For the Resolution of Biquadratic Equations.

PROBLEM 11.

For a given homogeneous term proposed numerically, for simple biquadratic equations of the form . . *aaaa* = *bbbb,* the value of the proposed root sought equal to *a* can be extracted analytically:

Let the equation proposed numerically be *aaa* = 19565295376.

Place . . . . $b + c = a$.

The equation becomes : . .

$$\left.\begin{array}{c} b+c \\ b+c \\ b+c \\ b+c \end{array}\right| = 19565295376.$$

Therefore the equation from the homogeneous products and for the particular order of the terms is :

It becomes . . . .

$$\underbrace{+bbbb}_{Ab} \quad \underbrace{\begin{array}{l} +\ 4.bbbc \\ +\ 6.bbcc \\ +\ 4.bccc \\ +\ ..cccc \end{array}}_{Bc} = 19565295376.$$

Moreover the form for that type of operation with two parts for the analytical canon or set of instructions is that the first part *Ab* correspond to the first digit of the root, while the second part *Bc* accommodates the lesser digits of the root, as is apparent in the scheme set out below

Therefore the canon for extracting the root from the homogeneity 19565295376, is as follows.

The whole root to be extracted successively		3 7 4
		• • •
The homogeneity to be resolved	*bbbb*	1 9 5 6 5 2 9 5 3 7 6
Divisor . . . . First	*bbb*	2 7
Sing. root *b* = 3. Taken away .		
. . . . .	*bbbb*	8 1
The first singular root		3
		• • •
The remaining homogeneity to be resolved		1 1 4 6 5 2 9 5 3 7 6
	4.*bbb* . .	1 0 8 0 0 0
	6.b*b* . .	5 4 0 0
	4.*b*. . .	1 2 0
Divisor	*B*. . .	1 1 3 5 2 0
Ten times the sing. root *b* = 30 .	*4.bbbc* . .	7 5 6 0 0 0
Second sing. root . . *c* = 7	6.bbcc	2 6 4 6 0 0
	4.bccc	4 1 1 6 0
	cccc	2 4 0 1
Taken away	*Bc* . .	1 0 6 4 1 6 1
The first singular root		3 7
		• • •
The remaining homogeneity to be resolved		8 2 3 6 8 5 3 7 6
	4.*bbb* . .	2 0 2 6 1 2 0 0 0
	6.b*b* . .	8 2 1 4 0 0
	4.*b*. . .	1 4 8 0
Divisor	*B*. . .	2 0 3 4 3 4 8 8 0
Ten times the sing. root *b* = 370 .	*4.bbbc* . .	8 1 0 4 4 8 0 0 0
Second sing. root . . *c* = 4	6.bbcc	1 3 1 4 2 4 0 0
	4.bccc	9 4 7 2 0
	cccc	2 5 6
Taken away	*Bc* . .	8 2 3 6 8 5 3 7 6
The whole root finally extracted		3 7 4
		• • •
The remaining homogeneity finally		0 0 0 0 0 0 0 0 0

Therefore the factor 374 is the root itself extracted from the given homogeneity 19565295376 by resolution in this manner, equal to the root sought for *a* which was to be extracted from the problem considered.

PROBLEM 12.

For the given homogeneous term proposed numerically, for the given biquadratic equation of the form . . . *aaaa - ggga = bbbb,* the value of the proposed root sought equal to *a* can be extracted analytically:

Let the equation proposed numerically be *aaa* - 426.*a* = 2068948.
Hence . . . . 426 = *ggg*, & 2068948 = *bbbb*.
Put . . . . $b + c = a.$

Hence the equation is : $+ \left| \begin{matrix} b+c \\ b+c \\ b+c \\ b+c \end{matrix} \right. - \left| \begin{matrix} ggg \\ b+c \end{matrix} \right. = 2068948.$

Therefore with the products from the homogeneous terms,

It become : . . . .
$$\begin{matrix} +bbbb & -gggb \\ +4.bbbc & -gggc \\ +6.bbcc & \\ +4.bccc & \\ +\,..cccc & \end{matrix} = 2068948.$$

And by the same way distributed in two parts,

it becomes : . . . .
$$\underbrace{\begin{matrix} -gggb \\ +bbbb \end{matrix}}_{Ab} \quad \underbrace{\begin{matrix} -\,..gggc & +6.bbcc & & \\ +4.bbbc & +4.bccc & +\,..cccc \end{matrix}}_{Bc} = 2068948$$

Moreover, the Canon for this equation has the algebraic part divided into the two terms *Ab, Bc*, in order that the analytical work can proceed, which is duly established in agreement with the following Lemma, to be established in the usual way.
Therefore the resolution is done entirely by the application of this Canon, as is seen from the ordered numbers set out in the example placed below, in agreement with the given homogeneity 91148512, for the root to be extracted from this as follows.

Therefore the factor 38 is the root extracted from the given homogeneity 2068948 by resolution in this manner, equal to the root sought *a* which was to be extracted for the problem considered.

An example of anticipation.

Equation to be resolved . . . . $aaa - ggga = bbbb.$
$aaa - 43602354.a = 4172608.$
Hence . . . . $426 = ggg$, & $2068948 = bbbb$.

Canon of the resolution. . $\left| \begin{array}{l} - gggb \quad - ..gggc \quad + 6.bbcc \\ \underbrace{+ bbbb}_{Ab} \quad \underbrace{+ 4.bbbc \quad + 4.bccc \quad + ..cccc}_{Bc} \end{array} \right.$

The whole root to be extracted successively		3 5 2
The homogeneity to be resolved *bbbb*		004172608
	—ggg	—43602354
	bbb	27
Divisor	*—A*	—16602354
First sing. root *b* = 3	*—gggb*	—130807062
	bbbb	81
Taken away	*—Ab*	—49807062
Singular root		3
The remaining homogeneity to be resolved		4984878808
	—ggg	—43602354
	4.*bbb*	108000
Ten times the sing. root *b* = 30	6.*bb*	5400
	4.*b*	120
	+	113520
Divisor	*B*	69917646
	—gggc	—218010770
Second sing. root *c* = 5	4.*bbbc*	540000
	6.*bbcc*	135000
	4.*bccc*	15000
	cccc	625
	+	690625
Taken away	*Bc*	472613230
Root augmented		3 5
The remaining homogeneity to be resolved		258746508
	—ggg	—43602354
	4.*bbb*	171500000
Ten times the sing. root *b* = 350	6.*bb*	7350000
	4.*b*	1400
	+	172236400
Divisor	*B*	128534046
	—gggc	—87204708
Second sing. root *c* = 2	4.*bbbc*	343000000
	6.*bbcc*	2940000
	4.*bccc*	11200
	cccc	16
	+	345851216
Taken away	*Bc*	258746508
The whole root completely educed.		3 5 2
The remaining homogeneity finally.		000000000

Therefore the factor is itself the root 352 extracted for the root sought *a* from the given homogeneity 4172608 by resolution in this manner, which was to be extracted.

PROBLEM 13.

For a given homogeneous term proposed numerically, for biquadratic equations of the form . . *aaaa - ffaa + ggga = bbbb,* the value of the proposed root sought equal to *a* can be extracted analytically:

Let the equation proposed numerically be *aaa* - 1024.*aa* + 6254.*a* = 19633735875.
Hence . . . . 1024 = *ff*, 6254 = *ggg*, & 19633735875 = *bbbb*.
Place . . . . *b* + *c* = *a*.

Hence . . . $+\left|\begin{array}{l} b+c\\ b+c\\ b+c\\ b+c\end{array}\right. - \left|\begin{array}{l} ff\\ b+c\\ b+c\end{array}\right. + \left|\begin{array}{l} ggg\\ b+c\end{array}\right. = 19633735875.$

Therefore with the products from the homogeneous terms,

The equation becomes: . . . . . .
$$\begin{array}{lll} + ..bbbb & - ffbb & + gggb \\ + 4.bbbc & -2.ffbc & + gggc \\ + 6.bbcc & - ffcc & \\ + 4.bccc & & \\ + ..cccc & & \end{array} = 19633735875.$$

And by the same way distributed in two parts,

it becomes : . . . .
$$\underbrace{\begin{array}{l} + gggb \\ - ffbb \\ + ..bbbb \end{array}}_{Ab} \quad \underbrace{\begin{array}{llll} + gggc & + 4.bbbc & & \\ - ffcc & + 6.bbcc & & \\ -2.ffbc & + 4.bccc & + ..cccc & \end{array}}_{Bc} = 19633735875.$$

Moreover, the Canon for this equation has the algebraic part divided into the two terms *Ab, Bc*, in order that the analytical work can proceed, which is duly established in agreement with the following Lemma, to be established in the usual way.

Therefore the resolution is done entirely by the application of this Canon, as is seen from the order in the setting out of the figures of the example placed below, acting on the given homogeneity itself 19633735875, for the root to be extracted from this as follows .

The whole root to be extracted successively		3 7 5
The homogeneity to be resolved *bbbb*		1 9 6 3 3 7 3 5 8 7 5
	ggg	6 2 5 4
First sing. root *b* = 3 .	— *ff*	— 1 0 2 4
	bbb. . .	2 7
	+	2 7 0 0 6 2 5 4
Divisor	*A*	2 6 9 0 3 8 5 4
	gggb. . . .	1 8 7 6 2
	— *ffbb*. . .	— 9 2 1 6
	. *bbbb*. . .	8 1
	+. . . .	8 1 0 1 8 7 6 2
Taken away	*Ab* . .	8 0 0 9 7 1 6 2
Singular root		3
The remaining homogeneity to be resolved		1 1 6 2 4 0 1 9 6 7 5
	ggg .	6 2 5 4
	— *ff*	— 1 0 2 4
Ten times the sing. root *b* = 30 .	— 2.*ffb*	— 6 1 4 4 0
	4.*bbb* .	1 0 8 0 0 0
	6.*bb* . .	5 4 0 0
	4.*b*. . .	1 2 0
	+	1 1 3 5 2 6 2 5 4
	—	6 2 4 6 4
Divisor	*B*. . .	1 1 2 9 0 1 6 1 4
	gggc	4 3 7 7 8
Second sing. root . . *c* = 5	— *ffcc*	— 5 0 1 7 6
	— 2.*ffbc*	— 4 3 0 0 8 0
	4.*bbbc* . .	7 5 6 0 0 0
	6.*bbcc*	2 6 4 6 0 0
	4.*bccc*	4 1 1 6 0
	cccc	2 4 0 1
	+	1 0 6 4 2 0 4 7 7 8
	—	4 8 0 2 5 6
Taken away	*Bc* . .	1 0 5 9 4 0 2 2 1 8

Singular root		3 7
The remaining homogeneity to be resolved		1 0 2 9 9 9 7 4 9 5
	ggg	6 2 5 4
	— *ff*	— 1 0 2 4
Ten times the sing. root *b* = 370 .	— 2.*ffb*	— 7 5 7 7 6 0
	4.*bbb* .	2 0 2 6 1 2 0 0 0
	6.*bb* . .	8 2 1 4 0 0
	4.*b*. . .	1 4 8 0
	+	2 0 3 4 4 1 1 3 4
	—	7 5 8 7 8 4
Divisor	*B*. . .	2 0 2 6 8 2 3 5 0
	gggc	3 1 2 7 0
Second sing. root . . *c* = 5	— *ffcc*	— 2 5 6 0
	— 2.*ffbc*	— 3 7 8 8 8 0 0
	4.*bbbc* . .	1 0 1 3 0 6 0 0 0 0
	6.*bbcc*	2 0 5 3 5 0 0 0
	4.*bccc*	1 8 5 0 0 0
	cccc	6 2 5
	+	1 0 3 3 8 1 1 8 9 5
	—	3 8 1 4 4 0 0
Taken away	*Bc* . .	1 0 2 9 9 9 7 4 9 5
Whole root finally extracted :		3 7 5
The remaining homogeneity finally.		0 0 0 0 0 0 0 0 0

Therefore the root 375 is extracted for the root sought *a* from the given homogeneity 19633735875 in this manner by analysis, which was to be extracted.

PROBLEM 14.

For a given homogeneous term proposed numerically, for biquadratic equations of the form . . *aaaa - ffaa - ggga = bbbb,* the value of the proposed root sought equal to *a* can be extracted analytically:

Let the equation proposed numerically be *aaa - 1024.aa* - 6254.*a* = 19629045375.
Hence . . . . 1024 = *ff*, 6254 = *ggg*, & 19633735875 = *bbbb*.
Place . . . . *b* + *c* = *a*.

Hence . . . + $\left.\begin{array}{l} b+c \\ b+c \\ b+c \\ b+c \end{array}\right|$ - $\left.\begin{array}{l} ff \\ b+c \\ b+c \end{array}\right|$ - $\left.\begin{array}{l} ggg \\ b+c \end{array}\right|$ = 19629045375.

Therefore with the products from the homogeneous terms,
it becomes . . . .

+ ..*bbbb*	- *ffbb*	- *gggb*	= 19629045375.
+ 4.*bbbc*	-2.*ffbc*	- *gggc*	
+ 6.*bbcc*	- *ffcc*		
+ 4.*bccc*			
+ ..*cccc*			

And by the same way distributed in two parts,

it becomes . . . .

$$\underbrace{-gggb - ffbb + ..bbbb}_{Ab} \quad \underbrace{-gggc - ffcc - 2.ffbc + 4.bbbc + 6.bbcc + 4.bccc + ..cccc}_{Bc} = 19629045375.$$

Moreover, the Canon for this equation has the algebraic part divided into the two terms *Ab, Bc*, in order that the analytical work can proceed, which is duly established in agreement with the following Lemma, to be established in the usual way.

Therefore the resolution is done entirely by the application of this Canon, as is seen from the order in the setting out of the figures of the example placed below, acting on the given homogeneity itself 19629045375, for the root to be extracted from this as follows .

The whole root to be extracted successively		3 7 5
The homogeneity to be resolved *bbbb*		19629045375
	—*ggg*	— 6254
First sing. root *b* = 3 .	—*ff*	— 1024
	bbb. . .	27
	—. . . .	108654
Divisor	*A*	26891346
	—*gggb*. . ..	— 18762
	—*ffbb*. . .	— 9216
	. *bbbb*. . .	81
	—. . . .	940362
Taken away	*Ab* . .	80059638
Singular root		3
The remaining homogeneity to be resolved		11623081575
	— *ggg* .	— 6254
	—*ff*	— 1024
Ten times the sing. root *b* = 30 .	— 2.*ffb*	— 61440
	4.*bbb* .	108000
	6.*bb* . .	5400
	4.*b*. . .	120
	+	113520
	—	— 630894
Divisor	*B*. . .	112889106
	— *gggc*	— 43778
Second sing. root . . *c* = 7	— *ffcc*	— 50176
	— 2.*ffbc*	— 430080
	4.*bbbc* . .	756000
	6.*bbcc*	264600
	4.*bccc*	41160
	cccc	2401
	+	1064161
	—	4856338
Taken away	*Bc* . .	1059304662
Root augmented.		3 7
The remaining homogeneity to be resolved		1029934955

Singular root		3 7
The remaining homogeneity to be resolved		1 0 2 9 9 3 4 9 5 5
	—*ggg*	— 6 2 5 4
	—*ff*	— 1 0 2 4
Ten times the sing. root *b* = 370 .	— 2.*ffb*	— 7 5 7 7 6 0
	4.*bbb* .	2 0 2 6 1 2 0 0 0
	6.*bb* . .	8 2 1 4 0 0
	4.*b*. . .	1 4 8 0
	+	2 0 3 4 3 4 8 8 0
	—	7 6 5 0 3 8
Divisor	*B*. . .	2 0 2 6 6 9 8 4 2
	— *gggc*	— 3 1 2 7 0
Second sing. root . . *c* = 5	— *ffcc*	— 2 5 6 0 0
	— 2.*ffbc*	— 3 7 8 8 8 0 0
	4.*bbbc* . .	1 0 1 3 0 6 0 0 0 0
	6.*bbcc*	2 0 5 3 5 0 0 0
	4.*bccc*	1 8 5 0 0 0
	cccc	6 2 5
	+	1 0 3 3 7 8 0 6 2 5
	—	3 8 4 5 6 7 0
Taken away	*Bc* . .	1 0 2 9 9 3 4 9 5 5
Whole root finally extracted		3 7 5
The remaining homogeneity finally.		0 0 0 0 0 0 0 0 0

For the Resolution of Fifth Order Equations.

PROBLEM 15.

For a given homogeneous term proposed numerically, for quintic equations of the form . . *aaaaa* = *lllll,* the value of the proposed root sought equal to *a* can be extracted analytically:

Let the equation proposed numerically be *aaaaa* = 15755509298176.
Therefore *lllll* = 15755509298176.
Place . . . . *b* + *c* = *a.*
Hence . . . .

$$\left.\begin{array}{c} b+c \\ b+c \\ b+c \\ b+c \\ b+c \end{array}\right| = lllll.$$

For the products from the homogeneous terms therefore become . . . .

$$\begin{array}{l} +..bbbbb \\ +5.bbbbc \\ +10.bbbcc \\ +10.bbccc \\ +5.bcccc \\ +..cccc \end{array} = lllll = 15755509298176.$$

And by the same way distributed in two parts,

it becomes :

$$\underbrace{+..bbbbb}_{Ab} \quad \underbrace{\begin{matrix} +\ 5.bbbbc \\ +\ 10.bbbcc \\ +\ 10.bbccc \\ +\ 5.bcccc \\ +\ ..ccccc \end{matrix}}_{Bc} \quad = \quad 15755509298176.$$

Moreover, the Canon for this equation has the algebraic part divided into the two terms *Ab, Bc*, in order that the analytical work can proceed, which is duly established in agreement with the following Lemma, to be established in the usual way.

Therefore the resolution is done entirely by the application of this Canon, as is seen from the order in the setting out of the figures of the example placed below, acting on the given homogeneity itself 15755509298176, for the root to be extracted from this as follows .

The whole root to be extracted successively		4 • 3 • 6 •
The homogeneity to be resolved		1 5 7 5 5 5 0 9 2 9 8 1 7 6
Divisor	*bbbb*	2 5 6
First Sing. root *b* =4.		
Taken away	*bbbb*	1 0 2 4
The first singular root		4 • • •
The remaining homogeneity to be resolved		5 5 1 5 5 0 9 2 9 8 1 7 6
	5.*bbbb* . .	1 2 8 0 0 0 0 0
	10.*bbb* . .	6 4 0 0 0 0
Ten times the sing. root *b* = 40	10.*bb* . .	1 6 0 0 0
	5.*b*. . .	2 0 0
Divisor	*B*. . .	1 3 4 5 0 0 0 0
.	5.*bbbbc* .	3 8 4 0 0 0 0 0
Second sing. root . . *c* = 3	10.*bbbcc* .	5 7 6 0 0 0 0
	10.*bbccc* . .	4 3 2 0 0 0
	5.*bcccc*	1 6 2 0 0
	..*ccccc*	2 4 3
Taken away	*Bc* . .	4 4 6 0 8 4 4 3
The first singular root		4 • 3 • •
The remaining homogeneity to be resolved		1 0 5 4 6 6 4 9 9 8 1 7 6
	5.*bbbb* . .	1 7 0 9 4 0 0 5 0 0 0 0
Ten times the sing. root *b* = 430 .	10.*bbb* . .	9 9 5 0 7 0 0 0 0
	10.*bb* . .	1 8 4 9 0 0 0
	5.*b*. . .	1 8 8 0
Divisor	*B*. . .	1 7 1 7 . . . .
	5.*bbbbc* .	1 0 2 5 6 4 0 3 0 0 0 0 0
Third sing. root . . *c* = 6	10.*bbbcc* .	2 8 6 2 2 5 2 0 0 0 0
	10.*bbccc* . .	3 9 9 3 8 4 0 0 0
	5.*bcccc*	2 7 8 6 4 0 0
	..*ccccc*	7 7 7 6
Taken away	*Bc* . .	1 0 5 4 6 6 4 9 9 8 1 7 6
The whole root finally drawn out		4 • 3 • 6 •
The remaining homogeneity finally		0 0 0 0 0 0 0 0 0 0 0 0 0

	ff	— 2 6 4 8
Sing. root times ten *b* = 450.	*3.bb*	6 0 7 5 0 0
	3.b . . .	1 3 5 0
	+	6 0 8 8 5 0
Divisor	*B*	6 0 6 2 0 2
	-ffc	— 5 2 9 6
	3.bbc . . .	1 2 1 5 0 0 0 0
Second sing. root c = 2.	*3.bcc* . . .	5 4 0 0
	ccc . . .	8
	+	1 2 2 2 0 4 0 8
Taken away	*Bc*	1 2 1 5 1 1 1 2
Whole completely educed root		4 5 2
The homogeneity finally remaining		0 0 0 0 0 0 0 0

Therefore the factor is the root 436 itself extracted for the root sought equal to *a* from the given homogeneity 15755509298176 by resolution in this manner, which was to be extracted.

PROBLEM 16.

For a given homogeneous term proposed numerically, for quintic equations of the form . . *aaaaa - ffaaa + hhhha= lllll,* the value of the proposed root sought equal to *a* can be extracted analytically:

Let the equation proposed numerically be *aaaaa* - 57.*aaa* + 5263.*a*= 900050558322.
Therefore . . . . *ff* = 57, & *hhhh* = 5263, & *lllll* = 900050558322.
Place . . . . *b + c = a.*

Hence	+	*b + c*	-	*ff*	+	*hhhh* = 900050558322.
		b + c		*b + c*		*b + c*
		b + c		*b + c*		
		b + c		*b + c*		
		b + c				

For the products from the homogeneous terms therefore

it becomes	+..*bbbbb*	-..*ffbbb*	+..*hhhhb*	= 900050558322.
	+ 5.*bbbbc*	- 3.*ffbbc*	+..*hhhhc*	
	+ 10.*bbbcc*	- 3.*ffbcc*		
	+ 10.*bbccc*	- .*ffccc*		
	+ 5.*bcccc*			
	+ ..*cccc*			

And by the same way distributed in two parts,

it becomes :	+..*hhhhb*	+..*hhhhc*	+ 3.*ffbbc*	+ 10.*bbccc*	= 15755509298176.
	-..*ffbbb*	- .*ffccc*	+ 5.*bbbbc*	+ 5.*bcccc*	
	+..*bbbbb*	- 3.*ffbcc*	+ 10.*bbbcc*	+ ..*cccc*	
	Ab	*Bc*			

Moreover, the Canon for this equation has the algebraic part divided into the two terms *Ab, Bc*, in order that the analytical work can proceed, which is duly established in agreement with the following Lemma, to be established in the usual way.

Therefore the resolution is done entirely by the application of this Canon, as is seen from the order in the setting out of the figures of the example placed below, acting on the given homogeneity itself 900050558322, for the root to be extracted from this as follows.

The whole root to be extracted successively		2 4 6
The homogeneity to be resolved *lllll*		9 0 0 0 5 0 5 5 8 3 2 2
	hhhh	0 5 2 6 3
	—*ff*	— 0 0 0 5 7
	bbbb	1 6
	+	1 6 0 0 0 0
Divisor	*A*	1 5 9 9 4
	hhhhb	1 0 5 2 6
First Sing. root *b* =2.	—*ffbbb*	— 0 0 4 5 6
	bbbbc	3 2
	+	3 2 0 0 0 1 0 5 2 6
Taken away	*Ab*	3 1 9 5 4 5 0 5 2 6
The first singular root		2
The remaining homogeneity to be resolved		5 8 0 5 0 5 5 0 5 7 2 2
	hhhh	0 5 2 6 3
	—*ff*	— 0 0 0 5 7
Ten times the sing. root *b* = 20	—3.*ffb*	— 3 4 2
	—3.*ffbb*.	— 6 8 4 2 0 0
Divisor	5.*bbbb*	8 0 0 0 0 0
.	10.*bbb*	8 0 0 0 0
Second sing. root *c* = 4	10.*bb*	4 0 0 0
	5.*b*.	1 0 0
	+	8 8 4 1 0 0
	—	7 1 8
	B.	8 8 3 3 8
	4.*hhhhc*	1 1 0 5 2
	—3.*ffccc*	— 3 6 4 8
	—3.*ffbcc*	— 5 4 7 2 0
	—3.*ffbbc*	— 2 7 3 6 0 0
	5.*bbbbc*	3 7 2 0 0 0 0
	10.*bbbcc*	1 2 8 0 0 0 0
	10.*bbccc*	2 5 6 0 0 0
	5.*bcccc*	2 5 6 0 0
	..*ccccc*	1 0 2 4
	+	4 7 6 2 6 2 6 1 0 5 2
	—	2 3 1 9 6 8
Taken away	*Bc*	4 7 5 9 3 0 6 4 2 5 2
The first singular root		2 4
The remaining homogeneity to be resolved		1 0 4 5 7 4 8 6 3 2 0 2
	hhhh	5 2 6 3
	—*ff*	— 5 7
Ten times the sing. root *b* = 240	—3.*ffb*	— 4 1 0 4 0
	—3.*ffbb*.	— 9 8 4 9 6 0 0
	5.*bbbb*	1 6 5 8 8 8 0 0 0 0 0
	10.*bbb*	1 3 8 2 4 0 0 0 0
	10.*bb*	9 7 6 0 0 0
	5.*b*.	1 2 0 0
	+	1 6 7 2 7 7 2 0 0 0 0
	—	9 8 9.
Divisor	*B*.	1 6 7 1 7
.	4.*hhhhc*	3 1 5 7 8
Second sing. root *c* = 6	—3.*ffccc*	— 1 2 3 1 2
	—3.*ffbcc*	— 1 4 7 7 4 4 0
	—3.*ffbbc*	— 5 9 0 9 7 6 0 0
	5.*bbbbc*	9 9 5 3 2 8 0 0 0 0 0
	10.*bbbcc*	9 9 7 6 6 4 0 0 0 0
	10.*bbccc*	1 2 4 4 1 6 0 0 0
	5.*bcccc*	1 5 5 5 2 0 0
	..*ccccc*	7 7 7 6
	+	1 0 4 6 3 5 4 5 0 5 5 4
	—	6 0 5 8 7 3 5 2
Taken away	*Bc*	1 0 4 5 7 4 8 6 3 2 0 2
The whole root finally extracted		2 4 6
The remaining homogeneity finally		0 0 0 0 0 0 0 0 0 0 0 0

	ff	— 2 6 4 8
Sing. root times ten *b* = 450.	*3.bb*	6 0 7 5 0 0
	3.b . . .	1 3 5 0
	+	6 0 8 8 5 0
Divisor	*B*	6 0 6 2 0 2
	-ffc	— 5 2 9 6
	3.bbc . . .	1 2 1 5 0 0 0
Second sing. root c = 2.	*3.bcc* . . .	5 4 0 0
	ccc . . .	8
	+	1 2 2 0 4 0 8
Taken away	*Bc*	1 2 1 5 1 1 2
Whole completely educed root		4 5 2
		• • •
The homogeneity finally remaining		0 0 0 0 0 0 0 0

Therefore the factor is the root 246 extracted for the root sought *a* from the given homogeneity 900050558322 resolved in this manner by analysis, which was to be extracted.

The artifice for approximation in the asymmetric case.

Since it commonly happens that the root to be extracted from the given homogeneity is irrational or unable to be expressed by a [finite] number, it is clear that it cannot be extracted by analysis using any artifice, as there is always some remainder at the end of the work, an indication of an imperfect resolution. In this case, which is most frequent, the terminal digit has to be extended by gradual multiplication with ciphers added to the right, with two for roots of the quadratic kind, three for the cubic, four for the biquadratic, etc., for whatever quantity is desired, and now without variation the root to be extracted in the course of the work to a tenth, a hundredth, a thousandth, etc., part of one can be continued to the true required approximation. When the process of the approximation is simple and uniform, the following example should suffice to show the process for these.

Quoniam accidit communiter radicem e dato homogeneo educendum irrationalem sive numero inexplicabilem esse, nullo scilicet analytice artificio educibilem, quin ad finem operis supersit, semper residium aliquod imperfectae resolutionis indicium. In ea casu qui frequentissimus est, residium finale per multiplicationem gradualem additis ad dextram cyphris producendum est, binariis in genere quadratico, ternariis in cubico, quaternatiis in biquadratico, &c. quotcunque libuerit, & radix iam educta invariato operis tenore ad decimas, centesimas, millesimas, &c., unitas partes, ad requisitam scilicet veritati proximitatem continuanda est. Approximationis huius processum quum simplex & uniformis sit, exemplis sequentibus ostendisse sufficiat.

Example of a Quadratic Approximation.

The equation to be resolved . . . $aa + da = ff$
$aa + 14.a = 7929.$

Canon of the resolution

$+db$	$+dc$
$+bb$	$+2.bc$
	$+.cc$
$\underbrace{\qquad}_{Ab}$	$\underbrace{\qquad}_{Bc}$

			8 2
The homogeneity to be resolved			7 9 2 9
		d	1 4
		b	8
Divisor		*A* . .	9 4
		db .	1 1 2
	b = 8	*bb* .	. 6 4
Taken away . .		*Ab* .	7 5 2
The singular root			8
The remaining homogeneity to be resolved			4 0 9
		d	1 4
	b = 80	2*b*	1 6 0
Divisor		*B*	1 7 4
		dc	2 8
	c = 2	2.*bc*	3 2 0
		cc	4
Taken away		*Bc* . . .	3 5 2
The root to unity extracted			8 2
The remaining homogeneity to be resolved			0 5 7 0 0 0 0 0 0

The root to unity extracted			8 2
The remaining homogeneity to be resolved			5 7 0 0 0 0 0 0
		d	1 4
	b = 820	2*b*	1 6 4 0
Divisor		*B*	1 7 8 0
		dc	4 2
	c =3	2.*bc*	4 9 2 0
		cc	9
Taken away		*Bc* . . .	5 3 4 9
			8 2 3
The remaining homogeneity to be resolved			3 5 2 0 0 0 0
		d	1 4
	b = 8230	2*b*	1 6 4 6 0
Divisor		*B*	1 7 8 6 0
		dc	1 4
	c =1	2.*bc*	1 6 4 0
		cc	1
Taken away		*Bc* . . .	1 7 8 6 6
			8 2 3 1
The remaining homogeneity to be resolved			1 7 2 3 9 0 0
		d	1 4
	b = 8230	2*b*	1 6 4 6 2 0
Divisor		*B*	1 7 8 6 2 0
		dc	1 2 6
	c =9	2.*bc*	1 4 8 1 5 8 0
		cc	8 1
Taken away		*Bc* . . .	1 6 0 7 6 6 1
The root continued to the 1000th part			8 2 3 1 9
The remaining homogeneity to be resolved			1 1 6 2 3 9

The root $\frac{82319}{1000}$ or $82\frac{319}{1000}$ is constructed from the given homogeneity 7929 in this manner from the proposed equation, truly within one thousandth part of unity.

Example of a Cubic Approximation.

The equation to be resolved . . . $aaa + ffa = ggg$,
$aaa + 135.a = 98754.$

The canon of the resolution $+\ ffb \quad +\ ..ffc \quad +\ 3.\ bcc$
$\underbrace{+\ bbb}_{Ab} \quad \underbrace{+\ 3.bbc \quad +\ ..ccc}_{Bc}$

The root.		4 5
		• •
The homogeneity to be resolved	*ggg*	9 8 7 5 4
		• •
	ff	. . 1 3 5
	bb	. 1 6
Divisor	*A*	1 7 3 5
	ffb	5 4 0
First Sing. root *b* =4.	*bbb*	6 4
Taken away	*Ab*	6 9 4 0
		4
		•
The remaining homogeneity to be resolved		2 9 3 5 4
		4
		•
The remaining homogeneity to be resolved		2 9 3 5 4
		• •
	ff . . .	. . 1 3 5
Sing. root times ten *b* = 40.	*3.bb*	. 4 8 0 0
	3.b . . .	1 2 0
Divisor	*B*	5 0 5
	ffc	6 7 5
	3.bbc	2 4 0 0 0
Second sing. root c = 5.	*3.bcc*	3 0 0 0
	ccc	2 2 5
Taken away	*Bc*	2 7 8 0 0
Root to one educed.		4 5
		• • • •
The remainder produced of the homogeneity		1 5 5 4 0 0 0 0 0 0
		• • • •
	ff . . .	. . 1 3 5
b = 450.	*3.bb*	. 6 0 7 5 0 0
	3.b . . .	1 3 5 0
Divisor	*B*	6 2 2
	ffc	2 7 0
	3.bbc	1 2 1 5 0 0 0
Second sing. root c = 2.	*3.bcc*	5 4 0 0
	ccc	8
Taken away	*Bc*	1 2 4 7 4 0 8
		4 5 2
		• • • •
Remainder of the homogeneity produced.		3 0 6 5 9 2 0 0 0
		• • • •
	ff . . .	. . 1 3 5
b = 4520.	*3.bb*	. 6 1 2 9 1 2 0 0
	3.b . . .	1 3 5 6 0
Divisor	*B*	6 2 6 5 4
	ffc	5 4 0
	3.bbc	2 4 5 1 6 4 8 0 0
Second sing. root c = 2.	*3.bcc*	2 1 6 9 6 0
	ccc	6 4
Taken away	*Bc*	2 5 0 7 8 1 8 2 4
The root to the 100th part from the continuation of the remainder of the homogeneity produced is taken away.		4 5 2 2
		• • • •
		5 5 8 1 0 1 7 6