LETTERS

ON

DIFFERENT SUBJECTS

IN

NATURAL PHILOSOPHY.

LETTER I.—OF MAGNITUDE, OR EXTENSION.

THE hope of having the honour to communicate, in person, to your Highness, my lessons in Geometry, becoming more and more distant, which is a very sensible mortification to me, I feel myself impelled to supply personal instruction by writing, as far as the nature of the subjects will permit.

I begin my attempt by assisting you to form a just idea of *Magnitude*; producing, as examples, the smallest as well as the greatest extensions of matter actually discoverable in the system of the Universe. And, first, it is necessary to fix on some one determinate division of measure, obvious to the senses, and of which we have an exact idea, that of a *Foot*, for instance. The quantity of this once established, and rendered familiar to the eye, will enable us to form the idea of every other quantity as to length, great or small; the former, by ascertaining how many feet it contains; and the latter, by ascertaining what part of a foot measures it. For having the idea of a *foot*, we have that also of its *half*, of its *quarter*, of its *twelfth part*, denominated an inch, of its *hundredth*, and of its *thousandth* part, which is so small as almost to escape the sight. But it is to be remarked, that there are animals not of greater extension than this last subdivision of a foot, which, however, are composed of members through which the blood circulates, and which again contain other animals, as diminutive compared to them as they are compared to us. Hence it may be concluded, that animals exist whose smallness eludes the imagination; and that these again are divisible into parts inconceivably smaller. Thus, for example, though the ten thousandth part of a foot be too small for sight, and, compared to us, ceases to be an object of sense, it nevertheless surpasses in magnitude certain complete animals; and must to one of those animals, were it endowed with the power of perception, appear extremely great.

Let us now make the transition from these minute quantities, in pursuing which the mind is lost, to those of the greatest magnitude. You have the idea of a mile; the distance from hence to Magdeburg is computed to be 83 English miles; a mile contains 5280 feet, and we employ it in measuring the distance of the different regions of the globe, in order to avoid numbers inconceivably great in our calculations, which must be the case if we used a foot instead of a mile. A mile then, containing 5280 feet, when it is said that Magdeburg is 83 miles from Berlin, the idea is much clearer than if the distance of these two cities were said to be 43,824 feet: A number so great almost overwhelms the understanding. Again, we shall have a tolerably just idea of the magnitude of the earth, when we are told that its circumference is about 25,020 miles. And the diameter being a

straight line passing through the centre, and terminating, in opposite directions, in the surface of the sphere, which is the acknowledged figure of the earth, for which reason also we give it the name of *globe*—the diameter of this *globe* is calculated to be 7964 miles; and this is the measurement which we employ for determining the greatest distances discoverable in the heavens. Of all the heavenly bodies the *Moon* is nearest to us, being distant only about 30 diameters of the earth, which amount to 240,000 miles, or 1,238,400,000 feet; but the first computation of 30 diameters of the earth is the clearest idea. The *Sun* is about 400 times farther from us than the moon; and when we say his distance is 12000 diameters of the earth, we have a much clearer idea than if it were expressed in miles or in feet.

You know that the earth performs a revolution round the sun in the space of a year, but that the sun remains fixed. Beside the *Earth*, there are ten other similar bodies, named planets, which revolve round the sun; two of them at smaller distances, *Mercury* and *Venus*; and eight at greater distances, namely, *Mars*, *Ceres*, *Pallas*, *Juno*, *Vesta*, *Jupiter*, *Saturn*, and the *Georgium Sidus*. All the other stars which we see, comets excepted, are called fixed; and their distance from us is incomparably greater than that of the sun. Their distances are undoubtedly very unequal, which is the reason that some of these bodies appear greater than others. But the nearest of them is, unquestionably, above 5000 times more distant than the sun: its distance from us, accordingly, exceeds 45,000,000 of times the earth's diameter, that is, 356,050,000,000 miles; and this again, multiplied by 5280, will give that prodigious distance expressed in feet. And this, after all, is the distance only of those fixed stars which are the nearest to us;—the most remote which we see are perhaps a hundred times farther off. It is probable, at the same time, that all these stars taken together constitute only a very small part of the whole universe, relatively to which these prodigious distances are not greater than a grain of sand compared to the earth. This immensity is the work of the Almighty, who governs the greatest bodies and the smallest.

Berlin, 19*th April* 1760.

Letter II.—Of Velocity.

Flattering myself that your Highness may be pleased to accept the continuation of my instructions, a specimen of which I took the liberty of presenting to you in a former letter, I proceed to unfold the idea of velocity, which is a particular species of extension, and susceptible of increase and of diminution. When any substance is transported, that is, when it passes from one place to another, we ascribe to it a velocity. Let two persons, the one on horseback, the other on foot, proceed from Berlin to Magdeburg, we have, in both cases, the idea of a certain velocity; but it will be immediately affirmed, that the velocity of the former exceeds that of the latter. The question then is, Wherein consists the difference which we observe between these several degrees of velocity? The road is the same to him who rides and to him who walks; but the difference evidently lies in the time which each employs in performing the same course. The velocity of the horseman is the greater of the two, as he employs less time on the road from Berlin to Magdeburg; and the velocity of the other is less, because he employs more time in travelling the same distance. Hence

it is clear, that in order to form an accurate idea of velocity, we must attend at once to two kinds of quantity—namely, to the length of the road, and to the time employed. A body, therefore, which in the same time passes through double the space which another body does, has double its velocity; if, in the same time, it passes through thrice the distance, it is said to have thrice the velocity, and so on. We shall comprehend, then, the velocity of a body, when we are informed of the space through which it passes in a certain quantity of time. In order to know the velocity of my pace, when I walk to Lytzow (about a league from Berlin), I have observed that I make 120 steps in a minute, and one of my steps is equal to two feet and a half. My velocity, then, is such as to carry me 300 feet in a minute, and a space 60 times greater, or 18,000 feet in an hour. Were I, therefore, to walk from hence to Magdeburg, it would take exactly 24 hours. This conveys an accurate idea of the velocity with which I am able to walk. Now it is easy to comprehend what is meant by a greater or less velocity. For if a courier were to go from hence to Magdeburg in 12 hours, his velocity would be the double of mine; if he went in eight hours, his velocity would be triple. We remark a very great difference in the degrees of velocity. The tortoise furnishes an example of a velocity extremely small. If she advances only one foot in a minute, her velocity is 300 times less than mine, for I advance 300 feet in the same time. We are likewise acquainted with velocities much greater. That of the wind admits of great variation. A moderate wind goes at the rate of 10 feet in a second, or 600 feet in a minute; its velocity therefore is the double of mine. A wind that runs 20 feet in a second, or 1200 in a minute, is rather strong; and a wind which flies at the rate of 50 feet in a second is extremely violent, though its velocity is only 10 times greater than mine, and would take two hours and twenty-four minutes to blow from hence to Magdeburg.

The velocity of sound comes next, which moves 1142 feet in a second, and 68,520 in a minute. This velocity, therefore, is 228 times greater than that of my pace; and were a cannon to be fired at Magdeburg, if the report could be heard at Berlin, it would arrive there in seven minutes. A cannon ball moves with nearly the same velocity; but when the piece is loaded to the utmost, the ball is supposed capable of flying 2000 feet in a second, or 120,000 in a minute. This velocity appears prodigious, though it is only 400 times greater than that of my pace in walking to Lytzow; it is at the same time the greatest velocity known upon earth. But there are in the heavens velocities far greater, though their motion appears to be extremely deliberate. You know that the earth turns round on its axis in 24 hours: every point of its surface, then, under the equator, moves 25,020 English miles in 24 hours, while I am able to get through only 83 miles. Its velocity is accordingly nearly 300 times greater than mine, and less notwithstanding than the greatest possible velocity of a cannon ball. The earth performs its revolution round the sun in the space of a year, proceeding at the rate of 589,950 English miles in 24 hours. Its velocity, therefore, is 18 times more rapid than that of a cannon ball. The greatest velocity of which we have any knowledge is undoubtedly that of light, which moves 9,200,000 English miles every minute, and exceeds the velocity of a cannon ball 400,000 times.

22d April 1760.

LETTER III.—OF SOUND, AND ITS VELOCITY.

THE elucidations of the different degrees of velocity, which I have had the honour to lay before your Highness, carry me forward to the examination of sound, or noise in general. It must be remarked, that a certain portion of time always intervenes before sound can reach our ears, and that this time is longer in proportion to our distance from the place where the sound is produced; a second of time being requisite to convey sound 1142 feet.

When a cannon is fired, those who are at a distance do not hear the report for some time after they have seen the flash. Those who are about 5 miles off, or 24,000 feet distant, do not hear the report till 21 seconds after they see the flash. You must no doubt have frequently remarked, that the noise of thunder does not reach the ear for some time after the lightning; and it is by this we are enabled to calculate our distance from the place where the thunder is generated. If, for example, we observe that 20 seconds intervene between the flash and the thunder-clap, we may conclude that the seat of the thunder is 22,840 feet distant, allowing 1142 feet of distance for every second of time. This primary property leads us to inquire, in what sound consists? Whether its nature is similar to that of smell—that is, whether sound issues from the body which produces it, as smell is emitted from the flower, by filling the air with subtile exhalations, calculated to affect our sense of smelling. This opinion was formerly entertained; but it is now demonstrated, that from a bell struck nothing proceeds that is conveyed to our ear, and that the body which produces sound loses no part of its substance. When we look upon a bell that is struck, or the string of an instrument when touched, we perceive that these bodies are then in a state of trembling, or agitation, by which all their parts are affected; and that all bodies, susceptible of such an agitation of their parts, likewise produce sound. These shakings or vibrations are visible in the string of an instrument when it is not too small; the tense string A C B passes alternately into the situation A M B and A N B. *(See* Plate I. *Fig.* 1., *in which I have represented these vibrations much more obvious to sense than they are in fact).* It must be observed, that these vibrations put the adjacent air into a similar vibration, which is successively communicated to the more remote parts of the air, till it come at length to strike our organ of hearing. It is the air, then, which receives these vibrations, and which transmits the sound to our ear. Hence it is evident, that the perception of sound is nothing else but the impression made on our ear by the concussion of the air, communicated to us through the organ of hearing; and when we hear the sound of a string touched, our ear receives from the air as many strokes as the string performs vibrations in the same time. Thus, if the string performs 100 vibrations in a second, the ear likewise receives 100 strokes in the same time; and the perception of these strokes is what we call sound. When these strokes succeed each other uniformly, or when their intervals are all equal, the sound is regular, and such as is requisite to music. But when the strokes succeed unequally, or when their intervals are unequal among themselves, an irregular noise, incompatible with music, is the result. On considering somewhat more attentively the musical sounds, whose vibrations take place equally, I remark first, that when the vibrations, as well as the strokes impressed on the ear, are more or less strong, no other difference of sound results from it, but that of

stronger or weaker, which produces the distinction termed by musicians, *forte et piano.* But there is a difference much more essential, when the vibrations are more or less rapid—that is, when more or fewer of them are performed in a second. When one string makes 100 vibrations in a second, and another string makes 200 vibrations in the same time, their sounds are essentially different; the former is lower or more flat, and the other higher or more sharp. Such is the real difference between the flat and sharp sounds, on which all music hinges, and which teaches how to combine sounds different in respect of flatness and sharpness, but in such a manner as to produce an agreeable harmony. In the flat sounds there are fewer vibrations in the same time than in the sharp sounds; and every key of the harpsichord contains a certain and determinate number of vibrations, which are completed in a second. Thus the note marked by the letter C makes nearly 100 vibrations in a second, and the note marked $\overset{\equiv}{c}$ makes 1600 vibrations in the same space of time. A string which vibrates 100 times in a second, will give precisely the note C; and if it vibrated only 50 times, the note would be lower or more flat. But with regard to our ear, there are certain limits beyond which sound is no longer perceptible. It would appear that we are incapable of determining either the sound of a string which makes less than 30 vibrations in a second, because it is too low; or that of a string which would make more than 7552 in a second, because such a note would be too high.

26th April 1760.

Letter IV.—Of Consonance and Dissonance.

I resume my remark, that on hearing a simple musical sound, our ear is struck with a series of strokes equally distant from each other, the frequency and number of which, in a given space of time, constitute the difference which subsists between low notes and high; so that the smaller the number of vibrations or strokes produced in a given time, say a second, the lower we estimate that note; and the greater the number of such vibrations, the higher is the note. The perception of a simple musical sound may, therefore, be compared to a series of dots equidistant from each other, as . . . . If the intervals between these dots be greater or smaller, the sound produced will be lower or higher. It cannot be doubted, that the perception of a simple sound is somewhat similar or analogous to the sight of such a series of dots equidistant from each other: we are enabled thus to represent to the eye what the ear perceives on hearing sound. If the distances between the dots were not equal, or were these dots scattered about confusedly, they would be a representation of a confused noise, inconsistent with harmony. This being laid down, let us consider what effect two sounds emitted at once must produce on the ear. First, it is evident, that if two sounds are equal, or if each performs the same number of vibrations in the same time, the ear will be affected in the very same manner as by a single note; and in music these two notes are said to be in unison, which is the simplest *accord:* we mean by the term *accord* the blending of two or more sounds heard at once. But if two sounds differ in respect of low and high, we shall perceive a mixture of two series of strokes, in

each of which the intervals are equal among themselves, but greater in the one than in the other; the greater intervals corresponding to the lower note, and the smaller to the higher. This mixture, or this accord of two notes, may be represented to the eye by two series of dots arranged on two lines A B and C D;

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    1    2    3    4    5    6    7    8    9    10   11
A   .    .    .    .    .    .    .    .    .    .    .    . B
C   .   .   .   .   .   .   .   .   .   .   .   .   .      . D
    1   2   3   4   5   6   7   8   9   10  11  12
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and in order to form a just idea of these two series, we must have a clear perception of the order which subsists among them—or, in other words, of the relation between the intervals of the one line and of the other. Having numbered and marked the dots of each line, and placed No. 1. under No. 1., those marked with the figure 2 will not exactly correspond, and still less those marked 3: but we find No. 11. exactly over No. 12.; from which we discover that the higher note makes 12 vibrations, and the other only 11. If we had not affixed the figures, the eye would hardly have perceived this order: it is the same with the ear, which would with much difficulty have traced it in the two notes which I have represented by two rows of dots. But in the following figure,

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. . . . . . . . . . . . .
.   .   .   .   .   .   .
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you discover at the first glance that the upper line contains twice as many dots as the under, or that the intervals in the under line are twice as great as those of the upper. This is undoubtedly next to unison, the simplest of all cases, in which you can at once discover the order which subsists between these two series of dots; and the same thing holds with respect to the two notes represented by these two lines of dots: the number of vibrations contained in the one will be precisely the double of the vibrations contained in the other, and the ear will easily perceive the pleasing relation of these two sounds; whereas, in the preceding case, it was extremely difficult, if not impossible, to discriminate. When the ear readily discovers the relation subsisting between two notes, their accord is denominated *consonance;* and if it be very difficult, or even impossible to catch this relation, the accord is termed *dissonance.* The simplest consonance, then, is that in which the high note produces precisely twice as many vibrations as the low note. This consonance, in the language of music, is called *octave*—every one knows what it means; and two notes which differ precisely an octave, harmonize so perfectly, and possess such a complete resemblance, that musicians mark them by the same letters. Hence it is that in church-music the women sing an octave higher than the men, and yet imagine they are uttering the same sounds. You may easily ascertain the truth of this by touching the keys of a harpsichord, when you will perceive with pleasure the delightful accord of all the notes which are just an octave distant; whereas any other two notes whatever will strike the ear less agreeably.

29th April 1760.

Letter V.—Of Unison and Octaves.

Your Highness has by this time remarked, that the accord which musicians call an *octave*, strikes the ear in a manner so decided, that the slightest deviation is easily perceptible. Thus, having touched the key marked F, that marked f, which is an octave higher, is easily attuned to it, by the judgment of

the ear only. If the string which is to produce this note be ever so little too high or too low, the ear is instantly offended; and nothing is easier than to put the two keys perfectly in tune. Thus we observe, that in singing the voice slides easily from one note to another, which is just an octave higher or lower. But were it required to pass immediately from the note F to the note d, for example, an ordinary singer might easily fall into a mistake, unless assisted by an instrument. Having fixed the note F, it is almost impossible all at once to make the transition to the note d. What then is the reason of this difference, that it is so easy to make note f harmonize with note F, and so difficult to make note d accord with it? The reason is evident from the remarks already made: It is this, that note F and note f make an octave, and that the number of vibrations of note f is precisely double that of note F. In order to have the perception of this accord, you have only to consider the proportion of one to two, which, as it instantly strikes the eye by the representation of the dots I formerly employed, affects the ear in a similar manner. You will easily comprehend, then, that the more simple any proportion is, or expressed by small numbers, the more distinctly it presents itself to the understanding, and conveys to it a sentiment of satisfaction. Architects likewise carefully attend to this maxim, as they uniformly employ in their works proportions as simple as circumstances permit. They usually make the height of doors and windows double the breadth, and endeavour to employ throughout proportions capable of being expressed by small numbers, because this is obvious and grateful to the understanding. The same thing holds good in music: Accords are pleasing only in so far as the mind perceives the relation subsisting between the sounds; and this relation is so much more easily perceptible, as it is expressed by small numbers. Now, next to the relation of equality, which denotes two sounds in unison, the ratio of two to one is undoubtedly the most simple, and it is this which furnishes the accord of an octave: hence it is evident, that this accord possesses many advantages above every other consonance. Having thus explained the accord, or interval of two notes denominated by musicians an octave, let us consider several notes, as F, f, $\bar{f}$, $\bar{\bar{f}}$, $\bar{\bar{\bar{f}}}$ each of which is an octave higher than the one immediately preceding: since then the interval of F from f, of f from $\bar{f}$, of $\bar{f}$ from $\bar{\bar{f}}$, of $\bar{\bar{f}}$ from $\bar{\bar{\bar{f}}}$, is an octave, the interval of F to $\bar{f}$ will be a double octave, that of F to $\bar{\bar{f}}$ a triple octave, and that of F to $\bar{\bar{\bar{f}}}$ a quadruple octave. Now, while note F makes one vibration, note f makes two, note $\bar{f}$ makes four, note $\bar{\bar{f}}$ makes eight, and note $\bar{\bar{\bar{f}}}$ makes sixteen: hence we see, that as an octave corresponds in the relation of 1 to 2, a double octave must be in the ratio of 1 to 4, a triple in that of 1 to 8, and a quadruple in that of 1 to 16. And the ratio of 1 to 4 not being so simple as that of 1 to 2, for it does not so readily strike the eye, a double octave is not so easily perceptible to the ear as a single; a triple is still less perceptible, and a quadruple still much less so. When, therefore, in tuning a harpsichord, you have fixed the note F, it is not so easy to attune the double octave $\bar{f}$ as the single f; it is still more difficult to attune the triple octave $\bar{\bar{f}}$ and the quadruple $\bar{\bar{\bar{f}}}$ without rising through the intermediate octaves. These accords are likewise comprehended in the term consonance; and as that of unison is most simple, they may be arranged according to the following gradations:—

I. Degree, unison, indicated by the relation of 1 to 1.
II. Degree, the immediate octave, in the ratio of 1 to 2.
III. Degree, the double octave, in that of 1 to 4.
IV. Degree, the triple octave, in that of 1 to 8.

V. Degree, the quadruple octave, in that of 1 to 16.
VI. Degree, the quintuple octave, in that of 1 to 32.

And so on as long as sound is perceptible. Such are the accords denominated consonances, to the knowledge of which we have been thus far conducted; but hitherto we know nothing of the other species of consonance, and still less of the dissonances employed in music. Before I proceed to the explanation of these, I must add one remark respecting the name *octave*, given to the interval of two notes, the one of which contains twice the vibrations contained in the other. You see the reason of it in the principal stops of the harpsichord, which rise by seven degrees before you arrive at the octave C, D, E, F, G, A, B, c, so that stop c is the eighth, reckoning C the first. And this division depends on a certain series of musical intervals, the nature of which shall be unfolded in the following letters.

3d May 1760.

Letter VI.—Of other Consonances.

It may be affirmed, that the relations of 1 to 2, of 1 to 4, of 1 to 8, of 1 to 16, which we have hitherto considered, and which contain the progression of octaves, are all formed by the number 2 only; since 4 is 2 times 2; 8, 2 times 4; 16, 2 times 8. Were we to admit, therefore, the number 2 alone into music, we should arrive at the knowledge of only the accords or consonances, which musicians call the single, double, or triple octave; and as the number 2, by its reduplication, furnishes only the numbers 4, 8, 16, 32, 64, the one being always double the preceding, all other numbers would remain unknown. Now, did an instrument contain octaves only, as the notes marked C, c, $\bar{c}$ $\bar{\bar{c}}$ $\bar{\bar{\bar{c}}}$ and were all others excluded, it could not produce an agreeable music, on account of its too great simplicity. Let us introduce, then, together with number 2, the number 3 likewise, and observe what accords or consonances would be the result. The ratio of 1 to 3 presents at once two sounds, the one of which makes 3 times more vibrations than the other in the same time. This ratio is undoubtedly the most easily to be comprehended, next to that of 1 to 2; it will, accordingly, furnish very pleasing consonances, but of a nature totally different from that of octaves. Let us suppose, then, that in the proportion of 1 to 3, number 1 corresponds to note C; since note c is expressed by number 2, number 3 gives a sound higher than c, but at the same time lower than note $\bar{c}$ which corresponds to number 4. Now, the note expressed by 3 is that to which musicians affix the letter g, and they denominate the interval from c to g, *a fifth*, because in the keys of a harpsichord that of g is the fifth from c, as c, d, e, f, g. If then number 1 produces the sound C, number 2 will give c; number 3 gives g, number 4 the note $\bar{c}$; and note $\bar{g}$ being the octave of g, the number corresponding to it will be 2 times 3, or 6. Rising still an octave, the sound $\bar{\bar{g}}$ will correspond to a number twice greater, that is 12. All the notes with which the two numbers 2 and 3 furnish us, indicating note C by 1, therefore, are,

C, c, g, $\bar{c}$, $\bar{g}$, $\bar{\bar{c}}$, $\bar{\bar{g}}$, $\bar{\bar{\bar{c}}}$
1. 2. 3. 4. 6. 8. 12. 16.

Hence it is clear, that the ratio of 1 to 3 expresses an interval, compounded of an octave and a fifth; and that this interval, on account of the simplicity of the numbers which represent it, must be, next to the octave, the most grateful to the ear. Musicians accordingly assign the second rank among consonances to the fifth; and the ear catches it so easily, that

there is no difficulty in tuning a fifth. For this reason, in violins, the four strings rise by fifths, the lowest being g, the second $\bar{d}$, the third $\bar{a}$, and the fourth $\bar{\bar{e}}$;* and every musician puts them in tune by the ear only. A fifth, however, is not so easily tuned as an octave; but the fifth above the octave, as from C to g, being expressed by the proportion of 1 to 3, is more perceptible than a simple fifth, as from C to G, or from c to g, which is expressed by the proportion of 2 to 3: and it is likewise known by experience, that having fixed the note C, it is easier to attune to it the higher fifth g, than the simple G. If unity had marked the note F, number 3 would mark the note $\bar{c}$, so that

F, f, $\bar{c}$, $\bar{f}$, $\bar{\bar{c}}$, $\bar{\bar{f}}$, $\bar{\bar{\bar{c}}}$, would be marked by
1, 2, 3, 4, 6, 8, 12, where, from f to c the interval is a fifth in the relation of 2 to 3; from $\bar{f}$ to $\bar{\bar{c}}$, from $\bar{\bar{f}}$ to $\bar{\bar{\bar{c}}}$, are also fifths, as the ratio of 4 to 6, and of 8 to 12, is the same as that of 2 to 3. For if two strings perform, in the same time, the one 4 vibrations, the other 6, the former string will make, in a time equal to half the first space of time, two vibrations; and the second, in the same time, will make three. Now the sounds emitted from these strings are the same in both cases; of consequence the relation of 4 to 6 expresses the same interval as that of 2 to 3, that is, a fifth. Hence we have arrived at the knowledge of another interval contained in the ratio of 3 to 4, which is that of $\bar{c}$ to $\bar{f}$, and consequently also of c to f, or of C to F. Musicians call it *a fourth*; and being expressed by greater numbers, it is not so agreeable, by a great deal, as the fifth, and still less so than the octave. Number 3 having furnished us new accords or consonances, namely, the fifth and the fourth, before we call in any others, let us take

* That is, in the language of *sol-faing, sol, re, la, mi.*

it again three times, in order to have the number 9, which will give a higher note than note 3, or $\bar{c}$ one octave and one fifth. Now, $\bar{\bar{c}}$ is the octave of $\bar{c}$, and $\bar{\bar{g}}$ the fifth of $\bar{\bar{c}}$; number 9 then gives the note $\bar{\bar{g}}$, so that $\bar{\bar{c}}$, $\bar{\bar{f}}$, $\bar{\bar{g}}$, $\bar{\bar{\bar{c}}}$, will be marked by
6. 8. 9. 12; and if these notes be taken in the lower octaves, the relations remaining the same, we shall have,

C, F, G; c, f, g; $\bar{c}$, $\bar{f}$, $\bar{g}$; $\bar{c}$, $\bar{\bar{f}}$, $\bar{\bar{g}}$; $\bar{\bar{\bar{c}}}$
6, 8, 9.; 12, 16, 18; 24, 32, 36; 48, 64, 72, 96;

which leads us to the knowledge of new intervals.

The first is that of F to G, contained in the ratio of 8 to 9, which musicians call a *second*, or *tone*. The second is that of G to f, contained in the ratio of 9 to 16, called a *seventh;* and which is one second, or one tone less than an octave. These proportions, being already expressed by very great numbers, are not reckoned among the consonances; and musicians call them *dissonances* or *discords*.

Again, if we take three times the number 9, or 27, it will mark a tone higher than $\bar{c}$, and precisely a fifth higher than g; it will be accordingly the tone $\bar{d}$, and its octave $\bar{\bar{d}}$ will correspond to twice the number 27, or 54, and its double octave $\bar{\bar{\bar{d}}}$ to twice the number, 54, or 108. Let us represent these tones some octaves lower, in the manner following:

C, D, F, G; c, d, f, g; $\bar{c}$, $\bar{d}$, $\bar{f}$,
24, 27, 32, 36; 48, 54, 64, 72; 96, 108, 128,
$\bar{g}$; $\bar{\bar{c}}$, $\bar{\bar{d}}$, $\bar{\bar{f}}$, $\bar{\bar{g}}$; $\bar{\bar{\bar{c}}}$.
144; 192, 216, 256, 288; 384.

Hence we see, that the interval from D to F is contained in the ratio 27 to 32, and that of F to d in the ratio of 32 to 54, the two terms of which are divisible by 2; and then, in place of this relation, we have that of 16 to 27. The first interval is called a *tierce minor*, or *lesser third*, and the other a *greater*

sixth. The number 27 might be still farther multiplied by 3; but music extends not so far, and we limit ourselves to number 27, resulting from 3, multiplied three times by itself: other musical tones still wanting are introduced by means of number 5, and shall be explained in my next Letter.

3d *May* 1760.

Letter VII.—Of the Twelve Tones of the Harpsichord.

The present subject of my correspondence with your Highness is so dry, that I begin to apprehend it may be growing tiresome. That I may not waste too much time on it, and be relieved from the necessity of recurring frequently to a topic so uninteresting, I send you by this conveyance three letters at once. My intention in undertaking it was to render visible the real origin of musical notes, with which musicians themselves are almost totally unacquainted. It is not to theory they are indebted for the knowledge of all these sounds, but rather to the secret power of genuine harmony, operating so efficaciously on their ears, that they have been constrained, if I may be allowed to say it, to receive tones actually in use, though they are not hitherto perfectly agreed respecting their just determination. The principles of harmony are ultimately reducible to numbers, as I have demonstrated; and it has been remarked, that the number 2 furnishes octaves only, so that having fixed, for example, the note F, we are conducted to the notes f, $\bar{f}$, $\bar{\bar{f}}$, $\bar{\bar{\bar{f}}}$. The number 3 afterwards furnishes C, c, $\bar{c}$, $\bar{\bar{c}}$, $\bar{\bar{\bar{c}}}$, which differ one-fifth from the preceding series; and the repetition of this same number 3 furnishes again the fifths of the first, namely, G, g, $\bar{g}$, $\bar{\bar{g}}$, $\bar{\bar{\bar{g}}}$; and finally, the third repetition of this number 3 adds farther the notes D, d, $\bar{d}$, $\bar{\bar{d}}$, $\bar{\bar{\bar{d}}}$. The principles of harmony then being attached to simplicity, seem to forbid our pushing farther the repetition of number 3; hitherto, accordingly, we have only the following notes for each octave:

F. G. c. d. f.
16. 18. 24. 27. 32. which certainly would not furnish a very copious music. But let us introduce, in addition to these, number 5, and observe the tone which shall emit five vibrations while F emits only one. Now f makes two vibrations in the same time, $\bar{f}$ makes four, and $\bar{\bar{c}}$ six. The note in question, then, is between $\bar{f}$ and $\bar{\bar{c}}$. It is that which musicians indicate by letter $\bar{a}$, the accord of which with note $\bar{f}$ is denominated a *greater third*, and is found to produce a very agreeable concord, being expressed by the very simple ratio of 4 to 5. Farther, note $\bar{a}$ with note $\bar{\bar{c}}$ produces an accord contained in the ratio of 5 to 6, which is almost as agreeable as the former, and which is denominated a *lesser third*, represented by the ratio of 27 to 32, and its difference from the first is almost imperceptible to the ear. This same number 5 being applied to the other notes G, c, d, will give us, in like manner, their greater thirds, taken in the second octave below, that is to say, the notes $\bar{b}$, $\bar{\bar{e}}$, and $\bar{\bar{\bar{f}}}$, which, being transposed, will give the following notes, with their corresponding numbers:

F. Fs. G. A. B. c. d. e. f.
128. 135. 144. 160. 180. 192. 216. 240. 256.

Take away the notes Fs, and you will have the principal touches of the harpsichord, which, according to the ancients, constitute the genus denominated *diatonic*, resulting from number 2, from number 3, thrice repeated, and from number 5. Admitting these sounds only, we are in a condition to compose harmonies very agreeable and various, the beauty of

which is founded on the simplicity alone of the numbers corresponding to the notes. Finally, upon applying a second time the number 5, we shall be furnished with the thirds of the four new tones, A, E, B, Fs, which we have just found, we shall have the notes Cs, Gs, Ds, and B, so that now the octave is completed of the 12 tones received in music. All these tones derive their origin from the three numbers, 2, 3, and 5, multiplying 2 by itself, as often as the octaves requires; but we carry the multiplication of 3 only to the third stage, and of five to the second. All the tones of the first octave are contained in the following table, in which you will see how the fundamental numbers 2, 3, and 5, enter into the composition of those which express the relation of these notes.

				Difference.
ut or	C	2, 2, 2, 2, 2, 2, 2, 3 . . .	384	
ut ※	Cs	2, 2, 2, 2, 5, 5	400	16
re	D	2, 2, 2, 2, 3, 3, 3	432	32
re ※	Ds	2, 3, 3, 5, 5	450	18
mi	E	2, 2, 2, 2, 2, 3, 5	480	30
fa	F	2, 2, 2, 2, 2, 2, 2, 2, 2 . .	512	32
fa ※	Fs	2, 2, 3, 3, 3, 5	540	28
sol	G	2, 2, 2, 2, 2, 2, 3, 3 . . .	576	36
sol ※	Gs	2, 2, 2, 3, 5, 5	600	24
la	A	2, 2, 2, 2, 2, 2, 2, 5 . . .	640	40
si b.	B♭	3, 3, 3, 5, 5	675	35
si n.	B♮	2, 2, 2, 2, 3, 3, 5	720	45
ut	c	2, 2, 2, 2, 2, 2, 2, 2, 3 . .	768	48

While note C makes 384 vibrations, the tone Cs gives 400, and the others as many as are marked by their corresponding numbers: note c will give, then, in the same time, double the number of vibrations marked by 384, that is 768. And for the following octaves, you have only to multiply these numbers by

2, by 4, or by 8. Accordingly note $\bar{c}$ will give twice 768, or 1536 vibrations; note $\bar{\bar{c}}$ twice 1536, or 3072 vibrations; and note $\bar{\bar{\bar{c}}}$ twice 3072, or 6144 vibrations. In order to comprehend the formation of sounds by means of these numbers 2, 3, and 5, it must be remarked, that the points placed between the numbers in the preceding number signify that they are multiplied into each other; thus, taking the tone Fs, for example, the expression 2, 2, 3, 3, 3, 5, signifies 2 multiplied by 2, that product by 3, that again by 3, that again by 3, and that by 5. Now 2 by 2 make 4, that by 3 make 12, that by 3 make 36, that by 3 make 108, and that by 5 make 540. Hence it is seen that the differences between these tones are not equal among themselves; but that some are greater, and others less. This is what real harmony requires. The inequality, however, not being considerable, we commonly look on all these differences as equal, denominating the interval from one note to another, *semitone;* and thus the octave is divided into 12 *semitones.* Many modern musicians make them equal, though this be contrary to the principles of harmony, because no one fifth or third is perfectly exact, and the effect is the same as if these tones were not perfectly in tune. They likewise admit, that we must give up exactness of accord in order to obtain the advantage of equality of semitones, so that the transposition from any one tone whatever to another may in no respect injure the melody. They acknowledge, however, that the same piece played in the tone C, or a half tone higher, that is Cs, must considerably affect its nature. It is evident, therefore, that in fact all semitones are not equal, whatever efforts may be made by musicians to render them such; because true harmony resists the execution of a design contradictory to its nature. Such, then, is the real origin of the musical notes already

in use; they are derived from the numbers 2, 3, and 5. Were we farther to introduce number 7, that of the tones of an octave would be increased, and the art of music carried to a higher degree of perfection. But here the mathematician gives up the musician to the direction of his ear.

3d May 1760.

Letter VIII.—Of the Pleasure derived from Fine Music.

It is a question as important as curious, whence is it that a fine piece of music excites a sentiment of pleasure? The learned differ on this subject. Some pretend that it is mere caprice, and that the pleasure produced by music is not founded on reason, because what is grateful to one is disgusting to another. Far from deciding the question, this renders it only more complicated. The very point to be determined is, how comes it that the same piece of music produces effects so different, since all admit that nothing happens without reason? Others maintain that the pleasure derived from fine music consists in the perception of the order which pervades it. This opinion appears at first sight sufficiently well founded, and merits a more attentive examination. Music presents objects of two kinds, in which order is essential. The one relates to the difference of the sharp or flat tones; and you will recollect, that it consists in the number of vibrations performed by each note in the same time. This difference, which is perceptible between the quickness of the vibrations of all sounds, is what is properly called harmony. The effect of a piece of music, of which we feel the relations of the vibrations of all the notes that compose it, is the production of harmony. Thus, two notes which differ an octave, excite a perception of the relation of 1 to 2; a fifth, of that of 2 to 3; and a greater third, of that of 4 to 5. We comprehend, then, the order which is found in harmony, when we know all the relations which pervade the notes of which it is composed; and it is the perception of the ear which leads to this knowledge. This perception more or less delicate, determines why the same harmony is felt by one, and not at all by another, especially when the relations of the notes are expressed by somewhat greater numbers. Music contains, beside harmony, another object equally susceptible of order, namely, the *measure*, by which we assign to every note a certain duration; and the perception of the measure consists in the knowledge of this duration, and of the relations which result from it. The drum and tymbal furnish the example of a music in which measure alone takes place, as all the notes are equal among themselves, and then there is no harmony. There is likewise a music consisting wholly in harmony, to the exclusion of measure. This music is the *choral*, in which all the notes are of the same duration; but perfect music unites harmony and measure. Thus the connoisseur who hears a piece of music, and who comprehends, by the acute perception of his ear, all the proportions on which both the harmony and the measure are founded, has certainly the most perfect knowledge possible of that music; while another, who perceives these proportions only in part, or not at all, understands nothing of the matter, or possesses at most a very slender knowledge of it. But the sentiment of pleasure excited by fine music must not be confounded with the knowledge of which I have been speaking, though it may be confidently affirmed, that a piece of music cannot produce any, unless the relations of it are perceived. For this knowledge alone is not sufficient

to excite the sentiment of pleasure; something more is wanting, which no one hitherto has unfolded. In order to be convinced that the perception alone of all the proportions of a piece of music is insufficient to produce pleasure, you have only to consider music of a very simple construction, such as goes in octaves alone, in which the perception of proportions is undoubtedly the easiest. Such music would be far from conveying pleasure, though you might have the most perfect knowledge of it. It will be said, then, that pleasure requires a knowledge not quite so easily attained—a knowledge that occasions some trouble; which must, if I may use the expression, cost us something. But, in my opinion, neither is this a satisfactory solution. A dissonance, the relations of which are expressed by the highest numbers, is caught with more difficulty; a series of dissonances, however, following without choice, and without design, cannot please. The composer must therefore have pursued in his work a certain plan, executed in real and perceptible proportions. Then a connoisseur, on hearing such a piece, and comprehending, beside the proportions, the very plan and design which the composer had in view, will feel that satisfaction which constitutes the pleasure procured by exquisite music to an ear accustomed to relish the beauties and delicacies of that enchanting art. It arises, then, from divining in some measure the views and feelings of the composer, whose execution, when fortunate, fills the soul with an agreeable sensation. It is a satisfaction somewhat similar to that which is derived from the sight of a well acted pantomime, in which you may conjecture, by the gesture and action, the sentiments and dialogue intended to be expressed, and which presents besides a well digested plan. The enigma of the chimney-