas maior fuerit quam haec, tum angulus *VLλ* fiet maior, contra vero si illa quantitas minor fuerit quam haec, angulus *VLλ* minor evadet.

COROLLARIUM 6

877. Nisi ergo talis motus navis sit producendus cuius directio *GLλ* cum directione venti *VL* constituat angulum *VLl* vel rectum vel acutum, eiusmodi motus facile producetur, efficiendo ut prima motus directio *rs* fiat in data directione, id quod fieri potest, atque tum ut vis normalis evanescat; quod fiet cursum directum instituendo.

COROLLARIUM 7

878. At si motus desideretur adversus ventum, ita ut angulus *VLA* fiat acutus id initio motus obtineri nequit. Quare post initium cursus ita dirigi debet ut sit perpetuo

$$784(r\sigma-s\rho)u^2\upsilon>(nr-ms)\left((n\mu+m\nu)\sqrt{c}-(ns+m\nu)\sqrt{\upsilon}\right)^2 g^2.$$

Hoc autem commodissime obtinebitur, si saltem circa motus initium vela *ef* teneantur normalia ad cursum praesentem *GL,* vel etiam reclinantia ut angulus *rtL* fiat obtusus. Quando vero iam aliquam celeritatem acquisiverit navis, tum vela pededentim in eum situm disponi poterunt, qui ad motum aequabilem requiritur.

COROLLARIUM 8

879. In hoc autem cursu, qui adversus ventum institui debet, probe notandum est, directionem prorae *GA* perpetuo intra directiones *VL* et *Lλ* esse conservandam, hincque cursum obliquum esse tenendum, quo angulus *RnG* fiat maximus.

COROLLARIUM 9

880. Si ponatur anguli *Vrf* sinus $=\zeta$ seu $n\mu+mv=\zeta$ atque anguli *rtL* sinus $=\eta$ et cosinus $=\vartheta$, ita ut sit $\eta=ns+mr$ et $\vartheta=nr-ms$, itemque anguli *RnG* sinus $=p=r\sigma-s\rho$ et cosinus $=q=s\sigma+r\rho$, erit vis tangentialis accelerans

$$T=\frac{M\eta(\zeta\surd c-\eta\surd \upsilon)^2 g^2}{784V}-\frac{Mqu^2\upsilon}{V}$$

et vis normalis cursum *GL* versus *l* deflectens

$$N=\frac{M\vartheta(\zeta\surd c-\eta\surd \upsilon)^2 g^2}{784V}-\frac{Mpu^2\upsilon}{V}.$$

COROLLARIUM 10

881. Quo igitur cursus navis adversus ventum quantum fieri potest dirigatur, oportet ut sit, quemadmodum iam monstravimus

$$784pu^2\upsilon>\vartheta(\zeta\surd c-\eta\surd \upsilon)^2 g^2;$$

simulque ut motus acceleretur debet esse $\eta(\zeta\surd c-\eta\surd \upsilon)^2 p^2>784qu^2\upsilon$: ex quibus multo magis esse debet $p\eta>q\vartheta$ seu ang. $RnG>rsG$. sive angulus *Rzr* debet esse acutus.

COROLLARIUM 11

882. Debebit igitur praeter angulum *Rzr* acutum quoque esse

$$\zeta\surd c-\eta\surd \upsilon>\frac{28u}{g}\sqrt{\frac{p\upsilon}{\vartheta}};$$

simulque

$$\zeta\surd c-\eta\surd \upsilon>\frac{28u}{g}\sqrt{\frac{q\upsilon}{\eta}}.$$

Utrique autem conditioni maxime satisfit si cursus obliquitas ea accipiatur, cui respondet maximus angulus *RnG,* tum enim *p* maximum, *q* vero minimum obtinet valorem.

SCHOLION

883. Pendet igitur cognitio motus, quem datus ventus datae navi dato modo directae praeter celeritatem venti, velorum quantitatem atque resistentiam praecipue a duobus angulis, primo scilicet ab angulo *Vrf* quem venti directio cum positione velorum *ef* constituit, secundo ab angulo *rtL,* quem directio velorum *ef* cum directione motus *GL* facit, quorum angulorum differentia est obliquitas cursus *AGL,* qua directio resistentiae *nRM* seu angulus *RnG* determinatur. Prouti igitur in cursu hi anguli manent constantes, vel alteruter, vel uterque variabilis existit, navis aliam viam describet aliamque in singulis locis habebit celeritatem. Maxime autem difficile erit motum definire si angulus obliquitatis cursus *AGL* fuerit variabilis, cum ab eius variatione non solum angulus *RnG* immutetur, sed etiam ipsa resistentiae quantitas absoluta quae per *uu* exprimetur, quorum utrumque difficile est assignare, quomodo mutata obliquitate cursus vel augeatur vel diminuatur. Ceterum intelligitur quantum expediat naves ita construere, ut centrum resistentiae pro quaque cursus obliquitate in rectam verticalem per centrum gravitatis transeuntem incidat, hoc enim si fuerit praestitum, atque insuper vela in malis ita disponantur, ut media directio vis, quam recipiunt in eandem verticalem cadat, non solum gubernaculi actio tantopere desiderabitur, sed etiam levi vi gubernaculum dirigetur, cum nunquam eiusmodi vis occurrat quae navem convertere conetur. Praeterea vero etiamsi accidat ut centra, tum resistentiae tum vis venti non praecise in eum locum cadant, tamen dummodo discrimen satis fuerit exguum, ope gubernaculum aliam vin sensibilem non exerit, praeter conversionem navis circa axem vertricalem per centrum gravitatis transeuntem, tamen ope gubernaculi directio motus vehementer immutatur. Dum enim ope gubernaculi cursus obliquitas mutatur, simil media directio resistentiae in aliam plagam convertitur, quo fit ut ipsa motus directio mox declinetur. Ita actione gubernaculi non solum positio vanis seu axis eius longitudinalis a puppi ad proram ducti afficitur, sed etiam ipsa directio motus in eam plagam, in quam prora dirigitur, deflectitur siquidem naves directe progredientes valde exiguam patiuntur resistentiam respectu eius quam sufferunt, si cursu obliquo feruntur. Hanc ob causam directionem venti instituendus, ope gubernaculi enim efficiendum est, ut ea cursus obliquitas conservetur, cui maximus angulus *RnG* respondeat. Deinde ut motus navis simul maxime acceleretur, atque adversus plagam ex qua ventus venit, inflectatur, vela ita sunt continuo disponenda, ut angulus *rts* tantus conservetur, quantum reliquae circumstantiae permittunt ; quo maior enim est angulus *rts* eo maior erit eius sinus η contraque eo minor eius consinus ϑ, ex quo tum acceleratio maxima obtinebitur, tum etiam maxima cursus declinatio adversus ventum. At si cursus iam angulum acutum constituat cum directione venti, tum utique angulum *rts* magis acutum esse oportebit, quoniam alioquin ventus vela non impelleret. Quo circa perpetuo effici debeat, ut angulua *Vre* tam sit acutus, quam quantitis vis venti permittit. Cum autem celeritas iam erit facta aequabilis, tum ea cursus directio instituenda estm quae supra pro motu aequabili est monstrata.

PROPOSITIO 84

PROBLEMA

884. *Si navis AEBF* (Fig. 110) *oblique moveatur in fluvio, cuius cursus sit CGD, ita ut axis navis AB cum directione fluvii CD constituat angulum obliquum AGC; determinare motus mutationem a vi fluvii ortam.*

SOLUTIO

Sit fluvii celeritas debita altitudini k atque anguli AGC quem axis seu spina navis AB cum cursu fluvii CD facit sinus $= \mu$, cosinus $= \nu$; habeat autem navis iam motum quo eius centrum gravitatis G celeritate altitudini v debita progrediatur in directione GL quae cum cursu fluminis GD constituat angulum LGD cuius sinus sit

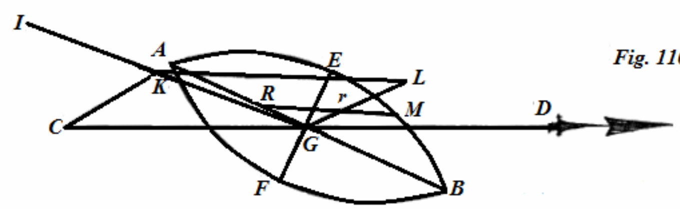


Fig. 110

$= m$ cosinus $= n$. Iam ut allisio aquae ad superficiem navis reperiatur, concipiatur totum systema ex fluvio et navi compositum celeritate $\surd v$ promoveri in directionem ipsi GL contrariam, quo fiet ut fluvius in navem quiescentem impingat. Sumta ergo $GC = \surd k$, et $GL = \surd \nu$, compleatur parallelogrammum $GCKL$, atque diagonalis GK repraesentabit tum directionem quam celeritatem, qua fluvius in navem quiescentem allidere est concipiendus. In triangulo autem GCK datur angulus C, cuius sinus $= m$ et cosinus $= n$, atque ambo latera $CK = \surd \upsilon$ et $CG = \surd k$, ex quibus oritur celeritas fluvii in directione IG incurrentis $= \surd\left(k - 2n\surd k\upsilon + \upsilon\right)$, atque anguli CGI sinus

$$= \frac{m\surd \upsilon}{\surd\left(k - 2n\surd k\upsilon + \upsilon\right)},'$$

eiusque cosinus

$$= \frac{\surd k - n\surd \upsilon}{\surd\left(k - 2n\surd k\upsilon + \upsilon\right)},$$

ex quibus angulus AGI innotescit; erit autem anguli AGI sinus

$$= \frac{\upsilon\surd k - (\mu n + \nu m)\surd \upsilon}{\surd\left(k - 2n\surd k\upsilon + \upsilon\right)},$$

eiusque cosinus

$$= \frac{\upsilon\surd k + (\mu m - \nu n)\surd \upsilon}{\surd\left(k - 2n\surd k\upsilon + \upsilon\right)}.$$

Concipiatur nunc navis in directione *GI* in aqua quiescente promoveri, sit *R* centrum resistentiae, et *Rm* media directio resistentiae, atque resistentia ipsa sit tanta, quantam pateretur figura plana *uu* eadem celeritate contra aquam mota. His positis navis ab allapsu fluvii urgebitur in directione *RM,* vi, quae aequalis est ponderi voluminis aquae $\left(k-2n\sqrt{k\upsilon}+\upsilon\right)u^2$, seu posito pondere navis $=M$ et volumine partis submersae $=V$, erit vis fluvii allidentis

$$=\frac{\left(k-2n\sqrt{k\upsilon}+\upsilon\right)u^2}{V},$$

qua vi navis secundum directionem *RM* propelletur. Haec igitur vis resoluta in binas, quarum altera in *GL* incidit, altera ad *GL* est normalis, dabit cum vim tangentialem, qua motus in directione *GL* accelerabitur, tum vim normalem, qua motus directio deflectetur a *GL* versus *GD.* Denique nisi punctum *R,* quod est centrum resistentiae respondens obliquitati cursus *AGI* in *G* incidat navis simul circa axem verticalem per centrum gravitatis *G* ductum convertetur et quidem in plagam *AEL* si *R* inter *A* et *G* fuerit situm, in plagam contrariam vero si punctum *R* intra *B* et *G* cadat; si quidem angulus *CGI* fuerit minor angulo *AGC,* nam si angulus *CGI* maior foret angulo *AGC* haec omnia contra se haberent. Q.E.I.

COROLLARIUM 1

885. Si navis vel quiescat vel motum habeat secundum directionem fluminis *GD* tum angulus *CGI* evanescet, atque *R* erit centrum resistentiae respondens obliquitati cursus *AGC.* Vis convertens igitur tendet ad obliquitatem *AGC* augendam, si *R* intra puncta *A* et *G* cadat, contra vero axem *AB* secundum cursus fluminis directionem *CD* disponet, si *R* intra *G* et *B* cadat.

COROLLARIUM 2

886. Si ergo navis initio directe secundum cursus fluminis directionem *CD* fuerit disposita, eaque aliquantillum declinetur, sponte situm pristinum recipiet, si centrum resistentiae *R* intra puncta *G* et *B* cadat. Contrario autem si *R* intra *G* et *A* fuerit situm, directio *AB* continuo magis a cursu fluminis *CD* declinabitur.

COROLLARIUM 3

887. Si igitur pars impulsui fluvii exposita *EAF* fuerit satis ponderosa, magnamque vim a fluvio patiatur, tum navis seu corpus quodcunque per fluvium recta descendere poterit, quia hoc casu punctum *G* ad *A* accedit, punctum *R* vero recedit.

COROLLARIUM 4

888. At si pars *EAF* fuerit perquam acuta et levis, ita ut centrum resistentiae *R* prope ad *A* collocetur, tum difficulter navis directe per fluvium descendet, sed a minima vi de directione *CD* depulsa, magis a situ directo elongabitur.

COROLLARIUM 5

889. Eodem vero casu, quo motus navis *GL* incidit in cursum fluvii *CD,* quoniam angulus *GRM* maior erit angulo *AGC,* navis a sua directione depelletur, atque ad ripam fluvii, in quam pro *A* declinat pelletur.

COROLLARIUM 6

890. Ponamus igitur navem iam motum accepisse versus ripam in directione *GL,* atque angulum *AGO* conservari; accelerabitur iste motus, siquidem angulus *LrM* fuerit acutus, simul vero de hac directione deflectetur, versus *GD,* siquidem angulus *LGD* maior fuerit quam *MRG.*

SCHOLION

891. Apparet igitur motum corporum a fluvio abreptorum maxime esse irregularem, etiamsi ea plano diametrali verticali sint praedita atque duas partes similes et aequales utrinque habeant. Primo enim nisi axis *AB* in ipsam fluvii directionem incidat, corpus versus eam ripam pellitur, in quam prora *A* vergit (convenit enim eam navis partem, quae cum aqua conflictatur proram appellari). Deinde variis modis corpus ab aqua circa axem verticalem per centrum gravitatis ductum circumagetur, pro diverso situ centri resistentiae *R,* quod durante rotatione continuo locum mutat, nisi pro omni obliquitate sit fixum. Tertio ob vim normalem ad motus directionem *GL* a vi fluvii ortam, ipsa motus directio afficitur eaque vel magis declinatur a cursu fluminis vel ad eum reducetur, prout angulus *MRG* vel maior fuerit vel minor *LGD.* At vero etiamsi motus versus fluvii cursum inflectatur, tamen eo recidere nequit, quia minuto angulo *LGD* eousque ut minor fieret quam *MRG,* deflexio orirétur. Denique tametsi semel directio *GL* parallela foret directioni *RM,* quo casu vis normalis evanesceret, tamen ob corporis conversionem vel a vi aquae, vel ab alia levissima causa ortam, statim alia media directio vis aquae aderit, quae motum turbabit. Interim tamen eiusmodi navis, in qua centrum resistentiae versus puppim *B* cadit directe a fluvio abripietur, si quidem cursu directo motum inceperit. Corpora autem irregularia, quae nequidem duabus partibus aequalibus et similibus gaudent motu maxime irregulari ferri oportet, ex quo mox ad ripam alterutram devolventur. Quod autem ad naves attinet, ope gubernaculi in *B* applicati motus regularis facile obtinebitur, atque cursus vel secundum flumen directe deorsum vel ad ripam institui poterit; semper autem motus descensus praevalebit, ex quo fluvius in directione, ad ipsius cursum normali traiici omnino non potest; sed navis inter traiectum eo magis deorsum abripietur quo celerior fuerit fluminis cursus. Cum vi fluvii nunc alia vis

sollicitans vel remorum vel venti coniungi posset, atque determinari, quomodo quisque intentus cursus quam commodissime sit instituendus, sed cum haec propius ad alterum librum respiciant, in quo navigationem naviumque dispositionem ex professo pertractare est constitutum, hic plura afferre de istiusmodi motibus non est visum, praesertim cum facile sit ex methodo tradita huiusmodi quaestiones, quae proponi queant, ad calculum revocare atque resolvere. Superest igitur ut paucis exponamus motum navium quae non sunt liberae, sed alicubi alligatae, quae doctrina in traiectu fluviorum sine vi aliena praecipue est utilis.

PROPOSITIO 85

PROBLEMA

892. *Si navis AD* (Fig. 111) *in fluvio secundum directionem ZV fluente, ope funis IZ puncto firmo Z ita fuerit alligata, ut perpetuo respectu funis IZ eandem positionem teneat, id quod factum concipiatur per funem BC, punctum navis B cum C connectentem; determinare motum, quem cursus fluminis isti navi imprimet.*

SOLUTIO

Ponemus ante omnia partem navis aquae submersam esse parallelepipedum, quia alioquin vires quas fluvius in diversis positionibus exerit, difficulter inter se comparari z possent. Sit itaque *ABDC* sectio aquae quae erit rectangulum, cuius latus *AB* sit $= a$, latus $AC = b$, et profunditas navis in aqua $= c$, erit superficies in *AB* vim fluvii excipiens $= ac$, et superficies sub *AC* infra aquam versans $= bc$. Sit porro fluvii celeritas debita altitudini k atque longitudo funis $ZI = f$, quae vehementer magna sit respectu quantitatum a, b, et c; sitque anguli *ZIA* sinus $= m$ et cosinus $= n$. Quod nunc ad

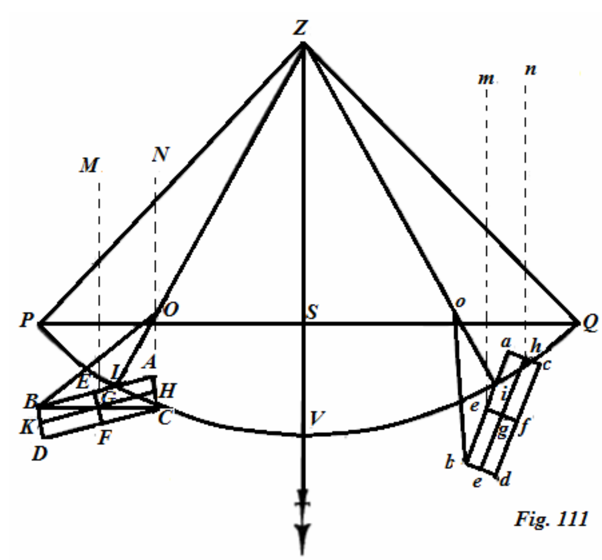


Fig. 111

motum huius navis attinet, intelligitur navem alium motum habere non posse praeter gyratorium circa punctum *Z,* quo punctum *I* in arcu circulari *PVQ,* cuius centrum est *Z,* feretur. Ponamus autem navem motum in *P* inchoasse, atque percurrendo arcum *PVQ* pervenisse in situm *ABDC,* in quo punctum *I* celeritatem habeat altitudini v debitam, qua conabitur per arcum *IVQ* progredi. Ponatur insuper anguli *IZV,* quem positio funis *IZ* cum directione fluvii *ZV* constituit, sinus $= x$ et cosinus $= y$. His praemissis consideremus vim fluvii, quam in navem exercebit. Ac primo quidem irruet in latus *AB* cuius area est $= ac$ in directione *ME* sub angulo $MEA = ZIA + IZV$, cuius anguli igitur sinus

erit $= my + nx$, cosinus vero $= ny - mx$ qui cosinus simul est sinus anguli *NHA* sub quo fluvius in latus *AC* impingit. Iam si navis quiesceret, vis fluvii sponte haberetur, at cum navis iam motum habere ponatur, reperietur vis, quam fluvius in latus $AB = ac$ exeret, in directione *EF* normali ad *AB* in eius puncto medio *E*, $= ac\left((my+nx)\surd k - n\surd v\right)^2$;
vis vero quam lateri *AC* imprimet in directione *HG* simili modo erit
$= bc\left((ny-mx)\surd k + m\surd \upsilon\right)^2$.
Posito autem pondere navis $= M$ et volumine partis submersae $= V$, expressiones inventae, si per $\frac{M}{V}$ multiplicentur, dabunt pondera his viribus aequivalentia. Hinc igitur orietur momentum harum duarum virium coniunctim ad navem circa polum *Z* circumagendam in plagam *IVQ*

$$= \frac{Macnf}{V}\left((my+nx)\surd k - n\surd \upsilon\right)^2 - \frac{Mbcnf}{V}\left((ny-mx)\surd k + m\surd \upsilon\right)^2,$$

quod divisum per momentum ipsius navis respectu poli *Z*, quod ob longitudinem f maximam est $= Mff$ dabit vim gyratoriam. Ponamus ergo navem per elementum arcus *ds* versus *V* progredi, erit $ds = -fdx$; atque interea ita motus accelerabitur ut sit

$$Vd\upsilon = \frac{-nacdx}{y}\left((my+nx)\surd k - n\surd \upsilon\right)^2 + \frac{mbcdx}{y}\left((ny-mx)\surd k + m\surd \upsilon\right)^2,$$

Ad motum autem ipsum cognoscendum sufficit advertisse conversionem in plagam *VQ* continuare quamdiu fuerit $ank(my+nx)^2 > bmk(ny-mx)^2$
seu tangens ang. $MEA > \sqrt{\frac{bm}{an}}$; qui terminus oritur si ponatur *v* = C. Nam si navis habet motum in directione *IV*, eo progredietur sive accelerato sive retardato; sin autem alicubi quiescat, tum si fluvii propelletur in eandem directionem *aP* ad *Q*. Si igitur ubique inter *P* et *Q* fuerit tang. $MEA > \sqrt{\frac{bm}{an}}$, tum navis ex *P* egressa ad *Q* usque pertinget, quae omnia abunde sufficiunt ad motum cognoscendum. Q. E. I.

COROLLARIUM 1

893. Si navis perveniat ultra *V* in situm *abdc* omnia manebunt ut ante, praeterquam quod anguli *VZi* sinus poni debeat $= -x$. Ita vis navem in *i* ulterius versus *Q* urgens erit ut

$$= ac\left((my+nx)\surd k - n\surd \upsilon\right)^2$$

COROLLARIUM 2

894. Motum ergo hunc ultra V versus Q continuabit, quoad fuerit

$$na\left(my-nx\right)^2 > mb\left(ny+mx\right)^2$$

seu tang. $mea > \sqrt{\frac{mb}{na}}$. Sit angulus cuius tangens est $\sqrt{\frac{mb}{na}}, = \alpha$, fiet
$mea > \alpha$ seu $ZIA - VZi > \alpha$.
Quo igitur navis ad Q usque pervenire queat, oportet ut angulus VZQ non maior sit quam $ZIA - \alpha$.

COROLLARIUM 3

895. Si ponamus porro motum navis circa polum Z in P inchoasse, quo eiusmodi motus per V ad Q usque continuetur necesse est ut sit $PZV + ZIA > \alpha$, denotante α angulum cuius tangens est $\sqrt{\frac{mb}{na}}$: nisi enim hoc fuerit, motus nequidem incipere poterit; hic autem omnes angulos ponimus acutos, alias tangentes accipere oportet.

COROLLARIUM 4

896. Si recta ZV per medium fluvium transeat, atque requiratur ut navis ita a ripa P sponte ad ripam Q fluvium transeundo appellat, oportebit ut sit
$\text{ang.}VZQ = \text{ang.}ZIA - \text{ang.}\alpha$ Si enim ang. VZQ minor foret, tum navis ad Q ingenti vi appelleret, quod est evitandum.

COROLLARIUM 5

897. Si ergo detur fluvii latitudo PQ, atque tam ipsa species navis, quam angulus AIZ, ex illa aequalitate inventa reperietur longitudo funis ZV ad traiectum requisita, indeque punctum Z in medio flumine accipiendum, cui navis est alliganda.

COROLLARIUM 6

898. Si ponatur semissis latitudinis fluvii $PS = QS = h$, et intervallum $Zs = z$, angulique QZV sinus x, cosinus y, erit $\frac{h}{z} = \frac{x}{y}$. Cum igitur sit

$$\sqrt{\frac{mb}{na}} = \frac{my-nx}{ny+mx},$$

erit $mz\sqrt{na} - nh\sqrt{na} = nz\sqrt{mb} + mh\sqrt{mb}$, hincque

$$ZS = z = \frac{h\left(n\sqrt{na} + m\sqrt{mb}\right)}{m\sqrt{na} - n\sqrt{mb}};$$

ac longitudo funis

$$ZV = f = \frac{h\sqrt{na+mb}}{m\sqrt{na} - n\sqrt{mb}}.$$

COROLLARIUM 7

899. Perspicuum autem est ante omnia esse debere

$$m\sqrt{na} > n\sqrt{mb}, \text{ seu } \frac{a}{b} > \frac{n}{m} \text{ sive } \frac{b}{a} < \frac{m}{n}.$$

Ducta ergo in rectangulo *ABDC* diagonali *BC* angulus *ZIA* maior esse debebit quam angulus *ABC.* Ceterum perinde est in quonam puncto rectae *AB* funis alligetur.

COROLLARIUM 8

900. Quo navis ceteris paribus celerrime a ripa *P* ad ripam *Q* pertingat, efficiendum est ut acceleratio in puncto medio *V* fiat maxima. At si navis in *V* quiesceret, foret vis eam versus *Q* pellens ut $m^2na - mn^2b$, quae quantitas fit maxima si posito $\frac{m}{n} = t$ fuerit

$$at^3 - 2bt^2 - 2at + b = 0.$$

COROLLARIUM 9

901. Quoniam autem longitudo navis *a* multum excedit latitudinem *b,* erit circiter

$$t = \frac{m}{n} = \sqrt{2} + \frac{3b}{4a}.$$

Hancobrem expediet angulum *AIZ* accipere 60° circiter, si quidem propemodum latitudo *b* fuerit triens vel quadrans longitudinis *a.* Hocque ipso multum excedet angulus *AIZ* angulum *ABC.*

COROLLARIUM 10

902. Si figura navis fuerit quadrata ut sit $b = a$, fiet $t = \frac{m}{n} = \frac{3+\sqrt{5}}{2}$

seu angulus *AIZ* erit $69°\ 6'$; unde fit

$m = \frac{1+\sqrt{5}}{2\sqrt{3}}$ et $n = \frac{\sqrt{5}-1}{2\sqrt{3}}$.

Hincque porro prodit longitudo funis $= f = h\sqrt{\left(\frac{15}{2}+3\sqrt{5}\right)} = 3,8h,$

angulusque *VZQ* fit $10°\ 48'$.

COROLLARIUM 11

903. Si angulus *AIZ* seu eius tangens *t* detur, reperietur commodissima ratio longitudinis navis *a* ad latitudinem *b,* ex hac aequatione

$$\frac{b}{a} = \frac{t^3-2t}{2tt-1},$$

ex qua intelligitur necessario esse debere $tt > 2$ seu angulum *AIZ* maiorem quam $54°,\ 45',$ fit autem sponte $t > \frac{b}{a}$ uti requiritur. Erit vero

$$f = \frac{h\sqrt{(1+tt)}\sqrt{(tt-1)}}{t\sqrt{(2tt-1)} - t\sqrt{(tt-2)}}$$

COROLLARIUM 12

904. Si constituatur angulus *AIZ* sexaginta graduum seu $t = \sqrt{3}$, satis commoda figura navis prodibit, orietur enim $\frac{b}{a} = \frac{\sqrt{3}}{5}$ seu *b* ad *a* ut 53 ad 153 proxime. Deinde longitudo funis *ZV* reperietur

$$= f = \frac{4h\sqrt{2}}{\sqrt{15}-\sqrt{3}} = \frac{h\left(\sqrt{5}+1\right)\sqrt{2}}{\sqrt{3}}$$

seu proxime $f = 2,64222h$ sive $\frac{f}{h} = \frac{37}{14}$. Angulus vero *PZQ* erit $44°,\ 28'$.

SCHOLION

905. Modus hic fluvium sine remis et velis traiiciendi admodum est commodus et ad usum accommodatus, cum facile sit eas conditiones adimplere,quibus traiectio non solum possibilis, sed etiam celerrima reddatur. Imprimis autem commendandus est casus in ultimo corollario erutus, pro quo funis non adeo longus requiritur atque proportio inter longitudinem navis $AB = a$ et latitudinem $AC = b$, valde commode reperta est; fere scilicet longitudo tripla prodiit latitudinis. Facile autem perspicitur cum traiectus a ripa *P* usque ad ripam *Q* fuerit peractus, quomodo vicissim a *Q* ad *P* cursus sit instituendus;

funes scilicet *ZI* et *OB* in altero latere *CD* affigendi. Verum quo haec transmutatio facilius fieri queat, expediet punctum *I* in ipso puncto *H* assumere quo obtinebitur, ut in reditu non opus sit hunc funem deligari alioque loco firmare; perinde enim est quo loco *I* capiatur. Deinde notandum est funem *BC* tam longum esse accipiendum, ut angulus quem directio funis *ZI* cum *KH* producta constituet sit 60 graduum. Punctum *O* autem neque nimis propinquum puncto *I* neque nimis ab eo remotum accipi debet, ne minima funis *CB* elongatione angulus *AIZ* notabiliter diminuatur; minime autem angulus *AIZ* immutabitur si capiatur $OI = IB$. Denique etiamsi elongatione funis *BO* angulus *AIZ* quantitatem assignatam ammittat, tamen statim remedium afferri poterit de quo non opus est plura monere. Progredior itaque ad motum navium in fluvio determinandum, quae quidem ut ante circa punctum fixum sunt mobiles, sed positionem suam respectu funis non tenent constantem; scilicet mobiles eas ponam circa ambo puncta *Z* et *I* libere.

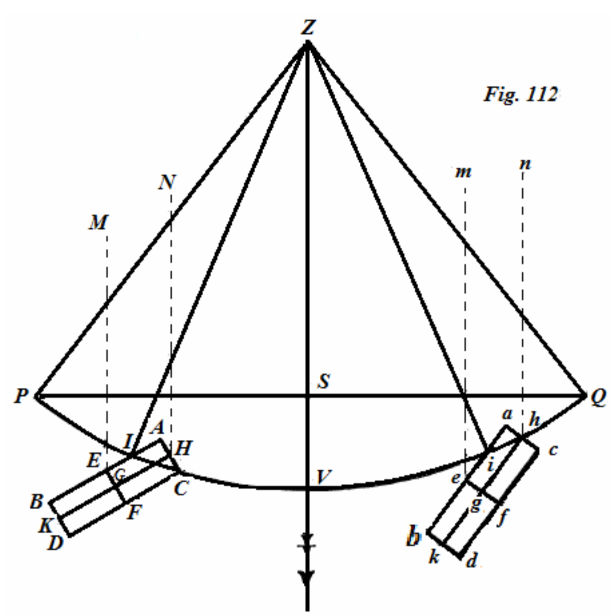
PROPOSITIO 86

PROBLEMA

906. *Si navis ABDC* (Fig. 112) *in fluvio ope funis IZ ita puncto fixo Z sit alligata, ut non solum circa punctum Z sed etiam circa punctum I in quo funis navi est annexus, sit mobilis; determinare motum huius navis ab allisione fluvii oriundum.*

SOLUTIO

Repraesentet recta *ZV* directionem fluvii, sitque celeritas fluvii debita altitudini *k;* navi vero iterum tribuamus figuram ut ante cuius sectio quaevis horizontalis sit parallelogrammum rectangulum *ABDC.* Sitque longitudo $AB = a$, latitudo $AC = b$, et profunditas sub aqua $= c$. Punctum *I* vero, in quo funis navi est affixus sit in latere *AB,* ponaturque $EI = i$. Versetur nunc navis in situ *ABCD,* quo anguli *AIZ* sinus sit $= m$ cosinus $= n$, hicque motum habeat progrediendi per arcum *IVQ* centro *Z* descriptum, cuius radius



seu longitudo funis *IZ* sit $=f$; angulique *IZV,*quem directio funis cum cursu fluminis constituit sinus sit $=x$, cosinusque $=y$, ac celeritas puncti *I* debita sit altitudini *v.* His praemissis erit ex solutione praecedentis problema *tis* vis quam fluvius lateri *AB* in directione *EF* imprimit $=ac\left((my+nx)\sqrt{k}-n\sqrt{v}\right)^2$, vis vero, quam latus *AC* in directione *HK* excipiet, erit $=bc\left((ny-mx)\sqrt{k}+m\sqrt{v}\right)^2$. Nunc cum navis sit mobilis circa punctum *I* navis eiusmodi situm acceperit necesse est, quo momenta virium respectu puncti *I* sese destruant. Est vero momentum prioris vis respectu puncti $I=\ aci\left(\left(my+nx\right)\sqrt{k}-n\sqrt{v}\right)^2$, et momentum alterius contrarie agens $=\dfrac{bbc}{2}-\left((ny-mx)\sqrt{k}+m\sqrt{v}\right)^2$.

Quamobrem habebitur ista aequatio

$$\frac{\left(my+nx\right)\sqrt{k}-n\sqrt{v}}{(ny-mx)\sqrt{k}+m\sqrt{v}}=\frac{b}{\sqrt{2ai}},$$

ex qua aequatione valor ipsarum m et *n* indeque angulus *AIZ,* ad quem navis sponte se composuit. Momentum porro harum virium ad navem circa polum *Z* convertendam per arcum *IVQ* erit ut ante

$$\frac{Macfn}{V}=\left((my+nx)\sqrt{k}-n\sqrt{v}\right)^2-\frac{Mbcfm}{V}\left((ny-mx)\sqrt{k}+m\sqrt{v}\right)^2$$

posito pondere navis $=M$ et volumine partis submersae $=V$. Per priorem vero conditionem haec formula simplicior evadit, proditque ob $V=abc$ momentum conversionis circa *Z*
$=\dfrac{MF}{bb}\left(bn-2im\right)\ \left(\left(my+nx\right)\sqrt{k}-n\sqrt{\upsilon}\right)^2$;

quamdiu igitur erit $bn>2im$ conversio secundum *IVQ* accelerabitur. At prior aequatio evoluta dat

$$m=\frac{by\sqrt{k}-x\sqrt{2aik}+\sqrt{2ai\upsilon}}{\sqrt{(b^2+2ai)(k-2x\sqrt{kv}+\upsilon)}}$$

atque

$$n=\frac{\sqrt{y}\sqrt{2aik}+bx\sqrt{k}-b\sqrt{v}}{\sqrt{\left(b^2+2ai)(k-2x\sqrt{kv}+\upsilon\right)}}.$$

Ex his fit

$$\left((my+nx)\sqrt{k}-n\sqrt{\upsilon}\right)^2=\frac{b\sqrt{\left(k-2x\sqrt{k\upsilon}+\upsilon\right)}}{b^2+ai};$$

atque

$$bn-2im=\frac{\left(b^2+2i\sqrt{2ai}\right)\left(x\sqrt{k}-\sqrt{v}\right)+by\left(\sqrt{2ai}-2i\right)\sqrt{k}}{\sqrt{(b^2+2ai)(k-2x\sqrt{kv}+\upsilon)}}.$$

Navis igitur tamdiu in arcu *IVQ* progredi perget quoad fuerit

$$b^2x+2ix\sqrt{2ai}+by\sqrt{2ai}-2biy>0,$$

id quod per se ita se habet, antequam navis ad *V* pertingit. At cum ultra medium fluvii erit progressa, perveneritque in situm *abdc,* anguli *VZi* sinus erit negativus. Quare si ponatur etiam anguli *VZ i* sinus $=x$ et cosinus $=y$, navis in arcu *VQ* eousque progredietur, quoad fuerit

$$by\sqrt{2ai}-2biy>b^2x+2ix\sqrt{2ai}.$$

Ultimus igitur terminus erit punctum *Q,* existente anguli *QzV* tangente

$$=\frac{b\sqrt{2ai}-2bi}{b^2+2i\sqrt{2ai}},$$

sive angulus *QZV* erit excessus duorum angulorum, quorum maioris tangens est $\frac{b}{2i}$ minoris vero tangens $=\frac{b}{\sqrt{2ai}}$; ; quae ad motum cognoscendum sufficiunt.

Q. E. I.

COROLLARIUM 1

907. Si punctum *I* in ipso puncto *A* capiatur ita ut sit $i=\frac{a}{2}$, navis e puncto *P* egressa non ultra punctum *V* pertinget; fiet enim hoc casu anguli *QZV* tangens $=0$.

COROLLARIUM 2

908. Simili modo navis non ultra punctum *V* progredietur, si fuerit $i=0$, seu si punctum *I* in quo funis alligatur in puncto medio lateris *AB* capiatur: ex quo manifestum est dari punctum *I* inter *A* et *E* quo navis funi alligata ultra *V* maxime progrediatur.

COROLLARIUM 3

909. Si autem quaeratur locus puncti *I,* quo angulus *VZQ* fiat maximus, reperitur ista aequatio $\left(bb+4ii\right)\sqrt{a}=\left(\left(2bb+4ai\right)\sqrt{2i}\right)\sqrt{2i}$ seu haec
$l6ai^4-32a^2i^3-24ab^2i^2-8b^4i+ab^4=0$ quae dat

$$b^2=\frac{12ai^2\ \pm\ 4i(2i-a)\sqrt{2ai}}{a-8i};$$

debet igitur esse $a>8\text{i}$ seu $EI<\frac{1}{8}AB$.

COROLLARIUM 4

910. Si sumatur $a=\text{l}8i$ fiet $b^2=\dfrac{648}{10}i^2$ seu $b=\dfrac{18i}{\sqrt{5}}=8,05i,$ [corrected: $b^2=60i^2$ so that the following is incorrect;] unde fit $a:b=\sqrt{5}:1,$ et $EI=\frac{1}{9}AE,$ hincque anguli *VIQ* prodit tangens $=\dfrac{3\sqrt{5}}{16}$, seu angulus *VZQ* erit $22°, 45'$: unde ex data fluvii latitudine longitudo funis determinabitur.

COROLLARIUM 5

911. Quo minor accipitur *EI* pars ipsius *AB* eo minor prodit latitudo navis *AC;* atque si punctum *I* prorsus in *E* incidat, tum latitudo *Aa* omnino evanescet. Omnia igitur puncta *I* quibus navis ultra *V* progredi poterit, continentur inter *E* et punctum quoddam intra *E* et *A* situm cuius distantia ab *E* est octava pars longitudinis *AB.*

SCHOLION

912. Hoc igitur etiam modo sine remis et velis traiectus per fluvium institui poterit; at ingentem cautionem adhiberi oportet tam in figura navis idonea eligenda quam in punctis *Z* et *I* inveniendis, in quibus funis *ZI* est firmandus. Ponamus enim eiusmodi navem eligi cuius longitudo *AB* se habeat ad latitudinem ut $\sqrt{5}$ ad 1, punctum *I* in quo funis alligari debet, ita assumendum erit ut distet a puncto medio *E* longitudinis *AB* intervallo $AI=\frac{1}{18}AB.$ Deinde si ponatur latitudo fluvii $PQ=2h$ haec latitudo bisecari debet in *S* ut sit $PS=QS=h;$ anchoraque in *Z* firmanda est, ut *ZS* sit perpendicularis ad *PQ,* atque intervallum *ZS* tantum accipi oportet ut angulus *SZP* seu *BZQ* fiat $22°,\ 45'$. Erit igitur *ZS* $=h\cdot 2,3847=\frac{31}{13}h$ proxime; atque longitudo funis $ZP=h\cdot 2,5859=\frac{75}{29}h$ proxime. Absoluto autem transitu per fluvium ex *P* in *Q* contra ex *Q* in *P* pervenietur, alligando funem in lateris *CD* puncto e regione sito inter *C* et *F* quod ab *F* distet intervallo $=\frac{1}{18}CD$. Hicque modus praxi videtur esse convenientissimus.