DEFINITIONS.

Some definitions, which first establish the names in common use by the art itself, so that the special interpretations of words presented in the discussion can be easily understood .

Definitiones quaedam, quae praemissae [sic : proaemii] loco vocabulis tam ipsius artis communibus, quam praesentis tractatus peculiaribus interpretandis utiliter inservire possint.

[Note: Some of these definitions, as indicated, are augmented forms of those in Viete's *In Artem Analyticem Isagoge,* or *Introduction to the Analytic Art*; others are 'word only' introductions to processes of an algebraic nature that may at first appear obscure. A translation of some of Viete's works is given by T. Richard Witmer in his *The Analytic Art*, to be abbreviated here as AR. The present translation is more liberal than rigid, as an attempt has been to present the material of a mathematical nature in an understandable form, rather than adhering to a literal translation. Certain words and phrases are of course no longer in use, as they refer to defunct processes or ideas.]

First Definition.

[S*ymbolic logistic* is a form of the art of calculation that reasons with symbols, as opposed to *Logistice numerosa*, the *numerical logistic*, where the calculation is done using numbers. This definition is taken from Ch. IIII of *The Analytic Art*; see also AR, p.17.]

Logistice speciosa, which often occurs in Analysis, and the use of which is essential in all Arithmetic, is of the same kind as in Arithmetic, and is used in a the same way. In fact in arithmetical processes it is called *Logistice numerosa*. The only difference between these two methods is the following, and this has much to do with the reason for the name: In Arithmetic, the magnitudes of measurable things are computed and expressed by a number from a measurement by means of the special characters or figures of the art. But here in the speciosa it is done with written letters or symbols, taken from the alphabet of course, and as regards the kind of the quantity, all sorts can be handled. The name *speciosa* has been obtained from its legal use, from the word 'speciei'.

[This assertion about the word *speciosa* either by Harriot himself, or by the editor of his work, Walter Warner, is contentious, as the word has a Diophantine origin, according to Klein: see Jacob Klein: *Greek Mathematical Thought and the Origin of Algebra*, (Dover), p. 281. A commentary on the diverging viewpoints of Viete and Harriot is given in this book: the former was an outstanding 'pure' mathematician, with a philosophical interest in the structures and elements considered, while the latter was more of a practitioner, with an interest in presenting Viete's work in an accessible form - this appears to be Klein's view. The purpose of the *speciosa* is a means of obtaining geometrical quantities such as proportions, areas, etc. in terms of symbols which are to be letters of the alphabet, that can be manipulated in the same way as ordinary numbers that refer to lengths, etc.]

Logistice speciosa, cuius in Analytics istis frequens, & omnino necessarius est usus Arithmeticae, generis eiusdem participatione germana est. Est namque Arithmetica Logistica numerosa. Hoc tantum inter eas (quantum ad nominis rationem attinet) discriminis est. In Arithmetica rerum mensurabilium quantitates per characteres seu figures artis proprias, numero, ut generali mensura exprimuntur & computantur. In hac autem per notas literales, elementa scilicet alphabetica, re ipsae tanquam in specie (ex usu forensi recepto speciei vocabulo) significantur & omnimode tractantur, unde speciosa appellationem obtinuit.

Definition 2.

[This gives the definition of an equation; it is a summary of Chapter VIII of the *Isogoge*: see AR, pp.29 -30].

An equation is used to indicate the general equality of two or more quantities of any kind. But in fact the equality is the special name of the art, whereby a quantity sought is compared with any given quantity, with the products of the one with the other set out in a distinct manner for the comparison. The sought part of the equation is the pure or affected power [*i.e.* the equation may consist of a pure power only of the unknown, or can entail additional terms of lesser powers of the same unknown], the homogeneous part truly given is usually called the given part of this comparison or equation [i. e. the term independent of the unknown].

[Viete had the mistaken or unnecessary belief that all products in equations had to be of the same power or dimensions. One may look at some of Viete's equations to see what he had in mind, such as the expansion of sine of a multiple of an angle in terms of the products of the sine of the angle, where one takes the sine of the angle as the unknown. The presence of extra terms of lower powers of the unknown was called the affected part of an equation by Viete.]

Aequatio communi significatione pro quantitatum duarum vel plurium qualicunque aequalitate usurpatur, sed [ut est proprium huius artis vocabulum] est quantitatis quaesitae cum quantitate aliqua data, facta alterius ad alteram comparatione, distincte ordinata aequalitas. Cuius [pars quaesititia est potestas pura vel affecta], pars vero data homogeneum [comparationis seu aequationis] datum nominare solet.

Definition 3.

The [Analytical] method can be shown to be the most powerful tool for [the resolution of] propositions of any kind, either for theorems or problems put in place in a knowledgeable manner: and the entirely natural method of doing this is to start from the principles and elements of the method. [The resolution of an equation] proceeds through an individual continued succession of steps to the confirmation of the preposition, according to its properties; thus the method put together has been called Synthetic, in the idiom of the method from antiquity.

In propositionibus cuiuscunque generis iisque sive theorematis sive problematis scientifice constituendis, potissima demonstranda methodus, & via omnino naturalis est, qua a principiis & elementis doctrinae, cuiusque; propriis per continuatas consequentias componendo proceditur ad propositi confirmationem, unde methodus compositiua & veterum artificum idiomate Syntheticae appellata est.

Definition 4.

Moreover as often as not it happens that certain problems have to be solved by a process involving trial and error, for suitable Logistic means are not available for providing a natural synthesis of a proposition from a knowledge of the elements and principles. Indeed a way cannot be found for the solution of the problem to be sought rationally from the first principles and natural synthesis of the elements. For in the case of this kind of ignorance which often occurs, it is necessary to carry out the steps backwards from a trial solution, contrary to the natural [way] of proceeding. For indeed with a quantity chosen in the beginning pertinent to the solution sought for the problem, as noted and given by assumption, the progress is continued for as long as you wish; with the assumed quantity given (either in single steps or in steps with affections implicated) until it coincides with the other true quantity. [This is the method of *numeric logistic* or numerical logic.] Out of which indeed, from the equality found by the art of this method, and quickly put in place, from which either the sought quantity is seen to emerge, or the solution may be elicited finally from further skills, until thus the problem is solved at last.

Quoniam autem saepissime accidit in problematis praesertim fortuito oblatis soluendis, ut Logista mediis ad propositum arguendum idoneis destitutus, a scientiae principiis & elementis naturali synthetices via ad problematis solutionem ratiocinio exquirendam & concludendam procedere nequeat. In huiusmodi ignorationis casu qui sere perpetuus est, necessitate edoctus viam capessit retrogradam & naturali contrariam. Facto namque initio ab ignota aliqua & quaesita ad problema pertinente quantitate, tanquam nota & data assumpta, continuatis consequentiis resoluendo progreditur quousque; in assumptae illius quantitatis tanquam datae (sive simplicis sive graduatae & affectionibus implicatae) cum quantitate aliqua vere data aequalitatem incidat. Ex qua quidem aequalitate artificio huiusmodi inventa & rite constituta ipsa de qua quaeritur quantitas, vel per se manifesta prodeat, vel ulteriore artificio ervatur unde problema tandem solvatur. Atque methodus ista resolutiva est quam vocabulo significante veteres Analyticen appellarunt.

Definition 5.

You can call the processes introduced by these two definitions composition and resolution [*i. e.* definition 3 or *symbolic* and definition 4 or *numeric logistic*], which are generally introduced by Mathematicians as the two different ways of reasoning by which their demonstrations have been accomplished, and by means of which the solutions are usually expressly set out. [The first method involves the solution of an equation by algebraic means, while the second relies on a numerical procedure to generate the root place by place]. Of these the first, by composition, is initially simple and proceeds to be more complicated; following the natural structure and order of what has been given, and from the first situation it is allowed to be reduced to the last [*i. e.* to the solution]. With the other, truly the working goes from being more complicated to being less complicated and more simple, and this process is called resolution: the order is backwards and contrary to the natural one, from the end evidently to be ascending to the beginning to reach a conclusion. For if the elements and axioms of the skills have been properly set up; the methods should be interchangeable, and thus, what is naturally happening in the first case, can be achieved by reasoning in the second. And thus the Logical progress consequently is equally firm for either part, as is required for a clear proof.

Componendi & resolvendi voces quae duabus hisce definitionibus inseruntur Mathematicorum solennes sunt, quibus in demonstrationibus suis conficiendis contrarias ratiocinandi vias, ubi ipsius fuerit, expresse significare solent. In earum altera a simplicioribus & minus compositis ad magis composita, quod est componendo, secundum naturalem scientiarum structuram & ordinationem, a priore scilicet ad posterius descenditur. In altera vero a magis compositis ad minus composita simplicia, quod est resoluendo, ordine retrogrado & naturali contrario, a posteriore scilicet ad prius ascenditur ad conclusionem. Nam scientiarum elementa & axiomata si legitime constituta sint, convertibilia esse debent; unde sit, ut quae natura antededentia sunt, ratione consequentia esse possint. Atque id circo processum Logicum per consequentia in utramque partem aeque firmum & apodicticum esse necesse est.

Definition 6.

There are two distinct opposing parts of the above definition of Analysis that need to be considered. The first part is involved in the establishment of the whole equation, which of course - as the saying goes, is expanded from the answer to one assumed question by a series of consequences to the answers for numerous questions sought, as the equation for some given quantity is to be found and set up, and there it remains on completion. Moreover, the ancients gave the name *Zetetics* to an investigation or inquiry where an equation of this kind is established for investigation by some skilful construction.

[Thus, according to Plato, *zetetics* is the art of establishing an equation along theoretical lines; *porisms* constitute the art of finding relations between theorems, while *exegesics* is the art of actually solving equations. See Klein, page 320.]

Ex superiore Analytices definitione duas illius officio distinctas partes esse colligitur. Quarum prima in aequationum constitutione tota versatur, quae scilicet ut dictum est, ab assumpto quaesito tantum dato per consequentia tendit ad assumpti quaesiti cum quantitate aliqua data aequationem inveniendam & constituendam. & in constituta terminatur & acquiescit. Cum autem aequationum huiusmodi constitutio in artificiosa quaesiti investigatione consistat, veteres artem istam, Zeteticen, quasi investigatoriam seu inquisitoriam nominarunt.

Definition 7.

The other part of Analysis is where, from an equation now put in place by Zetetics, a quantity sought can be shown to be brought forth, either by a continuation to the solution of the equation, or by some change. Assuming that it is possible to produce a solution, either of course, in the form of a species [i.e. a symbol], or by a number - if the equation can be solved by a number, then a perfect solution finally exists for the problem proposed . To this part of Analysis the name *Exegetices* has been imposed by *Francis Viete*, that great teacher of Analysis, as if the solution was about to be declared or given up.

Analytices pars altera est, qua ex aequatione per Zeteticen iam constituta, quaesita quantitas continuato vel mutato resolutionis genere exhibetur, vel specie scilicet si reipsa exhiberi possit, vel numero, si numero explicanda sit, unde propositi problematis perfecta tandem existit solutio. Huic Analytices parti a *Francisco Viete,* magno artis Analyticae magistro, Exegetics, quasi declaratorae seu exhibitoriae nomen impositum est.

Definition 8.

The old Analysis brought in another kind of Analysis as well as *Zetetic*, on account of it being pertinent to the solution of problems, which was called *Porism*: according to this, the truth of a proposition in doubt is examined as it were by bringing in [or deducing from the proposition] the truth of some other fortuitous [known] theorem. Indeed the method of this other analysis is, from the assumed approval of the theorem, by means of the [logical] consequences, to concede the truth of a known theorem. These forms of reasoning are nevertheless different from each other: for with *Zetetics* that which is sought in an equality can be deduced as it were; or give rise to a conceded theorem, for a *porism* applied to a given question, as one can see in examples of each kind. Thus there is a different origin between these processes, for with *porisms* one theorem may be shown to be the equivalent to another, or concedes the truth of another theorem, as is seen to be the case in examples of both kinds. Thus a difference arises between these methods since with porisms, the process of deduction results in the identification of one theorem with another, or in conceding the truth of a theorem; for a final resolution is not the task for the verification of a theorem, as is the case with zetetics.

Veteres Analystse praeter Zeeicen quae ad problematum solutionem proprie pertinet aliam Analycices speciem fecerunt poristicen, quasi illatoriam qua theorematum fortuito propositorum dubitata veritas examinatur. Methodus enim utriusque Analytica est, ab assumpto probando tanquam concesso per consequentia ad verum concessum. In hoc tamen inter se differunt quos Zetitice quaestionem deducit ad aequale, datum scilicet quaesito, poristice autem ad idem, vel concessum, ut in exemplis utriusque generis videre licet. Unde & altera inter eas oritur differentia quod in poristice, cum processus eius terminetur in identitate vel concesso, ulteriore resolutione non sit opus (ut sit in Zetetice) ad propositi finalem verificationem.

Definition 9.

From the definition of *Exegetic* established, the twofold [nature] of this process ought to be apparent along with the two kinds of logistic: the numeric and the symbolic, which are permitted to draw on the same material, to the same end. For now we can proceed to examine the solutions of equations that have been set up, and to note that in practise moreover, the whole way of operating with these kinds of processes is different. For with symbolic exegetic, the equation is set up by first using zetetics, and by a continuation of the reasoning, the equation can be solved accurately by progressing in kind or form, to be reduced in an orderly manner, and from that order the quantity sought is itself produced by the art, in a sure and simple manner. To this extent, exegetic is at present used with equations which do not exceed ordinary and quadratic [forms], for in these easy cases, as the ancients said, they are sustained on account of the exact method, which is uniform and sure, and hence the solution sought is known perfectly. But according to some authors of modern Algebra, this fundamental method invented for these simple cases is deficient from the start for the resolution of equations of higher degree, such as cubics, bi-quadratics, and so on. Hence the method is curtailed, as it is fundamentally unable to correct higher forms from their own imperfections, and they are abandoned by us, as equations derived from these higher forms have in the greater part been found to be insoluble.

Ex praemissa Exegetices definitione, duplicem eam esse debere apparet iuxta duplicem Logistices naturam, numerosam & speciosam, quae licet eandem tractent materiam, & ad eundem finem, aequationum scilicet iam constitutarum resolutionem spectent, in praxi tamen & operationis modo toto genere inter se diderunt. Nam Exegetice speciosa aequationem Zetetices opera prima constitutam, continuato ratiocinationis processu ad speciem sive formam resolutioni proprie ordinatam reducit, & ex ordinata quaesitam quantitatem in specie sua reali certo ac simplici artificio exhibet. Adeo ut Exegetice ista dum in aequationibus exercetur quae ordinem quadraticum non excedunt, sed in loco plano, ut loquuntur vereres, subsistunt, propter exactam methodi uniformitatem ac certudinem pro perfecte scientifica habenda sit. Cum autem tentatum est a modernis quibusdam Algebrae authoribus deficiente priore est methodo alio invento fundamento ab sublimiorum graduum aequationes, cubicas scilicet & biquadraticas resolvendas, artem promovere, mutilam eam & imperfectam ex inemendabli fundamenti sui imperfectionem: nobis reliquerunt, ut aequationum quae in altiores illas formulas incidunt, pars magna pro irresolubilibus habitae sint.

Definition 10.

Hence only by having devised that other method, the *Exegetic numerosa*, which is extended to all orders of every equation, by a method of generality and infallibility, does the ability exist for the resolution of any equation established by zetetics. The number found is the quantity sought, and the number is resolved with the help of small secondary changes. The singular art of Exegetics, the rules and instructions for its practise, are expounded in the present exposition, which covers all aspects of Exegetics.

Exogitata est igitur tandem altera illa Exegetice numerosa, quae ad omnes omnium ordinum aequationes resolvendas extenditur, methodo generali ac infalllibili, qua scilicet ex aequatione quacunque resolutorio Zetetices processu constituta & ad numeros revocatas, quaesita quantitas, secundaria mutatae resolutionis opera numero exhibetur. Peculiaris est Exegetices huius ars, regulis suis & praeceptis ad praxim instructa, quae in praesenti tractatu, qui totus Exegeticus est, traduntur.

Definition 11.

The resolution of an equation by the *Exegetic numerosa* in comparison with the resolution by the before-mentioned zetetics is seen to be inferior: for by putting together the two methods under a common name for an analysis of an equation by the two different methods to be in agreement, the resolution by Zetetics is indeed logical or carried out according to a plan, while Exegetices is truly operational, [like arithmetic]. For there is no other operation present in this Exegetices other than the Arithmetic of the ancients, which in simple form has continued hitherto only to be of use in the common extraction of roots, while the new version is somewhat extended beyond the ancients in a generalisation to be an elevated method.

Resolutionem Exegetices numerosam ad praeviam zetetices resolutionem comparatam, secundariam appellare visum est, ut ex adiecta communi nomini differentia utramque licet diversi generis, analysim tamen dive resolutionem esse constet. Zeteticen quidem Logicam sive discursiuam, Exegeticen vero operativum. Non est enim aliud exegetice ista numerosa quam veteris Arithmeticae operationis, quae in simplicium tantum potestatum radicubus extrahendis communiter usitata hucusque permansit, nova quaedam & a veteribus intentata ad methodigeneralitatem exaltatio.

Definition 12.

The term '*root*' is used with a two-fold meaning in the following. For in the establishment of equations by zetetic itself, that which is assumed or supposed initially can always be sought by reasoning . Hence, from what has now been arranged [in an equation in this way], the unknown is folded into an equation either as a power or signified in some manner ; in any case the root of the equation sought or substituted into the equation can be named. But in the resolution of equations by exegetics, this is done either by *symbolic logistic*, where the real unknown has been found by analytical reasoning from the equation set up [in terms of symbols], or by *numeric logistic,* where an expressed number has been found from the given homogeneity of the equation, by analytical operations. Thus the root is exhibited or it can be generated or resolved by a number of means, with the method used to be given any name you please - and which also generally gives rise to the value of the root sought. And besides, concerning the root now deduced, which is shown either symbolically or numerically from the equation, as it is customary for an explanation of the means of verification to be given, [the derivation of] the root of the equation can also be said to be the explanation. For it is not from a single power that the means of resolution is shown, either for the root extracted, or for the root now shown or drawn, as the manner of composition can be explained from the equation itself. Whereby it is allowed in the exegetics of *Viete,* where the problems concerning quadratics, cubics, and higher powers have been set forth in a skilful manner in terms of the powers of the root sought: but yet in the analysis that follows, what is revealed is not concerned with individual powers, as the whole equation itself is set up for examination, in order that an unknown quantity can be found. [In such an equation], the root sought is no larger than the smallest step of the power for the required root of the equation to be found; or the leap from the root sought to the required established value, with the declared root found seen to be in good agreement. But having explained how the root of the equation is found, the same too is the case for the generation of powers of the root, if the analysis is itself composed in a sensible manner by some method, for it is manifest that there is nothing else between these things that needs to be taken care of.

Radicis vocubulum duplici significatione insequentibus usurpatur. Nam in aequationibus zetetice constituendis ipsum perpetuo est quaesitum quod in ratiocinationis initio pro concesso assumitur seu supponitur, unde in iam constitutis dum adhuc in aequationis involueris ignota latet vel potestatis vel suo modo significanter, etiam aequationis radix quaesititia vel supposititia nominari potest. In aequationibus autem resolvendis sive iam resolutis, cum per exegeticen, seu speciosam ex ipsa aequatione iam constituta ratiocinio analytico in specie sua reali exhibita est , seu numerosam e dato aequationis homogeneo operatione anaytica in numero expresso educta est, radicem exhibititiam, vel eductitiam eam pro vario resolutionis genere, variato nomine appellare liceat, quae etiam radicis quaesititae valor communiter nominari obtinuit. Ac praeterea quia de ea iam exhibita vel educta aequatio ad operis vel ratiocinii verificationem explicari solet, radix aequationis explicatoria dici quoquae potest. Non est enin potestas e qua resolutionis via exhibita vel educta est radix, vel quae de radice iam exhibita vel educta via compositionis explicatur, sed ipsa aequatio. Quare licet in *Vieta* exegeticis ubi diserte de quadratis & cubis & reliquis potestatibus problemata enunciata sunt, de potestatis radice necessario quaerendum erat: Tamen in sequentibus quae non se potestatibus sed de ipsis aequationius subiective & integra specie acceptis enunciantur, in quibus non magis summae potestatis generatoria sit, sive hoc sive illo nomine inscribatur rem ipsam intelligentibus nihi interesse posse se quo curandum sit, manifestum est.

Definition 13.

The definition of an equation duly set up initially by zetetic is to be accepted, and since a common equation is generally deduced or found from the terms of any proposed problem or question, this method should be given a suitable name. Such a name certainly should be distinguishable from the names associated with other kind of equations; hence sets of equations are described in terms of canons, about which we will say more later.

Praemissa aequationis definitio de aequatone per zeteticen rite constituta accipienda est, quae quoniam a problematum seu quaestionum utcunque propositarium terminis communiter deducitur aequationem communem sive aduentitiam nuncupare licet, ut ipso nomine ab alio aequationis genere prorsus diverso distinguatur, quam cononicam appellare visum est, de qua postea.

Definition 14.

$$\begin{array}{l|l} a+b & aa+ba \\ a-c & \quad -ca-bc \end{array}$$

$$\begin{array}{l|l} a+b & aaa+baa+bca \\ a+c & \quad +caa-bda \\ a-d & \quad -daa-cda-bcd \end{array}$$

$$\begin{array}{l|l} a+b & aaaa+baaa+bcaa \\ a+c & \quad +caaa+bdaa \\ a+d & \quad +daaa+cdaa+bcda \\ a-f & \quad -faaa-bcaa-bcfa \\ & \quad\quad -cfaa-bdfa \\ & \quad\quad -dfaa-cdfa-bcdf \end{array}$$

There is also another certain sort of equation, which cannot be classified in terms of canons; nevertheless for these, just as canons are derived from their own origins in the following, so these can be designated according to their own canons. Equations of this kind are generated by multiplication from binomial roots [meaning a term consisting of two symbols that are either added or subtracted] in a recursive manner in the same location. The product of these roots by multiplication is equal to the roots themselves multiplied as

in ordinary multiplication with numbers, as is seen from this example.
[We may note the special form of these products, where the root in the last factor is positive while all the others are negative.]

The minimal form of these individual equations is defined to be that which gives rise to a square. From which nevertheless two canonical forms can be derived: the first form and of course the second form, that have a similar definition, then to be used in their particular applications, [as we now show].

Est etiam aliud quoddam aequationum genus; quae licet canonicae non sint, cum tamen ab iis, ut ab originalibus suis aequationes canonicae deriventur, canonicarum originales in sequentibus denominantur. Huius generis aequationes per genesim sive muliplicationem e radicibus binomiis absque alio discursu immediat conficiuntur. In quibus facta ex radicibus multiplicatis, raducibus ipsis multiplicandis sub forma tantum multiplicationis ordinatis aequantur, ut in exemplis hisce videre est. Harum aequationum forma propriae aequationis definitioni minime quadrat. Quae tamen ab iis derivantur duae cononicarum species, primaria scilicet et secundaria, tum definitioni conformes sunt, tum usui proprie applicabiles.

Definition 15.

$$\begin{aligned} aa &+ ba \\ &- ca = + bc \end{aligned}$$

$$\begin{aligned} aaa &+ baa + bca \\ &+ caa - bda \\ &- daa - cda = + bcd \end{aligned}$$

$$\begin{aligned} aaaa &+ baaa + bcaa \\ &+ caaa + bdaa \\ &+ daaa + cdaa + bcda \\ &- cfaa - bdfa \\ &- dfaa - cdfa = + bcdf \end{aligned}$$

The first kind of the canonical form is constructed from the original derivation. For by rearranging the order of the roots, by changing a part of the original equation, the first canonical form is made by moving the given homogenous part opposite, with the other affected homogenous part remaining in place. The reasoning of the derivation is set out in section 2, but here are some examples of this kind. [The product of the roots is set on the r.h.s. of the equation as a positive number in this canonical form.]

Primaria canoricarum species est earum quae ab originalibus per derivationem constituuntur. Nam reiecta formali radicum ordinatione, quae aequationis originalis pars altera est, & ex alterius partis homogeneis homogeneo dato per affectionis mutationem in situm reliquis oppositum translato, sit aequatio canonica primaria. Derivationes ratio in sectione 2, proponitur, speciei autem exempla hec sunt.

Definition 16.

$$aa = + bb$$

$$\begin{aligned} aaa &- bba \\ &- bca \\ &- cca = + bbc \\ &\qquad + bcc \end{aligned}$$

$$\begin{aligned} aaaa &- bbaa - bbca \\ &- bcaa - bcca \\ &- ccaa - bbda \\ &- bdaa - bdda \\ &- cdaa - cdda \\ &- 2.bcda = + bbcd \\ &\qquad + bccd \\ &\qquad + ccda \end{aligned}$$

The second kind of canonical form is that which can be established from the first by reduction. For the second canonical or reduced form is prepared by removing one power from the steps in the powers established by the first form, and forms of this kind are prepared in this way, as is established in the third section; moreover some examples of this method are shown here. [The sum of the roots is made zero, as by setting $b - c = 0$ or $b + c - d = 0$, etc., in these examples.]

Secundaria canonicarum species est earum quae a primariis per reductionem constituuntur. Nam tollendo unum aliquem ex primariae gradibus parodicis sit secundaria seu reductitia varii sunt huius speciei reductinis modi, de quibus in tertia sectione agitur, speciei autem exempla huiusmodi sunt.

Definition 17.

These two kinds of equations are established as canons, as through their application, just as by canons or rules, the number of roots in equations is general equations is determined, (that may be seen in Section four). Thus these are called canons, not from the manner in which they have been set up, but because of their useful nature.

Haec duae aequatinum species pro canonicis habentur, quia per earum applicationem tanquam per canones sive regulas, radicum numerus in aequationibus communibus determinatur, (quod in quarta Sectione videre licet) unde non a constitutionis forma sed ab instrumentario huiusmodi usu canonicarum nomen illis debetur.

Definition 18.

The reciprocal equation is called that for which the given homogenous [term] is equal to the product of the coefficients from the neighbouring powers: and vice versa the powers from the product from the similar steps are equal. The amount is

Aequatio reciproca appellatur, cuius homogeneous datum facto e coefficientibus & reciproce potestas facto e gradibus parodicis aequatur. Qualis est

$aaa - caa - bba \;=\; + bbc.$ For bbc is equal to $\left.\begin{matrix} bb \\ c \end{matrix}\right|$ and aaa is equal to $\left.\begin{matrix} aa \\ a \end{matrix}\right|$

The wrong sign is given in the last term on the left hand side of the equation in the original work.]

A management summary for the division of this Analysis.

The work is set out in two parts:

- The first part is prepared according to the rules of *exegetics*, set out in six chapters or sections.
  - First Part : *Logistices speciosa*, in which the four kinds of operation are made clear from examples.
  - Second Part : in which the derivation of the equations of the first canonical form are established [by multiplication] from their binomial factors. These are set out in the order of the powers.
  - Third Part : in which the equations of the second canonical form are deduced or reduced from the first [from some relation between the roots], by the removal of some power, with the remaining roots of the equation remaining unchanged.
  - Fourth Part : in which the roots of the equations of the first canonical form, as of the reduced equations, are estalished. [Thus, the *Praxis* assumed the roots, and shows the canonical equation they satisfy.]
  - Fifth Part : in which the number of the roots of common equations are determined from their equivalent equipollant canonical form.
  - Sixth Part : in which the reduction of common equations by the parodic exclusion of some power is shown, and roots can be changed by substitution.

- The second part, the *Exegetes numerosae* itself, contains the practise of the rules and regulations explained in the first part, the instruction from the main part of which should make the material that follows easily understood.