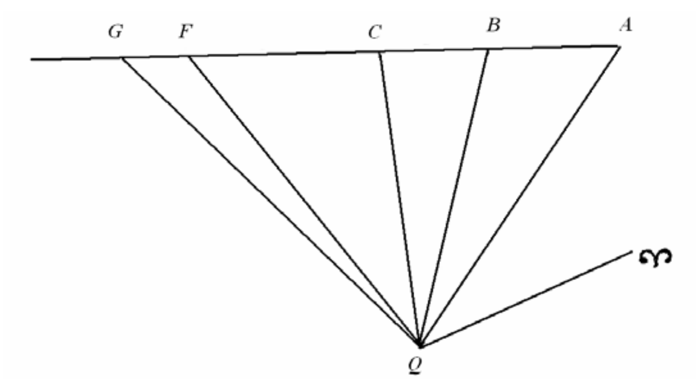
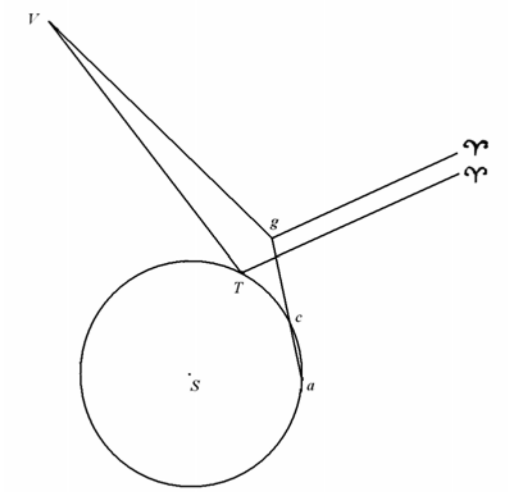
LEMMA IV.
Comets occupy places beyond the moon, moving in the region of the planets.

Just as the lack of diurnal parallax raises comets beyond sub lunar regions, so one is convinced from the annual parallax that they fall in the regions of the planets. For comets, which are progressing along the order of the signs [of the Zodiac], are all at the end of their appearance, either going more & more slowly or backwards, if the earth is between them & the sun ; but just as much faster if the earth turns in opposition. And on the contrary, those which go against the order of the signs are equally faster at the end of their appearance, if the earth is present between them & the sun ; & equally they go slower or backwards, if the earth is situated on the other side [of its orbit] . This is in maximum agreement with the motion of the earth : in the variation of its position, in the same way as it shall be with the planets, which for the motion of the earth are either in agreement, or in opposition, in the one case seen to be progressing slower, in the other quicker. If the earth travels in the same direction as the comet, & in the angular motion about the sun it goes a little faster, so that a right line drawn through the earth & the comet always meet at places beyond the comet, the comet seen from the earth on account of its slower motion appears to be moving backwards ; but if the earth were carried forwards slower [than the comet], the motion of the comet (with the motion of the earth subtracted) shall be at least slower. But if the earth proceeds in the opposite direction, the comet will appear a great deal faster. Moreover either from the acceleration or retardation or from the backwards motion, the distance of the comet can be deduced in this manner. Let ♈*QA*, ♈*QB*, ♈*QC* be three observations of the longitudes of the comet from the initial motion, & let ♈ *QF* be the final longitude observed, when the comet ceases to be



seen. The right line *ABC* is drawn, the parts of which *AB*, *BC* lying between *QA* & *QB*, *QB* & *QC* , shall be in turn as the times between the first three observations. *AC* is produced to *G* so that *AG* shall be to *AB* as the time between the first observation & the final to the time between the first observation & the second, & *QG* may be joined. And if the comet may be moving uniformly in a straight line, & the earth is either at rest or also may be progressing along a line with a uniform motion; the angle ♈*QG* will be the final longitude of the comet observed in the final time of observation. Therefore the angle *FQG*, which is the difference of the longitudes, arises from the inequality of the motion of the comet & the earth. But this angle, if the earth & the comet are moving in opposite directions, is added to the angle ΥQG & thus the apparent motion of the comet is returned faster : but if the comet goes in the same direction as the earth, the same is



subtracted, & the motion of the comet is returned either slower, or perhaps backwards ; as I have just explained. Therefore here the angle arises particularly from the motion of the earth, & therefore consequently can be taken for the parallax of the comet, clearly with some increment or decrement of this ignored – because an inequality in the motion of the comet in its own orbit may arise. Truly the distance of the comet thus can be deduced from this parallax.

Let S designate the sun, acT the great orbit, a the position of the earth at the first observation, c the place of the earth at the third observation, T the place of the earth at the final observation, & $T\Upsilon$ a right line drawn towards the first star in *Aries*. The angle ΥTV is taken equal to the angle ΥQF, that is, equal to the longitude of the comet when the earth is present at T. Join ac, & that is produced to g, so that ag shall be to ac as AG to AC, & g will be the position that the earth will reach in the time of the final observation, by continuing uniformly in a straight line. And thus if gr is drawn parallel to Tr itself, & the angle ΥgV is taken equal to the ΥQG, this will be the angle ΥgV equal to the longitude of the comet seen from the place g ; & the angle TVg will be the parallax, which arises from the translation of the earth from the place g to the place T: & hence V will be the position of the comet in the plane of the ecliptic. Moreover this place V is accustomed to be within the orbit of Jupiter.

Likewise the paths of comets is deduced from the curvature. These bodies go almost in large circles while they travel with their greatest speed ; but at the end of their course, when that part of the apparent motion which arises from parallax, has a greater proportion to the whole apparent motion, they are accustomed to be deflected from these circles, & just as often as the earth is moving in one direction, to go off in the opposite direction. This deflection arises mainly from the parallax, because that corresponds to the motion of the earth ; & the amount of this is signified, by my calculation, to have deduced the disappearance of comets well enough far beyond Jupiter. From which it follows that in the perigees & perihelions, when they are present closer, often fall within the orbit of Mars & of the inner planets.

Also the nearness of comets is confirmed by the light of the heads. For the light of a heavenly body illuminated by the sun, & going off into distant regions, is diminished in splendour in the square of the distance : clearly in the square ratio on account of the increase of distance from the sun, & in another square ratio on account of the diminution of the apparent diameter. From which if the apparent diameter & the quantity of light of the comet is given, the distance of the comet will be given, by saying that the distance will be to the distance of a planet, in the ratio of the diameter [of the comet] to the diameter of the planet directly & inversely in the inverse square root ratio of the light [of the comet] to the light of the planet. Thus if the smallest diameter of the hairs of the comet of the year 1682, observed by *Flamsted* with an optical tube 16 feet long & measured by a micrometer, was equal to $2'.0''$; but the nucleus or the star in the middle of the head was occupying scarcely the tenth part of this width, & thus the width was only 11" or 12". Truly with the light & clarity of the head it exceeded the head of the comet of the year 1680, & was emulating a star of the first or second order magnitude. We may consider Saturn & its ring to be as if four times brighter : & because the light of the ring will be in the same manner equal to the light of the globe in the centre, & the apparent diameter of the globe shall be as if 21", & thus the light of the globe & of the ring jointly will be equal to the light of a globe, the diameter of which shall be 30": the distance of the comet to the distance of Saturn will be as 1 to $\sqrt{4}$ inversely, & as 12" to 30" directly, that is, as 24 to 30 or 4 to 5. Again the comet of the year 1665 in the month of April, as *Helvelius* is the authority, by its clarity surpassed almost all the fixed stars, even Saturn, on account of the vividness of its colour for a long while. Certainly this comet was brighter than that other one, which had appeared at the end of the preceding year, & might be compared with the stars of the first magnitude. The width of the hairs was around 6', but the nucleus compared with the planets with the aid of a telescope clearly was less than Jupiter, & now smaller than the intermediate body of Saturn, & at other times judged to be equal. Again since the diameter of the hairs of the comet rarely exceeded 8' or 12', truly the diameter of the nucleus, or of the central star shall be around a tenth or perhaps a fifteenth part of the diameter of the hairs, it is apparent that these stars & most of the same kind are of the same apparent magnitudes as the planets. From which since the light of these can often be compared with the light of Saturn, & which may exceed that a little ; it is evident, that all comets taken together at the perihelion are either below Saturn, or not much more. Therefore, those who relegate comets to the region of the fixed stars err completely: by which account certainly comets would be illuminated no more by our sun, than the planets, which are here, can be illuminated by the fixed stars.

We have discussed these things without considering the obscurity of comets through all that most dense & abundant smoke, by which the head is surrounded, as if it were always shining through a dense cloud. For as much as the body is rendered obscure by this smoke, so it is necessary that it must approach much closer to the sun, so that the abundance of light reflected from them emulates the planets. Thence it appears to be the case that comets fall far below Saturn's globe, just as we have shown from parallax. The same indeed as may be confirmed fully from the tails; these arise either from reflection from the smoke scattered through the ether, or from the light of the head. In the first case it is to be lessened with the distance of the comets, lest the smoke from the head always arises & by expanding may be propagated with an incredible speed through an exceedingly great distance, in the latter case all the light for both the tails as well as the hairs are to be attributed to the nucleus of the head. [It is of course the solar wind that is responsible for the tail.] Therefore if we may consider all this light to be gathered together & to be encircled by the disc of the nucleus, certainly that nucleus now, whenever it sends off a great & most brilliant tail, will exceed the splendour of Jupiter itself many times. Therefore with an apparent smaller diameter emitting more light, the comet must be much more illuminated by the sun, & thus much closer to the sun. Furthermore, when their head is hidden under the sun, & with great smoking tails sometimes emitted in the image of tree trunks on fire, by the same argument such comets must be deduced to be within the orbit of Venus. For all that light if it be supposed to be gathered together in a star, I may say that it may surpass Venus itself & lest whenever several Venuses are in conjunction.

Finally the same may be deduced from the increasing light of the heads of comets receding from the earth towards the sun, & in the decrease in their recession from the sun. For since the comet at the end of the year 1665 (from the observations of *Hevelius*) from when it began to be seen, always maintained its apparent motion, & thus went beyond perigee ; truly the splendour of the head likewise was increasing from day to day, until it ceased to be evident on account of being hidden by the rays of the sun. The comet of the year 1683 (from the observations of the same *Hevelius*) at the end of the month of July, when first it was evident, was moving most slowly, completing around 40 or 45 minutes of arc in its orbit. From that time its diurnal motion always increased as far as Sept. 4. when it became around five degrees. Therefore in this whole time the comet was approaching the earth. Which was also deduced from the diameter of the head measured with a micrometer : evidently as Hevelius found on Aug. 6. to be as much as 6'. 5" with the hair included, but on Sept. 2 to be 9'. 7". Therefore the head appeared less long than at the end of the motion, but yet initially in the vicinity of the sun it was more visible in length than around the end, as the same *Hevelius* reports. Hence in this whole time, the light decreased on account of its receding from the sun, not withstanding its approach to the earth. The comet of the year 1618 about the middle of the month of December, & that one of the year 1680 around the end of the same month, were moving with great speed, & thus soon they were in perigees. The true maximum splendour of the heads was seen almost two weeks before, but only when they emerged from the rays of the sun, & the maximum splendour of the tails a little before, in the near vicinity of the sun. The head of the first comet, beside the observations *Cysat* on December 1, was seen to be greater than stars of the first order magnitude, & on December 16. (now present in perigees) with a small magnitude, with the splendour or clarity of the light a great deal reduced. On Jan. 7, with the head uncertain, *Kepler* finished his observations.

On the 12th day of December *Flamsteed* observed & noted the head of the latter comet at a separation of 9 degrees from the sun ; it was conceded to be a star of scarcely the third order magnitude. On December the 15th & 17th is appeared the same as a star of the third order magnitude, certainly diminished compared to the splendour of the clouds of the setting sun. December 26th , the day of the greatest motion, with that present near the perigee, it dropped to the mouth of Pegasis, a star of the third magnitude. Jan. 3rd it appeared as a star of the fourth magnitude, Jan. 9th as a star of the fifth, Jan.13th it disappeared on account of the brilliance of the rising moon. Jan. 25. scarcely being equal to a star of the seventh magnitude.

Thence if the times are taken equally from the perigee, then the heads put in more distant regions at these times [from the earth], on account of the equal distances from the earth, ought to shine equally with maximum brilliance; but it is of maximum brightness in the direction of the sun, & nearly disappearing from the other side of the perigee. Therefore from the great difference of the light in each situation, it must be concluded that the maximum light comes from the comet & the sun nearby in the first situation. For the light of comets is accustomed to be regulated, & the maxima to appear when the heads are moving the fastest, & thus they are at the perigees; unless perhaps that is greater in the vicinity of the sun.

Corol. 1. Therefore comets shine by the light of the sun reflected from them.

Corol. 2. Also from what has been said it is understood why comets frequent the region of the sun so much. If they may be discerned in regions far beyond Saturn, often they must appear on opposite sides of the sun. Indeed they must be closer to the earth, which is present in these parts; & the sun interposed obscures the rest. Truly by running through the history of comets, I have found that four or five times more have been detected in the hemisphere towards the sun, than in the opposite hemisphere, besides, without doubt a few others, that the light of the sun has concealed. Without doubt in the descent to our regions neither do they emit tails, nor thus are they illuminated by the sun, as required so that they may be found in the first place by the naked eye, as they shall be closer to Jupiter itself. But of the space described in the small interval around the sun, the greater length is situated on the side of the earth, which looks at the sun ; & in that greater part comets are usually illuminated more by the sun, as they are very much closer.

Corol. 3. Hence also it is evident, that the heavens are free from resistance. For comets have followed oblique paths sometimes contrary to the course of the planets, are moving most freely in all cases, & their motion, even considered for a long time contrary to the course of the planets. I am mistaken if they shall not be bodies of the same kind as planets, & may be moving in orbit, returning perpetually. For as writers wish to regard comets as some kind of meteors, the argument following from the constantly changing heads, may be seen to be without foundation. The heads of comets are surrounded by huge atmospheres ; & the atmospheres below must be denser. From which it is from these clouds, not from the bodies of comets themselves, that these changes are seen. Thus the earth if it may be viewed from the planets, without doubt brilliant from the light of its clouds, & the firm body beneath the clouds almost hidden. Thus belts have been formed by the clouds of the Jovian planet, in which the positions may change among themselves, & the firm body of Jupiter is discerned with difficulty by these clouds. And the bodies of comets must be buried under both much more deep & thicker atmospheres.

PROPOSITION XL. THEOREM XX.

Comets move in conic sections having foci in the centre of the sun, & with rays drawn to the sun describe areas proportional to the times.

This is apparent by Corol. 1. Prop. X11I. Book I, together with Prop. V11I, X11, & X11I. of Book 11I.

Corol. 1. Hence if comets return in orbit; the orbits will be ellipses, & the periodic times will be to the periodic times of the planets in the three on two ratio of the principle axes. And thus comets are present for the most part beyond the planets, & by that account describing orbits with greater axes, revolving slower. So that if the axis of the orbit of a comet shall be four times greater than the orbit of Saturn, the time of revolution of the comet shall be to the time of revolution of Saturn, that is, to 30 years, as $4\sqrt{4}$ (or 8) to 1, & thus it will be of 240 years.

Corol 2. But the orbits will be thus so close to parabolas, that parabolas may be taken in turn without sensible error.

Corol. 3. And therefore (by Corol. 7, Prop. XVI, Book 1.) the velocities of all comets, will be to the velocity of any planet revolving in a circle around the sun, in the square root ratio of twice the distance of the planet from the centre of the sun to the distance of the comet from the centre of the sun approximately. We may consider the radius of the great orbit, or the maximum radius of the ellipse in which the earth is revolving to be of 100000000 parts: & the earth from its daily motion describes on average 1720212 parts, & in its hourly motion $71675\frac{1}{2}$ parts. And thus a comet at the same mean distance from the sun, with that velocity which shall be to the velocity of the earth as $\sqrt{2}$ to 1, describes in its daily motion 2432747 parts, & in the hourly motion $101364\frac{1}{2}$ parts. But in the greater or lesser distances, the daily as well as the hourly motion will be inversely in the square root ratio of the distances, & thus is given.

[Chandrasekhar points out the formula on page 480 from which this constant is derived, correct to 6 places, which is now commonly called the *Gaussian constant of gravitation*.]

Corol. 4. From which if the major latus rectum of the parabola shall be four times the radius of the great orbit, & the square of that radius is put to be of 100000000 : the area that will be described by the radius drawn from the comet to the sun in single days will be of $1216373\frac{1}{2}$ parts, & the area of the individual hours will be of $50682\frac{3}{4}$ parts. But the major latus rectum may be either less or greater in some ratio, the diurnal area & hourly areas will be greater or less inversely in the square root of that ratio.

LEMMA V.
To find the curved line of the parabola generated, that will pass through any number of given points.

[This is Newton's theory of interpolation. An excellent article on this can be found by Duncan C. Fraser, *Newton & Interpolation*, in the commemorative volume published in 1927 for the *Mathematical Gazette* by Bell & Co. This has subsequently been set out by Chandrasekhar in his book, *Newton's Principia for the Common Reader*, p.481 (Oxford).]

Let these be the points *A, B, C, D, E, F*, etc, and from the same send some number of perpendiculars *AH, BI, CK, DL, EM, FN* to some right line in the given position *HN*.

Case. 1. If the intervals *HI, IK, KL* of the points *H, I, K, L, M, N* shall be equal, gather together the first differences b, $2b$, $3b$, $4b$, $5b$, of the perpendiculars *AH, BI, CK*, and the second differences c, $2c$, $3c$, $4c$; the third d, $2d$, $3d$; that is, so that thus there shall be

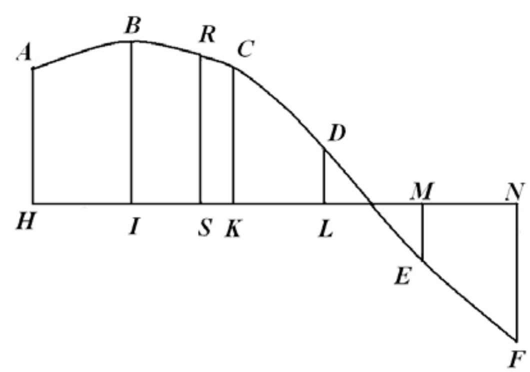
$b \quad 2b \quad 3b \quad 4b \quad 5b$

$c \quad 2c \quad 3c \quad 4c$

$d \quad 2d \quad 3d$

$e \quad 2e$

f



$AH - BI = b,\ BI - CK = 2b, CK - DL = 3b,\ DL + EM = 4b,\ -EM + FN = 5b$; then $b - 2b = c$, thus it may go on to the last difference, which is f.
Then erect some perpendicular *RS*, which shall be the applied ordinate to the curve sought : in order that the length of this may be found, put the intervals *HI, IK, KL, LM,* etc equal to one, & call

$$AH = a, -HS = p\ , \tfrac{1}{2}p \times -IS = q, \tfrac{1}{3}q \times +SK = r, \tfrac{1}{4}r \times +SL = s, \tfrac{1}{5}s \times +SM = t\,;$$

but it is clear that by proceeding as far as to the second last perpendicular *ME*, & by putting negative signs in front of the terms *HS, IS*, which lie on the one side of the point *S* towards *A*, & with a positive sign for the terms *SK, SL*, etc which lie towards the other direction of the point *S*. And with the signs observed properly, there will be
$RS = a + bp + cq + dr + es + ft$,

Case 2. However if the intervals *HI, IK*, etc of the points *H, I, K, L*, etc shall be unequal, take the first differences b, $2b$, $3b$, $4b$, $5b$ etc to be the difference of the perpendiculars *AH, BI, CK*, etc , divided by the intervals of the perpendiculars ; the second differences *c*, 2*c*, 3*c*, 4*c*, etc to be the previous differences divided by the interval between every two intervals; the third *d*, 2*d*, 3*d*, etc those divided by the interval between every three intervals; the fourth *e*, 2*e*, the third divided by every four intervals, & thus henceforth; that is, so that there shall be thus :

$b = \frac{AH-BI}{HI}, 2b = \frac{BI-CK}{IK}, 3b = \frac{CK-DL}{KL}$, & then $c = \frac{b-2b}{HK}, 2c = \frac{2b-3b}{IL}, 3c = \frac{3b-4b}{KM}, \&c.$

then again, $d = \frac{c-2c}{HL}, 2d = \frac{2c-3c}{IM},$

With the differences found call

$$AH = a, -HS = p,\ p\times -IS = q,\ q\times +SK = r,\ r\times +SL = s,\ s\times +SM = t\ ;$$

clearly by proceeding as far as the penultimum perpendicular *ME*, & the applied ordinate [*i.e.* the *y* coordinate] $RS = a + bp + cq + dr + es + ft$,

Corol. Hence the areas of all curves can be found approximately. For if from some points of any curve are found of which the quadrature is required, & a parabola is understood to be drawn through the same points : the area of this same parabola will be approximately equal to the quadrature sought. Moreover the quadrature of the parabola can always be found geometrically by very well–known methods.

LEMMA VI.

By observing some number of places of a comet to find its place at some given intermediate time.

In fig. preceding figure, *HI, IK, KL, LM* may designate the times between observations, *HA, IB, KC, LD, ME* are five observations of the longitude of the comet, *HS* the given time between the first observation & the longitude sought. And if the regular curve *ABCDE* is understood to be drawn through the points *A, B, C, D, E* ; then by the above lemma, its applied ordinate *RS* can be found, & *RS* is the longitude sought.

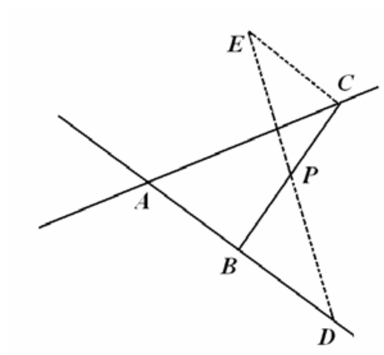
By the same method from five latitudes observed the latitude at a given time can be found.

If the differences of observed longitudes shall be small, for example only of 4 or 5 degrees ; three or four observations will suffice to find a new longitude & latitude. But if the differences shall be greater, for example of 10 or 20 degrees, five observations must be used.

LEMMA V11.

To draw a right line BC through a given point P, the parts of which PB, PC have a given ratio in turn, with the two given positions of the abscissa AB, AC.

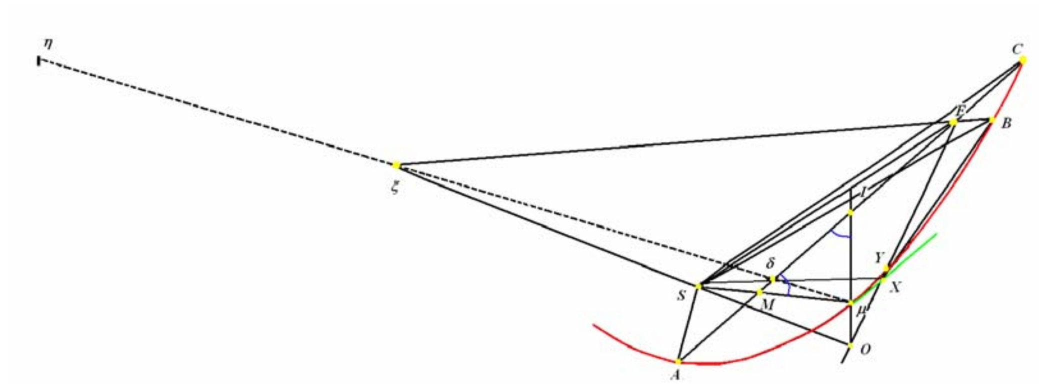
From that point *P* to one of the lines *AB* some line *PD* is drawn, & the same is produced towards the other right line *AC* as far as to *E*, so that *PE* shall be to *PD* in that given ratio; *EC* shall be drawn parallel to *AD* itself ; & if *CPB*, is drawn, *PC* will be to *PB* as *PE* to *PD*. *Q.E.F.*



LEMMA V11I.

Let ABC be a parabola having the focus S. The chord AC, bisected in I, cuts the segment ABCI, of which the diameter shall be $I\mu$ and the vertex μ. On $I\mu$ produced there is taken μO equal to half of $I\mu$. OS is joined, & that is produced to ξ, so that $S\xi$ shall be equal to 2SO. And if the comet B is moving in the arc CBA, & ξB is drawn cutting AC in E: I say that the point E will cut a segment AE from the chord AC, approximately proportional to the time.

[From the construction, $\mu O = \frac{1}{2} I\mu = \frac{1}{3} IO$, and $SO = \frac{1}{3} O\xi$.]



For *EO* is joined cutting the parabolic arc *ABC* in *Y* & μX may be drawn, which is a tangent to the same arc at the vertex μ, & crossing *EO* in *X*; & therefore the curvilinear area $AEX\mu A$ will be to the curvilinear area $AEY\mu A$ as *AE* to *AC*. And therefore since the triangle *ASE* shall be in the same ratio to the triangle *ASC* as *AE* to *AC*, the total area $ASEX\mu A$ will be to the total area $ASCY\mu A$ as *AE* to *AC*. But since, ξO shall be to *SO* as 3 to 1, & *EO* to *XO* shall be in the same ratio, *SX* will be parallel to *EB* : & therefore if *BX* is joined, the triangle *SEB* will be equal to the triangle *XEB*. From which if the triangle *EXB* is added to the area $ASEX\mu A$, & from the sum the triangle *SEB* is taken, there will remain the area $ASBX\mu A$ equal to the area $ASEX\mu A$, & thus to the area $ASCY\mu A$ as *AE* to *AC*. But the area $ASBY\mu A$ is approximately equal to the area $ASBX\mu A$, & this area $ASBY\mu A$ is to the area $ASCY\mu A$, as the time of the arc described *AE* to the time of the whole arc described *AC*. And thus *AE* is to *AC* in the approximate ratio of the times. *Q. E. D*.

[Chandrasekhar had trouble with this proof on account of the now obscure theorem, (the basis of which *L* & *S* attribute to *Archimedes* : *de Parabola* , see Heath, *The Works of Archimedes*, *Quadrature of the Parabola*, Prop. XXIV & prior to this, available in Dover), & the poor quality of the diagram, which he has taken the trouble to have re–drawn, & he has finally tracked down a more modern proof in Salmon's *Treatise on Conic Sections*, p.372 in the 6th edition. I have added some colours to the diagram, which is almost completely illegible in the original *Principia*, & which is hard to see even in Cohen's redrawn version, though of course the idea behind it is readily understood, when everything else is tidied up. Thus, in Newton's day, the works of Archimedes were generally well-known, so he felt disinclined to elaborate on a result that presumably was obvious to him.

However, here is the note produced by L. & J., showing that $ASBY\mu A : ASCY\mu A = AE : AC$:

Since the chord AC has been bisected in *I*, the semi-segment $A\mu I = \mu IC$. Likewise because μX is a tangent to the parabola at μ, μX will be parallel to the chord *AC* (by Lem. IV, *de Conics*, *Apoll*., Book I.), & hence the triangle *OIE* is similar to the triangle $O\mu X$, & thus, on account of $IO = 3\times \mu O$, $\Delta IOE = 9\times \Delta\mu OX$ and $\Delta IOE = \frac{9}{8}\times \text{trap.}I\mu XE$. Therefore the triangle $\Delta IAO : \Delta IA\mu = \frac{3}{2}$, on omitting lines in the diagram to avoid confusion, since likewise *A* shall be a vertex of each triangle, & the base *OI* shall be $\frac{3}{2}\times \mu I$; truly $\Delta A\mu I = \frac{3}{4}\times\text{semi-segment } A\mu I$, from Prop. XXIV of Archimedes quoted above. Whereby $\Delta AOI = \frac{9}{8}\times\text{semi-segment } A\mu I$, that is, in the ratio composed from $\frac{3}{2}\times\frac{3}{4}$, & hence $\Delta AOI : \text{semi-segment } A\mu I = \frac{9}{8} = \Delta IOE : \text{trapezium } \mu XEI$, just as in turn we have the trapezium $\mu XIE : \text{semi-segment } A\mu I = IE : AI$, & hence on compounding, since the curvilinear area $A\mu XE = \text{half-segment } A\mu I + \text{trap.}\mu XEI$, then the curvilinear area $A\mu XE : \text{semi-segment } A\mu I = AE : AI$, & hence the curvilinear area $A\mu XE : \text{total segment } A\mu C = AE : AC$.]

Corol. When the point *B* falls upon the vertex μ of the parabola, *AE* is to *AC* in the accurate ratio of the times.

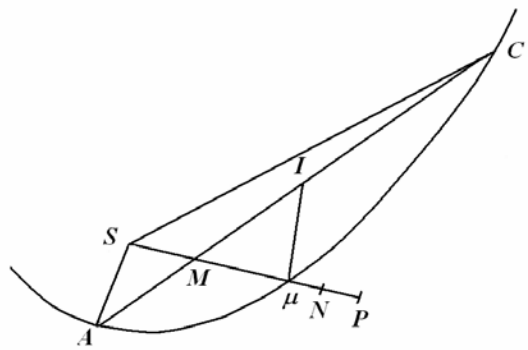
Scholium.

If $\mu\xi$ is joined cutting *AC* in δ, & on that there is taken ξn, which shall be to μB as $27MI$ to $16M\mu$: with *Bn* drawn cutting the chord *AC* in the ratio of the times more accurately than before. But the point *n* may be placed beyond the point ξ, if the point *B* is more distant from the principle vertex of the parabola than the point μ; & closer, if the point *B* is at a lesser distance from the same vertex.

LEMMA IX.
The right lines $I\mu$, μM, and the length $\frac{AI\times IC}{4S\mu}$ are equal to each other.

For $4S\mu$ is the latus rectum of the parabola pertaining to the vertex μ.

LEMMA X.
If $S\mu$ is produced to N & P, so that μN shall be the third part of μI, & SP shall be



to SN as SN to $S\mu$. A comet, in which time it will describe the arc $A\mu C$, if it may be progressing in that time with the velocity that it has at the altitude equal to SP, describes a length equal to the chord AC.

For if the comet with the velocity that it has at μ, may be progressing in the same time uniformly in a right line, which will be a tangent to the parabola at μ; the area, that it will describe by a radius drawn to the point *S*, shall be equal to the area of the parabola $ASC\mu$. & thus the product formed from the length described by the tangent & the length $S\mu$ shall be to the product formed from the lengths *AC* & *SM*, as the area $ASC\mu$ to the triangle *ASC*; that is, as *SN* to *SM*. Whereby *AC* is to the length of the tangent described, as $S\mu$ to *SN*. But since the velocity of the comet at the altitude *SP* [*i.e.* distance from the sun at *S*] shall be (by Corol. 6, Prop. XVI, Book I.) to its velocity at the altitude $S\mu$ inversely in the square root ratio of *SP* to $S\mu$, that is, in the ratio $S\mu$ to *SN*; the length described with this velocity in the same time will be to the length described in the tangent, as $S\mu$ to *SN*. Therefore *AC* & the length described with this new velocity, since they shall be in the same ratio to the length described by the tangent, are equal to each other. *Q.E.D.*

Corol. Therefore a comet with that velocity, that it has at the altitude $S\mu + \frac{2}{3}I\mu$, will describe the chord *AC* in approximately the same time.

LEMMA XI.

If a comet deprived of all its motion may be dropped from the altitude SN or $S\mu + \frac{2}{3} I\mu$, so shat it falls into the sun, & it is continued always to be acted on by force, by which it was acted on from the beginning; the same in half the time, in which it described the arc AC in its orbit, will describe a distance equal to the length $I\mu$ in its descent.

For the comet, in the time during which it describes the arc of the parabola *AC,* will describe the chord *AC* with that velocity which it did from the altitude *SP* (by the most recent lemma), & thus (by corol. 7, Prop. XVI, Book I.) in the same time in a circle, its diameter would be *SP*, & by the force of gravity by its revolving, would describe an arc, the length of which would be to the parabolic chord of the arc *AC*, in the square root ratio of 1 to 2. And therefore with that weight, which it had at the altitude *SP* to the sun, by falling from that altitude into the sun, describe in half of that time (by corol. 9, Prop. IV, Book I.) a distance equal to the square of half the chord applied to the square of the altitude *SP*, that is, the distance $\frac{AI^2}{4SP}$. From which since the weight of the comet towards the sun at the altitude *SN* shall be to its weight towards the sun at the altitude *SP*, as *SP* to $S\mu$: the comet with the weight that is has at the altitude *SN*, by falling towards the sun in the same time, describes the interval $\frac{AI^2}{4S\mu}$, that is, a distance equal to the length $I\mu$ or $M\mu$. *Q.E.D.*

PROPOSITION XLI. PROBLEM XXI.

To determine the trajectory of a comet moving in a parabola from three given observations.

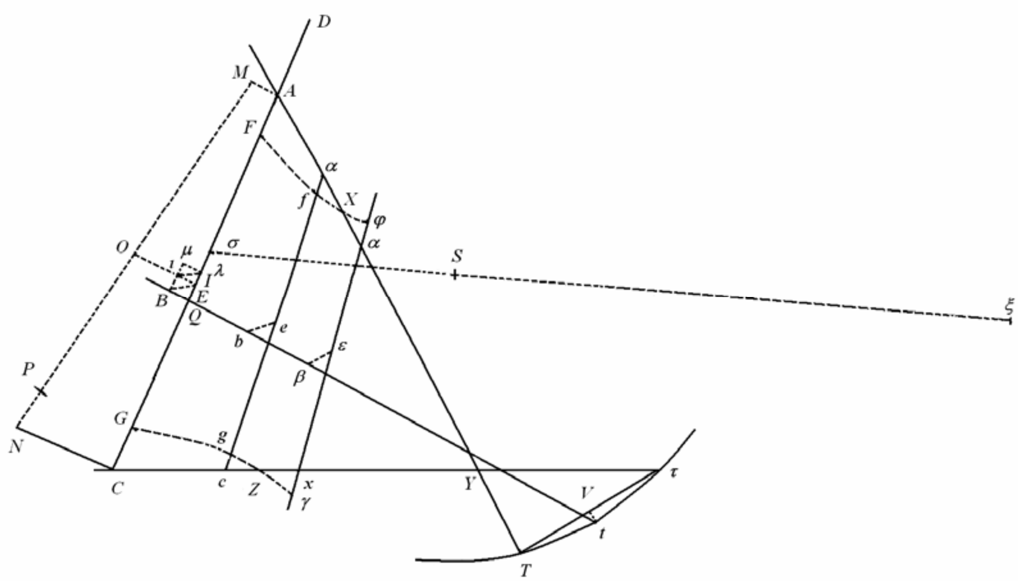
This most difficult problem has been attacked for a long time in many ways, & I have set out certain problems in Book I, which are concerned with its solution. Afterwards I have thought out the following simpler solution.

Three observations are selected with equal time intervals from the approximate intervals one after the other. Moreover let that time interval of the time, when the comet is moving the slowest, be a little greater than the other, thus so that it is apparent that the difference of the times shall be to the sum of the times, as the sum of the times to around 600 days ; or so that the point *E* (in the figure of Lemma VIII.) may fall nearly on the point *M*, & thence it wanders towards *I* rather than towards *A*. If such observations shall not be on hand, the place of a new comet can be found by Lemma VI.

Let *S* designate the sun, T, t, τ three places of the earth in its great orbit ; *TA, tB*, τC three observations of the longitudes of the comet ; *V* the time passing between the observations of the first & the second, *W* the time passing between the second & the third ; *X* the longitude, that the comet in that whole time may be able to describe, with that velocity that it has at the mean earth to sun position, & which (by Corol. 3, Prop. XL, Book 11I) is required to be found, & *tV* the perpendicular to the chord $T\tau$.

In the middle longitude observed *tB* , some point *B* is taken as the location of the comet in the plane of the ecliptic, & thence the line *BE* is drawn towards the sun *S*, which shall be to the sagittam *tV*, as the product $SB \times St^2$ to the cube of the hypotenuse of the right angled triangle, the sides of this are *SB* & the tangent of the latitude of the comet in the second observation for the radius *tB*. And the right line *AEC* is drawn through the point *E* (by Lemma V11 of this section), the parts of which *AE, EC* terminated by the lines *TA* & τC , shall be in turn as the times *V* & *W*: & both *A* & *C* will be locations of the comet in the plane of the ecliptic approximately in the first & third observations, but only if *B* shall be the place correctly assumed in the second observation.

To *AC* bisected at *I* erect the perpendicular *Ii*. Through the point *B* draw the hidden line *Bi* parallel to *AC* itself. Join the hidden line *Si* cutting *AC* in λ , & completer the



parallelogram $iI\lambda p$. Take $I\sigma$ equal to $3I\lambda$, & through the sun *S* draw the hidden line $\sigma\xi$ equal to $3S\sigma + 3i\lambda$. And now with the letters *A, E, C,* & *I* deleted ; from the point *B* towards the point ξ draw a new hidden line *BE*, which shall be to the first *BE* in the square ratio of the distance *BS* to the quantity $S\mu + \frac{1}{3}i\lambda$. And through the point *E* again draw the right line *AEC* from the same rule as before, that is, so that the parts of this *AE* & *EC* shall be in turn, as the times between the observations *V* & *W*; both *A* & *C* will be more accurate places of the comet.

The perpendiculars *AM, CN, IO* are erected to *AC* bisected at *I*, of which *AM* & *CN* shall be the tangents of the latitudes in the first & third observations to the radii *TA* & *TC* ; *MN* is joined cutting *IO* in *O* ; the rectangle $iI\lambda\mu$ is put in place as before. In *IA* produced *ID* is taken equal to $S\mu + \frac{2}{3}i\lambda$. Then on *MN*, *MP* is taken towards *N*, which shall be to the above longitude found *X*, in the square root ratio of the mean distance of the earth from the sun (or the radius of the great orbit) to the distance *OD*. If the point *P* falls on the point *N*; *A, B & C* will be three places of the comet, through which it must describe its orbit in the plane of the ecliptic. But if the point *P* is not incident in the point *N*; then on the right line *AC* there is taken *CG* equal to *NP*, thus so that the points *G* & *P* may lie in the same direction as the right line *NC*.

By the same method, by which the points *E, A, C, & G* have been found from the assumed point *B*, with whatever other assumed points *b & G,* new points *e, a, c, g*, & $\varepsilon, \alpha, \chi, \gamma$ may be found. Then if through *G, g,* γ the circumference of the circle $Gg\gamma$ may be drawn, cutting the right line τC in *Z*: *Z* will be the position of the comet in the plane of the ecliptic. And if on *AC, ac,* $\alpha\chi$, the right lines $AF, af, \alpha\varphi$ are taken respectively equal to $CG, cg, \chi\gamma$, & through the points F, f, φ the circumference of the circle $Ff\varphi$ is drawn, cutting the right line *AT* in *X* ; *X* will be another position of the comet in the plane of the ecliptic. To the points *X* & *Z* the tangents of the latitudes of the comet may be erected to the radii *TX* & *TZ*; & two places of the comet will be found in its proper orbit. Finally, (by Prop. XIX, Book I.) with the focus *S*, a parabola is described through two places, & this will be the trajectory of the comet. *Q. E. I.*

The demonstration of this construction follows from the lemmas : certainly since the right line *AC* may be cut at *E* in the ratio of the times, by Lemma V11, as required by Lemma V11I ; & *BE* by Lemma XI would be the part of the right line *BS* or $B\xi$ in the plane of the ecliptic, put in place between the arc *ABC* & the chord *AEC* ; & *MP* (by Corol. Lem. X.) would be the length of the chord of the arc, that the comet in its proper orbit would describe between its first & third observations, & thus it should be equal to *MN*, but only if *B* were a true place of the comet in the plane of the ecliptic.

The remainder of the points *B, b,* β are not taken as you wish, for it is agreed to choose them near each other. If the angle *AQt* may be known roughly, in which projection of the orbit described in the plane of the ecliptic cuts the line *tB* ; in that angle it will be necessary to draw the hidden line *AC*, which shall be to $\frac{4}{3}T\tau$ in the square root ratio of *SQ* to *St*. And with the right line *SEB* acting, the part of which *EB* is equal to the length *Vt*, the point *B* will be determined that it was allowed to use in the first place. Then with the right line *AC* deleted & with the following preceding construction drawn again, & with the above length *MP* found ; on *tB* the point *b* is taken, by that rule, so that if *TA, TC* shall mutually cut each other in *T*, the distance *Tb* shall be to the distance *TB*, in the ratio composed from the ratio *MP* & *MN* & the ratio of the square root of *SB* to *Sb*. And by the same method the third point β can be found but only if there is a desire to repeat the operation a third time. Moreover by this method the two operations at most will suffice. For if the distance *Bb* becomes very small, after the points *F, f* & *G, g* have been found the right lines *Ff* & *Gg* drawn will cut *TA* & τC in the points sought *X* & Z.

Example.

The comet of the year 1680 may be proposed. The motion of this has been observed by *Flamsted* , & from the observations the following table shows the computation, & with the observations from the same corrected by *Halley*.

	Apparent time in hours.	True time in hours.	Longitude of the sun.	Longitude of the comet.	Northern Latitude of the comet.
1680.*Dec*. 12	4.46'	4.46'. 0"	♑ 1^0.51'. 23"	♑ 6^0.32'.30"	8^0. 28'.0"
21	6.32$\frac{1}{2}$	6.36.59	11. 6. 44	♒ 5. 8 .12	21. 42.13
24	6.12	6.17.52	14. 9. 26	18.49. 23	25. 23. 5
26	5.14	5.20.44	16. 9. 22	28.24. 13	27. 0.52
29	7.55	8. 3. 2	19.19. 43	♓ 13.10. 41	28. 9.58
30	8.02	8.10.26	20. 21. 9	17.38. 20	26. 15. 7
I681.*Jan*. 5	5.51	6. 1.38	26. 22.18	♈ 8.48. 53	24. 11.56
9	6.49	7. 0.53	♒ 0. 29. 2	18.44. 4	23. 43.52
10	5.54	6. 6.10	1. 27.43	20.40. 50	16. 42.18
13	6.56	7. 8.55	4. 33.20	25.59. 48	16. 4. 1
25	7.44	7.58.42	16. 45.36	♉ 9.35. 0	17. 56.30
30	8.07	8.21.53	21. 49.58	13.19. 51	16. 42.18
Feb. 2	6.20	6.34.51	24. 46.59	15.13. 53	16. 4. 1
5	6.50	7. 4.41	27. 49.51	16.59. 6	15. 27. 3

		Longitude of sun.		Longitude of comet.		Latitude of Comet	
		M, E_1	$E_1a = E_2, E_3$	M, E_1, E_1a, E_2	E_3	M, E_1, E_2	E_3
1680.*Dec*.	12	♑ 1^0.53'. 2"	♑ 1^0.51'.23"	♑ 6^0.33'.0" $_{M, E_1}$ 6.31.21 $_{E_1aE_2}$	♑ 6^0.32'.30"	8^0. 26'.0"	8^0. 28'.0"
	21	11.8.10 [E_1] 11.8.10$\frac{1}{3}$ [M]	11. 6.44	♒ 5. 8 .12	♒ 5. 8 .12	21. 45.13	21. 42.13
	24	14.10.49	14. 9.26	18.49. 10	18.49. 23	25. 23.24	25. 23. 5
	26	16.10.38	16. 9.22	28.24. 6	28.24. 13	27. 0. 57	27. 0.52
	29	19.20.56	19.19.43	♓ 13.11. 45	♓ 13.10. 41	28. 10.05	28. 9.58
	30	20.22.20	20.21. 9	17.37. 5	17.38. 20	28. 11.12	28. 11.53
1682.*Jan*.	5	26.23.19	26.22.18	♈ 8.49. 10	♈ 8.48. 53	26. 15. 26	26. 15. 7
	9	♒0.29.54	♒0.29. 4	18.43. 18	18.44. 4	24. 12.42	24. 11.56
	10	1.28.34	1.27.43	20.40. 57	20.40. 50	23. 44. 0	23. 43.52
	13	4.34. 6	4.33.20	25.59. 34	25.59. 48	22. 17. 36	22. 17. 28
	25	16.45.58	16.45.36	♉ 9.55. 48 $_{M, E_1}$ 9.55. 48 $_{E_1aE_2}$	♉ 9.35. 0	17. 56. 54	17. 56.30
	30	21.50. 9	21.49.58	13.19. 36	13.19. 51	16. 40. 57	16. 42.18
Feb.	2	24.47. 4	24.46.59	15.13. 48	15.13. 53	16. 2. 2	16. 4. 1
	5	27.49.51	27.49.51	16.59. 52	16.59. 6	15. 27. 23	15. 27. 3

To these observations add several made myself :

	Apparent time.	Longitude of comet.	Northern latitude of comet
1682.*Feb*. 25	$8^h.30'$	♉ $26^0.18'.35''$	$12^0.46'.46''$
27	8. 15	27. 4 .30	12. 36. 12
Mar. 1	11. 0	27. 52. 42	12. 23. 40
2	8. 0	28. 12. 48	12. 19. 38
5	11.30	29. 18. 0	12. 3. 16
7	9.30	♊ 0. 4. 0	11. 57. 0
9	8.30	0. 43. 4	11. 45. 52

I have completed these observations with a seven foot telescope, & that I have carried out with a micrometer with threads located at the focus of the telescope : with which instruments we have determined the positions of the fixed stars amongst themselves & the position of the comet to these fixed stars.

	Longitude of comet			Latitude of comet		
	M, E_1	E_2	E_3	M, E_1	E_2	E_3
1682.*Feb*. 25	♉ $26^0.19'.22''$	♉ $26^0.18'.17''$	♉ $26^0.18'.35''$	$12^0.46\frac{7}{8}$	$12^0.46'\frac{7}{8}$	$12^0.46'.46''$
27	27. 4. 28	27. 4.24	27. 4.30	12. 36	$12.36\frac{1}{5}$	12. 36.12
Mar. 1	27.53. 8	27.53. 6	27.52.42	$12.24\frac{3}{7}$	$12.24\frac{6}{7}$	12. 23.40
2	28.12.29	28.12.27	28.12.48	$12.19\frac{1}{2}$	12. 20	12. 19.38
5	29.20.51	29.20.51	29.18. 0	$12.19\frac{2}{3}$	$12.3\frac{1}{2}$	12. 3.16
7			♊ 0. 4. 0			11. 57. 0
9	♊ 0. 43. 2	♊ 0. 43. 4	♊ 0. 43. 4	$11.44\frac{3}{5}$	$11.45\frac{7}{8}$	11. 45.52

A shall describe the star of the fourth magnitude in the left heel of *Perseus* (by *Bayer* o) *B* the following star of the third magnitude in the left foot (Bayer ζ) & *C* a star of the sixth magnitude (Bayer n) in the heel of the same foot, & *D, E, F, G, H, I, K, L, M, N, O, Z,* $\alpha, \beta, \gamma, \delta$ other lesser stars in the same foot. And *p, P, Q, R, S, T, V, X* shall be the places of the comet in the observations described above : & with the distance present *AB* of $80\frac{7}{12}$, & there was :

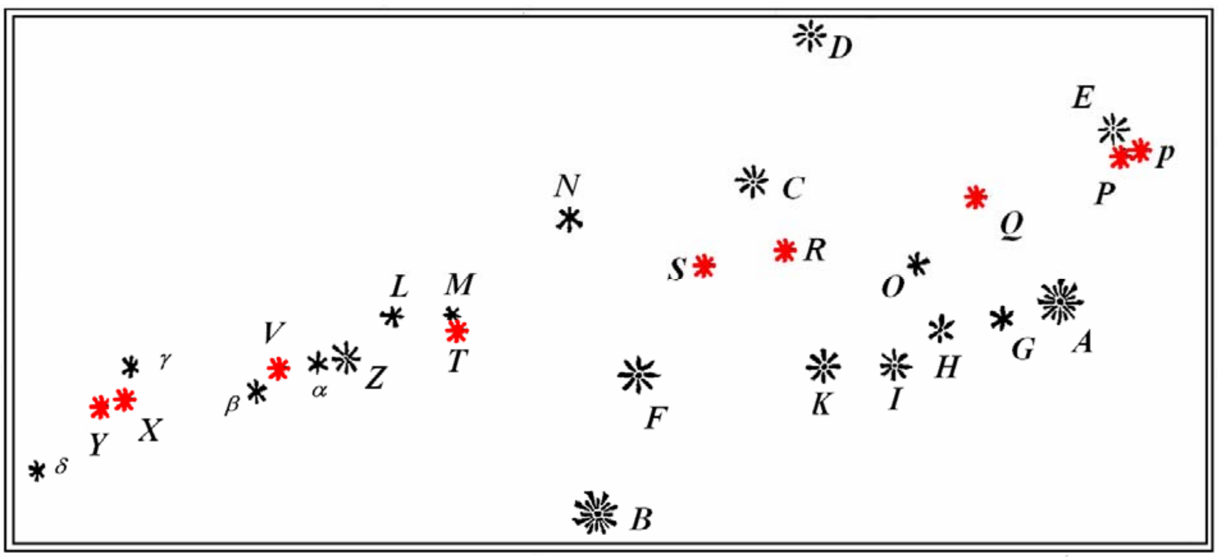
AC of the parts $52\frac{1}{4}$, $BC\ 58\frac{5}{6}$, $AD\ 57\frac{5}{12}$, $BD\ 82\frac{6}{11}$, $CD\ 23\frac{2}{3}$, $AE\ 29\frac{4}{7}$, $CE\ 57\frac{1}{2}$, $DE\ 49\frac{11}{12}$, $AI\ 27\frac{7}{12}$ $BI\ 52\frac{1}{6}$, $CI\ 36\frac{7}{12}$, $DI\ 53\frac{5}{12}$, $AK\ 38\frac{2}{3}$, $BK\ 43$, $CK\ 31\frac{5}{9}$, $FK\ 29$, $FB\ 23$, $FC\ 36\frac{1}{4}$, $AH\ 18\frac{4}{7}$, $DH\ 50\frac{7}{8}$, $BN\ 46\frac{5}{12}$, $CN\ 31\frac{1}{3}$, $BL\ 45\frac{5}{12}$, $NL\ 31\frac{5}{7}$;

HO was to *HI* as 7 to 6 & produced passed between the stars *D* & E, thus so that the distance of the star *D* from this right line was $\frac{1}{6}CD$. *LM* was to *LN* as 2 to 9, & produced passed through the star *H*. From these the positions of the fixed stars were determined among themselves.

Finally our countryman *Pound* in turn observed the positions of these fixed stars between themselves, & produced the longitudes & latitudes of these in the following table.

Fixed Stars	Longitude.	Latitude North.	Fixed Stars	Longitude.	Latitude North.
A	♉ 26. 41. 50	12. 8.36	*L*	♉ 29.33.34	12. 7.48
B	28 .40. 23	11.17.54	*M*	29.18.54	12. 7.20
C	27.58.30	12.40.25	*N*	28.48.29	12.31.91
E	26.27. 17	12.52. 7	*Z*	29.44.48	11.57.13
F	28.28.37	11.52.22	*a.*	29.52. 3	11.55.48
G	26.56 . 8	12. 4.58	β	11 0. 8.23	11.48.56
H	27.11 .45	12. 2. 1	γ	0.40.10	11.55.18
I	27.25. 2.	11.53.11	δ	1. 3.20	11. 30.42
K	27.42. 7	11.53.26			

Now I have observed the positions of the comet to these fixed stars as follows. On Friday *Feb*. 25. (old calendar) at 8.30 p.m. the distance of the comet present at *p* was a little less from the star *E* than $\frac{3}{13}AE$, & greater than $\frac{1}{5}AE$, & thus approximately equal to $\frac{3}{14}AE$; & the angle *ApE* was somewhat obtuse, but almost right. Certainly if a perpendicular is sent from *A* to *pE*, the distance of the comet from that perpendicular was $\frac{1}{5}pE$.



In the same night at the hour of 9.30 p.m., the distance of the comet present at *P* from the star *E* was greater than $\frac{1}{4\frac{1}{2}}AE$, & less than $\frac{1}{5\frac{1}{4}}AE$, & thus equal to $\frac{1}{4\frac{7}{8}}AE$, or approximately $\frac{8}{39}AE$. But with a perpendicular sent from the star *A* to the right line *PE* , the distance of the comet was $\frac{4}{5}PE$.

On Sunday, *Feb*. 27 at 8.15 p.m., the distance of the comet present at *Q* from the star *O* was equal to the distance of the stars *O* & *H*, & the right line *QO* produced passed between the stars *K* & *B*. The position of this right line on account of intervening clouds could not be defined more accurately.

On Tuesday, *March* 1 at 11 p.m., the comet was present at *R*, accurately placed between the stars *K* & *C* , & the part *CR* of the right line *CRK* a little greater than $\frac{2}{3}CK$, & a little less than $\frac{1}{3}CK+\frac{1}{8}CR$, & thus equal to $\frac{1}{3}CK+\frac{1}{16}CR$ or $\frac{16}{45}CK$.

On Wednesday, *March* 2, at 8 p.m. , the distance of the comet present at *S* was approximately $\frac{4}{9}FC$ from the star *C*. The distance of the star *F* from the right line *CS* produced was $\frac{1}{24}SC$; & the distance of the star *B* from the same right line, was five times greater than the distance from the star *F*. Likewise the right line *NS* produced passed between the stars *H* & *I*, being five or six times closer to the star *H* than to the star I.

On Saturday, *March* 5, at 11.30 p.m., with the comet present at *T*, & with the right line *MT* equal to $\frac{1}{2}ML$, & with the right line *LT* produced passing between *B* & *F* four or five times closer to *F* than *B*, taking from *BF* a fifth or sixth part of that towards *F*. And *MT* produced passed beyond the distance *BF* in the direction of the star *B*, being four times closer to the star *B* than the star *F*. Star *M* was exceedingly close which scarcely could be seen with the telescope, & *L* a star perhaps of magnitude greater than eight.

On Monday, *March* 7, at 9.30 p.m., with the comet present at *V*, with the right line $V\alpha$ produced, it passes between *B* & *F*, bearing off from *BF* towards *F* $\frac{1}{10}$ by *BF*, & it was to the right line $V\beta$ as 5 to 4. And the separation of the comet from the right line $\alpha\beta$ was $\frac{1}{2}V\beta$.

On Wednesday, *March* 9, at 8.30 p.m., with the comet present at *X*, the right line γX was equal to $\frac{1}{4}\gamma\delta$, & a perpendicular sent from the star δ to the right line γX was $\frac{2}{5}\gamma\delta$.

On the same night at midnight, the comet was present at *Y*, the right line γY was equal to $\frac{1}{3}\gamma\delta$, or a little less, perhaps $\frac{5}{16}\gamma\delta$, and a perpendicular sent from the star δ to the right line γY was equal to around $\frac{1}{6}\gamma\delta$ or $\frac{1}{7}\gamma\delta$. But the comet on account of the vicinity of the horizon was scarcely able to be seen, nor its place to be defined as distinctly as with the preceding.

And from the observations of this kind by constructing figures & doing computations I derived the longitudes & latitudes of the comet, & our countryman *Pound* from the correct locations of the fixed stars corrected the places of the comet, & the corrected places are given above. I used a micrometer constructed very poorly, but still the errors in the latitude & longitude scarcely exceeded one minute (as far as they arose from our observations). Moreover the comet (according to our observations) at the end of its motion began to be noticeably deflected towards the North, from the parallel that it had held at the end of February.

Now towards determining the orbit of the comet, I have selected three from the observations just described, which *Flamsted* had determined on *Dec*. 21, *Jan*. 5, & *Jan*. 25. From these I have found *St* of 9842,1 parts & *Vt* of 455 parts, such that 10000 is the radius of the great orbit. Then according to the first operation on assuming *tB* to be of 5657 parts, I have found $SB = 9747$, *BE* at first in turn 412, $S\mu = 9503$, $i\lambda = 413$: *BE* in the second case becomes 421, while ['=' signs have been introduced for convenience,] $OD = 10186, X = 8528,4\ MP = 8450, MN = 8475, NP = 25$. From which according to the second operation I have deduced the distance $tb = 5640$. And by this operation I have found at last that the distances $TX = 4775$ and $\tau Z = 11322$. From which by defining the orbit, I have found the descending node in ♋ 1^0. 53' & the ascending node in ♑ 1^0. 53'; the inclination of its plane to the plane of the ecliptic to be 61^0. 20'$\frac{1}{3}$; its vertex (or the perihelion of the comet) to stand at 8^0. 38' from the node, & to be in ♐ 27^0. 43' with the southern latitude $7^0.34'$; & its latus rectum to be 236,8, & the area described in single days by the radius drawn to the sun to be 93585, on putting 100000000 for the square of the radius of the great orbit ; truly the comet was progressing along the signs of the zodiac, & on the 8th day of *December* at 4 minutes past midday was in the vertex of its orbit or perihelion. All these things I have determined graphically from a scale of equal parts & the chords of the angles taken from a table of natural sines ; by constructing a large enough diagram, in which as it were the radius of the great orbit of 10000 parts was equal to $16\frac{1}{3}$ English inches.

	Comet to sun distance .	Long. deduced.	Lat. deduced.	Long.obs.	Lat.obs.	Differ. Long.	Differ. Lat.
Dec.12	2792	♑ $6^0.32'$	♑ $8^0.18'\frac{1}{2}$	$6^0.31'\frac{1}{3}$	$8^0.26'$	+1'	$-7'\frac{1}{2}$
29	8403	♓ $13.13\frac{2}{3}$	28. 0	♓ $13.11\frac{3}{4}$	$28.10\frac{1}{12}$	+2	$-10\frac{1}{12}$
Feb. 5	16669	♉ 17. 0	$15.29\frac{2}{3}$	♉ $16.59\frac{7}{8}$	$15.27\frac{2}{5}$	+0	$+2\frac{1}{4}$
Mar. 5	21737	$29.19\frac{3}{4}$	12. 4	$29.20\frac{6}{7}$	$12.3\frac{1}{2}$	–1	$+\frac{1}{2}$

Finally so that is could be agreed that the comet truly was moving in such an orbit found, I deduced partially through arithmetical operations & partially graphically the locations of the comet in its orbit according to the required times of observation : as can be seen in the following table.

Certainly after our countryman *Dr. Halley* had determined the orbit more accurately by arithmetical calculations than one could do graphically by described lines ; indeed the position of the nodes was retained in ♋ (*Cancer*) & ♑ (*Capricornus*) as $1^0.53'$, & the inclination of the plane of the orbit to the ecliptic $61^0.53'$, & so that the time of the perihelion of the comet was on the 8th day of *December* at 4 minutes past noon, truly the distance of the perihelion from the ascending node of the comet measured in the orbit of the comet was found to be $9^0.20'$, & the latus rectum or the parabola was of 2430 parts, with the mean distance of the earth from the sun being 100000 parts. And from these given, with an accurate arithmetical calculation put in place, the places of the comet computed at the times of observation were as follows.

True time	Comet sun distance.	Long.computed	Lat.computed	Differ. Long.	Differ. Lat.
Dec. $12^d\ 4^h.46'$	28028	♑ $6^0.29'.25''$	$8^0.26'.0''$ Bor.	$-3'.5''$	$-2'.0''$
21. 6.37	61076	♒ 5. 6.30	21.43.20	–1.42	+ 1. 7
24. 6.18	70008	18.48.20	25.22.40	–1. 3	– 0.25
26. 5.21	75576	28.22.45	27. 1.36	–1.28	+ 0.44
29. 8. 3	14021	♓ 13.12.40	28.10.10	+1.59	+ 0.12
30. 8.10	84021	17.40. 5	28.10.20	+1.45	– 0.33
Jan. 5. 6. $1\frac{1}{2}$	101440	♈ 8.49.49	26.15.15	+0.56	+ 0. 8
9. 7. 0	110959	18.44.36	24.12.54	+0.32	+ 0. 8
10. 6. 6	113162	20.41.02	23.44.10	+0.10	+0.18
13. 7. 9	120000	26. 0.21	22.17.30	+0·33	+0.25
25. 7.59	145370	♉ 9.33.40	17.57.55	–1.20	+ 1.25
30. 8.22	155303	13.17.41	16.42. 7	–2.10	–0.11
Feb. 2. 6.35	160951	15.11.11	16. 4.15	–2.42	+0.14
5. 7. $4\frac{1}{2}$	166686	16.58.25	15.29.13	–0.41	+2.10
25. 8.41	202570	26.15.46	12.48. 0	–2·49	+1.14
Mar. 5.11.39	216205	29.18.35	12. 5. 40	+ 0·35	+ 2.24

Also this comet appeared in the preceding month of *November* in *Coburg* in Saxony, & was observed by Mr. *Gottfried Kirch* in the days of the month I have shown, on the 4^{th}, the 6^{th} & on the 11^{th}, in the old style ; & from its position to nearby fixed stars observed first with a two foot long telescope & then more accurately with a ten foot telescope, & the difference of the longitudes of *Coburg* & *London* of 11^0 & with the places of the fixed stars observed by our countryman *Pound*, our *Halley* has determined the locations of the comet as follows.

On *November* 3^{rd} at $17^h.2'$, at the apparent time in *London*, comet was in ♌ $29^0.51'$ with latitude North $1^0.17'.45''$.

On *November* 5^{th} at $15^h.58'$, comet was in ♍ (*Virgo*) $3^0.\ 23'$ with latitude North $1^0.6'$.

On *November* 10^{th} at $16^h.31'$, the comet was equally distant from the stars of *Leo* σ and τ according to *Bayer* ; indeed it did not touch the right line joining these, but was a little away from that. In *Flamsteed's* catalogue of stars σ then had the position ♍ $14^0.15'$ with almost the latitude North $1^0.41'$, τ truly ♍$17^0.39'\frac{1}{4}$, with latitude North $0^0.34'$. And the mean point between these stars was ♍$15^0.39'\frac{1}{4}$ with latitude North $0^{gr}.\ 33\frac{1}{2}$. The distance of the comet from that right line shall be around 10' or 12', & the difference of the longitudes of the comet & the mean of these points was 7', & the difference of the latitudes around $7'\frac{1}{2}$. And thence comet was in ♍$15^0.32'$ with latitude North around 26'.

The first observation from the position of the comet to some small fixed stars was clearly accurate enough. The second also was accurate enough. In the third, which was less accurate, the error was under 6 or 7 minutes, & scarcely greater. Truly the longitude of the comet in the first observation, which was more accurate than the others, computed in the predicted parabola, was in ♌ (*Leo*) $29^0.30'.22''$ with a latitude of $1^0.\ 25'.7''$ North, & its distance from the sun 115546.

Again *Halley,* by observing that a conspicuous comet had appeared four times with an interval of 575 years, evidently in the month of *September* after the death of *Julius Caesar* ; in the year of *Christ* 531 under the consulate of *Lampadius* & *Orestes ;* in the year of *Christ* 1106 in the month of *February*, & during the end of the year 1680, & that with a long & conspicuous tail (except that during the death of *Caesar*, the tail was less apparent on account of the inconvenient position of the earth:) sought an elliptic orbit the major axis of which would be of 1382957 parts, with a mean distance of the earth from the sun being 10000 parts: certainly in which orbit the comet would be able to revolve in 575 years. And on putting the ascending node in ♋ at $2^0.\ 2'$; the inclination of the plane of the orbit to the plane of the ecliptic $61^0\ .\ 6'.48''$; the perihelion of the comet in this plane ♐ (*Sagittarius*) $22^0.\ 44'.\ 25''$;the equal time of the perihelion *December* $7^d.\ 23^h.\ 9'$; the distance of the perihelion from the ascending node in the plane of the ecliptic $9^0.\ 17'.\ 35''$; & the conjugate axis 184081,2: I have computed the motion of the comet in this elliptic orbit. Moreover its places in this orbit both deduced from observations as well as by computation are shown in the following table.

True time		Long. observed	Lat. N. obs.	Long.computed.	Lat.computed.	Differ. Long.	Differ. Lat.
Nov.	3.16.47	♌ 29^0.51'. 0"	1^0.17'. 45"	♌ 29^0.51'. 22"	1^0.17'. 32"	+ 0'.22"	– 0'.13"
	5.15.37	♍ 3.23. 0	1. 6. 0	♍ 3.24. 32	1. 6. 9	+ 1.32	+ 0. 9
	10.16.18	15.32. 0	0.27. 0	15.33. 2	0.25. 7	+ 1. 2	– 1.53
	16.17. 0	[no readings]	[no readings]	♎ 8.16.45	0.52. 7 A		
	18.21.34	"	"	18.52.15	1.26.54		
	20.17. 0	"	"	28.10.36	1.53.35		
	23.17. 5	"	"	♏ 13.22.42	2.29. 0		
Dec.	12^d 4^h.46'	♑ 6^0.32'.30"	8.28. 0	♑ 6.32.20	8.29. 6 B	– 1.10	+ 1. 6
	21. 6.37	♒ 5. 8.11	21.42.13	♒ 5. 6.14	21.44.42	– 1.58	+ 2.29
	24. 6.18	18.49.23	25.23. 5	18.47.30	25.23.35	– 1.53	+ 0.30
	26. 5.21	28.24.13	27. 0.52	28.21.42	27. 2. 1	– 2.31	+ 1. 9
	29. 8. 3	♓ 13.10.41	28. 9.58	♓ 13.11.14	28.10.38	+ 0.33	+ 0.40
	30. 8.10	17.38.20	28.11.53	17.38.27	28.11.38	+ 0. 7	– 0.16
Jan.	5. 6. 1$\frac{1}{2}$	♈ 8.48.53	26.15. 7	♈ 8.48.51	26.14.57	– 0. 2	– 0.10
	9. 7. 1	18.44. 4	24.11.56	18.43.51	24.12.17	– 0.13	+ 0.21
	10. 6. 6	20.40.50	23.43.32	20.40.23	23.43.25	+ 0.10	– 0. 7
	13. 7. 9	25.59.48	22.17.28	26. 0. 8	22.16.32	+ 0·20	– 0.56
	25. 7.59	♉ 9.35. 0	17.56.30	♉ 9.34.11	17.56. 6	– 0.49	– 0.24
	30. 8.22	13.19.51	16.42.18	13.18.28	16.40. 5	– 1.23	– 2.13
Feb.	2. 6.35	15.13.53	16. 4. 1	15.11.59	16. 2. 7	– 1.54	– 1.54
	5. 7. 4$\frac{1}{2}$	16.50. 6	15.27. 3	16.59.17	15. 27. 0	+ 0.11	+ 2.10
	25. 8.41	26.18.35	12.46.46	26.16.59	12.45. 22	– 1·36	– 1.24
Mar.	1.11.10	27.52.42	12.23.40	27.51.47	12.22.28	– 0·55	– 1.12
	5.11.39	29.18. 0	12. 3.16	29.20.11	12. 2.50	+ 2.11	– 0·26
	9. 8.38	0.43. 4	11.45.52	♊ 0.42.43	11.45.35	– 0·21	– 0·17

The observations of this comet from the beginning to the end are not in less agreement with the motion of the comet in the orbit now described, since the motions of the planets are accustomed to agree with their theories, & by agreeing they prove that the comet was one & the same, which appeared in this whole time, & its orbit was correctly designated here.

In the preceding table we have set aside the observations from the 16th , 18th , 20th & 23rd days of *November* as less accurate. For the comet was also observed in these times. Certainly *Ponteo* & associates, on *November* 17th old time, at the hour of 6 in the morning in *Rome*, that is, at 5 hours & 10 minutes *London* time, by threads pointing towards fixed stars, had observed the comet in ♎ (*Libra*) 8^0.30' with the southern latitude 0^0. 40'. These observations can be found in a tract that *Ponteo* published about this comet. *Cellio*, who was present there & who sent his observations to *Mr. Cassini* in a letter, saw the same comet at the same hour in ♎ 8^0.30' with a southern latitude of 0^0. 30'. At the same hour *Gallet* of *Avenion* (that is, at the hour of 5. 41 a.m. *London* time) saw the comet in ♎ 8^0. without latitude; but the comet by the theory now was in ♎ 8^0. 16'. 45" with the southern latitude of 0^0.53'. 7",

On *November* 18th in the morning at 6. 30 a.m. *Rome* time (that is, at 5.40 a.m. *London* time) *Pontio* saw the comet in ♎ 13^0. 30' with the southern latitude 1^0. 20'. *Cellio* observed it in ♎ 13^0.30', with the southern latitude 1^0. 00'. But *Gallet* at the early morning hour of 5^h.30' in *Avenion* saw the comet in ♎ 13^{gr}. 00', with the southern latitude 1^0. 00'.

And *R.P. Ango* in the university of *La Fleche* in *France*, at five o'clock in the morning (that is, at 5^h. 9' London time) saw the comet half-way between two small stars, of which one is the middle star of three in line in the southern hand of Virgo, *Bayer's* ψ, & the other is the end of the wing, *Bayer's* θ. Thence the comet then was in ♎ 12^0 .46' with the latitude 50' South. In the same day in *Boston* in *New England* at a latitude of $42\frac{1}{2}{}^0$, in the hour of five in the morning, (that is at the *London time* of 9^h.44' a.m.) the comet was seen near ♎ 14^0, with the southern latitude 1^0. 30', as I have been informed by the most illustrious *Halley*.

On *Nov.* 19. at the morning hour of $4\frac{1}{2}$ in Cambridge, the comet (observed by a certain young man) was separated from Spica in ♍ by about 2^0 towards the North-West [There is some confusion here : Madame du Chatelet considers *spica* to be an ear – as in ear of corn or wheat, or a tuft of hair, in a constellation, while Cohen considers *Spica* to be the star of this name, which he now writes with a capital *S,* belonging to Virgo; Motte makes the best of both worlds, & sometimes calls it by one name, then by the other.] Or it was in ♎ 19^{gr}.23'. 47'' with the southern latitude 2^0. 1'. 59''. In the same day at the hour of 5 in the morning in *Boston* in *New England*, the comet was distant from Spica in ♍ by one degree, a difference of the latitudes present of 40'. On the same day on the island of *Jamaica*, the comet was distant from Spica by an interval of around one degree. On the same day *Mr. Arthur Storer* at the river *Patuxent*, near *Hunting Creek* in *Maryland*, on the border of Virginia at the latitude $38\frac{1}{2}{}^0$, at the hour of five in the morning (that is, at 10^h *London* time) saw the comet above the star Spica in ♍ & almost coincident with Spica, with the separation between the same around $\frac{1}{4}{}^0$. And from these observations collated among themselves I deduced that at the hour 9^h. 44' of *London* time that the comet was in ♎ 18^0. 50' with a latitude of around 1^0 .25'. Moreover the comet by theory now was in ♎ 18^0. 52'. 15'' with a latitude of 1^0. 26'. 54'' South.

Nov 20. *Dr. Montanari,* professor of astronomy at Padua, at 6 o'clock in the morning in *Venice* (that is, at 5^h.10' *London* time) saw the comet in ♎ 18^0 with the latitude 1^0.30' South. On the same day in *Boston*, the comet separation from Spica ♍ was 4^0 longitude to the East, & thus it was about 23^0. 24' in ♎.

Nov 21. *Ponteo* & his companions on the morning at $7\frac{1}{4}$ hours observed the comet in ♎ 27^0. 50' with a southern latitude of 1^0. 16', *Cellius* in ♎ 28^0. *Ango* at 5 o'clock in the morning in ♎ at 27^0. 45'', *Montanari* in ♎ at 27^0.51'. On the same day on the island of *Jamaica* saw the comet near the principle star of *Scorpio*, & that had around the same latitude as *Spica* in *Virgo*, that is, 2^0. 2'. On the same day at 5 o'clock in the morning, at *Balasore* in *India*, (that is at the hour of the preceding night in *London* of 11^h. 20') took the distance of the comet from the ear in ♍ as 7^0. 35' to the East. In the right line between the ear & the scales, & thus it was present in ♎ 26^0. 58' with a southern latitude of around 1^0.11' ; & after 5^h & 40' (which corresponds to the hour of around 5 a.m. in *London*) it was in ♎ 28^0. 10' with a southern latitude of 1^0. 16'. By the theory the comet now truly was in ♎ 28^0.12' .36'', with a southern latitude of 1^0. 53'. 35''.

Nov. 22. The comet was seen by *Montanari* in ♏(*Scorpio*) 2^0. 33', but in *Boston* in *New-England* it appeared in ♏ around 3^0 , with almost the same latitude as before, that is, 1^0. 30'. In the same day at 5 o'clock in the morning at *Balasore* the comet was observed in ♏ 1^0. 50'; & thus at the hour of five in the morning in *London* the comet was in ♏ around

3^0. 5'. In the same day at $6\frac{1}{2}$ hours in the morning in *London* our countryman Hook saw the comet in ♏ at around 3^0. 30', & that in a right line which passed through the [corn] ear of *Virgo* [i.e. Spica] & the heart of *Leo*, not exactly however, but in a line a little deflected to the North. *Montanari* likewise noted that the line from the comet drawn through the ear, on this day & in the following days passed through the southern side of the heart of *Leo*, with a very small interval interposed between the heart of the lion & this line. A right line passing through the heart of *Leo* & the ear of *Virgo*, cut the ecliptic in ♍ 3^0. 46'; at an angle 2^0. 51'. And if the comet were located on this line in ♏ 3^0 its latitude would have been 2^0. 26'. But since the comet from the agreement of *Hook* & *Montanari*, to some extent was separated from this line a little towards the North, its latitude was a little less. On the 20^{th} day, from the observation of *Montanari*, its latitude was almost equal to the latitude of the ear in ♍, & that was around 1^0. 30', & by the agreement of *Hook*, *Montanari,* & *Father Alga* it was always increasing, & now significantly greater than 1^0. 30'. Now between these two limits found: 2^0. 26' & 1^0. 30', the magnitude of the average latitude was around 1^0.58'.

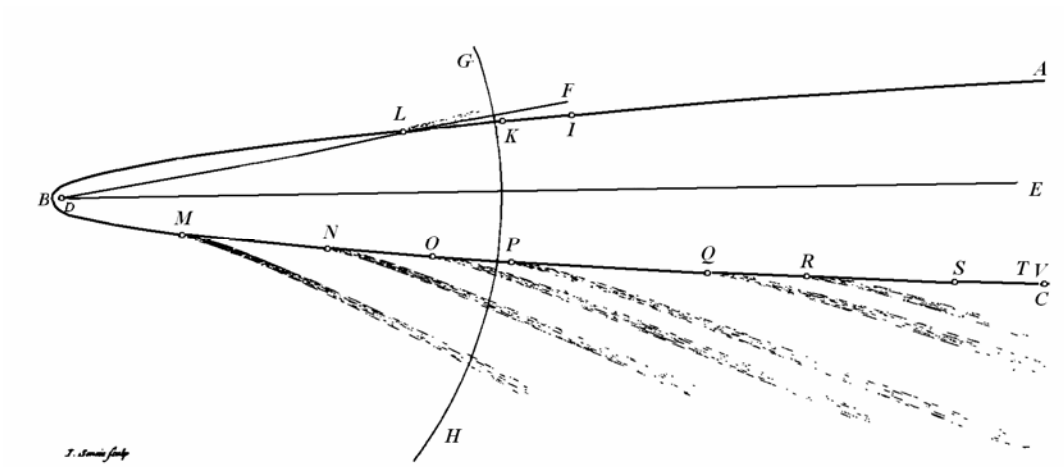
The comet's tail, from the agreement of *Hook* & *Montanari*, was directed towards the ear in ♍, falling a little from that star, nearly to the south according to *Hook*, & nearly to the north, according to *Montanari* ; & thus that decline was scarcely sensible, & the tail to be present almost parallel to the equator, was being deflected a little by the opposition of the sun towards the North.

Nov. 23. old time, at the hour of 5 in the morning in *Noreberg* (that is at $4\frac{1}{2}$ hours in *London*) *D. Zimmerman* saw the comet in ♏ 8^0. 8', with a latitude of 2^0. 31' South, evidently with its separation taken from the fixed stars.

Nov. 24. Before sunrise the comet was seen by *Montanari* in ♏ 12^0. 52', towards the northern side of the line drawn through the heart of *Leo* & the ear of *Virgo*, & thus it had a latitude a little less than 2^0. 38'. This latitude, as we have said, from the observations of *Montanari, Ango* & *Hook*, was always increasing ; & now thus it was a little greater than 18^0.58' & with a mean magnitude, without noticeable error, to be put in place at 2^0.18'. *Ponteo* & *Galtetius* now wished to decease the latitude, both *Cellio* & an observer in *New-England* retained the same magnitude, evidently one degree or one & a half degrees. The observations of *Ponteo* & *Cellio* were coarser, especially as those were taken by means of azimuths & altitudes, & as those of *Gallet* ; those are better which are taken through the positions of the comet relative to the fixed stars by *Montanari, Hook, Ango* & by the observer in *New-England*, & some that *Ponteo* & *Cellio* have made. On the same day at 5 o'clock in the morning the comet was seen in *Balasore* in ♏11^0. 45'; & thus at the fifth hour of the early morning in *London* it was in ♏ at around 13^0. Truly by the theory the comet now was in ♏13^0. 22'. 42''.

Nov. 25. Before sunrise *Montanari* observed the comet in ♏$17\frac{3}{4}^0$ approximately. And Cellius observed at the same time that the comet was in a right line drawn between the bright star in the right thigh of *Virgo* & the southern scale of *Libra*, & this right line cut the path of the comet in ♏ 18^0. 36'. Truly by the theory the comet was now approximately in ♏$18\frac{1}{3}^0$.

Therefore these observations agree with the theory just as they agree amongst themselves, & by agreeing they prove it to be one & the same comet, which appeared in the whole time from the fourth day of *November* as far as to the ninth of *March*. The trajectory of this comet cut the plane of the ecliptic twice, & therefore was not a straight line. It did not cut the ecliptic in opposite directions of the heavens, but at the end of *Virgo* & at the beginning of *Capricorn*, through an interval of around 98 degrees; & thus the course of the comet was deflected greatly from a great circle. For in the month of *November* its course dropped by at least three degrees below the plane of the ecliptic to the South, & after the month of *December* it moved away from the ecliptic to the North by 29^0, & the two parts of the orbit, the one with the comet approached the sun & the other with it receded from the sun, appearing to have an angle between each other of more than 30^0, as *Montanari* had observed. This comet went through nine signs, clearly from the last degree of *Leo* as far as to the first point of *Gemini*, besides the sign of *Leo*, through which it had gone before it began to be seen; & no other theory is extant that by which the comet traverses so much of the heavens with a regular motion. Its motion was greatly uneven. For around the 20th day of *November* it described around 5^0 each day ; then with the motion retarded between *Novemb*er 26th & *Decemb*er 12th, evidently in a time of 15 & a half days, it described only 40^0; truly again with an accelerated motion it



describes almost 5^0 per individual day, before the motion again began to be retarded. And the theory, which corresponded properly to so much of the unequal motion through the great part of the sky, & which observed the same laws as the theory of the planets, as well as agreeing accurately with accurate astronomical observations, cannot be otherwise than true.

The rest of the trajectory that the comet described, & indeed the tail that it projected at individual places, has been shown traced out in the adjoining diagram in the plane of the trajectory : where *ABC* denotes the trajectory of the comet, *D* the sun, *DE* the axis of the trajectory, *DF* the line of the nodes, *GH* intersection of the sphere of the great orbit with the plane of the trajectory, *I*, the position of the comet on *Nov*. 4th of the year 1680, *K* the place of the same on *Nov*. 11th , *L* the place on *Nov*. 19th, *M* the place on *Dec*. 12th, *N* the place on *Dec*. 21st, *O* the place on *Dec*. 29th, *P* the place on *Jan*. 5th, the following *Q* the place on Jan. 25th, *R* the place on *Feb*. 5th, *S* the place on *Feb*. 25th, *T* the place on *Mar*. 5th, & *V* the place on *Mar*. 9th.

Now the following observations are used in determining the tail:

Nov. 4th & 6th. The tail had not yet appeared.

Nov. 11th. A tail now appeared of half a degree, not visible except with a telescope ten feet long.

Nov. 17th. The tail appeared to *Ponteo* more than 15 degrees long.

Nov. 18th. The tail was 30^0 long, & was seen in *New-England* to be directly away from the sun, & and was extended as far as to the planet [*Mars*] ♂, which then was in ♍ 9^0. 54'.

Nov. 19 th. In *Mary-land* , the tail was seen to be 15^0 or 20^0 long.

Dec. 10th. The tail (from *Flamsteed's* observation) was passing through the middle of the interval between the tail of the serpent of *Ophiuchus* & the star δ in the south wing of *Aquilae*, & stopped close to the stars A,ω,b in *Bayer's* tables. Therefore its end was in ♑ $19\frac{1}{2}$ gr, with a northern latitude of around $34\frac{1}{4}{}^0$.

Dec. 11th. The tail increased to as far as the head of *Sagittarius* (*Bayer* α,β) ending in ♑ 26^0. 43', with a latitude of 38^0. 34' North.

Dec. 12th. The tail was passing through the middle of *Sagittarius*, not being extended any longer, stopping in ♒ 4^0, with a latitude North of around $42\frac{1}{2}{}^0$. These are understood to be concerned with the clearer part of the tail. For in the more obscure light, perhaps in the heavens in a more clear weather, the tail on *Dec*.12 th, at the hour of 5. 40' in *Rome* (observed by *Pontio*) rose to 10^0 above the rump of the swan ; & from this star its edge stopped at 45' to the North-West. But the tail was 3^0 wider in these days towards the upper extremity, and thus its middle was 2^0. 15' distant from that star towards the South, and the upper end was in (*Pisces*) ♓ 22^0, with the latitude of $6l^0$ North. And hence the length of the tail was around 70^0.

Dec. 21st. The same increased almost to the chair of *Cassiopeia*, being equally distant from β and *Schedir* [the brightest star in this constellation], and its distance from each of these two stars was equal to the distance between themselves, and thus stopping in ♈ 24^0, with the latitude of $47\frac{1}{2}{}^0$.

Dec. 29 th. The tail was touching *Scheat* placed on the left, and the interval of the two stars in the northern foot of *Andromeda* was filled up completely, and the length of the tail was 54^0 ; and thus was defined in ♉ 19^0, with the latitude 35^0.

Jan. 5 th. The tail touched the star π in Andromeda's breast on its right side, and the star μ in the girdle of this at the left side ; and (on a par with our observations) the length was 40^0; but it was a curve and with the convex side seen towards the South. And on one side it made an angle of 4^0 with a circle passing through the sun and the head of the comet; but near to the other boundary is was inclined to that circle at an angle of 10^0 or 11^0 and the chord of the tail contained an angle of 8^0 with the circle.

Jan. 13th. With enough light the tail appeared to be finished sensibly between *Alamech* and *Algol*, and with a more tenuous light it was defined to the region of the star χ in the side of *Perseus*. The distance of the end of the tail from the circle joining the sun and the comet was 3^0.50', and the inclination of the chord of the tail to that circle was $8\frac{1}{2}{}^0$.

Jan. 25 th and 26 th. The tail shone with a rather feeble light to a length of 6^0 or 7^0 ; and with a night or two following when the sky was very clear, with the light most tenuous and barely visible, it reached a length of 12 degrees and a little beyond. But its axis was directed accurately to the bright star in the eastern shoulder of *Auriga*, and thus fell from opposing the sun towards the North at an angle of 10^0. And then on *Feb*. 10 th with the naked eye I viewed the tail two degrees long. For the aforesaid light was more tenuous and not apparent through a glass. But *Ponteo* on *Feb*. 7 th wrote that he had seen the tail at a length of 12^0.

Feb. 25 th and after that the comet appeared without a tail.

On examining the orbit just described above, and on considering the other phenomena of this comet, it will be agreed without difficulty, that the bodies of comets are solid, compact, fixed [in shape] and durable in the image of the bodies of planets. For if they were nothing other than vapours or exhalations of the earth, sun and planets, here a comet in its passing in the vicinity of the sun ought to be dissipated at once. For the heat of the sun is as the intensity of the rays, that is, reciprocally as the square of the distances of the places from the sun. And thus since the distance of the comet from the centre of the sun on *December* 8th, when it was passing through the perihelion, was to the distance of the earth from the centre of the sun almost as 6 to 1000, the heat of the sun at the comet at this time was to the heat of our summer sun as 1000000 to 36, or 28000 to 1. But the heat of boiling water is around three times greater than the heat that the dry earth receives on the summer sun, as I have found out : and the heat of incandescent iron (if I conjecture rightly) will be around three or four times greater than the heat of the boiling water; and thus the heat, that dry earth present on the comet passing through the perihelion, is able to experience from the rays of the sun, is around 2000 times more than the heat of the glowing iron. But with so much heat, vapours and exhalations and all volatile matter ought to be at once consumed and dissipated.

Therefore the comet in its perihelion receives an immense amount of heat from the sun, and that heat can be conserved for a very long time. For an incandescent iron globe one inch wide present in air, can scarcely sent off all its heat in the space of an hour. Moreover a greater globe will conserve the heat in the ratio of the diameters, because the surface area (according to the measure of this cooled by the surrounding air) is less in that ratio to the quantity of its included warm matter. And thus a glowing iron globe equal to this earth, that is, more or less 40000000 feet wide, or in just as many days, and thus in 50000 years, will scarcely be cooled. Yet I suspect that the duration of the heat, on account of hidden causes, will be increased in a smaller ratio than that of the diameter: and I would choose to have the true ratio found by experiment.

Again it is to be observed that the comet in the month of *December*, but only when it had been heated by the sun, sent off a much longer and splendid tail than in the month of *November* before, when it had not yet reached the perihelion. And generally, all the greatest and most brilliant tails arise suddenly from comets after their passage through the region of the sun. Therefore the heating of the comet leads to the magnitude of the tail : and thence I seem to deduce that the tail shall be nothing other than the most tenuous vapour, that the head or nucleus of the comet emits by its heat.

Otherwise there are three opinions concerning the nature of the tails of comets ; either to be the light of the sun passing through the transparent heads of comets, or to arise from the refraction of light in passing from the head of the comet to the earth, or finally the cloud to be either vapour from the head of the comet arising and passing away in directions opposite to the sun.

The first opinion is that of those who have not yet imbued the science of optical matters. For the light of the sun cannot be discerned in a darkened chamber, unless the light is certainly reflected from the dust and smoke of particles always flying about in the air : and thus in air filled with thicker smoke it is more brilliant, and strikes the senses more strongly; in clearer air it is more tenuous and scarcely perceived: but in the heavens nothing is able to be reflected without reflecting matter being present. The light certainly cannot be discerned that is in a heavenly body, except that from thence it is reflected into our eyes. For vision can only be from rays which impinge on the eyes. Therefore matter of some kind is required reflecting in the tail region, lest the whole heavens be illuminated uniformly in a brilliant light from the sun.

The second opinion is overwhelmed by many difficulties. The tails are never variegated with colours : which yet are accustomed to be found whenever refraction occurs. The light of the fixed stars and of the planets transmitted distinctly to us demonstrates the medium of the heavens is not influenced by any refractive force. For as it is said that the fixed stars sometimes were seen as comets by the Egyptians, but as that happened most rarely, it can be ascribed to fortuitous refraction of the clouds.
[It may be of course be that they were viewing a cosmic event such as a supernova.]
The radiations and scintillation of the fixed stars [*i.e.* twinkling] also are accustomed to refractions both within the eyes as well as trembling of the air : which certainly vanish with the eye applied to a telescope. The rising of vapours in the air may cause a tremble, so that the rays are easily turned away in turn from the narrow space of the pupil, but by no means from the side of the wider glass in the aperture of the objective. Thus it is that such a scintillation arises in the first case, and ceases in the second: and the cessation in the second case demonstrates the regular transmission of light through the heavens without any sensible refraction. It has been said, incorrectly, that one cannot always see the tails of comets, because their light is not strong enough, as then the secondary rays do not have enough strength to affect the eyes, and it is for this reason that we cannot see the tails of fixed stars, as [likewise] they do not have enough strength to affect the eyes, and therefore the tails of fixed stars are not seen: but it is known that the light of fixed stars can be increased more than 100 times by means of telescopes, yet still tails are not discerned. The light of planets is more plentiful too, truly without tails: but often comets have the greatest tails, when the light of the head is feeble and very dull. Thus indeed the comet of the year 1680, in the month of *December*, in which time the head by its light was scarcely equal to a second order magnitude star, was sending out a magnificent tail as far as to 40^0, 50^0, 60^0 or 70^0 of longitude and beyond : after *January* 17th and the18th the head appeared as a star of only the 7 order magnitude, truly the tail by a certain weak light was enough to be seen 6^0 or 7^0 long, and with the most obscure light, scarcely made it possible to see, it was stretched out to as far as 12^0 or a little further: as has been said above. But on both *February* 9th and 10th when one had abandoned looking at the head with the naked eye, I considered the tail to be 2^0 in length with a telescope. Again if the tail was arising from the refraction of celestial matter, and by virtue of the heavens the figure as deflected in the opposite direction to the sun, that deflection must always be into the same region of heavens and always to be made in the same direction. And the comet of the year 1680 *December* 28, in the hour $8\frac{1}{2}$ p.m. *London* time, was present in ♓ 8^0.41', with a latitude North of 28^0. 6', with the sun present in ♑ 18^0.26'. And the comet of the year 1577*,* on *December* 29 was moving through ♓ 8^0. 41' with a latitude North of 28^0.40', with the sun present in ♑ 18^0.26' approximately. In each case the earth was present at the same place, and the comet appeared in the same part of the heavens : yet in

the first case the tail of the comet (from my own and from the observations of others) declined at an angle of $4\frac{1}{2}^0$ from the opposition of the sun towards the North ; truly in second case (from the observations of Tycho) the declination was of 21^0 to the South. Therefore the refraction of the heavens is rejected, and it remains that the phenomena of tails is derived from some matter reflecting the light.

The laws which the tails observe confirm that they arise from the heads of comets, and that they ascend into regions away from the sun. So that since the tails which the heads leave behind as they progress in these orbits, are in the planes of the orbits of comets passing the sun, these always deviate in directions opposite to the sun. Which appear to a spectator established in these planes to be in directions directly away from the sun ; moreover to an observer in these planes the deviation would appear to change little by little, and in days would become greater and greater. Because the deviation, with all else being equal, is less when the tail is more oblique to the orbit of the comet, as when the head of the comet approaches closer to the sun; especially if the angle of deviation of the tail is viewed near the head of the comet. Besides, since the non-deviating tails appear in straight lines, and moreover those deviating are curved. So that the curvature is greater where the deviation is greater, and more noticeable where the tail is longer, with all things being equal : for with shorter tails the curvature is scarcely noticed.

In addition, the angle of deviation is less near the comet's head, and greater towards the other extremity , and thus as a consequence the convex side of the tail is turned towards the parts that have been carried away by its deviation, and which are in right lines from the sun drawn through the head of the comet to infinity. And because the tails which are more prolix and wider, and shine by light more vigorously, shall be a little more brilliant on the convex side and terminate less indistinctly than at the concave side. Therefore the phenomena of the tail depends on the motion of the head, but not in the region of the heavens in which the head is seen ; and therefore they do not come into being through refraction of the heavens, but arise from the material supplied by the head. And indeed just as in our air the top of any body on fire tries to become higher, and that either perpendicularly if the body is at rest, or obliquely if the body is moving to the side: thus in the heavens, where bodies gravitate towards the sun, smoke and vapours must rise from the sun (as has now been said) and reaches higher and straight up if the body is at rest, or obliquely, if the body by progressing always leaves behind places from which the higher parts of the vapour rise. And that obliquity will be less when the ascent of the vapour is faster : without doubt in the vicinity of the sun and near smoking bodies. But from the differences of the obliquity the column of vapour appears curved : and because the vapour in the preceding side of the column is a little more recent, thus also it likewise will be a little denser, and the light reflected on that account more abundant, and it will be terminated more distinctly. Concerning the sudden and uncertain agitations of the tails and with regard to the irregular figures of these, which some have described occasionally, here I add nothing; therefore as either from the changes in our air, and from the motions of clouds sometimes the parts of tails arise from obscurity ; or perhaps from parts of the milky way, which disregarded may be confused with the tails, and only parts of that are seen.

But it is possible to understand from the rarity of our air, how vapours from which the atmospheres of comets are able to arise, which are sufficient to fill such immense spaces.

For the air adjacent to the surface of the earth occupies around 850 more parts of space than water of the same weight, and thus a cylindrical column of air 850 feet high is of the same weight as a column of water of the same width measuring a foot in height. Moreover a column of air rising to the top of the atmosphere equals by its weight a column of water around 33 feet high; and therefore if the lower part of the whole column of air 850 feet high is taken away, the part remaining above will equal by its weight a column of water to of height 32 feet. Thus truly (by a rule confirmed by many experiments, that the compression of air shall be as the weight of the incumbent atmosphere, and that the gravity shall be inversely as the square of the distance of places from the centre of the earth), by putting in place the computation according to the Corollary, Prop. XX11, Book II, I have found that the air, if it may rise from the surface of the earth to a height of one radius of the earth, shall be rarer than with us in a ratio greater by far than the distance greater than all of the distance between the orbit of Saturn and a globe of diameter one inch. And thus a globe of our air one inch wide, that with the rarity which it would have at a height of one radius of the earth, would fill all the planetary regions as far as to Saturn's sphere and more beyond. Hence since the air higher besides is rarefied indefinitely ; and the hair or atmosphere of a comet, by ascending from its centre, almost ten times higher shall be than the surface of the nucleus, then the tail may ascend higher than this altitude, and it must become the even more rarefied. And since on account of the great thickness of the atmosphere of the comet, and the great size of the gravitation of the bodies towards the sun, and the gravitation of the particles of the air and vapour between themselves, it can happen that the air in the celestial distances and in the tails of comets themselves thus may not be rarefied ; but yet the exceedingly small quantity of air and vapour suffices in abundance for all that phenomena of the tails, and is evident from this computation. For the signs of rarity of the tails is deduced from the stars by their translucence. With the brilliant light of the sun through the earth's atmosphere, with its thickness of a few miles, and the stars and the moon itself obscured and thoroughly extinguished : equally illuminated by the light of the sun, through the immense thickness of the tails, the stars are known to shine through without detriment and with minimum loss of clarity. Nor is the splendour of many tails usually greater than the air in our darkened chamber with the light from the sun in a space of one or two inches reflecting in the sunshine.

The interval of time in which the vapour ascends from the head to the tail, can almost be known by drawing a line from the end of the tail to the sun, and by noting the place where that line cuts the trajectory. For the vapour at the end of the tail, if it ascends straight from the sun, starts to ascend from the head, at which time the head was at the point of the intersection. But the vapour does not ascend straight from the sun, for by retaining the motion of the comet, that it had before it had its ascent, and by adding the motion of the ascent to the motion of the same, it ascends obliquely. From which the solution of the problem will be more realistic, so that the straight line, which cuts the orbit, shall be parallel to the length of the tail, or rather (on account of the curved motion of the comet) as the same may diverge from the line of the tail. With this agreed upon I found that the vapour, which was at the end of the tail on *January* 25th , had began to rise from the head before *December* 11th , and thus in its whole ascent it took more than 45 days. But all that tail which appeared on *December* 10th , ascended in an interval of these two days, which had elapsed from the time of the perihelion of the comet. Therefore the vapour from the beginning in the vicinity of the sun rose most quickly, and afterwards from the motion retarded by its own gravitation proceeded always to ascend more slowly ; and in ascending increased the length of the tail : but the tail, as soon as it appeared, was made nearly completely from vapour, which arose from the time of the perihelion ; nor did that first part vanish, the vapor which first ascended, and of which the end of the tail was composed, until on account of its excess distance even though being illuminated by the sun, it is ceased to be seen by our eyes. Also the tails of other comets, which are shorter and do not ascent with a rapid continual motion from the heads and soon vanish, but instead the tails are permanent columns of vapours and exhalations, propagated from the heads most slowly over many days with a motion which, by sharing that motion with the head that it had at the start, together with the head they are able to move through the heavens. And hence again it is gathered that the space of the heavens is free of resistance ; just as in which not only the solid bodies of the planets and of comets, but also the rarest vapours of the tails go forwards most freely, and conserve the most rapid velocities of their motion for a long time.

Kepler ascribed the ascent of the tails from the atmospheres of the heads of comets, and their progression in directions away from the sun, to the action of the rays of light dragging the material of the tail along with it. And it is not absurd to reason that the finest vapours are able to be carried along in spaces free from all resistance by the action of the rays, and it cannot therefore be by any other reason, although dense vapours on being impeded are unable to be propelled sensibly by the rays of the sun in our regions. Another astronomer has considered that it is possible to be given both light as well as heavy particles, and the matter of the tails to be light, and by their lightness to ascend from the sun. But since the weight of terrestrial bodies shall be as the matter in the bodies, and thus as the quantity of matter remains the same [thus, particles that levitate do not exist], the weight cannot be made greater or less. I suspect that ascent to arise rather from the rarefaction of the matter in the tail. Smoke ascends in a furnace by the force of the air on which it floats. That air rarefied by heat ascends, on account of the diminution of the specific gravity, and the mixed-up smoke rises with it. Indeed, why may the tail of the comet not rise from the sun in the same way? For the sun's rays do not disturb the medium, through which they pass, except by reflection and refraction. The reflecting particles are heated by this action and in turn will heat the aether wind with which they are mixed up. It in turn is rarefied by that communicated heat, and from that rarefaction on account of the diminished specific gravity by which it is drawn towards the sun, it rises and takes the reflecting particles with it from which the tail is composed. The vapours which compose the tails of comets turn around the sun, and tend as a consequence to travel away from that star, which again contributes to their ascension, for the atmosphere of the sun and the material of the heavens either clearly remains at rest, or turns more slowly than the matter in the tails, and because it turns by that motion which it receives from the rotation of the sun.

These are the causes for the ascent of the vapours which form the tails of comets when they are in the vicinity of the sun's atmosphere, or where their orbits are more curved, and where the comets are present within the denser and for that reason thicker part of the sun's atmosphere, and as a consequence the more heavy, and they will send out the longest tails. For the tails, which then begin to appear conserve their motion and yet gravitate towards the sun, moving around the sun in ellipses in the manner of the heads, and by that motion they will always accompany the heads although they adhere most freely to these. For the gravity of these vapours acting towards the sun can no more make the tail longer by falling later, any more than the weight of the head can effect an increase in the length of the tail by falling towards the sun. Thus falling at the same time with the common [acceleration of] gravity either into the sun, or being retarded in their ascent in the same manner ; and thus gravity hardly impedes the heads and tails of comets ; so that they easily take and afterward most freely observe some position of the head and of the tail in turn from the causes described, or from any others.

Therefore the tails, which arise in the perihelions of comets, will go off with their heads into remote regions, and either thence after a long series of years will return to us with the same, or perhaps there little by little they vanish by rarefaction. For later in the descent of the heads to the sun new and very short tails must be propagated from the heads in a slow motion, and immediately in the perihelions of these comets, which descent as far as the atmosphere of the sun, are increased into an immensity. For the vapour will be perpetually rarefied and dilated in these free spaces. From which account it comes about that every tail shall be wider at the upper extremity than next to the head of the comet. But from that rarefaction the vapour perpetually spreads out wider and is scattered through the whole heavens, then little by little it is attracted to the planets by their gravity, and it seems likely to think that it may become mixed with their atmospheres. For just as the seas are required for the entire constitution of this earth, and so that from these by the heat of the sun enough vapours may be excited, which either begins to fall as rain from clouds, and all the earth may be watered and nourished for the growing of vegetables ; or to be condensed onto the freezing tops of mountains (as some conjecture with reason) running off in springs and rivers : thus comets may seem to be required for the preservation of the seas and of the moisture on the planets, and from the exhalations and condensed vapours of which, whatever is consumed by vegetation and decay from the condensed vapours and is changed into dry ground, will be continually supplied and replaced. For all vegetation generally grows by means of humidity, then that in the large part is return to dry ground by rotting, and slime always falls to the bottom of fluids that are putrefying. Hence the mass of the dry earth thence is increased, and the liquids, unless they are supposed to be increased from elsewhere, must always decrease, and at last cease to be. Again I suppose that the spirit, which is the smallest but the most subtle and the most excellent part of our air, and that is necessary to give life to all things, comes especially from comets.

The atmospheres of comets are diminished in their descent towards the sun by the tails extending out, and (that certainly in the direction facing the sun) are rendered narrower : and in turn in the recession of these from the sun, when now extends less in tails, they may be made bigger ; but only if *Hevelius* noted these phenomena correctly. But they appear smallest, when the heads now heated by the sun and which send off the longest and brightest tails ; and the nuclei are surrounded by smoke perhaps thicker and darker in the lower parts of the atmosphere. For all the smoke generated by the great heat is usually thicker and darker. Thus the head of the comet, that we are discussing (*i.e.*, the one of 1680), at equal distances from the sun and from the earth appeared more obscure after it perihelion rather than before. For in the month of *December* when it was accustomed to be compared to a star of the third order magnitude, but in the month of *November* with stars of the first and second orders. And those who saw both, described the first as the greater comet. For on *November* 19th, to a certain young man of Cambridge*,* this comet with its light appearing dull and somewhat obtuse, was equal to the light of *Spica Virgo*, and was shining clearer than later. And to *Montanari* on *November* 20th in the old style, the comet appeared greater than a star of the first magnitude, with a tail of two degrees length. And from letters of Mr. *Storer*, that have come into our hands, he remarked that its head in the month of *December* was very small, when the maximum and most brilliant tail was sent off, and that it departed in grandeur from that who had seen the comet in the month of *November,* before the rising of the sun. It was conjectured that the reason for this was because in the head at the start was more plentiful, and it was being used up little by little.

In the same way it seemed to be seen, that the heads of other comets, which had emitted great and most brilliant tails, had appeared somewhat obscure and very small. For in the year 1668 on *March* 5th in the new style, in the seventh hour in the evening, the *R. P. Valentinus Estancius,* viewing in *Brazil*, saw the comet almost horizontal at sunset on a wintry day, with a very small head and barely conspicuous, but truly with the tail above shining in such a manner, that standing on the shore an image of this could be easily discerned reflected from the sea. The image truly produced was that of a splendid beam 23^0 in length, going towards the south west, and almost parallel to the horizontal.

But yet the splendour lasted only three days, suddenly perceptibly decreasing ; and meanwhile with the decrease in splendour with an increase in the magnitude of the tail. From which also in *Portugal* it was said to have occupied almost a quarter of the sky (that is, 45^0) from the west to the east with a most significant extension: for still the whole appeared, with the head always hidden in these regions below the horizon. From the increase of the tail and the decrease of the splendour it was evident that the head was receding from the sun, and it was nearest to it at the beginning of its appearance, in the manner of the comet of the year 1680.

One reads in the *Saxon Chronicle* that a similar comet appeared in the year 1106, *the star of which was small and obscure (like that in the year 1680) but the tail of which had a great brilliance, and which extended from that like a giant tree trunk towards the North-East*, as *Hevelius* had also from the monk *Simeon* of *Durham*. Initially it appeared in the month of *February*, and then around the evening, at the setting of the winter sun. Then truly from the position of the tail it is gathered that the head was in the vicinity of the sun. *Matthew of Paris* said, *that it was about a single cubit from the sun, from the third hour* [more correctly six (the square brackets here are Newton's comments] *until the ninth hour it was sending out a long ray from itself.* Such also was the most outstanding light from that comet described by *Aristotle* in Book I. Meteor, 6. *The head of which in the first day was not conspicuous, because that had set before the sun or perhaps appearing in the sun's rays, but in the following day so much of it was to be seen as possible. As it was able to leave the sun by a small distance, and soon set also. On account of the excessive brightness* [evidently of the tail] *the head did not appear covered in fire, but with time proceeding (according to Aristotle) with the tail now less ablaze, the head of the comet returned to its shape. And the splendour of the tail extended to a third part of the sky* [*i.e*., to 60^0]. *But it appeared in winter time* [in the fourth year of the 101st Olympiad] and ascending as high as the belt of *Orion*, *when it vanished.* With that comet of the year 1618, which appeared from the rays of the sun with the most splendid tail, was seen to be a star of the first magnitude or greater, but many larger comets appear, which have shorter tails. Of these, some are said to have been the equal of Jupiter, others of Venus, or even of the moon.

We have said that comets are a kind of planets revolving in very eccentric orbits around the sun: And just as with the planets which are not accustomed to have small tails, which are revolving closer to and around the sun in smaller orbits, but thus also comets, which fall closer to the sun at their perihelions, it may be seen to be agreed upon for this reason, are to be very much smaller, lest they disturb the sun too much by their attraction. Now the transverse diameters of the orbits and the periodic times of the revolutions, remain to be determined from the gathering together of comets returning in the same orbits after long time intervals. Meanwhile the following proposition may shed some light on this business.

PROPOSITION XLLI. PROBLEM XX11.

To correct the trajectory found of a comet.

[Chandrasekhar gives explanations of these operations from p.530 onwards.]

Operation 1. The position of the plane of the trajectory is assumed, by the above proposition found ; and three places are selected from the observations of the comet designated the most accurate, and which are as far apart from each other as possible ; and let *A* be the time between the first and the second, and *B* the time between the second and the third. Moreover it is agreed that the comet has turned through its perigee at one of these places, or at least not to be far away from the perigee. From these apparent positions, three true locations of the comet are found by trigonometrical operations in that assumed plane of the trajectory. Then with these locations found, around the centre of the sun or the focus, by arithmetical operations, put in place with the aid of Prop. XXI. Book I, a conic section is described: and the areas of this, terminated by the rays drawn from the sun to the places found shall be *D* and *E*; clearly *D* shall be the area between the first and second observations, and *E* area between the second and the third. And *T* shall be the whole time, in which the whole area $D+E$ it must describe with the velocity found by Prop. XVI. Book 1.

Operation. 2. The longitude of the nodes of the plane of the trajectory is increased, with 20' or 30' added to that longitude, which are called *P*; and the inclination of that to the plane of the ecliptic is maintained. Then from the three aforementioned observed places of the comet, three true points may be found in this new place, as above : then also with the orbit passing through these places, and the two areas of the same described between the observations, which shall be *d* and *e*, so that the total time in which the whole area $d+e$ must be describe shall be *t*.

Operation 3. The longitude of the nodes is used in the first operation, and the inclination of the plane of the trajectory is augmented to the plane of the ecliptic, with 20' or 30' added to that inclination, which is called *Q*. Then from the aforesaid observations with the three apparent places of the comet found in this new plane with the three true places, and with the orbit passing through these places, and so that the two areas of the same described between the observations, which shall be δ & ε, and the whole time τ, in which the whole area $\delta+\varepsilon$ must be described.

Now let $\frac{C}{1}=\frac{A}{B}, \frac{G}{1}=\frac{D}{E}, \frac{g}{1}=\frac{d}{e}$, and $\frac{\gamma}{1}=\frac{\delta}{\varepsilon}$; and let S be the true time between the first and third observations ; and with the signs + and – properly observed the numbers m and n are sought from the observations, from this rule, that there shall be $2G-2C=mG-mg+nG-n\gamma$, and $2T-2S=mT-mt+nT-n\tau$. And if in the first operation I designates the inclination of the plane of the trajectory to the plane of the ecliptic, and K the longitude of one or the node, $I+nQ$ will be the true inclination of the plane of the trajectory to the plane of the ecliptic, and $K+mP$ the true longitude of the node. And finally if in the first operation, and in the second, and third, the quantities R, r and ρ designate the latera recta of the trajectory, and the quantities $\frac{1}{L}, \frac{1}{l}, \frac{1}{\lambda}$ the transverse widths [or diameters] of the same respectively : $R+mr-mR+n\rho-nR$ will be the true latus rectum, and $\frac{1}{L+ml-mL+n\lambda-nL}$ the true width of the trajectory that the comet will describe. But with the transverse diameter given also the periodic time of the comet is given. *Q.E.I.*

Otherwise the periodic times of revolution of comets, and the transverse diameters of the orbits, cannot be determined with enough accuracy, except by collating the comets among themselves, which appear at different times. If more comets, after equal intervals of time, are found to describe the same orbit, it will be concluded that all these are one and the same comet, revolving in the same orbit. And then at last from the given times of revolution the transverse diameters will be given, and from the diameters the orbits of the ellipses will be determined.

To this end therefore the trajectories of several comets are required to be computed, from the hypothesis that they shall be parabolas. For trajectories of this kind always agree approximately with the phenomena. That made clear, not only from the parabolic trajectory of the comet of the year 1680, as well as with the observations brought together above, but also from the particular features of that comet, which appeared in the years 1664 and 1665, and had been observed by *Hevelius*. This he had worked out from the observations of the longitude and latitude of this comet, but less accurately. From the same observations our countryman *Halley* computed the places of this comet anew, and then finally from such positions found determined the trajectory of the comet. Moreover he found its ascending node in ♊ $21^0.13'. 55''$, the inclination of the orbit to the plane of the ecliptic $21^0. 18'. 40''$, the distance of the perihelion from the node in the orbit $49^0.17'. 30''$. The perihelion in ♌ $8^0.40'. 30''$ with the latitude south from the centre of the sun $16^0.1'.45''$. The comet in perihelion on *Novemb*er 24th at $11^h. 52'$. p.m. *London* time, or *London*, or $13^h.8'$ *Danzig* [*Gdansk*], in the old style, and latus rectum of the parabola 410286, with the mean distance of the earth from the sun being at the distance 100000. Since the proper places of the comet in this orbit agree with the computation, it will be apparent from the following table computed by *Halley*.

Apparent time in *Gdansk*, old style.	Observed distances of comet.		Positions observed.		Positions computed in Orbit.
December.					
$3^d.18^h.29'\frac{1}{2}$	from heart of *Leo* " *Spica Virginis*	$46^{gr}.24'.20''$ 22.52.10	Long.♎ Lat. south.	7^{gr}. 1'. 0" 21.39. 0	♎ 7^{gr}.1' .29" 21.38.50
$4.18.1\frac{1}{2}$	" heart of *Leo* " *Spica Virginis*	46. 2.45 23.52.40	Long.♎ Lat. south.	16.15. 0 22.24. 0	♎ 0.16. 5 22.24. 0
7.17.48	" heart of *Leo* " *Spica Virginis*	44.48. 0 27.50.40	Long.♎ Lat. south.	3. 6. 0 25.22. 0	♎ 3. 7. 33 25.21.40
17.14.43	" heart of *Leo* " right arm of *Orion*	53.15.15 45.43.30	Long.♎ Lat. south.	2.50. 0 49.25. 0	♌ 2.50. 0 49.25. 0
19.9.25	" *Procyone* " Bright star jaw *Cetus*	35.13.15 52.56. 0	Long.♊ Lat. south.	28.40.30 25.48. 0	♊ 28.43. 0 45.46. 0
$20.9.53\frac{1}{2}$	" *Procyone* " Bright star jaw *Cetus*	40.49. 0 40. 4. 0	Long.♊ Lat. south.	13. 3. 0 39.54. 0	♊ 3. 5. 0 39.53. 0
$21.9.9\frac{1}{2}$	" right arm of *Orion* " Bright star jaw *Cetus*	26.21.25 29.28. 0	Long.♊ Lat. south.	2.16. 0 33.41. 0	♊ 2.18.30 33.39.40
22.9. 0	" right arm of Orion " Bright star jaw *Cetus*	29.47. 0 30.29.30	Long.♉ Lat. aust.	24.24. 0 27.45. 0	♉ 24.27. 0 27.46. 0
26.7.58	" Bright star *Aries* " *Aldebaran*	23.20. 0 26.44. 0	Long.♉ Lat. south.	9. 0. 0 12.36. 0	♉ 9. 2.28 12.34.13
27.7.58	" Bright star *Aries* " *Aldebaran*	20.45. 0 28.10. 0	Long.♉ Lat. south.	7. 5.40 10.23. 0	♉ 7. 8.45 10.23.13
28.7.58	" Bright star *Aries* " *Hyades*	18.29. 0 29.37. 0	Long.♉ Lat. south.	5.24.45 8.22.50	♉ 5.27.52 8.23.37
31.6.45	" Girdle *Androm*. " *Hyades*	30.48.10 32.53.30	Long.♉ Lat. south.	2. 7.40 4.13. 0	♉ 2. 8.20 4.16.25
Jan. 1665 $7.7.37\frac{1}{2}$	" Girdle *Androm*. " *Hyades*	25.11. 0 37.12.25	Long.♈ Lat. bor.	28.24.47 0.54. 0	♈ 28.24. 0 0.53. 0
13.7. 0	" Girdle Androm. " *Hyades*	28. 7.10 38.55.20	Long.♈ Lat. north.	27. 6.54 3. 6.50	♈ 27. 6.39 3. 7.40
24.7. 29	" Girdle *Androm*. " *Hyades*	20.32.15 40. 5. 0	Long.♈ Lat. north.	26.29.15 5.25.50	♈ 26.28.50 5. 26. 0
Feb. 7.8. 37			Long.♈ Lat. north.	27. 4.46 7. 3.29	♈ 27.24.55 7. 3.15
22.8.46			Long.♈ Lat. north.	28.29.46 8.12.36	♈ 28.29.58 8.10.25
Mar. 1.8. 16			Long.♈ Lat. north.	29.18.15 8.36.26	♈ 29.18.20 8.36.26
7.8.37			Long.♈ ♉ Lat. north.	0. 2.48 8.56.30	♉ 0. 2.42 8.56.56

In the month of *February* at the start of the year 665, the first star of *Aries*, that in what follows I will call γ, was at ♈ 18^0. 30'. 15" with the latitude to the North 7^0. 8'. 58". The second star of *Aeries* was at ♈ 29^0. 17'.18" with the latitude to the North 8^0. 18'.16". And a certain other star of the seventh magnitude, that I will call *A*, was at ♈ 28^0.14'.45" with the latitude to the North 8^0.28'. 33". Now the comet on *Feb*. $7^d.7^h$. 30' *Paris* time (that is *Feb*. 7^d. 8^h. 37 *Gdansk*) old style, made a triangle with the stars γ and *A* right-angled at γ. And the distance of the comet from the star γ was equal to the distance of the stars γ and *A*, that is 1^0.19'. 46" on a great circle, and therefore that was 1^0. 20'.26" in the parallel of the latitude of the star γ. Whereby if from the longitude of the star γ the longitude 1^0.20'. 26" were taken away, the longitude of the comet ♈ 27^0. 9'. 49" will remain. *Auzout* from his observation put this comet in ♈ 27^0. 0' roughly. And from the diagram, from which *Hook* had traced out its motion, that now was in ♈ 26^0.59'. 24". From the mean ratio I have put the same in ♈ 27^0. 4'. 46". From the same observation

Auzout now put the latitude of the comet to be 7^0 and 4' or 5' towards the North. The same would be more correctly 7^0. 3'.29", clearly with the difference of the latitude of the comet and the star γ, being equal to the difference of the longitudes of the stars γ and A.

Feb. 22^d. 7^h. 30' *London*, that is Feb. 22^d. 8^h. 46' *Gdansk*, the distance of the comet from the star *A*, according to the observation of *Hook* from his own delineated scheme, and according to the figure of *Petit* traced after the observations of *Auzout*, it was the fifth part of the distance between the star *A* and the first of *Aries*, or 15" 57". And the distance of the comet from the line joining the star *A* and the first of *Aries* was the fourth part of the same fifth part, that is 4'. And thus the comet was in ♈ 28^0.29'. 46", with the latitude North 8^0. 12'. 36".

March 1^d. 7^h. 0' *London* time, that is *March* 1^d. 8^h. 16' *Gdansk*, the comet was observed near the second star of *Aries*, with the distance between the same being to the distance between the first and the second of *Aries*, that is to 1^0. 33', as 4 to 45 according to *Hook*, or as 2 to 23 following *Gottignies*. From which the distance of the comet from the second star of *Aries* was 8'. 16" following *Hook*, or 8' 5" following *Gottignies*, or in the ratio of the mean 8'. 10". Now the comet according to *Gottignies* had just progressed beyond the second star of *Aries* around a distance of a quarter or a fifth part of the journey completed in a single day, that is, around 1'. 35" (with which *Auzout* agreed well enough) or a little less following *Hook*, thing of 1'. Whereby if to the first longitude of *Aries* there is added 1', and to its latitude 8'. 10", the longitude of the comet will be found ♈ 29^0. 18', and the latitude to the North 8^0. 36'. 26".

March 7^d. 7^h.30' *Paris* time (that is *March* 7^d.8^h. 37' *Gdansk*) from the observations of *Anzout* the distance of the comet from the second star of *Aries* was equal to the distance of the second star of *Aries* from the star *A*, that is 52'.29". And the difference of the longitudes of the comet and of the second star of *Aries* was 45' or 46', or in the mean ratio of 45'. 30". And thus the comet was in ♉ 0^0. 2'.48". From the diagram of observations of *Auzout*, that *Petit* constructed, *Hevelius* deduced the latitude of the comet to be 8^0.54'. But the engraver curved the path of the comet irregularly at the end of the motion, and *Hevelius* in the diagram of *Auzout's* observations by himself corrected the irregular curvature, and thus the latitude of the comet was made to be 8^0. 55'. 30". And by correcting the irregularity a little more, the latitude emerged to be 8^0. 56', or 8^0. 57'.

Here the comet was also seen on the 9th day of *March*, and then it must have been located in ♉ 0^0.18', with a latitude North of about 9^0.3'$\frac{1}{2}$.

This comet appeared for three months, and to have described almost six signs, and in one day it completed almost 20^0. Its course was deflected a great deal from a great circle, curved in the North ; and its backwards motion at the end was made direct. And not standing in the way of such an unusual course, the theory agreed with the observations no less accurately from start to finish, as the theory of the planets is accustomed to agree with the observations of these, as will be apparent on inspecting the table. Yet around two minutes are required to be taken away at first, when the comet was the travelling the fastest; that which can be done by taking away 12' from the angle between the ascending node and the perihelion, or by putting that angle in place to be 49^0. 27'. 18". The annual parallax of each comet has been most noticeable (both of this and of the above), and thus the annual motion of the earth in a great circle.

Also the theory is confirmed by the motion of the comet, which appeared in the year l683. This was backwards in orbit, its plane with the plane of the ecliptic contained almost a right angle. The ascending node (from *Halley's* computation) was in ♍ 23^0.23'; the inclination of the orbit to the ecliptic 83^0.11'; the perihelion in ♊ 25^0. 29'. 30"; the distance of the perihelia from the sun, 56020, with the radius of the great orbit being of magnitude 100000, and with the time of the perihelion *July* $2^d.3^h$.50'. Moreover the locations of the comet in this orbit computed by *Halley*, and collated with the places observed by *Flamsted*, are shown in the following table.

1683 Observed mean time.	Place of Sun.	Comp. Long. of comet	Comp.Lat. North.	Long.obs. of comet	Lat. North Observ.	Differ. Long.	Differ. Lat.
d. h.	gr. ' "	gr. ' "	gr. ' "	gr. ' "	gr. ' "		
Jul. 13.12. 5'	♌ 1. 2.30	♋ 13. 5.42	29.28.13	♋ 13. 6.42	29.28.20	+1'.0"	+0'.7"
15.11.15	2.53·12	11.37. 4	29.34. 0	11.39.43	29.34.50	+1.55	+0.50
17.10.20	4.45.45	10. 7. 6	29.33.33	10. 8.40	29.34. 0	+1.34	+0.30
23.13.40	10·38.21	5.10.27	28.51.41	5.11.30	28.50.28	+1. 3	–1.14
25.14. 5	12.35.28	3.27.53	24.24.47	3.27. 0	28.23.40	–0.53	–1. 7
31. 9.42	18. 9.22	♊ 27.55. 3	26.22.52	♊ 27.54.24	26.22.25	–0.39	–0.27
31.14.55	18.21.53	27.41. 7	16.16.57	27.41. 8	26.14.50	+0. 1	–2. 7
Aug. 2.14.56	20.17.16	25.29.32	25.16.19	25.28.46	25.17.28	–0.46	+1.9
4.10.49	22. 2.50	23.18.20	24.10.49	23.16.55	24.12.19	–1.25	+1.30
6.10. 9	23.56.45	20.42.23	22.47. 5	20.40.32	22.49. 5	–1.51	+2. 0
9.10.26	26.50.52	16. 7.57	20. 6.37	16. 5.55	20. 6.10	–2. 2	–0.27
15.14. 1	♍ 2.47.13	3.30.48	11.37.33	3.26.18	11.32. 1	–4.30	–5.31
16.15.10	3.48. 2	0.43. 7	9.34.16	0.41.55	9.34.13	–1.12	–0. 3
18.15.44	5.45.33	♉24·52.53	5.11.15	♉ 24.49. 5	5. 9.11	–3.48	–2. 4
			South.		South.		
22.14.44	9.35.49	11. 7.14	5.16.58	11. 7.12	5.16.58	–0. 2	–0. 3
23.15.52	10.36.48	7. 2.18	8.17. 9	7. 1.17	8.16.41	–1. 1	–0.28
26.16. 2	13·31.10	♈ 24.45.31	16.38. 0	♈ 24.44. 0	16.38.20	–1.31	+0.20

Also the theory is confirmed for the retrograde motion of the comet, which appeared in the year 1682. The ascending node of this (from the computation of *Halley*) was in ♉ 21^0.16'. 30". The inclination of the orbit to the plane of the ecliptic 17^{gr}. 6'. 0". The perihelion in ♒ 2^0. 52'. 50". The distance from the perihelion to the sun 58328, with the radius of the great orbit proving to be 100000. And the time equal to the perihelion *September* $4^d.7^h$. 39'. Truly the places computed from the observations of *Flamsted*, and collated with the places computed by the theory, are shown in the following table.

1682. Observed mean time.	Position of the sun.	Long.Comp. of comet.	Comp. Northern Lat.	Obs. Long. of comet.	Observ. Northern Lat.	Differ. Long.	Differ. Lat.
d. h.	gr. ' "	gr. ' "	gr. ' "	gr. ' "	gr. ' "	' "	' "
Aug. 19.16.38	♍ 7. 0. 7	18.14.28	25.50. 7	18.14.40	25.49.55	–0.12	+0.12
20.15.38	7.55.52	24.46.23	26.14.42	24.46.22	26.12.52	+0. 1	+1.50
21. 8.21	8.36.14	29.37.15	26.20. 3	29.38. 2	26.17.37	–0.47	+2.26
22. 8. 8	9.33.55	♍ 6.29.53	26. 8.42	♍ 6.30. 3	26. 7.12	–0.10	+1.30
29. 8.20	16.22.40	♒12.37.54	18.37.47	♒12.37·49	18.34. 5	+0. 5	+3.42
30. 7.45	17.19.41	15.36. 1	17.26.43	15.36.18	17.27.17	+0.43	–0.34
Sept. 1. 7.33	19.16. 9	20·30·53	15.13. 0	20·27· 4	15. 9.49	+3.49	+3.11
4. 7.22	22.11.28	25.42. 0	12.23.48	25.40.58	12.22. 0	+1. 2	+1.48
5. 7.32	23.10.29	27. 0.46	11.33. 8	26.59.24	11.33.51	+1.22	–0.43
8. 7.16	26. 5.58	29.58.44	9.26.46	29.58.45	9.26.43	–0. 1	+0. 3
9. 7.26	27. 5. 9	♏ 0.44.10	8.49.10	♏ 0.44. 4	8.48.25	+0. 6	+0.45

Also the theory for the retrograde motion of the comet is confirmed, which appeared in the year 1723. The ascending node of this (being computed by Mr. *Bradley*, *Savilian* professor of astronomy at *Oxford*) was in ♈ 14^0.16'. The inclination of the orbit to the plane of the ecliptic 49^0. 59'. The perihelion in ♉ 12^0. 15'. 20''. The distance of the perihelion from the sun 998651, with the radius of the great orbit taken to be 1000000, and the corrected time of the perihelion to be *September*. 16^d. 16^h. 10'. Indeed the places calculated in this orbit by *Bradley*, and the places collated with the observations by our compatriots *Dr*. *Pound* and *Dr*. *Halley* are shown in the following table.

1723 Observed mean time.	Observed. Long. of comet.	Observ.Lat.North.	Comput. Long. of comet.	Comput. Lat. of comet..	Differ. Long.	Differ. Latit.:
Octob. 9^d. 8^h. 5'	♒ 7^0.22'.15''	5^0. 2'. 0''	♒ 7^0.21'.26''	5^0. 2'.47''	+49''	– 47''
10. 6 .21	6.41.12	7.44.13	6.41.42	7.43.18	– 50	+ 55
12. 7.22	5.39.58	11.55. 0	5.40.19	11.54.55	– 21	+ 5
14. 8.57	4.59.49	14.43.50	5. 0.37	14.44. 1	– 48	– 11
15. 6.35	4.47.41	15.40.51	4.47.45	15.40.55	– 4	– 4
21. 6.22	4. 2.32	19.41.49	4. 2.21	19.42. 3	+ 11	– 14
22. 6.24	3.59. 2	20. 8.12	3.59.10	20. 8.17	– 8	– 5
24. 8. 2	3.55.29	20.55.18	3.55.11	20.55. 9	+ 18	+ 9
29. 8.56	3.56.17	22.20.27	3.56.42	22.20.10	– 25	+ 17
30. 6.20	3.58. 9	22.32.28	3.58.17	22.32.12	– 8	+ 16
Nov. 5. 5.53	4.16.30	23.38.33	4.16.23	23.38. 7	+ 7	+ 26
8. 7. 6	4.29.36	24. 4.30	4.29.34	24. 4.40	– 18	– 10
14. 6.20	5. 2.16	24.48.46	5. 2.51	24.48.16	– 35	+ 30
20. 7.45	5.42.20	25.24.45	5.43.13	25.25.17	– 53	– 32
Dec. 7.6.45	8. 4.13	26.54.18	8. 3.55	26.53.42	+ 18	+ 36

From these examples it is made abundantly clear that the motion of comets set out by our theory are shown no less accurately, than the motion of the planets are accustomed to be shown by the same theories. And therefore the orbits of comets can be enumerated by this theory, and the periodic time of the comet revolving in some orbit finally becomes known, and then at last the transverse width of the elliptic orbits and the altitudes of the aphelions become known.

The retrograde comet, which appeared in the years 1607, described an orbit, the ascending node of which (from *Halley's* computation) was in ♉12^0. 21' ; the inclination of the plane of the orbit to the plane of the ecliptic was 17^0. 2 ' ; the perihelion was in ♒ 2^0.16'; and the distance of the perihelion from the sun was 58680, with the radius of the great circle taken to be 100000. And the comet was in perihelion in *October* 16^d.3^h.50'. This agreed approximately with the orbit of the comet which appeared in the year 1682. If these two were one and the same, this comet was revolving in a time of 75 years, and the major axis of its orbit will be to the major axis of the great orbit of the earth, as $\sqrt[3]{75\times 75}$ to 1, or around 1778 to 100. And because the distance of the aphelion of this comet from the sun, will be to the mean distance of the earth from the sun, almost as 35 to 1. With which understood, it would not be difficult to determine the elliptic orbit of this comet. And thus this will itself come about if the comet, in the interval of 75 years, may be returned hereafter in this orbit. The remaining comets are seen to be revolving with a greater time and to ascend higher.

The remaining comets, on account of the great magnitude of their numbers, and the great distance of the aphelion from the sun, and the long delay at the aphelions, by gravity must disturb each other , and both the eccentricities and times of revolution at some times are increased a little, at other times diminished. Hence it cannot be expected that the same comet will return accurately in the same orbit, and with the same periodic time. It will suffice if no greater changes come about, than may arise from the aforesaid causes.

And hence the reason is provided, why comets are not restrained to the Zodiac in the manner of planets, but depart thence and by various motions be carried to all regions of the heavens. Evidently to that end, so that at their aphelions, where they are moving the slowest, as they shall be at great distances from each other so that their mutual attraction will not be experienced. It is by that reason why the comets which descend from the highest, and which as a consequence moving the most slowly at their aphelions, must return to the highest places.

The comet which appeared in the year 1680, was at a shorter distance from the sun at its perihelion than a sixth part of the diameter of the suns ; and therefore because of the maximum velocity in that vicinity, and some density of the sun's atmosphere, it must have experienced some resistance, and be retarded by a small amount, and to approach closer to the sun : and by getting closer to the sun in the individual revolutions, finally it will fall into the body of the sun. But at the aphelions where it is moving the slowest, by the attraction of other comets it can be retarded, and suddenly fall into the sun [at its next approach]. Thus also the fixed stars, which exhaust themselves little by little from light and vapours, are able to renew themselves by the comets that fall into them, and rekindled by new fuel arise as new stars. Fixed stars are of this kind, which suddenly appear, and at the start with maximum brilliance, and subsequently vanish little by little. Such was the star that *Cornelius Gemma* barely saw on the quiet night of 8th of *November* 1572 in the chair of *Cassiopeia*, illuminating that part of the heavens ; but the following night (*November* 9th) it was seen most brilliant among all the fixed stars, and with its light scarcely conceding to the light from *Venus*. *Tycho Brahe* saw this on the eleventh day of the same month when it was maximally brilliant ; and from that time decreasing little by little and in the space of the sixteen months it was observed to be vanishing. In the month of *November*, when it first appeared, it was equal to the light from *Venus*. In the month of *December* diminished a certain amount it was seen to equal the light of Jupiter. In the year 1573, in the month of *January* it was less than Jupiter and greater than *Sirius*, to which at the end of *February* and the beginning of *March* it emerged equal. In the month of *April* and in *May* it was a star of the second magnitude, in *June, July* and *August* it was equal to stars of the third magnitude, *September, October* and *November* to stars of the fourth magnitude, *December* and in the month of *January* of the year 1574, to stars of the fifth magnitude, and in the month of *February* it was seen equal to stars of the sixth magnitude, in the month of *March* it vanished from sight. In the beginning the colour was clear, white and very bright, afterwards yellow, and in the month of March in the year 1573 reddish in the image of *Mars* or of the star *Aldebaran*; but in *May* it took on a bluish- white appearance, such as we see in Saturn, which colour it maintained until the end, when it was made more obscure, yet always becoming more obscure.

Such also was the star in the right foot of the *Serpent*, that students of Kepler saw to appear initially on the 30th day of *September* in the year 1604, in the old style, and by its light surpassed *Jupiter*, when in the preceding night it had been barely apparent.
From that truly in a little time it decreased, and in the space of fifteen or sixteen months it had vanished from view. It was a new star of this kind which appeared so brilliant to the

fixed stars, in the time of *Hipparchus*, that persuaded him, it is said [as reported by *Pliny*], to observe the fixed stars, and to give them in a catalogue. But the fixed stars, which in turn appear and vanish, and which increase in brightness little by little, and by their light scarcely surpass any fixed stars of the third magnitude, are seen to be stars of another kind and are seen in turn revolving with one part bright and another part obscure. But vapours, which arise from the sun and from the fixed stars and from the tails of comets, can fall by their gravity into the atmospheres of planets and there to be condensed and converted into water and humid spirit, and in the end by a slow heat to be changed little by little into salts and sulphur, tinctures, slime and mud, clay and sand, stones and coral, and other terrestrial substances.

GENERALE SCHOLIUM .

The hypothesis of vortices is beset with many difficulties. Since each and every planet by a ray drawn to the sun describes areas proportional to the times, the periodic times of the parts of the vortex must be in the square ratio of the distances from the sun. In order that the periodic times of the planets shall be in the three on two proportion of the distances from the sun, the periodic times of the parts of the vortices must be in the three on two proportion of the distances. In order that the smaller vortices around *Saturn*, *Jupiter*, and the other planets are able to be kept rotating and quietly swimming in the vortex of the sun, the periodic times of the parts of the vortex of the sun must be equal. The revolutions of the sun and planets about their axes, which must agree with the motions of the vortices, disagree with all these proportions. The motions of comets are the most regular, and they observe the same laws of motion as the planets, and are unable to be explained by vortices. Comets are carried in extremely eccentric motions into all parts of the heavens, which cannot happen, unless vortices are renounced.

Projectiles, in our air, only experience the resistance of the air. In the most subtle air, as it becomes in a *Boyle* vacuum, the resistance stops, accordingly a fine feather and a gold solid fall together with the equal velocity in this vacuum. And equal is the account of celestial spaces, which are above the atmosphere of the earth. All bodies must be able to move most freely in these spaces ; and therefore planets and comets to be revolving perpetually in orbits, given in kind and position, following the laws set out above. Indeed they will persevere in their orbits by the laws of gravity, but the original situation of the orbits cannot be accounted for by these laws.

The six principal planets are revolving around the sun in concentric circles, moving in the same direction, approximately in the same plane. Ten moons are revolving around the earth, Jupiter, and Saturn in concentric circles, moving in the same direction, in approximately the planes of the planets. And all these regular motions do not have their origins from the causes of mechanics ; if indeed the comets are in very eccentric orbits, and are sent freely into all parts of the heavens. By which motions comets generally cross the orbits of the planets quickly and easily, and into their aphelions where they are slower and they remain there for a long time ; since they are most distant from each other, so that they attract each other minimally. This most elegant structure of the sun, planets and comets could only arise from the planning and in the dominion of an intelligent and powerful being. And if the fixed stars shall be the centres of similar systems, all these likewise constructed under the same plan in the dominion of *One*: especially since the light of the fixed stars shall be of the same nature as the light from the sun, and all the light is sent from all the systems into every system in turn. And so that the systems of fixed stars do not fall into each other by gravitation, here in turn the same immense distance has been put in place.

This universal presence rules not as the soul of the world, but as the master of everything. And therefore he is usually called *Παντοχράτωρ* the master of all his dominion. For *deus* [god] is a relative word and is referring to servants: and the deity is the master of the gods, not of a special body, as they believe in which *deus* is the soul of the world, but only of servants. The Master is an eternal, infinite, absolutely perfect being: but some perfect being is not the master god without a dominion. For we may say my *deus* [god], your *deus* [god], the god of *Israel*, god of the gods, and master of the masters : but we cannot say my eternal one, your eternal one, the eternal one of *Israel*, the eternal one of the gods ; nor can we say my infinite one, your perfect one. These names do not have a relation to servants. The word *deus* actually signifies master [*dominum*]: but every master is not a god. [I have omitted Newton's note on the origin of this: see *Cohen* p.941 if interested.] The dominion of a spiritual being constitutes god; indeed the true god come from true gods, the greatest god from the greatest, an imaginary god from the imaginary. And from the true domination it follows that the true god is alive, intelligent and powerful ; and from his remaining perfections to be the greatest, or the peak of perfection. He is eternal, infinite, omnipotent, omnipresent, and omniscient, that is, enduring from infinity to infinity, and present from the infinite to the infinite : ruling all; and knowing everything, which can arise or not able to arise. Not eternity and infinite but eternal and infinite ; not duration or space, but enduring and present. He always, and is present everywhere, and by existing always and everywhere, constitutes duration and space. Since any small particle of space shall be *always*, and any indivisible instant *everywhere*, certainly the maker and master of all things cannot be missing at any time or place. Every perceiving soul who experiences in different times, with different senses, and by the motion of several organs, is still always the same indivisible person. There are these successive parts in the duration [of a person's experiences of his existence], and these parts coexisting in space : there is nothing [in ordinary space and time] that has any resemblance to that which constitutes the person that is the man, nor in his thinking principle [I have relied here on Madame du Chatelet's translation]; and there will be much less than in the thinking substance of god. Every man, whatever thing he is thinking about, is one and the same man during his life from all the individual sense organs. God is one and the same god always and everywhere. Omnipresence is not by *virtue* alone, but also by *substance* : for virtue without substance cannot exist. Everything is moved generally and containing the person himself, but not without the some action from other to be experienced by him. God suffers nothing from the motions of bodies : they experience no resistance from the omnipresence of god. It has been confessed that it is necessary that the supreme god must exist : and by the same necessity it is *always* and *everywhere*. From which also the whole being the same to him, all eyes, all ears, all mind, all arms, all the strength of feeling, of intelligence, and of action, but in a manner not human, still less than with a body, and in a manner completely unknown to us. As a blind man has no idea of colours, thus we have no ideas of the ways in which the wisest god experiences and knows all things. Without any body and bodily shape, and thus unable to be seen, nor to be heard, touched, or worshipped under any kind of thing related to the body. We have ideas of his attributes, but that shall be of other substances unknown to us. We can see only the figures and colours of bodies, we hear only their sounds, we touch only their external surfaces, we smell only their odours, and we taste only their flavours: but as for the insides of substances, we know them not by any sense, nor by any reflection ; and we have far less idea about the substance of god. This we know only by his properties and attributes, and by the wisest and most optimal structures of things, and by their final causes, these we admire on account of their perfections ; moreover they are venerated and worshipped on account of his dominion. For we adore as servants, and god without a dominium, providence, final causes, is nothing else but fate and nature. From blind necessity metaphysics, which certainly is the same always and everywhere, no variation of things arises. The diversity which reigns over everything, whatever the times and the places , by necessity can come only from the ideas and will of an existing being. But it is said that god allegorically can see, hear, and that he can laugh, love, hate, desire, give, take, enjoy, be angry, fight, fabricate. For everything that one can say about god is taken from some comparison with human things; but these comparisons, although they are very imperfect, yet have some likeness. And thus concerning god, from which certainly from the difference of phenomena, pertain to natural philosophy.

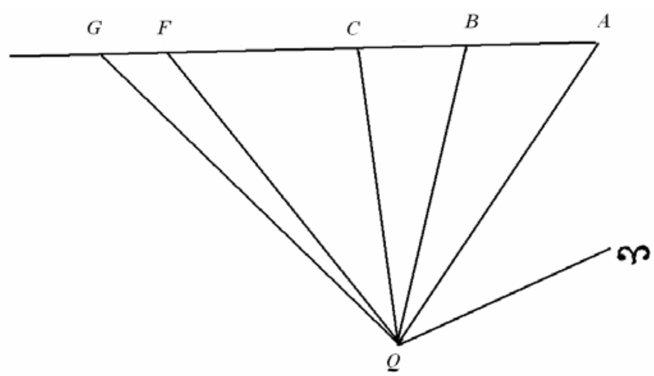
At this point I have established the phenomena of the heavens and of our seas through the force of gravity, but I have not assigned the cause of gravity. Certainly this force arises from some cause, which penetrates as far as to the centre of the sun and of the planets without diminution of strength; and which acts not only on the quantity of particles on the *surface*, on which it acts, (as they are the customary mechanical causes) but also on the quantity of the *solid* material ; and the action of this is extended thence over immense distances, always by decreasing in the inverse square of the distances. Gravity in the sun is composed from the gravity of the individual particles of the sun, and by receding from the sun it decreases accurately in the square ratio of the distances as far as to the orbit of *Saturn*, as that is evident from the quiet of the aphelion of the planets, and as far as to the final aphelions of comets, but only if these aphelions are at rest. Indeed I have not yet been able to deduce an account of these pleasing properties from the nature of the phenomena, and I devise no hypothesis. For whatever cannot be deduced from phenomena, it is required to call *hypothesis* ; and hypothesis, whether it be of some metaphysical, physical, occult, or mechanical qualities, have no place in *experimental philosophy*. In this philosophy the propositions are deduced from the phenomena, and rendered general by induction. Thus the impenetrabilities, mobilities, and the impetus of bodies and the laws of the motions and of gravity have become known. And it is enough that gravity actually exists, and acts according to the laws set forth by us, and it is sufficient to explain all the motions of heavenly bodies and those of our seas.

Now this will be the place to add something on this most subtle kind of spirit that penetrates through all solid bodies, and which is hidden in their substance ; it is by this force and the action of this spirit that the particles of bodies attract each other mutually to the smallest distances, and are made to stick together ; and it is by this same means that electrified bodies are acted on at greater distances, both by repelling as well as attracting small bodies in the vicinity ; and the light is emitted , reflected, refracted, inflected [*i.e.* internally reflected], and the bodies heated; all the sensations are excited, and the members of animals are moved according to its will, evidently by the vibrations of this spirit through the solid filaments of the nerves from the external sense organs to the brain and propagated to the muscles. But these are things that cannot be explained in a few words; nor do we have a sufficient supply of experiments, by which the laws of action of this spirit must be accurately determined and shown.

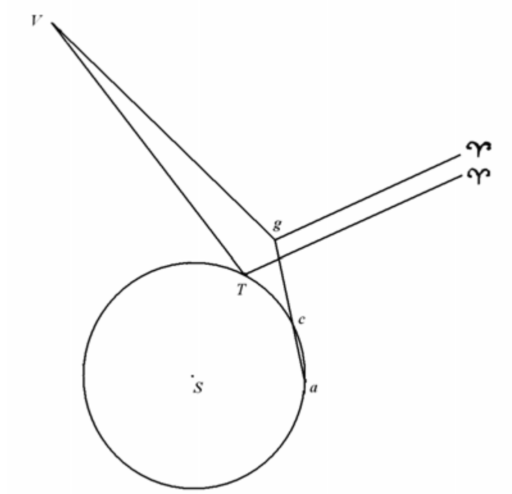
The End.

LEMMA IV.
Cometas esse luna superiores & in regione planetarum versari.

Ut defectus & parallaxeos diurnae: extulit cometas supra regiones sublunares, sic ex parallaxi annua convincitur eorum descensus in regiones planetarum. Nam cometae, qui progrediuntur secundum ordinem signorum, sunt omnes sub exitu apparitionis aut solito tardiores aut retrogradi, si terra est inter ipsos & solem; at iusto celeriores si terra vergit ad oppositionem. Et contra, qui pergunt contra ordinem signorum sunt iusto celeriores in fine apparitionis, si terra versatur inter ipsos & solem; & iusio tardiores vel retrogradi, si terra sita est ad contrarias partes. Contingit hoc maxime ex motu terrae in vario ipsius situ, perinde ut sit in planetis, qui pro motu terrae vel conspirante vel contrario nunc retrogradi sunt, nunc tardius progredi videntur, nunc vero celerius. Si terra pergit ad eandem partem cum cometa, & motu angulari circa solem tanto celerius fertur, ut recta per terram & cometam perpetuo ducta convergat ad partes ultra cometam, cometa e terra spectatus ob motum suum tardiorem apparet esse retrogradus; sin terra tardius fertur, motus cometae (detracto motu terrae) sit saltem tardior. At si terra pergit in contrarias partes, cometa exinde velocior apparet. Ex acceleratione autem vel retardatione vel motu retrogrado distantia cometae in hunc modum colligitur. Sunto ♈*QA*, ♈*QB*, ♈*QC* observatae tres longitudines cometae sub initio motus, sitque ♈ *QF* longitudo ultimo observata, ubi cometa videri desinit. Agatur recta *ABC*, cuius partes *AB*, *BC* rectis *QA* & *QB*, *QB* & *QC* interjectae, sint ad invicem ut tempora inter observationes tres primas. Producatur *AC* ad *G*, ut sit *AG* ad *AB* ut tempus inter observationem primam



& ultimam ad tempus inter observationem primam & secundam, & jungatur *QG*. Et si cometa moveretur uniformiter in linea recta, atque terra vel quiesceret, vel etiam in linea recta uniformi cum motu progrederetur; foret angulus ♈*QG* longitudo cometae tempore observationis ultimae. Angulus igitur *FQG*, qui longitudinum differentia est, oritur ab inaequalitate motuum cometae ac terrae. Hic autem angulus, si terra & cometa in contrarias partes moventur, additur angulo ♈*QG* & sic motum apparentem cometae velociorem reddit: sin cometa pergit in easdem partes cum terra, eidem subducitur, motumque cometae vel tardiorem reddit, vel forte retrogradum; uti modo exposui. Oritur igitur hic angulus praecipue ex motu terram, & idcirco pro parallaxi cometae merito habendus est, neglecto videlicet eius incremento vel decremento nonnullo; quod a cometae motu inaequabili in orbe proprio oriri possit. Distantia vero cometae ex hac parallaxi sic colligitur. Designet *S* solem, *acT* orbem magnum, *a* locum terrae in



observatione prima, *c* locum terrae in observatione tertia, *T* locum terrae in observatione ultima, & *T*♈ lineam rectam versus principium arietis ductam. Sumatur angulus ♈*TV* aequalis angulo ♈*QF*, hoc est, aequalis longitudini cometae ubi terra versatur in *T*. Jungatur *ac*, & producatur ea ad *g*, ut sit *ag* ad *ac* ut *AG* ad *AC*, & erit *g* locus quem terra tempore observationis ultimae, motu in recta ac uniformiter continuato, attingeret. Ideoque si ducatur *gr* ipsi *Tr* parallela, & caplatur angulus ♈*gV* angulo ♈*QG* aequalis, erit hic angulus ♈*gV* aequalis longitudini cometae e loco *g* spectati; and angulus *TVg* parallaxis erit, quae oritur a translatione terrae de loco *g* in locum *T*: ac proinde *V* locus erit cometae in plano eclipticae. Hic autem locus *V* orbe *Jovis* inferior esse solet.

Idem colligitur ex curvatura vim cometarum. Pergunt haec corpora propemodum in circulis maximis quamdiu moventur celerius; at in fine cursus, ubi motus apparentis pars illa, quae a parallaxi oritur, majorem habet proportionem ad motum totum apparentem, deflectere solent ab his circulis, & quoties terra movetur in unam partem, abire in partem contrariam. Oritur haec deflexio maxime ex parallaxi, propterea quod respondet motui terrae; & insignis eius quantitas, mea computo, collocavit disparentes cometas satis longe infra jovem. Unde consequens est quod in perigaeis & periheliis, ubi propius adsunt, descendunt saepius infra orbes martis & inferiorum planetarum.

Confirmatur etiam propinquitas cometarum ex luce capitum. Nam corporis coelestis a sole illustrati & in regiones longinquas abeuntis, diminuitur splendor in quadruplicata ratione distantiae: in duplicata ratione videlicet ob auctam corporis distantiam a sole, & in alia duplicata ratione ob diminutam diametrum apparentem. Unde si detur & lucis quantitas & apparens diameter cometae, dabitur distantia, dicendo quod distantia sit ad distantiam planetae, in ratione diametri ad diametrum directe & ratione subduplicata lucis ad lucem inverse. Sic minima capillitii cometae anni 1682 diameter, per tubum opticum sexdecim pedum a *Flamstedio* observata & micrometra mensurata, aequabat $2'.0''$; nucleus autem seu stella in media capitis vix decimam partem latitudinis huius occupabat, ideoque lata erat tantum 11" vel 12". Luce vera & claritate capitis superabat caput cometae anni 1680, stellasque primae vel secundae magnitudinis aemulabatur. Ponamus saturnum cum annulo suo quasi quadrupla lucidiorem suisse: & quoniam lux annuli propemodum aequabat lucem globi intermedii, & diameter apparens globi sit quasi 21", ideoque lux globi & annuli coniunctim aequaret lucem globi, cuius diameter esset 30": erit distantia cometae ad distantiam saturni ut 1 ad $\sqrt{4}$ inverse, & 12" ad 30" directe, id est, ut 24 ad 30 seu 4 ad 5. Rursus cometa anni 1665 mense aprili, ut auctor est *Helvelius*, claritate sua pene fixas omnes superabat, quinetiam ipsum saturnum, ratione coloris videlicet longe vividioris. Quippe lucidior erat hic cometa altero illo, qui in fine anni praecedemis apparuerat, & cum stellis primae magnitudinis conferabatur. Latitudo capillitii erat quasi 6', at nucleus cum planetis ope tubi optici collatus plane minor erat jove, & nunc minor corpore intermedio saturni, nunc ipsi aequalis judicabatur. Porro cum diameter capillitii cometarum raro superet 8' vel 12', diameter vero nuclei, seu stellae centralis sit quasi decima vel forte decima quinta pars diametri capillitii, patet stellas hasce ut plurimum eiusdem esse apparentis magnitudinis cum planetis. Unde cum lux earum cum luce saturni non raro conferri possit, eamque aliquando superet; manifestum est, quod cometae omnes in periheliis vel infra saturnum collocandi sint, vel non longe supra. Errant igitur toto coelo, qui cometas in regionem fixarum prope ablegant: qua certe ratione non magis illustrari deberent a sole nostro, quam planetae, qui hic sunt, illustrantur a stellis fixis.

Haec disputavimus non considerando obscurationem cometarum per fumum illum maxime copiosum & crassum, quo caput circundatur, quasi per nubem obtuse semper lucens. Nam quanto obscurius redditur corpus per hunc fumum, tanto propius ad solem accedat necesse est, ut copia lucis a se reflexae planetas aemuletur. Inde verisimile sit cometas longe infra sphaeram saturni descendere, uti ex parallaxi probavimus. Idem vero quam maxime confirmatur ex caudis. Hae vel ex reflexione fumi sparsi per aethera, vel ex luce capitis oriuntur. Priore casu minuenda est distantia cometarum, ne fumus a capite scmper ortus per spatia nimis ampla incredibili cum velocitate & expansione propagetur in posteriore referenda est lux omnis tam caudae quam capillitii ad nucleum capitis. Igitur si concipiamus lucem hanc omnem congregati & intra discum nuclei coarctari, nucleus ille jam certe, quoties caudam maximam & fulgentissimam emittit, jovem ipsum splendore suo multum superabit. Minore igitur cum diametro apparente plus lucis emittens, multo magis illustrabitur a sole, ideoque erit soli multo propior. Quinetiam capita sub sole delitescentia, & caudas cum maximas tum fulgentissimas instar trabium ignitarum nonnunquam emittentia, eodem argumento infra orbem veneris collocari debent. Nam lux illa omnis si in stellam congregati supponatur, ipsam venerem ne dicam veneres plures conjunctas quandoque superaret.

Idem denique colligitur ex luce capitum crescente in recessu cometarum a terra solem versus, ac decrescente in eorum recessu a sole versus terram. Sic enim cometa posterior anni 1665 (observante *Hevelio*) ex quo conspici coepit, remittebat semper de motu suo apparente, ideoque praeterierat perigaeum; splendor vero capitis nihilominus indies crescebat, usque dum cometa radiis solaribus obtectus desiit apparere. Cometa anni 1683 (observante eodem Hevelio) in sine mensis julii, ubi primum conspectus est, tardissime movebatur, minuta prima 40 vel 45 circiter singulis diebus in orbe suo conficiens. Ex eo tempore motus eius diurnus perpetuo augebatur usque ad Sept. 4. quando evasit graduum quasi quinque. Igitur toto hoc tempore cometa ad terram appropinquabat. Id quod etiam ex diametro capitis micrometro mensurata colligitur:

quippe quam Hevelius reperit Aug. 6. esse tantum 6'. 5" inclusa coma, at Sept. 2.esse 9'. 7". Caput igitur initio longe minus apparuit quam in fine motus, at initio tamen in vicinia solis longe lucidius extitit quam circa finem, ut refert idem *Hevelius*. Proinde toto hoc tempore, ob recessum ipsius a sole, quoad lumen decrevit, non obstante accessu ad terram. Cometa anni 1618 circa medium mensis *Decembris*, & iste anni 1680 circa finem eiusdem mensis, celerrime movebantur, ideoque tunc erant in perigaeis. Verum splendor maximus capitum contigit ante duas fere septimanas, ubi modo exierant de radiis solaribus; & splendor maximus caudarum paulo ante, in majore vicinitate solis. Caput cometae prioris, iuxta observationes *Cysati*, *Decem*. 1. majus videbatur stellis primae magnitudinis, & *Decemb.* 16. (jam in perigaeo existens) magnitudine parum, splendore seu claritate luminis plurimum defecerat. *Jan*. 7. *Keplerus* de capite incertus finem fecit observandi. Die 12 mensis *Decemb*. conspectum & a *Flamstedio* observatum est caput cometae posterioris in distantia novem graduum a sole; id quod stellae tertiae magnitudinis vix concessum fuisset. *Decemb*. 15 & 17 apparuit idem ut stella tertiae magnitudinis, diminutum utique splendore nubium iuxta solem occidentem. *Decemb*. 26. velocissime motus, inque perigaeo propemodum existens, cedebat ori pegasi, stella tertiae magnitudinis Jan. 3. apparebat ut stella quartae, *Jan*. 9. ut stella quintae, *Jan*.13. ob splendorem lunae crescentis disparuit. *Jan*. 25. vix aequabat stellas magnitudinis septima:. Si sumantur aequalia a perigreo hinc inde tempora, capita quae temporibus illis in longinquis regionibus posita, ob aequales a terra distantias, aequaliter lucere debuissent, in plaga solis maxime splenducre, ex altera perigaei parte evanuere. Igitur ex magna lucis in utroque situ differentia, concluditur magna solis & cometae vicinitas in situ priore. Nam lux cometarum regularis esse solet, & maxima apparere ubi capita velocissime moventur, atque ideo sunt in perigaeis; nisi quatenus ea major est in vicinia solis.

Corol. 1. Splendent igitur cometae luce solis a se reflexa.

Corol. 2. Ex dictis etiam intelligitur cur cometae tantopere frequentant regionem solis. Si cernerentur in regionibus longe ultra saturnum, deberent saepius apparere in partibus soli oppositis. Forent enim terrae viciniores, qui in his partibus versarentur; & sol interpositus obscuraret caeteros. Verum percurrendo historias cometarum, reperi quod quadruplo vel quintuplo plures detecti sunt in hemisphaerio solem versus, quam in hemisphaerio opposito, praeter alios proculdubio non paucos, quos lux solaris obtexit. Nimirum in descensu ad regiones nostras neque caudas emittunt, neque adeo illustantur a sole, ut nudis oculis se prius detegendos exhibeant, quam sint ipso jove propiores. Spatii autem tantillo intervallo circa solem descripti pars longe major sita est a latere terrae, quod solem respicit; inque parte illa majore cometae, soli ut plurimum viciniores, magis illuminari solent.

Corol. 3. Hinc etiam manifestum est, quod coeli resistentia destituuntur. Nam cometa vias obliquas & nonnunquam cursui planetarum contrarias secuti, moventur omnifariam liberrime, & motus suos, etiam contra cursum planetarum diutissime conservant. Fallor ni genus planetarum sint, & motu perpetuo in orbem redeant. Nam quod scriptores aliqui meteora esse volunt, argumentum a capitum perpetuis mutationibus ducentes, fundamento carere videtur. Capita cometarum atmosphaeris ingentibus cinguntur; & atmosphaerae inferne densiores esse debent. Unde nubes sunt, non ipsa cometarum corpora, in quibus mutationes illae visuntur. Sic terra si e planetis spectaretur, luce nubium suarum proculdubio splenderet, & corpus firmum sub nubibus prope delitesceret. Sic cingula jovis in nubibus planetae illius formata sunt, qua situm mutant inter se, & firmum jovis corpus per nubes illas difficillus cernitur. Et multo magis corpora cometarum sub atmosphaeris & profundioribus & crassioribus abscondi debent.

PROPOSITIO XL. THEOREMA XX.

Cometas in sectonibus conicis umbilicos in centro solis habentibus moveri, & radiis ad solem ductis areas temporibus proportionales describere.

Patet per corol. 1. Prop. X11I. Libri primi, collatum cum Prop. V11I, X11, & X11I. Libri tertii.

Corol. 1. Hinc si cometae in orbem redeunt; orbes erunt ellipses, & tempora periodica erunt ad tempora periodica planetarum in axium principalium ratione sesquiplicata. Ideoque cometae maxima ex parte supra planetas versantes, & eo nomine orbes axibus majoribus describentes, tardius revolventur. Ut si axis orbis cometae sit quadruplo major axe orbis saturni, tempus revolutionis cometae erit ad tempus revolutionis saturni, id est, ad annos 30, ut $4\sqrt{4}$ (seu 8) ad 1, ideoque erit annorum 240.

Corol 2. Orbes autem erunt parabolis adeo finitimi, ut eorum vice parabolae sine erroribus sensibilibus adhiberi possint.

Corol. 3. Et propterea (per Corol. 7. Prop. XVI. Lib. 1.) velocitas cometae omnis, erit temper ad velocitatem planetae cuiusvis circa solem in circulo revolventis, in subduplicata ratione duplae distantiae planetae a centro solis, ad distantiam cometae a centro solis quamproxime. Ponamus radium orbis magni, seu ellipseos in qua terra revolvitur semidiametrum maximam esse partium 100000000: & terra motu suo diurno mediocri describet partes 1720212, & motu horario partes $71675\frac{1}{2}$. Ideoque cometa in eadem telluris a sole distantia mediocri, ea cum velocitate quae sit ad velocitatem telluris ut $\sqrt{2}$ ad 1, describet motu suo diurno partes 2432747, & motu horario partes $101364\frac{1}{2}$. In majoribus autem vel minoribus distantiis, motus tum diurnus tum horarius erit ad hunc motum diurnum & horarium in subduplicata ratione distantiarum reciproce, ideoque datur.

Corol. 4. Unde si latus rectum parabolae quadruplo majus sit radio orbis magni, & quadratum radii illius ponatur esse partium 100000000 :area quam cometa radio ad solem ducto singulis diebus dcscribit, erit partium $1216373\frac{1}{2}$, & singulis horis area illa erit partium $50682\frac{3}{4}$. Sin latus rectum majus sit vel minus in ratione quavis, erit area diurnae and horaria major vel minor in eadem ratione subduplicata.

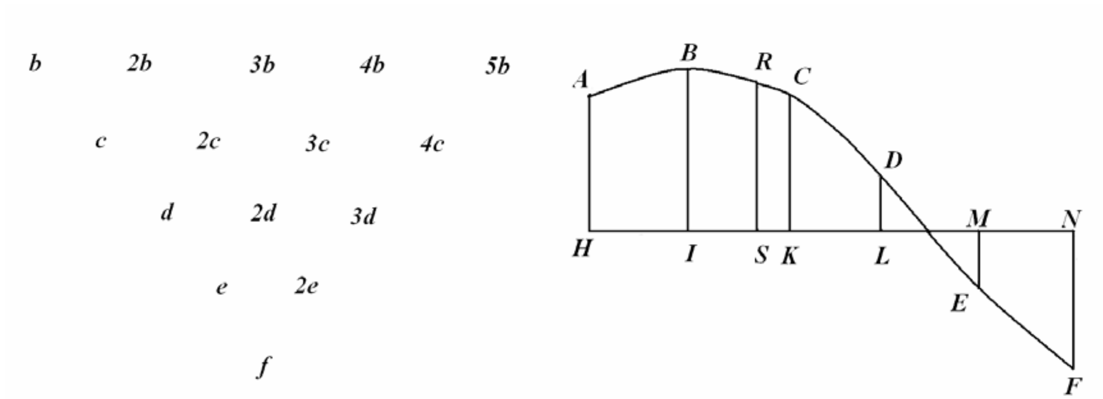
LEMMA V.

Invenire lineam curvam generis parabolici, quae per data quotcunque puncta transibit.

Sunto puncta illa *A, B, C, D, E, F*, & ab iisdem ad rectam quamvis positione datam *HN* demitte perpendicula quotcunque *AH, BI, CK, DL, EM, FN*.

Cas. 1. Si punctorum *H, I, K, L, M, N* aequalia sunt intervalia *HI, IK, KL*, collige perpendiculorum *AH, BI, CK*, differentias primas *b*, 2*b*, 3*b*, 4*b*,5*b*, secundas *c*, 2*c*, 3*c*, 4*c*, tertias *d*, 2*d*, 3*d*, id est, ita ut sit
$AH - BI = b,\ BI - CK = 2b, CK - DL = 3b,\ DL + EM = 4b,\ -EM + FN = 5b$, dein
$b - 2b = c$, & sic pergatur ad differentiam ultimam, quae hic est *f*.
Deinde erecta quacunque perpendiculari *RS*, quae fuerit ordinatim applicata ad curvam quaesitam: ut invenlatur huius longitudo, pone intervalia *HI, IK, KL, LM,* unitates esse, & dic
$AH = a, -HS = p\ , \frac{1}{2}p \times -IS = q, \frac{1}{3}q \times +SK = r, \frac{1}{4}r \times +SL = s, \frac{1}{5}s \times +SM = t$; pergendo videlicet ad usque penullimum perpendiculum *ME*, & praeponendo signa negativa terminis *HS, IS*, qui jacent ad partes puncti *S* versus *A*, & signa affirmativa terminis *SK, SL*, qui jacent ad alteras partes puncti *S*. Et signis probe observatis,
erit $RS = a + bp + cq + dr + es + ft$,

*Ca*s. 2. Quod si punctorum *H, I, K, L*, inaequalia sint intervalia *HI, IK*, collige perpendiculorum *AH, BI, CK*, differentias primas per intervalia perpendiculorum divisas *b*, 2*b*, 3*b*, 4*b*, 5*b*; secundas per intervalia bina divisas *c*, 2*c*, 3*c*, 4*c*, tertias per intervalia terna divisas *d*, 2*d*, 3*d*, quartas per intervalla quaterna divisas *e*, 2*e*, & sic deinecps; id est, ita ut sit
$b = \frac{AH-BI}{HI}, 2b = \frac{BI-CK}{IK}, 3b = \frac{CK-DL}{KL}$, & dein $c = \frac{b-2b}{HK}, 2c = \frac{2b-3b}{IL}, 3c = \frac{3b-4b}{KM}$, &c.
postea $d = \frac{c-2c}{HL}, 2d = \frac{2c-3c}{IM}$, Inventis differentiis,
dic $AH = a, -HS = p,\ p$ in $-IS = q,\ q$ in $+SK = r,\ r$ in $+SL = s,\ s$ in $+SM = t$; pergendo scilicet ad usque perpendiculum penullimum *ME*, & erit ordinatim applicata
$RS = a + bp + cq + dr + es + ft$,



Corol. Hinc areae curvarum omnium inveniri possunt quamproxime. Nam si curvae euiusvis quadrandae inveniantur puncta aliquot, & parabola per eadem duci intelligatur: erit area parabolae huius eadem quamproxime cum area curvae illius quadrandae. Potest autem parabola per methodos notissimas semper quadrari Geometrice.

LEMMA VI.

Ex observatis aliquot locis cometae invenire locum eius ad tempus quodvis intermedium datum.

Designent *HI, IK, KL, LM* tempera inter observationes (*in fig. praeced*.) *HA, IB, KC, LD, ME* observatas quinque longitudines cometae, *HS* tempus datum inter observationem primam & longitudinem quaesitam. Et si per puncta *A, B, C, D, E* duci intelligatur curva regularis *ABCDE*; & per lemma superius invenlatur eius ordinatim applicata *RS*, erit *RS* longitudo quaesita.

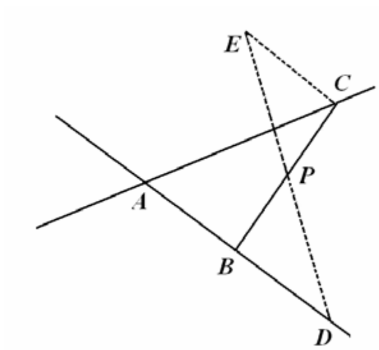
Eadem methodo ex observatis quinque latitudinibus invenitur latitudo ad tempus datum.

Si longitudinum observatarum parvae sint differentiae, puta graduum tantum 4 vel 5 ; suffecerint observationes tres vel quatuor ad inveniendam longitudinem & latitudinem novam. Sin majores sint differentiae, puta graduum 10 vel 20, debebunt observationes quinque adhiberi.

LEMMA V11.

Per datum punctum P ducere rectam lineam BC, cuius partes PB, PC, rectis duabus positione datis AB, AC abscissae, datam habeant rationem ad invicem.

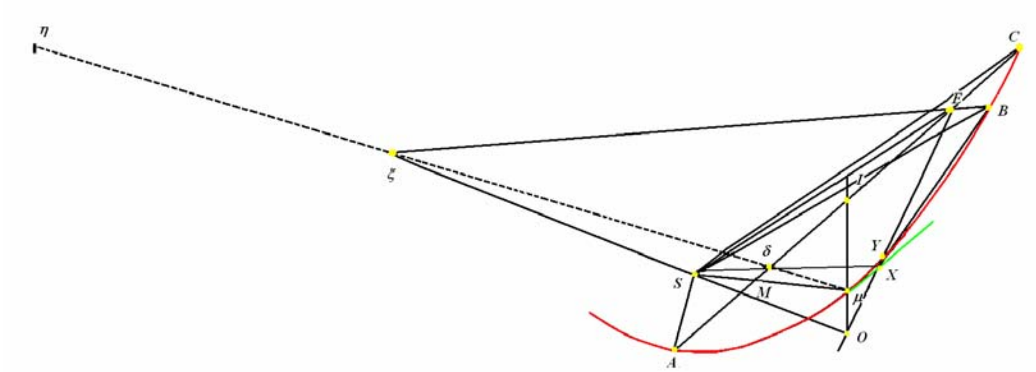
A puncto illo *P* ad rectarum alterutram *AB* ducatur recta quaevis *PD*, & producatur eadem versus rectam alterant *AC* usque ad *E*, ut sit *PE* ad *PD* in data illa ratione. Ipsi *AD* parallela sit *EC*; & si agatur *CPB*, erit *PC* ad *PB* ut *PE* ad *PD*. *Q.E.F.*



LEMMA V11I.

Sit ABC parabola umbilicum habens S. Chorda AC bisecta in I abscindatur segmentum ABCI, cuius diameter sit $I\mu$ *and vertex* μ. *In* $I\mu$ *producta caplatur* μO *aequalis dimidio ipsius* $I\mu$. *Jungatur OS, & producatur ea ad* ξ, *ut sit* $S\xi$ *aequalis 2SO. Et si cometa B moveatur in arcu CBA, & agatur* ξB *secans AC in E: dico quod punctum E abscindet*
de chorda AC segmentum AE tempori proportionale quamproxime.

Jungatur enim EO secans arcum parabolicum ABC in *Y* & agatur μX, quae tangat eundem arcum in vertice μ, & actae *EO* occurrat in X; & erit area curvilinea $AEX\mu A$ ad aream curvilineam $AEY\mu A$ ut *AE* ad *Ae*. Ideoque cum triangulum *ASE* sit ad triangulum *ASC* in eadem ratione, erit area tota $ASEX\mu A$ ad aream totam $ASCY\mu A$ ut *AE* ad *AC*. Cum autem , ξO sit ad *SO* ut 3 ad 1, & *EO* ad *XO* in eadem ratione, erit *SX* ipsi *EB* parallela: & propterea si jungatur *BX*, erit triangulum *SEB* triangula *XEB* aequale. Unde si ad aream $ASEX\mu A$ addatur triangulum *EXB*, & de summa auferetur triangulum *SEB*, manebit area $ASBX\mu A$ areae $ASEX\mu A$ aequalis, atque ideo ad aream $ASCY\mu A$



ut *AE* ad *AC*. Sed areae $ASBX\mu A$ aequalis est area $ASBY\mu A$ quamproxime, & haec area $ASBY\mu A$ est ad aream $ASCY\mu A$, ut tempus descripti arcus *AE* ad tempus descripti arcus totius *AC*. Ideoque *AE* est ad *AC* in ratione temporum quamproxime. *Q. E. D.*

Corol. Ubi puntium *B* incidit in parabolae verticem μ, est *AE* ad *AC* in ratione temporum accurate.

Scholium.

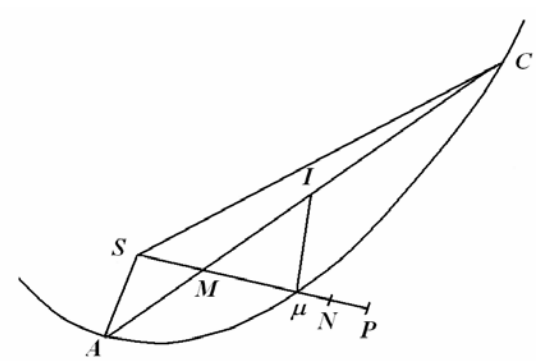
Si jungatur $\mu\xi$ secans *AC* in δ, & in ea caplatur ξn, quae sit ad μB ut 27*MI* ad $16M\mu$: acta *Bn* secabit chordam *AC* in ratione temporum magis accurate quam prius. Jaceat autem punctum *n* ultra punctum ξ, si punctum *B* magis distat a vertice principali parabolae quam punctum μ; & citra, si minus distat ab eadem vertice.

LEMMA IX.
Rectae $I\mu$ & μM & longitudo $\frac{AIC}{4S\mu}$ aequantur inter se.

Nam $4S\mu$ est latus rectum parabolae pertinens ad verticem μ.

LEMMA X.
Si producatur $S\mu$ ad N & P, ut μN sit pars tertia ipsius μI, & SP sit ad SN ut SN ad $S\mu$. Cometa, quo tempore decribit arcum $A\mu C$, si progrederetur ea semper cum velocitate quam habet in altitudine ipsi SP aequali, desciberet longitudinem aequalem chordae AC.

Nam si cometa velocitate, quam habet in μ, eodem tempore progrederetur uniformiter in recta, quae parabolam tangit in μ; area, quam radio ad punctum *S* ducto describeret, aequalis esset areae parabolicae $ASC\mu$. Ideoque contentum sub longitudine in tangente descripta & longitudine $S\mu$ esset ad contentum sub longitudinibus *AC* & *SM*, ut area $ASC\mu$ ad triangulum *ASC*; id est, ut *SN* ad *SM*. Quare *AC* est ad longitudinem in tangente descriptam, ut $S\mu$ ad *SN*. Cum autem velocitas cometae in altitudine *SP* sit (per Corol. 6. Prop. XVI. Lib. I.) ad eius velocitatem in altitudine $S\mu$ in subduplicata ratione *SP* ad $S\mu$ inverse, id est, in ratione $S\mu$ ad *SN*; longitudo hac velocitate eodem tempore descripta, erit ad longitudinem in tangente descriptam, ut $S\mu$ ad *SN*. Igitur *AC* & longitudo hac nova velocitate descripta, cum sint ad longitudinem in tangente descriptam in eadem



ratione, aequantur inter se. *Q.E.D.*

Corol. Cometa igitur ea cum velocitate, quam habet in altitudine $S\mu + \frac{2}{3}I\mu$, eodem tempore describeret chordam *AC* quamproxime.

LEMMA XI.

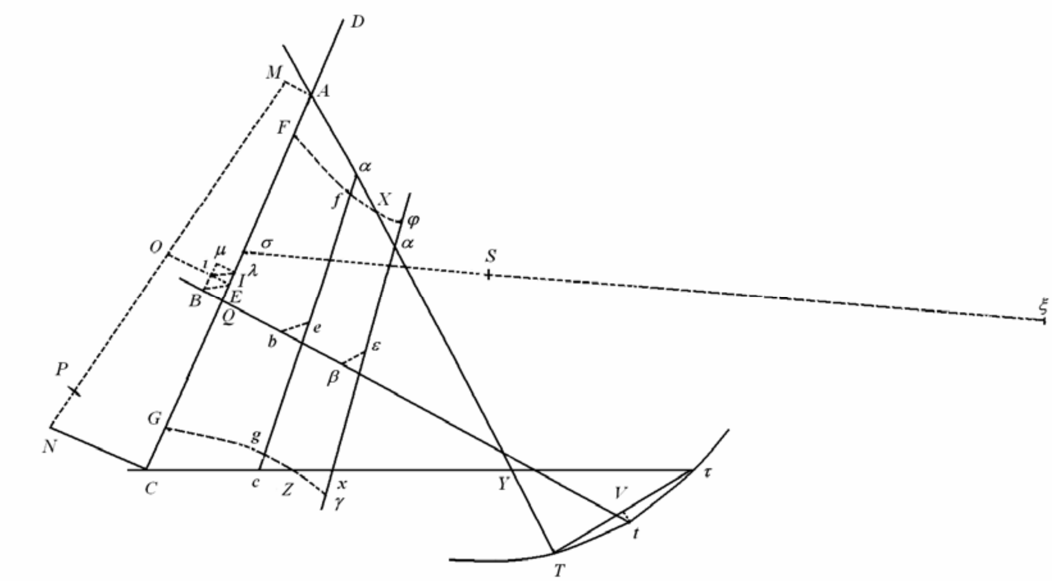
Si cometa motu omni privatus de altitudine SN seu $S\mu + \frac{2}{3}I\mu$ *demitteretur, ut caderet in solem, & ea semper vi uniformiter continuata urgeretur in solem, qua urgetur sub initio; idem semisse temporis, quo in orbe suo destribat arcum AC, descensu suo describeret spatium longitudini* $I\mu$ *aequale.*

Nam cometa, quo tempore describat arcum parabolicum *AC* eodem tempore ea cum velocitate, quam habet in altitudine *SP* (per lemma novissimum) describet chordam *AC*, ideoque (per corol. 7. Prop. XVI. Lib. I.) eodem tempore in circulo, cuius semidiameter esset *SP*, vi gravitatis suae revolvendo, describeret arcum, cuius longitudo esset ad arcus parabolici chordam *AC*, in subduplicata ratione unitatis ad binarium. Et propterea eo cum pondere, quod habet in solem in altitudine *SP*, cadendo de altitudine illa in solem, describeret semisse temporis illius (per corol. 9. Prop. IV. Lib. I.) spatium aquale quadrato semissis chordae illius applicato ad quadruplum altitudinis *SP*, id est, spatium $\frac{AI^2}{4SP}$. Unde cum pondus cometae in solem in altitudine *SN* sit ad ipsius pondus in solem in altitudine *SP*, ut *SP* ad $S\mu$: cometa pondere quod habet in altitudine *SN* eodem tempore, in solem cadendo, describet spatium $\frac{AI^2}{4S\mu}$, id est, spatium longitudini $I\mu$ vel $M\mu$ aequale. *Q.E.D.*

PROPOSITIO XLI. PROBLEMA XXI.

Cometae in parabola moti trajectoriam ex datis tribus observationibus determinare.

Problema hocce longe difficillimum multimode aggressus, composui problemata quaedam in libro primo, quae ad eius solutionem spectant. Postea solutionem



sequentem paulo simpliciorem excogitavi.

Seligantur tres observationes aequalibus temporum intervaliis ab invicem quamproxime distantes. Sit autem temporis intervallum illud, ubi cometa tardius movetur, paulo majus altero, ita videlicet ut temporum differentia sit ad summam temporum, ut summa temporum ad dies plus minus sexcentos; vel ut punctum *E* (in fig. Lem. VllI.) incidat in punctum *M* quamproxime, & inde aberret versus *I* potius quam versus *A*. Si tales observationes non praesto sint, inveniendus est novus cometae locus per lemma sextum.

Designent *S* solem $T,\ t,\ \tau$ tria loca terrae in orbe magno, *TA,tB*, τC observatas tres longitudines cometae, *V* tempus inter observationem primam & secundam, *W* tempus inter secundam ac tertiam, *X* longitudinem, quam cometa toto illa tempore ea cum velocitate, quam habet in mediocri telluris a sole distantia, describere posset, quaeque (per corol. 3. Prop. XL. Lib. 11I.) invenienda est, & *tV* perpendiculum in chordam $T\tau$. In observata longitudine media *tB* sumatur utcunque punctum *B* pro loco cometae in plano eclipticae, & inde versus solem *S* ducatur linea *BE*, quae sit ad sagittam *tV*, ut contentum sub *SB* & *St quad*. ad cubum hypotenusae trianguli rectanguli, cuius latera sunt *SB* & tangens latitudinis cometae in observatione secunda ad radium *tB*. Et per punctum *E* agatur (per huius Lem. V11.) recta *AEC*, cuius partes *AE, EC*, ad rectas *TA* & τC terminatae, sint ad invicem ut tempora *V* & *W*: & erunt *A* & *C* loca cometae in plano eclipticae in observatione prima ac tertia quamproxime, si modo *B* sit locus eius recte assumptus in observatione secunda.

Ad *AC* bisectam in I erige perpendiculum *Ii*. Per punctum *B* age occultam *Bi* ipsi *AC* parallelam. Junge occultam *Si* secantem *AC* in λ, & comple parallelogrammum $iI\lambda p$. Cape $I\sigma$ aequalem $3I\lambda$, & per solem *S* age occultam $\sigma\xi$ aequalem $3S\sigma+3i\lambda$. Et deletis jam literis *A, E, C, I*, a puncto *B* versus punctum ξ duc occultam novam *BE*, quae sit ad priorem *BE* in duplicata ratione distantiae *BS* ad quantitatem $S\mu+\frac{1}{3}i\lambda$. Et per punctum *E* iterum duc rectam *AEC* eadem lege ac prius, id est, ita ut eius partes *AE* & *EC* sint ad invicem, ut tempora inter observationes *V & W*. Et erunt *A* & *C* loca cometae magis accurate.

Ad *AC* bisectam in *I* erigantur perpendicula *AM, CN, IO*, quorum *AM* & *CN* sint tangentes latitudinum in observatione prima ac tertia ad radios *TA* & *TC*. Jungatur *MN* secans *IO* in *O*. Constituatur rectangulum $iI\lambda\mu$ ut prius. In *IA* producta caplatur *ID* aequalis $S\mu+\frac{2}{3}i\lambda$. Deinde in *MN* versus *N* caplatur *MP*, quae sit ad longitudinem supra inventam *X*, in subduplicata ratione mediocris distantiae telluris a sole (seu semidiametri orbis magni) ad distantiam *OD*. Si punctum *P* incidat in punctum *N*; erunt *A, B, C* tria loca cometae, per quae orbis eius in plano eclipticae describi debet. Sin punctum *P* non incidat in punctum *N*; in recta *AC* caplatur *CG* ipsi *NP* aequalis, ita ut puncta *G* & *P* ad easdem partes rectae NC jaceant.

Eadem methodo, qua puncta *E, A, C, G*, ex assumpto puncto *B* inventa sunt, inveniantur ex assumptis utcunque punctis aliis *b & G* puncta nova *e,a, c, g*, & $\varepsilon,\alpha,\chi,\gamma$. Deinde si per *G, g*, γ ducatur circumferentia circuli $Gg\gamma$, secans rectam τC in Z: erit Z locus cometae in plano eclipticae. Et si in $AC,\ ac,\ \alpha\chi$ capiantur $AF,af,\alpha\varphi$ ipfis $CG,cg,\chi\gamma$ respective aequales, & per puncta $F,\ f,\varphi$ ducatur circumferentia circuli $Ff\varphi$, secans rectam *AT* in *X*; erit punctum *X* alius cometae locus in plano eclipticae. Ad puncta *X* & *Z* erigantur tangentes latitudinum cometae ad radios *TX*

& *TZ* ; & habebuntur loca duo cometae in orbe proprio. Denique (per Prop. XIX. Lib. I.) umbilico *S*, per loca illa duo describatur parabola, & haec erit trajectoria cometae. *Q. E. I.*

Constructionis huius demonstratio ex lemmatibus consequitur: quippe cum recta *AC* secetur in *E* in ratione temporum, per Lemma V11, ut oportet per Lem. V11I: & *BE* per Lem. XI. sit pars rectae *BS* vel $B\xi$ in plano eclipticae arcui *ABC* & chordae *AEC* interjecta; & *MP* (per Corol. Lem. X.) longitudo sit chordae arcus, quem cometa in orbe proprio inter observationem primam ac tertiam describere debet, ideoque ipsi *MN* aequalis fuerit, si modo *B* sit verus cometae locus in plano eclipticae.

Caeterum puncta $B,\ b,\ \beta$ non quaelibet, sed vero proxima eligere convenit. Si angulus *AQt*, in quo vestigium orbis in plano eclipticae descriptum secat rectam *tB*, praeterpropter innotescat; in angulo illo ducenda erit recta occulta *AC*, quae sit ad $\frac{4}{3}T\tau$ in subduplicata ratione *SQ* ad *St*. Et agendo rectam *SEB*, cuius pars *EB* aequetur longitudini *Vt*, determinabitur punctum *B* quod prima vice usurpare licet. Tum recta *AC* deleta & secundum praecedentem constructionem iterum ducta, & inventa insuper longitudine *MP*; in *tB* caplatur punctum *b*, ea lege, ut si *TA, TC* se mutuo secuerint in *T*, sit distantia *Tb* ad distantiam *TB*, in ratione composita ex ratione *MP* ad *MN* & ratione subduplicata *SB* ad *Sb*. Et eadem methodo inveniendum erit punctum tertium β si modo operationem tertio repetere lubet. Sed hac methodo operationes duae ut plurimum suffecerint. Nam si distantia *Bb* perexigua obvenerit; postquam inventa sunt puncta *F, f* & *G, g*, actae rectae *Ff* & *Gg* secabunt *TA* & τC in punctis quaesitis *X* & Z.

Exemplum.

Proponatur cometa anni 1680. Huius motum a *Flamstedio* observatum & ex observationibus computatum, atque ab *Halleio* ex iisdem observationibus correctum, tabula sequens exhibet.

	Tem.appar. h.	Tem. verum. h.	Long. Solis.	Cometae Longitudo.	Cometae Lat. bor.
1680.*Dec*. 12	4.46'	4.46'. 0"	♑ 1^0.51'. 23"	♑ 6^0.32'.30"	8^0. 28'.0"
21	6.32$\frac{1}{2}$	6.36.59	11. 6. 44	♒ 5. 8 .12	21. 42.13
24	6.12	6.17.52	14. 9. 26	18.49. 23	25. 23. 5
26	5.14	5.20.44	16. 9. 22	28.24. 13	27. 0.52
29	7.55	8. 3. 2	19.19. 43	♓ 13.10. 41	28. 9.58
30	8.02	8.10.26	20. 21. 9	17.38. 20	26. 15. 7
1681.*Jan*. 5	5.51	6. 1.38	26. 22.18	♈ 8.48. 53	24. 11.56
9	6.49	7. 0.53	♒ 0. 29. 2	18.44. 4	23. 43.52
10	5.54	6. 6.10	1. 27.43	20.40. 50	16. 42.18
13	6.56	7. 8.55	4. 33.20	25.59. 48	16. 4. 1
25	7.44	7.58.42	16. 45.36	♉ 9.35. 0	17. 56.30
30	8.07	8.21.53	21. 49.58	13.19. 51	16. 42.18
Feb. 2	6.20	6.34.51	24. 46.59	15.13. 53	16. 4. 1
5	6.50	7. 4.41	27. 49.51	16.59. 6	15. 27. 3

		Long.Solis		Long.Cometae		Lat.Cometae	
		M, E_1	$E_1a = E_2, E_3$	M, E_1, E_1a, E_2	E_3	M, E_1, E_2	E_3
1680.*Dec.*	12	♑ 1^0.53'. 2"	♑ 1^0.51'.23"	♑ 6^0.33'.0" M, E_1 6.31.21 E_1aE_2	♑ 6^0.32'.30"	8^0. 26'.0"	8^0. 28'.0"
	21	11.8.10 [E_1] 11.8.10$\frac{1}{3}$ [M]	11. 6.44	♒ 5. 8 .12	♒ 5. 8 .12	21. 45.13	21. 42.13
	24	14.10.49	14. 9.26	18.49. 10	18.49. 23	25. 23.24	25. 23. 5
	26	16.10.38	16. 9.22	28.24. 6	28.24. 13	27. 0. 57	27. 0.52
	29	19.20.56	19.19.43	♓ 13.11. 45	♓ 13.10. 41	28. 10.05	28. 9.58
	30	20.22.20	20.21. 9	17.37. 5	17.38. 20	28. 11.12	28. 11.53
1682.*Jan.*	5	26.23.19	26.22.18	♈ 8.49. 10	♈ 8.48. 53	26. 15. 26	26. 15. 7
	9	♒0.29.54	♒0.29. 4	18.43. 18	18.44. 4	24. 12.42	24. 11.56
	10	1.28.34	1.27.43	20.40. 57	20.40. 50	23. 44. 0	23. 43.52
	13	4.34. 6	4.33.20	25.59. 34	25.59. 48	22. 17. 36	22. 17. 28
	25	16.45.58	16.45.36	♉ 9.55. 48 M, E_1 9.55. 48 E_1aE_2	♉ 9.35. 0	17. 56. 54	17. 56.30
	30	21.50. 9	21.49.58	13.19. 36	13.19. 51	16. 40. 57	16. 42.18
Feb.	2	24.47. 4	24.46.59	15.13. 48	15.13. 53	16. 2. 2	16. 4. 1
	5	27.49.51	27.49.51	16.59. 52	16.59. 6	15. 27. 23	15. 27. 3

His adde observationes quasdam e nostris

	Tem.appar.	Cometae Longitudo.	Cometae Lat. bor.
1682.*Feb.* 25	8^h.30'	♉ 26^0.18'.35"	12^0. 46'.46"
27	8. 15	27. 4 .30	12. 36.12
Mar. 1	11. 0	27. 52. 42	12. 23.40
2	8. 0	28. 12. 48	12. 19.38
5	11.30	29. 18. 0	12. 3.16
7	9.30	♊ 0. 4. 0	11. 57. 0
9	8.30	0. 43. 4	11. 45.52

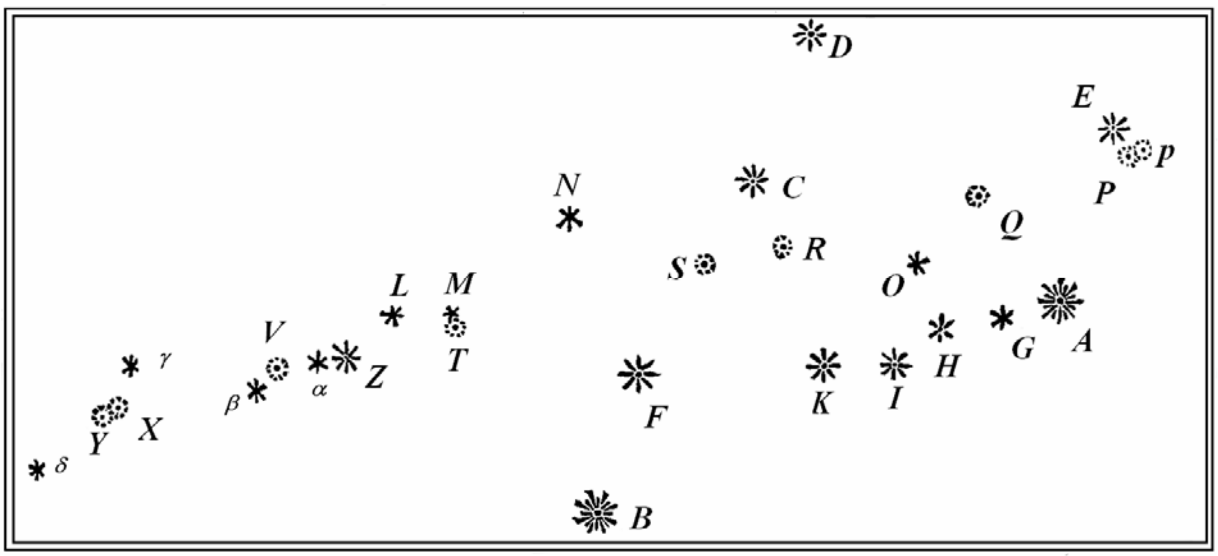
Hae observationes telescopio septupedali, & micrometro filisque in foco telescopii locatis peractae sum: quibus instrumentis & positiones fixarum inter se & positiones cometae ad fixas determinavimus.

		Long.Cometae			Lat.Cometae		
		M, E_1	E_2	E_3	M, E_1	E_2	E_3
1682.*Feb.*	25	♉ 26^0.19'. 22"	♉ 26^0.18'.17"	♉ 26^0.18'.35"	12^0.46$\frac{7}{8}$	12^0.46'$\frac{7}{8}$	12^0. 46'.46"
	27	27. 4. 28	27. 4.24	27. 4.30	12. 36	12. 36$\frac{1}{5}$	12. 36.12
Mar.	1	27.53. 8	27.53. 6	27.52.42	12.24$\frac{3}{7}$	12. 24$\frac{6}{7}$	12. 23.40
	2	28.12.29	28.12.27	28.12.48	12. 19$\frac{1}{2}$	12. 20	12. 19.38
	5	29.20.51	29.20.51	29.18. 0	12. 19$\frac{2}{3}$	12. 3$\frac{1}{2}$	12. 3.16
	7			♊ 0. 4. 0			11. 57. 0
	9	♊ 0. 43. 2	♊0. 43. 4	♊ 0. 43. 4	11. 44$\frac{3}{5}$	11. 45$\frac{7}{8}$	11. 45.52

Designet *A* stellam quartae magnitudinis in sinistro calcaneo Persei (Bayero o) *B* stellam sequentem tertiae magnitudinis in sinistro pede (Bayero ζ) & *C* stellam sextae magnitudinis (Bayero *n*) in talo eiusdem pedis, ac *D, E, F, G, H, I, K, L, M, N, O, Z,* $\alpha, \beta, \gamma, \delta$ stellas alias minores in eodem pede. Sintque *p, P, Q, R, S, T, V, X*, loca cometae in observationibus supra descriptis: & existente distantia *AB* partium $80\frac{7}{12}$ erat *AC* partium

$52\frac{1}{4}$, BC $58\frac{5}{6}$, AD $57\frac{5}{12}$, BD $82\frac{6}{11}$, CD $23\frac{2}{3}$, AE $29\frac{4}{7}$, CE $57\frac{1}{2}$, DE $49\frac{11}{12}$, AI $27\frac{7}{12}$ BI $52\frac{1}{6}$, CI $36\frac{7}{12}$, DI $53\frac{5}{12}$, AK $38\frac{2}{3}$, BK 43, CK $31\frac{5}{9}$, FK 29, FB 23, FC $36\frac{1}{4}$, AH $18\frac{4}{7}$, DH $50\frac{7}{8}$, BN $46\frac{5}{12}$, CN $31\frac{1}{3}$, BL $45\frac{5}{12}$, NL $31\frac{5}{7}$.

HO erat ad *HI* ut 7 ad 6 & producta transibat inter stellas *D* & E, sic ut distantia stellae *D* ab hac recta esset $\frac{1}{6}CD$. *LM* erat ad *LN* ut 2, ad 9, & producta transibat per stellam *H*.



His determinabantur positiones fixarum inter se.

Tandem *Poundius* noster iterum observavit positiones harum fixarum inter se, & earum longitudines & latitudines in tabulam sequentem retulit.

Fixarum	Longitudines.	Lat. boreae.	Fixarum	Longitudines.	Lat. boreae.
A	♉ 26. 41. 50	12. 8.36	*L*	♉ 29.33.34	12. 7.48
B	28 .40. 23	11.17.54	*M*	29.18.54	12. 7.20
C	27.58.30	12.40.25	*N*	28.48.29	12.31.91
E	26.27. 17	12.52. 7	*Z*	29.44.48	11.57.13
F	28.28.37	11.52.22	*a.*	29.52. 3	11.55.48
G	26.56 . 8	12. 4.58	β	11 0. 8.23	11.48.56
H	27.11 .45	12. 2. 1	γ	0.40.10	11.55.18
I	27.25. 2.	11.53.11	δ	1. 3.20	11. 30.42
K	27.42. 7	11.53.26			

Positiones vero cometae ad has fixas observabam ut sequitur. Dic veneris *Feb*. 25. st vet. hor. $8\frac{1}{2}$ p.m. cometae in *p* existentis distantia a stella *E* erat minor quam $\frac{3}{13}A$E, major quam $\frac{1}{5}A$E, ideoque aequalis $\frac{3}{14}AE$ proxime; & angulus *ApE* nonnihil obtusus erat, sed fere rectus. Nempe si demitteretur ad *pE* perpendiculum ab *A*, distantia cometae a perpendiculo illo erat $\frac{1}{5}pE$.

Eadem nocte hora $9\frac{1}{2}$, cometae in *P* existentis distantia a stella *E* erat major quam $\frac{1}{4\frac{1}{2}}AE$, minor quam $\frac{1}{5\frac{1}{4}}AE$, ideoque aequalis $\frac{1}{4\frac{7}{8}}AE$, seu $\frac{8}{39}AE$ quamproxime A perpendiculo autem a stella *A* ad rectam *PE* demisso distantia cometae erat $\frac{4}{5}PE$.

Die solis *Feb*. 27. hor. $8\frac{1}{4}$ p.m. cometae in *Q* existentis distantia a stella *O* aequabat distantiam stellarum *O* & *H*, & rectae *QO* producta transibat inter stellas *K* & *B*. Positionem huius rectae ob nubes intervenientes magis accurate definire non potui.

Die martis *Mart.* 1. hor. 11 p.m. cometa in *R* existens, stellis *K* & *C* accurate interjacebat, & rectae *CRK* pars *CR* paulo major erat quam $\frac{2}{3}CK$, & paulo minor quam $\frac{1}{3}CK+\frac{1}{8}CR$, ideoque aequalis $\frac{1}{3}CK+\frac{1}{16}CR$ seu $\frac{16}{45}CK$.

Die mercurii *Mart*. 2. hor.8. p.m. cometae existentis in *S* distantia a stella *C* erat $\frac{4}{9}FC$ quamproxime. Distantia stellae *F* a recta *CS* producta erat $\frac{1}{24}SC$; & distantia stellae *B* ab eadem recta, erat quintuplo major quam distantia stellae *F*. Item recta *NS* producta transibat inter stellas *H* & *I*, quintuplo vel sextuplo propior existens stellae *H* quam stellae I.

Die saturni *Mart*.5. hor.$11\frac{1}{2}$ p. m. cometa existente in *T*, recta *MT* aequalis erat $\frac{1}{2}ML$, & recta *LT* producta transibat inter *B* & *F*, quadruplo vel quintuplo propior *F* quam *B*, auferens a *BF* quintam vel sextam ejus partem versus *F*. Et *MT* producta transibat extra spatium *BF* ad partes stellae *B*, quadruplo propior existens stellae *B* quam stellae *F*. Erat *M* stella perexigua quae per telescopium videri vix potuit, & *L* stella major quasi magnitudinis octavae.

Die lunae *Mart*. 7 hor. $9\frac{1}{2}$ p.m. cometa existente in *V*, recta *Va*, producta transibat inter *B* & *F*, auferens a *BF* versus *F* $\frac{1}{10}$ *BF*, & erat ad rectam $V\beta$ ut 5 ad 4. Et distantia cometae a recta $\alpha\beta$ erat $\frac{1}{2}V\beta$.

Die mercurii *Mart.* 9. hora $8\frac{1}{2}$ p.m. cometa existente in *X*, recta γX aequalis erat $\frac{1}{4}\gamma\delta$, & perpendiculum demissum a stella δ ad rectam γX erat $\frac{2}{5}\gamma\delta$.

Eadem nocte hora 12, cometa existente in *Y*, recta γY aequalis erat$\frac{1}{3}\gamma\delta$, aut paulo minor, puta $\frac{5}{16}\gamma\delta$, & perpendiculum demissum a stella δ ad rectam γY aequalis erat $\frac{1}{6}\gamma\delta$ vel $\frac{1}{7}\gamma\delta$ circiter. Sed cometa ob viciniam horizontis cerni vix potuit, nec locus eius tam distincte ac in praecedensibus definiri.

Ex huiusmodi observationibus per constructiones figurarum & computationes derivabam longitudines & latitudines cometae, & *Poundius* noster ex correctis fixarum locis loca cometae correxit, & loca correcta habentur supra. Micrometro parum affabre constructio usus sum, sed longitudinum tamen & latudinum errores (quatenus ex observationibus nostris oriantur) minutum unum primum vix superant. Cometa autem (iuxta observationes nostras) in fine motus sui notabiliter deflectere coepit boream versus, a parallelo quem in fine mentis *Februarii* tenuerat.

Jam ad orbem cometae determinandum; selegi ex observationibus hactenus descriptis tres, quas *Flamstedius* habuit *Dec*. 21, *Jan*. 5, & *Jan*. 25. Ex his inveni *St* partium 9842,1 & *Vt* partium 455, quales 10000 sunt semidiameter orbis magni. Tum ad operationem primam assumendo *tB* partium 5657, inveni *SB* 9747, *BE* prima vice 412., $S\mu$ 9503, $i\lambda$ 413: *BE* secunda vice 421, *OD* 10186, *X* 8528,4 *MP* 8450, *MN* 8475, *NP* 25. Unde ad operationem secundam collegi distantiam *tb* 5640. Et per hanc operationem inveni tandem distantias *TX* 4775 & τZ 11322. Ex quibus orbem definiendo, inveni nodos eius descendensem in ♋ & ascendensem in ♑ 1^{gr}. 53'; inclinationem plani eius ad planum eclipticae 61^{gr}. 20'$\frac{1}{3}$; verticem eius (seu perihelium cometae) distare a nodo 8^{gr}. 38', and esse in ♐ 27^{gr}. 43' cum latitudine australi 7^{gr}.34'; & eius latus rectum esse 236,8, areamque radio ad solem ducto singulis diebus descriptam 93585, quadrato semidiametri orbis magni posito 100000000; cometam vera in hoc orbe secundum seriem signorum processisse, & *Decemb*. 8^{d}. 0^{h}. 4'. p. m. in vertice orbis seu perihelio fuisse. Haec omnia per scalam partium aequalium & chordas angulorum ex tabula sinuum naturallum collectas determinavi graphice; construendo schema satis amplum, in quo videlicet semidiameter orbis magni (partium 10000) aequalis esset digitis $16\frac{1}{3}$ pedis *Anglicani*.

Tandem ut constaret an cometa in orbe sic invento vere moveretur, collegi per operationes partim arithmeticas partim graphicas loca cometae in hoc orbe ad observationum quarundam tempora : uti in tabula sequente videre licet.

	Distant Comet a Sole	Long.Collect	Lat.Collect	Long.Obs.	Lat.Obs.	Differ. Long.	Differ. Lat.
Dec.12	2792	♑ 6^{0}. 32'	♑ 8^{0}.18'$\frac{1}{2}$	6^{0}.31'$\frac{1}{3}$	8^{0}.26'	+1'	$-7'\frac{1}{2}$
29	8403	♓ 13 .13$\frac{2}{3}$	28. 0	♓13.11$\frac{3}{4}$	28. 10$\frac{1}{12}$	+2	$-10\frac{1}{12}$
Feb. 5	16669	♉ 17. 0	15.29$\frac{2}{3}$	♉16.59$\frac{7}{8}$	15. 27$\frac{2}{5}$	+0	$+2\frac{1}{4}$
Mar. 5	21737	29.19$\frac{3}{4}$	12. 4	29.20$\frac{6}{7}$	12. 3$\frac{1}{2}$	–1	$+\frac{1}{2}$

Postea vero Halleius noster orbitam per calculum arithmeticum accuratius determinavit, quam per descriptiones linearum fieri licuit; & retinuit quidem locum nodorum in ♋ & ♑ 1^{gr}. 53', & inclinationem plani orbitae ad eclipticam 61^{gr}. 53', ut & tempus perihelii cometae *Decemb*. 8^{d}.0^{h}.4' distantiam vero perihelii a nodo ascendense in orbita cometae mensuratam invenit esse 9^{gr}.20' & latus rectum parabolae esse 2430 partium, existente mediocri solis a terra distantia partium 100000. Et ex his datis, calculo itidem arithmetico accurate instituto, loca cometae ad observationum tempora computavit, ut sequitur.

Tempus verum	Distant Comet a Sole	Long.comp.	Lat.comp.	Differ. Long.	Differ. Lat.
Dec. 12^{d} 4^{h}.46'	28028	♑ 6^{0}.29'.25"	8^{0}.26'.0" Bor.	–3'.5"	– 2'. 0"
21. 6.37	61076	♒ 5. 6.30	21.43.20	–1.42	+ 1. 7
24. 6.18	70008	18.48.20	25.22.40	–1. 3	– 0.25
26. 5.21	75576	28.22.45	27. 1.36	–1.28	+ 0.44
29. 8. 3	14021	♓ 13.12.40	28.10.10	+1.59	+ 0.12
30. 8.10	84021	17.40. 5	28.10.20	+1.45	– 0.33
Jan. 5. 6. $1\frac{1}{2}$	101440	♈ 8.49.49	26.15.15	+0.56	+ 0. 8
9. 7. 0	110959	18.44.36	24.12.54	+0.32	+ 0. 8
10. 6. 6	113162	20.41.02	23.44.10	+0.10	+0.18
13. 7. 9	120000	26. 0.21	22.17.30	+0·33	+0.25
25. 7.59	145370	♉ 9.33.40	17.57.55	–1.20	+ 1.25
30. 8.22	155303	13.17.41	16.42. 7	–2.10	–0.11
Feb. 2. 6.35	160951	15.11.11	16. 4.15	–2.42	+0.14
5. 7. $4\frac{1}{2}$	166686	16.58.25	15.29.13	–0.41	+2.10
25. 8.41	202570	26.15.46	12.48. 0	–2·49	+1.14
Mar. 5.11.39	216205	29.18.35	12. 5. 40	+ 0·35	+ 2.24

Apparuit etiam hic cometa mense *Novembri* praecedente & *Coburgi* in Saxonia a D^{no}. *Gottfried Kirch* observatus est diebus mensis huius quarto, sexto & undecimo, stylo veteri; & ex positionibus eius ad proximas stellas fixas ope telescopii nunc bipedalis nunc decempedalis satis accurate observatis, ac differentia longitudinum *Coburgi* & *Londini* graduum undecim & locis fixarum a *Poundio* nostro observatis, *Halleius* noster loca cometae determinavit ut sequitur.

Novem. 3^{d}. 17^{h}.2', tempore apparente *Londini*, cometa erat in ♌ 29^{gr}.51' cum lat. bor. 1^{gr}.17'.45".
Novem. 5^{d}. 15^{h}.58', cometa erat in ♍ 3^{gr}. 23' cum lat. bor. 1^{gr}.6'.
Novem. 10^{d}. 16^{h}.31', cometa aequaliter distabat a stellis leonis σ ac τ *Bayero* ; nondum vero attigit rectam easdem jungentem, sed parum abfuit ab ea. In stellarum catalogo *Flamstediano* σ tunc habuit ♍ 14^{gr}.15' cum lat. bor. 1^{gr}.41' fere, τ vero ♍17^{gr}.39'$\frac{1}{4}$, cum lat. bor. 0^{gr}.34'. Et medium punctum inter has stellas fuit ♍15^{gr}.39'$\frac{1}{4}$ cum lat. bor. 0^{gr}. 33$\frac{1}{2}$. Sit distantia cometae a recta illa 10' vel 12' circiter, & differentia longitudinum cometae & puncti illius medii erit 7', & differentia latitudinum 7'$\frac{1}{2}$, circiter. Et inde cometa erat in ♍15^{gr}.32' cum lat. bor. 26' circiter.

Observatio prima ex situ cometae ad parvas quasdam fixas abunde satis accurata fuit. Secunda etiam satis accurata fuit. In tertia, quae minus accurata fuit, error minutorum sex vel septem subesse potuit, & vix major. Longitudo vero cometae in observatione prima, quae caeteris accuratior fuit, in orbe praedicto parabolico computata erat ♌ 29^{gr}.30'.22" latitudo borealis 1^{gr}. 25'.7". & distantia eius a sole 115546.

Porro *Halleius* observando quod cometa insignis intervallo annorum 575 quater apparuisset, scilicet mense *Septembri* post caedem *Julii Caesaris*, anno *Christi* 531 *Lampadio* & *Oreste Coss*. anno *Christi* 1106 mense mense *Februario*, & sub finem anni 1680, idque cum cauda longa & insigni (praeterquam quod sub mortem *Caesaris*, cauda ob incommodam telluris positionem minus apparuisset:) quaesivit orbem ellipticum cuius axis major esset partium 1382957, existente mediocri distantia telluris a sole partium 10000: in quo orbe utique cometa annis 575 revolvi possit. Et ponendo nodum ascendentem in ♋ 2^{gr}. 2'; inclinationem plani orbis ad planum eclipticae 61^{dg} 6'.48"; perihelium cometae in hoc plano ♐ 22^{gr}. 44'. 25"; tempus aequatum perihelii *Decem*. 7^{d}. 23^{h}. 9' ; distantiam perihelii a nodo ascendente in plano eclipticae 9^{gr}. 17'. 35"; & axem conjugatum 184081,2: computavit motum cometae in hoc orbe elliptico. Loca autem eius tam ex observationibus deducta quam in hoc orbe computata exhibentur in tabula sequente.

Tempus verum		Long. obs.	Lat. Bor. obs.	Long.comp.	Lat.comp.	Differ. Long.	Differ. Lat.
Nov.	3.16.47	♌ 29^{0}.51'. 0"	1^{0}.17'. 45"	♌ 29^{0}.51'. 22"	1^{0}.17'. 32"	+ 0'.22"	– 0'.13"
	5.15.37	♍ 3.23. 0	1. 6. 0	♍ 3.24. 32	1. 6. 9	+ 1.32	+ 0. 9
	10.16.18	15.32. 0	0.27. 0	15.33. 2	0.25. 7	+ 1. 2	– 1.53
	16.17. 0			♎ 8.16.45	0.52. 7 A		
	18.21.34			18.52.15	1.26.54		
	20.17. 0			28.10.36	1.53.35		
	23.17. 5			♏ 13.22.42	2.29. 0		
Dec.	12^{d} 4^{h}.46'	♑ 6^{0}.32'.30"	8.28. 0	♑ 6.32.20	8.29. 6 B	– 1.10	+ 1. 6
	21. 6.37	♒ 5. 8.11	21.42.13	♒ 5. 6.14	21.44.42	– 1.58	+ 2.29
	24. 6.18	18.49.23	25.23. 5	18.47.30	25.23.35	– 1.53	+ 0.30
	26. 5.21	28.24.13	27. 0.52	28.21.42	27. 2. 1	– 2.31	+ 1. 9
	29. 8. 3	♓ 13.10.41	28. 9.58	♓ 13.11.14	28.10.38	+ 0.33	+ 0.40
	30. 8.10	17.38.20	28.11.53	17.38.27	28.11.38	+ 0. 7	– 0.16
Jan.	5. 6. 1½	♈ 8.48.53	26.15. 7	♈ 8.48.51	26.14.57	– 0. 2	– 0.10
	9. 7. 1	18.44. 4	24.11.56	18.43.51	24.12.17	– 0.13	+ 0.21
	10. 6. 6	20.40.50	23.43.32	20.40.23	23.43.25	+ 0.10	– 0. 7
	13. 7. 9	25.59.48	22.17.28	26. 0. 8	22.16.32	+ 0·20	– 0.56
	25. 7.59	♉ 9.35. 0	17.56.30	♉ 9.34.11	17.56. 6	– 0.49	– 0.24
	30. 8.22	13.19.51	16.42.18	13.18.28	16.40. 5	– 1.23	– 2.13
Feb.	2. 6.35	15.13.53	16. 4. 1	15.11.59	16. 2. 7	– 1.54	– 1.54
	5. 7. 4½	16.50. 6	15.27. 3	16.59.17	15. 27. 0	+ 0.11	+ 2.10
	25. 8.41	26.18.35	12.46.46	26.16.59	12.45. 22	– 1·36	– 1.24
Mar.	1.11.10	27.52.42	12.23.40	27.51.47	12.22.28	– 0·55	– 1.12
	5.11.39	29.18. 0	12. 3.16	29.20.11	12. 2.50	+ 2.11	– 0·26
	9. 8.38	0.43. 4	11.45.52	♊ 0.42.43	11.45.35	– 0·21	– 0·17

Observationes cometae huius a principio ad finem non minus congruunt cum motu cometae in orbe jam descripto, quam motus planetarum congruere solent cum eorum theoriis, & congrendo probant unum & eundem fuisse cometam, qui toto hoc tempore apparuit, eiusque orbem hic recte desinitum fuisse.

In tabula praecedente omisimus observationes diebus *Novembris* 16, 18, 20 & 23 ut minus accuratas. Nam cometa his etiam temporibus observatus fuit. *Ponthaeus* utique & socii, *Novem*. 17. st. vet. hora sexta matutina *Romae*, id est, hora 5. 10' *Londini*, filis ad fixas applicatis, cometam observarunt in ♎ 8^{gr}.30' cum latitudine australi 0^{gr}. 40'. Extant eorum observationes in tractatu, quem *Ponthaeus* de hoc cometa in lucem edidit. *Cellius*, qui aderat & observationes suas in epistola ad D. Cassimum misit, cometam eadem hora vidit in ♎ 8^{gr}.30' cum latitudine australi 0^{gr}. 30'. Eadem hora *Galtetius Avenioni* (id est, hora matutina 5. 41. *Londini*) cometam vidit in ♎ 8^{gr}. fine latitudine, Cometa autem per theoriam jam fuit in ♎ 8^{gr}. 16'. 45'' cum latitudine australi 0^{gr} 53'. 7'',

Nov. 18. hora matutina 6. 30' Romae (id est, hora 5, 40' *Londini*) *Ponthaus* cometam vidit in ♎ 13^{gr}. 30' cum Latitudine australi 1^{gr}. 20'. *Cellius* in ♎ 13^{gr}.30' cum latitudine australi 1^{gr}. 00'. *Gallet* autem hora matutina 5.30' *Avenioni* cometam vidit in ♎ 13^{gr}. 00', cum altitudine australi 1^{gr}. 00'. Et *R.P. Ango* in academia *Flexiensi* apud *Gallos*, hora quinta matutina (id est, hora 5. 9' *Londini*) cometam vidit in media inter stellas duas parvas, quarum una media est trium in recta linea in virginis australi manu, *Bayero* ψ, & altera est extrema alae *Bayero* θ. Unde cometa tunc fuit in ♎ 12^{gr} .46' cum latitudine australi 50'. Eodem die *Bostoniae* in *Nova Anglia* in latitudine $42\frac{1}{2}$ graduum, hora quinta matutina, (id est Londini hora matutina 9. 44') cometa visus est prope ♎ 14^{gr}, cum latitudine australi 1^{gr}. 30', uti a cl. *Halleio* accepi.

Nov. 19. hora mat.$4\frac{1}{2}$, Cantabrigiae, cometa (observante juvene quodam) distabat a spica ♍ quasi 2^{gr} boreazephyrum versus. Erat autem spica in ♎ 19^{gr}.23'. 47'' cum lat. aust. 2^{gr}. 1'. 59''. Eodem die hor. 5. mat. *Bostoniae* in *Nova Anglia*, cometa distabat a spica ♍ gradu uno, differentia latitudinum existente 40'. Eodem die in insula *Jamaica*, cometa distabat a spica intervallo quasi gradus unius. Eodem die *D. Arthurus Storer* ad fluvium *Patuxent*, prope *Hunting Creek* in *Maryland*, in confinio Virginiae in lat. $38\frac{1}{2}^{gr}$, hora quinta matutina (id est, hora 10^{h} *Londini*) cometam vidit supra spicam ♍ & cum spica propemodum coniunctum, existente distantia inter eoidem quasi $\frac{1}{4}^{gr}$. Et ex his observationibus inter se collatis colligo quod hora 9. 44' *Londini* cometa erat in ♎ 18^{gr}. 50' cum latitudine australi 1^{gr} .25' circiter. Cometa autem per theoriam jam erat in ♎ 18^{gr}. 52'. 15'' cum latitudine australi 1^{gr}. 26'. 54''.

Nov 20. D. *Montenarus* astronomiae professor Paduensis, hora sexta matutina *Venetiiis* (id est, hora 5. 10' *Londini*) cometam vidit in ♎ 18^{gr}. cum latitudine australi 1^{gr}.30'. Eodem die *Bostoniae*, distabat cometa a spica ♍ 4^{gr}. longitudinis in orientem, ideoque erat in ♎ 23^{gr}. 24' circiter.

Nov 21. *Pontheaus* & socii hor. mat. $7\frac{1}{4}$ cometam observarunt in ♎ 27^{gr}. 50' cum latitudine australi 1^{gr}. 16', *Cellius* in ♎ 28^{gr}. *Ango* hora quinta matutina in ♎ 27^{gr}. 45'', *Montenarus* in ♎ 27^{gr}.51'. Eodem die in insula Jamaica cometa visus est prope principium scorpii, eandemque circiter latitudinem habuit cum spica virginis, id est, 2^{gr}. 2'. Eadem die ad horam quintam matutinam *Ballasorae* in *India Orientali*, (id est ad horam noctis praecedensis 11. 20' *Londini*) capta est distantia cometae a spica ♍ 7^{gr}. 35' in orientem. In linea recta erat inter spicam & lancem, ideoque versabatur in ♎ 26^{gr}. 58' cum lat. austr. 1^{gr}. 11' circiter; & post horas 5 & 40' (ad horam scilicet quintam matutinam *Londini*) erat in ♎ 28^{gr}. 10' cum lat. australi 1^{gr}. 16'. Per theoriam vero cometa jam erat in ♎ 28^{gr}.10'. 36'', cum latitudine australi 1^{gr}. 53'. 35''.

Nov. 22. Cometa visus est a *Montenaro* in ♏ 2^{gr}. 33', *Bostoniae* autem in *Nova-Anglia* apparuit in ♏ 3^{gr} circiter, eadem fere cum latitudine ac prius, id est, 1^{gr}. 30'. Eodem die ad horam quintam matutinam *Ballasorae* cometa observabatur in ♏ 1^{gr}. 50'; ideoque ad horam quintam matutinam *Londini* cometa erat in ♏ 3^{gr}. 5' circiter, Eodem die

Londini hora mat. 6½ Hookius noster cometam vidit in ♏ 3^{gr}. 30' circiter, idque in linea recta quae transit per spicam virginis & cor leonis, non exacte quidem, sed a linea illa paululum deflectemem ad boream. *Montenarus* itidem notavit quod linea a cometa per spicam ducta, hoc die & sequentibus transibat per australe latus cordis leonis, interposito perparvo intervallo inter cor leonis & hanc lineam. Linea recta per cor leonis & spicam virginis transiens, eclipticam secuit in ♍ 3^{gr}. 46'; in angulo 2 ^{gr}. 51'. Et si cometa locatus suisset in hac linea in ♏ 3^{gr} eius latitudo fuisset 2^{gr}. 26'. Sed cum cometa consentientibus *Hookio* & *Montenaro*, nonnihil distaret ab hac linea boream versus, latitudo eius fuit paulo minor. Die 20. ex observatione *Montanari*, latitudo eius propemodum aequabat latitudinem spicae ♍, eratque 1^{gr}. 30' circiter, & consentientibus *Hookio*, *Montenaro* & *Algone* perpetuo augebatur, ideoque jam sensibiliter major erat quam 1^{gr}. 30'. lnter limites autem jam constitutos 2 ^{gr}. 26' & 1^{gr}. 30', magnitudine mediocri latitudo erit 1^{gr}. 58' circiter. Cauda cometae, consentientibus *Hookio* & *Montenaro*, dirigebatur ad spicam ♍, declinans aliquantulum a stella ista, iuxta *Hookium* in austrum, iuxta *Montenarum* in boream; ideoque declinatio illa vix fuit sensibilis, & cauda aequatori fere parallela existens, aliquantulum deflectebatur ab oppositione solis boream versus.

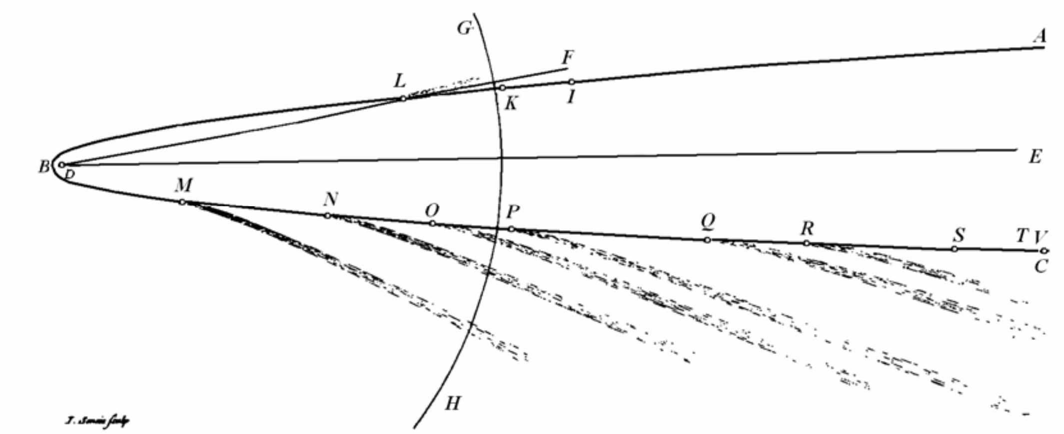
Nov. 23. st. vet. hora quinta matutina *Noriburgi* (id est hora 4½ *Londini*) *D. Zimmerman* cometam vidit in ♏ 8^{gr}. 8', cum latitudine australi 2^{gr}. 31', captis scilicet eius distantiis a stellis fixis.

Nov. 24. Ante ortum solis cometa visus est a *Montenaro* in ♏ 12^{gr}. 52', ad boreale latus rectae quae per cor leonis & spicam virginis ducebatur, ideoque latitudinem habuit paulo minorem quam 2^{gr}. 38'. Haec latitudo, uti diximus, ex observationibus *Montanari, Angonis* & *Hookii*, perpetuo augebatur; ideoque jam paulo major erat quam 18^{gr}. 58' & magnitudine mediocri, sine notabili errore, statui potest 2^{gr}.18'. Latitudinem *Ponthaeus* & *Galtetius* jam decrevisse volunt, & *Cellius* & observator in *Nova-Anglia* eandem fere magnitudinern retinuisse, scilicet gradus unius vel unius cum semisse. Crassiores sunt observationes *Ponthaei* & *Cellii*, eae praesertim quae per azimuthos & altitudines capiebantur, ut & eae *Galletii*: meliores sunt eae quae per positiones cometae ad fixas a *Montenaro, Hookio, Angone* & observatore in *Nova-Anglia*, & nonnunquam a *Ponthaeo & Cellio* sunt factae. Eadem die ad horam quintam matutinam Ballasorae cometa observabatur in ♏ 11^{gr}. 45'; ideoque ad horam quintam matutinam *Londini* erat in ♏ 13^{gr} circiter. Per theoriam vero cometa jam erat in ♏ 13^{gr}. 22'. 42".

Nov. 25. Ante ortum solis *Montenarus* cometam observavit in ♏ $17\frac{3}{4}$ ^{gr} circiter. Et Cellius observavit eadem tempore quod cometa erat in linea recta inter stellam lucidam in dextro femore virginis & lancem australem librae, & haec recta secat viam cometae in ♏ 18^{gr}. 36'. Per theoriam vero cometa jam erat in ♏ $18\frac{1}{3}$ gr. circiter.

Congruunt igitur hae observationes cum theoria quatenus congruunt inter se, & congruendo probant unum & eundem suisse cometam, qui toto tempore a quarto die *Novembris* ad usque nonum *Martii* apparuit. Trajectoria cometae huius bis secuit planum eclipticae, & propterea non fuit rectilinea. Eclipticam secuit non in oppositis coeli partibus, sed in fine virginis & principio capricorni, intervallo graduum 98 circiter; ideoque cursus cometae plurimum deflectebatur a circulo maximo. Nam & mense *Novembri* cursus eius tribus saltem gradibus ab ecliptica in austrum declinabat, & postea mense *Decembri* gradibus 29 vergebat ab ecliptica in septentrionem partibus duabus orbitae, in quibus cometa tendebat in solem & redibat a sole, angulo apparente graduum plus triginta ab invicem declinantibus, ut observavit *Montenarus*. Pergebat hic cometa per signa novem, a leonis scilicet ultimo gradu ad principium geminorum, praeter signum leonis, per quod pergebat antequam videri coepit; & nulla alia extat theoria, qua cometa tantam coeli partem motu regulari percurrat. Motus eius fuit maxime inaequabilis. Nam circa diem vigesimum *Novembris* descripsit gradus circiter quinque singulis diebus; dein motu retardato inter *Novemb*. 26 & *Decemb*. 12, spatio scilicet dierum quindecim cum semisse, descripsit gradus tantum 40; postea vero motu iterum accelerato, descripsit gradus fere quinque singulis diebus, antequam motus iterum retardari coepit. Et theoria, quae motui tam inaequabili per maximam coeli partem probe respondet, quaeque easdem observat leges cum theoria planetarum, & cum accuratis observationibus astronomicis accurate congruit, non potest non esse vera.

Caeterum trajectoriam quam cometa descripsit, & caudam veram quam singulis in locis projecit, visum est annexo schemate in plano trajectoriae delineatas exhibere: ubi *ABC* denotat trajectoriam cometae, *D* solem, *DE* trajectoriae axem, *DF* lineam nodorum, *GH* intersectionem sphaerae orbis magni cum plano trajectoriae, I, locum cometae *Nov*. 4.



Ann. 1680, *K* locum eiusdem *Nov*. 11, *L* locum *Nov*. 19, *M* locum *Dec*. 12., *N* locum Dec. 21, *O* locum *Dec*. 29, *P* locum *Jan*. 5 sequent. *Q* locum Jan. 25, *R* locum *Feb*. 5, *S* locum *Feb*. 25, *T* locum *Mar*.5, & *V* locum *Mar*. 9.

Observationes vero sequentes in cauda definienda adhibui.

Nov. 4 & 6. Cauda nondum apparuit. Nov. 11. Cauda jam coepta non nisi semissem gradus unius longa tubo decempedali visa fuit.

Nov. 17. Cauda gradus amplius quindecim longa *Ponthtaeo* apparuit.

Nov. 18. Cauda 30^{gr} longa, solique directe opposita in *Nova-Anglia* cernebatur, & protendebatur usque ad stellam ♂, quae tunc erat in ♍ 9^{gr}. 54'. *Nov*. 19. In *Mary-land* cauda visa fuit gradus 15 vel 20 longa. Dec. 10. Cauda (observante *Flamstedio*) transibat per medium distantiae inter caudam serpentis *Ophiuchi* & stellam δ in aquilae australi ala, & desinebat prope stellas A,ω,b in tabulis *Bayeri*. Terminus igitur erat in ♑ $19\frac{1}{2}^{gr}$, cum latitudine boreali $34\frac{1}{4}^{gr}$ circiter. *Dec*. 11. Cauda surgebat ad usque caput sagittae (*Bayero* α,β) desinens in ♑ 26^{gr} . 43', cum latitudine boreali 38^{gr}. 34'. *Dec*. 12, Cauda transibat per medium sagittae, nec longe ultra protendebatur, desinens in ♒ 4^{gr}, cum latitudine boreali $42\frac{1}{2}^{gr}$ circiter. Intelligenda sunt haec de longitudine caudae clarioris. Nam luce obscuriore, in coelo forsan magis sereno, cauda *Dec*.12, hora 5. 40' *Romae* (observante *Ponthaeo*) supra cygni uropygium ad gradus 10 sese extulit; atque ab hac stella eius latus ad occasum & boream min. 45 destitit. Lata autem erat cauda his diebus gradus 3, iuxta terminum superiorem, ideoque medium eius distabat a stella illa 2^{gr}. 15' austrum versus, & terminus superior erat in ♓ 22^{gr}, cum latitudine boreali 61^{gr}. Et hinc longa erat cauda 70*gr* circiter. *Dec*. 21. Eadem surgebat fere ad cathedram *Cassiopeiae*, aequaliter distans a β & *Schedir*, & distantiam ab utraque distantiae earum ab invicem aequalem habens, ideoque desinens in ♈ 24^{gr}, cum latitudine $47\frac{1}{2}^{gr}$. *Dec*. 29. Cauda tangebat *Scheat* sitam ad sinistram, & intervallum stellarum duarum in pede boreali *Andromedae* accurate complebat, & longa erat 54^{gr} ; ideoque definebat in ♉ 19^{gr}, cum latitudine 35^{gr}. *Jan*. 5. Cauda tetigit stellam π in pectore Andromedae ad latus eius dextrum, & stellam μ in eius cingulo ad latus sinistrum; & (iuxta observationes nostras) longa erat 40^{gr}; curva autem erat & convexo latere spectabat ad austrum. Cum circulo per solem & caput cometae transeunte angulum confecit graduum 4 iuxta caput cometae; at iuxta terminum alterum inclinabatur ad circulum illum in angulo 10 vel 11 graduum & chorda caudae cum circulo illo continebat angulum graduum octo. *Jan*. 13. Cauda luce satis sensibili terminabatur inter *Alamech* & *Algol*, & luce tenuissima definebat e regione stellae in χ latere *Persei*. Distantia termini caudae a circulo solem & cometam jungente erat 3^{gr}. 50', & inclinatio chordae caudae ad circulum illum $8\frac{1}{2}^{gr}$. *Jan*. 25 & 26. Cauda luce tenui micabat ad longitudinem graduum 6 vel 7; & nocte una & altera sequente ubi coelum valde serenum erat, luce tenuissima & aegerrime sensibili attingebat longitudinem graduum duodecim & paulo ultra. Dirigebatur autem eius axis ad lucidam in humero orientali aurigae accurate, ideoque declinabat ab oppositione solis boream versus in angulo graduum decem. Denique *Feb*. 10 caudam oculis armatis aspexi gradus duos longam. Nam lux praedicta tenuior per vitra non apparuit. *Ponthaeus* autem *Feb*. 7. se caudam ad longitudinem graduum 12. vidisse scribit. *Feb*. 25 & deinceps cometa sine cauda apparuit.

Orbem jam descriptum spectanti & reliqua cometae huius phaenomena in animo revolventi, haud difficulter constabit, quod corpora cometarum sunt solida, compacta, fixa ac durabilia ad instar corporum planetarum. Nam si nihil aliud essent quam vapores vel exhalationes terrae, solis & planetarum cometa hicce in transitu suo per viciniam solis statim dissipari debuisset. Est enim calor solis ut radiorum densitas, hoc est, reciproce ut quadratum distantiae locorum a sole. Ideoque cum distantia cometae a centro solis *Decemb*. 8. ubi in perihelia versabatur, esset ad distantiam terrae a centro solis ut 6 ad 1000 circiter, calor solis apud cometam eo tempore erat ad calorem solis aestivi apud nos. ut 1000000 ad 36, seu 28000 ad 1. Sed calor aquae ebullientis est quasi triplo major quam calor quem terra arida concipit ad aestivum solem, ut expertus sum: & calor ferri candentis (si recte conjector) quasi triplo vel quadruplo major quam calor aquae ebullientis; ideoque calor, quem terra arida apud cometam in perihelio versantem ex radiis solaribus concipere posset, quasi 2000 vicibus major quam calor ferri candentis. Tanto autem calore vapores & exhalationes, omnisque materia volatilis statim consumi ac dissipari debuissent.

Cometa igitur in perihelio suo calorem immensum ad solem concepit, & calorem illum diutissime conservare potest. Nam globos ferri candentis digitum unum latus, calorem suum omnem spatio horae unius in aere consistens vix amitteret. Globus autem major calorem diutius conservaret in ratione diametri, propterea quod superficies (ad cuius mensuram per contactum aeris ambientis refrigeratur) in illa ratione minor est pro quantitate materiae suae calidae inclusae. Ideoque globus ferri candentis huic terrae aequalis, id est, pedes plus minus 40000000 latus, diebus totidem, & idcirco annis 50000, vix refrigesceret. Suspicor tamen quod duratio caloris, ob causas latentes, augeatur in minore ratione quam ea diametri: & optarim rationem veram per experimenta investigari.

Porro notandum est quod cometa mense *Decembri*, ubi ad solem modo incaluerat, caudam emittebat longe majorem & splendidiorem quam antea mense *Novembri*, ubi perihelium nondum attigerat. Et universaliter caudae omnes maximae & fulgentissimae e cometis oriuntur statim post transitum eorum per regionem solis. Conducit igitur calefactio cometae ad magnitudinem cauda: Et inde colligere videor quod cauda nihil allud sit quam vapor longe tenuissimus, quem caput seu nucleus cometae per calorem suum emittit.

Caeterum de cometarum caudis triplex est opinio; eas vel jubar esse solis per translucida cometarum capita propagatum, vel oriri ex refractione lucis in progressu ipsius a capite cometae in terram, vel denique nubem esse seu vaporem a capite cometae jugiter surgentem & abeuntem in partes a sole aversas. Opinio prima eorum est qui nondum imbuti sunt scientia rerum opticarum. Nam jubar solis in cubiculo tenebroso non cernitur, nisi quatenus lux reflectitur e pulverum & fumotum particulis per aerem semper volitantibus: ideoque in aere fumis crassioribus infecto splendidius est, & sensum fortius ferit; in aere clatiore tenuius est & aegrius sentitur: in coelis autem sine materia reflectente nullum esse potest. Lux non cernitur quatenus in jubare est, sed quatenus inde reflectitur ad oculos nostros. Nam visio non sit nisi per radios qui in oculos impingunt. Requiritur igitur materia aliqua reflectens in regione caudae, ne coelum totum luce solis illustratum uniformiter splendeat. Opinio secunda multis premitur difficultatibus. Caudae nunquam variegantur coloribus: qui tamen refractionum solent esse comites inseparabiles. Lux fixarum & planetarum distincte ad nos transmissa demonstrat medium coeleste nulla vi refractiva pollere. Nam quod dicitur fixas ab *Aegyptiis* cometas nonnunquam visas suisse, id, quoniam rarissime contingit, ascribendum est nubium refractioni fortuitae. Fixarum quoque radiatio & scintillatio ad refractiones tum oculorum tum aeris tremuli referendae sunt: quippe quae admotis oculo telescopiis evanescunt. Aeris & ascendemium vaporum tremore sit, ut radii facile de angusto pupillae spatio per vices detorqueantur, de latiore autem vitri objectivi apertura neutiquam. Inde est quod scintillatio in priori casu generetur, in posteriore autem cesset: & cessatio in posteriore casu demonstrat regularem transmissionem lucis per coelos sine omni refractione sensibili. Nequis contendat quod caudae non soleant videri in cometis, cum eorum lux non est satis fortis, quia tunc radii secundarii non habent satis virium ad oculos movendos, & propterea caudas fixarum non cerni: sciendum est quod lux fixarum plus centum vicibus augeri potest mediantibus telescopiis, nec tamen caudae cernuntur. Planetarum quoque lux copiosior est, caudae vero nullae: cometae autem saepe caudatissimi sunt, ubi capitum lux tenuis est & valde obtusa. Sic enim cometa anni 1680, mense *Decembri*, quo tempore caput luce sua vix aequabat stellas secundae magnitudinis, caudam emittebat splendore notabili usque ad gradus 40, 50, 60 vel 70 longitudinis & ultra: postea *Jan*. 17 & 18 caput apparebat ut stella septimae tantum magnitudinis, cauda vero luce quidem pertenui sed satis sensibili longa erat 6 vel 7 gradus, & luce obscurissima, quae cerni vix posset, porrigebatur ad gradum usque duodecimum vel paulo ultra: ut supra dictum est. Sed & *Feb.* 9 & 10 ubi caput nudis oculis videri desierat, caudam gradus duos longam per telescopium contemplatus sum. Porro si cauda oriretur ex refractione materiae coelestis, & pro figura coelestis deflecteretur de solis oppositione, deberet deflexio illa in iisdem coeli regionibus in eandem semper partem fieri. Atqui cometa anni 1680 *Decemb.* 28. hora $8\frac{1}{2}$ p.m. *Londini*, versabatur in ♓ $8^{gr}.41'$, cum latitudine boreali $28^{gr}. 6'$, sole existente in ♑ $18^{gr}.26'$. Et cometa anni 1577*, Dec*. 29 versabatur in ♓ $8^{gr}. 41'$ cum latitudine boreali $28^{gr}.40'$, sole etiam existente in ♑ $18^{gr}.26'$ circiter. Utroque in casu terra versabatur in eadem loco, & cometa apparebat in eadem coeli parte: in priori tamen casu cauda cometae (ex meis & aliorum observationibus) declinabat angulo graduum $4\frac{1}{2}$ ab oppositione solis aquilonem versus ; in posteriore vera (ex observationibus Tychonis) declinatio erat graduum 21 in austrum. Igitur repudiata coelorum refractione, superest ut phaenomena caudarum ex materia aliqua lucem reflectente deriventur.

Caudas autem a capitibus oriri & in regiones a sole aversas ascendere confirmatur ex legibus quas observant. Ut quod in planis orbium cometarum per solem transeuntibus jacentes, deviant ab oppositione solis in eas semper partes, quas capita in orbibus illis progredientia relinquunt. Quod spectatori in his planis constituto apparent in partibus a sole directe aversis; digrediente autem spectatore de his planis, deviatio paulatim sentitur, & indies apparet major. Quod deviatio caeteris paribus minor est ubi cauda obliquior est ad orbem cometae, ut & ubi caput cometae ad solem propius accedit; praesertim si spectetur deviationis angulus iuxta caput cometae. Praeterea quod caudae non deviantes apparent rectae, deviantes autem incurvantur. Quod curvatura major est ubi major est deviatio, and magis sensibilis ubi cauda caeteris paribus longior est: nam in brevioribus curvatura aegre animadvertitur. Quod deviationis angulus minor est iuxta caput cometae, major iuxta caudae extremitatem alterant, atque ideo quod cauda convexo sui latere partes respicit a quibus fit deviatio, quaeque in recta sunt linea a sole per caput cometae in infinitum ducta. Et quod caudae quae prolixiores sunt & latiores, & luce vegetiore micant, sint ad latera convexa paulo splendidiores & limite minus indistincto terminatae quam ad concava. Pendens igitur phaenomena caudae a motu capitis, non autem a regione coeli in qua caput conspicitur; & propterea non fiunt per refractionem coelorum, sed a capite suppeditante materiam oriuntur. Etenim ut in aere nostro sumus corporis cuiusvis igniti petit superiora, idque vel perpendiculariter si corpus quiescat, vel oblique si corpus moveatur in latus: ita in coelis, ubi corpora gravitant in solem, fumi & vapores ascendere debent a sole (uti jam dictum est) & superiora vel recta petere, si corpus fumans quiescit; vel oblique, si corpus progrediendo loca semper deserit a quibus superiores vaporis partes ascendetant. Et obliquitas ista minor erit ubi ascensus vaporis velocior est: nimirum in vicinia solis & iuxta corpus fumans. Ex obliquitatis autem diversitate incurvabitur vaporis columna: & quia vapor in columnae latere praecedente paulo recentior est, ideo etiam is ibidem aliquanto densior erit, lucemque propterea copiosius reflectet,and limite minus indistincto terminabitur. De caudarum agitationibus subitaneis & incertis, deque earum figuris irregularibus, quas nonnulli quandoque describunt, hic nihil adjicio; propterea quod vel a mutationibus aeris nostri, & motibus nubium caudas aliqua ex parte obscurantium oriantur; vel forte a partibus viae lacteae, quae cum caudis praetereuntibus confundi possint, ac tanquam earum partes spectari.

Vapores autem, qui spatiis tam immensis implendis sufficiant, ex cometarum atmosphaeris oriri posse, intelligetur ex raritate aeris nostri. Nam aer iuxta superficiem terrae spatium occupat quasi 850 partibus majus quam aqua eiusdem ponderis, ideoque aeris columna cylindrica pedes 850 alta eiusdem est ponderis cum aquae columna pedali latitudinis eiusdem. Columna autem aeris ad summitttem atmosphaerae assurgens aequat pondere suo columnam aquae pedes 33 altam circiter; & propterea si columnae totius aereae pars inferior pedum 850 altitudinis dematur, pars reliqua superior aequabit pondere suo columnam aquae altam pedes 32. Inde vero (per regulam multis experimentis confirmatum, quod compressio aeris sit ut pondus atmosphaerae incumbentis, quodque gravitas sit reciproce ut quadratum distantiae locorum a centro terrae) computationem per Corol. Prop. XX11. Lib. 11. ineundo, inveni quod aer, si ascendatur a superficiae terrae ad altitudinem semidiametri unius terrestris, rarior sit quam apud nos in ratione longe majori, quam spatii omnis infra orbem saturni ad globum diametro digiti unius descriptum. Ideoque globus aeris nostri digitum unum latus, ea cum raritate quam haberet in altitudine semidiametri unius terrestris, impleret omnes planetarum regiones usque ad sphaeram saturni & longe ultra. Proinde cum aer adhuc altior in immensum rarescat; & coma seu atmosphaera cometae, ascendendo ab illius centro, quasi decuplo altior fit quam superficies nuclei, deinde cauda adhuc altius ascendat, debebit cauda esse quam rarissima. Et quamvis ob longe crassiorem cometarum atmosphaeram, magnamque corporum gravitationem solem versus, & gravitationem particularum aeris & vaporum in se mutuo, fieri possit ut aer in spatiis coelestibus inque cometarum caudis non adeo rarescat; perexiguam tamen quantitatem aeris & vaporum ad omnia illa caudarum phaenomena abunde sufficere, ex hac computatione perspicuum est. Nam & caudarum insignis raritas colligitur ex astris per eas translucentibus. Atmosphaera terrestris luce solis splendens, crassitudine sua paucorum milliarium, & asta omnia & ipsam lunam obscurat & extinguit penitus: per immensam vero caudarum crassitudinem, luce pariter solari illustratam, asta minima sine claritatis detrimento translucere noscuntur. Neque major esse solet caudarum plurimarum splendor, quam aeris nostri in tenebroso cubiculo latitudine digiti unius duorumve lucem solis in jubare reflectentis.

Quo temporis spatio vapor a capite ad terminum caudae ascendit, cognosci fere potest ducendo rectam a termino caudae ad solem, & notando locum ubi recta illa trajectoriam secat. Nam vapor in termino caudae, si recta ascendat a sole, ascendere coepit a capite, quo tempore caput erat in loco intersectionis. At vapor non recta ascendit a sole, sed motum cometae, quem ante ascensum suum habebat, retinendo, & cum motu ascensus sui eundem componendo, ascendit oblique. Unde verior erit problematis solutio, ut recta illa, quae orbem secat, parallela sit longitudini caudae, vel potius (ob motum curvilineum cometae) ut eadem a linea caudae divergat. Hoc pacto inveni quod vapor, qui erat in termino caudae *Jan*. 25, ascendere creperat a capite ante *Dec*. 11, ideoque ascensu suo toto dies plus 45 consumpserat. At cauda illa omnis quae *Dec*. 10 apparuit, ascenderat spatio dierum illorum duorum, qui a tempore perihelii cometae elapsi fuerant. Vapor igitur sub initio in vicinia solis celerrime ascendebat, & postea cum motu per gravitatem suam semper retardato ascendere pergebat; & ascendendo augebat longitudinem caudae : cauda autem, quamdiu apparuit, ex vapore fere omni constabat, qui a tempore perihelii ascenderat; & vapor, qui primus ascendit, & terminum caudae composuit, non prius evanuit quam ob nimiam suam tam a sole illustrante quam ab oculis nostris distantiam videri desiit. Unde etiam caudae cometarum aliorum, quae breves sunt, non ascendunt motu celeri & perpetuo a capitibus & mox evaneseunt, sed sunt permanentes vaporum & exhalationum columnae, a capitibus lentissimo multorum dierum motu propagatae, quae, participando ; motum illum capitum quem habuere sub initio, per coelos una cum capitibus moveri pergunt. Et hinc rursus colligitur spatia coelestia vi resistendi destitui; utpote in quibus non solum solida planetarum & cometarum corpora, sed etiam rarisssimi caudarum vapores motus suos velocissimos liberrime peragunt ac diutissime conservant.

Ascensum caudarum ex atmosphaeris capitum & progressum in partes a sole aversas Keplerus ascribit actioni radiorum lucis materiam caudae secum rapientium. Et auram longe tenuissimam in spatiis liberrimis actioni radiorum cedere, non est a ratione prorsus alienum, non obstante quod substantiae crassae impeditissimis in regionibus nostris a radiis solis sensibiliter propelli nequeant. Alius particulas tam leves quam graves dari posse existimat, & materiam caudarum levitare, perque levitatem suam a sole ascendere. Cum autem gravitas corporum terrestrium sit ut materia in corporibus, ideoque servata quantitate materiae intendi & remitti nequeat, suspicor ascensum illum ex rarefactione materiae caudarum potius oriri. Ascendit fumus in camino impulsu aeris cui innatat. Aer ille per calorem rarefactus ascendit, ob diminutam suam gravitatem specificam, and fumum implicatum rapit secum. Quidni cauda cometae ad eundem modum ascendetit a sole? Nam radii solares non agitant media, quae permeant, nisi in reflexione & refractione. Particulae reflectentes ea actione calefactae calefacient auram aethereqm cui implicantur. Illa calore sibi communicato rarefiet, & ob diminutam ea raritate gravitatem suam specificam, qua prius tendebat in solem, ascendet & secum rapiet particulas reflectentes ex quibus cauda componitur: Ad ascensum vaporum conducit etiam, quod hi gyrantur circa solem & ea actione conatur a sole recedere, at solis atmosphaera & materia coelorum vel plane quiescit, vel motu solo quem a solis rotatione acceperit, tardius gyrator. Hae sunt causae ascensus caudarum in vicinia solis, ubi orbes curviores sunt, & cometae intra densiorem & ea ratione graviorem solis atmosphaeram consistunt, & caudas quam longissimas mox emittunt. Nam caudae, quae tunc nascuntur, conservando motum suum & interea versus solem gravitando, movebuntur circa solem in ellipsibus pro more capitum, & per motum illum capita semper commitabuntur & iis liberrime adhaerebunt. Gravitas enim vaporum in solem non magis efficiet ut caudae postea decidant a capitibus solem versus, quam gravitas capitum efficere possit, ut haec decidant a caudis. Communi gravitate vel simul in solem cadens, vel simul in ascensu suo retardabuntur; ideoque gravitas illa non impedit; quo minus caudae & capita positionem quamcunque ad invicem a causis jam descriptis, aut aliis quibuscunque facillime accipiant & postea liberrime servent.

Caudae igitur, quae in cometarum periheliis nascuntur, in regions longinquas cum eorum capitibus abibunt, & vel inde post longam annorum seriem cum iisdem ad nos redibunt, vel potius ibi rarefactae paulatim evanescent. Nam postea in descensu capitum ad solem caudae novae breviusculae lento motu a capitibus propagati debebunt, &

subinde in periheliis cometarum illorum, qui ad usque atmosphaeram solis descendunt, in immensum augeri. Vapor enim in spatiis illis liberrimis perpetuo rarescit ac dilatatur. Qua ratione sit ut cauda omnis ad extremitatem superiorem latior sit quam iuxta caput cometae. Ea autem rarefactione vaporem perpetuo dilatatum diffundi tandem & spargi per coelos universos, deinde paulatim in planetas per gravitatem suam attracti, & cum eorum atmosphaeris misceri rationi consentaneum videtur. Nam quemadmodum maria ad constitutionem terrae huius omnino requiruntur, idque ut ex iis per calorem solis vapores copiose satis excitentur, qui vel in nubes coacti decidant in pluviis, & terram omnem ad procreationem vegetabilium irrigent & nutriant ; vel in frigidis montium verticibus condensati (ut aliqui cum ratione philosophantur) decurrant in fontes & flumina: sic ad conservationem marium & humorum in planetis requiri videntur cometae, ex quorum exhalationibus & vaporibus condensatis, quicquid liquoris per vegetationem & putrefactionem consumitur & in terram aridam convertitur, continuo suppleri & refici possit. Nam vegetabilia omnia ex liquoribus omnino crescunt, dein magna ex parte in terram aridam per putrefactionem abeunt, & limus ex liquoribus putrefactis perpetuo decidit. Hinc moles terrae aridae indies augetur, & liquores, nisi aliunde augmentum sumerent, perpetuo decrescere deberent, ac tandem deficere. Porro suspicor spiritum illum, qui aeris nostri pars minima est sed subtilissima and optima, & ad rerum omnium vitam requiritur, ex cometis praecipue venire.

Atmosphaerae cometarum in descensu eorum in solem excurrendo in caudas, diminuuntur, & (ea certe in parte quae solem respicit) angustiores redduntur: & vicissim in recessu eorum a sole, ubi jam minus excurrunt in caudas, ampliantur; si modo phaenomena eorum *Hevelius* recte notavit. Minimae autem apparent, ubi capita jam modo ad solem calefacta in caudas maximas & fulgentissimas abiere; & nuclei fumo forsan crassiore & nigriore in atmosphaerarum partibus infimis circundantur. Nam fumus omnis ingenti calore excitatus crassior & nigrior esse solet. Sic caput cometae, de quo egimus, in aequalibus a sole ac terra distantiis obscurius apparuit post perihelium suum quam antea. Mense enim *Decembri* cum stellis tertiae magnitudinis conferri solebat, at mense *Novembri* cum stellis primae & secundae. Et qui utrumque viderant, majorem describunt cometam priorem. Nam juveni cuidam *Cantabrigiensi, Novem*. 19, cometa hicce luce sua quantumvis plumbea & obtusa, aequabat spicam virginis, and clarius micabat quam postea. Et *Montenaro Nov*. 20 st. vet. cometa apparebat major stellis primae magnitudinis, existente cauda duorum graduum longitudinis. Et D. *Storer* literis, quae in manus nostras incidere, scripsit caput eius mense *Decembri*, ubi caudam maximam & fulgentissimam emittebat, parvum esse & magnitudine visibili longe cedere cometae, qui mense *Novembri* ante solis ortum apparuerat. Cuius rei rationem esse conjectabatur, quod materia capitis sub initio copiosior esset, & paulatim consumeretur.

Eodem spectare videtur, quod capita cometarum aliorum, qui caudas maximas & fulgentissimas emiserunt, apparuerint subobscura & exigua. Nam anno 1668 *Mart*. 5. st. nov. hora septima vespertina *R. P. Valentinus Estancius, Brasiliae* agens, cometam vidit horizonti proximum ad occasum solis brumalem, capite minimo & vix conspicuo, cauda vero supra modum fulgente, ut stantes in littore speciem eius e mari reflexam facile cernerent. Speciem utique habebat trabis splendentis longitudine 23 graduum, ab occidente in austrum vergens, & horizonti fere parallela. Tantus autem splendor tres solum dies durabat, subinde notabiliter decrescens; & interea decrescente splendore aucta est magnitudine cauda. Unde etiam in *Lusitania* quartam fere coeli partem (id est, gradus 45) occupasse dicitur ab occidente in orientem splendore cum insigni protensa: nec tamen tota apparuit, capite semper in his regionibus infra horizontem delitescente. Ex incremento caudae & decremento splendoris manifestum est, quod caput a sole recessit, eique proximum fuit sub initio, pro more cometae anni 1680. Et in chronico *Saxonico* similis legitur cometa anni 1106, *cuius stella erat parva & obscura (ut ille anni 1680) sed splendor qui ex ea exivit valde clarus & quase ingens trabs ad orientem & aquilonem tendebat*, ut habet etiam *Hevelius* ex *Simeone Dunelmensi* Monacho. Apparuit initio mensis *Februarii*, ac deinceps circa vesperam, ad occasum solis brumalem. lnde vero & ex situ caudae colligitur caput suisse soli vicinum. *A sole*, inquit *Matthaeus Parisienis, distabat quasi cubito uno, ab hora tertia* [rectius sexta] *usque ad horam nonam radium ex se longum emittens.* Talis etiam erat ardentissimus ille cometa ab *Aristotele* descriptus Lib. I. Meteor, 6. *cuius caput primo die non conspectum est, eo quod ante solem vel saltem sub radius solaribus occidisset, sequente vero die quantum potuit visum est. Nam quam minima fieri potest distantia solem reliquit, & mox occubuit. Ob nimium ardorem* [caudae scilicet] *nondum apparebat capitis sparsus, sed procedente tempore (ait Aristoteles) cum* [cauda] *jam minus flagraret, reddita est* [capiti] *cometae sua facies. Et splendorem suum ad tertiam usique coeli partem* [id est, ad 60^{gr}] *extendit.Apparuit autem tempore hyberno* [an. 4. olymp. 101.) & *ascendens usque ad cingulum orionis, ibi evanuit.* Cometa ille anni 1618, qui e radiis solaribus caudatissimus emersit, stellas primae magnitudinis aequare vel paulo superare videbatur, sed majores apparuere cometae non pauci, qui caudas breviores habuere. Horum aliqui jovem, alii venerem vel etiam lunam aequasse traduntur.

Diximus cometas esse genus planetarum in orbibus valde eccentricis circa solem revolventium: Et quemadmodum e planetis non caudatis minores esse solent, qui in orbibus minoribus & soli propioribus gyrantur, sic etiam cometas, qui in periheliis suis ad solem propius accedunt, ut plurimum minores esse, ne solem attractione sua nimis agitent, rationi consentaneum videtur. Orbium vero transversas diametros & revolutionum tempora periodica, ex collatione cometarum in iisdem orbibus post longa temporum intervalia redeuntium, determinanda relinquo. Interea huic negotio propositio sequens lumen accendere potest.

PROPOSITIO XLLI. PROBLEMA XX11.
Inventam cometae trajectoriam corrigere.

Operatio 1. Assumatur positio plani trajectoriae, per propositionem superiorem inventa; & seligantur tria loca cometae observationibus accuratissimis desinita, & ab invicem quam maxime distantia; sitque *A* tempus inter primam & secundam, ac *B* tempus inter secundam ac tertiam. Cometam autem in eorum aliquo in perigaeo versari convenit, vel saltem non longe a perigaeo abesse. Ex his locis apparentibus inveniantur, per operationes trigonometricas, loca tria vera cometae in assumpto illo plano trajectoriae. Deinde per loca illa inventa, circa centrum solis seu umbilicum, per operationes arithmeticas, ope Prop. XXI. Lib. I. instituras, describatur sectio conica: & eius areae, radiis a sole ad loca inventa ductis terminatae, sunto *D* & *E*; nempe *D* area inter observationem primam & secundam, & E area inter secundam ac tertiam. Sitque T tempus totum, quo area tota $D+E$ velocitate cometae per Prop. XVI. Lib.1. inventa describi debet.

Oper. 2. Augeatur longitudo nodorum plani trajectoriae, additis ad longitudinem illam 20' vel 30', quae dicantur P; & servetur plani illius inclinatio ad planum eclipticae. Deinde ex praeditus tribus cometae locis observatis, inveniantur in hoc novo plano loca tria vera, ut supra: deinde etiam orbis per loca illa transiens, & eiusdem areae duae inter observationes descriptae, quae sint d & e, nec non tempus totum t, quo area tota $d+e$ describi debeat.

Oper. 3. Servetur longitudo nodorum in operatione prima, & augeatur inclinatio plani trajectoriae ad planum eclipticae, additis ad inclinationem illam 20' vel 30', quae dicantur Q. Deinde ex observatis praedictis tribus cometae locis apparentibus inveniantur in hoc novo plano loca tria vera, orbisque per loca illa transiens, ut & eiusdem areae duae inter observationes descriptae, quae sint δ & ε, & tempus totum τ, quo area tota $\delta+\varepsilon$ describi debeat.

Jam sit C ad 1 ut A ad B, & G ad 1 ut D ad E, & g ad 1 ut d ad e, & γ ad 1 ut δ ad ε; sitque S tempus verum inter observationem primam ac tertiam; & signis + & – probe observatis quaerantur numeri m & n, ea lege, ut sit $2G-2.C=mG-mg+nG-n\gamma$, & $2T-2S$ aequale $mT-mt+nT-n\tau$. Et si in operatione prima I designet inclinationem plani trajectoriae ad planum eclipticae, & K longitudinem nodi alterutrius, erit $I+nQ$ vera inclinatio plani trajectoriae ad planum eclipticae, & $K+mP$ vera longitudo nodi. Ac denique si in operatione prima, secunda ac tertia, quantitates R, r & ρ designent latera recta trajectoriae, & quantitates $\frac{1}{L}, \frac{1}{l}, \frac{1}{\lambda}$ eiusdem latera transversa respective: erit $R+mr-mR+n\rho-nR$ verum latus rectum, & $\frac{1}{L+ml-mL+n\lambda-nL}$ verum latus transversum trajectoriae quam cometa describit. Dato autem latere transverso datur etiam tempus periodicum cometae. *Q.E.I.*

Caeterum cometarum revolventium tempora periodica, & orbium latera transversa, haud satis accurate determinabuntur, nisi per collationem cometarum inter se, qui diversis temporibus apparent. Si plures cometae, post aequalia temporum intervalla, eundem orbem descripsisse reperiantur, concludendum erit hos omnes esse unum & eundem cometam, in eodem orbe revolventem. Et tum demum ex revolutionum temporibus dabuntur orbium latera transversa, & ex his lateribus determinabuntur orbes elliptici.

In hunc finem computandae sunt igitur cometarum plurium trajectoriae, ex hypothesi quod sint parabolicae. Nam huiusmodi trajectoriae cum phaenomenis semper congruent quamproxime. Id liquet, non tantum ex trajectoria parabolica cometae anni 1680, quam cum observationibus supra contuli, sed etiam ex ea cometae illius insignis, qui annis 1664 & 1665, apparuit, & ab *Hevelio* observatus fuit. Is ex observationibus suis longitudines & latitudines huius cometae computavit, sed minus accurate. Ex iisdem observationibus Halleius noster loca cometae huius denuo computavit, & tum demum ex locis sic inventis trajectoriam cometae determinavit. Invenit autem eius nodum ascendensem in ♊ 21^{gr} 13'. 55'', inclinationem orbitae ad planum eclipticae 21^{gr}. 18'. 40'', distantiam perihelii a nodo in orbita 49^{gr}.17'. 30''. Perihelium in ♌ 8^{gr}.40'. 30'' cum latitudine austrina heliocentrica

$16^{gr}.1'.45''$. Cometam in perihelio *Novem*. 24^{d}. 11^{h}. 52'. p. m. tempore aequato *Londini*, vel $13^{h}.8'$ *Gedani*, stylo veteri, & latus rectum parabolae 410286, existente mediocri terrae a sole distantia 100000. Quam probe loca cometae in hoc orbe computata congruunt cum observationibus, patebit ex tabula sequente ab *Halleio* supputata.

Temp. Appar. Gedani, st.vet.	Observatae Cometae distantiae.		Loca observata.		Loca computata in Orbe.
Decemb.					
$3^{d}.18^{h}.29'\frac{1}{2}$	a Corde Leonis a Spica Virginis	$46^{gr}.24'.20''$ 22.52.10	Long.♎ Lat. aust.	$7^{gr}.\ 1'.\ 0''$ 21.39. 0	♎ $7^{gr}.\ 1'.29''$ 21.38.50
$4.18.1\frac{1}{2}$	a Corde Leonis a Spica Virginis	46. 2.45 23.52.40	Long.♎ Lat. aust.	16.15. 0 22.24. 0	♎ 0.16. 5 22.24. 0
7.17.48	a Corde Leonis a Spica Virginis	44.48. 0 27.50.40	Long.♎ Lat. aust.	3. 6. 0 25.22. 0	♎ 3. 7. 33 25.21.40
17.14.43	a Corde Leonis ab Hum.Orionis dext	53.15.15 45.43.30	Long.♎ Lat. aust.	2.50. 0 49.25. 0	♌ 2.50.0 49.25. 0
19.9.25	a Procyone a Lucid.Mandib.Ceti	35.13.15 52.56. 0	Long.♊ Lat. aust.	28.40.30 25.48. 0	♊ 28.43. 0 45.46. 0
$20.9.53\frac{1}{2}$	a Procyone a Lucid.Mandib.Ceti	40.49. 0 40. 4. 0	Long.♊ Lat. aust.	13. 3. 0 39.54. 0	♊ 3. 5. 0 39.53. 0
$21.9.9\frac{1}{2}$	ab Hum.dext. Orionis a Lucid.Mandib.Ceti	26.21.25 29.28. 0	Long.♊ Lat. aust.	2.16. 0 33.41. 0	♊ 2.18.30 33.39.40
22.9. 0	ab Hum.dext. Orionis a Lucid.Mandib.Ceti	29.47. 0 30.29.30	Long.♉ Lat. aust.	24.24. 0 27.45. 0	♉ 24.27. 0 27.46. 0
26.7.58	a Lucida Arietis ab Aldebaran	23.20. 0 26.44. 0	Long.♉ Lat. aust.	9. 0. 0 12.36. 0	♉ 9. 2.28 12.34.13
27.7.58	a Lucida Arietis ab Aldebaran	20.45. 0 28.10. 0	Long.♉ Lat. aust.	7. 5.40 10.23. 0	♉ 7. 8.45 10.23.13
28.7.58	a Lucida Arietis a Palilicio	18.29. 0 29.37. 0	Long.♉ Lat. aust.	5.24.45 8.22.50	♉ 5.27.52 8.23.37
31.6.45	a Cing. Androm. a Palilicio	30.48.10 32.53.30	Long.♉ Lat. aust.	2. 7.40 4.13. 0	♉ 2. 8.20 4.16.25
Jan. 1665 $7.7.37\frac{1}{2}$	a Cing. Androm. a Palilicio	25.11. 0 37.12.25	Long.♈ Lat. bor.	28.24.47 0.54. 0	♈ 28.24. 0 0.53. 0
13.7. 0	a Cing. Androm. a Palilicio	28. 7.10 38.55.20	Long.♈ Lat. bor.	27. 6.54 3. 6.50	♈ 27. 6.39 3. 7.40
24.7. 29	a Cing. Androm. a Palilicio	20. 32.15 40. 5. 0	Long.♈ Lat. bor.	26.29.15 5.25.50	♈ 26.28.50 5. 26. 0
Feb. 7.8. 37			Long.♈ Lat. bor.	27. 4.46 7. 3.29	♈ 27.24.55 7. 3.15
22.8.46			Long.♈ Lat. bor.	28.29.46 8.12.36	♈ 28.29.58 8.10.25
Mar. 1.8. 16			Long.♈ Lat. bor.	29.18.15 8.36.26	♈ 29.18.20 8.36.26
7.8.37			Long.♈ ♉ Lat. bor.	0. 2.48 8.56.30	♉ 0. 2.42 8.56.56

Mense *Februario* anni ineuntis 1665, stella prima arietis, quam in sequentibus vocabo γ, erat in ♈ 18^{gr}. 30'. 15'' cum latitudine boreali 7^{gr}. 8'. 58''. Secunda arietis erat in ♈ 29^{gr}. 17'.18'' cum latitudine boreali 8^{gr}. 18'.16''. Et stella quaedam alia septimae magnitudinis, quam vocabo *A*, erat in ♈ 28^{gr}.14'.45'' cum latitudine boreali 8^{gr}.28'. 33''. Cometa vero *Feb*. $7^{d}.7^{h}$. 30' *Parisiis* (id est *Feb*. 7^{d}. 8^{h}. 37' *Gedani*) st. vet. triangulum constituebat cum stellis illis γ and *A* rectangulum ad γ. Et distantia cometae a stella γ aequalis erat distantiae stellarum γ and *A*, id est 1^{gr}.19'. 46'' in circulo magno, atque ideo ea erat 1^{gr}. 20'.26'' in parallelo latitudinis stellae γ. Quare si de longitudine stellae γ detrahatur longitudo 1^{gr}.20'. 26'', manebit longitudo cometae ♈ 27^{gr}. 9'. 49''. *Auzoutius* ex hae sua observatione cometam posuit in ♈ 27^{gr}. 0' circiter. Et ex schemate, quo *Hookius* motum eius delineavit, is jam erat in ♈ 26^{gr}.59'. 24". Ratione mediocri posui eundem in ♈ 27^{gr}. 4'. 46". Ex eadem observatione *Auzoutius* latitudinem cometae jam posuit 7^{gr} & 4' vel 5' boream versus. Eandem rectius posuisset 7^{gr}. 3'.29", existente scilicet differentia latitudinum cometae & stellae γ, aequali differentiae longitudinum stellarum γ and *A*.

Feb. 22^{d}. 7^{h}. 30' *Londini*, id est Feb. 22^{d}. 8^{h}. 46' *Gedani*, distantia cometae a stella *A*, iuxta observationem *Hookii* a seipso in schemate delineatam, ut & iuxta observationes *Auzoutii* a *Petito* in schemate delineatas, erat pars quinta distantiae inter stellam *A* & primam arietis, seu 15" 57". Et distantia cometae a linea jungente stellam *A* & primam arietis erat pars quarta eiusdem partis quintae, id est 4'. Ideoque cometa erat in ♈ 28^{gr} .29'. 46", cum lat. bor. 8^{gr}. 12'. 36".

Mart. 1^{d}. 7^{h}. 0' *Londini*, id est *Mart*. 1^{d}. 8^{h}. 16' *Gedani*, cometa observatus fuit prope secundam arietis, existente distantia inter eosdem ad distantiam inter primam & secundam arietis, hoc est ad 1^{gr}. 33', ut 4 ad 4; secundum *Hookium*, vel ut 2 ad 23 secundum *Gottignies*. Unde distantia cometae a secunda arietis erat 8'. 16" secundum *Hookium*, vel 8' 5" secundum *Gottignies*, vel ratione mediocri 8'. 10". Cometa vero secundum *Gottignies* jam modo praetergressus fuerat secundam arietis quasi spatio quartae vel quintae partis itineris uno die confecti, id est 1'. 35" circiter (quocum satis consentit *Auzoutius*) vel paulo minorem secundum *Hookium*, puta 1'. Quare si ad longitudinem primae arietis addatur 1', & ad latitudinem eius 8'. 10", habebitur longitudo cometae ♈ 29^{gr}. 18', & latitudo borealis 8^{gr}. 36'. 26".

Mart. 7^{d}. 7^{h}.30' *Parisiis* (id est *Mart*. 7^{d}.8^{h}. 37' *Gedani*) ex observationibus *Anzoutii* distantia cometae a secunda arietis aequalis erat distantiae secundae arietis a stella *A*, id est 52'.29". Et differentia longitudinum cometae & secundae arietis erat 45' vel 46', vel ratione mediocri 45'. 30". Ideoque cometa erat in ♉ 0^{gr}. 2'.48". Ex schemate observationum *Auzoutii*, quod *Petitus* construxit, *Hevelius* deduxit latitudinem cometae 8gr.54'. Sed sculptor viam cometae sub finem motus eius irregulariter incurvavit, & *Hevelius* in schemate observationum *Auzoutii* a se constructo incurvationem irregularem correxit, & sic latitudinem cometae fecit esse 8^{gr}. 55'. 30". Et irregularitatem paulo magis corrigendo, latitudo evadere potest 8^{gr}. 56', vel 8^{gr}. 57'.

Visus etiam fuit hic cometa *Martii* die 9, & tunc locari debuit in ♉ 0^{gr}.18', cum lat. bor. 9^{gr}.3'$\frac{1}{2}$ circiter.

Apparuit hic cometa menses tres, signaque fere sex descripsit, & uno die gradus fere viginti confecit. Cursus eius a circulo maximo plurimum deflexit, in boream incurvatus; & motus eius sub finem ex retrogrado factus est directus. Et non obstante cursu tam insolito, theoria a principio ad finem cum observationibus non minus accurate congruit, quam theoriae planetarum cum eorum observationibus congruere solent, ut inspicienti tabulam patebit. Subducenda tamen sunt minuta duo prima circiter, ubi cometa velocissimus fuit; id quod fiet auferendo duodecim minuta secunda ab angulo inter nodum ascendentem & perihelium, seu constituendo angulum illum 49^{gr}. 27'. 18", Cometae utriusque (and huius & superioris) parallaxis annua insignis fuit, & inde demonstratur motus annuus terrae in orbe magno.

Confirmatur etiam theoria per motum cometae, qui apparuit anno l683. Hic fuit retrogradus in orbe, cuius planum cum plano eclipticae angulum fere rectum continebat. Huius nodus ascendens (computante *Halleio*) erat in ♍ $23^{gr}.23'$; inclinatio orbitae ad eclipticam $83^{gr}.11'$; perihelium in ♊ 25^{gr}. 29'. 30"; distantia perihelia a sole, 56020, existente radio orbis magni 100000, & tempore perihelii *Julii* $2^{d}.3^{h}.50'$. Loca autem cometae in hoc orbe ab *Halleio* computata, & cum locis a *Flamstedio* observatis collata, exhibentur in tabula sequente.

1683 Temp.Aequit.	Locus Solis	Cometae Long.Comp.	Lat. Bor. Comp.	Cometae Long.Obs.	Lat. Bor. Observ.	Differ. Long.	Differ. Lat.
d. h.	gr. ' "	gr. ' "	gr. ' "	gr. ' "	gr. ' "		
Jul. 13.12. 55'	♌ 1. 2.30	♋ 13. 5.42	29.28.13	♋ 13. 6.42	29.28.20	+1'.0"	+0'.7"
15.11.15	2.53·12	11.37. 4	29.34. 0	11.39.43	29.34.50	+1.55	+0.50
17.10.20	4.45.45	10. 7. 6	29.33.33	10. 8.40	29.34. 0	+1.34	+0.30
23.13.40	10·38.21	5.10.27	28.51.41	5.11.30	28.50.28	+1. 3	–1.14
25.14. 5	12.35.28	3.27.53	24.24.47	3.27. 0	28.23.40	–0.53	–1. 7
31. 9.42	18. 9.22	♊ 27.55. 3	26.22.52	♊ 27.54.24	26.22.25	–0.39	–0.27
31.14.55	18.21.53	27.41. 7	16.16.57	27.41. 8	26.14.50	+0. 1	–2. 7
Aug. 2.14.56	20.17.16	25.29.32	25.16.19	25.28.46	25.17.28	–0.46	+1.9
4.10.49	22. 2.50	23.18.20	24.10.49	23.16.55	24.12.19	–1.25	+1.30
6.10. 9	23.56.45	20.42.23	22.47. 5	20.40.32	22.49. 5	–1.51	+2. 0
9.10.26	26.50.52	16. 7.57	20. 6.37	16. 5.55	20. 6.10	–2. 2	–0.27
15.14. 1	♍ 2.47.13	3.30.48	11.37.33	3.26.18	11.32. 1	–4.30	–5.31
16.15.10	3.48. 2	0.43. 7	9.34.16	0.41.55	9.34.13	–1.12	–0. 3
18.15.44	5.45.33	♉ 24·52.53	5.11.15	♉ 24.49. 5	5. 9.11	–3.48	–2. 4
			Austr.		Austr.		
22.14.44	9.35.49	11. 7.14	5.16.58	11. 7.12	5.16.58	–0. 2	–0. 3
23.15.52	10.36.48	7. 2.18	8.17. 9	7. 1.17	8.16.41	–1. 1	–0.28
26.16. 2	13·31.10	♈ 24.45.31	16.38. 0	♈ 24.44. 0	16.38.20	–1.31	+0.20

Confirmatur etiam theoria per motum cometae retrogradi, qui apparuit anno 1682. Huius nodus ascendens (computante *Halleio*) erat in ♉ $21^{gr}.16'$. 30". Inclinatio orbitae ad planum eclipticae 17^{gr}. 6'. 0". Perihelium in ♒ 2^{gr}. 52'. 50". Distantia perihelia a sole 58328, existente radio orbis magni 100000. Et tempus aequatum perihelii *Sept*. $4^{d}.7^{h}$. 39'. Loca vera ex observationibus *Flamstedii* computata, & cum locis per theoriam computatis collata, exhibentur in tabula sequente.

1682. Temp. Appar.	Locus Solis	Cometae Long.Comp.	Lat. Bor. Comp.	Cometae Long.Obs.	Lat. Bor. Observ.	Differ. Long.	Differ. Lat.
d. h.	gr. ' "	gr. ' "	gr. ' "	gr. ' "	gr. ' "	' "	' "
Aug. 19.16.38	♍ 7. 0. 7	18.14.28	25.50. 7	18.14.40	25.49.55	–0.12	+0.12
20.15.38	7.55.52	24.46.23	26.14.42	24.46.22	26.12.52	+0. 1	+1.50
21. 8.21	8.36.14	29.37.15	26.20. 3	29.38. 2	26.17.37	–0.47	+2.26
22. 8. 8	9.33.55	♍ 6.29.53	26. 8.42	♍ 6.30. 3	26. 7.12	–0.10	+1.30
29. 8.20	16.22.40	♒ 12.37.54	18.37.47	♒ 12.37·49	18.34. 5	+0. 5	+3.42
30. 7.45	17.19.41	15.36. 1	17.26.43	15.36.18	17.27.17	+0.43	–0.34
Sept. 1. 7.33	19.16. 9	20·30·53	15.13. 0	20·27· 4	15. 9.49	+3.49	+3.11
4. 7.22	22.11.28	25.42. 0	12.23.48	25.40.58	12.22. 0	+1. 2	+1.48
5. 7.32	23.10.29	27. 0.46	11.33. 8	26.59.24	11.33.51	+1.22	–0.43
8. 7.16	26. 5.58	29.58.44	9.26.46	29.58.45	9.26.43	–0. 1	+0. 3
9. 7.26	27. 5. 9	♏ 0.44.10	8.49.10	♏ 0.44. 4	8.48.25	+0. 6	+0.45

Confirmatur etiam theoria per motum retrogradum cometae, qui apparuit anno 1723. Huius nodus ascendens (computante D. *Bradleo*, astronomiae apud *Oxonienses* professore *Saviliano*) erat in ♈ $14^{gr}.16'$. Inclinatio orbitae ad planum eclipticae 49^{gr}. 59'. Perihelium in ♉ 12^{gr}. 15'. 20". Distantia perihelia a sole 998651, existente radio orbis magni 1000000, & tempore aequato perihelii *Septem*. 16^{d}. 16^{h}. 10'. Loca vero cometae in hoc orbe a *Bradleo* computata, & cum locis a seipso & patruo suo D. *Poundio*, & a D. *Halleio* observatis collata exhibentur in tabula sequente.

1723 Temp. Aequat. Observat.	Comet. Long. Observat.	Lat. Bor. Observat.	Comet. Long.Comput.	Comet. Long. Comput.	Differ. Long.	Differ. Latit:
Octob. 9d. 8h. 5'	♒ 7°.22'.15"	5°. 2'. 0"	♒ 7°.21'.26"	5°. 2'.47"	+49"	– 47"
10. 6 .21	6.41.12	7.44.13	6.41.42	7.43.18	– 50	+ 55
12. 7.22	5.39.58	11.55. 0	5.40.19	11.54.55	– 21	+ 5
14. 8.57	4.59.49	14.43.50	5. 0.37	14.44. 1	– 48	– 11
15. 6.35	4.47.41	15.40.51	4.47.45	15.40.55	– 4	– 4
21. 6.22	4. 2.32	19.41.49	4. 2.21	19.42. 3	+ 11	– 14
22. 6.24	3.59. 2	20. 8.12	3.59.10	20. 8.17	– 8	– 5
24. 8. 2	3.55.29	20.55.18	3.55.11	20.55. 9	+ 18	+ 9
29. 8.56	3.56.17	22.20.27	3.56.42	22.20.10	– 25	+ 17
30. 6.20	3.58. 9	22.32.28	3.58.17	22.32.12	– 8	+ 16
Nov. 5. 5.53	4.16.30	23.38.33	4.16.23	23.38. 7	+ 7	+ 26
8. 7. 6	4.29.36	24. 4.30	4.29.34	24. 4.40	– 18	– 10
14. 6.20	5. 2.16	24.48.46	5. 2.51	24.48.16	– 35	+ 30
20. 7.45	5.42.20	25.24.45	5.43.13	25.25.17	– 53	– 32
Dec. 7.6.45	8. 4.13	26.54.18	8. 3.55	26.53.42	+ 18	+ 36

His exemplis abunde satis manifestum est, quod motus cometarum per theoriam a nobis expositam non minus accurate exhibentur, quam solent motus planetarum per eorum theorias. Et propterea orbes cometarum per hanc theoriam enumerari possunt, and. tempus periodicum cometae in quolibet orbe revolventis tandem sciri, & tum demum orbium ellipticorum latera transversa & apheliorum altitudines innotescent.

Cometa retrogradus, qui apparuit anno 1607, descripsit orbem, cuius nodus ascendens (computante *Halleio*) erat in ♉ 12gr. 21' ; inclinatio plani orbis ad planum eclipticae erat 17gr. 2 ' ; perihelium erat in ♒ 2gr.16'; & distantia perihelia a sole erat 58680, existente radio orbis magni 100000. Et cometa erat in perihelio *Octob*. 16d.3h.50'. Congruit hic orbis quamproxime cum orbe cometae, qui apparuit anno 1682. Si cometae hi duo fuerint unus & idem, revolvetur hic cometa spatio annorum 75, & axis major orbis eius erit ad axcm majorem orbis magni, ut $\sqrt{c}$: 75×75 ad 1, seu 1778 ad 100 circiter. Et distantia aphelia cometae huius a sole, erit ad distantiam mediocrem terrae a sole, ut 35 ad 1 circiter. Quibus cognitis, haud difficile fuerit orbem ellipticum cometae huius determinare. Atque haec ita se habebunt si cometa, spatio annorum septuaginta quinque, in hoc orbe posthac redierit. Cometae reliqui majori tempore revolvi videntur & altius ascendere.

Caeterum cometae, ob magnum eorum numerum, & magnam apheliorum a sole distantiam, & longam moram in apheliis, per gravitates in se mutuo nonnihil turbari debent, & eorum eccentricitates & revolutionum tempora nunc augeri aliquantulum, nunc diminui. Proinde non est expectandum ut cometa idem in eodem orbe, & iisdem temporibus periodicis accurate redeat. Sufficit si mutationes non majores obvenerint, quam quae a causis praedictis oriantur.

Et hinc ratio redditur, cur cometae non comprehendantur zodiaco more planetarum, sed inde migrent & motibus variis in omnes coelorum regiones ferantur. Scilicet eo fine, ut in apheliis suis, ubi tardissime moventur, quam longissime distent ab invicem, & se mutuo quam minime tractant. Qua de causa cometae, qui altius descendunt, ideoque tardissime moventur in apheliis, debent altius ascendere.

Cometa, qui anno 1680 apparuit, minus distabat a sole in perihelio suo quam parte sexta diametri solis; & propter summam velocitatem in vicinia illa, & densitatem aliquam atmosphaerae solis, resistentiam nonnullam sentire debuit, & aliquantulum retardari, and propius ad solem accedere: & singulis revolutionibus accedendo ad solem, incidet is tandem in corpus solis. Sed & in aphelia ubi tardissime movetur, aliquando per attractionem aliorum cometarum retardari potest, & subinde in solem incidere. Sic etiam stellae fixae, quae paulatim expirant in lucem & vapores, cometis in ipsas incidentibus refici possunt, & novo alimento accensae pro stellis novis haberi. Huius generis sunt stellae fixae, quae subito apparent, & sub initio quam maxime splendens, & subinde paulatim evanescunt. Talis fuit stella in cathedra Cassiopeiae quam *Cornelius Gemma* octavo *Novembris* 1572, lustrando illam coeli partem nocte serena minime vidit; at nocte proxima (*Novem*. 9.) vidit fixis omnibus splendidiorem, & luce sua vix cedentem veneri. Hanc *Tycho Brahaeus* vidit undecimo eiusdem mensis ubi maxime splenduit; & ex eo tempore paulatim decrescentem & spatio mensium sexdecim evanescentem observavit. Mense *Novembri*, ubi primum apparuit, venerem luce sua aequabat. Mense *Decembri* nonnihil diminuta jovem aequare videbatur. Anno 1573, mense *Januario* minor erat jove & major Sirio, cui in fine *Februarii* & *Martii* initio evasit aequalis. Mense *Aprili* & *Maio* stellis secundae magnitudinis, *Junio, Julio* & *Augusto* stellis tertiae magnitudinis, *Septembri, Octobri* & *Novembri* stellis quartae, *Decembri* & anni 1574, mense *Januario* stellis quinta, & mense *Februario* stellis sextae magnitudinis aequalis videbatur, mense *Martio* ex oculis evanuit. Color illi ab initio clarus, albicans ac splendidus, postea flavus, & anni 1573 mense Martio rutilans instar martis aut stellae aldebaran; Maio autem albitudinem sublividam induxit, qualem in saturno cernimus, quem colorem usque in finem servavit, semper tamen obscurior facta. Talis etiam fuit stella in dextro pede serpentarii, quam Kepleri discipuli anno 1604, die 30 *Septembris* st vet. apparere coepisse observarunt, & luce sua stellam jovis superasse, cum nocte praecedente minime apparuisset. Ab eo vero tempore paulatim decrevit, & spatio mensium quindecim vel sexdecim ex oculis evanuit. Tali etiam stella nova supra modum splendente *Hipparchus* ad fixas observandas & in catalogum referendas excitatus fuisse dicitur. Sed fixae, quae per vices apparent & evanescunt, quaeque paulatim crescunt, & luce sua fixas tertiae magnitudinis vix unquam superant, videntur esse generis alterius and revolvendo partem lucidam & partem obscuram per vices ostendere. Vapores autem, qui ex sole & stellis fixis & caudis cometarum oriuntur, incidere possunt per gravitatem suam in atmosphaeras planetarum & ibi condensari & converti in aquam & spiritus humidos, & subinde per lentum calorem in sales, & sulphura, & tincturas, & limum, & lutum, & argillam, & arenam, & lapides, & coralia, & substantias alias terrestres paulatim migrare.

SCHOLIUM GENERALE.

Hypothesis vorticum multis premitur difficultatibus. Ut planeta unusquisque radio ad solem ducto areas describat tempori proportionales, tempora periodica partium vorticis deberent esse in duplicata ratione distantiarum a sole. Ut periodica planetarum tempora sint in proportione sesquiplicata distantiarum a sole, tempora periodica partium vorticis deberent esse in sesquiplicata distantiarum proportione. Ut vortices minores circum saturnum, jovem & alios planetas gyrati conserventur & tranquille natent in vortice solis, tempora periodica partium vorticis solaris deberent esse aequalia. Revolutiones solis & planetarum circum axes suos, quae cum motibus vorticum congruere deberent, ab omnibus hisce proportionibus discrepant. Motus cometarum sunt summe regulares, & easdem leges cum planetarum motibus observant, & per vortices explicari nequeunt. Feruntur cometae motibus valde eccentricis in omnes coelorum partes, quod fieri non potest, nisi vortices tollantur.

Projectilia, in aere nostro, solam aeris resistentiam sentiunt. Sublato aere, ut sit in vacuo Boyliano, resistentia cessat, siquidem pluma tenuis & aurum solidum aequali cum velocitate in hoc vacuo cadunt. Et par est ratio spatiorum coelestium, quae sunt supra atmosphaeram terrae. Corpora omnia in illis spatiis liberrime moveri debent; & propterea planetae & cometae in orbibus specie & positione datis secundum leges supra expositas perpetuo revolvi. Perseverabunt quidem in orbibus suis per leges gravitatis, sed regularem orbium situm primitus acquirere per leges hasce minime potuerunt.

Planetae sex principales revolvuntur circum solem in circulis soli concenctricis, eadem motus directione, in eodem plano quamproxime. Lunae decem revolvuntur circum terram, jovem & satumum in circulis concentricis, eadem motus directione, in planis orbium planetarum quamproxime. Et hi omnes motus regulares originem non habent ex causis mechanicis; siquidem cometae in orbibus valde eccentricis, & in omnes coelorum partes libere feruntur. Quo motus genere cometae per orbes planetarum celerrime & facillime transeunt, & in apheliis suis ubi tardiores sunt & diutius morantur; quam longissime distant ab invicem, ut se mutuo quam minime tractant. Elegantissima haecce solis, planetarum & cometarum compages non nisi consilio & dominio entis intelligentis & potentis oriri potuit. Et si stellae fixae sint centra similium systematum, haec omnia simili consilio constructa suberunt *Unius* dominio: praesertim cum lux fixarum sit eiusdem naturae ac lux solis, & systemata omnia lucem in omnia invicem immittant. Et ne fixarum systemata per gravitatem suam in se mutuo cadant, hic eadem immensam ab invicem distantiam posuerit.

Hic omnia regit non ut anima mundi, sed ut universorum dominus. Et propter dominium suum, dominus deus *Παντοχράτωρ* dici solet. Nam deus est vox relativa & ad servos refertur: & deitas est dominatio dei, non in corpus proprium, uti sentiunt quibus deus est anima mundi, sed in servos. Deus summus est ens aeternum, infinitum, absolute perfectum: sed ens utcunque perfectum sine dominio non est dominus deus. Dicimus enim deus meus, deus vester, deus *Israelis*, deus deorum, & dominus dominorum: sed non dicimus aetemus meus, aetemus vester, aetemus *Israelis*, aeternus deorum; non dicimus infinitus meus, vel perfectus meus. Hae appellationes relationem non habent ad servos.

Vox deus passim significat dominum: sed omnis dominus non est deus. Dominatio entis spirituailis deum constituit, vera verum, summa summum, ficta fictum. Et ex dominatione vera sequitur deum verum esse vivum, intelligentem & potentem; ex reliquis perfectionibus summum esse, vel summe perfectum. Aeternus est & infinitus, omnipotens & omnisciens, id est, durat ab aeterno in aeternum,and adest ab infinito in infinitum : omnia regit; & omnia cognoscit, quae fiunt aut fieri possunt. Non est aeternitas & infinitas, sed aeternus & infinitus; non est duratio & sparium, sed durat & adest. Durat semper, & adest ubique, & existendo semper & ubique, durationem & spatium constituit. Cum unaquaeque spatii particula sit *semper*, & unumquodque durationis indivisibile momentum *ubique*, certe rerum omnium fabricator ac dominus non erit *nunquam*, *nusquam*. Omnis anima sentiens diversis temporibus, & in diversis sensuum, & motuum organis eadem est persona indivisibilis. Partes dantur successivae in duratione, coexistentes in spatio, neutrae in persona hominis seu principio eius cogitante; & multo minus in substantia cogitante dei. Omnis homo, quatenus res sentiens, est unus & idem homo durante vita sua in omnibus & singulis sensuum organis. Deus est unus & idem deus semper & ubique. Omnipraesens est non per *virtutem* solam, sed etiam per *substantiam*: nam virtus sine substantia subsistere non potest.

In ipso continentur & moventur universa, sed sine mutua passione. Deus nihil patitur ex corporum motibus: illa nullam sentiunt resistentiam ex omnipraesentia dei. Deum summum necessario existere in confesso est: Et eadem necestitate semper est & ubique. Unde etiam totus en sui similis, totus oculus, totus autis, totus cerebrum, totus bractiium, totus vis sentiendi, intelligendi, & agendi, sed more minime humano, more minime corporeo, more nobis prorsus incognito. Ut caecus non habet ideam colorum, sic nos ideam non habemus modorum, quibus deus sapientissimus sentit & intelligit omnia. Corpore omni & figura corporea prorsus destituitur, ideoque videri non potest, nec audiri, nec tangi, nec sub specie rei alicuius corporei coli debet. Ideas habemus attributorum eius, sed quid sit rei alicuius substantia minime cognoscimus. Videmus tantum corporum figuras & colores, audimus tantum sonos, tangimus tantum superficies externas, olfacimus odores solos, & gustamus sapores: intimas substantias nullo sensu, nulla actione reflexa cognoscimus; & multo minus ideam habemus substantiae dei. Hunc cognoscimus solummodo per proprietates eius & attributa, & per sapientissimas & optimas rerum structuras & causas finales, & admiramur ob perfectiones.; venerantur autem & colimus ob dominium. Colimus enim ut servi, & deus sine dominio, providensia, & causis finalibus nihil aliud est quam fatum & natura. A caeca necessitate metaphysita, quae utique eadem est semper & ubique, nulla oritur rerum variatio. Tota rerum conditarum pro locis ac temporibus diversitas, ab ideis & voluntate entis necessario existentis solummodo oriri potuit. Dicitur autem deus per allegoriam videre, audire, loqui, ridere, amare, odio habere, cupere, dare, accipere, gaudere, irasci, pugnare, fabricare, condere, construere. Nam sermo omnis de deo a rebus humanis per similitudinem aliquam desumitur, non perfectam quidem, sed aliqualem tamen. Et haec de deo, de quo utique ex phaenomenis differere, ad philosophiam naturalem pertinet. Hactenus phaenomena coelorum & maris nostri per vim gravita exposui, sed causam gravitatis nondum assignavi Oritur utique haec vis a causa aliqua, quae penetrat ad usque centra solis & planetarum sine virtutis diminutione; quaeque agit non pro quantitate *superficierum* particularum, in quas agit (ut solent causae mechanicae) sed pro quantitate materiae *solidae* ; & cuius actio in immensas distantias undique extenditur, decrescendo semper in duplicata ratione distantiarum. Gravitas in solem componitur ex gravitatibus in singulas solis particulas, & recedendo a sole decrescit accurate in duplicata ratione distantiarum ad usque orbem saturni, ut ex quiete apheliorum planetarum manifestum est, & ad usque ultima cometarum aphelia, si modo aphelia illa quiescant. Rationem vero harum gratitatis proprietorum ex phaenomenis nondum potui deducere, & hypotheses non fingo. Quicquid enim ex phaenomenis non deducitur, *hypothesis* vocanda est ; & hypotheses seu metaphysitae, seu physitae, seu qualitatum occultarum, seu mechanicae, in *philosophia experimentali* locum non habent. In hac philosophia propositiones deducuntur ex phaenomenis, & redduntur generales per inductionem. Sic impenetrabilitas, mobilitas, & impetus corporum & leges motuum & gravitatis innotuerunt. Et satis est quod gravitas revera existat, & agat secundum leges a nobis expositas, & ad corporum coelestium & maris nostri motus omnes sufficiat.

Adjicere jam liceret nonnulla de spiritu quodam ssubtilissimo corpora crassa pervadente, & in iisdem latente; cuius vi & actionibus particulae corporum ad minimas distantias se mutuo attractunt, & contiguae factae cohaerent; & corpora electrica agunt ad distantias majores, tam repellendo quam attractendo corpuscula vicina; & lux emittitur, reflectitur, refringitur, inflectitur, & corpora calefacit; & sensatio omnis excitatur, & membra animalium ad voluntatem moventur, vibrationibus scilicet huius spiritus per solida nervorum capillamenta ab externis sensuum organis ad cerebrum & a cerebro in musculos propagatis. Sed haec paucis exponi non possunt; neque adest sufficiens copia experimentorum, quibus leges actionum huius spiritus accurate determinari & monstrari debent.

FINIS.