EXEGETICE
NUMEROSA.

For quadratic equations to be resolved.

PROBLEM 1.

For a given homogeneous term proposed numerically, for simple quadratic equations [of the form] . . $aa = ff$, the value of the proposed root of a sought can be extracted analytically:

Edato aequationis quadraticis simplicis : $aa == ff$, in numeris propositae homogeneo radicem radicis quaesititiae a valorem analyticie reducere.

Let the equation proposed numerically be aa = 48233025.

Then . . . . 48233025 = ff.

Substitute . . . . $b + c = a$.

Hence . . . . $\left.\begin{matrix} b+c \\ b+c \end{matrix}\right|$ = 48233025.

Therefore the equation from the homogeneous products and for the particular order of the terms is :

$$\underbrace{+bb}_{Ab} \quad \underbrace{+\ 2.bc \ + \ ..cc}_{Bc} \quad = \quad 48233025.$$

Factis igitur et ordinatis ut oportet homogeneis particularibus sit.

Moreover, the Canon [or procedure] for the resolution of this equation has the algebraic part divided into the two terms Ab, Bc, in order that the analytical work can proceed, which is duly established in agreement with the Lemma to follow.

Est autem aequationis huius pars speciosa bipartita Ab, Bc resolutionis Canon cuius applicationis operis analytici processus dirigendus, quem rite constitutum esse ex sequenti Lemmate constabit.

Therefore the resolution is done entirely by the application of this Canon, as is seen from the order of the figures set out in the example placed below, in agreement with the given homogeneity 48233025, in order that the root can be deduced as follows from this number.

The whole root to be drawn successively		6 9 4 5
		• • • •
The homogeneity to be resolved	ff	4 8 2 3 3 0 2 5
Divisor	b	6
First Sing. root $b = 6$. . .		
Taken away	bb	3 6
The singular root		6
		• • • •
The remaining homogeneity to be resolved		1 2 2 3 3 0 2 5
Divisor	$2.b$ . . .	. 1 2 0
Ten times the sing. root b = 60 . .	$2.bc$ . . .	1 0 8 0
Second sing. root . . c = 9	cc . . .	. . . 8 1
Taken away	Bc . . .	1 1 6 1
The root augmented		6 9
		• • • •
The remaining homogeneity to be resolved		6 2 3 0 2 5
Divisor	$2.b$ . . .	1 3 8 0
Ten times the sing. root b = 690 . .	$2.bc$ . . .	5 5 2 0
Second sing. root . . c = 4	cc . . .	. . 1 6
Taken away	Bc . . .	5 5 3 6
The root augmented		6 9 4
		• • • •
The remaining homogeneity to be resolved		6 9 4 2 5
Divisor	$2.b$ . . .	. . 1 3 8 8 0
Ten times the sing. root b = 6940 .	$2.bc$ . . .	. . 6 9 4 0 0
Second sing. root . . c = 5	cc . . .	. . . . .2 5
Taken away	Bc . . .	. . . . 6 9 4 2 5
The whole root finally drawn out		6 9 4 5
		• • • •
The remaining homogeneity finally		0 0 0 0 0

Therefore a factor equal to the root 6945 of the root a sought has been extracted from the given homogeneity [or constant term in the equation] 48233025, through resolution in this manner.

Lemma.

Since for the proposed equation *aa* = 48233025 with the root 6945, a way of explaining the composition is to proceed backwards through the resolution. Hence for the converse, an explanation of how the product for the original equation is produced is sufficient for the demonstration of the resolution, which is obvious enough in the current example, as with the direction the Canon acts, in order that the truth is established, as required.

[Thus, the algebraic argument of completing the square of the finite power series can be performed in either a forwards or backwards direction using 'if and only if statements, or '⇔' statements, which can be gone through in a rigorous manner in either direction. There is no need here to be concerned with convergence as a finite expansion is used.]

Quoniam proposita aequatio *aa* = 48233025 de radice 6945 per resolutionem educta retrogrado compositionis via explicabilis est; E converso igitur per aequationis explicationem compositivae factam, quae in praesenti exemplo satis obvia est, tam ipsius resolutionis quam Canonis cuius directione est resolutio, veritas ut oportuit comprobatur.

Note 1.

So with these problems which are observed in numerical exegetics, the equation is understood to have proposed numbers, which either pertain to the given homogeneity or constant term only, if the equation is simple as above, or if the equation is affected then the equation will be expressed together with another number for the given coefficient.

In Problematis hisce quae ad Exegesin numerosam spectant, in numeris proposita intellegenda est aequatio, cuius homogeneum datum, si simplex sit vel si affecta una cum homogeneo coefficientia data numeris exprimuntur.

Note 2.

Besides this it should also be noted initially for the two terms noted *Ab, Bc* which are appointed to the kind of Canon set up to distinguish between the two particular terms; the first, namely *Ab* is understood to denote the constituents which pertain to the extraction of the first digit of the singular root [as propounded by Viete]; the second *Bc* truly is applied to the particular terms which serve for the extraction of the following digits of the individual root, which is to be applied with continuous repetition.

Because if it is to be taken again, [as for an equation with affected terms] then these two terms *Ab, Bc*, can be divided again into four parts, namely *A,* for the first significant divisor, *Ab* itself for the part taken away; *B* for the secondary divisors, and *Bc* for the remainders. And from these four terms found in the parts of the canon, the places of the root are to be inserted in a sequence.

Moreover it is necessary to note the significance and use of these parts for particular different equations, to which a different significant number for the affection are added [according to Viete]. For although in the resolution of simple equations in which divisors and the first parts are taken away from a single particular number are in agreement, you observe that further places in the canon, that may designate the remainders and divisors *A* and *Ab* to be adscribed are empty; nevertheless generally when divisors and remainders from more terms are to be added together, if they are of the same affections *A, Ab, B ,Bc* ; and if indeed they are of contrary affections then the differences of these are taken, usually without trouble by rewriting the total of the canon (as it must become different), to be denoted in this way is the most suitable use. These in the most elaborately or minimally set out of examples are shown in the following text, without elaboration from this note.

Praeteria hoc quoque imprimis notadum est, duarum notarum A*b*, *Bc*, quae in Canone constituendo ad speciei canonicae particularia bifariam distinguenda adscribuntur, primam, scilicet *Ab* particularia ea quae ad primae radicis singularis eductionem pertinent, denotare intelligendum est, secundam vero *Bc* ad particularia quae secundariarum radicum singulorum eductioni inserviunt denotanda, perpetua iteratione adhibendam esse.

Quod sic porro accipiendum est, duas istas notas Ab, Bc quadripartito rursus discribuendas esse, videlicet A pro divisore primario significando, ipsam vero Ab pro ablatitio, atque B pro divisoribus secundariis, Bc pro ablatitiis. Et quatuor hisce notas inter ipsius canonis particularia seriatim suis locis inserendas.

Notarum autem istarum significationem & usum pro diversa particularum quibus significandis adiiciuntur affectione vel numero diversificari necesse est. Nam etsi in aequationibus simplicibus resolvendis in quibus divisores & ablatitia primaria ex unico particulari constant ulterior praeter ipsius canonis notas, notarum *A* & *Ab* ad divisores & ablatitia designanda adscriptio supervacanea foret, generaliter tamen ubi divisores & ablatitia ex pluribus particularibus componuntur, si sint eiusdem affectionis notarum *A, Ab, B, Bc* ad particularium summas, si vero contrariarum affectionù ad eorum differentias, absque molesta totius fere canonis rescriptione (quod alias fieret) denotandas admodum commodus est usus. Haec licet in sequentium exemplorum schematismis vel minimum advertenti manifesta dint, non tamen abs re visum est hoc loco adnotasse.

PROBLEM 2.

For a given homogeneous term proposed numerically, for quadratic equations [of the form] . . $aa + da = ff$, the value of the root of a sought can be deduced analytically.

Let the equation proposed numerically be $aa + 432.a = 13584208$.

Hence . . . . $432 = d$ and $13584208 = ff$.

Substitute . . . . $b + c = a$.

Hence . . . . $\left.\begin{matrix} b+c \\ b+c \end{matrix}\right| \quad \left.\begin{matrix} d \\ b+c \end{matrix}\right| = 13584208$

Therefore with the products & two parts set out in this way for the particular homogeneity: the equation becomes :

$$\begin{matrix} +db & +dc \\ +bb & +2.bc \\ \hline Ab & +.cc \\ & \hline Bc \end{matrix} = 13584208.$$

Moreover, the Canon for this equation has the algebraic part divided into the two terms *Ab, Bc*, in order that the analytical work can proceed, which is duly established in agreement with the following Lemma. Therefore the resolution is done entirely by the application of this Canon, as is seen from the ordered numbers set out in the example placed below, in agreement with the given homogeneity 13584208, in order that the root can be deduced as follows from this number.

The whole root to be drawn out successively			3 4 7 6
			• • • •
The homogeneity to be resolved		*ff*	1 3 5 8 4 2 0 8
			• • • •
		d	. 0 4 3 2
Sing. root . .	*b* = 3	*b*	. 3
Divisor		*A*	. . 3 4 3 2
		db	. . 1 2 9 6
First sing. root.	*b* = 3	*bb*	. . 9
Taken away . .		*Ab*	. 1 0 2 9 6
The singular root			3
			• • • •
The remaining homogeneity to be resolved			3 2 8 8 2 0 8
			• • • •
		d	. . . . 4 3 2
Sing. root times ten	*b* = 30	2*b*	. . 6 0
Divisor		*B*	. . 6 4 3 2
		dc	. . 1 7 2 8
Second sing. root.	*c* = 4	2.*bc*	. . 2 4 0
		cc	. . . 1 6
Taken away		*Bc*	. . . 2 7 3 2 8
The root augmented			3 4
			• • • •
The remaining homogeneity to be resolved			5 5 5 4 0 8
			• • • •
		d	4 3 2
Aug. root times ten	*b* = 340	2*b*	6 8 0
Divisor		*B*	7 2 3 2
		dc	3 0 2 4
Third sing. root.	*c* = 7	2.*bc*	4 7 6 0
		cc	. 4 9
Taken away		*Bc*	. . . 5 1 1 1 4
The root augmented			3 4 7
			• • • •
The remaining homogeneity to be resolved			4 4 2 6 8
			• • • •
		d	4 3 2
Aug. root times ten	*b* = 3470	2*b*	6 9 4 0
Divisor		*B*	7 3 7 2
		dc	2 5 9 2
Third sing. root.	*c* = 6	2.*bc*	4 1 6 4 0
		cc	. 3 6
Taken away		*Bc*	. . . 4 4 2 6 8
The whole root finally drawn out			3 4 7 6
			• • • •
The remaining homogeneity finally			0 0 0 0 0
			• • • •

Therefore a factor equal to the root 3476 of the root *a* sought has been extracted from the given homogeneity or constant term 3584208, on resolution in this manner.

Lemma.

If, for the given homogeneity or constant term of the proposed equation: $aa + 432.a = 13584208$, the root a is found by the mode of extraction to be equal to 3476, then the equation can be solved as follows:

The equation is : . . . . $aa = \begin{array}{c|}3476\\3476\end{array}$ & $432.a = \begin{array}{c|}432\\3476\end{array} = 48233025.$

But . . . . $\begin{array}{c|}3476\\3476\end{array} = 12082576$ & $\begin{array}{c|}432\\3476\end{array} = 1501632.$

And . . . . $\begin{array}{r|}+\ 12082576\\+\ 1501632\end{array} = 13584208$

Hence. . . . . $aa + 432.a = 13584208$

But this is the equation itself proposed

There is hence a congruence of the equation with the root 3476, arising from the retrograde manner of composition of the equation, for the root from the extraction 3476 is equal to the root *a* to be found. The resolution by means of which the extraction has been done truly leads to the root, & consequently the use of the canon in this direction, by means of which the resolution has been done, is duly proven true.

The case of devolution.

For equations of the form $aa + da = ff$ sometimes the coefficient falls on numbers to be taken from the previous level of resolution as far as they extended into that, otherwise the subtraction from the homogeneity cannot be performed. [This is the case where *d* is a large number, and so has an effect on determining the initial and succeeding values of *b*; in the previous problem, the affection has only had a small effect, as *d* is small.] In such a case, the evaluation of the coefficient is to be moved to the second or the third place, or further, if it should be necessary. Hence the place is chosen in this manner for the division of the working into two parts and a start can be made, [in determining the gross value of b by a single digit] as is shown by the two examples following.

Devolution example 1.

The equation to be resolved . . . . . $\left\{\begin{array}{l} aa + da = ff. \\ aa + 75325.a = 41501984. \end{array}\right.$

The canon of the resolution . . . $\left\{\begin{array}{ll} +db & +\ ..db \\ +bb & +\ 2.bc \\ \hline Ab & +\ ..cc \\ & \overline{\quad Bc \quad} \end{array}\right.$

The whole root to be drawn out successively			5 4 7
The homogeneity to be resolved	*ff*		4 1 5 0 1 9 8 4
		d	. 7 5 3 2 5
Sing. root . .	*b* = 5	*b*	. . 5
Divisor		*A*	. . 7 5 8 2 5
		db	. 3 7 6 6 2 5
First sing. root.	*b* = 5	*bb*	. 2 5
Taken away . .		*Ab*	. 3 7 9 1 2 5
The singular root			5
The remaining homogeneity to be resolved			3 5 8 9 4 8 4
		d	. . 7 5 3 2 5
Sing. root times ten	*b* = 50	2*b*	. . 1 0 0
Divisor		*B*	. . 7 6 3 2 5
		dc	. . 3 0 1 3 0 0
Second sing. root.	*c* = 4	2.*bc*	. . 4 0 0
		cc	. . . . 1 6
Taken away		*Bc*	. . 3 0 5 4 6 0
The root augmented			5 4
The remaining homogeneity to be resolved			5 3 4 8 8 4
		d	. . . 7 5 3 2 5
Aug. root times ten	*b* =540	2*b*	. . 1 0 8 0
Divisor		*B*	. . 7 6 4 0 5
		dc	. 5 2 7 2 7 5
Third sing. root.	*c* = 7	2.*bc*	. . 7 5 6 0
		cc	. . . . 4 9
Taken away		*Bc*	. 5 3 4 8 8 4
The whole root finally drawn out			5 4 7
The remaining homogeneity finally			0 0 0 0 0 0

Therefore for the given homogeneity 369701984, from the working done, a root 547 equal to the root *a* sought has been extracted through being resolved in this manner, which was the intention of the exercise.

Devolution example 2.

The equation to be resolved . . . . . . $\left\{ \begin{array}{l} aa + da = ff. \\ aa + 675325.a = 369701984. \end{array} \right.$

The canon of the resolution . . . $\left\{ \begin{array}{ll} +db & + ..db \\ \underline{+bb} & + 2.bc \\ Ab & \underline{+ ..cc} \\ & Bc \end{array} \right.$

The whole root to be drawn out successively			5 4 7
The homogeneity to be resolved		*ff*	3 6 9 7 0 1 9 8 4
		d	6 7 5 3 2 5
Sing. root . .	*b* = 5	*b*	5
Divisor		*A*	6 7 5 8 2 5
		db	3 3 7 6 6 2 5
First sing. root.	*b* = 5	*bb*	2 5
Taken away . .		*Ab*	3 3 7 9 1 2 5
The singular root			5
The remaining homogeneity to be resolved			3 1 7 8 9 4 8 4
		d	6 7 5 3 2 5
Sing. root times ten	*b* = 50	2*b*	1 0 0
Divisor		*B*	6 7 6 3 2 5
		dc	2 7 0 1 4 0 0
Second sing. root.	*c* = 4	2.*bc*	4 0 0
		cc	1 6
Taken away		*Bc*	2 7 0 5 4 6 0
The root augmented			5 4
The remaining homogeneity to be resolved			4 7 3 4 8 8 4
		d	.6 7 5 3 2 5
Aug. root times ten	*b* =540	2*b*	1 0 8 0
Divisor		*B*	6 7 6 4 0 5
		dc	4 7 2 7 2 7 5
Third sing. root.	*c* = 7	2.*bc*	7 5 6 0
		cc	4 9
Taken away		*Bc*	4 7 3 4 8 8 4
The whole root finally drawn out			5 4 7
The remaining homogeneity finally			0 0 0 0 0 0

Therefore from the given homogeneity 369701984, the root 547 has been extracted from the working by resolving in this way. This is equal to the root *a* sought, which was the intention of the exercise.

PROBLEM 3.

For a given homogeneous term proposed numerically, for quadratic equations [of the form. . . . *aa - da = ff,* the value of the proposed root sought of *a* can be extracted analytically.

Let the equation proposed numerically be *aa* - 624.*a* = 16305156.
Hence . . . . 624 = *d* and 16305156 = *ff.*
Putting . . . . *b + c = a.*
Hence . . . . $\left.\begin{matrix} b+c \\ b+c \end{matrix}\right| \quad \left.\begin{matrix} -d \\ b+c \end{matrix}\right| = 16305156.$

[This is the beginning of the difficulty Viete and Harriot and their successors had with negative signs. As stated elsewhere, only positive roots of equations were allowed, as they were considered useful; negative solutions were discarded as not being useful. There is hence a confusion between the uses of the signs + and -, which is more apparent with the "-" sign; in these days this sign meant 'subtract of take away the following number from the previous number, and a number such as -10 had no meaning; + 10 was of course just 10, and the problem did not exist. We now of course use these signs in two contexts; on the one hand, -10 specifies this number on the real number line, while on the other hand 20 - 10 means 'take 10 from 20'. Hence, in the following derivation of the canon, there is a definite move to avoid negative numbers, or to use them so that there is always a positive outcome; as is the case where negative outcomes for A*b* are added to the other side of the equation. In particular, the initial digit of the root is always chosen to that positive contributions are made to it for the smaller digits.]

Therefore the equation becomes, with the homogeneous terms set out in this way in two parts

. . . .

$$\begin{array}{ll} -db & -dc \\ +bb & +2.bc \\ \hline Ab & +\ldots cc \\ & \hline Bc \end{array} = 16305156.$$

The whole root to be drawn out successively			4 3 6 2
The homogeneity to be resolved	*ff*		1 6 3 0 5 1 5 6
		— *d*	— 6 2 4
		b	4
Divisor		*A*	. . 3 3 7 6
First sing. root.	*b* =4	— *db*	. . - 2 4 9 6
Taken away . .		*bb*	. 1 6
		Ab	. 1 3 5 0 4
The singular root			4
The remaining homogeneity to be resolved			2 8 0 1 1 5 6
		—*d*	—6 2 4
Sing. root times ten	*b* = 40	2*b*	8 0
Divisor		*B*	7 3 7 6
		dc	- 1 8 7 2
Second sing. root.	*c* = 3	2.*bc*	2 4 0
		cc	9
Taken away		*Bc*	. . . 2 3 0 2 8
The root augmented			4 3
The remaining homogeneity to be resolved			4 9 8 3 5 6
		—*d*	— 6 2 4
Aug. root times ten	*b* = 430	. . .	8 6 0
Divisor		2*b*	7 9 7 6
		B	- 3 7 4 4
Third sing. root.	*c* = 6	*dc*	5 1 6 0
		2.*bc*	. 3 6
	. . .	*cc*	4 8 2 1 6
Taken away . . .		*Bc*	
The root augmented			4 3 6
The remaining homogeneity to be resolved			1 6 1 9 6
		— *d*	—6 2 4
Aug. root times ten	*b* = 4360	2*b*	8 7 2 0
Divisor		*B*	8 0 9 6
		— *dc*	—1 2 4 8
Third sing. root.	*c* = 2	2.*bc*	1 7 4 4 0
		cc	. . 4
Taken away		*Bc*	. . . 1 6 1 9 6
The whole root finally drawn out			4 3 6 2
The remaining homogeneity finally			0 0 0 0 0

Moreover, the Canon [or procedure] for this equation has the algebraic part divided into the two terms *Ab, Bc*, in order that the analytical work can proceed, which is duly established in agreement with the Lemma to follow. Therefore the resolution is done entirely by the application of this Canon, as is seen from the order of the figures set out in the example placed below, in agreement with the given homogeneity 16305156, in order that the root can be deduced as follows from this number.

Therefore the root 4362 is extracted from the given homogeneity 16305156, resolved in this way from the working. This number is equal to the root *a* sought, which was the aim of the exercise.

The case of anticipation [or, looking ahead].

In the equation $aa - da = ff$, with the number proposed to be resolved, it occasionally happens that the coefficient of the divisor has more than double the individual figures of the homogeneity to be resolved. And thus the place for the resolution to start is put a number of places to the left of that homogeneity, in order that it receives as many squared places as the coefficient has simple figures. For the work of resolution begins in this first empty place by inspection, which has the advantage that the first figure of the coefficient, for the single digit estimate of the root to be extracted, is either equal or closest to that digit.

The equation to be resolved . . . . .
$$\begin{cases} aa - da = ff. \\ aa - 6253.a = 6254. \end{cases}$$

The canon of the resolution . . .
$$\left\{ \begin{array}{ll} -db & -..dc \\ +bb & +2.bc \\ Ab & +..cc \\ & Bc \end{array} \right.$$

An example of anticipation.

The whole root to be drawn out successively			6 2 5 4
The homogeneity to be resolved	*ff*		0 0 0 6 2 5 4
		—*d*	—6 2 5 3
		b	. . 6
Divisor		*A*	. . 2 5 3
First sing. root.	*b* = 6	—*db*	—.3 7 5 1 8
		bb	. 3 6
Taken away . .		*Ab*	. [-] 1 5 1 8
The singular root			6
The remaining homogeneity to be resolved			1 5 2 4 2 5 4
		—*d*	. . — 6 2 5 3
Sing. root times ten	*b* = 60	2*b*	. . 1 2 0
Divisor		*B*	. . 5 9 4 7
		—*dc*	. . —1 2 5 0 6
Second sing. root.	*c* = 2	2.*bc*	. 2 4 0
		cc	. 4
Taken away		*Bc*	. . 1 1 8 9 4
The root augmented			6 2
The remaining homogeneity to be resolved			3 3 4 8 5 4
		— *d*	. —6 2 5 3
Aug. root times ten	*b* = 620	2*b*	. 1 2 4 0
Divisor		*B*	. . 6 1 4 7
		—*dc*	. . — 3 1 2 6 5
Third sing. root.	*c* = 5	2.*bc*	. 6 2 0 0
		cc	. . 2 5
Taken away		*Bc*	. . 3 0 9 8 5
The root augmented			6 2 5
The remaining homogeneity to be resolved			2 5 0 0 4
		—*d*	. . . —6 2 5 3
Aug. root times ten	*b* = 6250	2*b*	. . 1 2 5 0 0
Divisor		*B*	. . 6 2 4 7
		—*dc*	. . —2 5 0 1 2
Third sing. root.	*c* = 4	2.*bc*	. . 5 0 0 0 0
		cc	. . . . 1 6
Taken away		*Bc*	. . . 2 5 0 0 4
The whole root finally drawn out			6 2 5 4
The remaining homogeneity finally			0 0 0 0 0

Therefore the root 6254 has been extracted from the given homogeneity 6254, resolved in this way from the working. This number is equal to the root *a* sought, which was the aim of the problem. [The other root is of course equal to -1.]

Case of rectification.

Besides this, often in the proposed equation , $aa - da = ff$, the coefficient may give confusion as concerning the doubt of the first singular [digit] of the root to be chosen. Accordingly, it is clearer in this case for the square of the coefficient to be added to the homogeneity and the first singular root of this sum [i.e. the first digit] is taken, as for the first singular root for the homogeneity to be resolved, for which either there will be agreement, or agreement with the next smaller digit. It is permitted to call this method of drawing out [the singular root], 'artificium Epanorthosin' [lit. the artifice or trick of the second square] (by the author Viete) or rectification.

Even with positive equations, if a case of similar doubt occurs in the first of the roots to be chosen, it is possible to apply the same remedy of rectification. But for these the difference and not the sum of the squares of the coefficients to be resolved is taken, which either will also give agreement, or from the next smaller choice of digit.

The equation to be resolved . . . . . $\begin{cases} aa - da = ff. \\ aa - 732 .a = 86005. \end{cases}$

The canon of the resolution . . . $\left\{ \begin{array}{ll} -db & -..dc \\ +bb & +2.bc \\ \hline Ab & +..cc \\ & \overline{Bc} \end{array} \right.$

Example of resolution.

Given homogenuity	*ff*	8 6 0 0 5
Square of coefficient	*dd*	5 3 5 8 2 4
Sum	*ff* + *dd*	6 2 1 8 2 9
First singular root		8

To continue the resolution.

The whole root to be drawn out successively			8 3 5
The homogeneity to be resolved *ff*			8 6 0 0 5
		— *d*	— 7 3 2
b =8		— *db*	—5 8 5 6
		bb .	6 4
Taken away . .		*Ab* .	5 4 4
The singular root			8
The remaining homogeneity to be resolved			3 1 6 0 5
		—*d*	— 7 3 2
Sing. root times ten	*b* = 80	2*b*	1 6 0
Divisor		*B*	8 6 8
		—*dc*	—2 1 9 6
Second sing. root.	*c* = 3	2.*bc*	4 8 0
		cc	9
Taken away		*Bc* . .	2 6 9 4
The root augmented			8 3
The remaining homogeneity to be resolved			4 6 6 5
		— *d*	—7 3 2
Aug. root times ten	*b* = 830	. .	1 6 6 0
Divisor		2*b*	9 2 8
		B	—3 6 6 0
Third sing. root.	*c* = 5	—*dc*	8 3 0 0
		2.*bc*	2 5
Taken away . . .	. . .	*cc*	4 6 6 5
		Bc	
The whole root finally drawn out			8 3 5
The remaining homogeneity finally			0 0 0 0

PROBLEM 4.

For equations of the given form . . . . $aa + da = ff$, procedures for finding both roots are to be established. The given homogeneous term is proposed numerically, and the value of a proposed root of a sought can be extracted analytically, as well as the other root.

The equation to be resolved $\begin{cases} -aa + da = ff. \\ -aa + 370.a = 9261. \end{cases}$

The canon for the resolution

$$\begin{array}{c} +db \\ -bb \\ \hline Ab \end{array} \qquad \begin{array}{c} +dc \\ -2.bc \\ -.cc \\ \hline Bc \end{array}$$

For extracting the smaller root.

The root			2 7
The homogeneity to be resolved			9 2 6 1
		d	3 7 0
	b = 2	— *b*	—2
Divisor		*A* . .	3 5 0
	b = 2	*db* . .	7 4 0
Taken away . .		—*bb* .	—4
		Ab .	7 0 0
The singular root			2
The remaining homogeneity to be resolved			2 2 6 1
		d	3 7 0
Sing. root times ten	*b* = 20	— 2*b*	—4 0
Divisor		*B*	3 3 0
		dc	2 5 9 0
Second sing. root.	*c* = 7	—2.*bc*	—2 8 0
		—*cc*	— 4 9
Taken away		*Bc* . . .	2 2 6 1
The root			2 7
The remaining homogeneity nothing			0 0 0 0
The root augmented			4 3 6
The remaining homogeneity to be resolved			1 6 1 9 6

The greater root is extracted by anticipation.

The root			3 4 3
The homogeneity to be resolved			0 9 2 6 1
		d	3 7 0
	b = 3	— *b*	— 3
Divisor		*A*	7 0
	b = 3	*db*	1 1 1 0
Taken away		—*bb*	—9
		Ab	2 1 0
The singular root			3
The remaining homogeneity to be resolved			1 1 7 3 9
		d	3 7 0
Sing. root times ten	*b* = 30	— 2*b*	— 6 0
Divisor		*B*	— 2 3 0
		dc	1 4 8 0
Second sing. root.	*c* = 4	—2.*bc*	—2 4 0
		— *cc*	— 1 6
Taken away		— *Bc*	- 1 0 8 0
The root augmented			3 4
The remaining homogeneity to be resolved			9 3 9
		d	3 7 0
Sing. root times ten	*b* = 340	—2*b*	—6 8 0
Divisor		*B*	— 3 1 0
		dc	1 1 1 0
Second sing. root.	*c* = 3	—2.*bc*	—2 0 4 0
		— *cc*	— 9
Taken away		—*Bc*	— 9 3 9
The root			3 4 3
The final remaining homogeneity			0 0 0 0

Therefore from the given homogeneity 9261, the roots of *a* sought have been extracted from the working. These are the two roots 27 & 243, which were to be extracted, with the resolution procedure applied twice as shown.

Compendium.

By Theorem 2, Section 5, the sum of the two roots for the proposed equation - $aa + da = ff$ has been established, for a given coefficient, and the product is equal to the given homogeneity , as in the numerical example 27 + 343 = 370, & 27 | = 6261.
343 |
Therefore with one root found, the other root can be found without the help of analysis.

Note 1.

It has been clearly noted in examples in ancient works how it was possible to find the solutions of quadratic equations. Nevertheless Viete was not of the opinion that his own work should be diminished on account of these methods for cutting short his method for the numerical solutions of quadratics, and he wished the whole general art of numerical analysis to appear in public suitably arranged from the general order of things, and with the whole method set out in an orderly fashion. For our example of this, the quadratics, the Analyst also proposed in his writings on numerical Exegetics. Hence the rules for Devolution, Anticipation, and Rectification from the general art are observed in quadratics, and it is by necessity that instruction of the method are given with these initially.

Aequationum quadraticarum resolutionem in veteri praxi apodictice tractare posse notum est. Vieta tamen ne propriae inventionis existimationi quadraticarum mutilatione derogaret, exegeticen suam numerosam artem natura generalem generali ac integra methodo concinnatam in publicum prodire voluit. Cuius exemplo Analysta noster quadricarum quoque Exegesin numerosam in scriptis suis proposuit. Hinc est, quod de Devolutionis & Anticipationis & Rectificationis regulis quae artem generalem spectant in quadraticis hisce primo ex methodi necessitate praecipiendum erat.

Note 2.

It is to be noted here also, since the root is to be constructed from the secondary divisors by using multiples of ten (as observed in the previous schemes, and in the those that follow), so the divisors [have an associated power of 10], as do the particular terms which are subtracted [for the place in the analysis under consideration], which suitably designated terminate uniformly with the powers of 10. From which the order of the smaller terms is easy brought out, which are presented in the prescribed and useful form prior to this, with the various endings for the terms, made with much care and concern.

Notandum hic quoque venit, quod per radicum quae ad divisores secundarios constituendos adhibentur decuplationem (ut in antecedentibus schematismis sit, & in sequentibus observandum est) tam divisoris quam ablatitii particularis ad puncta directoria aequationis gradualibus congrue designata uniformiter terminantur. Unde particularum ordinatio prompta & facilis evadit, quae in forma antehac praescripta & usitata, particularum terminationibus variatis, nimium curiosa facta est & anxia.

Note 3.

Besides there is this to be turned to, the titles in the schematics of the examples ascribed in the margins, which are not a necessary part of the work of the Analyst, as he is concerned with the application of the canon, and the places of the designated root only are to be used. At the start, this was made an opportunity for instruction in the art. But in real practice, with the simplicity of the canon for serious continuation, the application is sufficiently guided [by the order present], without the necessity for words of this kind to make an appearance, as this should be sufficient in place of these words to suggest the next step along the way.

Praeterea advertendum est, titulos in exemplorum schematismis ad marginem adscriptos, non ad operis Analytici necessitatem sed ad canonis applicati, & radicum notas designandas solummodo adhibitos esse. Quod in novitia hac artis instructione opportune faciendum erat. In praxi autem reali, quae simplici canonis, continuata serie, applicatione sufficientissime dirigitur, verboso huiusmodi apparatu non erit opus. Quod hoc loco obiter admonuisse sufficiat.