For the Resolution of Cubic Equations.

PROBLEM 5.

For a given homogeneous term proposed numerically, for simple cubic equations [of the form] . . $aa = ggg$, the value of the proposed root of a sought can be extracted analytically:

Let the equation proposed numerically be $aaa = 105689636352$.
And put . . . . $b + c = a$.

Hence . . . .
$$\left.\begin{array}{r} b+c \\ b+c \\ b+c \end{array}\right| = 105689636352.$$

Therefore the equation from the homogeneous products and for the particular order of the terms is :

is: . . . .
$$\underbrace{+bbb}_{Ab} \quad \underbrace{\begin{array}{l} +\ 3.bbc \\ +\ 3.bcc \\ +\ ..ccc \end{array}}_{Bc} = 105689636352.$$

Moreover the form for that type of operation with two parts for the analytical canon or set of instructions is that the first part Ab correspond to the first digit of the root, while the second part Bc accommodates the lesser digits of the root, as is apparent in the scheme set out below
Therefore the canon for extracting the root from the homogeneity 105689636352 is as follows.

The whole root to be extracted successively		4 7 2 8
		• • • •
The homogeneity to be resolved		1 0 5 6 8 9 6 3 6 3 5 2
Divisor	*bb*	1 6
. First Sing. root $b = 4$.		
. . Taken away	*bbb*	6 4
. . .		
The first singular root		4
		• • • •
The remaining homogeneity to be resolved		4 1 6 8 9 6 3 6 3 5 2
Divisor	3.*bb* .	4 8 0 0
Ten times the sing. root $b = 40$.	3.*bbc* . .	3 3 6 0 0
Second sing. root . . $c = 7$	.3.*bcc*	5 8 8 0
	ccc . . .	3 4 3
Taken away	*Bc* . .	3 9 8 2 3
The root augmented		4 7
		• • • •
The remaining homogeneity to be resolved		1 8 6 6 6 3 6 3 5 2
Divisor	3.*bb* .	6 6 2 7 0 0
Ten times the sing. root $b = 470$.	3.*bbc* . .	1 3 2 5 4 0 0
Second sing. root . . $c = 2$	.3.*bcc*	5 6 4 0
	ccc . . .	8
Taken away	*Bc* . .	1 3 3 1 0 4 8
The root augmented		4 7 2
		• • • •
The remaining homogeneity to be resolved		5 3 5 5 8 8 3 5 2
Divisor	3.*bb* .	6 6 8 3 5 2 0 0
Ten times the sing. root $b =$ 4720 .	3.*bbc* . .	5 3 4 6 8 1 6 3 0
	.3.*bcc*	9 9 6 2 4 0
Second sing. root . . $c = 8$	*ccc* . . .	5 1 2
	Bc . .	5 3 5 5 8 8 3 5 2
Taken away		
Whole root completely extracted		4 7 2 8
		• • • •
The remaining homogeneity finally		0 0 0 0 0 0 0 0 0

Therefore the factor 4728 is the root itself extracted from the given homogeneity 105689636352 by resolution in this manner, equal to the root a sought which was to be extracted from the problem considered.

PROBLEM 6.

For the given homogeneous term proposed numerically, for the given cubic equation [of the form] . . . $aaa + daa + ffa = ggg$, the value of the proposed root of a sought can be extracted analytically:

Let the equation proposed numerically be $aaa + 68.aa + 4352.a = 186394079$.
And put . . . . $b + c = a$.

Hence . . . .
$$\left.\begin{array}{c} b+c \\ b+c \\ b+c \end{array}\right| \left.\begin{array}{c} +d \\ b+c \\ b+c \end{array}\right| \left.\begin{array}{c} +ff \\ b+c \end{array}\right| = 105689636352.$$

Therefore with the products from the homogeneous terms,
The equation become . . . .
$$\begin{array}{lll} + \,..bbb & + \,.\,dbb & + ffb \\ + \,3.bbc & + 2.\,dbc & + ffc \\ + 3.bcc & + \,.\,dcc & \\ + \,..ccc & & \end{array} = 105689636352.$$

And in the same way distributed in two parts,

it becomes . . . .
$$\begin{array}{lll} + ffb & + ffc & + 3.bbc \\ + dbb & + dcc & + 3.bcc \\ \underbrace{+ bbb}_{Ab} & \multicolumn{2}{l}{\underbrace{+ 2.dbc \quad + ..ccc}_{Bc}} \end{array} = 105689636352.$$

Moreover, the Canon for this equation has the algebraic part divided into the two terms *Ab, Bc*, in order that the analytical work can proceed, which is duly established in agreement with the following Lemma, to be established in the usual way.
Therefore the resolution is done entirely by the application of this Canon, as is seen from the ordered numbers set out in the example placed below, in agreement with the given homogeneity 186394079, for the root to be extracted from this as follows .

The whole root to be extracted successively		5 4 7
The homogeneity to be resolved *ggg*		1 8 6 3 9 4 0 7 9
	ff	. . 0 4 3 5 2
	d	. . 0 6 8
	bb	2 5
Divisor	*A*	2 6 1 1 5 2
First Sing. root $b = 5$. .	*ffb*	2 1 7 6 0
	dbb	1 7 0 0
	bbb	1 2 5
Taken away	*Ab*	1 4 4 1 7 6 0
The first singular root		5
The remaining homogeneity to be resolved		4 2 2 1 8 0 7 9

	ff	. . 4 3 5 2
	d	. . 6 8
	2.db	6 8 0 0
Sing. root times ten *b* = 50.	*3.bb* . . .	7 5 0 0
	3.b . . .	1 5 0
Divisor	*B*	8 3 8 0 3 2
	ffc	1 7 4 0 8
	dcc	1 0 8 8
Second sing. root c = 4.	*2.dbc*	2 7 2 0 0
	3.bbc	3 0 0 0 0
	3.bcc	2 4 0 0
	ccc	6 4
Taken away	*Bc*	3 5 4 6 6 8 8
Augmented root		5 4
The remaining homogeneity to be resolved		6 7 5 1 1 9 9

	ff	. . 4 3 5 2
	d	. . 6 8
	2.db	7 3 4 4 0
Sing. root times ten *b* = 540.	*3.bb* . . .	8 7 4 8 0 0
	3.b . . .	1 6 2 0
Divisor	*B*	9 5 4 2 8 0
	ffc	3 0 4 0 6
	dcc	3 3 3 2
Second sing. root c = 7.	*2.dbc*	5 1 4 0 8 0
	3.bbc	6 1 2 3 6 0 0
	3.bcc	7 9 3 8 0
	ccc	3 4 3
Taken away	*Bc*	6 7 5 1 1 9 9
Whole root completely extracted		5 4 7
The homogeneity finally remaining		0 0 0 0 0 0 0

Therefore the factor 547 is the root itself extracted from the given homogeneity 186394079. by resolution in this manner, equal to the root *a* sought which was to be extracted from the problem considered.

Lemma.

Since for the proposed equation . . . . *aaa* + 68. *aa* + 4352. *a* = 186394079, with the root 547 extracted by resolution, the backward-step method of composition is used to verify the process. Therefore from the converse of the resolution, the explanation for the composition follows by means of which equations are produced. Hence, for the resolution of this cubic, as the Canon acts in one direction to achieve the resolution, the truth is established for the resolution process. The general computation is hence the verification of this method, and in the problems to follow either from being acted upon, referred to, or understood to be necessary.

[Note : Thus, the solving of an equation is the opposite process from that of its composition, the latter we regard as true beyond any doubt, as the process or canon starts with a simple equality of a number and a symbol, which are acted on by other numbers and the symbol itself to generate the equation finally; if this generation canon is run in the opposite direction, acting on the equation, it hence solves the equation, which is hence also a true process. The canon referred to in the text is the one that solves the equation. This is the argument given, in so many words; however, the process is not quite this simple, as it depends on the initial choice of the first digit of the root, and of the subsequent smaller contributions. It is known from such numerical work that the initial choice of the first digit may result in divergence, or even oscillation about a fixed number of steps, as well as the simple convergence considered.]

E converso igitur per aequationis explicationem compositive factam, tam ipsius resolutionis quam canonis cuius directione facta est resolutio veritas oportuit asseritur. Generalis est verificationis huiusmodi ratio, & ad sequentia Problemata vel actu adscribenda vel ut necessaria subintelligenda.

Another schematic variety of problem & example 6,
from a somewhat different ordering of the canon.

The whole root to be drawn out successively		5 4 7
The homogeneity to be resolved *ggg*		1 8 6 3 9 4 0 7 9
	ff	4 3 5 2
	db	3 4 0
	bb	2 5
Divisor	*A*	2 8 8 3 5 2
First Sing. root $b = 5$. .	*ffb*	2 1 7 6 0
	dbb	1 7 0 0
	bbb	1 2 5
Taken away	*Ab*	1 4 4 1 7 6 0
The first singular root		5
The remaining homogeneity to be resolved		4 2 2 1 8 0 7 9
	ff	4 3 5 2
	d	6 8
	2.db	6 8 0 0
Sing. root times ten $b = 50$.	*3.bb* . . .	7 5 0 0
	3.b . . .	1 5 0
Divisor	*B*	8 3 8 0 3 2
	ffc	1 7 4 0 8
	dcc	1 0 8 8
Second sing. root $c = 4$.	*2.dbc*	2 7 2 0 0
	3.bbc	3 0 0 0 0
	3.bcc	2 4 0 0
	ccc	6 4
Taken away	*Bc*	3 5 4 6 6 8 8
Augmented root		5 4
The remaining homogeneity to be resolved		6 7 5 1 1 9 9
	ff	4 3 5 2
	d	6 8
	2.db	7 3 4 4 0
Sing. root times ten $b = 540$.	*3.bb* . . .	8 7 4 8 0 0
	3.b . . .	1 6 2 0
Divisor	*B*	9 5 4 2 8 0
	ffc	3 0 4 0 6
	dcc	3 3 3 2
Second sing. root $c = 7$.	*2.dbc*	5 1 4 0 8 0
	3.bbc	6 1 2 3 6 0 0
	3.bcc	7 9 3 8 0
	ccc	3 4 3
Taken away	*Bc*	6 7 5 1 1 9 9
Whole root completely extracted		5 4 7
The homogeneity finally remaining		0 0 0 0 0 0 0

Note.

For these two examples which are concerned with the resolution of the same equation, a difference can be discerned in the arrangement of the canons applied to the divisors [By divisor is meant in this context, of course, how the equation is divided up by the powers in the equation, in terms of the first and subsequent lesser powers of ten]. In the first canon, on account of a true construction of the divisor to be established in the approximation, a particular number of heterogeneous terms are commonly taken in the division. That form of the division is called the climax by Viete, from the steps from the terms used in a climbing aggregate or sum of powers [The greater part]. In the second canon, truly the law of homogeneity is to be served with a little more difficulty in preceding. In the following examples the climactic form is used with a little more expediency. For except now saying that the distinction for the climactic forms is only a difference in the components taken for the division from the available components of the divisor, for the number and order calculated from these are seen to be the same, independent of how they have been prepared; for any gain from the counterpart is only seen in practise.

In duobus hisque exemplis quae ad eiusdem aequationis resolutionem pertinent diversa canonis ordinatio divisoribus applicata cernitur. In priore propter nonnullam in divisore constituendo veritati approximationem particularia heterogenea ad divisorem componendum promiscue assumuntur. Quam divisionis formam divisore ex partibus gradualibus seu scansoriis aggregato *Vieta* climatican appellat. In secundo vero ad homogeniae legem servandam paulo difficiliore tentamine proceditur. In sequentibus igitur climatica forma ut magis expedita usurpatur. Nam praeter iam dictum discrimen in climatica divisione divisoris componentia ablatii componendibus, numero & ordine iis respondenta, quodammodo praeparoria sunt. Quod compendii alicuius instar, in praxi esse reperitur.

[Viete's algorithm starts by finding the single digit approximation b to the number ggg when substituted into the left-hand side of the cubic equation: the corresponding divisor $A = ff + d + bb$ in the first climatic form; while in the second form just introduced, $A = ff + db + bb$; barring numerous typographical errors, the product Ab is the same in each case. Similarly, there are a few small changes made in the terms making up the divisor B, from divisor $B = ff + d + 2.db + 3.bb + 3b$ in the first form, to divisor B = $ff + 2.db + 3.bb$ in the second case. The final product Bc is adjusted to equal the true product in the end in each case. Thus, a 'trial' sum of terms is taken for A and B initially, acting as a pointer, before the correct product is finally worked out, having selected a or b correctly.]

PROBLEM 7.

For the given homogeneous term proposed numerically, for the given cubic equation [of the form] . . . $aaa + ffa = ggg$, the value of the proposed root of a sought can be extracted analytically:

Let the equation proposed numerically be $aaa + 45796.a = 449324752$.
Hence . . . .45796 = ff, 449324752 = ggg.
And put . . . . $b + c = a$.

Hence . . . .
$$\begin{array}{l|l|l} b+c & +ff & \\ b+c & +b+c & = 44932752. \\ b+c & & \end{array}$$

Therefore with the products from the homogeneous terms,
The equation becomes: . . . .
$$\begin{array}{ll} + \;..bbb & +ffb \\ + \;3.bbc & +ffc \\ + \;3.bcc & \\ + \;..ccc & \end{array} = 449324752.$$

And by the same way distributed in two parts,

it becomes : . . . .
$$\underbrace{\begin{array}{l} +\,ffb \\ +\,bbb \end{array}}_{Ab} \quad \underbrace{\begin{array}{ll} +\,..ffc & +\,3.\,bcc \\ +\,3.bbc & +\,..ccc \end{array}}_{Bc} = 449324752.$$

Moreover, the Canon for this equation has the algebraic part divided into the two terms Ab, Bc, in order that the analytical work can proceed, which is duly established in agreement with the following Lemma, to be established in the usual way.
Therefore the resolution is done entirely by the application of this Canon, as is seen from the ordered numbers set out in the example placed below, in agreement with the given homogeneity 449324752, for the root to be extracted from this as follows.

The root.		7 4 6
The homogeneity to be resolved	*ggg*	4 4 9 3 2 4 7 5 2
	ff	. . 4 5 7 9 6
	bb	. 4 9
Divisor	*A*	5 3 5 7 9 6
	ffb	3 2 0 5 7 2
First Sing. root *b* = 7.	*bbb*	3 4 3
Taken away	*Ab*	3 7 5 0 5 7 2
The first singular root		7
The remaining homogeneity to be resolved		7 4 2 6 7 5 5 2
	ff . . .	. . 4 5 7 9 6
Sing. root times ten *b* = 70.	*3.bb*	.1 4 7 0 0
	3.b . . .	2 1 0
Divisor	*B*	1 5 3 6 7 9 6
	ffc	1 8 3 1 8 4
	3.bbc	5 8 8 0 0
Second sing. root c = 4.	*3.bcc*	3 3 6 0
	ccc	6 4
Taken away	*Bc*	6 4 0 5 5 8 4
Augmented root		7 4
The remaining homogeneity to be resolved		1 0 2 1 1 7 1 2
	ff	. . . 4 5 7 9 6
Sing. root times ten *b* = 740.	*3.bb* . . .	. 1 6 4 2 8 0 0
	3.b . . .	2 2 2
Divisor	*B*	1 6 9 0 8 1 6
	ffc	2 7 4 7 7 6
	3.bbc . . .	9 8 5 6 8 0 0
Second sing. root c = 6.	*3.bbc*	3 0 4 0 6
	3.bcc . . .	7 9 9 2 0
	ccc . . .	2 1 6
Taken away	*Bc*	1 0 2 1 1 7 1 2
Whole root completely extracted		7 4 6
The homogeneity finally remaining		0 0 0 0 0 0 0 0

Therefore the factor itself is the root 4728 equal to the root of *a* sought to be extracted by resolution in this manner from the given homogeneity 449324752, which was to be extracted from the problem considered.

Lemma.

For the proposed equation *aaa* + 45796.*a* = 449324752, with the root 746 extracted by resolution, equal to the root of *a* sought, the validation of the equation is as follows :

The equation is : . . . *aaa* = 746 · 746 · 746 & 45796. *a* = 45796. · 746 = 48233025.

But . . . . 746 · 746 · 746 = 415160936& 45796. · 746 = 34163816.

And . . . . 415160936 + 34163816. = 449324752.

Hence. . . . . *aaa* +45796. *a* = 449324752.

But this is the equation itself proposed

The explanation of the method of composition of the equation set out in backward steps is coincident therefore with the root 746, and in the same manner the root from the extraction 746 is the equal of the root sought *a,* and the resolution through which the extraction is done truly is the root, & consequently the canon, by the direction of which the resolution is done, duly is considered correct.

The case of devolution.

The equation to be resolved $aaa + ffa = ggg$,

$$aaa + 95400.a = 1819459.$$

The canon of the resolution . . . .

$$\underbrace{\begin{matrix} +ffb \\ +bbb \end{matrix}}_{Ab} \quad \underbrace{\begin{matrix} +..ffc & +3.bcc \\ +3.bbc & +..ccc \end{matrix}}_{Bc}$$

The whole root to be extracted successively.		1 9
The homogeneity to be resolved	*ggg*	1 8 1 9 4 5 9
	ff	9 5 4 0 0
	bb	1
Divisor	*A*	9 5 5 0 0
First Sing. root $b = 1$. .	*ffb*	9 5 4 0 0
	bbb	1
Taken away	*Ab*	9 5 5 0 0
The first singular root		1
The remaining homogeneity to be resolved		8 6 4 4 5 9
	ff	9 5 4 0 0
Sing. root times ten $b = 10$.	*3.bb* .	3 0 0
	3.b . . .	3 0
Divisor	*B*	9 5 7 3 0
	ffc	8 5 8 6 0 0
	3.bbc	2 7 0 0
Second sing. root c = 9.	*3.bcc*	2 4 3 0
	ccc	7 2 9
Taken away	*Bc*	8 6 4 4 5 9
Whole root completely extracted		1 9
The homogeneity finally remaining		0 0 0 0 0 0 0

Therefore root 19 is extracted by analysis from the given homogeneity 1819459, equal to the root of *a* sought which was to be found.

An example of rectification.

The equation to be resolved $aaa + ffa = ggg$,

$$aaa + 274576.a = 301163392.$$

The canon of the resolution . . . .

$$\underbrace{\begin{matrix} +ffb \\ +bbb \end{matrix}}_{Ab} \quad \underbrace{\begin{matrix} +..ffc & +3.bcc \\ +3.bbc & +..ccc \end{matrix}}_{Bc}$$

The given homogeneity	ggg	3 0 1 1 6 3 3 9 2
The coefficient from the cube of the steps	$-fff$	- 1 4 3 8 7 7 8 2 4
The difference	$ggg - fff$	1 5 7 2 8 5 5 6 8
First singular root b		5

The continuing resolution.

The whole root to be extracted successively.		5 3 6
The homogeneity to be resolved	ggg	3 0 1 1 6 3 3 9 2
	ff	. 2 7 4 5 7 6
	ffb	1 3 7 2 8 8 0
First Sing. root b = 5. .	bbb	1 2 5
Taken away	Ab	2 6 2 2 8 8 0
The first singular root		5
The remaining homogeneity to be resolved		3 8 8 7 5 3 9 2
	ff . . .	. 2 7 4 5 7 6
Sing. root times ten b = 50.	$3.bb$	. 7 5 0 0
	$3.b$ . . .	1 5 0
Divisor	B	
	ffc	1 0 3 9 5 7 6
	$3.bbc$	8 2 3 7 2 8
Second sing. root c = 3.	$3.bcc$	2 2 5 0 0
	ccc	1 3 5 0
Taken away	Bc	2 7
		3 2 1 1 4 2 8
The augmented root		5 3
The remaining homogeneity to be resolved		6 7 6 1 1 1 2
	ff . . .	. 2 7 4 5 7 6
Sing. root times ten b = 530.	$3.bb$	. 8 4 2 7 0 0
	$3.b$ . . .	1 5 9 0
Divisor	B	1 1 1 8 8 6 6
	ffc	1 6 4 7 4 5 6
	$3.bbc$	5 0 5 6 2 0 0
Second sing. root c = 6.	$3.bcc$	5 7 2 4 0
	ccc	2 1 6
Taken away	Bc	6 7 6 1 1 1 2
The whole root completely extracted.		5 3 6
The homogeneity finally resolved		0 0 0 0 0 0 0 0 0

Therefore the root 536 is extracted by the analysis from the given homogeneity 305163392, equal to the root of *a* sought, which was to be performed.

PROBLEM 8.

For a given homogeneous term proposed numerically, for cubic equations [of the form] . . $aaa - ffa = ggg$, the value of the proposed root of *a* sought can be extracted analytically:

Let the equation proposed numerically be $aaa - 2648.a = 91148512$.

Hence . . . . $2648 = ff$, $91148512 = ggg$.

And place . . . . $b + c = a$.

Hence . . . .
$$\left.\begin{array}{c} b+c \\ b+c \\ b+c \end{array}\right| - \left|\begin{array}{c} ff \\ \hline b+c \end{array}\right. = 91148512$$

Therefore with the products from the homogeneous terms,
The equation becomes : . . .

$$\begin{array}{ll} +\,..bbb & -ffb \\ +\,3.bbc & -ffc \\ +\,3.bcc & \\ +\,..ccc & \end{array} = 91148512$$

And by the same way distributed in two parts,

it becomes : . . . .

$$\underbrace{\begin{array}{c} -\,ffb \\ +\,bbb \end{array}}_{Ab}\quad \underbrace{\begin{array}{cc} -\,..ffc & +\,3.\,bcc \\ +\,3.bbc & +\,..ccc \end{array}}_{Bc} = 91148512$$

Moreover, the Canon for this equation has the algebraic part divided into the two terms *Ab, Bc*, in order that the analytical work can proceed, which is duly established in agreement with the following Lemma, to be established in the usual way.

Therefore the resolution is done entirely by the application of this Canon, as is seen from the order in the setting out of the figures of the example placed below, acting on the given homogeneity itself 91148512, for the root to be extracted from this as follows.

The whole root to be extracted successively.		4 5 2
The homogeneity to be resolved		9 1 1 4 8 5 1 2
	- *ff*	— 2 6 4 8
	bb	1 6
Divisor	*A*.	1 5 7 3 5 2
	ffb	— 1 0 5 9 2
First Sing. root *b* = 4.	*bbb*	6 4
Taken away	*Ab*	6 2 9 4 0 8
The first singular root		4
The remaining homogeneity to be resolved		2 8 2 0 7 7 1 2
	ff . . .	— 2 6 4 8
Sing. root times ten *b* = 40.	*3.bb*.	4 8 0 0
	3.b . . .	1 2 0
	+.	4 9 2 0
Divisor	*B*.	4 8 9 3 5 2
	- ffc	— 1 3 2 4 0
	3.bbc	2 4 0 0 0
Second sing. root c = 5.	*3.bcc*	3 0 0 0
	ccc	1 2 5
	+	2 7 1 2 5
Taken away	*Bc*	2 6 9 9 2 6 0
Augmented root		4 5
The remaining homogeneity to be resolved		1 2 1 5 1 1 2

	ff	— 2 6 4 8
Sing. root times ten b = 450.	*3.bb*	6 0 7 5 0 0
	3.b . . . -	1 3 5 0
	+	6 0 8 8 5 0
Divisor	*B*	6 0 6 2 0 2
	-ffc	— 5 2 9 6
	3.bbc . . .	1 2 1 5 0 0 0 0
Second sing. root c = 2.	*3.bcc* . . .	5 4 0 0
	ccc . . .	8
	+	1 2 2 2 0 4 0 8
Taken away	*Bc*	1 2 1 5 1 1 1 2
Whole completely educed root		4 5 2
The homogeneity finally remaining		0 0 0 0 0 0 0 0

Therefore the root 452 is extracted from the given homogeneity 91148512 by analysis, equal to the root of a sought, which had to be extracted.

Lemma.

If from the given homogeneity of the proposed equation $aaa - 2648.a = 91148512$, the root 452 has been extracted by the method of resolution, equal to the value of a required, then the verification of the method is as follows :

The equation is : . . . . $aaa = 452 \cdot 452 \cdot 452$ & $2648.\ a = 2468. \cdot 452$

But . . . . $452 \cdot 452 \cdot 452 = 92345408$ & $2468. \cdot 452 = 1196896.$

And . . . . $92345408 - 1196896. = 91148512$

Hence. . . . . $aaa - 2648.a = 91148512$

But this is the equation itself proposed

The explanation done by the retrograde way of composition of the equation is coincident therefore with the root 452, and in the same manner the root from the extraction 452 is the equal of the root sought *a,* and the resolution by which the extraction has been done truly is the root, & consequently the canon, acting in the direction in which the resolution is done, is duly verified.

Note.

To be noted here, that the common signs of the affections + and — are to be inserted, which in the series of terms of the canon are to be lined up with the remaining entries arranged together either having negative affections in equations, or with the sums of preceding nearby terms where many of the same affection occur (either of the other or of both). These are to be designated separately: and for this end, in order that the totals of positive and negative differences, for the divisors and subtractands to be put in position, should appear distinctly. See Note *a* to the first problem.

Notandum hic est, communia affectionis signa + & — quae in lineari canonis serie cum reliquis notis coordinata in aequationibus negative affectis habentur, ad particularium proxime praecedentium Summas, ubi plura eiusdem affectionis (sive alterius seve utriusque) occurrunt, separatim significandas, ifserta esse: idque eo sine, ut affirmatorum & negatorum differentiae totales, pro divisoribus atque ablatitiis constituendis, distincte appareant. Vide Notam a ad Problema primum.

The case of Anticipation.

In the equation *aaa* - *ffa* = *ggg* , with the number proposed to be resolved, it occasionally happens that the coefficient of the division has more double figures than the individual places of the homogeneity resolved in triple figures And thus in order that the resolution can be done, the starting point is put a number of zeros to the left of the first digit of the homogeneity in order that the number of places in *aaa* is the same as the number of places in *ffa* . Thus, in the first empty place, just by inspection, the work of resolution begins; in which this is the advantage, that the first square of the root is found from given term *ff*, the first figure or digit of the root can be extracted either from the given homogeneity or from the next smaller.

[Thus, the coefficient times *a* contributes a larger number than the homogeneity to the estimate of the cube for the first figure of the root; we should also be aware of the erroneous thought current at the time, due to Vieta, that all the numbers in an equation should have the same power, 3 in the present case, which explains the rather odd habit of writing coefficients as *ggg, ffa*, etc. This was in tune with areas and volumes, a practise still adhered to in physics, where all the terms in an equation must have the same dimensions.]

The equation to be resolved *aaa* - *ffa* = *ggg*,

aaa - 116620.*a* = 352947.

The canon of the resolution . . . .

$$\underbrace{-\,ffb \quad + bbb}_{Ab} \qquad \underbrace{-\,..ffc \quad + 3.bcc \quad + 3.bbc \quad + ..ccc}_{Bc}$$

The whole root to be extracted successively.		3 4 3
The homogeneity to be resolved		0 3 5 2 9 4 7
	- *ff*	— 1 1 6 6 2 0
	bb	. 9
Divisor	— *A*.	— 2 6 6 2 0
	— *ffb*	— 3 4 9 8 6 0
First Sing. root *b* = 3. .	*bbb*	2 7
Taken away	— *Ab*	— 7 9 8 6 0
The first singular root		3
The remaining homogeneity to be resolved		8 3 3 8 9 4 7
	ff . . .	— 1 1 6 6 2 0
Sing. root times ten *b* = 30.	*3.bb*.	. 2 7 0 0
	3.b . . .	9 0
	+.	2 7 9 0
Divisor	*B.*	1 6 2 3 8 0
	- *ffc*	— 4 6 6 4 8 0
	3.bbc	1 0 8 0 0
Second sing. root c = 4.	*3.bcc*	1 1 4 0
	ccc	6 4
	+	1 2 3 0 4
Taken away	*Bc*	7 6 3 9 2 0
Augmented root		5 4
The remaining homogeneity to be resolved		6 9 9 7 4 7
	ff . . .	— 1 1 6 6 2 0
Sing. root times ten *b* = 340.	*3.bb*.	. 3 4 6 8 0 0
	3.b . . .	1 0 2 0
	+.	2 7 9 0
Divisor	*B.*	2 3 1 2 0 0
	- *ffc*	— 3 4 9 8 6 0
	3.bbc	1 0 1 4 0 0 0
Second sing. root c = 3.	*3.bcc*	9 1 8 0
	ccc	2 7
	+	1 0 4 9 6 0 7
Taken away	*Bc*	6 9 9 7 4 7
Augmented root		3 4 3
The remaining homogeneity to be resolved		0 0 0 0 0 0

Therefore the root 543 has been extracted from the given homogeneity by analysis 352947 from the working, equal the value of *a* sought, which was to be extracted.

An example of rectification.

The equation to be resolved $aaa - ffa = ggg$,

$$aaa - 127296.a = 85760000.$$

The canon of the resolution . . . . $\quad - ffb \quad - ..ffc \quad + 3.\,bcc$

$$\underbrace{+\ bbb}_{Ab} \quad \underbrace{+\ 3.bbc \quad + ..ccc}_{Bc}$$

For the extraction of the first singular root by rectification.

The given homogeneity	*ggg*	8 5 7 6 0 0 0 0
The coefficient from the cubic scale	*fff*	4 5 4 4 4 6 7 2
The sum	*ggg* + *fff*	1 3 1 2 0 4 6 7 2
First singular root	*b*	5
		• • •

With the resolution continued.

The whole root to be extracted successively.		5 3 6
		• • •
The homogeneity to be resolved	*ggg*	8 5 7 6 0 0 0 0
		• • •
	—*ff*	— 1 2 7 2 9 6
	—*ffb*	— 6 3 6 4 8 0
First Sing. root *b* = 5. .	*bbb*	1 2 5
Taken away	—*Ab*	6 1 5 3 2 0
The first singular root		5
		• • •
The remaining homogeneity to be resolved		2 4 4 0 8 0 0 0
		• • •
	—*ff* . . .	— 1 2 7 2 9 6
Sing. root times ten *b* = 50.	*3.bb*	. 7 5 0 0
	3.b . . .	1 5 0
	+	7 6 5 0
Divisor	*B*	6 3 7 7 0 4
	—*ffc*	— 3 8 1 8 8 8
	3.bbc	2 2 5 0 0
Second sing. root c = 3.	*3.bcc*	1 3 5 0
	ccc	2 7
Taken away	*Bc*	2 0 0 5 8 1 2
The root augmented		5 3
		• • •
The remaining homogeneity to be resolved		4 3 4 9 8 8 8
		• • •
	—*ff* . . .	— 1 2 7 2 9 6
Sing. root times ten *b* = 530.	*3.bb*	. 8 4 2 7 0 0
	3.b . . .	1 5 9 0
Divisor	+	8 4 4 2 9 0
	B	7 1 6 6 9 4
	—*ffc*	— 7 6 3 6 7 6
Second sing. root c = 6.	*3.bbc*	5 0 5 6 2 0 0
	3.bcc	5 7 2 4 0
Taken away	*ccc*	2 1 6
	+	5 1 1 3 6 5 6
	Bc	4 2 4 9 8 8 8
The whole root completely extracted.		5 3 6
		• • •
The homogeneity finally resolved		0 0 0 0 0 0 0

Therefore the root 536 has been extracted from the given homogeneity 85760000 by analysis, equal to the value of *a* sought, which was to be extracted.

PROBLEM 9.

For a given homogeneous term proposed numerically, for the cubic equation $-aaa + ffa = ggg$, which is to be solved for the two roots, both the values of a equal to the root are to be extracted numerically by analysis.

Let the equation proposed numerically be $-aaa + 52416.a = 12244160$.
Hence . . . . $52416 = ff$, $12244160 = ggg$.
And place . . . . $b + c = a$.

Hence . . . . $- \left.\begin{array}{l} b+c \\ b+c \\ b+c \end{array}\right| + \left.\begin{array}{c} ff \\ b+c \end{array}\right| = 1244160.$

Therefore with the products from the homogeneous terms,
The equation becomes : . . . .

$$\begin{array}{ll} -\ ..bbb & +ffb \\ -\ 3.bbc & +ffc \\ -\ 3.bcc & \\ -\ ..ccc & \end{array} = 1244160.$$

And by the same way distributed in two parts,

it becomes : . . . .

$$\underbrace{\begin{array}{l} +ffb \\ -bbb \end{array}}_{Ab} \quad \underbrace{\begin{array}{lll} +..ffc & -3.bcc \\ -3.bbc & -..ccc \end{array}}_{Bc} = 1244160.$$

Moreover, the Canon for this equation has the algebraic part divided into the two terms *Ab, Bc*, in order that the analytical work can proceed, which is duly established in agreement with the following Lemma, to be established in the usual way.

Therefore the resolution is done entirely by the application of this Canon, as is seen from the order in the setting out of the figures of the example placed below, acting on the given homogeneity itself 1244160, for the root to be extracted from this as follows.

For the extraction of the larger root.

The whole root to be extracted successively.		2 1 6
The homogeneity to be resolved		1 2 4 4 1 6 0
	ff	5 2 4 1 6
	—*bb*	— 4
Divisor	*A*.	1 2 4 1 6
	ffb	1 0 4 8 3 2
First Sing. root *b* = 2. .	—*bbb*	— 8
Taken away	— *Ab*	— 2 4 8 3 2
The first singular root		2
The remaining homogeneity to be resolved		-1 2 3 9 0 4 0
	ff . . .	5 2 4 1 6
Sing. root times ten *b* = 20.	— *3.bb.*	— 1 2 0 0
	— *3.b* . . .	— 6 0
	—.	— 1 2 6 0
Divisor	— *B.*	— 7 3 5 8 4
	ffc	5 2 4 1 6
	— *3.bbc*	— 1 2 0 0
Second sing. root c = 1.	— *3.bcc*	— 6 0
	— *ccc*	— 1
	—	— 1 2 6 1
Taken away	— *Bc*	— 7 3 6 8 4
Augmented root		2 1
The remaining homogeneity to be resolved		- 5 0 2 2 0 0
	ff . . .	5 2 4 1 6
Sing. root times ten *b* = 210.	— *3.bb.*	— 1 3 2 3 0 0
	— *3.b* . . .	— 6 3 0
	—.	— 1 3 2 9 3 0
Divisor	— *B.*	— 8 0 5 1 4
	ffc	3 1 4 4 9 6
	— *3.bbc*	— 7 9 3 8 0 0
Second sing. root c = 6.	— *3.bcc*	— 2 2 6 8 0
	— *ccc*	— 2 1 6
	—	— 8 1 6 0 9 6
Taken away	— *Bc*	— 5 0 2 2 0 0
The whole root completely extracted.		2 1 6
The homogeneity finally resolved		0 0 0 0 0 0

For the extraction of the smaller root accomplished by devolution.

The whole root to be extracted successively.		2 1 6
The homogeneity to be resolved		1 2 4 4 1 6 0
	ff	5 2 4 1 6
	—*bb* . . . . . —	4
Divisor	*A*.	5 2 0 1 6
	ffb	1 0 4 8 3 2
First Sing. root *b* = 2.	—*bbb* . . . . —	8
Taken away	— *Ab* . . . . —	2 4 8 3 2
The first singular root		2
The remaining homogeneity to be resolved		2 0 3 8 4 0
	ff . . .	5 2 4 1 6
Sing. root times ten *b* = 20.	— 3.*bb*.	— 1 2 0 0
	— 3.*b* . . .	— 6 0
	—.	— 1 2 6 0
Divisor	*B*.	5 1 1 5 6
	ffc	2 0 9 6 6 4
	— 3.*bbc*	— 4 8 0 0
Second sing. root c = 4.	— 3.*bcc* . . . . —	9 6 0
	— *ccc* . . . . —	6 4
	— —	5 8 2 4
Taken away	— *Bc* . . . . —	2 0 3 8 4 0
The whole root completely extracted.		2 4
The homogeneity finally resolved		0 0 0 0 0 0

Therefore from the given homogeneity 1244160, from the working itself the two roots 216 & 24, have been extracted, both equal to the value of *a* sought for a root, by being resolved in this manner, which had to be extracted.

Compendium.

If the roots *b* & *c* of the proposed equation - *aaa* + *ffa* = *ggg* are put in place: *b* the larger, *c* truly the smaller, then the coefficient is *ff* = *bb* + *bc* + *cc* (by prop.6, Sect. 4.).
Therefore if the root *b* is given, then the equation is *cc* + *bc* = *ff* - *bb*, of which the root sought *c* shall be the smaller.
or if the minor root c is given, the equation shall be *bb* + *bc* = *ff* - *cc,* of which the root sought *b* is the larger.
Therefore for the proposed cubic equation with one root found, by the analysis of the quadratic equation the other is shown, which can be used for a short cut.

PROBLEM 10.

For a given homogeneous term proposed numerically, the cubic equation of the form *aaa* - *daa* = *ggg,* the value of the proposed root of *a* sought can be extracted analytically:

Let the equation proposed numerically be *aaa* - 68.*aa* = 134454528.
Hence . . . . 68 = *d*, 134454528 = *ggg*.
Place . . . . *b* + *c* = *a*.

Hence . . . . $+ \left| \begin{array}{l} b+c \\ b+c \\ b+c \end{array} \right. - \left| \begin{array}{l} d \\ b+c \\ b+c \end{array} \right. = 134454528.$

Therefore with the products from the homogeneous terms,

The equation becomes : . .
$$\begin{array}{lll} + \;..bbb & -\, dbb & = 134454528. \\ + \;\; 3.bbc & -\, 2.dbc & \\ + \;\; 3.bcc & -\, .dcc & \\ + \;..ccc & & \end{array}$$

And by the same way distributed in two parts,

it becomes . . . .
$$\begin{array}{l} -\; dbb \;-\; ..dcc \;+\; 3.\,bbc \qquad = 134454528. \\ \underbrace{+\; bbb}_{Ab} \;-\; \underbrace{2.dbc \;+\; 3.\,bcc \;+\; ..ccc}_{Bc} \end{array}$$

Moreover, the Canon for this equation has the algebraic part divided into the two terms *Ab, Bc*, in order that the analytical work can proceed, which is duly established in agreement with the following Lemma, to be established in the usual way.

Therefore the resolution is done entirely by the application of this Canon, as is seen from the order in the setting out of the figures of the example placed below, acting on the given homogeneity itself 134454528, for the root to be extracted from this as follows.

The whole root to be extracted successively.		5 3 6
The homogeneity to be resolved	*ggg*	1 3 4 4 5 4 5 2 8
	—*d*	— 6 8
	bb	2 5
Divisor	*A*.	2 4 3 2
	— *dbb* . . .	— 1 7 0 0
First Sing. root *b* = 5.	*bbb*	1 2 5
Taken away	*Ab*	1 0 8 0 0
The first singular root		5
The remaining homogeneity to be resolved		2 6 4 5 4 5 2 8
	— *d* . . .	— 6 8
Sing. root times ten *b* = 50.	2*db*.	— 6 8 0 0
	3.*bb*. . . .	7 5 0 0
	3.*b*. . . .	1 5 0
	+.	7 6 5 0
Divisor	—. . . .	6 8 6 8
	B.	6 9 6 3 2
	—*dcc*	— 6 1 2
Second sing. root c = 3.	— 2.*dbc*	— 2 0 4 0 0
	3.*bbc*	2 2 5 0 0
	3.*bcc*	1 4 5 0
	ccc	2 7
	+.	2 3 8 7 7
Taken away	—. . . .	2 1 0 1 2
	Bc	2 1 7 7 5 8
The root augmented.		5 3
The residual homogeneity to be resolved		4 6 7 8 7 2 8

	— *d* . . . —						6	8	
Sing. root times ten *b* = 530.	*2db*. . . . . —			7	2	0	8	0	
	3.bb . . .		8	4	2	7	0	0	
	3.b . . .				1	5	9	0	
	+		8	4	4	2	9	0	
Divisor	—. . . .		—	7	2	1	4	8	
	B.		7	7	2	1	4	2	
	—*dcc* . . . . . —				2	4	2	8	
Second sing. root c = 6.	— *2.dbc* . . . . —		4	3	2	4	0	0	
	3.bbc	5	0	5	6	2	0	0	
	3.bcc			5	7	2	4	0	
	ccc					2	1	6	
	+	5	1	1	3	6	5	6	
Taken away	—. . . .		4	3	4	8	4	8	
	Bc	4	6	7	8	7	2	8	
The root augmented.			5			3			3
The residual homogeneity to be resolved			0	0	0	0	0	0	0

Therefore the root 536 has been extracted by resolution from the given homogeneity 134454528, equal to the value of *a* sought, which had to be extracted.

Lemma.

If from the given homogeneity of the proposed equation *aaa* - 68.*a* = 134454528, the root 536 by resolution is extracted, equal to the value of *a*, and the explanation of the equation is as follows:

It is . . . . *aaa* = 536 536 536 & 68.*a* = 68. 536 536 = 134454528

But . . . . 536 536 536 = 15399656 & 68. 536 536 = 19536128.

And . . . . 134454528 + 19536128 = 134454528

Hence. . . . . *aaa* + 68. *a* = 134454528

But this is the equation itself proposed

The explanation done by the retrograde way of composition of the equation is coincident therefore with the root 536, and in the same manner the root from the extraction 536 is the equal of the value of the root sought for *a,* and the resolution through which the extraction is done truly results in the root, & consequently the canon, by the direction of which the resolution has been done, is duly verified.