CHAPTER FOUR

THE EFFECTS

OF FORCES ACTING ON FLOATING BODIES

PROPOSITION 40

THEOREM

400. *If in some ship or vessel AB, the weight of which shall be = M, the load P, of which the weight shall be* $=m$*, shall be moved through the distance Pp to p, the centre of gravity G of the whole vessel will be transferred along the direction Gg to g, parallel to Pp, so that there shall become*

$$Gg=\frac{m\cdot Pp}{M}\text{ (Fig. 66).}$$

DEMONSTRATIO

Z shall be the centre of gravity of the ship or with the cargo of the vessel taken *P*, the points *Z, G* and *P* shall be placed in a right line, thus so that there shall become $ZG:PG=m:M-m$ or $ZG:ZP=m:M$.

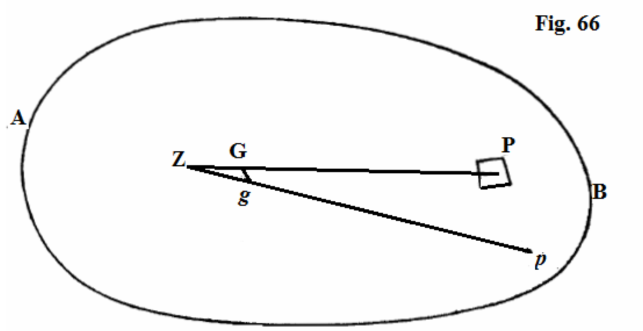

Fig. 66

Now with the cargo *m* moved from *P* to *p*, the whole body which I will consider composed from the two parts $M-m$ and *m*, the centre of gravity of the one part $M-m$ will be had as before at Z, truly the centre of gravity of the other part *m* now will be at *p*. On account of which the centre of gravity of the whole body *M* now will be found at the point *g* on the right line *Zp*, thus so that there shall become $Zg:\ pg=m:\ M-m$ or $Zg:Zp=m:M$; from which it is seen the right line *Gg* to become parallel to the right line *Pp*, and the triangles *ZGg* and *ZPp* to be similar to each other. Therefore there will become :

$$Gg:Pp=ZG:ZP=m:M\ ,$$

from which there becomes

$$Gg=\frac{m\cdot Pp}{M}\ .\qquad \text{Q.E.D.}$$

COROLLARY 1

401. Because in the movement of the cargo of the ship, or of a vessel of any kind, the weight of the displaced water is not changed before and after the translation of the cargo, it will be immersed in an equal volume of water.

COROLLARY 2

402. Therefore the body will retain the same situation in the water if the cargo may be moved in a certain way, if the centre of gravity may be moved upwards or downwards; which happens if the cargo may be moved vertically upwards or downwards.

COROLLARY 3

403. Moreover it is agreed from the preceding chapter, with the centre of gravity of the body moved upwards the stability of the state of equilibrium to be diminished ; the same truly to be increased with the centre of gravity moved downwards.

COROLLARY 4

404. Therefore if the cargo m shall be transferred vertically either upwards or downwards through the distance s, the centre of gravity will ascend or descend through the distance $\frac{ms}{M}$, and therefore the stability will be increased or decreased by the amount ms.

COROLLARY 5

405. But if the cargo may be advanced in some manner either horizontally or obliquely, then its state of equilibrium will not be conserved, but the body will be inclined from that state ; because by this same motion of the cargo, the centre of gravity of the body will be displaced from the vertical line drawn through the centre of magnitude of the submerged part.
[*i.e.* in modern terms, the centre of gravity has changed relative to the upthrust through the unchanged centre of buoyancy.]

COROLLARY 6

406. Hence also the change in the position of the centre of gravity will be deduced easily, if several loads may be transported to other locations. Indeed for that there is a need for the motion of each of the loads to be themselves to be considered.

SCHOLIUM

407. In this chapter before all else it is agreed to inquire, how much the state of equilibrium of a body floating in water may be changed, while only the centre of gravity may be moved from its location; indeed in the first place it will be agreed to enquire into the effect of these external forces, it will be agreed to have these changes set out, in which are able to arise in ships themselves with no alien external forces acceding ; even if changes of this kind may be unable to happen without outside forces. On this account I will consider first the centre of gravity to be moved from its location, with the whole weight of the body remaining the same, and I shall scrutinize what kind of change may arise in the state of equilibrium; then truly not only shall I inspect the change of the centre of gravity, but also I may put the weight of the body to have increased or decreased, which shall become either from new loads imposed or from these which were present being removed. Indeed from these cases not only will the ship be inclined, but also either will be more immersed in the water, or will be raised up from the water. On account of which with a change of this kind made, not only is it required to be defined, whether or not the state of the body shall be going to be changed, but also after the change how great shall the stability be going to become. But I shall consider only these minimal changes, since the reason for the calculation needs to be raised, since then nevertheless from that a judgment can be formed concerned with major changes ; since greater changes are able to be estimated from minimal changes successively run together.

PROPOSITION 41

PROBLEM

408. *If the centre of gravity of a ship or of some vessel floating in water shall have its centre of gravity moved a small amount from its position by the shifting of some load, to find the inclination of the vessel from its initial equilibrium state, and the stability, which it will then have acquired.*

SOLUTION

When the centre of gravity either rises or falls directly, the state of equilibrium suffers no change, except that its stability may either be diminished or increased. But if the centre of gravity may be moved obliquely, then this same motion will be able to be resolved into vertical and horizontal components, of which that state is not affected, but here generally depends on the inclination for the vessel from its first state. On account of which the vertical motion shall have no difficulty, again so that *AB* shall be the section of the body floating on the water *AFB* (Fig. 67) and *O* the centre of the magnitude of the submerged part, *G* the centre of gravity to be transferred horizontally through *Gg* at *g*; with which done the vessel will be inclined normal to the plane *OGg* about the axis, so that the section of the water shall become *ab* constituting the *ACa* with the first section, which is the angle of inclination which we sought; the sine of which shall be $= w$ on

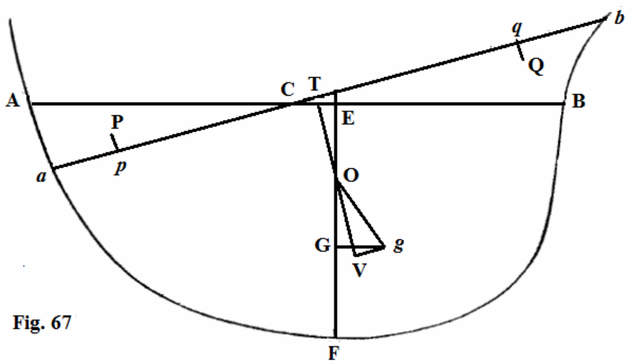

Fig. 67

putting the whole sine $=1$. Therefore with the section of the water *ab* and the centre of gravity *g*, equilibrium will be present, from which condition *w*, and thence the angle of inclination may be defined. But in place of the whole body, I will consider only the plane vertical figure AFB, since from these, which are represented from the plane figure, the conclusion shall be able to be formed easily from these bodies, if the relationship between the formulas showing the stability for plane figures and solids may be examined more attentively.

Therefore the triangle $ACa = BCb$, and since each shall be the minimum, C shall be the at the mid point of the line *AB*. Now since the submerged part shall be

$$aFb = AFB - ACa + BCb,$$

the moments of the water forces corresponding to these parts with the centre of gravity *g*, must cancel each other out. *TOV* is drawn through *O* normal to the section of the water *ab*, on which equally *gV* will fall normally from g . From these premises the moment of the part *AFB* with respect of g will be $= AFB \cdot gV$, because *O* if the centre of the magnitude *AFB*. Then the area of the triangle AC*a* is

$$= \frac{AC \cdot aC \cdot w}{2} = \frac{w \cdot AC^2}{2};$$

and its centre of gravity will be at *P* so that there shall be $Cp = \frac{2}{3}AC$. Therefore the moment hence arising is

$$= \frac{w \cdot AC^2}{2}\left(\frac{2}{3}AC + CT + gV\right);$$

but on account of part of the moment arising from the area *BCb* being negative, and

$$= \frac{w \cdot AC^2}{2}\left(\frac{2}{3}AC - CT - gV\right)$$

on putting *AC* for *BC*. Therefore the total moment arising from the area *aFb* will become:

$$AFB \cdot gV - \frac{2w \cdot AC^3}{3},$$

which there must be equilibrium present, will be $=0$. Now this may be transferred to the bodies, and in the section of the water body the right line shall be drawn through its centre of gravity *O* normal to the plane *OGg*, which will be the axis about which the body will be inclined ; and to this axis in the water section the sum of the cubes of the orthogonal applied lines are added together, or $\int\left(y^3+z^3\right)dx$ from prop. 29. which quantity I shall call *Q*, must be substituted in place of $2AC^3$. But in place of the area *AFB* it will be required to write the volume of the submerged part, which shall be $=V$. From these for the body or some vessel, of which the centre of gravity *G* is transferred horizontally to *g*, this equation $3gV\cdot V=wQ$ will be had, to be defined for the inclination thence arising. Now truly on account of the minimum angles to *O* and *g*, $gV=Gg-w\cdot GO$; which since there shall be $3V\cdot Gg=3wV\cdot GO+wQ$, the sine of the angle, which the vessel will be inclined about the horizontal axis normal to the plane *OGg*, evidently:

$$w=\frac{3V\cdot Gg}{3V\cdot GO+Q}.$$

But for the stability requiring to be found, only the plane figure to be considered, the distance of the centre of gravity of the area *aFb* from the right horizontal line *gV* is to be investigated, which is found

$$=OV-\frac{AC^2\cdot w^2}{3AFB} \text{ on account of } Pp=Qq=\frac{1}{3}w\cdot AC$$

and likewise the interval for the solid body will be consequently

$$=OV-\frac{w^2Q}{6V}.$$

But there is

$$OV=OG+w\cdot Gg-\frac{w^2\cdot OG}{2};$$

from which the distance of the centre of magnitude of the submerged part and of the centre of gravity after the inclination will become

$$=OG+w\cdot Gg-\frac{w^2\cdot OG}{2}-\frac{w^2\cdot Q}{6V}=OG+w\cdot Gg-\frac{w^2\cdot F}{2M},$$

with *M* denoting the weight of the body, and *F* the stability of the same before the inclination; for there becomes

$$F=M\left(GO+\frac{Q}{3V}\right).$$

Whereby after the inclination the stability [*i.e.* the restoration torque; initially called the *firmness*] will become

$$= M\left(OG + w\cdot Gg - \frac{w^2\cdot F}{2M} - \frac{Q}{3V}\right) = F + wM\cdot Gg - \frac{w^2\cdot F}{2},$$

clearly with respect of that axis about which the inclination has been made. Q. E. I.

COROLLARY 1

409. Since, before the centre of gravity is displaced from its location, the stability of the equilibrium state with respect of the horizontal axis normal to the plane *OGy* shall be

$$= M\left(GO + \frac{Q}{3V}\right);$$

if this stability may be called $= F$, the sine of the angle of inclination which is produced from the translation of centre of gravity *G* to *g*,

$$w = \frac{M\cdot Gg}{F};$$

with the whole sine $= 1$.

COROLLARY 2

410. Therefore the sine of the angle, by which the vessel is inclined about the horizontal axis normal to the plane *OGg*, while the centre of gravity *G* may be moved through the horizontal distance *Gg*, is directly as this distance *Gg* and the weight of the vessel taken together, and inversely as the stability of the vessel with respect of the same axis.

COROLLARY 3

411. Therefore, where the stability of a body is greater, there also it resists that inclination more, which arises from the translation of the load from one place to another; for which reason the maximum stability is to be acquired.

COROLLARY 4

412. Since the stability found after the inclination made is

$$= F + wM\cdot Gg - \frac{w^2 F}{2},$$

and

$$w = \frac{M\cdot Gg}{F};$$

that stability will become $= F + \dfrac{M^2\cdot Gg^2}{2F}$.

Therefore here the increment in the stability, which arises on account of the squared dimensions of the infinitely small *Gg,* is to be ignored entirely.

COROLLARY 5

413. Therefore when the centre of gravity may be moved straight up or down, the inclination shall be zero but only the stability may be changed; on the other hand, truly, when the centre of gravity may be moved horizontally, the stability is not affected, but the state of the equilibrium may be changed by the inclination alone.

COROLLARY 6

414. Therefore when the centre of gravity may be moved obliquely, then both the stability, as well as the state of the equilibrium of the body in the water, will be changed. But when a change in both places may occur, it can be understood well enough from the preceding propositions.

SCHOLIUM 1

415. So that these become known more easily for ships, we may put these in place in which the stability may be able to be adapted with respect of the two horizontal axes only, the one of the length extended from prow to stern, the other of the width normal to that; the motion of the centre of gravity, unless it may be made either along the length or width, must be resolved into two sides, the one made in the length and the other in the width, which will be required to be considered separately. Indeed that translation of the centre of gravity along the axis of the longitude will generate an inclination about the axis of the latitude or width, the sine of which will be composed from the weight of the ship in the path of the centre of gravity along the length, divided by the stability, with respect to the axis of the latitude. Truly the path of the centre of gravity along the axis of the latitude multiplied by the weight of the ship, and divided by the stability with respect of the axis of longitude expresses the sine of the angle of the inclination, by which the ship shall inclined about the axis of the longitude. Therefore these two inclinations together will provide the inclination of the ship arising from the translation of the centre of gravity arising from some horizontal area. But if the centre of gravity likewise shall either rise or fall, as before the decrease or increase of the stability is required to be investigated, just as the inclination may be investigated. Indeed the stability, on which the inclination depends, is not required initially to be introduced into the calculation, but there, which on account of the ascent or descent, now the centre of gravity is either diminished or increased.

SCHOLIUM 2

416. The solution of these problems, although it may consider some bodies floating on water, is made much easier from a consideration made free of bodies, so that expressions arise free it is possible to adapt to the nature of extended bodies. Moreover this same translation follows from the comparison of the formulas of plane figures to bodies, which we have found in the preceding chapter for the stability both of plane figures as well as of some bodies. Indeed since for the plane figure the stability shall be

$$= M\left(GO + \frac{2AC^3}{3AFB}\right),$$

moreover, for the body that shall be found

$$= M\left(GO + \frac{Q}{3V}\right),$$

where Q specifies the sum of the cubes of all the applied lines in the water section for the axis passing through its centre of gravity and parallel to the axis of the inclined normal; V truly shows the volume of the submerged part. Therefore as often as expressions of this kind arise, the translation from the plane figure to the solid will be made, if in place of the area AFB present under the water, the volume V of the parts of the body submerged may be written, and for $2AC^3$ there may be put Q, or, $\frac{1}{2}Q$ for AC^3. Therefore since problems of this kind may be resolved much easier by considering plane figures only, solutions of the same problem benefit from the comparisons of the same problem without trouble, and likewise whatever bodies will be able to be reduced; of which use is to be made in this same problem, and thus it will succeed in the following and in many others.

PROPOSITIO 42

PROBLEMA

417. *If a new load may be imposed on a vessel or ship of some kind floating on water, to find both its state of equilibrium as well as the change in the stability, which will arise from this new load.*

SOLUTION

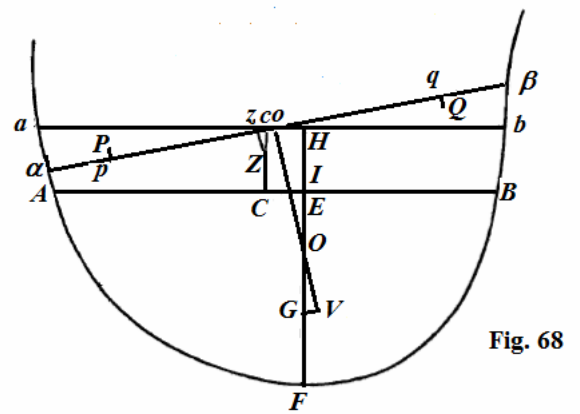

Fig. 68

AB shall be the section of the water, and *AFB* the part of the body submerged in water, of which the centre of the magnitude shall be at O, truly the centre of gravity of the whole body shall be at *G* (Fig. 68). Now with the whole weight of the body or ship put $= M$, some weight *m* may be added to that at some place. It is required therefore to investigate the change arising from this new load imposed, in the first place that sent to the centre of gravity *G* may be considered. Now since the weight of the body shall increase, a greater part of the body will be immersed, than before. Therefore the centre of gravity may sink straight downwards, so that now *ab* shall become the section of the water and *aFb* just as great a part of the body, as the weight $M + m$ shall require. The area of the water section shall be $= E$, and the volume of the submerged part *AFB* shall be $= V$, the volume of the part *AabB* newly immersed $= E \cdot HI$, if indeed I may put the load *m* used to be exceedingly small with respect to *M*, so that it shall not be necessary to be considering the inequality between *AB* and *ab*. Therefore there will become:

$$M : m = V : E \cdot HI \text{, from which there becomes } HI = \frac{mV}{ME}.$$

But in whatever way the part of the body underwater now may be turned, yet this same state will not be of equilibrium, unless the centre of the magnitude of this submerged part even now may be present on the right line *FH*. But if the centre of the magnitude of the submerged part even now may remain as the right line *HF*, if the centre of the portion of the submerged part *AabB* may fall on the same, since that happens when the right line *GO* likewise may pass through the centre of gravity; therefore in this case it shall be agreed to search, since *aFb* shall be going to become the state of equilibrium. But we may cannot put the centre of gravity of the water section to fall at *I,* but it must be present at some other point *O*, and the centre of the magnitude *AabB* will fall at the point *Z* at the midpoint of the line $Cc = HI$; therefore in this case the position of the place of equilibrium *aFb* will not be by a predicted property, but the body will be inclined about the axis normal to the plane *CIG*, thus so that the section of the water shall be going to become $=\alpha\beta$ with the angle being established from the former *acα* of which the sine shall be $=w$; the angle which we will investigate for the body as before, we will seek in the plane figure *AFB*. Therefore since now $\alpha\beta$ shall be horizontal to that, through *O* the vertical *Oo* may be drawn, which shall cross the horizontal *GV* drawn from G at V; and since it is placed to be in this state of equilibrium, I will consider the submerged part $\alpha F\beta$, which will be agreed to be composed from these parts

$$AFB + AabB - ac\alpha + bc\beta\ ,$$

of which the moments of the parts with respect to *G* mutually cancel each other out. But the moment of the area $AFB = AFB\cdot GV$, truly the area *AabB*

$$= AabB\left(GV-\alpha o+\alpha z\right) = AabB(GV-co-cz).$$

But the moment of the triangle *acα,* the area of which is $=\frac{1}{2}w\alpha c\cdot ac=\frac{1}{2}w\cdot AC^2$, with its centre of gravity put at *P,*
so that there shall become $cp=\frac{2}{3}c\alpha=\frac{2}{3}CA$,
will become

$$\tfrac{1}{2}w\cdot AC^2\left(GV-\tfrac{2}{3}AC-co\right)$$

Moreover, in a similar manner, the moment of the triangle *bcβ* will become

$$\tfrac{1}{2}w\cdot AC^2\left(GV+\tfrac{2}{3}AC-co\right).$$

Therefore since the part submerged in the water shall be

$$= AFB + AabB - ac\alpha + bc\beta,$$

the moment of the whole submerged part will become

$$= AFB\cdot GV + AabB\left(GV-co-cz\right)+\tfrac{2}{3}w\cdot AC^3 = 0.$$

Now this formula may be transferred to the whole body, by putting the volume of the submerged part V in place of *AFB*, the volume $\frac{mV}{M}$ in place of *AabB*, and the sum of all the cubes of the normal applied lines in the section of the water drawn through the centre of gravity in place of $2AC^3$, which shall be normal to the plane *CIG*; truly this sum, which in problem 29 was $\int\left(y^3+z^3\right)dx$ here for the sake of brevity will be called *Q*. On account of which we will have this equation for the body:

$$V\cdot GV+\frac{mV}{M}\left(GV-co-cz\right)+\tfrac{1}{3}wQ=0;$$

which since there shall be

$$GV = w \cdot GO,\ co = CI - w \cdot HO = CI - w \cdot OI - \frac{wmV}{ME}$$

and

$$cz = w \cdot cZ = \frac{wmV}{2ME},$$

will be transformed into this:

$$wV \cdot GO + \frac{wmV \cdot GO}{M} - \frac{mV \cdot CI}{M} + \frac{wmV \cdot OI}{M} + \frac{wm^2V^2}{2M^2E} + \tfrac{1}{3}wQ = 0,$$

from which there is elicited

$$w = \frac{mV \cdot CI}{MV \cdot GO + \tfrac{1}{3}MQ + mV \cdot GI + \frac{m^2V^2}{2ME}}.$$

If now the stability with respect of the same axis about which the inclination is made, which is

$$= MV \cdot GO + \frac{MQ}{3V}$$

may be called F, there will become:

$$w = \frac{m \cdot CI}{F + m \cdot GI + \frac{m^2V}{2ME}}$$

for which equation on account of the weight *m* being very small with respect of *M*, this equation will be allowed to be used without risk:

$$w = \frac{m \cdot CI}{F + m \cdot GI}.$$

Moreover the stability of this equilibrium state does not differ from that, which shall agree with that in the situation *aFb* ; which on account of the centre of the magnitude raised above *O* by the interval

$$\frac{m}{M+m}\left(OI + \frac{mV}{2ME} \right)$$

will become

$$=(M+m)\left(GO+\frac{m}{M+m}\left(OI+\frac{mV}{2ME}\right)+\frac{MQ}{3(M+m)V}\right)$$
$$=M\cdot GO+m\cdot GI+\frac{m^2V}{2ME}+\frac{MQ}{3V}.$$

Therefore since before the increase of the load m the stability will be

$$F=M\cdot GO+m\cdot GI+\frac{MQ}{3V}.$$

now the stability will be

$$=F+m\cdot GI+\frac{m^2V}{2ME},$$

for which expression it will be allowed equally to use this one $F+m\cdot GI$. But since here the load m is not sent down with the centre of gravity G itself we have considered, now the load m actually may be been removed and put at that place where it now is, and since it is understood from the preceding proposition, new changes may eventuate. Q. E. I.

COROLLARY 1

418. Since

$$F+m\cdot GI+\frac{m^2V}{2ME}$$

shall be the stable state of the new equilibrium, which the ship arrives at with the load m placed on the centre of gravity, the sine of the angle of inclination will be equal with the load m made to be imposed directly on CI and divided by this new stability.

COROLLARY 2

419. The same inclination is produced if the load m may be located anywhere on the right vertical FH, since the motion of the load upwards or downwards does not disturb the equilibrium; but a larger or smaller equilibrium may arise.

COROLLARY 3

420. If the load m may be imposed at the point K of the vertical line FI , that stability which may arise, if the centre of gravity G were allowed to change, must be diminished by the factor $m\cdot GK$. Therefore in this case the stability will become

$$= F + m \cdot IK + \frac{m^2 V}{2ME}.$$

COROLLARY 4

421. Therefore with the term $\frac{m^2 V}{2ME}$ rejected, it is observed the stability to be increased with the imposition of the new load, if the load may be put in place below the water section. On the other hand the stability will be diminished, if the load may be put above the water line.

COROLLARY 5

422. Therefore when the centre of gravity of the water section *C* lies on the vertical *FGI*, then nothing will happen to the inclination, from the imposed load, provided the centre of gravity of the load also shall lie on the right line *FGI*.

COROLLARY 6

423. But if the load m is not placed on the right line *FI* but beyond that, and the point *C* shall not lie on *I*, a twofold inclination will come about, the one evidently defined here and the other depending on the interval *CI*, truly the other arises from the preceding proposition and depending on the distance of the load from the right line *FI*.

SCHOLIUM 1

424. Therefore, from this proposition, it is understood more clearly how many ships of this kind, of which the centre of gravity of the water section, the centre of the magnitude of the submerged parts, and in which likewise the centre of gravity of the whole ship, should fall on the same vertical line, to excel over those ships which lack this property. For indeed with ships of this kind, with new loads, if the centres of gravity of these either may be placed freely, or they may be arranged by passing in vertical right lines through the centre of gravity, no inclination will arise, but the ship will become immersed more deeply by descending in the water. And also if the new load may not be drawn acting through the centre of gravity in a vertical line, only a single inclination will arise about some horizontal axis, since with other ships, in which this property is not to be found, a twofold inclination may eventuate. So that, moreover, if the ship may be immersed more deeply, a vertical right line drawn through the centre of gravity even now shall pass through the centre of gravity of the new section of water, it is necessary that the sections of the ship be parallel to the principal sections of the water, if not all yet perhaps their own centres of gravity may have a place approximately on the same vertical right line. But if all the horizontal sections of the ship, or perhaps these, which are immersed in the water, shall be prepared thus, so that the centres of gravity of these may be prepared, so that the centres of gravity of these shall be positioned on the same vertical right line, then

at once the centre of the magnitude of the submerged part falls on the same right line. On account of which this same rule, which tells us that a single right line passing through the centres of the individual horizontal sections, will provide a very useful addition for the most satisfactory construction of ships; indeed with a number of items observed required to be satisfied, which ought to be present for the completed ship ; provided it may be understood from what has gone before, and in the following will be instructed further.

SCHOLIUM 2

425. These matters which have been discussed in this proposition concerned with the imposition of new loads, the same also have a place, if the forces may be applied vertically to the ship, if indeed the forces may urge the ship to move downwards, then similarly the same effect will arise as if a new load may be imposed, to which the weight of that force may be equivalent, in that same place, at which the force may be applied; but if the force may pull upwards, then the effect will be opposite, and will be determined from the solution of a no more difficult problem, by putting the force of the load negative, or by writing $-m$ in place of m. Indeed a difference consists in the difference between loads and forces, as with the imposition of loads then both the size of the inertial force of the ship as well as its moment will be changed, and in addition the centre of gravity may be changed from its location, which generally, if pure forces may act, allow for no changes are to be ruled out. On account of which it will be required to study carefully the effect of the forces themselves of this kind, evidently as far as for the application of forces no new material shall be added to the ship; so that the effect of some forces acting on ships to be determined is resolved, which follows in this next chapter. The treatment of which, so that it may be seen more clearly, shall be fivefold ; before all else to be directed towards every aspect of the forces acting on ships or other bodies floating on water, since the state of bodies of this kind may be disturbed in five different ways.

Indeed in the first place, a ship or body floating on water can be affected by some other force thus, so that it may be immersed deeper, or it may be raised from the water; which effect arises from the vertical force, and may be judged easily from the present proposition.

The second of the forces corresponds to this effect, so that the ship from its own location may be driven forwards in a horizontal motion, for which the forces accustomed to be used for the motion to be obtained through the water, to be either by the use of oars, from the wind, or by the motion of the water itself, or by some other forces.

Thirdly, the ship may be inclined by forces acting about some horizontal axis passing through the centre of gravity, which shall be drawn normal to the vertical plane through the keel.

Fourthly, an inclination can be made about the horizontal axis drawn along the length of the ship through the centre of gravity ; indeed every inclination made about some axis, can be reduced to a twofold inclination about this axis.

Finally in the fifth place, a ship may be turned about a vertical axis passing through the centre of gravity, and such an effect is produced especially by the rudder in ships.

Moreover, how these five effects are connected between themselves, so that more and more effects may arise from a single force to be considered separately and can be elicited by calculation, in whatever way it may be allowed to be understood from the principles treated before. Indeed each effect is produced likewise from forces and is to be determined, whether the remaining effects are produced likewise or otherwise; and on that account if several forces may be acting on the ship, the total effect will be known, and if separately, how much may be brought about by each of the five kinds mentioned, which will be investigated carefully. So that I shall be concerned with each one of these five forces separately, and I will show how an individual effect may be produced by the forces acting, without thinking about the others. But before all else if a force were proposed acting on a ship, or on a body of some kind floating on water, then the magnitude of its effect is required to be examined, whether an effect of this kind may be able to be produced, concerning that which is required, or not; indeed not any force is suitable for producing some effect. Then if the strength of an effect of this kind were ascertained, then the magnitude of its effect would be able to be determined. And in this way since the effect of the five kinds of effects will be treated, it will be easy to judge, whichever forces shall going to be effective on a given ship.

PROPOSITION 43

PROBLEM

426. *If a ship may be acted on by some forces, to determine the effect of each force, which is exert on the ship being immersed more or less in the water.*

SOLUTION

In order to be making a judgment, by how much more or less a ship, on being immersed, may be acted on by forces, it will be required to be with respect of its centre of gravity, and to be investigated, whether that may be urged either up or down by the forces, or otherwise. On account of which, so that thus it is required to make known the motion of the centre of gravity, all the force ought to be considered to be applied in directions parallel to the centre of gravity itself; and the individual forces to be resolved into vertical and horizontal directions, of which only these will produce that effect, concerning which we examine here. Therefore if there were some force p the direction of which shall make an angle with the horizontal, the sine of which shall be m, on putting the total sine $=1$, mp will be that force, by which the ship will be urged downwards; evidently it will act upwards on the ship, if the direction of the force may be inclined upwards, downwards truly if downwards. Whereby if the values mp may be elicited from the forces acting, and collected together into one, the total force will be had either lifting the ship upwards, or pressing it downwards. This resultant force shall be equivalent to the weight P, and it shall act downwards; if indeed it may act upwards, only a negative value P will be required to be taken. Therefore the ship will be immersed more by this force P, if indeed P may have had a positive value; how much deeper moreover may be defined thus. The mass or weight of the whole ship shall be $=M$; the volume of the submerged part $=V$, and the [cross-] section of the water $=E$, truly the vertical depth by which the ship will be immersed in water, shall be put $=Z$, which I put to be exceedingly small, so that the vertical forces or the weight *P* generally shall be accustomed to be very small with respect to the weight of the ship *M*. Therefore after the increase the volume of the immersed part of the ship shall be $=V+Ez$, and hence this proportion shall arise from the principles of hydrostatics $M:V=M+P:V+Ez$, or this equation : $PV=MEz$, from which there becomes $z=\frac{PV}{ME}$. And thus the centre of gravity of the ship subsides more deeply, from the force *P*, and descends through the interval $\frac{PV}{ME}$. But if the forces acting may raise the ship, and the total lifting force shall be equivalent to the weight *P*, then the centre of gravity will rise through the interval $\frac{PV}{ME}$, or what is the same, it will fall through the interval $-\frac{PV}{ME}$, which expression arises from that on putting -*P* in place of *P*, as we have now found. Q. E. I.

COROLLARY 1

427. Therefore if all the forces acting shall be in horizontal directions, or if the vertical forces arising from these may cancel each other out, then the ship neither will be sunk lower in the water, nor will it be raised.

COROLLARY 2

428. If several forces may be acting on the ship, then it will be allowed to conclude from the individual ascent or descent of the centre of gravity, clearly which individual effects combined together indeed will indicate whether the centre of gravity will either rise or fall.

COROLLARY 3

429. Because the ascent or descent of the centre of gravity is done through the distance $\frac{PV}{ME}$, but truly since the volume of the submerged part *V* shall be proportional to the weight of the ship *M*, it follows that the greater or smaller immersion of the ship shall be directly proportional to the force *P* acting, and inversely to the section of the water *E*.

COROLLARY 4

430. Therefore where the section of the water were greater, there the change of the part submerged arising from the same force acting will be smaller. On account of which where ships shall be subjected to the smallest change of this kind, the greatest water section will be brought into effect.

COROLLARY 5

431. If the volume of water may be put to be *u*, the weight of which shall be equal to the weight *P*, and this volume may be used for expressing the magnitude of the force acting, on this account there will become $V : M = u : P$; therefore the distance, by which the centre of gravity is forced upwards or downwards $= \frac{u}{E}$; from which expression it can be understood easily, by how much more or less the ship may be immersed in the water.

SCHOLION

432. There is a need for the effects, which the forces acting on any body may produce, to become known by a twofold investigation; the first by which the progressive motion of the centre of gravity is defined, the second by which the rotational motion of the body about the centre of gravity is sought. Therefore in effect the same account of the forces will be required to be used for the forces acting on ships or other floating bodies, which must be taken jointly with the nature of water, by which any of the bodies being moved are restricted. Indeed in the first place so that it may pertain to the motion of the centre of gravity, it will be agreed to be considered for floating bodies in two ways, just as its direction shall be either horizontal or vertical ; for indeed the horizontal motion if it were one started, shall be conserved indefinitely, unless in as much as it may be retarded by the resistance of the water, but the vertical motion is stopped at once, and just as great a part may be moved underwater, as we have defined the size here ; and on this account we have undertaken to determine in this proposition not only the rise or fall of the centre of gravity, but also that term by which it comes to rest, and with which the centre of gravity agrees. Nevertheless the centre of gravity, when it begins to move, indeed cannot actually come to rest suddenly, yet we have agreed by defining an oscillatory motion of this kind restrained from coming to rest, since from that state the water may be stopped at once. Therefore I go on to the other case of the motion of the centre of gravity progressing requiring to be investigated, whereby the motion produced by the forces acting in a horizontal direction may be arrived at.

PROPOSITION 44

PROBLEM

433. *If a ship may be acted on by some forces, to determine the effect of these in the progression of the horizontal motion, either on being generated or altered.*

SOLUTION

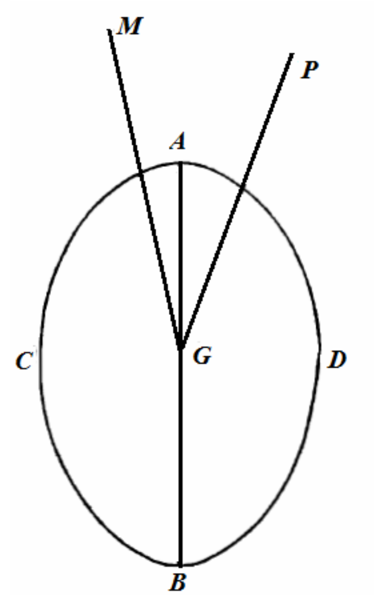

Fig. 69

Because the progression is sought concerned with the motion of the centre of gravity, I have considered all the forces must be applied in directions parallel to the centre of gravity, and since only horizontal motion is investigated, all the forces are required to be resolved into the vertical and horizontal components, which only later shall be adapted to this principle. Therefore if p were some force, the direction of which shall make an angle with the horizontal of which the sine shall be *m*, $p\sqrt{(1-mm)}$ will be the effective horizontal force sought producing this. Therefore of all the forces acting the horizontal forces may be sought in this way, and of these, if they shall be applied to the centres of gravity themselves, then the mean direction as well as the equivalent forces, which may be expressed by the weight *P*. Now ACBD (Fig. 69) shall be a horizontal section of the ship made through the centre of gravity *G*, and *GP* shall be the mean direction of all the forces acting, as far as they affect the horizontal motions. Therefore the ship will be acted on in the direction *GP* by the force *P*, and of which the force shall be equivalent to the weight *P.* If now the whole weight of the ship may be put $=M$, the accelerating force will be $=\frac{P}{M}$, by which it will be accelerated in the direction *GP*. Evidently if the ship now may be moved in the direction *GP* with the speed *v* according to the depth, while the element of the length of the ship *dx* will be absolved,

$$dv=\frac{Pdx}{M}.$$

But if the motion, which the ship may not have in the direction *GP*, but in some other *GM*, which shall make an angle *MGP* with *GP*, the sine of which shall be $=m$, truly the cosine $=n$ on putting the whole sine $=1$, so that there shall become $m^2+n^2=1$, then from the force acting P since the speed of the ship, which must be appropriate for the depth v will be changed, as well as that direction *MG*.

Moreover the speed will be increased by the force $\frac{nP}{M}$ thus so that while the ship runs through the element dx, there shall become:

$$dv = \frac{nPdx}{M}.$$

But the force from another normal $\frac{mP}{M}$ will force the ship from the right path GM to be deflected towards the direction GP, and while it will traverse the increment dx, it will be turned aside from the direction GM towards GP by an angle, of which the sine will be $= \frac{mPdx}{2Mv}$. And from this both the generation as well as the other motion of the ship from whatever forces will arise, becomes known. Q. E. I.

COROLLARY 1

434. Therefore if the ship may have been at rest, the motion follows in that mean direction *GP* from the forces acting; and it will begin to move more quickly, where the force P were greater, and likewise where the weight *M* of the ship itself were less.

COROLLARY 2

435. In a like manner if the ship now may have a motion in that very direction *GP*, it will be accelerated there more strongly, where the force *P* were greater, and where the weight of the ship *M* were less. Indeed the strength of the acceleration will be directly as the force *P*, and inversely as the weight of the ship.

COROLLARY 3

436. But if the motion, which the ship now has, may become in another direction *GM*, which the ship now has, then the acceleration there will be greater, where the angle *MGP* were smaller; indeed it is proportional to the cosine of the angle *MGP* with all else being equal.

COROLLARY 4

437. But so that in this case it may attain the declination from the direction *GM* , that will be directly proportional to the force *P* and to the sine of the angle *MGP* taken jointly, truly inversely proportional to the weight of the ship and to the square of the speed taken jointly.

COROLLARY 5

438. Therefore so that the ship may now be moved faster, there it is deflected less from its path by the same force acting obliquely; and with all else the same the deflection will be in the inverse ratio of twice the speed.

SCHOLION

439. Thus not only may all these things be found, but also the total motion of the ship for some given time will be able to be defined from these, but only if the water may offer no resistance, nor may it retard the motion itself to any extent, nor may the effect of the forces disturb the water. On account of the resistance of the water the initial motion of the ship is markedly reduced, and that thus more where the ship is moving faster; for indeed it is estimated from experimental consultation that the water resistance is proportional to the square of the speed. Then truly the direction of the resistance is required to be attended to especially, which unless it may be incident in the same direction as the motion of the ship, likewise also it will affect its direction, and it may deflect the ship from its path. But nevertheless in the following final chapter, we shall be going to investigate the effect of resistance, since it has a place among the forces, and since its magnitude and direction will have to be defined, its effect must be determined from these principles. Whereupon if a ship may now have a motion, likewise the resistance is required to be taken conjointly with the forces acting, and the effect which it will exert, both acting on the motion of the ship, as well as on the inclination and rotation, which will be required to be derived from this chapter. Thus in this proposition *PM* will be able to express the mean direction not only of the forces acting, but also of the resistance arising from the water; and *P* to denote the force arising from the forces acting and the resistance taken jointly. Whereby with the magnitude as well as the direction of the forces known, from this proposition the speed and direction of the ship at any place, will be able to be defined accurately, while it may be moving.

PROPOSITION 45

PROBLEM

440. *If a ship may be acted on by some forces, to find the moments of the inclining forces on the ship, not only about the length of the horizontal axis, but also about the width.*

SOLUTION

ACBD (Fig. 70) shall be the horizontal section of the ship made through the centre of gravity *G*, in which the longitudinal axis of length *AB* axis stands out, truly the latitudinal axis *CD* axis, about each of which axes, here it will be required to be investigated by how much the ship will be inclined by a force of some given magnitude,. Moreover it is understood similar reasoning is required to be put in place for each axis, and the force of the ship around the axis *AB* as well as the inclination around the axis *CD* is required to be investigated in a similar manner; on account of which it will suffice for the question to be resolved for one or other of the axis, as for example *AB*. But in the first place it is required to note there is no force, concerning which the ship can rotate about this axis, the direction of which either may concur with the axis *AB*, or shall be parallel to the same, or only shall be placed in the same with this axis. On account of which only the forces here

are required to be considered, the directions of which are not put in the same plane as the axis *AB*. Therefore any force acting on the ship shall be equivalent to a weight p, the direction of which may be drawn through some point perpendicular to the axis *AB*, the length of which shall be f; then some plane through the axis *AB* may be taken, and this perpendicular drawn, and the angle is sought of which the sine shall be $= m$, which the direction of the force makes with this plane. With which done the moment of the force requiring the ship to be made to rotate around the axis *AB* will be $= mpf$. In a similar manner the moments of the same kind may be sought from the individual forces acting, and with all had with respect to the same action, each in the same turning effect, or may be trying to turn the ship in the contrary direction, may be added together, which will be reduced to a simple expression of this kind *Pa*, in which a certain weight *P*, will be denoted by some right line. Therefore in this manner some number of forces will have acted on the ship, the moments will be defined with respect both of the axis *AB* as well as *CD*.
Q.E.I.

COROLLARY 1

441. Since it is of no concern which point may be taken in the direction of the force acting, it will be pleasing to take some plane drawn through the axis, and that point to be noted, in which the direction of the force crosses that plane.

COROLLARY 2

442. Therefore it will be arranged for this to be performed most conveniently to take either a vertical or horizontal plane, drawn through the axis, about which the rotating moment is sought. But often one of these planes will be preferred to the other, which chosen will readily become apparent.

COROLLARY 3

443. Therefore it is apparent, if the direction of some force may pass through either the centre of gravity *G* itself, or shall be placed in the plane *ACBD*, then the ship is not going to be turned about either axis, nor therefore to be allowed for any inclination.

SCHOLIUM

444. Since the inclination about any axis shall be twofold, for the two directions along which the inclination can be made, this is required to be attended to with care in the examination of the moments, where it may become apparent from the definition of all the moments defined, whither they will all try to turn the ship in the same direction, or otherwise: indeed in that case all the moments will be gathered together into one sum, truly where these moments, which act in the opposite direction, must be subtracted. But where this distinction may be observed more easily by the eye, and may be acted on most

quickly, for each axis both directions can be distinguished properly between each other, and it will help for these to be called by suitable names. Thus the ship can be inclined about the latitudinal axis *CD* in two ways, either towards the prow or the stern, while either the prow or the stern may be immersed more. But the inclination shall be either to the right or to the left about the longitudinal axis *AB*, which denomination someone chooses from there, who looks on standing in the stern or prow. Therefore with the moments gathering together of several forces with respect either of the axes *AB* or *CD*, it is required to be noting these properly in each direction, by which each of the axis try to incline the ship, which taken together may become legitimate. Moreover with the total moment found from the sum of all the forces acting, that same inclination is required to be defined, which is outstanding in the following proposition.

PROPOSITION 46

PROBLEM

445. *If a ship may be acted on by some forces, to determine the angle, by which it shall be inclined about the latitudinal axis, as well as about the longitudinal axis*.

SOLUTION

Initially we will consider the latitudinal axis *CD* (Fig. 70), and the moment $= Pa$ arising from all the forces, for the ship shall be required to be inclined either towards the prow or towards the stern around this axis *CD*, and the sine of the angle of inclination, which it produces shall be $= w$, which I consider as being exceedingly small. Now, if the stability of the ship with respect of the same latitude *CD*, shall be $= F$, to be expressed in this manner, as we have done in the preceding chapter, Fw will be the moment, by which the ship attempting to restore itself to an erect situation. Therefore since we may put this situation as being acted on by forces to be conserved, it is necessary that there shall be $Pa = Fw$, from which the sine of the angle of incidence produced $w = \frac{Pa}{F}$. In a similar manner, if the said stability of the ship with respect of the axis of the longitude *AB* shall be Φ, and the total moment of the forces striving to incline about this axis were $= Qb$, the sine of the angle at which the ship actually will be inclined about the axis AB, $= \frac{Qb}{\Phi}$.

Q.E.I.

COROLLARY 1

446. Therefore it may be agreed, since indeed it is now evident from the preceding, the inclination, that the given forces produce, thus to become smaller, where the stability of the ship with respect of the axis will become greater, around which it shall be inclined.

COROLLARY 2

447. The stability according to our received manner is designated by the product from the weight of the ship by a certain right line: from which it is easily understood the fraction $\frac{Pa}{F}$ to denote a pure number, which will express the sine of the angle of inclination with the whole sine put $=1$.

COROLLARY 3

448. Therefore the sine of the angle of inclination to the total sine, shall be as the moment of the forces effecting the inclination, to the stability of the ship with respect to its axis around which it shall be inclined.

COROLLARY 4

449. Therefore if the stability of the ship with respect of a certain axis were ten times greater, than the moment of the forces of the angle of inclination, then the smaller inclination will be 6 degrees, indeed in this case will produce an angle of inclination around 5°, 45'.

COROLLARIUM 5

450. Also it is understood, where the inclining forces were moved further from the centre of gravity, thus the greater to become the inclining moment, and therefore thence from that the greater inclination to be produced.

SCHOLIUM

451. On account of which, a twofold effect arises from the centre of gravity forces acting on ships, of which the one corresponds to a greater or lesser immersion, the other truly depends on the horizontal forwards motion of the ship; thus from the forces, which are accustomed to rotate bodies about their centre of gravity, a threefold effect arises in ships, for the three axis, about which the ship can rotate. Indeed, if we may consider three axes passing through the centre of gravity, two horizontal, the one longitudinal and the other latitudinal, and one vertical, the ship can be rotated by forces around the individual axes, thus so that a rotation about one axis does not disturb the rotations about the remaining axes. Moreover the effect of these three rotations about these three axes, on account of the action of the water, are completely dissimilar amongst themselves, and on that account are required to be presented separately. For the forces, which strive to turn the ship about each horizontal axis, at once follow according to their own effect, which since once it will have been produce, no further change to the ship arises. For the effect of these forces consists in the inclination of the ship around an axis of this kind, as far as to a certain angle, as long as these forces shall be keeping the stability of the ship in

equilibrium; and if the inclination were made at a certain angle, then the ship will persist in this state, if indeed the same forces may remain; but as the first forces either will be increased, diminished, or ceased completely, then the inclination either will increase, decrease, or in short the ship will be returned to its natural state, unless perhaps on account of the motion now received, a similar oscillatory motion may be produced. But as far as the account of the rotation about the vertical axis being prepared in some other way, indeed by the forces which produce this kind of motion, no appropriate force resists this motion, and on this account the ship shall be turned about the vertical axis by such forces, as long as the forces act, nor will the turning stop until the force has ceased, as the forces finally will have stopped, and the rotational motion now may be considered to be absorbed by the resistance of the water. On account of which, for the effect of turning forces of this kind to become known, it is required to investigate the rotational motion itself.

PROPOSITION 47

PROBLEM

452. *If a ship or some body floating in water may be acted on by some forces, to determine the moment of the force, by which the body shall be going to be turning around about the vertical axis through the centre of gravity.*

SOLUTION

ACBD (Fig. 71) shall be a horizontal section of the ship made through the centre of gravity *G*, and *EGF* shall be the vertical axis drawn through the centre of gravity *G*, about which the rotational motion is examined. Moreover the ship is able to rotate in a twofold manner about this axis, with itself being turned either to the right or left ; but I say the ship itself is to be turned to the right, when its prow will be considered to stand at its stern, is seen to rotate to the right; the opposite motion of that rotation is said to happen to the left. Therefore by any kind of force the ship is required to be seen to be turned strongly about this axis, whether it is considered to rotate to the right or to the left, or that it may force the ship to rotate to the right [*i.e.* clockwise] or to the left [*i.e.* anticlockwise], so that, if several forces acting together the effects of the forces to be added together, on the other hand truly they may be able to be subtracted easily. But before everything else it is to be considered, no force, of which the direction either may pass through the centre of gravity, or shall be vertical, or may be placed with the axis *EF* in the same plane, will prevail to produce a motion of this kind of rotation. Therefore since all the forces may be required to be resolved into vertical and horizontal components, in a case of this kind with only horizontal forces it will be required to find the rotation about the *EF* axis with only

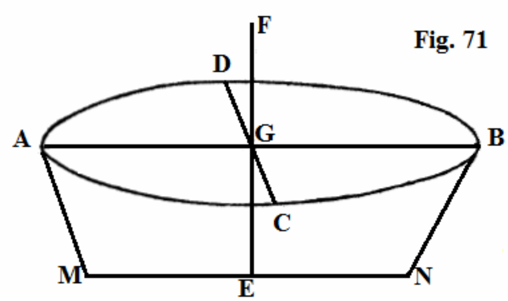

Fig. 71

horizontal forces. Thus moreover the turning force itself will be investigated. The horizontal plane may be considered in which the direction of some horizontal force shall be placed, and the point in which the axis *EF* will be cut by this plane shall be noted. Then from this point a normal may be drawn to the direction of the force, which multiplied by the same force will give the moment of this force for the rotational motion requiring to be produced about the axis *EF*. Or from above, I say, from a point of the axis *EF* some right line can be drawn for the direction of the horizontal force acting, and both the sine of the angle as well as the direction of the force is put in place by this right line, then the moment will be given by the product of the normal by the same force. Therefore if from the individual horizontal forces acting the moments of this kind may be elicited, and there, from the account of each had all will act together requiring to produce the same effect, or indeed if some shall be contrary, they may be gathered into one sum, it will produce an expression of this kind *Pa*, evidently made from some weight into some right line given, which made will show the whole moment of the forces, by which the motion of the rotation about the axis *EF* will be produced. Q. E. I.

COROLLARY 1

453. Therefore all the horizontal forces with these excepted, the directions of which pass through the axis of rotation *EF*, will strive to be rotating the ship about this axis, and actually will be rotating the ship, unless several forces of this kind may cancel each other out.

COROLLARY 2

454. Therefore whichever of the horizontal forces there may have a greater force for turning the ship around the vertical axis, where the greater were both the force itself as well as its distance from the axis of rotation *EF*, which distances may be measured both to the axis, as well as to the direction normal to the force.

COROLLARY 3

455. But since the ship likewise may be driven forwards by the horizontal forces, by which the ship may be moved, the ship also will be turned about the vertical axis, unless the mean direction of all the forces shall pass through the axis itself.

COROLLARY 4

456. Therefore, in order that the same forces which move the ship, shall not turn the ship around about its vertical axis, it is necessary either that the directions of the individual forces acting, or perhaps the mean direction of these, shall pass through the perpendicular drawn through the vertical line from the centre of gravity.

SCHOLIUM

457. This rotational motion about the vertical axis passing through the centre of gravity is of the greatest importance in ships, and with that being required to be produced, just as often as there is a need required for the rudder, with the help of which, if the ship were established in its motion, then it may be deflected either to the right or to the left. Indeed, since ships are accustomed to be progressing along the direction of the keel, either exactly or at least approximately, the course of the ship itself shall not be allowed to be changed, by the action of the rudder: evidently, when the rotation of the ship shall be to the right, then likewise the course of the ship also is to the right, that is from the North towards the East, or hence towards the South, or from the South towards the West, or hence back into the North again; but when the rudder shall be rotated to the left, the course will be deflected into the opposite region from the above. Therefore from these the effect of the rudder is understood thus to become greater, where that may be removed further from the centre of gravity; also, on account of which its position has been assigned to the end of the stern. Truly besides the rudder is exceedingly useful as well in leaving the course of the ship unchanged in direction, where there is a need for forces acting, which likewise are required to rotate the ship about the vertical axis, then indeed with the aid of the rudder this force has been required to be cancelled out. But lest that may not eventuate, which deservedly is considered to be a great inconvenience, in that the rudder especially is accustomed to be used, so that both the mean direction of the forces acting may pass through that vertical axis, as well as the resistance provided by the force by which the ship is required to be turned may be cancelled out. On account of which, where this same inconvenience may occur, both suitable placings of the masts, from which the forces are accustomed to be applied propelling the ship, are required to be selected carefully; indeed, as well as the anterior form of the ship, on which its resistance depends, is required to be determined with the greatest care; all of which we will set out further in the following chapters. But now it remains, that we may determine the rotational motion itself about the vertical axis, in which, since it cannot yet be agreed on concerning the magnitude of the resistance, we have decided not to consider the water resistance at all ; moreover, it may matter very little to know how much the rotational motion may be retarded by the water resistance ; provided indeed it will be agreed, and it may be able to judge the effect of the following, how its speed may be had for the various forces acting, with the condition of the ship varied, and that will suffice abundantly for the situation put in place. Also for this reason, in the following chapters we will investigate only, how great the resistance of the water may retard the forwards motion of the ship, nor will we be able to solicit, by how much it may hinder a rotation or deflection.

PROPOSITION 48

PROBLEM

458. *If a ship may be urged to perform a rotary motion about the vertical axis, to determine the motion of the ship, by which it may be rotated about this axis.*

SOLUTION

From these matters which have been demonstrated above concerned with the rotary motion around some axis passing through the centre of gravity, it is understood for these to be performed with the aid of two properties, evidently to the moment of these forces taken with respect of that axis, and the moment of the matter or inertia of the body with respect of the same axis. Therefore since in the preceding proposition I will have shown how to define the moment resulting from all the forces acting, which may strive to rotate the ship about the vertical axis drawn through the centre of gravity, which shall be $= Pa$ evidently performed by some weight P into the given right line a, it remains that the moment of inertia or of the whole material of the ship may be determined with respect of the same axis, which will be found by multiplying the individual particles of the ship by the squares of the distances of the same from that central axis, of which the sum of the products will have a form of this kind Mb^2, in which M shall denote the weight of the ship, b a right line with a given length. Therefore the rotational force, by which the ship actually may be turned about the vertical axis passing through the centre of gravity will be $=\frac{Pa}{Mb^2}$, from which the angular motion will be able to be defined. If now we may put the ship to have only an angular motion about the vertical axis, so that some point placed at a distance f from the axis may have a speed v appropriate for some depth, there will become, while that point of the infinitesimal arc dx shall be resolved : $dv=\frac{Pafdx}{Mb^2}$. Hence on integrating there will become $v=\frac{Pafx}{Mb^2}$, where x will denote the arc described from that initial point of the motion. Moreover now the angle s described, is

$$s=\frac{v}{f},\ \text{ and thus } \frac{v}{ff}=\frac{Pas}{Mb^2}.$$

Therefore the angular speed now acquired will be

$$=\frac{\sqrt{v}}{f}=\sqrt{\frac{Pas}{Mb^2}},$$

if indeed the resistance of the water may not be considered. Q. E. I.

COROLLARY 1

459. Therefore with the same rotational force remaining the ship thus will be turned about the vertical axis, where the moment of inertia of the ship were smaller with respect of the same axis.

COROLLARY 2

460. Therefore the rotation of that same ship will be performed more easily, when all the cargo may be arranged closer to the vertical axis passing through the centre of gravity. But on the other hand, if all the cargo were moved away as far as possible from this axis, the rotation will become most difficult.

COROLLARY 3

461. Therefore just as the ship either is allowed to be turned more easily, or must resist being turned maximally, thus a method of loading with respect to the vertical axis through the centre of gravity will require to be aimed at.

COROLLARY 4

462. Hence it is understood the ship thus to yield more easily to the action of the rudder, where the commodities and the rest of the cargo may be arranged closer to the vertical axis. Indeed with this agreed on, the moment of inertia of the ship thus will obtain a smaller value.

COROLLARY 5

463. Therefore from the same proposition, if the force of the water on the rudder were defined, the effect of the rudder for any ship will be able to be decided and determined.

SCHOLIUM

464. Therefore five effects are to be set out and defined, which the forces prevail to produce in any body floating on water, which thus are to be distinguished in turn from each other, so that each may be able to be present without the rest. On account of which if, while the body floating on the water may be acted on by some forces, these same five effects will be determined, and it will be agreed how the body may be affected by these forces. For in the first place it will be defined to what extent more or less the body may be immersed in the water; then from that, by how great a force may it be acted on for moving forwards and in what direction; thirdly and fourthly it will be learned, how much the body will be forced to be inclined about the horizontal and longitudinal axes ; and in the fifth finally it will be apparent, by how great a force shall the body be rotated, drawn about the vertical axis through the centre of gravity. Therefore since the action shall consistent with these five effects of all the forces, we will finish this chapter, and we will progress to the resistance of the water requiring to be defined, certainly there is a need from which the motion of the ship itself is required to be determined.

CAPUT QUARTUM

DE EFFECTU VIRIUM CORPORA AQUAE INSIDENTIA SOLLICITANTIUM

PROPOSITIO 40

THEOREMA

400. *Si in navi seu vase quocunque AB, cuius pondus sit = M, onus P cuius pondus sit* $= m$*, per spatium Pp transferatur in p, totius vasis centrum gravitatis G transferetur secundum directionem Gg ipsi Pp parallelam in g, ut sit*

$$Gg = \frac{m \cdot Pp}{M} \text{ (Fig. 66).}$$

DEMONSTRATIO

Sit Z centrum gravitatis navis seu vasis demto onere P, erunt puncta Z, G et P in linea recta posita, ita ut sit
$ZG : PG = m : M - m$ seu $ZG : ZP = m : M$.

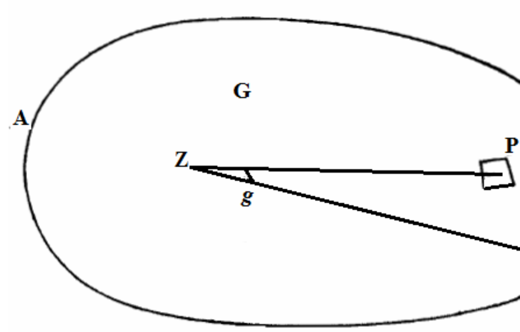

Fig. 66

Translato iam onere m ex P in p, totum corpus quod ex duabus partibus $M - m$ et m compositum considero, partis alterius $M - m$ centrum gravitatis ut ante habebit in Z, alterius vero partis m centrum gravitatis nunc erit in p. Quamobrem totius corporis M centrum gravitatis nunc reperietur in rectae Zp puncto g, ita ut sit $Zg : pg = m : M - m$ seu $Zg : Zp = m : M$; unde perspicitur rectam Gg parallelam fore rectae Pp, et triangula ZGg et ZPp inter se similia. Propterea erit

$$Gg : Pp = ZG : ZP = m : M ,$$

ex quo prodit

$$Gg = \frac{m \cdot Pp}{M}$$

Q.E.D.

COROLLARIUM 1

401. Quoniam onerum translatione navis seu vasis cuiusque aquae insidentis pondus non mutatur ante et post translationem oneris, aequale volumen aquae immergetur.

COROLLARIUM 2

402. Corpus igitur in aqua eundem retinebit situm onere quodam transposito, si centrum gravitatis verticaliter vel sursum vel deorsum transfertur; id quod evenit, si onus verticaliter vel sursum vel deorsum transportatur.

COROLLARIUM 3

403. Ex capite autem praecedente constat, centro gravitatis corporis sursum translato stabilitatem situs aequilibrii diminui; eandem vero augeri centro gravitatis deorsum translato.

COROLLARIUM 4

404. Si igitur onus m verticaliter vel sursum vel deorsum transfertur per spatium s, centrum gravitatis vel ascendet vel descendet per spatium $\frac{ms}{M}$, atque idcirco stabilitas vel diminuetur vel augebitur quantitate ms.

COROLLARIUM 5

405. Sin autem onus quodpiam vel horizontaliter vel oblique promoveatur, tum situs aequilibrii non conservabitur, sed corpus ex eo inclinabitur; quia isto oneris motu centrum gravitatis corporis de recta verticali per centrum magnitudinis partis submersae ducta depellitur.

COROLLARIUM 6

406. Hinc etiam facile mutatio situs centri gravitatis colligetur, si plura onera utcunque transponantur in alia loca. Ad hoc enim tantum opus est cuiusque oneris motum seorsim considerare.

SCHOLI ON

407. In hoc capite ante omnia visum est indagare, quantum situs aequilibri corporis aequae innatantis immutetur, dum centrum gravitatis solum de suo loco movetur; priusquam enim in effectum virium externarum inquiraturt convenit eas mutationes evoluisse, quae in ipsis navibus nullis accedentibus viribus alienis oriri possunt; etiamsi eiusmodi mutationes sine viribus alienis evenire nequeant. Hancobrem primum centrum gravitatis de suo loco moveri

considerabo, toto corporis pondere manente invariato, atque qualis mutatio in situ aequilibrii eveniat scrutabor; deinde vero non solum situm centri gravitatis mutatum spectabo, sed etiam ipsum pondus corporis augeri vel diminui ponam, quod fit vel novis oneribus imponendis, vel ab iis quae aderant auferendis. His enim casibus non solum navis inclinabitur, sed etiam aquae vel magis immergetur, vel ex aqua emerget. Quamobrem facta eiusmodi mutatione non solum definiendum est, quemnam situm corpus sit adepturum, sed etiam quanta post mutationem futura sit stabilitas. Omnes autem has mutationes tantum minimas contemplabor, cum calculi sublevandi causa, tum quod nihilominus inde iudicium de maioribus mutationibus formari potest; quia maiores mutationes ex minimis successive conflatae aestimari possunt.

PROPOSITIO 41

PROBLEMA

408. *Si navis vel cuiusvis vasis aquae insidentis per oneris cuiuspiam translationem centrum gravitatis aliquantillum de suo loco promoveatur, invenire declinationem vasis de pristino aequilibrii situ, atque stabilitatem, quam tum habebit.*

SOLUTIO

Cum centrum gravitatis recta vel ascendit vel descendit, situs aequilibrii nullam patitur mutationem, nisi quod eius stabilitas vel minuatur, vel augeatur. At si centrum gravitatis oblique promoveatur, tum iste motus resolvi poterit in verticalem et horizontalem, quorum ille situm non afficit, hic autem omnino ad vas de priore situ declinandum impenditur. Quamobrem cum promotio verticalis nil habeat difficultatis, porro corporis *AFB* (Fig. 67) aquae ita insidentis ut *AB* sit sectio aquae et *O* centrum magnitudinis partis submersae, centrum gravitatis G transferri horizontaliter per *Gg* in *g*; quo facto vas circa axem ad planum *OGg* normalem inclinabitur, ut sectio aquae fiat ab cum priore angulum *AO* a constituens, qui est angulus inclinationis, quem quaerimus; cuius sinus sit $= w$ posito sinu toto$=1$. Existente ergo ab sectione aquae et *g* centro gravitatis, aequilibrium aderit, ex cuius conditione *w*, indeque angulus inclinationis definietur. Loco

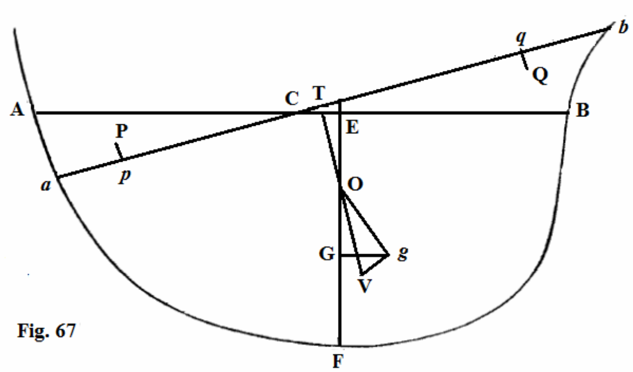

Fig. 67

totius corporis autem tantum considerabo figuram planam verticalem AFB, quum ex iis, quae pro figura plana reperientur, conclusio pro ipsis corporibus facile formari possit, si affinitas inter formulas stabilitatem exhibentes pro figuris planis et solidis attentius inspiciatur.

Erit igitur triangulum $ACa = BCb$, et cum utrumque sit minimum punctum C situm erit in medio rectae AB. Cum iam pars submersa sit

$$aFb = AFB - ACa + BCb,$$

momenta virium aquae his partibus respondentia respectu centri gravitatis g, se mutuo destruere debent. Per O ad sectionem aquae ab ducatur normalis TOV, in quam pariter ex g horizontalis gV normaliter cadet. His praemissis partis AFB momentum respectu g erit $= AFB \cdot gV$, quoniam O est centrum magnitudinis partis AFB. Deinde trianguli ACa area est

$$= \frac{AC \cdot aC \cdot w}{2} = \frac{w \cdot AC^2}{2};$$

eiusque centrum gravitatis erit in P ut sit $Cp = \frac{2}{3} AC$. Momentum igitur hinc ortum est

$$Cp = \frac{w \cdot AC^2}{2}\left(\frac{2}{3} AC + CT + gV\right);$$

pari autem ratione momentum ex area BCb ortum est negativum atque

$$C = \frac{w \cdot AC^2}{2}\left(\frac{2}{3} AC - CT - gV\right)$$

posito AC pro BC. Momentum igitur totale ex area aFb ortum erit

$$AFB \cdot gV - \frac{2w \cdot AC^2}{3},$$

quod cum aequilibrium adesse debeat, erit $= 0$. Transferantur iam haec ad corpora, atque in sectione aquae corporis per eius centrum gravitatis ducta sit recta ad planum OGg normalis, quae erit axis circa quem corpus inclinabitur; atque ad hunc axem in sectione aquae colligantur utrinque summae cuborum applicatarum orthogonalium, seu $\int \left(y^3 + z^3\right) dx$ ex prop. 29. quae quantitas, quam vocabo Q, substitui debet loco $2AC^3$. At loco area AFB scribendum erit volumen partis submersae quod sit $= V$. Ex his pro corpore seu vase quocunque, cuius centrum gravitatis G horizontaliter in g transfertur, habebitur ad inclinationem inde ortam definiendam haec aequatio $3gV \cdot V = wQ$. Est vero ob angulos ad O et g minimos $gV = Gg - w\cdot GO$; quare cum sit $3V\cdot Gg = 3wV\cdot GO + wQ$, erit sinus anguli, quo vas circa axem horizontalem ad planum OGg normalem inclinabitur, scilicet

$$w = \frac{3V\cdot Gg}{3V\cdot GO + Q}.$$

Ad stabilitatem autem inveniendam intervallum, figura tantum plana considerata, centri gravitatis areae aFb a recta horizontali gV est investigandum, quod reperitur

$$= OV - \frac{AC^2\cdot w^2}{3AFB} \quad ob\ Pp = Qq = \frac{1}{3}w\cdot AC$$

idemque intervallum pro corpore solido erit consequenter

$$= OV - \frac{w^2 Q}{6V}.$$

Est autem

$$OV = OG + w\cdot Gg - \frac{w^2\cdot OG}{2};$$

unde distantia centri magnitudinis partis submersae et centri gravitatis post inclinationem erit

$$= OG + w\cdot Gg - \frac{w^2\cdot OG}{2} - \frac{w^2\cdot Q}{6V} = OG + w\cdot Gg - \frac{w^2\cdot F}{2M},$$

denotante M pondus corporis, et F stabilitatem eiusdem ante inclinationem; est enim

$$F = M\left(GO + \frac{Q}{3V}\right).$$

Quare post inclinationem erit stabilitas

$$= M\left(OG + w\cdot Gg - \frac{w^2\cdot F}{2M} - \frac{Q}{3V}\right) = F + wM\cdot Gg - \frac{w^2\cdot F}{2},$$

respectu eius axis scilicet circa quem inclinatio est facta. Q. E. I.

COROLLARIUM 1

409. Cum, antequam centrum gravitatis de suo loco depellitur, stabilitas aequilibrii situs respectu axis horizontalis ad planum OGy normalis sit

$$= M\left(GO + \frac{Q}{3V}\right);$$

si haec stabilitas dicatur $= F$, erit sinus anguli inclinationis, qui ex translatione centri gravitatis *G* in *g* gignitur,

$$w = \frac{M \cdot Gg}{F};$$

existente sinu toto $= 1$.

COROLLARIUM 2

410. Sinus anguli ergo, quo vas circa axem horizontalem plano *OGg* normalem inclinatur, dum centrum gravitatis *G* motu horizontali per spatium *Gg* promovetur, est directe ut hoc spatium *Gg* et pondus vasis coniunctim, ac reciproce ut stabilitas vasis respectu eiusdem axis.

COROLLARIUM 3

411. Quo maior ergo corporis est stabilitas, eo magis id etiam ei inclinatione resistit, quae oritur a translatione onerum de loco alio in alium; quam ob causam etiam navibus maxima stabilitas est concilianda.

COROLLARIUM 4

412. Quia stabilitas post factam inclinationem inventa est

$$= F + wM \cdot Gg - \frac{w^2 F}{2},$$

atque est

$$w = \frac{M \cdot Gg}{F};$$

erit illa stabilitas

$$= F + \frac{M^2 \cdot Gg^2}{2F}.$$

Hoc igitur stabilitatis incrementum, quod post inclinationem accedit ob duas dimensiones intervalli quasi infinite parvi *Gg* omnino est negligendum.

COROLLARIUM 5

413. Quando ergo gravitatis centrum recta sursum deorsumve movetur, nulla fit inclinatio sed, sola stabilitas immutatur; contra vero quando centrum gravitatis horizontaliter movetur, stabilitas non afficitur, sed situs aequilibrii per inclinationem solum mutatur.

COROLLARIUM 6

414. Quando ergo centrum gravitatis oblique movetur, tum mutabitur tam stabilitas, quam situs corporis in aqua. Quanta autem mutatio in utroque accidat, ex propositionibus praecedentibus satis intelligere licet.

SCHOLION 1

415. Quo haec facilius ad naves, in quibus stabilitatem respectu duorum tantum axium horizontalium, alterius longitudinalis a prora ad puppim protensi, alterius latitudinalis ad illum normalis, cognitam esse ponimus, accommodari queant, motus centri gravitatis, nisi vel secundum longitudinem vel latitudinem fiat, resolvi debet in duos laterales, alterum in longitudine alterum in latitudine factum, quos seorsim considerari oportet. Illa enim centri gravitatis translatio secundum axem longitudinalem facta inclinationem circa axem latitudinalem generabit, cuius sinus aequalis erit facto ex pondere navis in viam centri gravitatis secundum longitudinem, diviso per stabilitatem respectu axis latitudinalis. Via vero centri gravitatis secundum latitudinem facta per pondus navis multiplicata, ac per stabilitatem respectu axis longitudinalis divisa exprimet sinum anguli inclinationis, quo navis circa axem longitudinalem inclinabitur. Hae igitur duae inclinationes coniunctae praebebunt inclinationem navis a translatione centri gravitatis per spatium quodcunque horizontale facta, ortam. At si centrum gravitatis simul vel ascendat vel descendat, ante decrementum vel augmentum stabilitatis est investigandum, quam in inclinationem inquiratur. Stabilitas enim, a qua inclinatio pendet, non prima in computum est ducenda, sed ea, quae ob ascensum vel descensum centri gravitatis iam est vel minuta vel aucta.

SCHOLION 2

416. Solutio huius problematis, quanquam id corpora quaecunque aquae insidentia spectabat, multo facilior et a corporum consideratione libera est facta, quod expressiones ex figurae planae contemplatione ortas ad naturam corporum extensorum accommodare licuit. Sequitur autem ista figurarum planarum ad corpora translatio ex comparatione formularum, quas in capite praecedente pro stabilitate tum figurarum planarum, tum corporum quorumque invenimus. Cum enim pro figura plana sit stabilitas

$$= M\left(GO + \frac{2AC^3}{3AFB}\right),$$

pro corpore autem ea reperta sit ubi Q denotat aggregatum cuborum omnium applicatarum in sectione aquae ad axem per eius centrum gravitatis transeuntem et axi inclinationis parallelum normalium; V vero exhibet volumen partis submersae. Quoties igitur ad eiusmodi expressiones pervenitur, a figura plana ad solidam fiet translatio, si loco areae AFB sub aqua versantis scribatur V volumen partis corporis submersae, atque pro $2AC^3$ ponatur Q, seu, $\frac{1}{2}Q$ pro AC^3. Cum igitur istius modi problemata multo facilius figuras tantum planas considerando resolvantur, huius comparationis beneficia solutiones eorundem problematum nullo negotio simul ad quaecunque corpora reduci poterunt; quod uti in isto problemate est factum, ita in sequente multisque aliis succedet.

PROPOSITIO 42

PROBLEMA

417. *Si vasi seu navi cuicunque aquae insidenti novum onus imponatur, invenire tum situs tum stabilitatis mutationem, quae ab hoc novo onere orietur.*

SOLUTIO

Sit AB sectio aquae, et AFB pars corporis aquae submersa, cuius centrum magnitudinis sit in O, totius vero corporis centrum gravitatis in G (Fig. 68). Nunc posito totius corporis seu navigii pondere $= M$ superaddatur illi in loco quocunque pondus m. Ad mutationem igitur ab hoc novo onere imposito ortam indagandam, concipiatur id primo ipsi centro gravitatis G immissum. Cum nunc pondus corporis sit auctum, maior corporis pars aquae immergetur, quam ante. Subsidat igitur centrum gravitatis recta deorsum, ut nunc ab fiat sectio aquae atque aFb tanta sit corporis portio, quantam pondus $M + m$ desiderat.

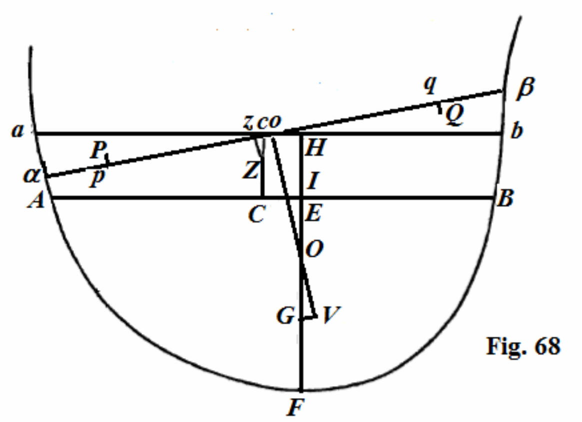

Fig. 68

Sit sectionis aquae area $= E$, atque volumen partis submersae AFB sit $= V$, erit volumen partis $AabB$ de novo immersae $= E \cdot HI$, si quidem uti pono onus m est vehementer paruum respectu M, quo inaequalitatem inter AB et ab considerasse non sit necesse. Erit ergo

$$M : m = V : E \cdot HI \text{, unde fit } HI = \frac{mV}{ME}.$$

Quamvis autem iam debita corporis pars sub aqua versetur, tamen iste situs non erit aequilibrii, nisi huius partis submersae centrum mangitudinis etiamnum in recta FH existat. Manebit autem partis submersae centrum mangitudinis ut recta HF, si portionis $AabB$ centrum magnitudinis in eandem cadat, id quod evenit quando recta GO simul per centrum gravitatis sectionis transeat; hoc ergo casu quaesitum iam constat, cum aFb

futurus sit aequilibrii situs. At ponamus sectionis aquae centrum gravitatis non in *I* cadere, sed in alio puncto *O* existere, atque portionis *AabB* centrum magnitudinis cadet in *Z* punctum medium rectae $Cc = HI$; hoc igitur casu situs *aFb* aequilibrii proprietate non erit praeditus, sed corpus inclinabitur circa axem ad planum *CIG* normalem, ita ut sectio aquae futura sit $= w$ cum priore angulum constituens a *acα* cuius sinus sit $= w$; quem angulum ante quam pro corpore investigemus, quaeremus pro figura plana *AFB*. Cum igitur nunc $\alpha\beta$ sit horizontalis ad eam, per *O* ducatur verticalis *Oo*, cui ex G horizontalis *GV* in *V* occurrat; atque quia hoc situ aequilibrium adesse ponitur, considerabo partem submersam $\alpha F\beta$, quam ex his partibus

$$AFB + AabB - ac\alpha + bc\beta$$

compositam esse considerari conveniet, quarum partium momenta respectu *G* se mutuo destruent. At est areae *AFB* momentum $= AFB \cdot GV$, areae vero *AabB*

$$= AabB\left(GV - \alpha o + \alpha z\right) = AabB(GV - co - cz).$$

Trianguli autem a *acα*, cuius area est $= \frac{1}{2} w\alpha c \cdot ac = \frac{1}{2} w \cdot AC^2,$ momentum, posito eius centro gravitatis in *P* ut sit

$$cp = \tfrac{2}{3} c\alpha = \tfrac{2}{3} CA \text{ erit } \tfrac{1}{2} w \cdot AC^2 \left(GV - \tfrac{2}{3} AC - co\right).$$

Trianguli autem *bcβ* simili modo momentum erit

$$\tfrac{1}{2} w \cdot AC^2 \left(GV + \tfrac{2}{3} AC - co\right).$$

Cum igitur sit pars aquae submersa

$$= AFB + AabB - ac\alpha + bc\beta,$$

erit momentum totius partis submersae

$$= AFB \cdot GV + AabB\left(GV - co - cz\right) + \tfrac{2}{3} w \cdot AC^3 = 0.$$

Transferatur nunc haec formula ad corpora, ponendo loco *AFB* volumen partis submersae *V*, loco *AabB* volumen $\frac{mV}{M}$, atque loco $2AC^3$ summam omnium cuborum applicatarum normalium in sectione aquae ad axem per eius centrum gravitatis ductum, qui axis normalis sit ad planum *CIG*; haec vero summa, quae in probl. 29 erat $\int\left(y^3 + z^3\right)dx$ hic nobis brevitatis causa vocetur *Q*. Quamobrem pro corpore hanc habebimus aequationem

$$V\cdot GV+\frac{mV}{M}\left(GV-co-cz\right)+\tfrac{1}{3}wQ=0;$$

quae cum sit

$$GV=w\cdot GO,\ co=CI-w\cdot HO=CI-w\cdot OI-\frac{wmV}{ME}$$

atque

$$cz=w\cdot cZ=\frac{wmV}{2ME}$$

transibit in hanc

$$wV\cdot GO+\frac{wmV\cdot GO}{M}-\frac{mV\cdot CI}{M}+\frac{wmV\cdot OI}{M}+\frac{wm^2V^2}{2M^2E}+\tfrac{1}{3}wQ=0,$$

ex qua elicitur

$$w=\frac{mV\cdot CI}{MV\cdot GO+\tfrac{1}{3}MQ+mV\cdot GI+\dfrac{m^2V^2}{2ME}}.$$

Si nunc stabilitas respectu eiusdem axis circa quem inclinatio est facta, quae est

$$=MV\cdot GO+\frac{MQ}{3V}$$

vocetur F, erit

$$w=\frac{m\cdot CI}{F+m\cdot GI+\dfrac{m^2V}{2ME}}$$

pro qua aequatione ob m pondus respectu *M* valde parvum tuto uti licebit hac

$$w=\frac{m\cdot CI}{F+m\cdot GI}.$$

Stabilitas autem huius aequilibrii situs ab eo non differet, quae in situm *aFb* competeret; quae ob centrum magnitudinis supra *O* elevatum intervallo

$$\frac{m}{M+m}\left(OI+\frac{mV}{2ME}\right)$$

erit

$$= (M+m)\left(GO + \frac{m}{M+m}\left(OI + \frac{mV}{2ME}\right) + \frac{MQ}{3(M+m)V}\right)$$
$$= M\cdot GO + m\cdot GI + \frac{m^2V}{2ME} + \frac{MQ}{3V}.$$

Cum igitur ante accessionem oneris *m* stabilitas esset

$$F = M\cdot GO + m\cdot GI + \frac{MQ}{3V}.$$

erit nunc stabilitas

$$= F + m\cdot GI + \frac{m^2V}{2ME},$$

pro qua expressione pariter licebit uti hac $F + m\cdot GI$. Quoniam autem hic onus *m* non ipsi centro gravitatis *G* immissum consideravimus, removeatur nunc onus m in eum locum ubi revera est positum, atque quaenam novae mutationes eveniant, ex propositione praecedente intelligitur. Q. E. I.

COROLLARIUM 1

418. Cum

$$F + m\cdot GI + \frac{m^2V}{2ME}$$

sit stabilitas novi aequilibrii situs, quem navis onere *m* in centro gravitatis collocato adipiscitur, sinus anguli inclinationis aequabitur facto ex onere imposito *m* ducto in *CI* et diviso per hanc novam stabilitatem.

COROLLARIUM 2

419. Eadem inclinatio prodit si onus m ubicunque in recta verticali *FH* collocetur, quia motu oneris sursum deorsumue facto situs aequilibrii non turbatur; at stabilitas minor maiorve evadet.

COROLLARIUM 3

420. Si onus *m* in rectae verticalis *FI* puncto *K* imponatur, stabilitas ea quae prodiret, si centro gravitatis *G* foret immissum, diminui debet facto $m\cdot GK$. Hoc ergo casu stabilitas erit

$$= F + m\cdot IK + \frac{m^2V}{2ME}.$$

COROLLARIUM 4

421. Reiecto ergo termino $\frac{m^2 V}{2ME}$ perspicuum est impositione oneris novi stabilitatem augeri, si onus infra sectionem aquae collocetur. Contra vero stabilitas diminuetur, si onus supra aquam ponatur.

COROLLARIUM 5

422. Quando ergo centrum gravitatis sectionis aquae *C* in ipsam verticalem *FGI* incidet, tum nulla fiet inclinatio, ab onere imposito, dummodo oneris centrum gravitatis quoque in rectam *FGI* incidat.

COROLLARIUM 6

423. Si autem onus *m* non in rectam *FI* ponitur sed extra eam, atque punctum *C* non incidat in *I*, duplex proveniet inclinatio, altera scilicet hic definita et ab intervallo *CI* pendens, altera vero ex praecedente propositione orta et a distantia oneris a recta *FI* pendens.

SCHOLION 1

424. Ex hac ergo propositione clarius intelligitur quantum eiusmodi navigia, quae sectionis aquae centrum gravitatis et partis submersae centrum magnitudinis in eadem recta verticali, in quam simul totius navis centrum gravitatis cadere debet; habeant posita aliis, quae hac proprietate carent, antecellant. In eiusmodi enim navibus, si nova onera, vel ipsi centro gravitatis immittantur, vel in recta verticali per id transeunte collocentur, nulla accidit inclinatio, sed navis tantum verticaliter descendendo aquae profundius immergetur. Atque etiam si onus novum non in rectam verticalem per centrum gravitatis ductam imponatur, unica tantum oritur inclinatio circa axem quendam horizontalem, cum in aliis navigiis, in quibus haec proprietas locum non invenit, duplex inclinatio eveniat. Quo autem, si navis profundius immergitur, recta verticalis per centrum gravitatis ducta etiamnum per novae sectionis aquae centrum gravitatis transeat, necesse est ut sectiones navis parallelae sectioni aquae principali, si non omnes, tamen proximae saltem sua gravitatis centra in eadem recta verticali habeant posita. At si omnes sectiones navis horizontales vel eae saltem, quae aquae immerguntur, ita sint comparatae, ut earum centra gravitatis sita sint in eadem recta verticali, tum sponte in eadem rectam cadet centrum magnitudinis partis submersae. Quamobrem ista regula, quae praecipit, ut una recta verticalis per singularum sectionum horizontalium centra gravitatis transeat, ingentem afferet utilitatem ad naves aptissime construendas; hac .enim observata pluribus satisfit requisitis, quae in perfecta navi inesse debent; prout ex antecedentibus intelligere licet, et in sequentibus fusius docebitur.

SCHOLION 2

425. Quae in hac propositione de impositione novi oneris sunt dicta, eadem quoque locum habent, si navi potentia verticalis applicetur, si enim potentia verticalis deorsum urgeat, tum idem orietur effectus ac si onus novum, cuius pondus illi potentiae aequivaleat, in eo ipso loco, in quo potentia applicatur, imponeretur; sin autem potentia sursum trahat, tum effectus erit contrarius, atque ex solutione problematis non difficilius determinabitur, ponendo oneris vim negativam, seu loco m scribendo m. In hoc vero discrimen inter onera et potentias consistet, quod impositione onerum tum vis inertiae navis tum etiam eius momentum immutetur, atque insuper centrum gravitatis de suo loco transferatur, quae omnia, si merae potentiae agant, nulli mutationi sunt obnoxia. Quamobrem oportebit effectum huiusmodi potentiarum seorsim scrutari, quatenus scilicet applicatione potentiarum nulla navi nova materia accedit; id quod mox in hoc capite, quod effectui quarumcunque potentiarum naves sollicitantium determinando est destinatum. Quae tractatio, quo dilucidius perspiciatur, ante omnia est advertendum omnem effectum, qui a potentiis navigia aliave corpora aquae innatantia sollicitantibus, quintuplicem esse posse; cum huiusmodi corporum status quinque diversis modis turbari queat. Primum enim navis vel corpus ·aquae insidens a vi aliena ita affici potest, ut vel profundius immergatur, vel ex aqua extrahatur; qui effectus a potentiis verticalibus oritur, atque ex propositione praesente facile iudicatur. Secundus potentiarum effectus in hoc constat, ut navis de suo loco ·motu horizontali propellatur, ad quem obtinendum remi, venti ipse aquae motus aliaeque vires adhiberi solent. Tertio navis a potentiis inclinatur circa axem quempiam horizontalem per centrum gravitatis transeuntem, qui ad planum verticale per spinam ductum sit normalis. Quarto inclinatio fieri potest circa axem horizontalem secundum navis longitudinem per centrum gravitatis ductum; ad duplicem enim hanc inclinationem omnis inclinatio, quae circa axem quemcunque fit, reduci potest. Quinto denique navis circa axem verticalem per centrum gravitatis transeuntem converti potest, talemque effectum in navibus gubernaculum praecipue producit. Hi autem quinque effectus, quanquam inter se ita sunt connexi ut plerumque plures ab una potentia oriantur, tamen singuli separatim considerari calculoque elici possunt, quemadmodum ex principiis ante traditis intelligere licet. Unusquisque enim effectus perinde producitur a potentiis atque determinatur, sive reliqui effectus simul producantur sive secus; et hancobrem si quaecunque potentiae navigium sollicitent, totalis effectus cognoscetur, si separatim, quantum in singulis memoratis quinque effectuum speciebus efficiatur, diligenter investigetur. Quo circa unumquemque de his quinque effectibus seorsim contemplabor, ac quomodo singuli a potentiis sollicitantibus producantur, animum a reliquis abstrahendo, ostendam. Ante omnia autem si proposita fuerit potentia navem corpusve quodcunque aquae innatans sollicitans, inquirendum est, utrum ea eiusmodi effectum, de quo quaeritur, producere valeat, an minus; non enim quaevis potentia ad quemvis effectum producendum est apta. Deinde si

compertum fuerit potentiam eiusmodi effectum exerere, tum quantitas istius effectus determinari debebit. Hocque modo cum singulae quinque effectuum species memoratae erunt pertractatae, facile erit iudicare, quid potentiae quaecunque in data navi sint effecturae.

PROPOSITIO 43

PROBLEMA

426. Si navis a quibuscunque potentiis sollicitetur, determinare effectum earum, quem exerent in navi magis minusve aquae immergenda.

SOLUTIO

Ad diiudicandum, quanto magis minusve navis a potentiis immergatur, ad eius centrum gravitatis respici oportet, et investigari, utrum id a potentiis deorsum vel sursum urgeatur, an secus. Quamobrem, sicut ad motum centri gravitatis cognoscendum facere oportet, omnes potentiae in directionibus sibi parallelis ipsi centro gravitatis applicatae concipi debebunt; singulaeque resolvi in verticales et horizontales, quarum illae solae eum effectum producent, in quem hic inquirimus. Si igitur fuerit quaecunque potentia p cuius directio cum horizonte angulum faciat, cuius sinus sit m, posito sinu toto $=1$, erit mp ea potentia, qua navis sursum deorsumve urgebitur; sursum scilicet sollicitabit navem, si directio potentiae sursum vergat, deorsum vero si deorsum. Quare si ex singulis potentiis sollicitantibus valores mp eliciantur, et in unum colligantur, habebitur totalis vis navem vel sursum elevans vel deorsum deprimens. Aequivaleat ista vis collecta ponderi P, tendatque deorsum; si enim sursum agat, tantum valorem P negativum accipere oportebit. Ab hac ergo vi P, si quidem P affirmativum habuerit valorem, navis magis immergetur; quanto profundius autem immergatur ita definietur. Sit massa seu pondus totius navis $=M$; volumen partis submersae $=V$, et sectio aquae $=E$, ponatur vero altitudo verticalis, qua navis aquae profundius immergetur $=Z$, quam pono vehementer exiguam, quod vires verticales seu pondus P plerumque valde exiguum respectu ponderis navis M esse soleat. Erit ergo post auctam navis immersionem volumen partis submersae $=V+Ez$, hincque nascetur per principia hydrostatica ista proportio $M:V=M+P:V+Ez$, seu ista aequatio $PV=MEz$, ex qua oritur $z=\frac{PV}{ME}$. . Profundius itaque centrum gravitatis navis subsidet, a vi P, et descendet per intervallum ~~. At si potentiae sollicitantes navem elevent, atque totalis vis elevans aequivaleat ponderi P, tum centrum gravitatis ascendet per intervallum $\frac{PV}{ME}$, seu quod idem est, descendet per intervallum $-\frac{PV}{ME}$, quae expressio ex illa nascitur ponendo -*P* loco *P*, uti iam invenimus. Q. E. I.

COROLLARIUM 1

427. Si ergo omnes potentiae sollicitantes directiones habeant horizontales, vel si potentiae verticales quae ex illis oriuntur se mutuo destruant, tum navis neque magis deprimetur in aquam, neque elevabitur.

COROLLARIUM 2

428. Si plures potentiae navem sollicitent, tum ex singulis ascensum descensumve centri gravitatis concludere licebit, quippe qui singuli effectus collecti verum centri gravitatis sive ascensum sive descensum indicabunt.

COROLLARIUM 3

429. Quia centri gravitatis ascensus descensusve fit per spatium $\frac{PV}{ME}$, at vero volumen partis submersae *V* semper sit proportionale ponderi navis *M*, sequitur maiorem minoremve navis immersionem proportionalem esse vi urgenti *P* directe, et sectioni aquae *E* inverse.

COROLLARIUM 4

430. Quo ergo maior fuerit sectio aquae, eo minor erit mutatio partis aquae immersae ab eadem vi sollicitante orta. Quamobrem quo naves eiusmodi mutationi minime sint obnoxiae, sectionem aquae amplissimam efficere expediet.

COROLLARIUM 5

431. Si aquae volumen, cuius pondus aequale sit ponderi *P*, ponatur *u*, atque hoc volumen adhibeatur ad quantitatem vis sollicitantis exprimendam, erit ob $V : M = u : P$ spatium, quo centrum gravitatis sursum deorsumve urgetur $= \frac{u}{E}$; ex qua expressione facillime, quanto navis magis minusve aquae immergatur, intelligi potest.

SCHOLION

432. Ad effectum in omni corpore, quem potentiae sollicitantes producunt, cognoscendum duplici investigatione est opus, altera qua motus progressivus centri gravitatis definitur, altera qua motus corporis gyratorius circa centrum gravitatis quaeritur. Eandem ergo rationem in effectu potentiarum navigia aliave corpora aquae innatantia sollicitantium inquirendo adhiberi oportet, quae cum natura aquae, qua libertas corporum sese quacunque movendi restringitur, coniungi debet. Quod enim primum ad motum centri

gravitatis attinet, eum in corporibus aquae innatantibus duplicem considerari convenit, prout eius directio vel horizontalis est vel verticalis; motus namque horizontalis si semel fuerit impressus, perpetuo conservatur, nisi quatenus ab aquae resistentia retardatur, motus verticalis autem statim sistitur, ac tanta pars sub aqua versatur, quantam hic definivimus; atque hancobrem in ista propositione non tam ipsum centri gravitatis ascensum descensumve determinare suscepimus, quam terminum illum quo sistitur, et in quo centrum gravitatis acquiescit. Quanquam enim revera gravitatis centrum, cum moveri incepit, subito non quiescit, tamen eiusmodi motu oscillatorio definiendo supersedendum censuimus, cum ab ipsa aqua statim sistatur. Pergo igitur ad alterum casum motus centri gravitatis progressivi investigandum, quo a potentiis sollicitantibus motum in directione horizontali adipiscitur.

PROPOSITIO 44

PROBLEMA

433. *Si navis a quibuscunque potentiis sollicitetur, determinare effectum earum in motu horizontali progressivo vel generando vel alterando.*

SOLUTIO

Quoniam de motu centri gravitatis progressivo quaestio est, omnes potentiae in directionibus parallelis ipsi centra gravitatis applicatae concipi debent, et quoniam motus tantum horizontalis investigatur, omnes potentiae resolvendae sunt in verticales et horizontales, quae posteriores tantum huic instituto sunt accommodatae. Si igitur quaecunque potentia fuerit *p*, cuius directio cum horizonte faciat angulum cuius sinus sit *m*, erit $p\sqrt{(1-mm)}$ potentia horizontalis effectum hic quaesitum producens. Omnium ergo potentiarum sollicitantium quaerantur hoc modo vires horizontales, earumque, si in ipso centra gravitatis sint applicatae, tum media directio tum potentia aequivalens, quae exprimatur pondere P. Iam sit ACBD (Fig. 69) sectio navis horizontalis per centrum gravitatis *G* facta, atque *GP* sit media directio omnium potentiarum sollicitantium, quatenus motum horizontalem afficiunt. Sollicitabitur ergo navis in directione *GP* a potentia *P*, cuius & vis aequivaleat ponderi *P*. Si nunc pondus totius navis ponatur $= M$, erit vis accelerans $= \dfrac{P}{M}$, qua in directione *GP* accelerabitur. Scilicet si navis iam

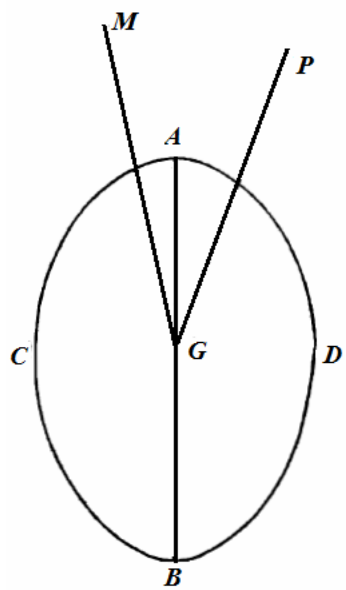

Fig. 69

moveatur in directione *GP* celeritate debita altitudini *v*, erit, dum navis elementum spatii *dx* absolvet,

$$dv = \frac{Pdx}{M}.$$

At si motus, quem navis iam habet non fiat in directione *GP*, sed in alia *GM*, quae cum *GP* angulum faciat *MGP*, cuius sinus sit $= m$, cosinus vero $= n$ posito sinu toto $= 1$, ita ut sit $m^2 + n^2 = 1$, tum a vi sollicitante *P* cum celeritas navis, quae debita sit altitudini v mutabitur, tum ipsa directio *MG*. Celeritas autem augebitur a vi $\frac{nP}{M}$ ita ut dum navis elementum *dx* percurrit, futurum sit

$$dv = \frac{nPdx}{M}.$$

A vi autem altera normali $\frac{mP}{M}$ cogetur navis a semita rectilinea *GM* deflectere versus directionem *GP*, ac dum percurret spatiolum *dx*, declinabit a directione *GM* versus *GP* angulo: cuius sinus erit $= \frac{mPdx}{2Mv}$. . Atque ex his tam generatio quam alteratio motus navium progressivi a potentiis quibuscunque orta, cognoscitur. Q. E. I.

COROLLARIUM 1

434. Si ergo navis quieverit, a potentiis sollicitantibus motum consequetur in ipsa directione media *GP*; eoque celerius moveri incipiet, quo maior fuerit vis *P*, simulque quo minus fuerit pondus ipsius navis *M*.

COROLLARIUM 2

435. Simili modo si navis iam habeat motum in ipsa directione *GP*, is accelerabitur eo fortius, quo maior fuerit potentia *P*, et quo minus fuerit pondus navis *M*. Momentum enim accelerationis est directe ut vis P, et reciproce ut pondus navis.

COROLLARIUM 3

436. Sin autem motus, quem navis iam habet, fiat in alia directione *GM*, tum acceleratio eo erit maior, quo minor fuerit angulus *MGP*; proportionalis enim est ceteris paribus cosinui *MGP*.

COROLLARIUM 4

437. Quod autem in hoc casu ad declinationem a directione *GM* attinet, ea proportionalis erit directe ipsi potentiae *P* et sinui anguli *MGP* coniunctim, inverse vero ponderi navis et quadrato celeritatis coniunctim.

COROLLARIUM 5

438. Quo celerius igitur navis iam movetur, eo minus ab eadem vi oblique agente de via sua deflectitur; atque ceteris paribus erit deflexio in reciproca ratione duplicata celeritatis.

SCHOLION

439. Haec omnia non solum ita se habent, sed etiam totus navis motus ad datum quodvis tempus ex iis posset definiri, si modo aqua nullam opponeret resistentiam, neque tam ipsum motum retardaret, nec effectum potentiarum turbaret. Propter aquae resistentiam enim primum motus navium insigniter retardatur, idque eo magis quo celerius navis movetur; aestimatur namque experientiam consulendo aquae resistentia quadrato celeritatis proportionalis. Deinde vero directio resistentiae praecipue est attendenda, quae nisi in ipsam motus navis directionem incidat, simul etiam eius directionem afficit, atque navem de via sua deflectit. Quamvis autem in sequente demum capite effectum resistentiae simus indagaturi, tamen inter potentias locum habet, et cum eius quantitas et directio fuerit definita, ipse effectus ex his ipsis principiis determinari debet. Quamobrem si navis iam habeat motum, cum potentiis sollicitantibus simul resistentia est coniungenda, et effectus, quem exerit tam in motu navis afficiendo, quam navem inclinando et convertendo ex hoc capite erit derivandus. Sic in hac propositione *PM* exprimere poterit mediam directionem non solum potentiarum sollicitantium sed etiam resistentiae ab aqua ortae; atque *P* denotare vim coniunctim ex potentiis sollicitantibus et resistentia natam. Quare cognitis resistentiae cum quantitate tum directione, ex hac propositione navis celeritas et directio quovis loco, dum promovetur, exquisite definiri poterit.

PROPOSITIO 45

PROBLEMA

440. *Si navis a potentiis quibuscunque sit sollicitata, invenire momenta virium navem tum circa axem horizontalem longitudinalem tum latitudinalem inclinantium.*

SOLUTIO

Sit *ACBD* (Fig. 70) sectio navis horizontalis per centrum gravitatis *G* facta, in qua extat *AB* axis longitudinalis, *CD* vero axis latitudinalis, circa quorum axium utrumque, quanta vi data potentia navem inclinet, hic inc

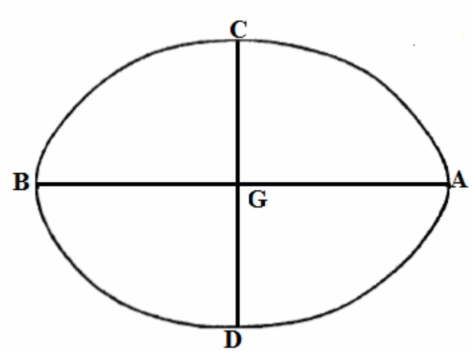

Fig. 70

vestigari oportet. Intelligitur autem pro utroque axe simile ratiocinium esse instituendum, similique modo vim navem tam circa axem *AB*, quam circa *CD* inclinantem esse indagandam; quamobrem sufficiet pro alterutro tantum axe puta *AB* quaestionem absolvisse. Primo autem notandum est, nullam potentiam, cuius directio vel cum axe *AB* concurrat vel eidem sit parallela vel tantum cum hoc axe in eodem plano sit sita, navem circa hunc axem convertere posse. Quamobrem eae tantum potentiae hic sunt considerandae, quarum directiones cum axe *AB* non in eodem plano sunt positae. Sit igitur quaecunque potentia navem sollicitans aequivaiens ponderi p, per cuius directionis punctum quodvis ad axem *AB* ducatur perpendicularis, cuius longitudo sit f; deinde concipiatur planum per axem *AB*, et hanc perpendicularem ductum, et quaeratur angulus, quem directio potentiae cum hoc plano constituit, cuius sinus sit $= m$. Quo facto momentum potentiae ad navem circa axem *AB* convertendam erit $= mpf$. Simili modo quaerantur eiusmodi momenta ex singulis potentiis sollicitantibus, et omnes, respectu habito ad ipsam actionem, utrum in eandem plagam, an in contrariam navem convertere conentur, in unum colligantur, quae reducentur ad eiusmodi simplicem expressionem Pa, in qua P pondus quodpiam, a vero rectam quandam denotabit. Hoc igitur modo quotcunque potentiae navem sollicitaverint, momenta respectu tam axis *AB* quam axis *CD* definientur.
Q.E.I.

COROLLARIUM 1

441. Quia indifferens est quodnam punctum in directione potentiae sollicitantis accipiatur, poterit planum quodpiam per axem ductum pro lubitu accipi, idque punctum notari, in quo directio potentiae illi plano occurrit.

COROLLARIUM 2

442. Expediet ergo ad hoc commodissime transigendum planum accipere vel verticale vel horizontale, per axem, circa quem momentum convertens inquiritur, ductum. Saepius autem horum planorum alterum alteri erit anteferendum, quae electio facillime cuique patefiet.

COROLLARIUM 3

443. Perspicitur ergo, si cuiuspiam potentiae directio vel per ipsum centrum gravitatis *G* transeat, vel in plano *ACBD* sita sit, tum navem circa neutrum axem conversum iri, neque propterea ullam inclinationem pati.

SCHOLION

444. Cum circa quemcunque axem inclinatio duplex sit, pro duplici plaga secundum quam inclinatio fieri potest, hoc in inquisitione momentorum diligenter est attendendum, quo omnibus momentis definitis appareat, an omnino in eandem plagam navem convertere conentur, an secus: illo enim casu omnia momenta in unam summam colligantur, hoc vero ea momenta, quae in plagam oppositam tendant, subtrahi debent. Quo autem facilius hoc discrimen oculis obversetur, atque citissime animadvertatur, pro utroque axe ambas plagas probe inter se dignovisse, et idoneis nominibus appellasse iuvabit. Ita navis circa axem latitudinalem *CD* duplici modo inclinari potest, vel proram vel puppim versus, inclinatur autem versus proram vel puppim, dum vel prora vel puppis magis immergitur. Circa axem longitudinalem *AB* autem inclinatio fit vel ad latus dextrum vel sinistrum, quae denominatio desumitur ab eo, qui in puppi stans proram aspicit. In colligendis igitur momentis plurium potentiarum respectu axis vel *AB* vel CD, probe est notandum in utram plagam quaeque potentia conetur inclinare navem, quo collectio fiat legitima. Invento autem momento totali ex omnibus potentiis sollicitantibus collecto, ipsa inclinatio est definienda, id quod sequente propositione praestabitur.

PROPOSITIO 46

PROBLEMA

445. *Si navis a potentiis quibuscunque sollicitetur, determinare angulum, quo ea tum circa axem latitudinalem tum longitudinalem inclinetur*.

SOLUTIO

Consideremus primo axem latitudinalem *CD* (Fig. 70), sitque momentum ex omnibus potentiis ortum ad navem circa hunc axem *CD* sive proram sive puppim versus inclinandam $=Pa$, atque anguli inclinationis, quem producit, sinus sit $=w$, quem tanquam vehementer parvum specto. Sit iam stabilitas navis respectu eiusdem axis latitudinalis $CD=F$, eo modo expressa, quo in capite praecedente fecimus, erit F w momentum, quo navis sese proprio conatu in situm erectum restituere annititur. Cum igitur hunc situm a potentiis sollicitantibus conservari ponamus, necesse est ut sit $Pa=Fw$, ex qua aequatione oritur anguli inclinationis productae sinus $w=\frac{Pa}{F}$ Simili modo, si dicatur stabilitas navis respectu axis longitudinalis $AB=\Phi$, atque momentum totale potentiarum navem circa hunc axem inclinare tendentium fuerit $=Qb$, erit anguli ad quem navis actu circa axem AB inclinabitur sinus $=\frac{Qb}{\Phi}$. Q.E.I.

COROLLARIUM 1

446. Constat igitur, quod quidem ex praecedentibus iam manifestum est, inclinationem, quam data potentia producit, eo fore minorem, quo maior fuerit stabilitas navis respectu axis, circa quem fit inclinatio.

COROLLARIUM 2

447. Stabilitas nostro recepto modo designatur per factum ex pondere vavis in lineam quandam rectam: unde facile intelligitur fractionem $\frac{Pa}{F}$ denotare merum numerum, qui exprimet sinum anguli inclinationis posito sinu toto $=1$.

COROLLARIUM 3

448. Erit ergo sinus anguli inclinationis ad sinum totum, uti momentum potentiarum inclinationem efficientium, ad stabilitatem navis respectu illius axis circa quem fit inclinatio.

COROLLARIUM 4

449. Si igitur stabilitas navis respectu cuiuspiam axis decies maior fuerit, quam momentum virium inclinantium, tum inclinatio minor erit 6 gradibus, prodit enim hoc casu angulus inclinationis circiter 5°, 45'.

COROLLARIUM 5

450. Intelligitur etiam, quo magis vires inclinantes a centro gravitatis fuerint remotae, eo maius fore momentum ad inclinandum, et propterea inde eo maiorem produci inclinationem.

SCHOLION

451. Quemadmodum a potentiis centrum gravitatis sollicitantibus in navigiis duplex nascitur effectus, quorum alter in maiore vel minore immersione consistit, alter vero in promotione navis horizontali, ita ex potentiis, quae corpora circa centrum gravitatis gyrari solent, in navibus triplex effectum oritur, pro tribus axibus, circa quos navis converti potest. Si enim in omni navi tres axes per centrum gravitatis transeuntes concipiamus, duos horizontales, alterum longitudinalem scilicet, alterum latitudinalem, et unum verticalem, navis a potentiis circa singulos converti poterit, ita ut conversio circa unum non turbet conversionem circa reliquos. Effectus autem harum conversionum circa tres istos axes propter actionem aquae inter se penitus sunt dissimiles, et hancobrem seorsim sunt evolvendi. Vires enim, quae tendunt ad navem circa alterutrum axem horizontalem convertendam, effectum suum statim consequuntur,

qui cum semel fuerit productus, nulla amplius mutatio in navi oritur. Consistit enim harum virium effectum in inclinatione circa eiusmodi axem ad certum angulum usque, quoad istae vires a stabilitate navis in aequilibrio conserventur; atque si inclinatio fuerit facta ad hunc angulum, tum navis in hoc statu persistit, si quidem vires eaedem maneant; at quam primum vires vel augentur vel diminuuntur vel penitus cessant, tum inclinatio vel augebitur vel deminuetur vel navis prorsus se in situm naturalem recipit, nisi forte ob motum iam receptum motus oscillatorio similis producatur. Longe aliter autem est comparata ratio conversionis circa axem verticalem, viribus enim quae eiusmodi conversionem producunt, nulla vis propria resistit, et hancobrem navis talibus viribus circa axem verticalem tamdiu convertitur, quamdiu vires agunt, neque conversio ante sistitur, quam vires penitus cessaverint, atque motus conversionis iam conceptus a resistentia aquae absorbeatur. Quocirca ad effectum eiusmodi virium convertentium cognoscendum ipsum motum conversionis indagari oportet.

PROPOSITIO 47

PROBLEMA

452. *Si navis vel corpus quodcunque aquae innatans sollicitetur a potentiis quibusvis, determinare vis momentum, quo corpus circa axem verticalem per centrum gravitatis transeuntem circumagetur.*

SOLUTIO

Sit *ACBD* (Fig. 71) sectio horizontalis . navis per centrum gravitatis *G* facta, atque *EGF* axis verticalis per centrum gravitatis *G* ductus, circa quem motus conversionis inquiritur.Duplici autem modo navis circa hunc axem gyrari potest, convertendo se vel ad dextram vel ad sinistram; dico autem navem se ad dextram convertere, quando prora ei, qui in puppi stat proramque intuetur, ad dextram rotari videtur; cui conversioni

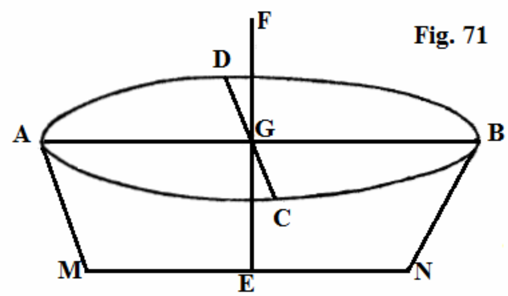

Fig. 71

contrarius motus ad sinistram fieri dicitur. De qualibet igitur potentia navem circa hunc axem verticalem circumagere valente videndum est, utrum ea navem versus dextram an versus sinistram rotari cogat, quo, si plures agant potentiae effectus conspirantium addi, contrariarum vero subtrahi facillime queant. Animadvertendum autem est ante omnia, nullam potentiam, cuius directio vel transeat per centrum gravitatis, vel sit verticalis, vel cum axe *EF* in eodem plano consistat, eiusmodi motum conversionis producere valere. Cum igitur omnes potentiae resolvantur in verticales et horizontales, in solis horizontalibus causa eiusmodi conversionis circa axem *EF* erit quaerenda. Ita autem ipsa vis convertens investigabitur. Concipiatur planum horizontale in quo sita sit directio potentiae cuiuspiam horizontalis, noteturque punctum in quo axis *EF* ab hoc plano secabitur. Deinde ex hoc puncto in directionem potentiae ducatur normalis, quae per ipsam potentiam multiplicata dabit momentum eius vis ad motum gyratorium circa axem *EF* producendum. Vel ex supra dicto axis *EF* puncto recta quaecunque duci potest ad directionem potentiae sollicitantis horizontalis, haecque recta tum in sinum anguli quem cum directione potentiae constituit, tum in ipsam potentiam ducta dabit momentum, Si igitur ex singulis potentiis sollicitantibus horizontalibus istiusmodi momenta eliciantur, eaque, ratione habita utrum omnia ad eundem effectum producendum conspirent, an quaedam sint contraria, in unam summam colligantur, prodibit eiusmodi expressio *Pa*, factum scilicet ex pondere quodam in quampiam rectam datam, quod factum exhibebit totale virium momentum, quo motus conversionis circa axem *EF* generabitur. Q. E. I.

COROLLARIUM 1

453. Omnes igitur potentiae horizontales exceptis iis, quarum directiones transeunt per axem conversionis *EF*, tendent ad navem circa hunc axem convertendam, atque actu convertent, nisi plures eiusmodi potentiae se mutuo destruant.

COROLLARIUM 2

454. Quaecunque igitur potentia horizontalis eo maiorem habebit vim ad navem circa axem verticalem circumagendam, quo maior fuerit tum ipsa vis, tum eius distantia ab axe conversionis *EF*, quae distantia mensuratur recta horizontali tam ad axem, quam ad directionem potentiae normali.

COROLLARIUM 3

455. Cum autem navis quoque a potentiis horizontalibus propellatur, iisdem potentiis, quibus navis promovetur, navis etiam circa axem verticalem convertetur. nisi media directio omnium per ipsum axem transeat.

COROLLARIUM 4

456. Ne igitur, quae potentiae navem promovent, eaedem navem declinent seu circa axem verticalem convertant, necesse est ut aut singularum potentiarum sollicitantium directiones, aut saltem earum media directio per verticalem e centro gravitatis eductam transeat.

SCHOLION

457. In navibus iste motus conversionis circa axem verticalem per centrum gravitatis transeuntem maximi est momenti, eique producendo, quoties opus est, gubernaculum est destinatum, cuius ope navis si in motu fuerit constituta, tum ad dextram tum ad sinistram potest deflecti. Quoniam enim naves vel exacte vel saltem proxime secundum directionem spinae progredi solent, actione gubernaculi ipse navis cursus immutatur: scilicet quando conversio fit ad dextram, tum simul cursus navis ad dextram hoc est a septentrione versus orientem, vel hinc versus austrum, vel ab austro versus occasum, hinve versus boream deflectitur; in plagas autem contrarias deflectitur cursus, quando conversio fit ad sinistram. Ex his igitur iam intelligitur effectum gubernaculi eo fore maiorem, quo longius id a centro gravitatis removeatur, quamobrem ipsi etiam in extrema puppi suus assignatus est locus. Praeterea vero etiam eximius est gubernaculi usus in cursu navis directo et immutato conservando, quo opus est quando potentiae sollicitantes simul vim habent navem convertendi circa axem verticalem, tum enim ope gubernaculi haec vis est destruenda. Ne autem hoc eveniat, quod ingens merito censetur incommodum, in id maxime incumbi solet, ut tam potentiarum sollicitantium media directio per illum axem verticalem transeat, quam resistentia a quae etiam vi careat navem convertendi. Quamobrem quo istud incommodum evitetur, tam idoneus malorum locus, quippe quibus vires applicari solent navem propellentes, diligenter eligendus, quam anterior navis figura, a qua resistentia eiusque directio pendet, summo studio est determinanda; quae omnia in sequentibus fusius evolventur. Nunc autem restat, ut ipsum motum rotationis circa axem verticalem determinemus, in quo, quia de resistentia nondum constat, animum ab aquae resistentia omnino abstrahemus; parum autem interest nosse quantum iste motus conversionis a resistentia aquae retardetur; dummodo enim constet, istum effectum sequi, atque iudicari queat, quomodo se habeat eius celeritas pro variis potentiis sollicitantibus, pro variaque navium conditione, ad institutum abunde sufficit. Quamobcausam etiam in sequentibus capitibus tantum investigabimus, quantum resistentia aquae motum navis progressivum retardet, neque erimus solliciti, quantum conversionem seu deflexionem impediat.

PROPOSITIO 48

PROBLEMA

458. *Si navis a potentiis ad motum rotatorium circa axem verticalem incitetur, determinare ipsum motum, quo circa hunc axem convertetur.*

SOLUTIO

Ex iis quae supra de motu rotatorio circa axem quempiam per centrum gravitatis transeuntem definiendo sunt demonstrata, intelligitur ad hoc negotium duabus opus esse rebus, momento scilicet potentiarum respectu illius axis sumto, atque momento materiae seu inertiae corporis respectu eiusdem axis. Cum igitur in praecedente propositione momentum ex omnibus potentiis sollicitantibus resultans definire docuerim, quod tendat ad navem circa axem verticalem per centrum gravitatis ductum convertendam, quod sit $= Pa$ facto scilicet ex pondere quopiam P in rectam datam a, superest ut momentum inertiae seu materiae totius navis respectu eiusdem axis determinetur, quod invenietur multiplicando singulas navis particulas per quadrata distantiarum suarum ab axe illo verticali, quorum productorum aggregatum huiusmodi habebit formam Mb^2, in qua M denotat pondus navis, b vero rectam longitudine datam. Vis igitur gyratoria, qua navis actu circa axem verticalem per centrum gravitatis transeuntem circumagetur erit $= \frac{Pa}{Mb^2}$, ex qua motum angularem definire licebit. Si nunc ponamus navem iam tantum habere motum angularem circa axem verticalem, ut punctum quodpiam in distantia f ab axe situm celeritatem habeat altitudini v debitam, erit, dum illud punctum arculum dx absolvit $dv = \frac{Pafdx}{Mb^2}$. Hinc integrando fiet $v = \frac{Pafx}{Mb^2}$, ubi x arcum ab illo puncto ab initio motus iam descriptum denotat. Sit autem s angulus iam descriptus, est

$$s = \frac{v}{f}, \quad \text{ideoque} \quad \frac{v}{ff} = \frac{Pas}{Mb^2}.$$

Celeritas igitur angularis iam acquisita erit

$$= \frac{\sqrt{v}}{f} = \sqrt{\frac{Pas}{Mb^2}},$$

siquidem resistentia aquae animo removeatur. Q. E. I.

COROLLARIUM 1

459. Manente igitur vi convertente eadem navis eo facilius circa axem verticalem convertetur, quo minus fuerit momentum inertiae navis respectu eiusdem axis.

COROLLARIUM 2

460. Ista ergo navis conversio eo facilius absolvetur, quo propius omnia onera ad axem verticalem per centrum gravitatis transeuntem collocentur. Contra autem, si omnia onera ab hoc axe maxime fuerint remota, conversio fiet difficillima.

COROLLARIUM 3

461. Prout ergo navis vel facillime conversionem admittere, vel conversioni maxime resistere debet, ita inde ratio onerationis respectu axis verticalis per centrum gravitatis ducti erit petenda.

COROLLARIUM 4

462. Hinc intelligitur navem eo citius actioni gubernaculi obsequi, quo propius merces reliquaque onera ad axem verticalem collocentur. Hoc enim pacto momentum inertiae navis eo minorem obtinebit valorem.

COROLLARIUM 5

463. Exista ergo propositione, si impulsus aquae in gubernaculum fuerit definitus, effectus gubernaculi pro quaque navi facile poterit diiudicari ac determinari.

SCHOLION

464. Expositi igitur atque definiti sunt quinque effectus, quos potentiae in corpore quocunque aquae innatante producere valent, qui ita a se invicem sunt disiuncti, ut quisque sine reliquis locum habere queat. Quamobrem si, dum corpus aquae innatans a potentiis quibuscunque sollicitetur, singuli isti quinque effectus determinentur, constabit quomodo corpus a potentiis afficiatur. Definitum enim erit primo, quanto corpus magis minusve aquae immergatur, deinde quanta vi ad motum progressivum et in quanam directione urgeatur; tertio et quarto cognoscetur, quantum corpus cum circa axem horizontalem longitudinalem inclinetur; ac quinto denique patebit, quanta vi corpus circa axem verticalem per centrum gravitatis ductum convertatur. Cum igitur in his quinque effectibus omnis potentiarum actio consistat, hoc caput finiemus, atque ad aquae resistentiam definiendam progrediemur, quippe qua opus est ad ipsum navium motum determinandum.