

## CHAPTER VIII

### CONCERNING MUSICAL MODES

1. Thus far we have established the nature of tones in general, and from these the precepts of the formation of harmonies are required to be formed, nor at this stage was there a place for the special precepts of the composition of music required to be treated. For before these precepts may be allowed to be treated in practice, it will be required to adapt [*i.e.* tuned to some scale] the musical instruments, and to discuss the manner in which these may be considered to be set up. For indeed since the tones, which are used for the production of musical works, are either with the help of the living voice, or to be heard being played by instruments; but before everything, both the voice as well as suitable instruments are required to render all the sounds required to be produced, for which there is a need in the musical work being expressed.

2. Therefore since the exponent of a musical work shall contain all the necessary tones, it will be observed from that same exponent, how many tones and of what kind must pertain to the musical instruments. Therefore the construction of musical instruments depends on the exponent of the musical works, with the aid of which they must be presented to the listeners; thus so that, if we may wish to represent musical works of other kinds, for those other kinds of musical instruments also may be required, which shall be adapted according to these exponents.

3. Therefore for the proposed exponent of a musical work, the instruments thus must be adapted for the tones required to be expressed, so that in these all the tones may be present, which that exponent includes within itself, unless perhaps certain tones shall be of too low or too high a pitch, so that they may not be perceived by the ear, which since they are superfluous, may safely be omitted. But the tones, which the proposed exponent may contain within itself, may be deduced from the divisors of this; on which account musical instruments are required to be constructed thus, so that all the perceptible tones may be understood to be expressed from the divisors of the exponent. Truly on the other hand it is understood for a given musical instrument, what kinds of musical works it may be suitable for producing.

4. Truly also the notes, which may be contained in a given musical instrument, may be indicated by the exponent most conveniently, which, at this point, is the smallest common divisor of all the notes contained in that instrument. Therefore from the exponent of the musical instrument it is understood, how it shall be adapted to a musical work of any kind requiring to be produced. Evidently other musical works are not able to be expressed by this instrument, unless the exponent of these shall be a divisor of the exponent of the instrument. Moreover for this it is required, that the tones may be available in all the

instruments, which arise from the divisors of its exponent; of these indeed if some are missing, the instrument may be disadvantaged nor to be of any suitable use.

5. Therefore for a well constructed musical instrument a suitable exponent is required to be selected, which may contain the exponents of all the musical works required to be produced by the instrument with the aid of its exponents. With which done, all the divisors and tones of this exponent which are to be investigated, and which may be expressed by these individual divisors, must be able to be played on the instrument, only with these excepted, which on account of being excessively deep or acute may be unable to be heard. But as well as these tones others for the sake of uniformity are able to be added, so that the sounds contained in individual octaves may become equal in number. And this not only happens in use, but also brings about more perfect instruments, so that they shall be adapted for more musical works being produced.

6. Therefore not only some divisor of the exponent may be assumed to produce a tone in the instrument, but also its double, quadruple, octuple etc., likewise its half, quarter, eighth, etc., parts. For with this agreed on it will happen, that all the diapason [*i.e.* octave] intervals may be said to be filled by an equal number of tones, and also they may become divided in a similar manner. From which also this convenience will be obtained, so that, if one octave were fitted correctly, from that the remaining octaves both higher as well as lower may be formed easily; so that there becomes of the single sounds contained in one octave of individual tones, others both higher as well as lower by one or more octaves may be put in place.

7. Therefore if the exponent of the instrument were  $A$ , its divisors shall be

$$1, a, b, c, d, e \text{ etc.},$$

in addition to the tones from these denoted divisors, also the tones

$$2, 2a, 2b, 2c, 2d \text{ etc.}, \text{ likewise } 4, 4a, 4b, 4c \text{ etc.},$$

then these also :

$$\frac{1}{2}, \frac{1}{2}a, \frac{1}{2}b, \frac{1}{2}c \text{ etc.}, \text{ likewise } \frac{1}{4}, \frac{1}{4}a, \frac{1}{4}b, \frac{1}{4}c \text{ etc.}$$

likewise must be produced by the instrument. But by multiplication with the fractions removes all the tones produced by the instrument will be :

$$2^n, 2^n a, 2^n b, 2^n c, 2^n d, \cdot \cdot \cdot 2^n A,$$

where  $n$  indicates some whole number. Therefore the exponent of an instrument constructed in this manner no longer will be  $A$ , but  $2^m A$  with  $m$  denoting some indefinite number neither too small or large, so that the tones shall be perceptible.

8. Therefore the instrument prepared thus will not only be suitable for producing musical works, the exponents of which may be contained in  $A$ , but also for such works, of which the exponents may be included in  $2^m A$ . From which it is understood with the tones from all the octaves partaking equally, the musical instruments to follow with greater perfection and to be able to perform more musical works. Then beginners too thence have this convenience, so that with the tones known present in one octave likewise easily know the tones of the remaining octaves.

9. Therefore we may assume for the exponents of musical works in the latter form of this kind  $2^m A$  and we will investigate, how many tones and of what kind any octave must contain. But for  $A$  only odd numbers may be agreed to be taken, since, if even numbers were assumed, it would be superfluous on account of the factor of two now contained in  $2^m$ . Therefore any exponent  $2^m A$  will give a particular division of the octave, both on account of the number of tones as well as in the ratio of the intervals, which the tones hold between themselves. But the division of the octaves of this kind by musicians is accustomed [formerly known as a *genus*] to be called a *musical mode*; and of such kinds three have been known for a long time, which are called the *diatonic*, *chromatic* and *enharmonic modes*.

10. If the lowest tone of the octave were  $E$ , the division of which from the given exponent  $2^m A$  is sought, the most acute will be  $2E$  and the remaining tones all will be contained between the limits  $E$  and  $2E$ . Whereby the individual divisors of  $A$  will require to be multiplied by binary powers of this kind, so that the products made shall be greater than  $E$ , but less than  $2E$ , and all these made will be tones contained in the octave. From which it is observed there are just as many tones contained in the octave, as  $A$  may have divisors, since each single divisor of  $A$  shall introduce a tone into such an octave.

11. Therefore if the exponent of the instrument, which hereafter we will call the exponent of the kind of music, were  $2^m a^p$  with  $a$  proving to be a prime number, one octave will contain  $p+1$  tones, since  $a^p$  has just as many divisors. But if the exponent were  $2^m a^p b^q$ , then  $(p+1)(q+1)$  or  $pq+p+q+1$  tones will be contained in the octave; for the number  $a^p b^q$  has just as many divisors, but no more, if indeed  $a$  and  $b$  were unequal prime numbers. In a similar manner the exponent of the kind  $2^m a^p b^q c^r$  will give  $(p+1)(q+1)(r+1)$  sounds held within the interval of one octave. Therefore it will be possible to judge at once from the kind of the exponent, how many sounds may be contained in a single octave.

12. But such will be these tones contained in each octave, which themselves may be called the divisors of  $A$  ; for each one must be multiplied by its kind of binary powers, so that the ratio of the maximum to minimum may be had less than two. Truly with these, more conveniently logarithms will be taken, evidently from these which we have used here, from which it will be apparent since the logarithm of two shall be 1, it will be evident at once, any divisor must be some multiple of a power of two, so that the logarithms of all the tones greater than one will not disagree with each other.

13. Therefore we will review the general musical genus [or mode] from the most simple as far as to the most composite, which indeed can be used, both known as well as unknown, and concerning which we will observe as we please any musical work which shall be appropriate. But the simplest without doubt is the kind  $2^m$ , which is had if there is  $A = 1$ . Therefore the single tone 1 is contained in the interval of the octave, which at once the tone 2 follows after a whole octave. Therefore all the tones held in the musical instrument will be  $1 : 2 : 4 : 8 : 16$ , since seldom do musical instruments include more than 4 octaves. But this kind is useless for any harmony being produced on account of the excessive simplicity.

14. If there may be put  $A = 3$ , of which the divisors are 1 and 3, the exponent  $2^m A$  therefore will give the following ordered music mode, and thence of which the divisors constituting the octave are tones in the ratio  $2:3:4$ ; [*i.e.*  $2^1 < 3 < 2^2$ ]. Therefore in this mode the octave is divided into two parts, of which the one is [commonly called] the fifth interval, the other the fourth. Also by putting 3 for the lowest tone of this, an octave thus can be represented so that the form of this octave will be  $3:4:6$ , where the lower interval is called the fourth, and the upper the fifth. Truly all the sounds following the exponent  $2^m \cdot 3$  provided by an instrument include all the ratios  $2 : 3 : 4 : 6 : 8 : 12 : 16 : 24 : 32$ . Moreover this mode is extremely simple, thus so that at no time will it have been in use.

15. In music for ever and a day musicians have not considered other consonants, unless the exponents of which consist of the customary prime numbers 2, 3 and 5, thus so that musicians will not proceed beyond the fifths in the formation of consonants. On this account here also, in the beginning, in place of  $A$  besides 3 and 5 and the powers of these I may not assume other numbers ; from these truly, which hence can arise, in the general exposition of music we will attempt to introduce 7 as well ; from which perhaps now some new kind of music to be composed and unheard musical works will be able to be performed.

16. Therefore the third musical mode will be  $2^m \cdot 5$ , [*i.e.*  $2^2 < 5 < 2^3$ ] in which the tones are contained in the octave shall be in the ratio  $4 : 5 : 8$ , of which the lower of the two intervals makes a major third, and the upper is a minor sixth. But this mode both, as it is exceedingly simple, as well as containing the number 5 but with the 3 omitted, and thus has the simpler arranged consonants missing, cannot have a use. Indeed it would be incongruous to be using greater prime numbers with the smaller ones ignored, because

there besides in this way the harmony will be rendered more intricate and less pleasing to the ear.

17. In these two modes, there has been the first power either of 3 or 5 in  $A$ . And thus now we assume the second power and there shall be an exponent  $2^m \cdot 3^2$  of four modes, in which the divisors of the quantity  $A$  or  $3^2$  are in the ratio 1:3:9. Therefore the octave will contain these tones 8:9:12:16 [were 9 and 12 are the only possible multiples of 3 lying in the octave] and consist of three intervals, of which the first is a major tone, with the two remaining truly fourths. And this was the first mode, which was brought into use, of which the inventor was Mercury in Greece, who expressed these four tones by the same number of strings, after which the instrument has been called the *tetrachord*. Also musicians following, expressing their gratitude towards the venerated Mercury, have become accustomed to divide their more complex modes into sets of four notes, from this instrument.

18. Therefore from this, the first musical mode, which agrees magnificently with the laws of harmony and also for this reason the listeners, who before this point were ignorant of harmony, in total admiration the intervals were extended as well to the fifth, fourth, major tone and octave, other intervals were not pleasing to the ear. And also after this time as far as to the time of Ptolemy, the consonant called the third remained unknown, clearly as Ptolemy first introduced it into music.

19. The exponent  $2^m \cdot 3 \cdot 5$  will be of the fifth mode of the music, so that on account of the divisors 3, 5, 15 of  $3 \cdot 5$  itself, the tones 8:10:12:15:16 will be contained in one octave. Therefore the music certainly will be extremely pleasing with the third major and minor intervals, with the sixth major and minor, with the fifth and fourth, with the major semitone and the seventh major. Yet meanwhile it cannot be agreed why this mode was never in use, even if several of the variations were as suitable as the preceding mode of Mercury. Concerning this matter without doubt the reason is, since both the third as well as the major and minor will have been unknown on account of the number 5 until Ptolemy, moreover here he would have introduced more modes of composition.

20. The sixth mode constitutes the exponent  $2^m \cdot 5^2$ , in which octave on account of the divisors 1, 5, 25 present of  $5^2$ , maintaining that ratio of the tones 16:20:25:32, by which the octave may be cut into three intervals, of which the two first are major thirds, truly the last a major third plus a diesis [*i.e.* quarter tone]. So that it is no wonder the mode at no time has been put into use, as, since at the most ancient times the thirds were unknown, then, since the consonants generally contained in this shall not be known modes; and for this it is agreed, because it may be lacking the most charming intervals, such as the fifth and fourth.

21. For us the seventh mode will be that for which the exponent is  $2^m \cdot 3^3$ . Therefore the divisors of  $3^3$  are 1, 3, 9, 27, from which the following octave is put in place 16:18:24:27:32, but at no time was it agreed to be put into use.

The exponent of the eighth mode is  $2^m \cdot 3^2 \cdot 5$ , of which the six odd divisors are 1, 3, 5, 9, 15, 45, from which the following tones will constitute an octave 32:36:40:45:48:60:64. And this mode may deserve the greatest influence to be received into use, unless it may be present now in accepted modes.

The ninth mode has the exponent  $2^m \cdot 3 \cdot 5^2$  and holds the tones in the following octave 64:75:80:96:100:120:128.

Moreover the tenth mode of the exponent  $2^m \cdot 5^3$  will have these tones 64:80:100:125:128.

22. The eleventh mode therefore will have the exponent  $2^m \cdot 3^4$  and hence it will contain the tones 64:72:81:96:108:128 in the octave. It is to be observed concerning this mode and in the preceding, that one finds some intervals and consonants which indeed we do not find in use today; whereby also the mode, which is now in use and which may be called the *diatonic-chromatic* mode, does not include these two latter modes, truly it includes within it all the preceding modes, thus so that all the modes shall be adapted to the preceding musical works, just as likewise too the mode accepted in use now may serve.

23. Again the twelfth mode with the exponent  $2^m \cdot 3^3 \cdot 5$  may be determined; therefore it will contain these tones in the octave

$$128:135:144:160:180:192:216:240:256.$$

And this mode agrees closely with the old diatonic mode, even if the ancients will have arranged only seven tones in this mode. Indeed with the tone 135 omitted this mode in the first place agrees with Ptolemy's *diatonic syntonon* mode, in which the octave may be divided into two tetrachords, of which each interval includes a fourth and thus may be divided into three intervals, so that the lowest shall be a greater semitone, the second a major tone and the third a minor tone.

24. Truly this same division and this mode of ours have dispensed with the tone 135; for with the tones beginning from the sound 120 this form will be had:

$$120:128:144:160 \mid 180:192:216:240,$$

of which two parts each is a fourth interval divided thus, so that the lowest intervals 120:128 and 180:192 shall be major semitones, the middle intervals 128:144 and 192:216 major tones and the highest tones 144:160 and 216:240 minor tones.

Therefore Ptolemy's diatonic mode was endowed with exceptional charm, as also may be testified from experience, since this mode even now shall be in use, while other ancient modes endowed with little or no favour were ignored.

25. But since this old diatonic mode shall be without the tone 135, which still relates equally to the octave and to the other tones, it is not generally considered to be had perfectly ; yet meanwhile, since there is so much similarity between this and our twelfth order mode, we will call the *corrected diatonic*. But it is understood from this, that the old musicians had held on tenaciously to that first invented by Mercury, thus so that musical instruments were in tetrachords and the individual tetrachords were divided into three parts, which indeed set up in this mode has agreed well enough with some harmonies, but truly has been the cause of unpleasant harmonies for the remaining.

26. Besides this diatonic syntonon mode of Ptolemy, several kinds of diatonic modes were being used by the ancients, of which the intervals thus may be contained in individual tetrachords :

Pythagoras' Diatonic	243:256, 8:9, 8:9 ;
Soft Diatonic	20:21, 9:10, 7:8;
Diatonic Toniaon	27:28, 7:8, 8:9;
Equal Diatonic.	11:12, 10:11, 9:10

In which everything was established by this, so that the first interval shall be almost a semitone, the other two almost tones, but all likewise may complete a musical fourth. Moreover it is seen easily, that these modes shall be imperfect and absurd, so that it shall be no wonder, that they shall be completely extinct.

27. Moreover just as at this time musical instruments to be divided according to octaves and all the octaves are accustomed to be divided equally, thus the ancients loved to divide their instruments into fourths and the individual fourths to be cut equally into three intervals, so that in this matter they were following Mercury's tetrachord rather than the harmony itself. And this division especially by Pythagorean musicians were performed with arbitrary numbers with no regard to the harmony, as appears clear enough from the examples produced; and in this manner they brought forth not a little harm with these numbers, thus so that they shall have deserved the criticism from Aristoxenos and his followers.

28. But the diatonic scale of Ptolemy, which happily emanated from this perverse manner of treating music, even now deservedly to be seen in use with cymbals, clavichords and with other manually played instruments, in which keys if two kinds are had, which the longer and shorter produce the sounds of the diatonic harmonic mode. In whatever way these keys are accustomed to be designated with letters. Hence therefore

there will be the sound *C* indicated by the number 192, the following 216 *D*, 240 *E*, 256 *F*, 288 *G*, 320 *A*, 360 *H* and 384 *c*.

29. Again with the same letters the sounds of the higher octaves may be indicated but in small type, and with numbers expressed twice as great ; and these with small print with one or more bars indicate higher octaves. Thus since 320 shall be *A*, there will be 640 *a*, 1280  $\bar{a}$ , 2560  $\bar{\bar{a}}$ , 5120  $\bar{\bar{\bar{a}}}$  etc. On this account matters of this kind with either smaller or larger letters will correspond to tones expressed by the following numbers. *C* evidently may be called all the tones expressed in this formulas  $2^n \cdot 3$ ; *D* the notes in  $2^n \cdot 3^3$ , *E* the tones in  $2^n \cdot 3 \cdot 5$ , *F* the tones in  $2^n$ , *G* the sounds in  $2^n \cdot 3^2$ , *A* the sounds in  $2^n \cdot 5$  and *H* the sounds in  $2^n \cdot 3^2 \cdot 5$ . But the tone in use with the mode  $2^n \cdot 3^3 \cdot 5$  omitted is called *F#*, that is *F* exceeded by a semitone.

30. The exponent  $2^m \cdot 3^2 \cdot 5^2$  hence constitutes the thirteenth mode, of which therefore these 9 tones fill the octave :

$$128:144:150:160:180:192:200:225:240:256,$$

to which mode the ancients appear to have set out in a straight line, while they worked out the *chromatic mode*, if indeed they perceived any harmony in this chromatic mode. For they established initially two consecutive semitones in the tetrachord of this mode each followed by minor third, or rather the complement of two semitones to a fourth. But in our mode two semitones are removed twice, which omitted some number of minor thirds follow. Yet meanwhile the old chromatic certainly was by necessity imperfect, and thus this mode of thirteen duly corrected by us is the corrected chromatic mode.

31. With the ancients , three of the most powerful chromatic modes were being investigated, which were being divided into two tetrachords, and indeed they were dividing a tetrachord into three intervals, which thus were being found in these three kinds:

The old chromatic mode	243:256, 67:76, 4864:5427;
The soft chromatic mode	27:28, 14:15, 5:6 ;
The chromatic syntone	21:22, 11:12, 6:7

It may be easily observed from any of these kinds of chromatic modes, how many may differ from the true principles of harmony. But our chromatic mode retained on division into tetrachords in the following manner with the tones 225 and 150 omitted may be able to be called into use, to be received into the octave with these tones :

$$120:128, 144:160 \mid 180:192, 200:240,$$



in which the division of the first tetrachord is indeed a diatonic syntone, truly the other a true chromatic.

[Recall that the modern chromatic scale in music has 12 notes in steps each half a step apart, while a diatonic scale has 7 notes with 5 whole steps with 2 half steps usually 2 or 3 steps apart.]

32. The fourteenth mode, the exponent of which is  $2^m \cdot 3 \cdot 5^3$ , will have these tones in an octave :

$$256 : 300 : 320 : 375 : 384 : 400 : 480 : 500 : 512;$$

so that we will call the mode the *corrected enharmonic*, since it may be seen to have an agreement in a certain way to the old enharmonic. Indeed the ancients had abandoned tetrachord divisions of this kind

Old Enharmonic	125 : 128, 243 : 250, 64 : 81
Ptolemaic Enharmonic	45 : 46, 23 : 24, 4 : 5,

of which neither can be in agreement with harmony. But the ancients scales may be able to be used with some gratification if in place of the enharmonic mode here the octave may be divided into tetrachords and with the division of the tetrachords

$$240 : 250 : 256 : 320 \mid 375 : 384 : 400 : 480,$$

evidently with the tone 300 omitted ; but even with this left out the mode can be considered deficient.

33. The fifteenth mode will be contained by the exponent  $2^m \cdot 5^4$  and it will have the following tones in the octave :

$$512 : 625 : 640 : 800 : 1000 : 1024 ,$$

but this mode is unable to be used because of the harsher intervals and the lack of pleasing consonants provided by the multiples of 3.

Truly the exponent  $2^m \cdot 3^5$  may constitute the sixteenth mode in the octave of which the following tones will be present :

$$128 : 144 : 162 : 192 : 216 : 243 : 256 ,$$

which mode on account of the lack of consonants arising from 5 will not provide a satisfactory variation.

But the seventeenth mode is expressed by the exponent  $2^m \cdot 3^4 \cdot 5$  with the minimum incongruence, so that it may be taken into use; for it will contain whatever of its tones progressing in the following ratio

$$256 : 270 : 288 : 320 : 324 : 360 : 384 : 405 : 432 : 480 : 512.$$

For contrary to this mode, any other cannot be chosen, evidently divided up with that having exceedingly small intervals, by which the changes occurring in that will scarcely able to be heard .

34. Therefore it is required to set out the eighteenth mode, of which the exponent is  $2^m \cdot 3^3 \cdot 5^2$ ; but truly, since this itself is the diatonic-chromatic mode being used by all musicians, it is worthwhile that a special chapter may be set out for its treatment. For the remainder, by which at this stage the established modes may be set out more clearly with their exponents, the following table is seen to be added, in which both the exponents of each mode as well as the tones contained in some octave and likewise the intervals between whatever neighbouring tones have been described. Also the names of the tones not commonly known to be noted by an asterix written nearby.

TABLE OF THE MUSICAL MODES

Signs of the tones	Tones	Intervals	Names of the Intervals
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Mode I. Exponent  $2^m$  .

<i>F</i>	1		
<i>f</i>	2	1 : 2	Diapason

Mode II. Exponent  $2^m \cdot 3$  .

<i>F</i>	2		
<i>c</i>	3	2 : 3	Diapente or Fifth
<i>f</i>	4	3 : 4	Diatessaron or Fourth.

Mode III. Exponent  $2^m \cdot 5$  .

<i>F</i>	4		
<i>A</i>	5	4 : 5	Third major
<i>f</i>	8	5 : 8	Sixth minor.

Mode IV. Exponent  $2^m \cdot 3^2$ .

<i>F</i>	8		
<i>G</i>	9	8:9	Tone major
<i>c</i>	12	3:4	Fourth
<i>f</i>	16	3:4	Fourth

}	Most ancient mode of music of Mercury.
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Mode V. Exponent  $2^m \cdot 3 \cdot 5$ .

<i>F</i>	8		
<i>A</i>	10	4:5	Major third
<i>c</i>	12	5:6	Minor third
<i>e</i>	15	4:5	Major third
<i>f</i>	16	15:16	Major semitone.

Mode VI. Exponent  $2^m \cdot 5^2$ .

<i>F</i>	16		
<i>A</i>	20	4:5	Major third
<i>c#</i>	25	4:5	Major third
<i>f</i>	32	25:32	Major third with diesis.

Mode VII. Exponent  $2^m \cdot 3^3$ .

<i>F</i>	16		
<i>G</i>	18	8:9	Major tone
<i>c</i>	24	3:4	Fourth
<i>d</i>	27	8:9	Major tone
<i>f</i>	32	27:32	Minor third less a comma.

Mode VIII. Exponent  $2^m \cdot 3^2 \cdot 5$ .

<i>F</i>	32		
<i>G</i>	36	8:9	Major tone
<i>A</i>	40	9:10	Minor tone
<i>H</i>	45	8:9	Major tone
<i>c</i>	48	15:16	Major semitone
<i>e</i>	60	4:5	Major third
<i>f</i>	64	15:16	Major semitone

Mode IX. Exponent  $2^m \cdot 3 \cdot 5^2$ .

<i>F</i>	64		
<i>G</i> <i>s</i>	75	64:75	Minor third less a diesis
<i>A</i>	80	15:16	Major semitone
<i>c</i>	96	5:6	Minor third
<i>c</i> <i>#</i>	100	24:25	Minor semitone
<i>e</i>	120	5:6	Minor third
<i>f</i>	128	15:16	Major semitone

Mode X. Exponent  $2^m \cdot 5^3$ .

<i>F</i>	64		
<i>A</i>	80	4:5	Major third
<i>cs</i>	100	4:5	Major third
<i>f</i> <sup>*</sup>	125	4:5	Major third
<i>f</i>	128	125:128	Enharmonic diesis

Mode XI. Exponent  $2^m \cdot 3^4$ .

<i>F</i>	64		
<i>G</i>	72	8:9	Major tone
<i>A</i> <sup>*</sup>	81	8:9	Major tone
<i>c</i>	96	27:32	Minor third less a comma
<i>d</i>	108	8:9	Major tone
<i>f</i>	128	27:32	Minor third less a comma

Mode XII. Exponent  $2^m \cdot 3^3 \cdot 5$ .

<i>F</i>	128			
<i>F</i> <i>#</i>	135	128:135	Limma minus	} Corrected diatonic mode of the ancients.
<i>G</i>	144	15:16	Major semitone	
<i>A</i>	160	9:10	Minor tone	
<i>H</i>	180	8:9	Major tone	
<i>c</i>	192	15:16	Major semitone	
<i>d</i>	216	8:9	Major tone	
<i>e</i>	240	9:10	Minor tone	
<i>f</i>	256	15:16	Major semitone	

Mode XIII. Exponent  $2^m \cdot 3^2 \cdot 5^2$ .

<i>F</i>	128			} Corrected chromatic mode of the ancients.
<i>G</i>	144	8:9	Major tone	
<i>G#</i>	150	24:25	Minor semitone	
<i>A</i>	160	15:16	Major semitone	
<i>H</i>	180	8:9	Major tone	
<i>c</i>	192	15:16	Major semitone	
<i>c#</i>	200	24:25	Major semitone	
<i>d#</i>	225	8:9	Major tone	
<i>e</i>	240	15:16	Major semitone	
<i>f</i>	256	15:16	Major semitone	

Mode XIV. Exponent  $2^m \cdot 3 \cdot 5^3$ .

<i>F</i>	256			} Corrected enharmonic mode of the ancients.
<i>G#</i>	300	64:75	Minor third less a diesis	
<i>A</i>	320	15:16	Major semitone	
<i>H*</i>	375	64:75	Minor third less a diesis	
<i>c</i>	384	125:128	Enharmonica diesis	
<i>c#</i>	400	24:25	Minor semitone	
<i>e</i>	480	5:6	Minor Third	
<i>f*</i>	500	24:25	Minor semitone	
<i>f</i>	512	125:128	Enharmonic diesis	

Mode XV. Exponent  $2^m \cdot 5^4$ .

<i>F</i>	512		
<i>A*</i>	625	512:625	Major third less a diesis
<i>A</i>	640	125:128	Enharmonca diesis
<i>c#</i>	800	4:5	Major third
<i>f*</i>	1000	4:5	Major third
<i>f</i>	1024	125:128	Enharmonca diesis .

Mode XVI. Exponent  $2^m \cdot 3^5$ .

<i>F</i>	128		
<i>G</i>	144	8:9	Major tone
<i>A*</i>	162	8:9	Major tone
<i>c</i>	192	27:32	Minor third less a comma
<i>d</i>	216	8:9	Major tone

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<i>e</i> *	243	8:9	Major tone
<i>f</i>	256	243:256	Pythagorean limma .

Mode XVII. Exponent  $2^m \cdot 3^4 \cdot 5$ .

<i>F</i>	256		
<i>F</i> #	270	128:135	Minor limma
<i>G</i>	288	15:16	Greater semitone
<i>A</i>	320	9:10	Minor tone
<i>A</i> *	324	80:81	Comma
<i>H</i>	360	9:10	Minor tone
<i>c</i>	384	15:16	Greater semitone
<i>c</i> #*	405	128:135	Minor limma
<i>d</i>	432	15:16	Greater semitone
<i>e</i>	480	9:10	Minor tone
<i>f</i>	512	15:16	Greater semitone.

## CAPUT VIII

### DE GENERIBUS MUSICIS

1. Hactenus in genere naturam sonorum et ex iis formandae harmoniae praecepta exposuimus neque adhuc locus fuit praecepta specialia compositionum musicarum tradendi. Antequam enim haec praecepta ad praxin accommodare liceat, instrumenta musica modumque ea attemperandi considerari oportet. Namque cum soni, qui ad opera musica edenda adhibentur, vel ope vivae vocis vel instrumentorum auditui offerantur, ante omnia tam vox quam instrumenta apta sunt reddenda ad omnes sonos, quibus ad opera musica exprimenda est opus, edendos.
2. Cum igitur exponens operis musici omnes sonos necessarios contineat, ex hoc ipso exponente perspicietur, quot et quales soni in instrumentis musicis inesse debeant. Pendet ergo instructio instrumentorum musicorum ab exponente operum musicorum, quae illorum ope auditui offerri debent; ita ut, si aliorum exponentium opera musica repraesentare voluerimus, ad ea quoque alia instrumenta musica requirantur, quae secundum illos exponentes sint accommodata.
3. Proposito ergo exponente operis musici sonis exprimendis instrumenta ita adaptari debent, ut in iis omnes soni, quos ille exponens in se complectitur, contineantur, nisi forte quidam soni sint vel nimis graves vel nimis acuti, ut auribus percipi nequeant, qui propterea tanquam superflui tuto omitti possunt. Soni autem, quos propositus exponens in se continet, colliguntur ex eius divisoribus; quocirca instrumenta musica ita sunt instruenda, ut omnes sonos perceptibiles divisoribus istius exponentis expressos comprehendant. Contra vero etiam ex dato instrumenta musico intelligitur, ad cuiusmodi opera musica edenda id sit idoneum.
4. Soni vero etiam, qui in dato instrumento musico continentur, commodissime per exponentem indicantur, qui, ut hactenus, est minimus communis dividiuus omnium sonorum in illo instrumenta contentorum. Ex exponente ergo instrumenti musici intelligitur, ad cuiusmodi opera musica edenda id sit aptum. Alia scilicet opera musica in hoc instrumenta exprimi non possunt, nisi quorum exponens sit divisor exponentis instrumenti. Ad hoc autem requiritur, ut in instrumenta omnes soni contineantur, qui ex divisoribus eius exponentis oriuntur; horum enim si qui deessent, instrumentum foret mancum nec ad usum satis idoneum.
5. Ad instrumentum ergo musicum bene instruendum idoneus exponens est eligendus, qui contineat omnium operum musicorum eius ope edendorum exponentes. Quo facto huius exponentis omnes divisores investigari sonique, qui his singulis divisoribus exprimuntur, in instrumentum induci debent, exceptis tamen iis, qui ob nimiam gravitatem et acumen percipi nequeunt. Praeter hos autem sonos commode alii

uniformitatis gratia adiungi possunt, ut soni in singulis octavis contenti fiant numero aequales. Hocque non solum est usu receptum, sed etiam instrumenta magis perfecta efficit, ut ad plura opera musica edenda sint apta.

6. Non solum igitur quilibet exponentis assumti divisor sonum in instrumentum inducet, sed etiam eius duplum, quadruplum, octuplum etc., item eius partes dimidia, quarta, octava etc. Hoc enim pacto fiet, ut omnia intervalla diapason dicta aequali sonorum numero replentur atque etiam simili modo fiant divisa. Unde quoque hoc obtinebitur commodum, ut, si una octava fuerit recte attemperata, ex ea reliquae octavae tam acutiores quam graviores facile efformentur; quod fit, dum singulorum sonorum in una octava contentorum alii una vel pluribus octavis tam acutiores quam graviores efficiantur.

7. Si igitur exponens instrumenti fuerit  $A$  eiusque divisores sint

$$1, a, b, c, d, e \text{ etc.},$$

praeter sonos his divisoribus denotatos etiam soni

$$2, 2a, 2b, 2c, 2d \text{ etc.}, \text{ item } 4, 4a, 4b, 4c \text{ etc.},$$

deinde quoque isti

$$\frac{1}{2}, \frac{1}{2}a, \frac{1}{2}b, \frac{1}{2}c \text{ etc.}, \text{ item } \frac{1}{4}, \frac{1}{4}a, \frac{1}{4}b, \frac{1}{4}c \text{ etc.}$$

item in instrumentum debent induci. Multiplicatione autem sublatis fractionibus omnes soni instrumento contenti erunt

$$2^n, 2^n a, 2^n b, 2^n c, 2^n d, \cdot \cdot \cdot 2^n A,$$

ubi  $n$  quemvis numerum integrum designat. Instrumenti ergo hoc modo instructi exponens non amplius erit  $A$ , sed  $2^m A$  denotante  $m$  numerum indefinitum tam parvum vel magnum, quoad soni sint perceptibiles.

8. Instrumentum igitur ita comparatum non solum erit idoneum ad opera musica edenda, quorum exponentes in  $A$  contineantur, sed etiam ad talia opera, quorum exponentes in  $2^m A$  comprehenduntur. Ex quo intelligitur omnibus octavis aequaliter sonis replendis instrumenta musica maiorem consequi perfectionem atque ad plura opera musica esse accommodata. Deinceps tirones quoque hoc inde habent commodum, ut cognitis sonis in una octava contentis simul facile reliquarum octavarum sonos cognoscant.

9. Pro exponentibus ergo operum musicorum in posterum huiusmodi formam  $2^m A$  assumemus atque investigabimus, quot et cuiusmodi sonos quaelibet octava continere



debeat. Pro  $A$  autem tantum numeros impares sumi conveniet, cum, si pares sumerentur, foret superfluum ob binarios iam in  $2^m$  contentos. Dabit ergo quivis exponens  $2^m A$  peculiarem octavae divisionem, tam ratione numeri sonorum quam ratione intervallorum, quae soni inter se tenent. Huiusmodi autem octavae divisio: a Musicis *genus musicum* appellari solet; taliumque generum tria a longo tempore sunt cognita, quae sunt *genus diatonicum, chromaticum et enharmonicum*.

10. Si octavae, cuius divisio ex dato exponente  $2^m A$  quaeritur, gravissimus sonus fuerit  $E$ , erit acutissimus  $2E$  reliquique soni omnes intra limites  $E$  et  $2E$  continebuntur. Quare singulos divisores ipsius  $A$  per eiusmodi binarii potestates multiplicari oportet, ut facta sint maiora quam  $E$ , minora vero quam  $2E$ , haecque facta omnia dabunt sonos in octava contentos. Ex quo perspicitur in octava tot contineri debere sonos, quot  $A$  habeat divisores, cum unusquisque divisor ipsius  $A$  sonum in quamque octavam inferat.

11. Si ergo exponens instrumenti, quem posthac exponentem generis musici vocabimus, fuerit  $2^m a^p$  existente  $a$  numero primo, una octava continebit  $p+1$  sonos, quia  $a^p$  totidem habet divisores. Sin autem exponens fuerit  $2^m a^p b^q$ , in octava  $(p+1)(q+1)$  seu  $pq+p+q+1$  continebuntur soni; numerus enim  $a^p b^q$  tot, non plures habet divisores, si quidem  $a$  et  $b$  fuerint numeri primi inaequales. Simili modo exponens generis  $2^m a^p b^q c^r$  dabit  $(p+1)(q+1)(r+1)$  sonos intra unius octavae intervallum contentos. Ex his ergo statim ex exponente generis iudicari licet, quot soni in una octava contineantur.

12. Quales autem sint isti soni in unaquaque octava contenti, ipsi divisores ipsius  $A$  declarabunt; singuli enim per eiusmodi binarii potestates debent multiplicari, ut maximus ad minimum minorem habeat rationem quam duplam. Hoc vero commodius sumendis logarithmis, iis scilicet, quos huc recepimus, patebit, ex quibus, cum binarii logarithmus sit 1, statim apparebit, per quamnam binarii potestatem quilibet divisor multiplicari debeat, ut omnium sonorum logarithmi plus unitate a se invicem non discrepent.

13. Genera ergo musica a simplicissimo usque ad maxime composita, quae quidem usum habere possunt, tam cognita iam quam incognita recensebimus atque de quolibet annotabimus, ad quaenam opera musica sit accommodatum. Simplicissimum autem sine dubio musicum genus est  $2^m$ , quod habetur, si est  $A=1$ . In intervallo ergo octavae unicus continetur sonus 1, quem statim sonus 2 integra octava superans sequitur. Omnes ergo soni in instrumento musico contenti erunt 1:2:4:8:16, quia raro instrumenta musica plures quam 4 octaves complectuntur. Hoc autem genus ob nimiam simplicitatem ineptum est ad ullam harmoniam producendam.

14. Exponens ergo  $2^m A$  dabit ordine sequens musicum genus, si ponatur  $A=3$ , cuius divisores sunt 1 et 3 indeque soni octavam constituentes 2:3:4. In hoc igitur genere octava

in duas partes dividitur, quarum altera est intervallum quinta, altera quarta. Forma etiam huius octavae infimum sonum ponendo 3 ita potest repraesentari 3:4:6, ubi intervallum inferius est quarta, superius vero quinta. Soni vero omnes instrumenti secundum exponentem  $2^m \cdot 3$  instructi erunt 2:3:4:6:8:12:16:24:32. Ceterum hoc genus est nimis simplex, ita ut nunquam fuerit in usu.

15. In musica ad hunc usque diem aliae consonantiae non sunt receptae, nisi quarum exponentes constant numeris primis solis 2, 3 et 5, adeo ut Musici ultra quinarium in formandis consonantiis non processerint. Hanc ob rem hic etiam, in initio, loco *A* praeter 3 et 5 eorumque potestates alios numeros non assumam; his vero, quae hinc oriri possunt, generibus musicis expositis tentabimus quoque 7 introducere; unde forte aliquando nova musicae genera formari novaque adhuc atque inaudita opera musica confici poterunt.

16. Erit ergo tertium musicae genus  $2^m \cdot 5$ , in quo soni in octava contenti sunt 4:5:8, quorum duorum intervallorum inferius tertiam maiorem, superius sextam minorem conficit. Hoc autem genus tam, quia est nimis simplex, quam, quod numerum 5 continet omisso ternario ideoque consonantias magis compositas omissis simplicioribus habet, usum habere nequit. Incongruum enim foret in consonantiis maiores numeros primos adhibere neglectis minoribus, eo quod hoc modo harmonia praeter necessitatem magis intricata minusque accepta redderetur.

17. In his duobus generibus in *A* unica fuit dimensio vel ipsius 3 vel 5. Nunc itaque sumamus duas dimensiones sitque quarti generis exponens  $2^m \cdot 3^2$ , in quo quantitatis *A* seu  $3^2$  divisores sunt 1:3:9. Octava ergo hos continebit sonos 8:9:12:16 et tribus constat intervallis, quorum primum est tonus maior, duo reliqua vero quartae. Hocque est primum genus, quod in usu fuisse perhibetur, cuius auctor erat primus musicae inventor in Graecia MERCURIUS, qui hos quatuor sonos totidem chordis expressit, unde instrumentum *tetrachordon* est appellatum. Ab hoc etiam instrumento sequentes Musici venerationis erga MERCURIUM ostendendae gratia sua magis composita genera in tetrachorda dividere sunt soliti.

18. In hoc ergo primo musicae genere, quod cum legibus harmoniae mirifice congruit atque etiam ob hanc causam auditores, qui ante nullam adhuc harmoniam cognoverant, in summam admirationem pertraxit, praeter quintam, quartam, tonum maiorem et octavam alia non inerant auribus grata intervalla. Atque etiam post hoc tempus usque ad tempora PTOLEMAEI incognita mansit consonantia tertia dicta, quippe quam PTOLEMAEUS primus in musicam introduxit.

19. Quinti generis musici exponens erit  $2^m \cdot 3 \cdot 5$ , quod ob divisores 1, 3, 5, 15 ipsius  $3 \cdot 5$  in una octava continebit sonos 8:10:12:15:16. Intervallis igitur gaudet tertia maiore et minore, sexta maiore et minore, quinta et quarta, hemitonio maiore et septima maiore utique perquam gratis. Interim tamen non constat hoc genus unquam fuisse in usu,

etiamsi plurium varietatum capax fuisset quam praecedens MERCURII genus. Cuius rei ratio procul dubio est, quod tertiam tam maiorem quam minorem propter numerum 5 usque ad PTOLEMAEUM ignoraverint, hic autem iam magis compositum genus introduxerit.

20. Sextum genus constituit exponens  $2^m \cdot 5^2$ , in cuius octava propter 1, 5, 25 divisores ipsius  $5^2$  insunt istam rationem tenentes soni 16 : 20 : 25 : 32, quibus octava in tria intervalla secatur, quorum duo priora sunt tertiae maiores, postremum vero tertia maior cum diesi. Quod genus mirum non est nunquam fuisse usu receptum, cum, quoniam antiquissimis temporibus tertiae fuerunt incognitae, tum, quod consonantiae in hoc genere contentae non ad modum sint suaves; atque ad haec accedit, quod hoc genus suavissimis intervallis, qualia sunt quinta et quarta, careat.

21. Septimum nobis genus erit, cuius exponens est  $2^m \cdot 3^3$ . Divisores ergo ipsius  $3^3$  sunt 1, 3, 9, 27, ex quibus sequens octava constituitur 16 : 18 : 24 : 27 : 32, quam autem unquam fuisse in usu non constat.

Octavi generis exponens est  $2^m \cdot 3^2 \cdot 5$ , cuius sex sunt divisores impares 1, 3, 5, 9, 15, 45, unde sequentes soni octavam constituent 32 : 36 : 40 : 45 : 48 : 60 : 64. Hocque genus summam continet gratiam merereturque in usum recipi, nisi iam in receptis generibus contineretur.

Nonum genus exponentem habet  $2^m \cdot 3 \cdot 5^2$  atque in octava sequentes sonos continet 64 : 75 : 80 : 96 : 100 : 120 : 128.

Decimum autem genus exponentis  $2^m \cdot 5^3$  octava hos habebit sonos 64 : 80 : 100 : 125 : 128.

22. Undecimum genus ergo exponentem habebit  $2^m \cdot 3^4$  hincque in octava continebit sonos 64 : 72 : 81 : 96 : 108 : 128. De quo genere uti et de praecedente est notandum, quod in iis intervalla et consonantiae insint, quae in genere hoc quidem tempore recepto non continentur; quare etiam genus, quod nunc est in usu et *diatonico-chromaticum* appellatur, haec duo postrema genera in se non complectitur, praecedentia vero genera omnia in se comprehendit, ita ut, ad quae opera musica praecedentia genera omnia sint accommodata, iisdem quoque genus nunc usu receptum inserviat.

23. Duodecimum genus porro exponente  $2^m \cdot 3^3 \cdot 5$  determinatur; in octava ergo continebit hos octo sonos

128 : 135 : 144 : 160 : 180 : 192 : 216 : 240 : 256.

Hocque genus proxima convenit cum veterum genere diatonico, etiamsi veteres septem tantum sonos in hoc genere collocaverint. Omisso enim sono 135 hoc genus apprime congruit cum genere *diatonico syntono* PTOLEMAEI, in quo octava in duo tetrachorda dividitur, quorum utrumque intervallum diatessaron complectitur et in tria intervalla ita dividitur, ut infimum sit hemitonium maius, sequens tonus maior et tertium tonus minor.

24. Hanc vero ipsam divisionem et nostrum hoc genus habet omisso sono 135; incipiendo enim octavam a sono 120 hanc habebit faciem

120:128:144:160:180:192:216:240,

quarum duarum partium utraque est intervallum diatessaron ita divisum, ut intima intervalla 120:128 et 180:192 sint hemitonia maiora, media vero 128:144 et 192:216 toni maiores atque suprema 144:160 et 216:240 toni minores. Eximia ergo suavitate PTOLEMAEI genus diatonicum erat praeditum, uti etiam experientia satis testatur, cum hoc genus etiamnum sit in usu, dum alia veterum genera minore vel nulla gratia praedita negligantur.

25. Cum autem hoc veterum genus diatonicum sono 135, qui tamen aequae in octavam pertinet ac reliqui, careat, non omnino pro perfecto est habendum; interim tamen, quia tanta est congruentia inter hoc nostrumque genus duodecimum, id *diatonicum correctum* vocabimus. Intelligitur autem ex hoc, quam pertinaciter veteres Musici primo MERCURII invento adhaeserint, ita ut instrumenta musica in tetrachorda singulaque tetrachorda in tres partes dividerint, quod quidem institutum in hoc genere satis cum harmonia constitit, in reliquis vero ingratae harmoniae causa fuit.

26. Praeter hoc vero genus diatonicum syntonum PTOLEMAEI apud veteres plures generis diatonici species in usu fuerunt, quarum intervalla in tetrachordis singulis contenta ita se habebant :

Diatonicum PYTHAGORAE	243:256, 8:9, 8:9;
Diatonicum molle	20:21, 9:10, 7:8;
Diatonicum toniacum	27:28, 7:8, 8:9;
Diatonicum aequale .	11:12, 10:11, 9:10

In quibus omnibus hoc erat institutum, ut prius intervallum sit fere hemitonium, reliqua duo fere toni, omnia autem simul diatessaron compleant. Facile autem perspicitur, quam imperfecta atque absurda sint haec genera, ita ut mirum non sit, quod penitus sint extincta.

27. Quemadmodum autem hoc tempore instrumenta musica secundum octaves dividi omnesque octavae aequaliter partiri solent, ita veteres sua instrumenta in quartas dividere singulasque quartas aequaliter in tria intervalla secare amabant, qua in re potius MERCURII tetrachordon quam ipsam harmoniam sequebantur. Hancque divisionem PYTHAGORICI praecipue Musici numeris arbitrariis nullo ad harmoniam respectu habito perfecerunt, uti ex allatis exemplis satis apparet; hocque modo istis numeris musicae non parvum damnum attulerunt, ita ut merito ab ARISTOXENO eiusque asseclis sint reprehensi.

28. Genus autem diatonicum syntonum PTOLEMAEI, quod feliciter ex perverso hoc musicam tractandi modo emanavit, etiamnum merito est in usu et in cymbalis, clavichordis aliisque instrumentis manualibus instructis conspicitur, in quibus duplicis generis claves habentur, quarum longiores et inferiores sonos generis diatonici syntoni edunt. Quemadmodum igitur hae claves litteris signari solent, ita etiam commode ipsi soni iisdem litteris denotantur. Hinc ergo erit sonus numero 192 indicatus *C*, sequentes 216 *D*, 240 *E*, 256 *F*, 288 *G*, 320 *A*, 360 *H* et 384 *c*.

29. Iisdem porro literis sed minusculis soni octava acutiores, seu numeris duplo maioribus expressi indicantur; haecque minusculae litterae cum una pluribusve lineis sonos octavis acutiores indicant. Ita cum 320 sit *A*, erit 640 *a*, 1280  $\bar{a}$ , 2560  $\bar{\bar{a}}$ , 5120  $\bar{\bar{\bar{a}}}$  etc. Hanc ob rem huiusmodi litteris sive maiusculis sive minusculis respondebunt soni sequentibus numeris expressi. *C* scilicet vocantur omnes soni in hac formula  $2^n \cdot 3$  contenti; *D* soni in  $2^n \cdot 3^3$  contenti, *E* soni in  $2^n \cdot 3 \cdot 5$  contenti, *F* soni in  $2^n$  contenti, *G* soni in  $2^n \cdot 3^2$  contenti, *A* soni in  $2^n \cdot 5$  contenti et *H* soni in  $2^n \cdot 3^2 \cdot 5$  contenti. Sonus autem in usitato genere omissus  $2^n \cdot 3^3 \cdot 5$  nuncupatur *Fs*, hoc est *F* cum hemitonio.

30. Decimum tertium genus deinceps constituet exponens  $2^m \cdot 3^2 \cdot 5^2$ , cuius ergo octavam isti 9 soni complent

$$128 : 144 : 150 : 160 : 180 : 192 : 200 : 225 : 240 : 256 ,$$

ad quod genus veteres collineasse videntur, dum *genus chromaticum* excogitaverunt, si quidem ullam harmoniam in hoc genere chromatico perceperunt. Constituerunt enim in huius generis tetrachordo primo duo hemitonia post eaque tertiam minorem seu potius complementum duorum hemitoniorum ad quartam. In nostro autem genere bis duo hemitonia se excipiunt, quae omissis aliquot sonis tertiae minores sequuntur. Interim tamen veterum genus chromaticum admodum imperfectum fuisse necesse est ideoque hoc genus decimum tertium nobis rite chromaticum correctum.

31. Apud veteres tres potissimum generis chromatici species versabantur, quas in duo tetrachorda, tetrachordum vero in tria intervalla dividebant, quae se in illis tribus speciebus ita habebant:

Chromaticum antiquum	243 : 256, 67 : 76, 4864 : 5427;
Chromaticum molle	27 : 28, 14 : 15, 5 : 6 ;
Chromaticum syntonum	21 : 22, 11 : 12, 6 : 7

Quae generis chromatici species, quantum veris harmoniae principiis repugnent, quilibet facile perspiciet. Genus autem hoc nostrum chromaticum retenta in tetrachorda divisione sequenti modo omissis sonis 225 et 150 in usum vocare potuissent recipiendis in octavam his sonis

$$120:128, 144:160 \mid 180:192, 200:240,$$

in quibus quidem prioris tetrachordi divisio est diatonica syntona, alterius vero chromatica genuina.

32. Decimum quartum genus, cuius exponens est  $2^m \cdot 3 \cdot 5^3$ , in octava habebit hos sonos

$$256:300:320:375:384:400:480:500:512;$$

quod genus vocabimus *enharmonicum correctum*, cum ad veterum genus enharmonicum quodammodo accedere videatur. Veteres quidem sequentes huius generis tetrachordi divisiones reliquerunt

Enharmonicum antiquum	125:128, 243:250, 64:81
Enharmonicum PTOLEMAICUM	45:46, 23:24, 4:5,

quarum neutra cum harmonia consistera potest. Potuissent autem veteres loco generis enharmonici cum aliqua gratia uti hac octavae in tetrachorda et tetrachordorum divisione

$$240:250:256:320 \mid 375:384:400:480,$$

omisso scilicet sono 300; sed hoc ipso deficiente genus imperfectum est censendum.

33. Decimum quintum genus continebitur isto exponente  $2^m \cdot 5^4$  habebitque in octava sequentes sonos

$$512:625:640:800:1000:1024,$$

quod autem genus propter duriora intervalla et defectum gratiorum consonantiarum ternario expositarum usum habere nequit.

Decimum sextum vero genus constituet exponens  $2^m \cdot 3^5$  in eiusque octava inerunt isti soni

$$128:144:162:192:216:243:256,$$

quod genus ob defectum consonantiarum ex 5 ortarum non satis varietatis continet.

Decimum septimum autem genus exponente  $2^m \cdot 3^4 \cdot 5$  expressum minime incongruum esse videtur, quod usu recipiatur; continebit enim eius quaelibet octava sonos sequenti ratione progredientes

$$256 : 270 : 288 : 320 : 324 : 360 : 384 : 405 : 432 : 480 : 512.$$

Contra hoc enim genus aliud quicquam excipi nequit, nisi quod nimis parva intervalla, comma scilicet, auditu vix percipienda in eo occurrant.

34. Sequeretur ergo exponendum genus decimum octavum, cuius exponens est  $2^m \cdot 3^3 \cdot 5^2$ ; quod vero, quia est ipsum genus diatonico-chromaticum hoc tempore apud omnes Musicos usu receptum, dignum est, ut peculiari capite pertractetur. Ceterum, quo hactenus exposita genera cum suis exponentibus clarius ob oculos ponantur, sequentem adiicere visum est tabulam, in qua tam exponentes cuiusque generis quam soni in quaque octava contenti itemque intervalla inter quosque sonos contiguos sunt descripta. Nomina etiam sonorum recepta apposui et sonos vulgo non cognitos asterisco notavi litterae proximae adscripto.

#### TABULA GENERUM MUSICORUM

Signa Sonorum	Soni	Intervalla	Nomina Intervallorum
Genus I. Exponens $2^m$ .			
<i>F</i>	1		
<i>f</i>	2	1:2	Diapason
Genus II. Exponens $2^m \cdot 3$ .			
<i>F</i>	2		
<i>c</i>	3	2:3	Diapente seu Quinta
<i>f</i>	4	3:4	Diatessaron seu Quarta.
Genus III. Exponens $2^m \cdot 5$ .			
<i>F</i>	4		
<i>A</i>	5	4:5	Tertia maior
<i>f</i>	8	5:8	Sexta minor.

Genus IV. Exponens  $2^m \cdot 3^2$ .

<i>F</i>	8			
<i>G</i>	9	8:9		Tonus major } Quarta } Quarta } Genus musicum } antiquissimum Mercurii.
<i>c</i>	12	3:4		
<i>f</i>	16	3:4		

Genus V. Exponens  $2^m \cdot 3 \cdot 5$ .

<i>F</i>	8			
<i>A</i>	10	4:5		Tertia maior
<i>c</i>	12	5:6		Tertia minor
<i>e</i>	15	4:5		Tertia maior
<i>f</i>	16	15:16		Hemitonium maius.

Genus VI. Exponens  $2^m \cdot 5^2$ .

<i>F</i>	16			
<i>A</i>	20	4:5		Tertia maior
<i>cs</i>	25	4:5		Tertia maior
<i>f</i>	32	25:32		Tertia maior cum diesi.

Genus VII. Exponens  $2^m \cdot 3^3$ .

<i>F</i>	16			
<i>G</i>	18	8:9		Tonus maior
<i>c</i>	24	3:4		Quarta
<i>d</i>	27	8:9		Tonus maior
<i>f</i>	32	27:32		Tertia minor commate minuta.

Genus VIII. Exponens  $2^m \cdot 3^2 \cdot 5$ .

<i>F</i>	32			
<i>G</i>	36	8:9		Tonus maior
<i>A</i>	40	9:10		Tonus minor
<i>H</i>	45	8:9		Tonus maior
<i>c</i>	48	15:16		Hemitonium maius
<i>e</i>	60	4:5		Tertia maior
<i>f</i>	64	15:16		Hemitonium maius



Genus IX. Exponens  $2^m \cdot 3 \cdot 5^2$ .

<i>F</i>	64		
<i>Gs</i>	75	64:75	Tertia minor diesi minuta
<i>A</i>	80	14:16	Hemitonium maius
<i>c</i>	96	5:6	Tertia minor
<i>cs</i>	100	24:25	Hemitonium minus
<i>e</i>	120	5:6	Tertia minor
<i>f</i>	128	15:16	Hemitonium maius

Genus X. Exponens  $2^m \cdot 5^3$ .

<i>F</i>	64		
<i>A</i>	80	4:5	Tertia maior
<i>cs</i>	100	4:5	Tertia maior
<i>f*</i>	125	4:5	Tertia maior
<i>f</i>	128	125:128	Diesis enharmonica

Genus XI. Exponens  $2^m \cdot 3^4$ .

<i>F</i>	64		
<i>G</i>	72	8:9	Tonus maior
<i>A*</i>	81	8:9	Tonus maior
<i>c</i>	96	27:32	Tertia minor commate minuta
<i>d</i>	108	8:9	Tonus maior
<i>f</i>	128	27:32	Tertia minor commate minuta

Genus XII. Exponens  $2^m \cdot 3^3 \cdot 5$ .

<i>F</i>	128			
<i>Fs</i>	135	128:135	Limma minus	} Genus diatonicum veterum correctum.
<i>G</i>	144	15:16	Hemitonium maius	
<i>A</i>	160	9:10	Tonus minor	
<i>H</i>	180	8:9	Tonus major	
<i>c</i>	192	15:16	Hemitonium maius	
<i>d</i>	216	8:9	Tonus major	
<i>e</i>	240	9:10	Tonus minor	
<i>f</i>	256	15:16	Hemitonium maius	

Genus XIII. Exponens  $2^m \cdot 3^2 \cdot 5^2$ .

<i>F</i>	128			
<i>G</i>	144	8:9	Tonus major	} Genus chromaticum cum veterum correctum.
<i>Gs</i>	150	24:25	Hemitonium minus	
<i>A</i>	160	15:16	Hemitonium maius	
<i>H</i>	180	8:9	Tonus major	
<i>c</i>	192	15:16	Hemitonium maius	
<i>cs</i>	200	24:25	Hemitonium minus	
<i>ds</i>	225	8:9	Tonus major	
<i>e</i>	240	15:16	Hemitonium maius	
<i>f</i>	256	15:16	Hemitonium maius	

Genus XIV. Exponens  $2^m \cdot 3 \cdot 5^3$ .

<i>F</i>	256			
<i>Gs</i>	300	64:75	Tertia minor diesi minuta	} Genus enharmonicum veterum correctum.
<i>A</i>	320	15:16	Hemitonium maius	
<i>H*</i>	375	64:75	Tertia minor diesi minuta	
<i>c</i>	384	125:128	Diesis enharmonica	
<i>cs</i>	400	24:25	Hemitonium minus	
<i>e</i>	480	5:6	Tertia minor	
<i>f*</i>	500	24:25	Hemitonium minus	
<i>f</i>	512	125:128	Diesis enharmonica	

Genus XV. Exponens  $2^m \cdot 5^4$ .

<i>F</i>	512			
<i>A*</i>	625	512:625	Tertia maior diesi minuta	
<i>A</i>	640	125:128	Diesis enharmonca	
<i>cs</i>	800	4:5	Tertia maior	
<i>f*</i>	1000	4:5	Tertia maior	
<i>f</i>	1024	125:128	Diesis enharmonca.	

Genus XVI. Exponens  $2^m \cdot 3^5$ .

<i>F</i>	128		
<i>G</i>	144	8:9	Tonus maior
<i>A</i> *	162	8:9	Tonus maior
<i>c</i>	192	27:32	Tertia minor commate minuta
<i>d</i>	216	8:9	Tonus maior
<i>e</i> *	243	8:9	Tonus maior
<i>f</i>	256	243:256	Limma Pythagoricum.

Genus XVII. Exponens  $2^m \cdot 3^4 \cdot 5$ .

<i>F</i>	256		
<i>Fs</i>	270	128:135	Limma minus
<i>G</i>	288	15:16	Hemitonium maius
<i>A</i>	320	9:10	Tonus minor
<i>A</i> *	324	80:81	Comma
<i>H</i>	360	9:10	Tonus minor
<i>c</i>	384	15:16	Hemitonium maius
<i>cs</i> *	405	128:135	Limma minus
<i>d</i>	432	15:16	Hemitonium maius
<i>e</i>	480	9:10	Tonus minor
<i>f</i>	512	15:16	Hemitonium maius.