Translator's Introduction.

For an elementary understanding of lunar theory, if such a thing is possible, a good place to start is the Wikipedia article of the same name, and related topics, followed perhaps for a brief account of the history of lunar theory, by reading if available J. F. Scott's note at the start of Vol. IV of *Newton's Letters*, and progressing to a text such as Godfray : *An Elementary Treatise on the Lunar Theory* (1853), the Appendix of which contains a nice non-technical summary of early developments, and then to the more compendious volume of Brown (1895) of the same name, etc. The *Lectures on the Lunar Theory* by J.C. Adams are also of interest, a little of which is quoted here below. All of these except the *Letters* can be downloaded from the Open Library website. Most books on classical mechanics including Whittaker, Synge & Griffith, do not mention the moon at all (!), an indication of the difficulty of the task, although Goldstein's work is an exception, and one can get some idea of the immense labour that individuals such as Hill and Delauney put into this study, since Newton's time, before the dawn of the computer age, whereby we may gain numerical prediction, but not necessarily physical understanding. A modern and highly relevant mathematical commentary is found in Chadrasekhar's delightful book, *Newton's Principia for the common reader*, Ch.13. In addition, the lecture notes of Prof. Richard Fitzpatrick on Newton's Dynamics, Ch. 14 are of special interest, where the problem is given a modern setting, and some of the inequalities resolved to some extent. Newton used the moon as a template for his theory of gravitation, in which endeavour he was highly successful, but he could not complete the task : indeed, John Couch Adams in the introduction to his lunar lectures, tells us that:

' Newton's *Principia* did not profess to be and was not intended to be a complete exposition of the Lunar Theory. It was fragmentary; its object was to shew that the more prominent irregularities admitted of explanation on his newly discovered theory of universal gravitation. He explained the Variation completely, and traced its effect in Radius Vector as well as in Longitude; and he also saw clearly that the change in eccentricity and motion of the apse that constitute the Evection could be explained on his principles, but he did not give the investigation in the *Principia*, even to the extent to which he had actually carried it. The approximations are more difficult in this case than that of the Variations, and require to be carried further in order to furnish results of the same accuracy as had already been obtained by Horrox from observation. He was more successful in dealing with the motion of the node and the law of the inclination. He shewed that when the Sun and Node were in conjunction, then for nearly a month the moon moved in a plane very approximately, and that the inclination of the orbit then reached a maximum, namely, 5^0 17' about; but as the Sun moved away from the Node the latter also began to move, attaining its greatest value when the separation was a quadrant, and that at this instant, the inclination was 5^0 very nearly. He also assigned the law of intermediate positions. The fact that the there was no motion when the Sun was at the Node, that is, in the plane of the moon's orbit, confirmed his theory that these inequalities were due to the Sun's action. When we spoke of Newton's results as being fragmentary and incomplete, let it not be understood that he gave only a very rude approximation to the truth. His results are far more accurate than those arrived at in elementary works of present day on the subject (*i.e*. towards the end of the 19th Century).'

It seems appropriate to insert here some definitions of astronomical terms and to quote relevant formulas for the various inequalities described by Newton. For this we follow Chandrasakher mainly: the diagram and the lettering used is that of Newton.

Some Descriptive Terms

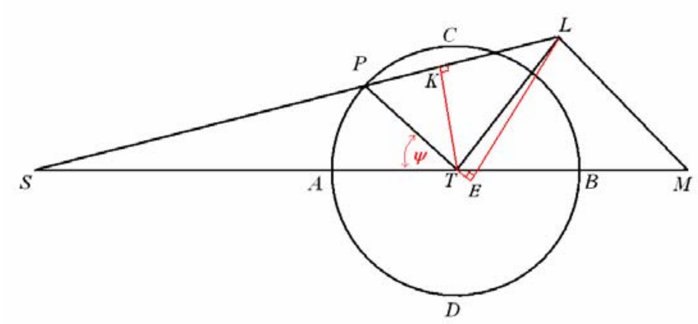
Celestial sphere : an imaginary spherical shell of very large radius concentric with the earth and rotating with the earth, onto which all heavenly bodies are projected.
Plane of the ecliptic: the plane of the earth's orbit around the sun, which changes slowly in time. Eclipses of the sun or moon can occur only in this plane, and hence the name.
Syzygy : when the sun *S*, the moon *P* (or a planet) & earth *T* lie on, or almost on, the same straight line, whether in conjunction or opposition.
Quadrature : when the moon (or a superior planet) lies in the first or last quarter position so that *PT* is perpendicular to *ST*.
Apogee: point in the moon's orbit furthest from the earth.
Perigee: point in the moon's orbit nearest to the earth.
Nodes : the two points were the moon's orbit cuts the plane of the earth's orbit or ecliptic, one rising and the other falling.
Vernal equinox : a reference point on the earth's orbit, when the sun rises in the first point of *Aries*, at which the tilt of the earth is at 90 degrees to the plane of the ecliptic, and the sun is directly overhead at the equator.
Synodic month : the time for a complete orbit of the moon around the sun measured by a line drawn from the earth to the sun. This is taken from a long time average to be 29.530589 days.

Main Phenomena

Evection (Ptolemy) : changes in the apparent size of the moon over monthly periods of time, measured by its subtended angle.
Elliptic Inequality, or, *Equation of the Centre* (Tycho.): changes in the rate of motion of the moon's centre or longitude as it rotates around the earth.
Variation (Tycho.): additional speeding up of the moon as it approaches full moon or new moon, and slowing down as it approaches quadrature.
Annual Equation : Gradual expansion of the moon's orbit as the earth gets closer to the sun at perihelion in January, and contraction as it goes through aphelion in July, due to the sun's perturbation.

Two effects that arise from the interaction with the sun are the precessions of the perigee and of the nodes. The perigee precesses in the same sense as the moon rotates about the earth, completing a complete cycle in 8.85 years, while the right ascending node precesses in a retrograde manner, and completes a cycle in 18.6 years.

The following notes relate mainly to Prop. XXV and Prop. XXVI. Initially we assume the moon's orbit is almost circular around the earth as centre, and lies in the plane of the ecliptic.



1. ***The perturbing function*** $\overrightarrow{TL}$:

The standard unit of acceleration, being that experienced by the earth in the Sun's gravity, is given by $\frac{GM_S}{ST^2}$, and $\frac{GM_S}{ST^2} = N^2 ST$, where N is the angular frequency of the earth about the sun; n is similarly defined as the mean angular frequency of the moon around the earth : and $\frac{GM_T}{PT^2} = n^2 PT = k$.

In the above diagram, we may note that $\overrightarrow{SL}$ denotes the force of the sun on the moon at P, while $\overrightarrow{ST}$ denotes the force of the sun on the earth at T; from which we note that the force $\overrightarrow{TL}$ must be added to $\overrightarrow{ST}$ to give $\overrightarrow{SL}$ and hence in the position shown, $\overrightarrow{TL}$ is the extra perturbing force due to the sun acting on the moon, all else being equal; on the other side of the quadrature line CD, the force acting on the moon is less by $\overrightarrow{TL}$ than the force acting on the earth; these forces thus distort the circular orbit of the moon. In the above diagram, $ST >> PT$, and it will be shown later that in this case $KL \simeq 2PK$, where K is the average distance of the moon from the sun in its orbit round the earth, and the figure $PLMT$ is assumed to be a parallelogram. The extra force $\overrightarrow{TL}$ can then be resolved into components TE and LE parallel and perpendicular to TP : the component of the perturbing force LE, or

$$F_\perp = \tfrac{3PK \times TK}{TP} = 3PK \sin\psi = 3PT \tfrac{PK}{TP} \cos\psi = 3PT \sin\psi \cos\psi = \tfrac{3}{2} PT \sin 2\psi$$

acting perpendicular to the radius TP; while the original component $PT + TE$ acts along TP; or the parallel perturbing force

$$TE = F_{||} = 3PK \tfrac{PK}{PT} - PT = PT\left(3\left(\tfrac{PK}{PT}\right)^2 - 1\right) = PT\left(3\cos^2\psi - 1\right) = PT\left(3\cos^2\psi - 1\right) = \tfrac{1}{2} PT\left(1 + 3\cos 2\psi\right).$$

Note finally that these lengths are to be multiplied by N^2 to give absolute forces as above, and the force $F_\perp = -\frac{3}{2} N^2.PT \sin 2\psi$, while $F_{||} = \frac{1}{2} N^2.PT\left(1 + 3\cos 2\psi\right)$.

The perturbing forces (or accelerations towards the sun) acting at the syzygies, when $\psi = 0$ or $\psi = \pi$ are both parallel to the axis AB, and given by $F_{||}(syzy.) = 2N^2.r_S$; in the position A, the moon P is pulled by an acceleration greater by this amount then the earth, while in the position *B*, in the above diagram, the force at *B* has the same value, but in this case, the earth is now accelerated more than the moon, by the same amount.

The perturbing forces acting at the quadratures *C* and *D* are given by $\psi = \frac{1}{2}\pi$ or $\psi = \frac{3}{2}\pi$, when the forces along *CD* are now both $F_{\|}(quad.) = -N^2.r_Q$.

2. ***The max. and min .centripetal acceleration***:

The initial circular orbit satisfies $\frac{GM_T}{PT^2} = n^2 PT$; this centripetal force becomes at the syzygies,

$$F_S = \frac{GM_T}{r_S{}^2} - F_{\|}(syzy.) = \left(n^2 - 2N^2\right)r_S = n^2\left(1 - 2\frac{N^2}{n^2}\right)r_S = \left(n^2 - 2N^2\right)r_S,$$

where *n* is the corresponding angular frequency of the moon, assumed to rotate in a circle about the earth as centre. At quadrature, the centripetal force becomes

$$F_Q = \frac{GM_T}{r_Q{}^2} - F_{\|}(quad.) = \left(n^2 + N^2\right)r_Q = n^2\left(1 + \frac{N^2}{n^2}\right)r_Q = \left(n^2 + N^2\right)r_Q$$

Thus,

$F_S = \left(n^2 - 2N^2\right)r_S = \left(n^2 - 2N^2\right)AT(1 - x)$ and $F_Q = \left(n^2 + N^2\right)r_Q = \left(n^2 + N^2\right)AT(1 + x)$,

where *x* is the departure from the initial radius. These accelerations can also be written in the form :

$F_S = n^2 r_S - 2N^2 r_S = \frac{GM_T}{(1-x)^2 \overline{PT}^2}\left(1 - 2m^2\right) = \frac{k}{(1-x)^2}\left(1 - 2m^2\right)$ and similarly,

$F_Q = n^2 r_Q + N^2 r_Q = \frac{GM_T}{(1+x)^2 \overline{PT}^2}\left(1 + m^2\right) = \frac{k}{(1+x)^2}\left(1 + m^2\right)$; results established by Chandrasekhar in his notation. Thus, the orbit is a prolate ellipse, and reverting to a general point on the orbit, the value of the radius becomes $r = AT(1 - x\cos 2\psi)$.]

3. ***Variation of the areas swept out:*** Recall that $F_{\perp} = -\frac{3}{2}N^2.PT\sin 2\psi$, then the torque exerted on the moon is given by $\frac{dH}{dt} = F_{\perp}.PT = -\frac{3}{2}N^2.PT^2\sin 2\psi$, which we can equate to a quantity proportional to the rate of change of the area swept out by the moon, relating to Kepler's Second Law. For the moon in a circular orbit, $n = \frac{d\psi}{dt}$, where *n* is the average time of the rotation of the moon for a synodic month, that is, following the phases of the moon. Thus, $dH = -\frac{3}{2}N^2.PT^2\sin 2\psi.dt = -\frac{3}{2}\frac{N^2}{n}.PT^2\sin 2\psi.d\psi$; recall that ψ is the angle between the earth-moon and the earth-sun radii, and on these being referred to the earth-moon ascending nodal angles of these radii, v and v' (or *U*) respectively, we have $\psi = v - v' = v\left(1 - \frac{v'}{v}\right) = v(1 - m)$, where $\frac{v'}{v}$ can also be expressed by the ratio $m = \frac{N}{n}$, where *N* is the angular frequency of the earth around the sun, and *n* the synodic angular frequency of the moon around the earth. Thus,

$$dH = -\frac{3}{2}N^2.PT^2\sin 2\psi.dt = -\frac{3}{2}m^2n^2.PT^2\sin 2v(1 - m).\frac{dv}{n} = -3m^2 n.PT^2\sin 2v(1 - m).dv$$

this can be integrated to give : $H = \text{constant} + \frac{3}{4}\frac{m^2 n}{1-m}.PT^2\cos 2\psi$. To turn this into a manageable form, recall that for an almost circular orbit, we have

$H = \overline{nr}^2 = n\overline{PT}^2 = H_{av.}$, and hence $H = \overline{PT}^2 \frac{dv}{dt} = H_{av.}\left(1+\frac{3}{4}\frac{m^2}{1-m}cos\,2\psi\right)$. As m is very small, in fact, $m \simeq \frac{1000}{178718}$, then the variation in the area between syzygies and quadrature is given approximately by $\frac{H_{syz.}}{H_{quad.}} = \frac{1+\frac{3}{4}\frac{m^2}{1-m}}{1-\frac{3}{4}\frac{m^2}{1-m}} \simeq \frac{1+\frac{3}{4}m^2}{1-\frac{3}{4}m^2} = \frac{11131}{11023}$.

4. Finding the other quantities.

Since the radial velocity is very small, the velocity V of the moon in orbit can be approximated by $V = \frac{2\pi}{t}\overline{PT} = n\overline{PT}$. Clearly this is related to the approximate rate at which the area A is swept out $\frac{dA}{dt} = \frac{1}{2}\overline{PT}.\overline{PT}\frac{dv}{dt} = \pi\overline{PT}^2 n$, and we have $\frac{dA}{dt} = \frac{1}{2}H = \frac{1}{2}\overline{PT}^2\frac{dv}{dt} = \frac{1}{2}H_{av.}\left(1+\frac{3}{4}\frac{m^2}{1-m}cos\,2\psi\right)$, while the velocity V is found from $V^2 = \frac{H^2_{av.}}{\overline{PT}^2}\left(1+\frac{3}{4}\frac{m^2}{1-m}cos\,2\psi\right)^2$.

Thus at the syzygies and at quadrature, we have $V_S{}^2 \simeq \frac{H^2_{av.}}{(1-x)^2\overline{PT}^2}\left(1+\frac{3}{2}\frac{m^2}{1-m}\right)$ and $V_Q{}^2 \simeq \frac{H^2_{av.}}{(1+x)^2\overline{PT}^2}\left(1-\frac{3}{2}\frac{m^2}{1-m}\right)$. Other quantities of interest can be found in the relevant chapters of Chandrasekhar.

As was his custom, Newton gives word summaries only in this initial proposition, for the quite extensive mathematical investigations that he had carried out in relation to the various inequalities of lunar motion, which he sketches out briefly in subsequent propositions, and which are treated here hopefully in more detail, with the help of Chadrasekhar's and the work of others.

PROPOSITION XXII. THEOREM XVIII.

All lunar motions, and all the inequalities of motions, follow from the principles put in place.

The major planets, while they are carried around the sun, meanwhile are able to have other smaller planets conveyed revolving around them, and these smaller ones must appear by Prop. LXV. Book I to revolve in ellipses having foci at the centres of the major planets. But the motions of these are disturbed in various ways by the action of the sun, and from these they will bring about the inequalities which are observed in our moon. Certainly this is moving faster at the syzygies than at the quadratures (by Corol. 2, 3, 4, & 5, *id.*), and with a radius drawn to the earth will describe a greater area in the time, and have an orbit less curved, and thus may approach closer to the earth, unless perhaps it may be prevented by an eccentric motion.

[Note that Newton's use of the term eccentricity referring to ellipses is not that of more modern times; he meant presumably merely the curvature of the approximately elliptic orbit at the point in question. The word of course has a long history in astronomy, dating

from Hipparcus, where it referred to eccentric circular orbits and the use of epicycles in describing inequalities in the motion of the moon and planets.]
For the eccentricity is a maximum (by Corol. 9. *id.*) where the apogee of the moon is situated at the syzygies, and the minimum where likewise it stands at the quadrature positions ; and thence the moon is faster at the perigee and closer to us , but slower at the apogee, and [in this case] further from us at the syzygies than when at the quadratures. In addition the apogee progresses forwards, and the nodes go backwards, but in unequal motions. And indeed the apogee is progressing faster at its syzygies, regressing slower at its quadratures (per corol. 7. & 8, *id.*), and as a consequence it is carried forwards annually by the excess of the progression over the regression. But the nodes (by Corol. 2, *id.*) are at rest in their quadratures and regressing the most quickly at the syzygies.
[We have inserted the correction made by Chandrasekhar on p. 422; by transposing the words quadratures and syzygies, thus correcting a mistake made in all three editions of the *Principia* & in all the translations.]

And also the latitude of the moon is greater at its quadratures (by corol.10, *id.*) then in its syzygies: and the mean motion is slower at the perihelion of the earth (by corol. 6. *id.*) than at its aphelion. And these are the most significant inequalities observed by astronomers. Also there are certain other inequalities not observed in earlier times by astronomers, by which the lunar motions thus may be disturbed, so that they are unable to be reduced by a law to some certain rule so far. For the velocities or the hourly motions of the moon at the apogee and nodes, and the equations of the same, so that both the difference between the maximum eccentricity at the syzygies and the minimum at the quadratures, and which inequality is called the variation, may be increased or diminished annually (by Corol. 14, *id.*) in the cubic ratio of the apparent diameter of the sun. And therefore the variation is increased or diminished in the square ratio of the time between the quadratures approximately (by Corol. 1 & 2, Lem. X. & Corol. 16, *idem*) but this inequality is usually referred to in astronomical calculations as part of the lunar prosthaphaeresis [*i.e*. the central equation of lunar theory], and combined with that.

PROPOSITION XXIII. PROBLEM V.

To derive the inequalities of the satellites of Jupiter and Saturn from lunar motions.

From the motions of our moon, the analogous motions of the moons or satellites of Jupiter thus may be derived. The mean motion of the nodes of the outermost of Jupiter's satellites [Callisto], is to the mean motion of the nodes of our moon, in a ratio composed from the square ratio of the periodic time of the earth around the sun to the periodic time of Jupiter around the sun, and in the simple ratio of the periodic time of the satellite around Jupiter to the periodic time of the moon around the earth (by Corol. 16. Prop. LXVI. Section XI. Book I.) and thus in 100 years the nodes are advanced by 8 ^{gr}. 24'.
[Following Chandrasakher, we have $\frac{\overline{\Delta\Omega_J}}{\overline{\Delta\Omega_E}} = \frac{T_E^2}{T_J^2} \times \frac{T_C}{T_E} = \frac{1}{11.86^2} \times \frac{16.69}{26.32}$; from the known average value for the moon, $\overline{\Delta\Omega_E} = 19.286^0$ per year, we have $\overline{\Delta\Omega_C} = 8^0.\ 22'$ for Callisto for 100 years, in close agreement with what is observed.]

The mean motions of the nodes of the inner satellites are to the motion of this outer satellite, as the periodic times of these inner satellites to the periodic time of this outer satellite (by the same corollary) and thence are given. Moreover the motion of the apogee of each satellite consequently is to the antecedent motion of its nodes [*i.e.* carried backwards], as the motion of the apogees of our moon to the antecedent motion of its nodes (by the same Corol.) and thence is given. Yet the motion of the perigees thus found must be diminished in the ratio 5 to 9 or approximately as 1 to 2, by a reason that cannot be explained here. The equations of the greatest nodes and of the apogee of each are almost to the greatest equations and of the apogee of the moon respectively, as the motion of the nodes and of the apogees of the satellites in the time of one revolution of the first equations, to the motion of the nodes and of the lunar apogee in the time of one revolution of the latter equations. The variation of the satellite seen from Jupiter, is to the variation of the moon, as in turn are the whole motion of the nodes in the times in which the satellites and the moon are rotating around the sun, by the same corollary ; and thus in the outer satellite does not exceed 5". 12"'.

PROPOSITION XXIV. THEOREM XIX.

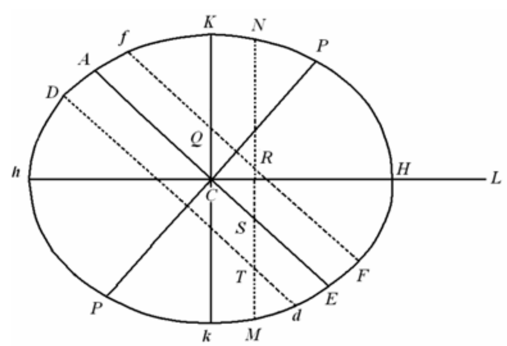
The flow and ebb of the sea arise from the actions of the sun and the moon.

[We may note that for this proposition, Chandrasakher has produced an annotated version on pp. 403–411 of his work on the *Principia*, setting out where he agrees and where he disagrees with what Newton has stated, from his modern knowledge concerning tides; the 3 hour rule introduced by Newton for the rising of the tide after the sun or moon has passed its meridian is thus discarded, as are other rather vague assertions, while the main body of the argument is retained.]

The sea must be raised up twice in each day both by the sun as well as by the moon, and it is apparent by Corol. 19. & 20. Prop. LXVI. Book I, that the maximum height of the water in deep and open seas, follows the influence of the luminous bodies at the meridian of a place, in an interval of less than six hours, as occurs in the seas of the *Atlantic* and of *Ethiopia* on being drawn eastwards between *France* and the *Cape of Good Hope*, and as in the seas of the *Pacific* along the coasts of *Chile* and *Peru* : on all the shores of which the tide comes in for around two, three or four hours, except where the motion propagated from the ocean depths may be retarded by shallow places as much as five, six, or seven hours or more. The number of hours I count from the influence of each luminous body at the meridian of the place, both below the horizon as well as above, and by the hours said of the moon I understand the twenty four parts of the time in which the moon by its apparent daily motion is returned to the place of the meridian. The force of the sun or of the moon required for the maximum elevation in the sea by the luminous body itself is at the meridian of the location. But the force from that remains acting on the sea for some time, and the influence may be increased by the new force, then the sea may rise to a new height, which happens in a space of one of two hours but more often in a space of around three hours on a shore, or even more if the sea shall be shallow.

The two motions, which the two luminous bodies excite, cannot be discerned separately, as they bring about a certain mixed motion. In the conjunction or opposition of the luminous bodies the effect of these will be added together, and the maximum flow and ebb will be put in place. At the quadratures, the sun may raise the water where the moon depresses it, or it may depress the water where the moon may raise it ; and from the difference of the effects the smallest of all the tides may arise. And because, by experience, it may be seen that the moon has a greater effect than the sun, the maximum height of the water occurs around the third lunar hour. Outside the conjunctions and the quadratures, the maximum tide which arises only from the force of the moon must always be in the third hour of the moon, and by the sun alone in the third hour of the sun, with the sum of the forces it is incident at some intermediate time that is close to the third hour of the moon ; and thus in the transition of the moon from the syzygies to the quadratures, where the third hour of the sun precedes the third hour of the moon, the maximum height of the water will also precede the third hour of the moon, and that in a maximum interval a little past the eight part of the lunar time ; and with equal intervals the maximum tide will follow the third lunar hour in the transition of the moon from quadrature to conjunction [or syzygies]. These are thus in the open sea. For at the estuaries of rivers the greatest flow with all else being equal reach a peak later.

But the effect of the luminous bodies depends on their distances from the earth. Indeed at the smallest distances the greater are the effects of these, at greater distances the lesser, and that in the cubic ratio of the apparent diameters. Therefore the sun in winter time, present at the perigee, produces a greater effect, and it comes about that the tide at the syzygies shall be greater, and at the quadratures less (with all else being equal) than in the summer time ; and the moon in perigee in individual months will disturb the tide more than before or after the fifteenth day, when it is moving into apogee [*i.e.* the half-cycle of the moon's orbit nearest the perigee]. From which two causes it happens that at successive syzygies the tides may not follow each other entirely as two high tides.



Also the effect of each luminous body depends on the declination of that, or the distance from the equator. For if a luminous body may be put in place at a pole, that may attract the individual parts of the water constantly, without an increase or decrease in the action, and thus it will generate no reciprocation of the motion. Therefore the luminaries by receding from the equator towards the pole, gradually lose their effect, and therefore they will raise smaller tides in the solstical syzygies than in the equinoxal syzygies. But they will raise greater tides in the solstical quadratures than in the equinoxal quadratures ; because now there the effect of the moon situated at the equator certainly exceeds the effect of the sun. Therefore the maximum tides arise in the syzygies and the minima at the quadratures of the luminaries, around the equinoxal times of each. And the maximum tide in the syzygies is always accompanied by the minimum in the quadrature, as is known from experience. But during the minimum distance of the sun from the earth, as in winter rather than in summer, it arises that the maximum and minimum tides often precede rather than follow the vernal equinox, and often follow rather than precede the autumnal equinox.

Also the effects of the luminous bodies depends on the latitude of the places. The figure *ApEP* designates the earth completely overwhelmed with deep water ; *C* is centre; *P, p* the poles; *AE* the equator ; *F* some place above the equator; *Ff* parallel to the place ; *Dd* parallel to that corresponding to the other side of the equator; *L* the place that the moon will occupy three hours before ; *H* the place on the earth directly under that ; *h* the place opposite to this; *K*, *k* places thus at 90 degrees distance; *CH, Ch* the maximum heights of the sea measured from the centre of the earth ; and *CK,* C*k* the minimum heights : and if with the axes *Hh, Kk* an ellipse is described, then by revolving about the major axis of this ellipse *Hh* the spheroid *HPKhpk* may be described; this will designate the figure of the sea approximately, and *CF, Cf, CD, Cd* will be the heights of the sea at the places *F, f, D, d*. Truly indeed if in the aforemade ellipse by revolving some point *N* it may describe the circle *NM,* cutting the parallel *Ff, Dd* at some points *R, T,* and the equator *AE* in *S; CN* will be the height of the sea at all the places *R, S, T,* situated on this circle. Hence in the daily revolution of some place *F,* the influx will be a maximum at *F,* three hours after the influence of the moon at the meridian above the horizon ; later the maximum outflow will be at *Q* three hours after the setting of the moon; then the maximum influx will be at *f* three hours after the influence of the moon at the meridian below the horizon ; the final maximum outflow will be at *Q* three hours after the rising of the moon ; and the last influx at *f* will be less than the first influx at *F.* For the whole sea may be separated into two completely hemispherical seas, one in the hemisphere *KHk* inclined to the north, the other towards the south in the opposite hemisphere *Khk;* which can be called the northern and southern floods. These seas always come in turn at the meridians of individual places mutually opposed to each other, between lunar intervals of twelve hours. And since the northern regions partake more in the northern seas, and the southern regions in the southern seas, thence the tides arise alternately greater and lesser, in the individual places beyond the equator, in which the luminous bodies rise and set. But the greater tide, with the moon declined at the vertical of the place, will happen at around three hours after the influence of the moon at the meridian above the horizon, and with the declination of the moon changed it becomes the lesser tide. And the greatest difference of the maximum flux arises at the time of the solstices ; especially if the ascending node of the moon is passing through the principle point of Aries. Thus it is found from experience, that in winter time the morning tide surpasses the evening tide, and in summer the evening tide surpasses the morning tide, indeed at *Plymouth* by a height of as much as one foot, at *Bristol* truly by a height of fifteen inches : from the observations of *Colepress* and *Sturmy.*

But the motions described up to this point are changed a little by that returning force of the waters, by which the tides of the sea, even with the cessation of the luminous bodies, may persevere for some time. This persistence of the impressed motion diminishes the difference of the alternate tides ; and the tide just after the syzygies is returned the most, and that diminished the most just after the quadratures. From which it shall be that the alternate tides at *Plymouth* and *Bristol* shall not be more different in turn than one foot or fifteen inches ; and so that the maxima of all the tides in the same ports shall not be the first from the syzygies, but the third. Also all the motions are retarded by passing through channels, thus so that the maximum of all the tides, in certain straits and estuaries of rivers, shall be even as the fourth or fifth day after the syzygies.

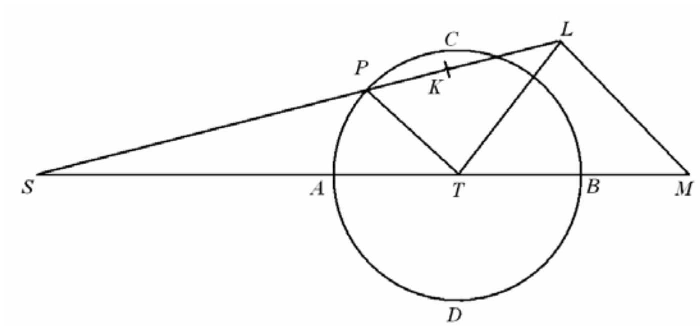
Again it can come about that the tide will be propagated from the ocean through different channels to the same port, and it may pass faster by one channel than by another : in which case the same tide, divided into two or more arriving successively, may consist of new motions of different kinds. We may consider two equal tides to arrive at the same port from different locations, the foremost of which precedes the other by a space of six hours, and may happen in the third hour after the influence of the moon at the meridian of the port. If the moon were turning at the equator, then by its own influence, equal flows would arrive every six hours, which being equal to the mutual reflux the same flows shall be equal, and thus in the space of that day the tides will effect that the still water remain at rest. If the moon then should decline from the equator, major and minor tides in turn are produced in the ocean, as has been said ; and then two major and two minor inflows are propagated in turn to this port. But the two major influxes compound the water to the highest level in the middle between each, the major and the minor influx act so that the water rises to the average height in the middle of these, and between the two minor influxes the water ascends to the minimum height in the middle of these. Thus in the space of twenty four hours, the water is accustomed to arrive not twice, but only once at the maximum height and once at the minimum ; and the maximum height, if the moon declines towards the pole above the horizon of the place, falls either in six or thirty hours after the influence of the moon at the meridian, and by changing the lunar declination it will become a deflux [*i.e.* an ebb-tide]. An example of all of which has been revealed from the observations of *Halley's* sailors at the port of *Batsham* in the kingdom of *Tunquin,* at the northern latitude of 20^0. 50'.
[This channel is now the Qiongzhou Strait, a region where unusual tides are found, separating the island of Hainan from the P.R. of China, and connecting the South China Sea to the Gulf of Beibu.]
There during the day following the transit of the moon through the equator, the water remains unmoved, then with the moon declining to the north the tides begins to flow and ebb, not twice, as in other ports, but only once in individual days ; and the tide rises on the setting of the moon, the maximum deflux on the moon rising. Here the tides increase with the declination of the moon, as far as the seventh or eighth day, then through another seven days falling by the same amounts, by which before it had risen ; and finally stops changing with the lunar declination [again passing through the equator], and soon it changes into deflux. For indeed at once the deflux starts at the setting of the moon, and the influx at its rising, then again it changes with the declination of the moon [until the whole process repeats itself].

The approach to this port is apparent from two narrow channels, the one from the Chinese Sea between the continent and the island of *Leuconia* [Hainan Island], the other from the Indian Sea between the continent and the island of *Borneo.* But the tide in the space of twelve hours arrives from the Indian Sea, and in the space of six hours from the Chinese Sea arrives through that narrow channel, and thus arising in the third and ninth hours of the moon, motions of this kind add together ; lest there shall be a condition imposed by other seas, I leave that determination from the observations of neighbouring shores. [For an early paper, see Whewell. *Phil. Trans*. 1833, p. 224.]

Up to this point I have given an account of the motions of the moon and of the seas. Now it is fitting to add a little on the quantity of the motions.

PROPOSITION XXV. PROBLEM VI.
To find the forces of the sun perturbing the motion of the moon.

Let *S* designate the sun, *T* the earth, *P* the moon, and *CADB* the orbit of the moon. On *SP* there is taken *SK* equal to *ST* [*i.e.* the mean Sun-Earth distance]; and let *SL* to *SK* be in the square ratio *SK* to *SP,*



[i.e. the forces $\overrightarrow{SL}$ and $\overrightarrow{SK}$ vary inversely with the distance from the sun : $\frac{|\overrightarrow{SL}|}{|\overrightarrow{SK}|}=\frac{SK^2}{SP^2}$;]

and *LM* is acting parallel to *PT* itself *;*
[i.e. resolve $\overrightarrow{LS}$ into components so that $\overrightarrow{LS}=\overrightarrow{LM}+\overrightarrow{MS}$;]
and if the gravitational acceleration of the earth on the sun may be put in place by the distance *ST* or *SK, SL* will be the gravitational acceleration of the moon on the sun. [Note that due to the inverse square law, the line *SL* is hence longer than the line *ST*, while the forces of attraction on the moon and the earth due to the sun act along *SL* and *ST*, while the ratio of the accelerations of these is $\frac{|\overrightarrow{SP}|}{|\overrightarrow{ST}|}=\frac{ST^2}{SL^2}$.]

This is composed from the parts *SM* and *LM,*
[on re-arranging the above forces per unit mass : $\overrightarrow{SL}=\overrightarrow{LM}+\overrightarrow{SM}$]
of which *LM* and the part *TM* of *SM* perturb the motion of the moon ; as established in Book I, Prop. LXVI., and in its corollaries.
[Thus, the earth and the moon form a sub-system, falling with almost the same centripetal acceleration a_{TS} towards the sun, while extra accelerations exist between these two bodies, so that $\vec{a}_{LS}=\vec{a}_{LT}+\vec{a}_{TS}$, where $\vec{a}_{LT}=\vec{a}_{LM}+\vec{a}_{MT}$]
Certainly the earth and the moon are revolving about a common centre of gravity, also the motion of the earth will be perturbed around that centre by similar forces [by the third law of motion] ; moreover both the sums of the forces as well as of the motions as referred to the moon [instead of the earth], and the sums of the forces designated by the analogous lines *TM* and *ML* themselves. The force *ML* in its mean magnitude is the centripetal force, by which the moon may rotate at a distance *PT* in its orbit around the earth at rest,
[*i.e.* with the sun's perturbation acting independently in the quadrature position, or $\vec{a}_{MT}=0$; or (according to a note of *L.* & *J.* on p. 2 of vol. IV) : on account of the great distance of

the sun from the earth, the figure *PTML* is a parallelogram, and thus *ML* is approximately equal to *PT*, and hence the force *ML* will be to the force by which the sun acts at the point *T*, as *PT* to *SK* or *ST*, but any central forces are between themselves as the radii of the circles which are described by these, and inversely as the square of the periodic times of these, and hence this force by which the sun acts at the point *T* is to the force by which the moon is retained in its orbit (supposing that to rotate about the earth at rest) as *PT* directly, and as the square of the periodic time of the moon around the earth to the square of the periodic time of the earth around the sun; hence from the composition of the ratios, the force *ML* is to the force holding the moon in its orbit, as the square of the periodic time of the moon to the square of the periodic time of the earth around the sun; that is, in the square ratio of .....],

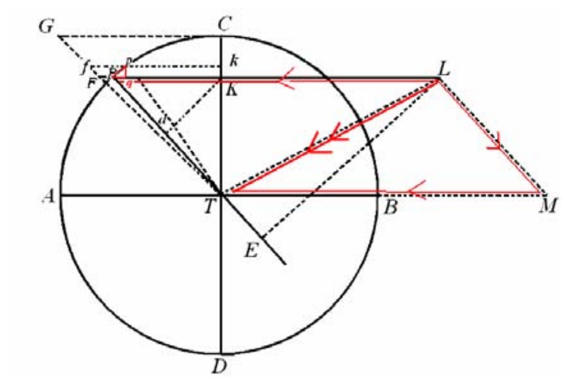
as the square ratio of the periodic times of the moon around the earth and of the earth around the sun (by Corol. 17. Prop. LXVI. Book I.) that is, in the square ratio of 27days, 7 hours, and 43 minutes to 365 days, 6 hours and 9 minutes, that is, as 1000 to 178725, or 1 to $178\frac{29}{40}$.

[*i.e.* recall that $\frac{|\overrightarrow{PT}|}{|\overrightarrow{ST}|} \propto \frac{PT}{ST} \times \left(\frac{T}{t}\right)^2$, where $|\overrightarrow{ST}| = ST$ has a scaling factor of 1 in the diagram, but other lengths such as *PT* have to be scaled as shown to become forces ; we can write this ratio of the centripetal forces in modern terms as $\frac{|\vec{F}_{PT}|}{|\vec{F}_{ST}|} = \frac{\left(\frac{2\pi}{t}\right)^2 \times PT}{\left(\frac{2\pi}{T}\right)^2 \times ST} = \frac{1000 \times PT}{178725 \times ST}$.]

But we have found in Proposition IV that, if the earth and the moon may be revolving about a common centre of gravity, the mean distance between these will be in turn as $60\frac{1}{2}$ mean earth radii approximately. And the force by which the moon can rotate around the earth at rest, to the distance *PT* of $60\frac{1}{2}$ earth radii, is to the force, by which it can revolve in the same time at a distance of 60 radii, as $60\frac{1}{2}$ to 60; and this force to the force of gravity with us on the earth as 1 to 60×60 approximately. And thus the average force *ML* compared to the force of gravity on the earth is as $1 \times 60\frac{1}{2}$ to $60 \times 60 \times 60 \times 178\frac{29}{40}$, or 1 to 638092,6. [*i.e*. the acceleration *ML* at the moon's orbit can be taken as $\frac{g}{638092{,}6}$.] Thence truly, and from the known proportion of the lines *TM, ML,* the force *TM* is also given : and these are the sun's forces disturbing the motion of the moon. *Q.E.I.*

PROPOSITION XXVI. PROBLEM VII.
To find the hourly increment to the area that the moon will describe in a circular orbit, by a radius drawn to the earth.

We have said the area that will be described by a radius drawn from the moon to the earth, to be proportional to the time, unless perhaps the motion of the moon may be disturbed by the action of the sun. Here we propose to investigate the inequality of the moments, or of the hourly increments. In order that the computation may be carried out more easily, we may consider the orbit of the moon to be circular, and we may neglect all the inequalities, with the sun excepted, by which this orbit is disturbed. On account of the truly huge distance of the sun, we may also consider the lines *SP, ST* to be parallel to each other. With this agreed on, the force *LM* will always be reduced to its average value



TP, and so that the force *TM* is reduced to its average value *3PK.*

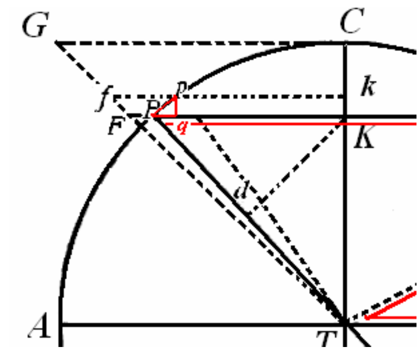
[For the distance *ST*, with the sun *S* imagined very far to the left in the diagram, is on average given by the distances $SP + PK$, and *SL* is given by $SP + PL$; while the magnitudes of the forces $\overrightarrow{SP}$ and $\overrightarrow{SL}$ vary inversely as their associated distances squared as above : $\frac{|\overrightarrow{SL}|}{|\overrightarrow{SK}|} = \frac{SK^2}{SP^2}$ or $SL = \frac{SK^3}{SP^2} = \frac{ST^3}{SP^2}$. Hence if we consider the constant distance *ST* to be given by $ST = SP + PK$, then

$$ST^3 - SP^3 = (ST - SP)\left(ST^2 + ST.SP + SP^2\right) = PK\left(SP^2 + 2.SP.PK + PK^2 + ST.SP + SP^2\right)$$

$$\simeq 3SP^2.PK + 2.SP.PK^2 + PK^3 = LS.SP^2 - SP^3 = SP^2\left(LS - SP\right) = SP^2.PL$$; or, on dividing, $PL \simeq 3PK\left(1 + \frac{2.PK}{3.SP} + \frac{PK^2}{3SP^2}\right)$; then, since *PK* is much less than *SP*, we have $PL \simeq 3PK$, as was required to be shown.]

These forces (by Corol. 2 of the laws) make the force *TL*; and this force, if the perpendicular *LE* may be sent to the radius *TP,* is resolved into the forces *TE, EL,* of which *TE,* by always acting along the radius *TP,* neither accelerates nor retards the description of the area *TPC* made by that radius *TP* ; and *EL* by acting along the perpendicular, does accelerate or retard the area described, at the same times as it

accelerates or retards the moon. That acceleration of the moon, in the transition of that from the quadrature C to the conjunction A, made in individual moments of time, is as the accelerating force EL, that is, as $\frac{3PK\times TK}{TP}$. [*i.e.*, from the similar triangles PEL and PKT.] The time may be established from the mean motion of the moon, or (because it returns almost the same thing) by the angle CTP, or also by the arc CP.
[*L. & J.* note 110 *a* : Thus, the perturbation produced by the sun is extremely small, and thus the arc or angle may be taken in proportion to the time.] The normal CG may be erected equal to CT. And with the arc of the quadrant AC divided into innumerable equal parts Pp, &c. by which just as many equal parts of the time shall be put in place, and by drawing pk perpendicular to CT, TG may be crossed by KP and kP produced to F and f; and FK will be equal to TK, and $\frac{Kk}{PK}=\frac{Pp}{Tp}$, that is, in a given ratio



[*L. & J.* note 110 *b* : this proportion follows from a most noteworthy property of the circle, for if from the point p the line increment pq may be drawn normal to PK, that will be parallel and equal to the line Kk, and the differential triangle Ppq will be formed similar to the triangle PKT, for since the angles pPK and KPT together make a right angle, and equally the angles KPT and PTK, the angles pPK and PTK are equal, from which $\frac{Kk}{PK}=\frac{Pp}{Tp}$.],

and thus $FK\times Kk$ or the area $FKkf$, will be as [the applied tangential force] $\frac{3PK\times TK}{TP}$, that is, as EL; and by adding the increments, the whole area $GCKF$ will be as the sum of all the forces EL impressed on the moon in the total time CP, and thus also as the velocity generated by this sum, that is, as the acceleration of the described area CTP, or the increment of the moment.

The force by which the moon can revolve about the earth at rest at the distance TP, in its periodic time $CADB$ of 27 days, 7 hours, and 43 minutes, effects that a body, by falling in the time CT, describes the distance $\frac{1}{2}CT$, and likewise it acquires a velocity equal to the velocity, by which the moon is moving in its orbit, which is apparent by Corol. 9. Prop. IV, Book I. But since the perpendicular Kd sent to TP shall be the third part of EL, and the half of TP itself or of ML at the octant, the force EL at the octant , where it is a maximum, will exceed the force ML in the ratio 3 to 2, and thus it will be to that force, by which the moon will be able to revolve about the quiescent earth in its periodic time, as 100 to $\frac{2}{3}\times 17872\frac{1}{2}$ or to 11915, and in the time [corresponding to] CT it must be able to generate a velocity which will be the $\frac{100}{22915}$ part of the lunar velocity, but in the time CPA it will generate a greater velocity in the ratio CA to CT or TP. The maximum force EL in the octant may be established by the area $FK\times Kk$ equal to the rectangular area $\frac{1}{2}TP\times Pp$. And the velocity, that the maximum force can generate in some time CP, will be to that velocity that all the smaller forces EL can generate in the same time, as the rectangle $\frac{1}{2}TP\times CP$ to the area $KCGF$: but in the total time CPA, the velocities arising will be in turn as the rectangle $\frac{1}{2}TP\times CA$ and the triangle TCG, or as the arc of the quadrant CA and the radius TP. And thus (by Prop. IX, Book V, *Euclid's*

Elements.) the latter velocity, generated in the total time, will be the $\frac{100}{22915}$ part, of the moon's velocity. To this velocity of the moon, which is analogous to the average moment [*i.e*. increment] of the area, there may be added and taken away half the other velocity ; and if the average moment may be expressed by the number 11915, the sum 11915+50 or 11965 will show the maximum moment of the area at the syzygies *A*, and the difference 11915 – 50 or 11865 the minimum moment of the same at quadrature. Therefore the areas described in equal times at the syzygies and quadratures, are in turn as 11965 to 11865. To the minimum moment 11865 there may be added the moment, which shall be to the difference of the moments 100 as the trapezium *FKCG* to the triangle *TCG* (or which is the same thing, as the square of the sine *PK* to the square of the radius *TP,* that is, as *Pd* to *TP*)*,* and the sum will show the moment of the area, when the moon is at some intermediate place *P.* Thus all these may themselves be had, from the hypothesis that the sun and the earth are at rest, and the moon is revolving in the synodic time of 27 days, 7 hours, and 43 min. But since the synodic period of the moon shall truly be 29 days, 12 hours and 44 min., the increments of the moments must be increased in the ratio of the times, that is, in the ratio 1080853 to1000000. With this agreed upon, the total increment, that was the $\frac{100}{11915}$ part of the average moment, now will become the $\frac{100}{11023}$ part of this. And thus the moment of the area of the moon at the quadrature will be to the moment of this at the syzygies as 11023–50 to 11023+50, or as 10973 to 11073; and to the moment of this, when the moon is moving through some intermediate place *P*, as 10973 to 10973+*Pd,* clearly with *TP* arising equal to 100. [See Chandrasakher, p. 239 onwards for a modern interpretation of the perturbing function (*a*) and the variation of the 'constant of areas' (*c*)]

Therefore the area, that the moon will describe with the radius drawn to the earth in individual instants of time, is as an approximation as the sum of the numbers 219,46 and the versed sine of double the distance of the moon from the nearest quadrature, in the circle the radius of which is one. Thus these themselves are found when the variation at the octant is of average magnitude. But if the variation there shall be greater or less, that versed sine has to be increased or diminished in the same ratio.

PROPOSITION XXVII. PROBLEM VIII.

From the hourly motion, to find the distance of the moon from the earth.

The area, that will be described by a radius drawn from the moon to the earth in individual instants of time, is as the hourly motion of the moon and the square of the distance of the moon from the earth jointly ; and therefore the distance of the moon from the earth is in a ratio composed from the square root of the area directly, and inversely as the ratio of the square root of the hourly motion. *Q.E.D.*

Corol. 1. Hence the apparent diameter of the moon is given: which certainly shall be inversely as its distance from the earth. Astronomers can test whether this rule agrees correctly with the phenomena.

Corol. 2. Hence also it is possible for the orbit of the moon to be defined more accurately than before from these phenomena.

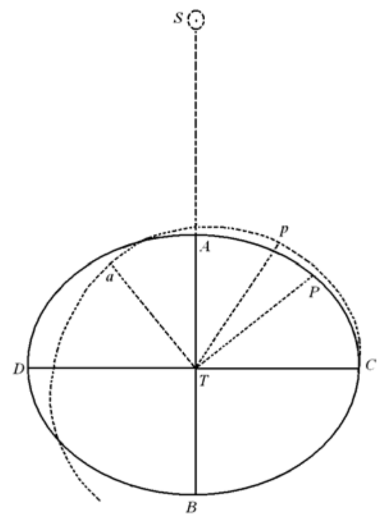
PROPOSITION XXVIII. PROBLEM IX.

To find the diameters of the orbit in which the moon can move without eccentricity.

[Here initially we give a modern view on solving this problem, following from the results expressed in Prop. XXVI : $H = H_{av.}\left(1+\frac{3}{4}\frac{m^2}{1-m}\cos 2\psi\right)$.

]

The curvature of the trajectory that will be described by a moving body, if it may be drawn along the perpendicular of that trajectory, is as the attraction directly and inversely as the square of the velocity. I consider the curvatures of lines to be between the final ratio of the sine or tangent of the angle of contact pertaining to equal radii, when these radii are reduced indefinitely. But the attraction of the moon towards the earth at the syzygies is the excess of its gravity *2PK* over the force of the sun (see fig. Prop.25.), by which the gravitating acceleration of the moon towards the sun exceeds the gravitating acceleration of the earth towards the sun, or may be increased by that amount. But in quadrature that attraction is the sum of the gravities of the moon towards the earth and of the force of the sun *KT,* by which the moon is drawn towards the earth. And these attractions, if $\frac{AT+CT}{2}$ is called N [the average radius], shall be as $\frac{178725}{AT^2}-\frac{2000}{CT\times N}$ and $\frac{178725}{CT^2}+\frac{1000}{AT\times N}$ approximately; or as



$$178725N\times CT^2-2000AT^2\times CT \text{ and } 178725N\times AT^2+1000CT^2\times AT.$$

For if the accelerative gravity of the moon towards the earth may be expressed by the number 178725, then the average force *ML,* which is *PT* or *TK* in quadrature, and is drawn towards the earth, will be 1000, then the average force *TM* in syzygies will be 3000; from which, if the average force *ML* may be removed there will remain 2000 by which the moon is drawn from the earth, formerly called 2*PK.* But the velocity of the moon in syzygies *A* and *B* is to its velocity in the quadratures *C* and *D,* as *CT* to *AT* and the moment of the area which the radius drawn from the moon to the earth will describe at the syzygies to the same moments conjointly, *i.e.* as 11073 *CT* to 10973 *AT.* This inverse ratio is taken squared and the first ratio taken once directly, and the curvatures of the orbit of the moon in syzygies to the same curvature in quadrature becomes as

$$120406729\times178725AT^2\times CT^2\times N-120406729\times2000AT^4\times CT$$

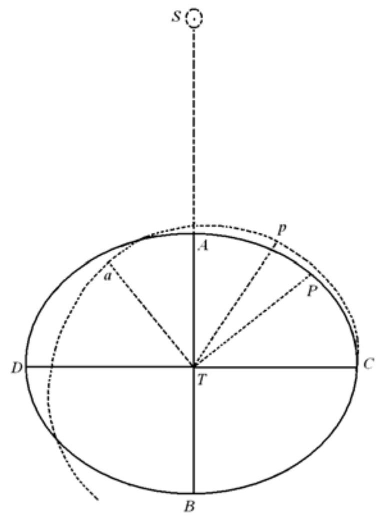
to

$$122611329\times1178725AT^2\times CT^2\times N+122611329\times1000CT^4\times AT,$$

i.e. as

$$2151969AT \times CT \times N - 24081AT^3 \text{ to } 2191371AT \times CT \times \mathrm{N} + 12261CT^3.$$

Because the figure of the moon's orbit is unknown, we may assume this in turn to be the ellipse *DBCA,* in the centre of which the earth *T* may be located, and the major axis of which is between the quadratures *DC*, and the minor axis *AB* may be placed between the syzygies . But since the plane of this ellipse is revolving in an angular motion about the earth, and the we may consider the curved trajectory of this that is completely free from any angular motion: the figure will be required to be considered, which the moon will describe in that ellipse by revolving in that plane, that is the figure *Cpa*, the individual points *p* of which may be found by taking some point *P* on the ellipse, that may represent the position of the moon, and by drawing *Tp* equal to *TP*, from that law that the angle *PTp* shall be equal to the apparent motion of the sun completed from the time of the quadrature *C* ; or (what returns almost the same) that the angle *CTp* shall be to the angle *CTP* as the synodic time of revolution of the moon to the periodic time of revolution or $29^d. 12^h. 43'$, to $27^d. 7^h. 43'$.



Therefore the angle *CTa* may be taken in the same ratio to the right angle *CTA,* and the longitude *Ta* shall be equal to the longitude *TA;* and *a* will be the inner apse and *C* the outer apse of this orbit *Cpa.* Because I find from the required ratio to be entered, that the difference between the curvature of the orbit *Cpa* at the vertex *a*, and the curvature of the circle described with centre *T* and with the radius *TA*, shall be to the difference between the curvature of the ellipse at the vertex *A* and the curvature of the same circle, in the square ratio of the angle *CTP* to the angle *CTp;* and because the curvature of the ellipse at *A* shall be to the curvature of that circle, in the square ratio *TA* to *TC;* and the curvature of that circle to the curvature of the circle with centre *T* described with radius *TC*, as *TC* to *TA* ; but the curvature of this to the ellipse curvature at *C,* in the square ratio *TA* to *TC*; and the difference between the ellipse curvature at the vertex *C* and the newest curvature of the circle , to the difference between the curvature of the figure *Tpa* at the vertex *C* and the same curvature of the circle, in the square ratio of the angle *CTp* to the angle *CTP.* Which ratios indeed are easily deduced from the sines of the angle of contact and the difference of the angles. But with these deduced between themselves, the curvature of the figure *Cpa* at *a* to the curvature of this at *C* can be produced, as $AT^3 + \frac{16824}{200000}CT^2 \times AT$ to $CT^3 + \frac{16824}{200000}AT^2 \times CT$. Where the number $\frac{16824}{200000}$ may designate the difference of the squares of the angle *CTP* and *CTp* applied to the square of the smaller angle *CTP,* or (which amounts to the same) the difference of the squares of the times $27^d. 7^h. 43'$, and $29^d. 12^h. 44'\ 1.9$, applied to the square of the time $27^d. 7^h. 43'$.

Therefore since *a* may designate the syzygies of the moon, and *C* the quadrature of this, the proportion now found must be the same as the proportion of the curvature of the orbit of the moon in syzygies to the same curvature at quadrature, as we have found above. Hence so that the proportion *CT* to *AT* may be found, I multiply the extremes and means between themselves. And the terms arising to the applied term $AT \times CT$, becomes

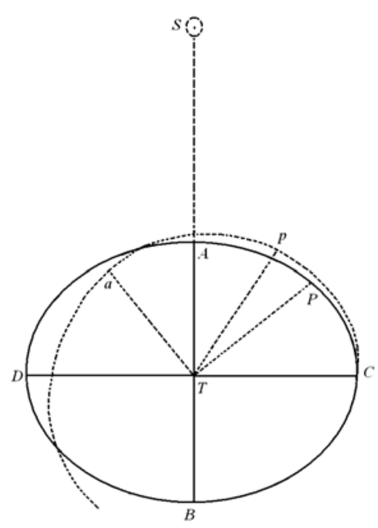
$$2062,79CT^4 - 215I969N \times CT^3 + 368676N \times AT \times CT^2$$

$$+36342AT^2 \times CT^3 - 362047N \times AT^2 \times CT + 2191371N \times AT^3 + 4051,4AT^4 = 0.$$

Here for the half sum of the terms *AT* and *CT* , *N* I write 1, and by putting *x* for the semi difference of the same, there shall be $CT = 1 + x,$ and $AT = 1 - x:$ with which written into the equation, and with the equation produced resolved, there will be obtained x equal to 0,00719, and thence the radius *CT* shall be 1,00719, and the radius *AT* 0,99281, which numbers are approximately as $70\frac{1}{24}$ and $69\frac{1}{24}$. Therefore the distance of the moon from the earth at syzygies is to the distance of this in quadrature as 69 to 70 (clearly with the consideration of the eccentricity disregarded), or with rounded numbers as 69 to 70.

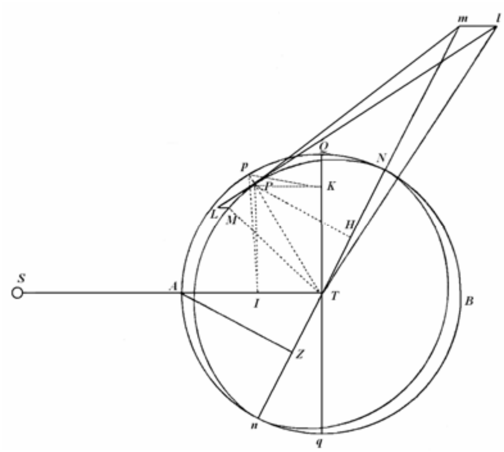
PROPOSITION XXIX. PROBLEM X.
To find the variation of the moon.

This inequality arises partially from the elliptic form of the lunar orbit, partially from the inequality of the moments of the area, that the moon describes by the radius drawn to the earth. If the moon *P* is moving in the ellipse *DBCA* around the earth at rest at the centre of the ellipse, and with the radius *TP* drawn to the earth describing an area *CTP* proportional to the time ; moreover the maximum radius *CT* of the ellipse shall be to the minimum radius *TA* as 70 to 69: the tangent of the angle *CTP* to the tangent of the mean motion at the quadrature *C* shall be computed, as the radius of the ellipse *TA* to the same radius *TC* or as 69 to 70. But the description of the area *CTP,* in the progression of the moon from quadrature to syzygies, to be accelerated in that ratio, so that the moment of this in the syzygies of the moon shall be to the moment of this in quadrature as 1073 to 10973, and so that the excess of the moment at some intermediate place *P* over the moment in quadrature shall be as the square of the sine of the angle *CTP.* As that comes about well enough, if the tangent of the angle *CTP* may be diminished in the square root ration of the number 10973 to the number 11073, that is, in the ratio of the number 68,6877 to the number 69, and with which agreed on, the tangent of the angle C*TP* now will be to the tangent of the mean motion as 68,6877 to 70, and the angle *CTP* in the octants, where the mean motion is 45^0, will be 44^0. 27'. 28", which taken from the angle of the mean motion 45^0 leaves the maximum variation 32' .32". These thus themselves are found if the moon, by going from the quadrature to the syzygy, may describe an angle *CTA* of almost 90^0. Truly on account



of the motion of the earth, by which the sun as a consequence by moving, has been apparently carried across, the moon, before it overtakes the sun, will describe the angle *CTA* greater than a right angle in the ratio of the synodic time of revolution of the moon to the periodic time of revolution, that is, in the ratio $29^d. 12^h. 44'$ to $27^d. 7^h. 43'$. And with this agreed on, all the angle about the centre *T* may be enlarged in the same ratio, and the maximum variation which otherwise shall be 32'. 32'', now increased in the same ratio shall be 35'. 10''.

This is its magnitude at the average distance of the sun from the earth, with the differences ignored which may arise from the curvature of the great orbit [of the earth] and from the greater attraction of the sun on the sickle-shaped moon at the new moon rather than at the gibbous and full moon [phases]. At the other distances of the sun from the earth, the maximum variation is in a ratio that is composed from the square of the ratio of the synodic time of the moon (at a given time of the year) directly, and in the inverse cubic ratio of the distance from the sun. And thus at the apogee of the sun, the maximum



variation is 33'. 14'', and at its perigee 37'.11'', but only if the eccentricity of the sun shall be to the transverse radius of the great orbit as $16\frac{15}{16}$ to 1000.

Up to the present we have investigated the variation in an orbit without eccentricity, in which the moon certainly in its octants is always at its average distance from the earth. If the moon because of its eccentricity, may be more or less distant from the earth than if located in this orbit, the variation can be a little more or a little less than produced here by this rule : but I leave the excess or deficiency requiring to be determined through the phenomena being observed by astronomers.

PROPOSITION XXX. PROBLEM XI.

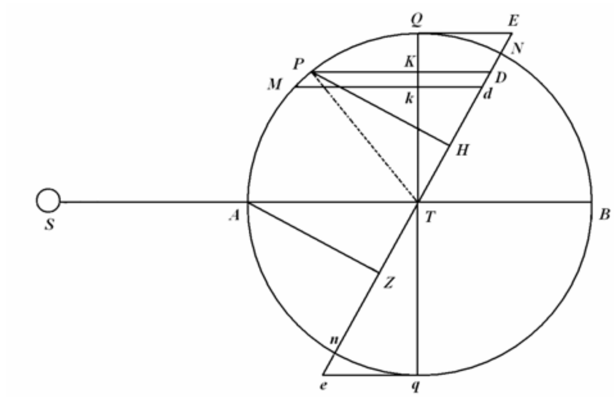
To find the hourly motion of the moon's nodes in a circular orbit.

[The Latin word *horarius* means 'related to the hours or seasons' or perhaps 'scheduled', as applied to a timetable.]

Let *S* represent the sun, *T* the earth, *P* the moon, *NPn* the orbit of the moon, *NPn* the projection [*lit*. footprint] of the orbit in the plane of the ecliptic; *N, n* the nodes, *nTNm* the nodal line produced indefinitely ; *PI, PK* the perpendiculars sent to the lines *ST, Qq; Pp* the perpendicular sent to the plane of the ecliptic; *A, B* the syzygies of the moon in the plane of the ecliptic ; *AZ* the perpendicular sent to the nodal line *Nn; Q, q* the quadratures of the moon in the plane of the ecliptic, & *pK* meeting the perpendicular to the line of *Qq*. The force of the sun perturbing the motion of the moon is twofold (by Prop. XXV.), the one proportional to the line *LM* in the diagram of that proposition, the other proportional to the line *MT*. And the moon is attracted to the earth by the first force, by the latter towards the sun along a right line drawn parallel to *ST* from the earth to the sun. The first force *LM* acts along the plane of the lunar orbit, and therefore changes nothing in the situation of the plane. This force therefore can be ignored. The latter force *MT* by which the plane of the orbit of the moon may be perturbed, may be expressed by the force 3*PK* or 3*IT.* And this force (by Prop. XXV.) is to the force by which the moon may be revolving uniformly almost in a circle in its periodic time about the earth at rest, as *31T* to the radius of the circle multiplied by the number178,725, or as *IT* to the radius multiplied by 59,575. For what remains in this calculation, and from that which follows generally, I consider all the lines drawn from the moon to the sun as parallel to the line which is drawn from the earth to the sun, therefore so that by diminishing all the effect in some cases almost as much as it may be increased in others ; and we seek the mean motion of the nodes, with minute details of this kind ignored, which may exceedingly impair the calculation.

Now let *PM* designate the arc, that the moon will describe in a given time taken as minimal, and *ML* the small line the half of which the moon, with the before mentioned force *3IT* impressed, may describe in the same time. *PL* and *MP* may be joined, and these may be produced to *m* and *l,* where they cut the plane of the ecliptic ; and the perpendicular *PH* may be sent to *Tm.* And because the right line *ML* is parallel to the plane of the ecliptic, and thus which cannot cross with the right line *ml* lying in that plane, and yet these lines lie in the common plane *LMPml* ; these right lines will be parallel and therefore the triangles *LMP* and *lmP* are similar. Now since *MPm* shall be in the plane of the orbit, in which the moon will be moving at the place *P*, it meets the line *Nn* drawn through the nodes *N, n* of this orbit at the point *m*. And because the force by which half of the line *LM* is generated, if likewise the whole force were impressed once at the place *P*, it would generate that whole line ; and puts into effect that the moon should move in an arc, of which the chord must be *LP,* and thus it should carry the moon from the plane *MPmT* into the plane *LPIT;* the motion of the angle of the nodes generated by that force, will be equal to the angle *mTl*. But *ml* is to *mP* as *ML* to *MP,* and thus since *MP* shall be given on account of the time given, *mt* is as the rectangle $ML \times mP$, that is, as the rectangle $IT \times mP$. And the angle *mTl,* but only if the angle *Tml* shall be right, is as $\frac{ml}{Tm}$, and therefore as $\frac{IT \times Pm}{Tm}$, that is (on account of the proportionals *Tm* and *mP, TP* and *PH*) as $\frac{IT \times PH}{TP}$, and thus on account of *TP* given, as $IT \times PH$. Because if the angle *Tml,* or *STN* shall be oblique, the angle *mTl* now becomes less, in the ratio of the sine of the angle *STN* to the radius, or *AZ* to *AT.* Therefore the velocity of the nodes is as $IT \times PH \times AZ$, or as contained by the product of the sines of the three angles *TP1, PTN* and *STN.*

If these angles may be right, with the nodes in quadrature and with the moon present in syzygies, then the small line *ml* will go off to infinity, and the angle *mTl* will emerge equal to the angle *mPl* . But in this case, the angle *mPl* is to the angle *PTM,* that the moon will describe in the same time by its apparent motion around the earth, as 1 to 59,575. For the angle *mPl* is equal to the angle *LPM,* that is, to the angle of deflection of the moon from a straight course, that only the aforementioned force of the sun 3*IT,* may generate in a given time, if the gravity of the moon should cease ; and the angle *PTM* is equal to the angle of deflection of the moon from a straight line course, by that force, by which the moon may be retained in its orbit, that could be generated in the same time, if the force



3*IT* of the sun should then cease. And these forces, as we have said above, are in turn as 1 to 59,515. Therefore since the mean hourly motion of the moon with respect to the fixed stars shall be $32'.56''.27'''.12^{\text{iv}}\frac{1}{2}$, the hourly motion of the node in this case will be $33''.10'''.33^{\text{iv}}.12^{\text{v}}$. Moreover in other cases this hourly motion will be to $33''.10'''.33^{\text{iv}}.12^{\text{v}}$ as the multiple of the three sines of the angles *TPI, PTN,* and *STN* (or of the distances of the moon from quadrature, of the moon from the node, and of the node from the sun) to the cube of the radius. And just as often as the sign of some angle changes from positive to negative, or from that in which a negative will be changed into a positive, the motion of regression is changed into progression, and progression is changed into regression. From which it shall arise that the nodes shall be progressing as often as the moon shall be moving between one quadrature and the node of the nearest quadrature. In other cases it will be regressing, and by the excess of the regression over the progression are carried forwards each month in advance.

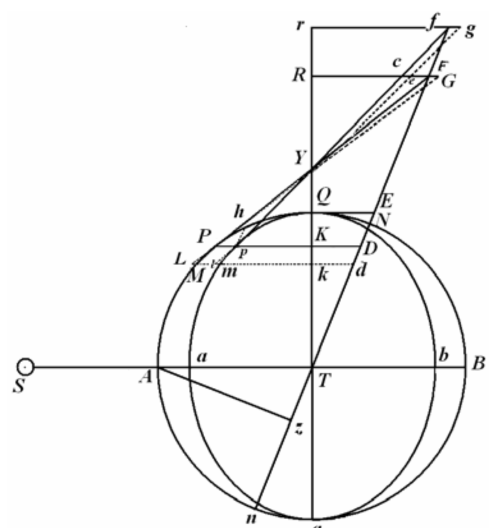
*Corol.*1. Hence if from a given minimal arc *PM* with the ends *P* and *M* there may be sent the perpendiculars *PK, M k* to the line *Qq* joining the quadratures, and the same may be produced cutting the nodal line *Nn* at *D* and *d;* the hourly motion of the nodes will be as the area *MPDd* and the square of the line *AZ* jointly. For indeed *PK, PH* and *AZ* become the three aforesaid sines. Clearly *PK* is the sine of the distance of the moon from the quadrature, *PH* the sine of the distance of the moon from the node, and *AZ* the sine of the distance of the node from the sun : and the velocity of the node will be as the product $PK \times PH \times AZ$. But *PT* is to *PK* as *PM* to *Kk,* and thus on account of the given *PT* and *PM* , there is *Kk* proportional to PK itself. Also *AT* is to *PD* as *AZ* to *PH,* and therefore *PH* is proportional to the rectangular $PD \times AZ$, and with the adjoining ratios, $PK \times PH$ it as the product $Kk \times PD \times AZ,$ and $PK \times PH \times AZ$ as $Kk \times PD \times AZqu\,;$ that is , as the area *PDdM* and AZ^2 jointly.
Q. E.D.

Corol. 2. At some given position of the nodes, the hourly mean motion is half the hourly motion at the syzygies of the moon, and therefore is to $16''. 35'''. 16^{iv}. 36^{v}$. as the square of the sine of the distance of the nodes from the syzygies to the square of the radius, or as AZ^2 to AT^2. For if the moon may be going around the semicircle *QAq* with a uniform motion, the sum of all the areas *PDdM,* in which time the moon goes from *Q* to *M,* will be the area *QMdE* which is terminated by the tangent to the circle *QE* ; and in which time the moon reaches the point *n,* that sum will be the total area *EQAn* that the line *PD* will describe, then with the moon going from *n* to *q,* the line *PD* falls outside the circle, and the area *nqe* described terminated by the tangent *qe* to the circle ; which, because the nodes at first were regressing, now indeed because they are progressing, the area must be taken from the first area, and since it shall equal the area *QEN*, there will remain the semicircle *NQAn.* Therefore since the sum of all the areas *PDdM,* in which time the moon will describe the semicircle, is the area of the semicircle ; and the whole sum in which time the moon will describe the circle is the whole area of the circle. But the area *PDdM,* when the moon is turning in the syzygies, is the rectangle under the arc *PM* and the radius *PT;* and the sum of all the areas equal to this area, in which time the moon will describe the circle, is the rectangle under the whole circumference and the radius of the circle ; and this rectangle, since it shall be equal to two circles, is more than twice the first rectangle. Hence the nodes, that by continuing with a uniform velocity as they have at the lunar syzygies, describe a distance twice as great as they actually describe ; and therefore the average motion by which, if it were continued uniformly, they would be able to describe a distance between themselves by the unequal motion they actually complete, is half the motion that they have in the moon's syzygies. From which since the maximum hourly motion, if the nodes are passing through quadrature, shall be $33''. 10'''. 33^{iv}. 12^{v}$, and the mean hourly motion in this case will be $16''.35'''.16^{iv}. 36^{v}$. And since the hourly motion of the nodes always shall be as AZ^2 and the area *PDdM* jointly, and therefore the hourly motion of the nodes at the syzygies of the moon shall be as AZ^2 and the area *PDdM* jointly, that is (on account of the area *PDdM* described at the syzygies) as AZ^2 also the mean motion will be as AZ^2, and thus this motion, when the nodes are moving beyond the quadratures, will be to $16''. 35'''.16^{iv}. 36^{v}$ as AZ^2 to AT^2. *Q. E. D.*

PROPOSITION XXXI. PROBLEM XII.
To find the hourly motion of the nodes of the moon in an elliptic orbit.

Let *Qpmaq* designate the ellipse described with the major axis *Qq*, and with the minor axes *ab*, *QAqB* the circumscribed circle, *T* the earth in the common centre of each, *S* the sun, *p* the moon moving in the ellipse, and *pm* the arc that will be described in a given particular element of time, *Nn* the line joining the nodes *N* and *n*, *pK* and *mk* perpendiculars sent to the axis *Qq* and hence produced from there until they cross the circle at *P* and *M,* and the line of the nodes at *D* and *d.* And if the moon, with the radius drawn to the earth, describes an area proportional to the time, the hourly motion of the nodes will be as the area *pDdm* and AZ^2 jointly.

For if *PF* is a tangent to the circle at *P,* and produced crosses *TN* in *F,* and *pf* is a tangent at *p* and produced meets the same line *TN* at *f,* moreover these tangents meet the axis *TQ* at *Y;* and if *ML* may designate the distance that the moon revolving in a circle, while meanwhile it describes the arc *PM,* urged and impelled by the aforementioned force 3*IT,* or can describe 3*PK* by a transverse motion, and *ml* may designate the distance that the moon revolving in the ellipse in the same time can describe, also urged by the force 3*IT* or 3*PK* ; and *LP* and *lp* produced cross the plane of the ecliptic at *G* and *g;* and *FG* and *fg* are joined, of which *FG* produced cuts *pf*, *pg* and *TQ* in *c, e* and *R* respectively, and *fg* produced cuts *TQ* in *r.* Because the force *31T* or *3PK* in the circle is to the force 3*IT* or *3pK* in the ellipse, as *PK* to *pK,* or *AT* to *aT;* the distance *ML* generated by the first force will be to the distance *ml* generated by the second force, as *PK* to *pK,* that is, on account of the similar figures *PYKp* and *FYRc,* as *FR* to *cR.* But *ML* is to *FG* (on account of the similar triangles *PLM, PGF*) as *PL* to *PG,* that is (on account of the parallel lines *Lk, PK, GR*) as *pl* to *pe,* that is to say, (on account of the similar triangles (*plm*, *cpe)* as *lm* to *ce* ; and inversely as *LM* is to *1m,* or *FR* to *cR,* thus *F*G is to *ce.* And therefore if *fg* shall be to *ce* as *fY* to *cY,* that is, as *fr* to *cR* (that is, as *sr* to *FR* and *FR* to *cR* jointly, that is, as *fT* to



FI and *FG* to *ce* jointly) because the ratio *FG* to *ce* with each taken away leaves the ratios *fg* to *FG* and *fT* to *FT,* there becomes *fg* to *FG* as *fT* to *FT;* and thus the angles, which

FG and *fg* subtend at the earth *T,* are equal to each other. But these angles (by that which we have put in place in the preceding proposition) are the motions of the nodes, by which in the time the moon travels in the arc of the circle *PM,* it travels in the arc of the ellipse *pm* : and therefore the motions of the nodes in the circle and in the ellipse are equal to each other. These thus are found, but only if *fg* shall be to *ce* as *fY* to *cY*, that is, if *fg* shall be eqaual to $\frac{ce\times fY}{cY}$. Truly on account of the similar triangles *fgp, cep,* there is *fg* to *ce* as *fp* to *cp;* and thus *fg* is equal to $\frac{ce\times fp}{cp}$; and therefore the angle, that *fg* actually subtends, is to the first angle, that *FG* subtends, that is, the motion of the nodes in the ellipse to the motion of the nodes in the circle, as this line *fg* or to the first value of *fg* or $\frac{ce\times fY}{cY}$, that is, as $fp\times cY$ to $fY\times cp$, or *fp* to *fT and cY* ad *cp,* that is to say, if *ph* crosses *FP* at *h* parallel to *TN* itself, so that *Fh* to *FY* and *FY* to *FP;* that is, as *Fh* to *FP* or *Dp* to *DP,* and thus as the area *Dpmd* to the area *DPMd.* And therefore, since (by Corol. I. Prop. XXX.) the latter area and AZ^2 jointly shall be proportional to the hourly motion of the nodes in the circle, the former area and AZ^2 jointly shall be proportional to the hourly motion of the nodes in the ellipse. *Q.E.D.*

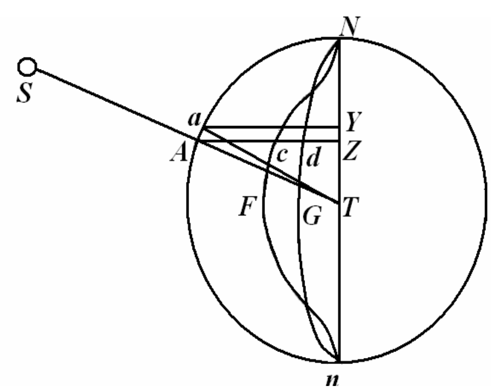
Corol. Whereby since, in the given position of the nodes, the sum of all the areas *pDdm,* in which time the moon goes through from quadrature to some place *m,* shall be the area *mpqQEd,* which is terminated by the tangent line of the ellipse *QE* ; and the sum of all those areas, in a whole revolution, shall be the area of the whole ellipse: the mean motion of the nodes in the ellipse will be to the mean motion of the nodes in the circle, as the [area of the] ellipse to the [area of the] circle; that is, as *Ta* to *TA,* or 69 to 70. And therefore, since (by Corol. 2. Prop. XXX.) the mean hourly motion of the nodes in the circle shall be to $16''.\ 35'''.\ 16^{iv}.\ 36^{v}.$ as AZ^2 to AT^2, if the angle $16''.21'''.3^{iv}.\ 30^{v}.$ may be taken to the angle $16''.\ 35'''.\ 16^{iv}.36^{v}.$ as 69 to 70, the mean hourly motion of the nodes in the ellipse will be to $16''.\ 21'''.\ 3^{iv}.\ 30^{v}.$ as AZ^2 to AT^2; that is, as the square of the sine of the distance of the node from the sun to the square of the radius.

For the remainder the moon, with the radius drawn to the earth, will describe areas faster at the syzygies than at the quadratures, and by that account the time is diminished at the syzygies, and augmented at the quadratures ; and together with the time the motion of the nodes is augmented or diminished. But the moment of the area at the quadrature of the moon to the moment of this in syzygies was as 10973 to 11073, and therefore the mean moment at the octants is to the excess in syzygies, and the defect at quadrature, as half the sum of the numbers 11023 to half the difference 50. From which since the time of the moon in equal individual particular orbits shall be inversely as its velocity, the mean time in the octants to the excess of the time at quadrature, and the defect in syzygies, arising from this effect, will be as 11023 to 50 approximately. Moreover in going from quadrature to syzygies, I find that the excess of the moments of the area in individual places, above the minimum moment at the quadratures, shall be as the square of the sine of the distance of the moon from the quadrature approximately ; and therefore the difference between the moments at some place and the average moment at the quadrant, is as the difference between the square of the sine of the distance of the moon from the square of the sine of 45 degrees, or half the square of the radius ; and the increment of the time in the individual places between the octants and the quadratures, and the decrement of this between the octants and the syzygies, is in the same ratio. But the motion of the nodes, in which time the moon traverses equal individual parts of the orbit, may be accelerated or retarded in the square ratio of the time. For this motion, while the moon passes through *PM* (with all else equal) is as *ML,* and *ML* is in the square ratio of the time. Whereby the motion of the nodes at syzygies, performed in that time in which the moon traversed a small part of the orbit, is diminished in the square ratio of the number 11073 to the number 11023 ; and the decrement to the motion of the rest as 100 to 10973, to the true total motion as 100 to 11073 approximately. But the decrement in places between the octants and the syzygies, and the increment in places between the octants and quadrature, is approximately to this decrement, as the ratio of the whole motion in these places to the whole motion at the syzygies, and the difference between the square of the sine of the distance of the moon from the quadrature and half the square of the radius to half the square of the radius taken jointly. From which, if the nodes are present in quadrature, and two places hence may be taken equally distant from the octant on each side, and two others may be taken, one equally distant from the syzygies and the other from the quadrature, then, from the decrements of the motion in the two places down between the syzygy and the octant, the increments of the motions in the two remaining places may be taken away, which are between the octant and quadrature ; the remaining decrement will be equal to the decrement in the syzygies : the reason for that will be easily seen on entering into a computation. And therefore the mean decrement, that must be taken from the mean motion of the nodes, is the fourth part of the decrement in the syzygy. The whole hourly motion of the nodes in syzygies, when the moon was supposed to describe equal areas by the radius drawn to the earth, was $32''.42.'''.7^{\text{iv}}$. And the decrement of the motion of the nodes, the moon in which time now moving faster will describe the same distance, we have said to be to this motion as 100 to 11073 ; and thus that decrement is $17'''43^{\text{iv}}.\ 11^{\text{v}}$, the fourth part of which $4'''.25^{\text{iv}}.\ 48^{\text{v}}$, of the mean hourly motion, taken from $16''.21'''.3^{iv}.\ 30^{\text{v}}$ found above, leaves $16''.16'''.37^{\text{iv}}.\ 42.^{\text{v}}$, the correction of the mean hourly motion.

If the nodes may be moving beyond the quadratures, and hence two places may be considered equally distant from the syzygies ; the sum of the motions of the nodes, when the moon is moving through these places, will be to the sum of the motions, when the moon may be present in the same places and the nodes may be at the quadratures, as AZ^2 to AT^2. And the decrements of the motions arising, from the causes now discussed, will be in turn as the motions themselves, and thus the remaining motions will be in turn as AZ^2 to AT^2 and the mean motions as the remaining motions. And thus the correct mean hourly motion, in some given situation of the nodes, is to $16''.16'''.37^{iv}.42^{\text{v}}$. as AZ^2 to AT^2, that is, as the square of the sine of the distance of the nodes from the syzygies to the square of the radius.

PROPOSITION XXXII. PROBLEM XIII.
To find the mean motion of the nodes of the moon.

The mean annual motion is the sum of all the mean horary motions during the year. Consider the node to be moving through *N,* and with an individual hour completed to be drawn back to its first position, so that with no obstacle to its proper motion, it may maintain a given situation with regard to the fixed stars. Meanwhile truly the sun *S,* by the motion of the earth, has moved from the node, and to complete uniformly its annual course. Moreover let *Aa* be that minimum given arc described in the minimum given time, by the direction of the right line *TS* always drawn to the sun and the circle *NAn*: and the mean hourly motion (by what has just been shown) will be to AZ^2, that is (on account of the proportionals *AZ, ZT*) as the rectangle under *AZ* and *ZY,* that is, as the area *AZTa.* And the sum of all the mean hourly motions from the beginning, is as the sum of all the areas *aTZA,* that is, as the area *NAZ.* But the maximum *AZTa* is equal to the rectangle under the arc *Aa* and the radius of the circle ; and therefore the sum of all the rectangles in the whole circle to the sum of just as many maximas, as the whole area of the circle to the rectangle under the whole circumference and the radius, that is, as 1 to 2. Moreover the hourly motion, corresponding to the maximum area, was $16''.16'''.\ 37^{iv}.42^{v}$. And this motion, in the whole sidereal year of 365 days. 6 hours. 9 min. shall be $39^0.\ 38'.\ 7''.\ 50'''$. And thus the half of this $19^{gr}.49'.\ 3''.55'''$. is the mean motion of the

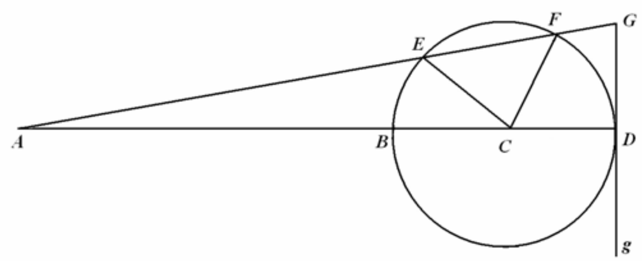


nodes corresponding to the whole circle. And the motion of the nodes, in which time the sun goes from *N* to *A,* is to $19^{gr}.\ 49'.\ 3''.\ 55'''$. as the area *NAZ* to the whole circle.

These results themselves thus are obtained from the hypothesis, that the node of the individual hour is redrawn into its former position, thus so that the sun by completing a whole year may return to the same node from which in the beginning it had set out. Truly by the motion of the node it shall arise that the sun shall return to the node quicker, and by computation now is in a shorter time. Since the sun in a whole year completes 360^0 and the node by the maximum motion completes $39^0.\ 38'.\ 7''.\ 50'''$ in the same time, or 39,6355 degrees ; and the mean motion of the node at some place *N* shall be to the mean motion of its quadrature, as AZ^2 to AT^2: the motion of the sun to the motion of the node at *N,* shall be as 360 AT^2 to 39,6355 AZ^2 ; that is, as 9,0827646 AT^2 to AZ^2. From which if the circumference of the whole circle *NAn* may be divided into equal small parts *Aa,* the time in which the sun travels through the element *Aa,* if the circle were at rest, will be to the time in which it traverses the same element, if the circle together with the nodes were revolving around the centre *T*, reciprocally as $9{,}0827646\,AT^2$ to $9{,}0877646AT^2 + AZ^2$. For the time is reciprocally as the velocity in which an element is traversed, and this velocity is the sum of the velocities of the sun and of the node. Therefore if the time, in which the sun without the motion of the node traversed the arc *NA,* may be expressed by the sector *NTA,* and the element of the time in which it traversed that element of the arc *Aa,* may be expressed by the element of the sector *ATa*; and (with the perpendicular *aY* sent to *Nn*) if *dZ* may be taken on *AZ* , its length shall be as the rectangle *dZ* by *ZY* to the element of the section *ATa* as AZ^2 to $9{,}0827646AT^2 + AZ^2$, that is, as *dZ* shall be to $\frac{1}{2}AZ$ as AT^2 to $9{,}0827646AT^2 + AZ^2$, the rectangle *eZ* into *ZY* will designate the decrement of the time arising from the motion of the node, in the total time in which the arc *Aa* is traversed. And if the point *d* touch the curve *NdGn,* the curved area *NdZ* will be the total decrement, in which time the whole arc *NA* is traversed; and therefore the excess of the sector *NAT* above the area *NdZ* will be that total time. And because the motion of the node in a smaller time is smaller in the ratio of the time, also the area *AaYZ* must be diminished in the same ratio. Because that arises if *eZ* may be taken in the length *AZ,* which shall be to the length *AZ* as AZ^2 to $9{,}0827646AT^2 + AZ^2$. Thus indeed the rectangle *eZ* in *ZY* will be to the area *AZYa* as the decrement of the time, in which the arc *Aa* is traversed, to the total time in which it may be traversed, if the node may be at rest : and therefore that rectangle will correspond to the decrement of the motion of the node. And if the point *e* may touch the curve *NeFn,* the total area *NeZ,* which is the sum of all the decrements, will correspond to the total decrement, in which time the arc *AN* is traversed ; and the area remaining *NAe* will correspond to the remaining motion, which is the true motion of the node, in which time the whole arc *NA* is traversed by the motion of the sun and node conjointly. Now truly the area of the semicircle is to the area of the figure *NeFn,* sought by the method of infinite series, as 793 to 60 approximately. But the motion which corresponds to the whole circle was $19^0.49'.\,3''.55'''$, and therefore the motion, which corresponds to duplicate figure *NeFn*, is $1^{gr}.\,29'.58''.2'''$. Which taken from the first motion leaves $18^0.19'.\,5''.53'''$, the total motion of this with respect to the fixed stars between conjunctions with the sun ; and this motion taken from the annual motion of the sun of 360^0, leaves $341^0.\,40'.\,54'.\,7'''$, the motion of the sun between the same conjunctions. But this motion is to the annual motion 360^0. So that the motion of the nodes is now found, $18^0.19'.\,5''.53'''$. to the motion of the year itself, which therefore will be $19^0.18'.\,1''.23'''$. This is the mean motion of the nodes in a sidereal year. The same from astronomical tables is $19^0.21'.21''.50'''$. The small difference is a one three hundredth part of the total motion, and may be seen to arise from the lunar eccentricity and from the inclination to the plane of the ecliptic. The motion of the nodes is accelerated by the eccentricity of the orbit, and by this in turn the inclination may be slowed down a little, and at the same time the velocity is reduced.

PROPOSITION XXXIII. PROBLEM XIV.
To find the true motion of the lunar nodes.

In the time which is as the area *NTA-NdZ,* (*in the fig. preced.*) the motion is as this area *NAe,* and thence is given. Truly on account of the excessive difficulty of the calculation, it is better to keep to the following construction of the problem. With the centre *C,* with some radius *CD,* the circle *BEFD* may be described. *DC* is produced to *A,* so that *AB* to *AC* shall be as the mean motion to half the true mean motion, when the nodes are in



quadrature, that is, as $19^0.18'.1''.23'''$. to $19^0.49'.3''.55'''$, and thus *BC* to *AC* as the difference of the motions $0^0.31'.2''.32.'''$, to the latter motion $19^0.49'.3''.55'''$, that is, as 1 to $38\frac{3}{10}$; then through the point *D* the indefinite line *Gg* is drawn, which is a tangent to the circle at *D;* and if the angle *BCE* or *BCF* may be taken equal to twice the distance of the sun from the place of the node, by the mean motion found; and *AE* or *AF* may be acting cutting the perpendicular *DG* in *G;* and the angle may be taken which shall be to total motion of the node between its syzygies (that is, to $9^0.11'.3''$.) as the tangent *DG* to the circumference of the whole circle *BED*; and this angle (for which the angle *DAG* can be used) may be added to the motion of the nodes when the nodes pass from quadrature to syzygies, and may be subtracted from the same mean motion when they pass from syzygies to quadratures; the true motion of these will be had. For the true motion thus found agrees approximately with the true motion which arises by putting in place the time by the area *NTA-NdZ,* and the motion of the node by the area *NAe;* so that the matter considered carefully will agree with the computations put in place. This is the half yearly equation of the motion of the nodes. And there is a monthly equation, but which is hardly necessary for finding the latitude of the moon. For since the variation of the inclination of the moon's orbit to the plane of the ecliptic shall be liable to a two-fold inequality, the one half-yearly, but the other monthly ; the monthly inequalities of this and the monthly equation of the nodes thus mutually modify and correct each other, so that both can be ignored in determining the latitude of the moon.

Corol. From this and from the preceding proposition it is evident that the nodes are at rest at their syzygies, but in the quadratures they are regressing by the hourly motion $16''.19'''.26^{iv}$. And because the equation of motion in the octants shall be $1^0.30'$. Which are rightly in order with all celestial phenomena.

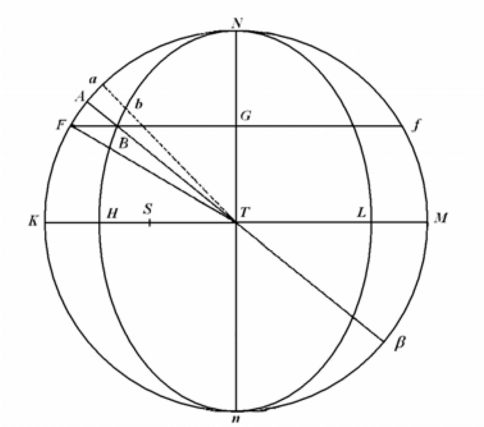
Scholium.

J. Machin, Prof. of Astronomy at Gresham College, and *Henry Pemberton* M. D. have separately discovered the motion of the nodes by another method. Certain mention of this method has been made elsewhere. And each paper that I have seen, contained two propositions, and they agreed between each other in each case. Truly *Machin's* paper is here adjoined, since it came first into my hand.

' Concerning the motion of the moon's nodes.

PROPOSITION I.

The mean motion of the sun from a node, is defined by the mean geometrical proportion, between the mean motion of the sun, and that mean motion by which the sun recedes most rapidly from the node in quadrature.



T shall be the place where the earth is, *Nn* the line of the lunar nodes at some given time, *KTM* a right angle drawn to this, *TA* a right line revolving around the centre with that angular velocity by which the sun and the nodes recede from each other in turn, thus so that the angle between the line *Nn* at rest and the revolving line *TA,* always becomes equal to the distance of the places of the sun and of the node. Now if some right line *TK* may be divided into the parts *TS* and *SK* which shall be as the motion of the mean hourly sun to the motion of the mean hourly node at quadrature, and the right line *TH* may be put as the mean proportional between the part *TS* and the whole *TK,* this right line between the rest of the proportions will be the mean motion of the sun from the node. For the circle *NKnM* with centre *T* and radius *TK* may be described, and with the same centre and radius *TH* and *TN* the ellipse *NHn* is described, and in the time in which the sun recedes from the node through the arc *Na,* if the right line *Tba* is drawn, the area of the sector *NTa* expresses the sum of the motions of the sun and of the node in the same time. Therefore by the afore mentioned law let the arc *aA* be the element the right line *Tba* will describe revolving in the given element of time, and that minimal sector *TAa* will be as the sum of the velocities by which the sun and the node are carried separately in that time. But the velocity of the sun is almost constant, just as the small inequality of this will lead to scarcely any variation in the mean motion of the nodes.

The other part of this sum, clearly the mean value of the velocity of the node, may be increased by the departure from the syzygies in the square ratio of the sine of its distance from the sun ; by the Corollary to Prop. 31, Book 3 *Princip*. and since it is a maximum at the quadrature to the sun at *K,* it will be obtained in the same ratio to the velocity of the sun which *SK* has to *TS,* that is as (the difference of the squares from *TK* and *TH* or) the rectangle *KHM* to the square *TH*. But the ellipse *NBH* divides the sector *ATa* which expresses the sum of these two velocities, into the two parts *ABba* and *BTb* proportional to these velocities. For *BT* may be produced to the circle at β , and from the point *B* the perpendicular *BG* may be sent to the major axis, which each produced cross the circle at the points *F* and *f,* and because the distance *ABba* it to the sector *TBb* as the rectangle $AB\beta$ to the square *BT* (for that rectangle is equal to the difference of the squares from *TA* and *TB* on account of the right line $A\beta$ cut equally and unequally at *T* and *B.*) Therefore this ratio, when the distance *ABba* is a maximum at *K,* will be the same as the ratio of the rectangle *KHM* to the square *HT*, but the maximum mean velocity of the nodes was in this ratio to the velocity of the sun. Therefore in the quadratures the sector *ATa* is divided into parts proportional to the velocities. And because the rectangle *KHM* is to the square *HT* as *FBI* to the square *BG* and the rectangle $AB\beta$ is equal to the rectangle *FBf.* Therefore the little area *ABba* when it is a maximum to the sector *TBb* of the remaining, as the rectangle $AB\beta$ to the square *BG*. But the ratio of these small areas always was as the rectangle $AB\beta$ to the square *BT* ; and therefore the element of the area A*Bba* at the place *A* is less than the similar element in quadrature, in the square ratio *BG* to *BT* that is in the square ratio of the sine of the distance of the sun from the node. And therefore the sum of all the elements of area *ABba* surely the interval A*BN* will be as the motion of the node in the time in which the sun is moving away from the node through the arc *NA.* And the distance of the remaining, surely the elliptic sector *NTB* will be as the mean motion of the sun in the same time. And therefore because annual mean motion of the node, that is which shall be in the time in which has completed its period, the mean motion of the node from the sun will be to the mean motion of the sun itself, as the area of the circle to the elliptic area, that is as the right line *TK* to the mean right line *TH* evidently the proportional between *TK* and *TS* ; or what amounts to the same, as the mean proportional *TH* to the right line *TS*.

PROPOSITION II.

From the given mean motion of the nodes, to find the true motion.

The angle *A* be the distance of the sun from the mean location of the node, or the mean motion of the sun from the node. Then if the angle *B* is taken, its tangent shall be to the tangent of the angle *A* as *TH* to *TK,* that is, in the square root ratio of the mean hourly motion of the sun with the node moving in quadrature ; the angle *B* will be the same distance of the sun from the true position of the node. For *FT* may be joined and from the demonstration of the ratio in the above proposition, the angle *FTN* will be the distance of the sun from the mean place of the node, but the true distance of the angle *ATN* from the node, and the tangents of these angles are to each other as *TK* to *TH.*

Corol. Hence the angle *FTA* is the equation of the lunar nodes, and the sine of this angle when it is a maximum in the octants, is to the radius as *KH* to $TK+TH$. But the sine of this equation at some other place *A* is to the maximum sine, as the sine of the sum of the angles $FTN+ATN$ to the radius : that is almost as the sine of twice the distance of the sun from the mean place of the node (clearly 2.*ETN*) to the radius.

Scholium.

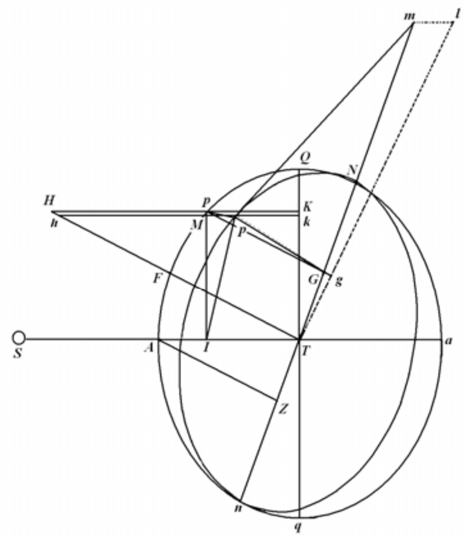
If the mean hourly motion of the nodes in quadrature shall be $16''.16'''.3^{\text{iv}}.42^{\text{v}}$. that is, $39°.38'.7''.50'''$ in the whole sidereal year, *TH* will be to *TK* in the square root ratio of the number 9,0827646 to the number 10,0827646, that is, as 18,6524761 to 19,6524761. And therefore *TH* shall be to *HK* as 18,6514761 to 1, that is, as the motion of the sun in one sidereal year to the mean motion of the nodes $19^0.18'.1''.23'''\frac{2}{3}$.
But if the mean motion of the nodes of the moon in the 20 Julian years shall be $386^0.50'.15''$. as thus from observations it is deduced in the theory of the moon used : the mean motion of the nodes in a sidereal year shall be $19^0.20'.31''.58'''$. And *TH* will be to *HK* as 360^0 to $19^0.20'.31''.58'''$, that is, as 18,61214 to 1, from which the mean hourly motion of the nodes in quadrature becomes $16''.18'''.48^{\text{iv}}$. And the maximum equation of the nodes in the octants will be $1^0.29'.57''$. '

PROPOSITION XXXIV. PROBLEMA XV.

To find the variation of the hourly inclination of the moon's orbit to the plane of the ecliptic.

Let *A* and *a* designate the syzygies; *Q* and *q* the quadratures; *N* and *n* the nodes; *P* the location of the moon in its orbit ; *p* the projection of that place into the plane of the ecliptic, and *mTl* the instantaneous motion of the nodes as above. And if the perpendicular *PG* may be sent to the line *Tm*, *pG* may be joined, and with that then produced then it may cross *Tl* in *g*, and also may be joined *PG*: the angle for the inclination to the orbit of the moon to the plane of the ecliptic will be *PGp*, when the moon may be passing through *P* ; and the angle *Pgp* will be the inclination of the same after a completed instant of time; and thus the angle *GPg* is the instantaneous variation of the inclination. But this angle *GPg* is to the angle *GTg* as *TG* to *PG* and *Pp* to *PG* jointly. And therefore if for the instant of time an hour may be substituted; since the angle *GTg* (by Prop. XXX.) shall be to the angle $33''.10'''.33^{\text{iv}}$. as $IT\times PG\times AZ$ to AT^3, the angle *GPg* (or the variation of the hourly inclination) shall be to the angle $33''.10'''.33^{\text{iv}}$. as $IT\times AZ\times TG\times\frac{Pp}{PG}$ to AT^3 *Q. E.I.*

Thus these may be had themselves from the hypothesis that the moon is gyrating



uniformly in its orbit. Because if that orbit is elliptical, the mean motion of the nodes may be diminished in the ratio of the minor axis to the major axis ; as has been explained above. And the variation of the inclination may be diminished in the same ratio also.

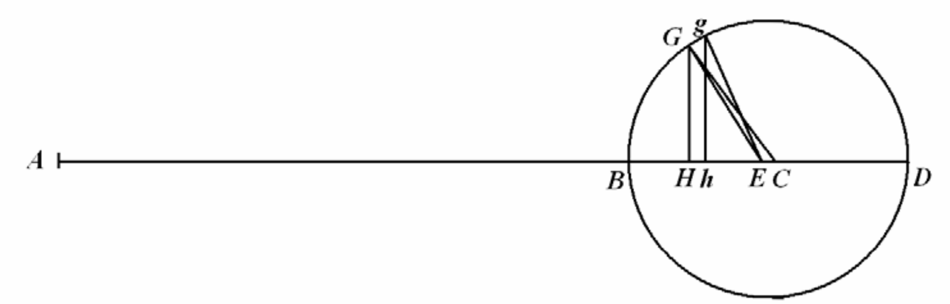
Corol. 1. If the perpendicular *TF* may be erected to *Nn*, and *pM* shall be the hourly motion of the moon in the plane of the ecliptic ; and the perpendiculars *pK, Mk* may be sent to *QT* and each produced crossing *TF* in *H* and *h*: *IT* will be to *AT* as *Kk* to *Mp*, and *TG* to *Hp* as *TZ* to *AT*, and thus $IT \times TG$ equals $\frac{Kk \times Hp \times TZ}{Mp}$, that is, equal to the area *HpMh* taken by the ratio $\frac{TZ}{Mp}$: and therefore the hourly variation of the inclination $33''.10'''.33^{\text{iv}}$. as *HpMh* multiplied by $AZ \times \frac{TZ}{Mp} \times \frac{Pp}{PG}$ to AT^3.

Corol. 2. And thus if the earth and the nodes of the individual completed hours may be withdrawn from the new places of these, and always reintroduced at once again into the first place, so that their position, through the period of a whole month, may remain in place ; the whole variation of the inclination in the time of a month of its inclinations will be to $33''.10'''.33^{\text{iv}}$. as the sum of all the areas *HpMh*, described in the revolution of the point *p*, and taken together with the appropriate signs $+$ and $-$, multiplied into $AZ \times TZ \times \frac{Pp}{PG}$ to $Mp \times AT^3$, that is to say, as the whole circle *QAqa* multiplied into $AZ \times TZ \times \frac{Pp}{PG}$ to $Mp \times AT^3$ that is, as the circumference *QAqa* multiplied into $AZ \times TZ \times \frac{Pp}{PG}$ to $2Mp \times AT^2$.

Corol. 3. Hence in the given situation of the nodes, the mean hourly variation, from which that monthly variation can be generated continually each month, is to

$33''.10'''.33^{iv}$. as $AZ \times TZ \times \frac{Pp}{PG}$ to $2AT^2$, or as $Pp \times \frac{AZ \times TZ}{\frac{1}{2}AT}$ to $PG \times 4AT$, that is (since Pp shall be to PG as the sine of the aforesaid inclination to the radius, and $\frac{AZ \times TZ}{\frac{1}{2}AT}$ shall be to $4AT$ as the sine of double the angle ATn to the radius quadrupled) as the sine of the same inclination multiplied into the sine of double the distance of the nodes from the sun, to four times the square of the radius.

Corol. 4. Because the variation of the hourly inclination, when the nodes are turning in



the quadratures, is (by this proposition) to the angle $33''.10'''.33^{iv}$. as $IT \times AZ \times TG \times \frac{Pp}{PG}$ to AT^3 , that is, as $\frac{IT \times TG}{\frac{1}{2}AT} \times \frac{Pp}{PG}$ to $2AT$; that is to say, as the sine of twice the distance of the moon from the quadrature multiplied into $\frac{Pp}{PG}$ to the radius doubled: the sum of all the hourly variations, in which time the moon in this situation of the nodes passes from quadrature to syzygies (that is, the space of $177\frac{1}{6}$ hours,) will be to the sum of just as many angles $33''.10'''.33^{iv}$, or 5878", as the sum of all the sines of twice the distances of the moon from quadrature multiplied into $\frac{Pp}{PG}$ to the sum of just as many diameters ; that is, as the diameter multiplied by $\frac{Pp}{PG}$ to the circumference; that is, if the inclination shall be $5^0.1'$, as $7 \times \frac{874}{10000}$ to 22 , or 278 to 10000. And hence the whole variation, from the sum of all the hourly variations in the aforementioned time put together, is 163", or 2'. 43".

PROPOSITION XXXV. PROBLEM XVI.

To find the inclination of the moon to the ecliptic arising at a given time.

Let AD be the sine of the maximum inclination, AB the sine of the minimum inclination ; BD is bisected in C, and the circle BGD may be described with centre C, and with the radius BC. On AC, CE may be taken in that ratio to EB that EB has to $2BA$: and if in a given time the angle AEG may be put in place equal to twice the distance of the nodes from the quadratures, and to AD there may be sent the perpendicular GH: AH will be the sine of the inclination sought.

For GE^2 is equal to

$$GH^2 + HE^2 = BHD + HE^2 = HBD +$$

$$HE^2 - BH^2 = HBD \ + BE^2 \ - 2BH \times BE$$

$$= BE^2 + 2EC \times BH = 2EC \times AB + 2EC \times BH = 2EC \times AH.$$

And thus since $2EC$ is given, GE^2 is as AH. Now AEg designates twice the distance of the nodes from the quadratures after some given instant of time completed, and the arc Gg on account of the given angle GEg will be as the distance GE. But Hh is to Gg as GH to GC and therefore Hh is as the product $GH \times Gg$, or $GH \times GE$; that is, as $\frac{GH}{GE} \times GE^2$ or $\frac{GH}{GE} \times AH$, that is, as AH and the sine of the angle AEG jointly. Therefore if AH in some case shall be the sine of the inclination, that will be increased in the same increments with the sine of the inclination, by Corol. 3 of the above proposition, and therefore it will always remain equal to that sine. But AH, when the point G falls on either the point B or D, is equal to the sine of this, and therefore always remains equal to the same. *Q. E. D.*

I have supposed in this demonstration that the angle BEG, which is twice the distance of the nodes from quadrature, is increased uniformly. For it would be superfluous on this occasion to consider all the small inequalities. Now consider the angle BEG to be right, and in this case Gg is to be the increase of the hours of twice the distance of the nodes and of the sun in turn ; and the hourly variation of the inclination in the same case (by Corol. 3 of the most recent Prop.) will be to $33''.10'''.\,33^{\text{iv}}$ as the product of the sine of the inclination AH and the sine of the right angle BEG, which is twice the distance of the nodes from the sun, to four times the square of the radius ; *i.e.* as the sine of the mean inclination AH to four times the radius ; that is (since that mean inclination shall be as $5^0.\,8'\frac{1}{2}$ as its sine 896 to the fourfold radius 40000, or as 224 to 10000. Moreover the whole variation, corresponding to the difference of the sines BD, is to that hourly variation as the diameter BD to the arc Gg; *i.e.*, as the diameter BD to the semi circumference BGD and the time of the hours $2079\frac{7}{10}$, in which a node goes from quadrature to syzygies, to one hour jointly ; *i.e.*, as 7 to 11 and $2079\frac{7}{10}$ to 1. Whereby if all the ratios may be joined together, the total variation BD becomes to $33''.10'''.\,33^{\text{iv}}$ as $224 \times 7 \times 2079\frac{7}{10}$ to 110000, *i.e.*, as 29645 to 1000, and thence that variation BD will be produced $16'.\,23''\frac{1}{2}$.

This is the maximum variation of the inclination as long as the position of the moon in its orbit may not be considered. For the inclination, if the nodes may be turning in syzygies, are not changed from the variation of the moon. But if the nodes are present in quadrature, the inclination is less when the moon turns in syzygies, as when that may be in quadrature, by the excess 2'. 43"; as we have indicated in corollary four of the above proposition. And the whole mean variation BD with the moon in quadrature, diminished by half of its excess $1'.21''\frac{1}{2}$, shall become 15'. 2", but in the syzygies increased by this amount to become 17'. 45". Therefore if the moon may be in place in syzygies, the whole variation of the nodes from quadrature to syzygies will be 17'. 45": and thus if the inclination, when the nodes are present in syzygies, shall be $5^0.17'.\ 20''$; the same, when the nodes are in quadrature, and the moon in syzygies, will be $4^0.\ 59'.\ 35''$. And this is thus confirmed from observations.

If now that inclination of the orbit may be desired, when the moon is turning in syzygies and the nodes anywhere ; *AB* becomes to *AD* as the sine of 4^0. 59'. 35" to the sine of 5^0. 17'. 20", and the angle *AEG* may be taken equal to twice the distance of the nodes from quadrature ; and *AH* will be the sine of the inclination sought. For the inclination of this orbit is equal to the inclination of the orbit , when the moon shall be set at 90^0 from the nodes, [as just found]. In other locations of the moon the monthly inequalities, that are allowed in the variation of the inclination, may be compensated in the calculation of the latitude of the moon, and in some manner may be corrected by the monthly inequality of the motion of the nodes (as we have said above), and thus it can be ignored in the calculation of the latitude.

Scholium.

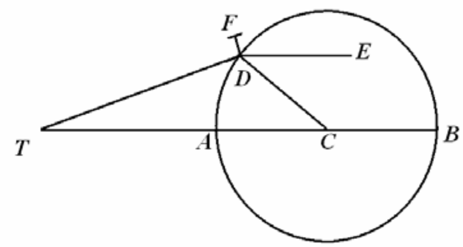
I wished to show from these computations of the moon's motions, how the causes of the lunar motions could be calculated from the theory of gravity. In addition by the same theory I found that the annul equation of the mean motion of the moon arose from the various dilations of the moon's orbit by the force of the sun, as in Corol. 6. Prop. LXVI. Book 1. This force is greater at the perigee of the sun, and dilates the moon's orbit ; at its apogee it is less, and it allows that orbit to be contracted. In a dilated orbit the moon is revolving slower, in the contracted orbit faster ; and the annual equation, by which this inequality may be balanced, is nothing at the apogee and perigee of the sun, at the mean distance of the sun from the earth it rises to around 11'. 50"', at other places it is proportional to the equation of the centre of the sun ; and it is added to the mean motion of the moon when the earth goes from the aphelion to its perihelion, and it is subtracted in the opposite part of the orbit. On assuming the radius of the orbit to be of size 1000 and the eccentricity of the earth to be $16\frac{7}{8}$, this equation, when it is a maximum, will produce 11'. 49", by the theory of gravity. But the eccentricity of the earth may be seen to be a little greater, and with increased eccentricity this equation must be increased in the same ratio. The eccentricity may be $16\frac{11}{12}$, and the maximum equation will become 11'. 51".

I have also found that at the perihelion of the sun, because of the greater force of the sun, the apogee and nodes of the moon move faster than at its aphelion, and that inversely in the cubic ratio of the distance from the earth. And thence the annual equations of these motions arise proportional to the equation of the sun's centre. But the motion of the sun is in the inverse square ratio of the distance from the sun to the earth, and the maximum equation of the centre, that these inequalities may generate, is 1^0. 56'. 20" agreeing with the predicated eccentricity of the sun $16\frac{11}{12}$. But if the sun's motion were inversely in the cubic ratio of the distance, this inequality would generate the maximum equation 2^0.54'. 30". And therefore the greatest equations, that the inequalities of the motions of the apogee, and of the nodes of the moon can generate, are to $2^0.\ 54'.\ 30''$ as the mean diurnal motion of the apogee and the mean diurnal motions of the nodes of the moon are to the mean diurnal motion of the sun. From which the greatest equation of the mean motion of the apogee becomes 19'.43", and the maximum mean motion of the nodes 9'. 24". Truly it is added to the first equation and subtracted from the second, when the earth travels from it perihelion to its aphelion : and it shall be the contrary on the opposite side of the orbit.

It will be agreed also by the theory of gravity that the action of the sun on the moon shall be a little greater, when the transverse diameter of the moon's orbit passes through the sun, than when the same is at a right angle to the line joining the earth and the sun : and therefore the orbit of the moon is a little greater in the first case than in the second. And hence another equation of the mean motion of the moon arises, depending on the position of the moon's apogee relative to the sun, which indeed is a maximum when the apogee of the moon is in the same octant as the sun ; and nothing when that arrives at the quadrature or syzygies : and it is added to the mean motion in the transition of the lunar apogee from the sun's quadrature to syzygies, and taken away in the transition of the apogee from the syzygy to the quadrature. This equation, that I will call half-yearly, when it is a maximum in the octants of the apogee, increases to around 3'. 45", as many times as I can gather from the phenomena. This is the quantity of this in the mean distance of the earth from the sun. Truly it may be increased or diminished in the inverse cubic ration of the distance of the sun, and thus at the maximum distance of the sun it is 3'. 34'" and at the minimum 3'.56" approximately: truly when the apogee of the moon is placed beyond the octants, it emerges smaller ; and is to the maximum equation, as the sine of twice the distance of the moon's apogee from the nearest syzygy or quadrature to the radius.

By the same theory of gravity the action of the sun on the moon it is a little greater when the right line drawn through the nodes of the moon pass through the sun, than when that line is at right angles with the right line joining the earth and the sun. And thence arises another equation of the mean motion of the moon, that I will call the second half-yearly, and which is a maximum when the nodes are turning in the octants of the sun, and which vanishes when they are in syzygy or in quadrature, and in other positions of the nodes proportional to the sine of twice the distance of each node from the nearest syzygy or quadrature: truly is added to the mean motion of the moon, if the sun is moving away ahead of its nearest node, subtracted if it is moving away behind its nearest node, and in the octants, when it is a maximum, it rises to 47" in the mean distance of the sun from the earth, as I have deduced from the theory of gravity. At other distances of the sun this maximum equation at the octants of the nodes is inversely as the cube of the distance of the sun from the earth, and thus at the perigee of the sun it rises to around 49" , and to around 45" at the apogee.

And by the same theory of gravity the moon's apogee is progressing at a maximum rate when it is either in conjunction with the sun or in opposition to the same, and recedes when it goes in quadrature with the sun. And the eccentricity shall be a maximum in the former case and a minimum in the latter, by Corol. 7, 8 and 9. of Prop. LXVI. Book I. And these inequalities by the same corollaries are very great, and they generate the principle equation of the apogees, that I have called half-yearly. And the maximum of this equation is around $12^0.18'$, as much as I have been able to gather from observations. Our *Horrox* first put in place that the moon revolved in an ellipse around the earth situated at its lower focus. *Halley* located the centre of the ellipse in an epicycle, the centre of which is revolving uniformly around the earth. And from the motion in the epicycle the inequalities now discussed arise in the progression and recession of the apogees as well as the amount of the eccentricity. It is supposed that the distance between the moon and the earth be divided into 100000, and *T* refers to the earth and *TC* the mean eccentricity of the of the moon from 5505 parts. *TC* may be produced to *B*, so that *CB* shall be the sine of the equation of the maximum half-yearly inequality $12^0.18'$ to the radius *TC*, and the circle *BDA* described with centre *C*, radius *CB* will be that epicycle in which the centre of the orbit of the moon is located and is revolving following the order of the letters *BDA*. The angle *BCD* is taken equal to twice the annual argument, or to twice the true distance of the place of the sun from the lunar apogee corrected once, and *CTD* will be the half-yearly first correction of the lunar apogee and *TD* the eccentricity of it orbit tending towards the apogee, corrected in the second place. Moreover, having the mean motion, the eccentricity, and the apogee of the moon, as well as with the major axis of the orbit of 200000 parts; and from these there is extracted the place in the orbit and its distance from



the earth, and that by well-known methods.

At the perihelion of the earth, on account of the greater force of the sun, the centre of the lunar orbit will move faster around the centre *C* than at the aphelion, and that in the inverse cubic ratio of distance of the earth from the sun. On account of the equation of the centre of the sun included in the annual argument, the centre of the moon's orbit will be moving faster in the epicycle *BDA* in the inverse square ratio of the distance of the earth from the sun. So that also at this time it will be moving faster in the simple inverse ratio of the distance ; from the centre *D* of the orbit there may be drawn the right line *DE* towards the lunar apogee, or parallel to the right line *TC* , and the angle *EDF* may be taken equal to the excess of the aforementioned angular argument over the distance of the lunar apogee from the following perigee of the sun; or what is the same, the angle *CDF* may be taken equal to the complement of the true anomaly of the sun to 360^0. And *DF* shall be in the ratio to *DC* as twice the eccentricity of the great orbit to the mean distance of the sun from the earth, and the mean diurnal motion of the sun from the lunar apogee to the mean diurnal motion of the sun from the appropriate apogee jointly, that is, as $33\frac{7}{8}$ to 1000 and 52'. 27''. 16''' to 59'. 8''. 10''' jointly, or as 3 to 100. And consider the centre of the moon to be located at the point *F*, and revolving on an epicycle whose centre is *D*, and the radius *DF*, and meanwhile the point *D* is progressing on the circumference of the circle *DABD*. For by this account, the velocity by which the centre of the moon will be moving along some curve described about the centre *C*, will be inversely as the cube of the distance of the sun from the earth approximately, as required.

The computation of this motion is difficult, but it may be rendered easier by the following approximation. If the mean distance of the moon from the earth shall be of 100000 parts, and the eccentricity *TC* shall be of 550 parts as above, the right line *CB* or

CD will be found of $1172\frac{3}{4}$ parts, and the right line *DF* of $35\frac{1}{5}$ parts. And this right line *DF* at the distance *TC,* subtends an angle at the earth, which the translation from the centre of the orbit from some place *D* to some place *F,* produced in the motion of its centre : and twice the same right line 2*DF* in a parallel position to the line which joins the earth and the upper focus of the lunar orbit, will subtend the same angle at the earth, that certainly that translation generates in the motion of the focus, and at the distance of the moon from the earth, it will subtend an angle that the same translation will generate in the motion of the moon, and which therefore can be said to be the second equation of the centre. And this equation, at the mean distance of the earth from the sun, is as the sine of the angle, which that right line *DF* contains approximately with the right line drawn from the point *F* to the moon, and when it is a maximum, 2'. 25'' emerges. But the angle that the right line *DF* makes with the right line drawn from the point *F* to the moon taken together, is found either by subtracting the angle *EDF* from the mean anomaly of the moon, or by adding the distance of the moon from the sun to the distance of the lunar apogee from the apogee of the sun. And as the radius is to the sine of the angle thus found, thus 2'. 25'', is required to be added to the second equation of the centre, if that sum shall be less than a semicircle, and to be subtracted if greater. Thus its longitude will be found in the syzygies of the luminous bodies themselves.

Since the atmosphere of the earth refracts the light of the sun as far as a height of 35 or 40 miles, and by refracting may scatter the same into the shadow of the earth, and on being scattered the light in the confines of the shadow dilates the shadow : to the diameter of the shadow, which is produced by parallax, I add 1 minute or $1\frac{1}{3}$ minutes in eclipses of the moon.

Truly lunar theory must be examined through the phenomena and firmly established first in syzygies, then in the quadratures, and finally in the octants. And with this need in mind, I have observed rather precisely the average movements of the moon and of the sun at the time of the meridian, in the royal observatory in Greenwich, and for the last day in December of the year 1700 in the old style , it will not be an inconvenience to use the following : truly the mean motion of the sun ♑ $20^0.43'.40'$, and of its apogee ♋ $7^0.44'.30''$, and the mean motion of the moon $15^0.21'.00''$, and of its apogee ♓ $8^0\ 20'.00''$, and of the ascending node ♌ $27^0.24'.20''$; and the difference of the meridians of this observatory and of the royal observatory in Paris to be $0^h.9'\ 20''$. but the mean motion of the moon, and of the apogees of this have not yet been found accurately enough.

PROPOSITIO XXII. THEOREMA XVIII.
Motus omnes lunares, omnesque motuum inaequalitates ex allatis principiis consequi.

Planetas majores, interea dum circa solem feruntur, posse alios minores circum se revolventes planetas deferre, & minores illos in ellipsibus, umbilicos in centris majorum habentibus, revolvi debere patet per Prop. LXV. Lib. I. Actione autem solis perturbabuntur eorum motus multimode, iisque adficientur inaequalitatibus quae in luna nostra notantur. Haec utique (per corol. 2, 3, 4, & 5, Prop. LXVI) velocius movetur, ac radio ad terram ducto describit aream pro tempore majorem, orbemque habet minus curvum, atque ideo propius accedit ad terram, in syzygiis quam in quadraturis, nisi quatenus impedit motus eccentricitatis. Eccentricitas enim maxima est (per corol. 9. Prop. LXVI.) ubi apogaeum lunae in syzygiis versatur, & minima ubi idem in quadraturis consistit; & inde luna in perigaeo velocior est & nobis propior, in apogaeo autem tardior & remotior in syzygiis quam in quadraturis. Progreditur insuper apogaeum, & regrediuntur nodi, sed motu inaequabili. Et apogaeum quidem (per corol. 7. & 8. Prop. LXVI.) velocius progreditur in syzygiis suis, tardius regreditur in quadraturis, & excessu progressus supra regressum annuatim fertur in consequentia. Nodi antem (per corol. 2. Prop. LXVI.) quiescunt in syzygiis suis & velocissime regrediuntur in quadraturis. Sed & major est lunae latitudo maxima in ipsius quadraturis (per corol.10. Prop. LXVI.) quam in syzygiis: & motus medius tardior in perihelio terrae (per corol. 6. Prop. LXVI.) quam in ipsius aphelio. Atque hae sunt inaequalitates insigniores ab astronomis notatae. Sunt etiam aliae quaediam a prioribus astronomis non observatae inaequalitates, quibus motus lunares adeo perturbantur, ut nulla hactenus lege ad regulam aliquam certam reduci potuerint. Velocitates enim seu motus horarii apogaei & nodorum lunae, & eorundem aequationes, ut & differentia inter eccentricitatem maximam in syzygiis & minimam in quadraturis, & inaequalitas quae variatio dicitur, augentur ac diminuuntur annuatim (per Corol. 14. Prop. LXVI.) in triplicata ratione diametri apparentis solaris. Et variatio praeterea augetur vel diminuitur in duplicata ratione temporis inter quadraturas quam proxime (per Corol. 1, & 2. lem. X. & Corol. 16. Prop. LXVI. Lib. I.) sed haec inaequalitas in calculo astronomico ad prosthaphaeresin lunae referri solet, & cum ea confundi.

PROPOSITIO XXIII. PROBLEMA V.
Motus inaequales satellitum jovis & *saturni a motibus lunaribus derivare.*

Ex motibus lunae nostrae motus analogi lunarum seu satellitum jovis sic derivantur. Motus medius nodorum satellitis extimi jovialis, est ad motum medium nodorum lunae nostrae, in ratione composita ex ratione duplicata temporis periodici terrae circa solem ad tempus periodicum jovis circa solem, & ratione simplici temporis periodici satellitis circa jovem ad tempus periodicum lunae circa terram (per Corol. 16. Prop. LXVI. Lib. I.) ideoque annis centum conficit nodus iste $8^{gr}. 24'$ in antecedentia. Motus medii nodorum satellitum interiorum sunt ad motum huius, ut illorum tempora periodica ad tempus periodicum huius (per idem corollarium) & inde dantur. Motus autem augis satellitis cuiusque in consequentia est ad motum nodorum ipsius in anticedentia, ut motus apogaei lunae nostrae ad huius motum nodorum, (per idem corol.) & inde datur. Diminui tamen debet motus augis sic inventus in ratione 5 ad 9 vel 1 ad 2 circiter, ob causam quam hic exponere non vacat. Aequationes maximae nodorum & augis satellitis cuiusque fere sunt ad aequationes maximas nodorum & augis lunae respective, ut motus nodorum & augis satellitum tempore unius revolutionis aequationum priorum, ad motus nodorum & apogaei lunae tempore unius revolutionis aequationum posteriorum. Variatio satellitis e jove spectati, est ad variationem lunae, ut sunt ad invicem toti motus nodorum temporibus quibus satelles & luna ad solem revolvuntur, per idem corollarium; ideoque in satellite extimo non superat 5".12'".

PROPOSITIO XXIV. THEOREMA XIX.

Fluxum & refluxum maris ab actionibus solis ac lunae oriri.

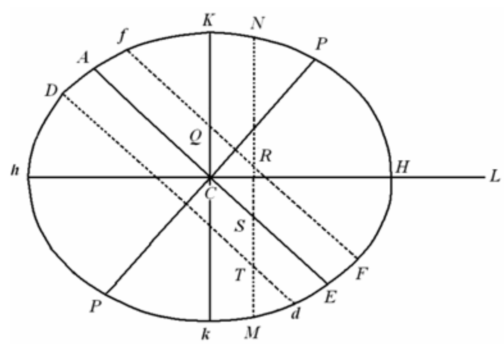
Mare singulis diebus tam lunaribus quam solaribus bis intumescere debere ac his defluere, patet per Corol. 19. & 20. Prop. LXVI. Lib. I. ut & aquae maximam altitudinem, in maribus profundis & liberis, appulsum lluminarium ad meridianum loci minori quam sex horarum spatio sequi, uti fit in maris *Atlantici* & *Aethiopici* tractu toto orientali inter *Galliam* & promontorium *Bonae Spei* ut & in maris *Pacifici* littore *Chilensi* & *Peruviano:* in quibus omnibus littoribus rectus in horam circiter secundam, tertiam vel quartam, incidit, nisi ubi motus ab oceano profundo per loca vadosa propagatus usque ad horam quintam sextam septimam aut ultra retardatur. Horas numero ab appulsu luminaris utriusque ad meridianum loci, tam infra horizontem quam supra, & per horas dici lunaris intelligo vigesimas quartas partes temporis quo luna motu apparente diurno ad meridianum loci revertitur. Vis solis vel lunae ad mare elevandum maxima est in ipso appulsu luminaris ad meridianum loci. Sed vis eo tempore in mare impressa manet aliquamdiu & per vim novam subinde impressam augetur, donec mare ad altitudinem maximam ascenderit, id quod fiet spatio horae unius duarumve sed saepius ad littora spatio horarum trium circiter, vel etiam plurium si mare sit vadosum.

Motus autem bini, quos luminaria duo excitant, non cernentur distincte, sed motum quendam mixtum efficient. In lluminarium coniunctione vel oppositione conjungentur eorum effectus, & componetur fluxus & refluxus maximus. In quadraturis sol attollet aquam ubi luna deprimit, deprimetque ubi luna attollit ; & ex effectuum differentia rectus omnium minimus orietur. Et quoniam, experientia teste, major est effectus lunae quam solis, incidet aquae maxima altitudo in horam tertiam lunarem circiter. Extra syzygias & quadraturas, rectus maximus qui sola vi lunari incidere semper deberet in horam tertiam lunarem, & sola solari in tertiam solarem, compositis viribus incidet in tempus aliquod intermedium quod tertiae lunari propinquius est ; ideoque in transitu lunae a syzygiis ad quadraturas, ubi hora tertia solaris praecedit tertiam lunarem, maxima aquae altitudo praecedet etiam tertiam lunarem, idque maximo intervallo paulo post octantes lunae; & paribus intervaliis rectus maximus sequetur horam tertiam lunarem in transitu lunae a quadraturis ad syzygias. Haec ita sunt in mari aperto. Nam in ostiis fluviorum fluxus majores caeteris paribus tardius ad *ἀκμήν* venient.

Pendens autem effectus lluminarium ex eorum distantiis a terra. In minoribus enim distantiis majores sunt eorum effectus, in majoribus minores, idque in triplicata ratione diametrorum apparentium. Igitur sol tempore hyberno, in perigaeo existens, majores edit effectus, efficitque ut rectus in syzygiis paulo majores sint, & in quadraturis paulo minores (caeteris paribus) quam tempore aestivo; & luna in perigaeo singulis mensibus majores ciet rectus quam ante vel post dies quindecim, ubi in apogaeo versatur. Unde fit ut rectus duo omnino maximi in syzygiis continuis se mutuo non sequantur.

Pendet etiam effectus utriusque luminaris ex ipsius declinatione seu distantia ab aequatore. Nam si luminare in polo constitueretur, tracteret illud singulas aquae partes constanter, sine actionis intensione& remissione, ideoque nullam motus reciprocationem cieret. Igitur luminaria recedendo ab aequatore polum versus, effectus suos gradatim amittent, & propterea minores ciebunt rectus in syzygiis solstitialibus quam in aequinoctialibus. In quadraturis autem solstitialibus majores ciebunt rectus quam in quadraturis aequinoctialibus; eo quod lunae jam in aequatore constituae effectus maxime superat effectum solis. Incidunt igitur rectus maximi in syzygias & minimi in quadraturas Lluminarium, circa tempora aequinocti utriusque. Et aestum maximum in syzygiis comitatur semper minimus in quadraturis, ut experientia compertum est. Per minorem autem distantiam solis a terra, tempore hyberno quam tempore aestivo, fit ut rectus maximi & minimi saepius praecedant aequinoctium vernum quam sequantur, & saepius sequantur autumnale quam praecedant.

Pendens etiam effectus lluminarium ex locorum latitudine. Designet *ApEP* tellurem aquis profundis undique coopertam; *C* centrum eius; *P, p* polos; *AE* aequatorem; *F* locum quemvis extra aequatorem; *Ff* parallelum loci; *Dd* parallelum ei respondentem ex altera parte aequatoris; *L* locum quem luna tribus ante horis occupabat; *H* locum telluris ei perpendiculariter subjectum; *h* locum huic oppositum; *K*, *k* loca inde gradibus 90 distantia, *CH, Ch* maris altitudines maximas mensuratas a centro telluris; & *CK,* C*k* altitudines minimas : & si axibus *Hh, Kk* describatur ellipsis, deinde ellipseos huius revolutione circa axem majorem *Hh* describatur sphaerois *HPKhpk;* designabit haec figuram maris quam proxime, & erunt *CF, Cf, CD, Cd* altitudines maris in locis *F, f, D, d*. Quinetiam si in praefata ellipseos revolutione punctum quodvis *N* describat circulum *NM,* secantem parallelos *Ff, Dd* in locis quibusvis *R, T,* & aequatorem *AE* in *S;* erit *CN* altitudo maris in locis omnibus *R, S, T,* sitis in hoc circulo. Hinc in revolutione diurna loci cuiusvis *F,* affluxus erit maximus in *F,* hora tertia poss appulsum lunae ad meridianum supra horizontem; postea defluxus maximus in *Q* hora tertia post occasum lunae; dein affluxus maximus in *f* hora tertia post appulsum lunae ad meridianum infra horizontem; ultimo defluxus maximus in *Q* hora tertia post ortum lunae; & affluxus posterior in *f* erit minor quam affluxus prior in *F.* Distinguitur enim mare totum in duos omnino fluctus hemisphaericos, unum in hemisphaerio *KHk* ad boream vergentem, alterum in hemisphaerio opposito *Khk;* quos igitur fluctum borealem & fluctum australem nominare licet. Hi fluctus semper sibi mutua oppositi veniunt per vices ad meridianos locorum singulorum, interposito intervallo horarum lunarium duodecim. Cumque regiones boreales magis participant fluctum borealem, & australes magis australem, inde oriuntur rectus alternis vicibus majores &



minores, in locis singulis extra aequatorem, in quibus luminaria oriuntur & occidunt. Aestus autem major, luna in verticem loci declinante, incidet in horam circiter tertiam post appulsum lunae ad meridianum supra horizontem, & luna declinationem mutante vertetur in minorem. Et fluxuum differentia maxima incidet in tempora solstitiorum; praesertim si lunae nodus ascendens versatur in principia arietis. Sic experientia compertum est, quod rectus matutini tempore hyberno superent vespertinos & verspertini tempore aestivo matutinos, ad *Plymuthum* quidem altitudine quasi pedis unius, ad *Bristoliam* vera altitudine quindecim digitorum: observantibus *Colepressio* & *Sturmio.*

Motus autem hactenus descripti mutantur aliquantulum per vim illam reciprocationis aquarum, qua maris rectus, etiam cessantibus luminarium actionibus, posset aliquamdiu perseverare. Conservatio haecce motus impressi minuit differentiam aestuum alternorum; & rectus proxime post syzygias majores reddit, eosque proxime post quadraturas minuit. Unde sit ut rectus alterni ad *Plymuthum* & *Bristoliam* non multo magis differant ab invicem quam altitudine pedis unius vel digitorum quindecim; utque rectus omnium maximi in iisdem portubus, non sint primi a syzygiis, sed tertii. Retardantur etiam motus omnes in transitu per vada, adeo ut rectus omnium maximi, in fretis quibusdam & fluviorum ostiis, sint quasii vel etiam quinti a syzygiis.

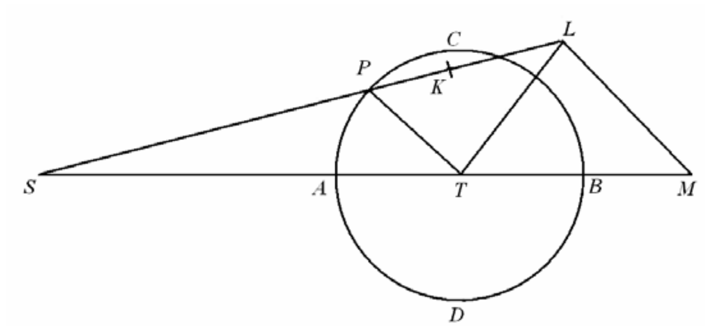
Porro fieri potest ut rectus propagetur ab oceano per freta diversa ad eundem portum, & citius transeat per aliqua freta quam per alia: quo in casu rectus idem, in duos vel plures successive advenientes divisus, componere possit motus novos diversorum generum. Fingamus rectus duos aequales a diversis locis in eundem portum venire, quorum prior praecedat alterum spatio horarum sex, incidatque in horam tertiam ab appulsu lunae ad meridianum portus. Si luna in hocce suo ad meridianum appulsu versabatur in aequatore, venient singulis horis senis aequales affluxus, qui in mutuos refluxus incidendo eosdem affluxibus aequabunt, & sic spatio dici illius efficient ut aqua tranquille stagnet. Si luna tunc declinabat ab aequatore, fient rectus in oceano vicibus alternis majores & minores, uti dictum est ; & inde propagabuntur in hunc portum affluxus bini majores & bini minores, vicibus alternis. Affluxus autem bini majores component aquam altissimam in medio inter utrumque, affluxus major & minor faciet ut aqua ascendat ad mediocrem altitudinem in medio ipsorum, & inter affluxus binos minores aqua ascendet ad altitudinem minimam. Sic spatio viginti quatuor horarum, aqua non bis ut fieri solet, sed semel tantum perveniet ad maximam altitudinem & semel ad minimam ; & altitudo maxima, si luna declinat in polum supra horizontem loci, incidet in horam vel sextam vel tricesimam ab appulsu lunae ad meridianum, atque luna declinationem mutante mutabitur in defluxum. Quorum omnium exemplum in portu regni *Tunquini* ad *Batsham,* sub latitudine boreali 20 gr. *50'. Halleius* ex nautarum observationibus patefecit. Ibi aqua die transitum lunae per aequatorem sequente stagnat, dein luna ad boream declinante incipit fluere & refluere, non bis, ut in aliis portubus, sed semel singulis diebus; & rectus incidit in occasum lunae, defluxus maximus in ortum. Cum lunae declinatione augetur hic rectus, usque ad diem septimum vel octavum, dein per alios septem dies iisdem gradibus decrescit, quibus antea creverat; & luna declinationem mutante cessat, ac mox mutatur in defluxum. Incidit enim subinde defluxus in occasum lunae & affluxus in ortum, donec luna iterum mutet declinationem. Aditus ad hunc portum fretaque vicina duplex patet, alter ab oceano *Sinesi* inter continentem & insulam *Luconiam,* alter a mari *Indico* inter continentem & insulam *Borneo.* An rectus spatio horarum duodecim a mari *Indico,* &

spatio horarum sex a mari *Sinensi* per freta illa venientes, & sic in horam tertiam & nonam lunarem incidentes, componant huiusmodi motus; sitne alia marium illorum conditio, observationibus vicinorum littorum determinandum relinquo.

Hactenus causas motuum lunae & marium reddidi. De quantitate motuum jam convenit aliqua subjungere.

PROPOSITIO XXV. PROBLEMA VI.
Invenire vires solis ad perturbandos motus lunae.

Designet *S* solem, *T* terram, *P* lunam, *CADB* orbem lunae.
In *SP* capiatur *SK* aequalis *ST;* sitque *SL* ad *SK* in duplicata ratione *SK* ad *SP,* & ipsi *PT*

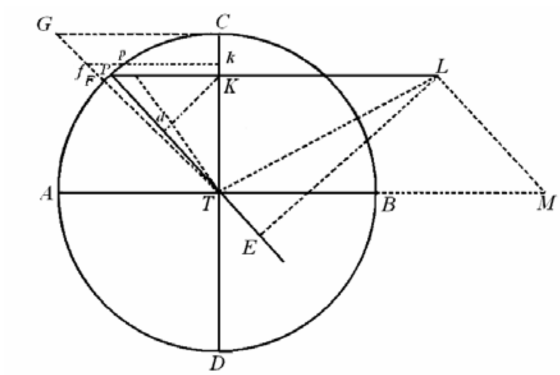


agatur parallela *LM;* & si gravitas acceleratrix terrae in solem exponatur per distantiam *ST* vel *SK,* erit *SL* gravitas acceleratrix lunae in solem. Ea componitur ex partibus *SM, LM,* quarum *LM* & ipsius *SM* pars *TM* perturbat motum lunae; ut in Libri primi Prop. LXVI. & eius corollariis expositum est. Quatenus terra & luna circum commune gravitatis centrum revolvuntur, perturbabitur etiam motus terrae circa centrum illud a viribus consimilibus; sed summas tam virium quam motuum referre licet ad lunam, & summas virium per lineas ipsis analogas *TM* & *ML* designare. Vis *ML* in mediocri sua quantitate est ad vim centripetam, qua luna in orbe suo circa terram quiescentem ad distantiam *PT* revolvi posset, in duplicata ratione temporum periodicorum lunae circa terram & terrae circa solem (per Corol. 17. Prop. LXVI. Lib. I.) hoc est, in duplicata ratione dierum 27. hor.7. min. 43. ad dies 365. hor. 6. min. 9. id est, ut 1000 ad 178725, seu 1 ad $178\frac{29}{40}$. Invenimus autem in propositione quarta quod, si terra & luna circa commune gravitatis centrum revolvantur, earum distantia mediocris ab invicem erit $60\frac{1}{2}$ semidiametrorum mediocrium terrae quamproxime. Et vis qua luna in orbe circa terram quiescentem, ad distantiam *PT* semidiametrorum terrestrium $60\frac{1}{2}$ revolvi posset, est ad vim, qua eodem tempore ad distantiam semidiametrorum 60 revolvi posset, ut $60\frac{1}{2}$ ad 60; & haec vis ad vim gravitatis apud nos ut 1 ad 60×60 quamproxime. Ideoque vis mediocris *ML* est ad vim gravitatis in superficie terra; ut $1 \times 60\frac{1}{2}$ ad $60 \times 60 \times 60 \times 178\frac{29}{40}$, seu 1 ad 638092,6. Inde vero & ex proportione linearum *TM, ML,* datur etiam vis *TM:* & hae sunt vires solis quibus lunae motus perturbantur. *Q.E.I.*

PROPOSITIO XXVI. PROBLEMA VII.

Invenire incrementum horarium areae quam luna, radio ad terram ducto, in orbe circulari describit.

Diximus aream, quam luna radio ad terram ducto describit, esse tempori proportionalem, nisi quatenus motus lunaris ab actione solis turbatur. lnaequalitatem momenti, vel incrementi horarii hic investigandam proponimus. Ut computatio facilior reddatur, fingamus orbem lunae circularem esse, & inaequalitates omnes negligamus, ea sola excepta, de qua hic agitur. Ob ingentem vero solis distantiam, ponamus etiam lineas



SP, ST sibi invicem parallelas esse. Hoc pacto vis *LM* reducetur semper ad mediocrem suam quantitatem *TP,* ut & vis *TM* ad mediocrem suam quantitatem *3PK.* Hae vires (per legum Corol. 2.) componunt vim *TL;* & haec vis, si in radium *TP* demittatur perpendiculum *LE,* resolvitur in vires *TE, EL,* quarum *TE,* agenda semper secundum radium *TP,* nec accelerat nec retardat descriptionem areae *TPC* radio illo *TP* factam ; *& EL* agendo secundum perpendiculum, accelerat vel retardat ipsam, quantum accelerat vel retardat lunam. Acceleratio illa lunae, in transitu ipsius a quadratura *C* ad conjunctionem *A*, singulis temporis momentis facta, est ut ipsa vis accelerans *EL,* hoc est, ut $\frac{3PK \times TK}{TP}$. Exponatur tempus per motum medium lunarem, vel (quod eodem fere recidit) per angulum *CTP,* vel etiam per arcum *CP.* Ad *CT* erigatur normalis *CG* ipsi *CT* aequalis. Et diviso arcu quadrantali *AC* in particulas innumeras aequales *Pp,* &c. per quas aequales totidem particulae temporis exponi possint, ductaque *pk* perpendiculari ad *CT,* jungatur *TG* ipsis *KP, kP* productis occurrens in *F* & *f;* & erit *FK* aequalis *TK,* & *Kk* erit ad *PK* ut *Pp* ad *Tp,* hoc est in data ratione, ideoque $FK \times Kk$ seu area *FKkf,* erit ut $\frac{3PK \times TK}{TP}$, id est, ut *EL;* & composite, area tota *GCKF* ut summa omnium virium *EL* tempore toto *CP* impressarum in lunam, atque ideo etiam ut velocitas hac summa genita, id est, ut acceleratio descriptionis areae *CTP,* seu incrementum momenti. Vis qua luna circa terram quiescentem ad distantiam *TP,* tempore suo periodico *CADB* dierum 27. hor.7. min. 43. revolvi posset, efficeret ut corpus, tempore *CT* cadendo, describeret longitudinem $\frac{1}{2}CT$, & velocitatem simul acquireret aequalem velocitati, qua luna in orbe suo movetur. Patet hoc per Corol. 9. Prop. IV. Lib. I. Cum autem perpendiculum *Kd* in *TP* demissum sit ipsius *EL* pars tertia, & ipsius *TP* seu *ML* in octantibus pars dimidia, vis *EL* in octantibus, ubi maxima est, superabit vim *ML* in ratione 3 ad 2, ideoque erit ad vim illam, qua luna tempore suo periodico circa terram quiescentem revolvi posset, ut 100 ad $\frac{2}{3}\times 17872\frac{1}{2}$ seu 11915, & tempore *CT* velocitatem generare deberet quae esset pars $\frac{100}{22915}$ velocitatis lunaris, tempore autem *CPA* velocitatem majorem generaret in ratione *CA* ad *CT* seu *TP.* Exponatur vis maxima *EL* in octantibus per aream $FK\times Kk$ rectangulo $\frac{1}{2}TP\times Pp$ aequalem. Et velocitas, quam vis maxima tempore quovis *CP* generare posset, erit ad velocitatem quam vis omnis minor *EL* eodem tempore generat, ut rectangulum $\frac{1}{2}TP\times CP$ ad aream *KCGF:* tempore autem toto *CPA,* velocitates genitae erunt ad invicem ut rectangulum $\frac{1}{2}TP\times CA$ & triangulum *TCG,* sive ut arcus quadrantalis *CA* & radius *TP.* Ideoque (per Prop. IX. Lib. v. Elem.) velocitas posterior, toto tempore genita, erit pars $\frac{100}{22915}$, velocitatis lunae. Huic lunae velocitati, quae areae momento mediocri analoga est, addatur & auferatur dimidium velocitatis alterius; & si momentum mediocre exponatur per numerum 11915, summa 11915+50 seu 11965 cxhibebit momentum maximum areae in syzygia *A*, ac differentia 11915 - 50 seu 11865 eiusdem momentum minimum in quadraturis. Igitur areae temporibus aequalibus in syzygiis & quadraturis descriptae, sunt ad invicem ut 11965 ad 11865. Ad momentum minimum 11865 addatur momentum, quod sit ad momentorum differentiam 100 ut trapezium *FKCG* ad triangulum *TCG* (vel quod perinde est, ut quadratum sinus *PK* ad quadratum radii *TP,* id est, ut *Pd* ad *TP*) & summa exhibebit momentum areae, ubi luna est in loco quovis intermedio *P.* Haec omnia ita se habent, ex hypothesi quod sol & terra quiescunt, & luna tempore synodico dierum 27. hor. 7. min. 43. revolvitur. Cum autem periodus synodica lunaris vere sit dierum 29. hor.12. & min. 44, augeri debent momentorum incrementa in ratione temporis, id est, in ratione 1080853 ad 1000000. Hoc pacto incrementum totum, quod erat pars $\frac{100}{11915}$ momenti mediocris, jam fiet eiusdem pars $\frac{100}{11023}$. Ideoque momentum areae in quadratura lunae erit ad eius momentum in syzygia ut 11023-50 ad 11023+50, seu 10973 ad 11073; & ad eius momentum, ubi luna in alio quovis loco intermedio *P* versatur, ut 10973 ad 10973+*Pd,* existente videlicet *TP* aequali 100.

Area igitur, quam luna radio ad terram ducto singulis temporis particulis aequalibus describit, est quam proxime ut summa numeri 219,46 & sinus versi duplicatae distantiae lunae a quadratura proxima, in circulo cuius radius est unitas. Haec ita se habent ubi variatio in octantibus est magnitudinis mediocris. Sin variatio ibi major sit vel minor, augeri debet vel minui sinus ille versus in eadem ratione.

PROPOSITIO XXVII. PROBLEMA VIII.

Ex motu horario lunae invenire ipsius distantiam a terra.

Area, quam luna radio ad terram ducto singulis temporis momentis describit, est ut motus horarius lunae & quadratum distantiae lunae a terra conjunctim ; & propterea distantia lunae a terra est in ratione composita ex subduplicata ratione areae directe & subduplicata ratione motus horarii inverse. *Q.E.D.*

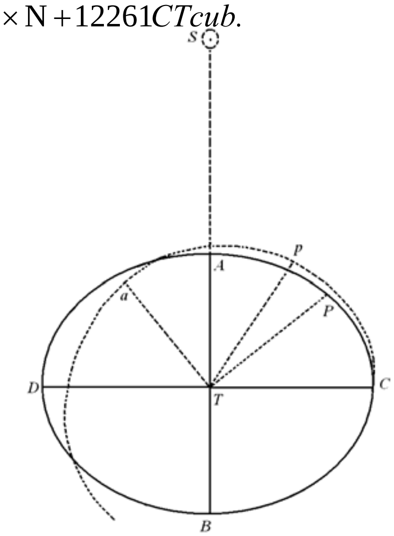
Corol. I. Hinc datur lunae diameter apparens: quippe quae sit reciproce ut ipsius distantia a terra. Tentent astronomi quam probe haec regula cum phaenomenis congruat.

Corol. 2. Hinc etiam orbis lunaris accuratius ex phaenomenis quam antehac definiri potest.

PROPOSITIO XXVIII. PROBLEMA IX.

Invenire diametros orbis in quo luna, sine eccentricitate, moveri deberet.

Curvatura trajectoriae, quam mobile, si secundum trajectoriae illius perpendiculum tractatur, describit, est ut attractio directe & quadratum velocitatis inverse. Curvaturas linearum pono esse inter se in ultima proportione sinuum vel tangentium angulorum contactuum ad radios aequales pertinentium, ubi radii illi in infinitum diminuuntur. Attractio autem lunae in terram in syzygiis est excessus gravitatis ipsius in terram supra vim solarem *2PK* (vide *fig. pag. 428.*) qua gravitas acceleratrix lunae in solem superat gravitatem acceleratricem terrae in solem vel ab ea superatur. In quadraturis autem attractio illa est summa gravitatis lunae in terram & vis solaris *KT,* qua luna in terram trahitur. Et hae attractiones, si $\frac{AT+CT}{2}$ dicatur *N*, sunt ut $\frac{178725}{ATq}-\frac{2000}{CT\times N}$ & $\frac{178725}{CTq}+\frac{1000}{AT\times N}$ quam proxime; seu ut $178725N\times CTq-2000ATq\times CT$ & $178725N\times ATq+1000CTq\times AT$.
Nam si gravitas acceleratrix lunae in terram exponatur per numerum 178725, vis mediocris *ML,* quae in quadraturis est *PT* vel *TK* & lunam tractit in terram, erit 1000, & vis mediocris *TM* in syzygiis erit 3000; de qua, si vis mediocris *ML* subducatur, manebit vis 2000 qua luna in syzygiis distrahitur a terra, quamque jam ante nominavi 2*PK.* Velocitas autem lunae in syzygiis *A* & *B* est ad ipsius velocitatem in quadraturis *C* & *D,* ut *CT* ad *AT* & momentum areae quam luna radio ad terram ducto describit in syzygiis ad momentum eiusdem areae in quadraturis conjunctim, i.e. ut 11073 *CT* ad 10973 *AT.* Sumatur haec ratio bis inverse & ratio prior semel directe, & fiet curvatura orbis lunaris in syzygiis ad ejusdem curvaturam in quadraturis ut
$120406729\times178725ATq\times CTq\times N-120406729\times2000ATqq\times CT$ ad
$122611329\times1178725ATq\times CTq\times N+122611329\times1000CTqq\times AT$, i. e. ut
$2151969AT\times CT\times N-24081ATcub.$ ad $2191371AT\times CT\times \mathrm{N}+12261CTcub.$



Quoniam figura orbis lunaris ignoratur, huius vice assumamus elipsin *DBCA,* in cuius centro *T* terra collocetur, & cuius axis major *DC* quadraturis, minor *AB* syzygiis interjaceat. Cum autem planum ellipseos huius motu angulari circa terram revolvatur, & trajectoria cuius curvaturam consideramus describi debet in plano quod omni motu angulari omnino destituitur: consideranda erit figura, quam luna in ellipsi illa revolvendo describit in hoc plano, hoc est figura *Cpa,* cuius puncta singula *p* inveniuntur capiendo punctum quodvis *P* in ellipsi, quod locum lunae repraesentet, & ducendo *Tp* aequalem*TP*, ea lege ut angulus *PTp* aequalis sit motui apparenti solis a tempore quadraturae *C* confecto ; vel (quod eodem fere recidit) ut angulus *CTp* sit ad angulum *CTP* ut tempus revolutionis synodicae lunaris ad

tempus revolutionis periodicae seu 29^d, 12^h. 43', ad 27^d. 7^h. 43' . Capiatur igitur angulus *CTA* in eadem ratione ad angulum rectum *CTA,* & sit longitudo *Ta* aequalis longitudini *TA;* & erit *a* apsis ima & *C* apsis summa orbis huius *Cpa.* Rationes autem ineundo invenio quod differentia inter curvaturam orbis *Cpa* in vertice *a*, & curvaturam circuli centro *T* intervallo
TA descripti, sit ad differentiam inter curvaturam ellipseos in vertice *A* & curvaturam eiusdem circuli, in duplicata ratione anguli *CTP* ad angulum *CTp;* & quod curvatura ellipseos in *A* sit ad curvaturam circuli illius, in duplicata ratione *TA* ad *TC;* & curvatura circuli illius ad curvaturam circuli centro *T* intervallo *TC* descripti, ut *TC* ad *TA* ; huius autem curvatura ad curvaturam ellipseos in *C,* in duplicata ratione *TA* ad TC; & differentia inter curvaturam ellipseos in vertice C & curvaturam circuli novissimi, ad differentiam inter curvaturam figurae *Tpa* in vertice *C* & curvaturam eiusdem circuli, in duplicata ratione anguli *CTp* ad angulum *CTP.* Quae quidem rationes ex sinubus angulorum contactus ac differentiarum angulorum facile colliguntur. His autem inter se collatis, prodit curvarura figurae *Cpa* in *a* ad ipsius curvaturam in *C,* ut $ATcub.+\frac{16824}{200000}CTq\times AT$ ad CT$cub.+\frac{16824}{200000}ATq\times CT$. Ubi numerus $\frac{16824}{200000}$ designat differentiam quadratorum angulorum *CTP & CTp* applicatam ad quadratum anguli minoris *CTP,* seu (quod perinde est) differentiam quadratorum temporum 27^d. 7^h. 43', & 29^d. 12^h. 44' 1.9, applicatam ad quadratum temporis 27^d. 7^h. 43'.

Igitur cum *a* designet syzygiam lunae, & *C* ipsius quadraturam, proportio jam inventa eadem esse debet cum proportione curvaturae orbis lunae in syzygiis ad eiusdem curvaturam in quadraturis, quam supra invenimus. Proinde ut inveniatur proportio *CT* ad *AT,* duco extrema & media in se invicem. Et termini prodeuntes ad $AT\times CT$ applicati, fiunt

$$2062,79CTqq-215I969N\times CTcub.+368676N\times AT\times CTq$$

$$+36342ATq\times CTq-362047N\times ATq\times CT+2191371N\times ATcub.+4051,4ATqq=0.$$

Hic pro terminorum *AT* & *CT* semisumma *N* scribo 1, & pro eorundem semidifferentia ponendo *x,* sit $CT=1+x,\ \&\ AT=1-x:$ quibus in aequatione scriptis, & aequatione prodeunte resoluta, obtinetur *x* aequalis 0,00719, & inde semidiameter *CT* sit 1,00719, & semidiameter *AT* 0,99281, qui numeri sunt ut $70\frac{1}{24}\ \ \&\ \ 69\frac{1}{24}$ quam proxime. Est igitur distantia lunae a terra in syzygiis ad ipsius distantiam in quadraturis (seposita scilicet eccentricitatis consideratione) ut 69 ad 70*,* vel numeris rotundis ut 69 ad 70.

PROPOSITIO XXIX. PROBLEMA X.

Invenire variationem lunae.

Oritur haec inaequalitas partim ex forma elliptica orbis lunaris, partim ex inaequalitate momentorum arem, quam luna radio ad terram ducto describit. Si luna *P* in ellipsi *DBCA* circa terram in centro ellipseos quiescentem moveretur, & radio *TP* ad terram ducto describeret aream *CTP* tempori proportionalem; esset autem ellipseos semidiameter maxima *CT* ad semidiametrum minimam *TA* ut 70 ad 69: foret tangens anguli *CTP* ad tangentem anguli motus medii a quadratura *C* computati, ut ellipseos semidiameter *TA* ad eiusdem semidiametrum *TC* seu 69 ad 70. Debet autem descriptio areae *CTP,* in progressu lunae a quadratura ad syzygiam, ea ratione accelerari, ut eius momentum in syzygia lunae sit ad eius momentum in quadratura ut 11073 ad 10973, utque excessus momenti in loco quovis imermedio *P* supra momentum in quadratura sit ut quadratum sinus anguli *CTP.* Id quod satis accurate fiet, si tangens anguli *CTP* diminuatur in subduplicata ratione numeri 10973 ad numerum 11073, id est, in ratione numeri 68,6877 ad numerum 69. Quo pacto tangens anguli C*TP* jam erit ad tangentem motus medii ut 68,6877 ad 70, & angulus *CTP* in octantibus, ubi motus medius est 45^{gr}. Invenietur 44^{gr}. 27'. 28''. qui subductus de angulo motus medii 45^{gr} relinquit variationem maximam 32' .32''. Haec ita se haberent si luna, pergendo a quadratura ad syzygiam, describeret angulum *CTA* graduum tantum nonaginta. Verum ob motum terrae, quo sol in consequentia motu, apparente transfertur, luna, priusquam solem assequitur, describit angulum *CTA* angula recto majorem in ratione temporis revolutionis lunaris synodicae ad tempus revolutionis periodicae, id est, in ratione 29^{d}. 12^{h}. 44' ad 27^{d}. 7^{h}. 43'. Et hoc pacto anguli omnes circa centrum *T* dilatantur in eadem ratione, & variatio maxima quae secus esset 32'. 32'', jam aucta in eadem ratione sit 35'. 10''.

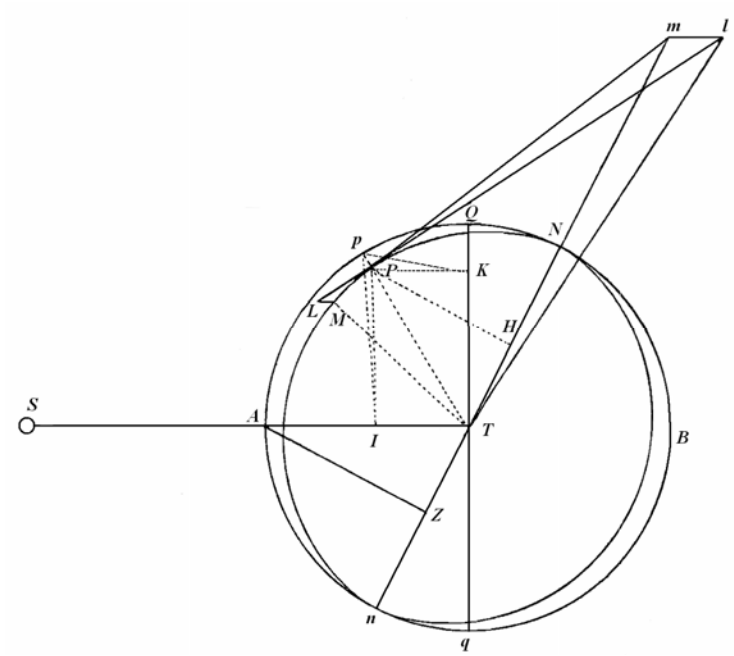
Haec est eius magnitudo in mediocri distantia solis a terra, neglectis differentiis quae a curvatura orbis magni majorique solis actione in lunam falcatam & novam quam in gibbosam & plenam, oriri possint. In aliis distantiis solis a terra, variatio maxima est in ratione quae componitur ex duplicata ratione temperis revolutionis synodicae lunaris (dato anni tempore) directe, & triplicata ratione distantiae solis a terra inverse. Ideoque in apogaeo solis, variatio maxima est 33'. 14'', & in eius perigaeo 37'.11'', si modo eccentricitas solis sit ad orbis magni semidiametrum transversam ut $16\frac{15}{16}$ ad 1000.

Hactenus variationem investigavimus in orbe non eccentrico, in quo utique luna in octanibus suis semper est in mediocri sua distantia a terra. Si luna propter eccentricitatem suam, magis vel minus distat a terra quam si locaretur in hoc orbe, variatio paulo major esse potest vel paulo minor quam pro regula hic allata: sed excessum vel defectum ab astronomis per phaenomena determinandum relinquo.

PROPOSITIO XXX. PROBLEMA XI.

Invenire motum horarium nodorum lunae in orbe circulari.

Designet *S* solem, *T* terram, *P* lunam, *NPn* orbem lunae, *NPn* vestigium orbis in plano eclipticae; *N, n* nodos, *nTNm* lineam nodorum infinite productam; *PI, PK* perpendicula demissa in lineas *ST, Qq; Pp* perpendiculum demissa in planum eclipticae; *A, B* syzygias lunae in plano eclipticae; *AZ* perpendiculum in lineam nodorum *Nn; Q q* quadraturas lunae in plano eclipticae, & *pK* perpendiculum in lineam *Qq* quadraturis interjacentem. Vis solis ad perturbandum motum lunae (per. Prop. xxv.) duplex est, altera

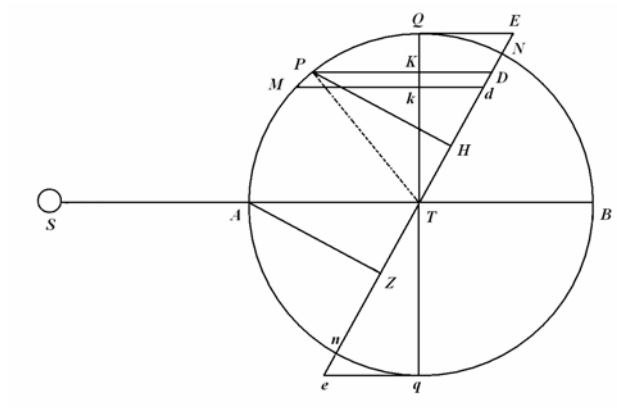


lineae *LM* in schemate propositionis illius, altera lineae *MT* proportionalis.
Et luna vi priore in terram, posteriore in solem secundum lineam rectae *ST* a terra ad solem ducta parallelam tractitur. Vis prior *LM* agit secundum planum orbis lunaris, & propterea situm plani nil mutat. Haec igitur negligenda est. Vis posterior *MT* qua planum orbis lunaris perturbatur eadem est cum vi 3*PK* vel 3IT Et haec vis (per Prop. XXV.) est ad vim qua luna in circulo circa terra aequiescentem tempore suo periodico uniformiter revolvi posset, ut *31T*ad radium circuli multiplicatum per numerum 178,725, sive ut *IT* ad radium multiplicatum per 59,575. Caeterum in hoc calculo, & eo omni qui sequitur, considero lineas omnes a luna ad solem ductas tanquam parallelas lineae quae a terra ad solem ducitur, propterea quod inclinatio tantum fere minuit effectus omnes in aliquibus casibus, quantum auget in aliis; & nodorum motus mediocres quaerimus, neglectis istiusmodi minutiis, quae calculum nimis impeditum redderent.

Designet jam *PM* arcum, quem luna dato tempore quam minimo describit, & *ML* lineaolam cuius dimidium luna, impellente vi praefata *3IT,* eodem tempore describere posset. Jungantur *PL, MP,* & producantur eae ad *m* & *l,* ubi secent planum eclipticae; inque *Tm* demittatur perpendiculum *PH.* Et quoniam recta *ML* parallela est plano eclipticae, ideoque cum recta *ml* quae in plano illa iacet concurrere non potest, & tamen iacent hae rectae in plano communi *LMPml* ; parallelae· erunt hae rectae & propterea similia erunt triangula *LMP, lmP.* Iam cum *MPm* sit in plano orbis, in quo luna in loco *P* movebatur, incidet punctum *m* in lineam *Nn* per orbis illius nodos *N, n* ductam. Et quoniam vis qua dimidium lineaolae *LM* generatur, si, tota simul & semel in loco *P* impressa esset, generaret lineam illam totam; & efficeret ut luna moveretur in arcu, cuius chorda esset *LP,* atque ideo transferret lunam de plano *MPmT* in planum *LPIT;* motus angularis nodorum a vi illa genitus, aequalis erit angulo *mTl*. Est autem *ml* ad *mP* ut *ML* ad *MP,* ideoque cum *MP* ob datum tempus data sit, est *mt* ut rectangulum $ML \times mP$, id est, ut rectangulum $IT \times mP$. Et angulus *mTl,* si modo angulus *Tml* rectus sit, est ut $\frac{ml}{Tm}$, & propterea ut $\frac{IT \times Pm}{Tm}$, id est (ob proportionales *Tm & mP, TP* & *PH*) ut $\frac{IT \times PH}{TP}$, ideoque ob datam *TP,* ut $IT \times PH$. Quod si angulus *Tml,* seu *STN* obliquus sit, erit angulus *mTl* adhuc minor, in ratione sinus anguli *STN* ad radium, seu *AZ* ad *AT.* Est igitur velocitas nodorum ut $IT \times PH \times AZ$, sive ut contentum sub sinubus trium angulorum *TP1,PTN & STN.*

Si anguli illi, nodis in quadraturis & luna in syzygia existentibus, recti sint, lineaola *ml* abibit in infinitum, & angulus *mTl* evadet angulo *mPl* aequalis. Hoc autem in casu, angulus *mPl* est ad angulum *PTM,* quem luna eodem tempore motu suo apparente circa terram describit, ut 1 ad 59,575. Nam angulus *mPl* aequalis est angulo *LPM,* id est, angulo deflexionis lunae a recto tramite, quem sola vis praefata solaris 3*IT,* fi tum cessaret lunae gravitas, dato illa tempore generare posset; & angulus *PTM* aequalis est angulo deflexionis lunae a recto tramite, quem vis illa, qua luna in orbe suo retinetur, si tum cessaret vis solaris 3*1T,* eodem tempore generaret. Et hae vires, ut supra diximus, sunt ad invicem ut 1 ad 59,515. Ergo cum motus medius horarius lunae rcspectu fixarum sit $32'.56''.27'''.12^{\text{iv}}\frac{1}{2}$, motus horarius nodi in hoc casu erit $33''.10'''.33^{\text{iv}}.12^{\text{v}}$. Aliis autem in casibus motus iste horarius erit ad $33''.10'''.33^{\text{iv}}.12^{\text{v}}$ ut contentum sub sinubus angulorum trium *TPI, PTN,* & *STN* (seu distantiarum lunae a quadratura, lunae a nodo, & nodi a sole) ad cubum radii. Et quoties signum anguli alicuius de affirmativo in negativum, deque negativo in affirmativum mutatur, debebit motus regressivus in progressivum & progressivus in regressivum mutari. Unde sit ut nodi progrediantur quoties luna inter quadraturam alterutram & nodum quadraturae proximum versatur. Aliis in casibus regrediuntur, & per excessum regressus supra progressum singulis mensibus feruntur in antecedentia.

Corol. I. Hinc si a dati arcus quam minimi *PM* terminis *P* & *M* ad lineam quadraturas jungentem *Qq* demittantur perpendicula *PK,M k,* eademque producantur donec secent lineam nodorum *Nn* in *D* & *d;* erit motus horarius nodorum ut area *MPDd* & quadratum linea *AZ* conjunctim. Sunto enim *PK, PH* & *AZ* praedicti tres



sinus. Nempe *PK* sinus distantiae lunae a quadratura, *PH* sinus distantiae lunae a nodo, & *AZ* sinus distantiae nodi a sole: & erit velocitas nodi ut contentum $PK \times PH \times AZ$. Est autem *PT* ad *PK* ut *PM* ad *Kk,* ideoque ob datas *PT& PM* est *Kk* ipsi PK proportionalis. Est & *AT* ad *PD* ut *AZ* ad *PH,* & propterea *PH* rectangulo $PD \times AZ$ proportionalis, & conjunctis rationibus, $PK \times PH$ est ut contentum $Kk \times PD \times AZ$, & $PK \times PH \times AZ$ ut $Kk \times PD \times AZqu$. id est, ut area *PDdM* & *AZ qu.* conjunctim.

Q. E.D.

Corol. 2. In data quavis nodorum positione, motus horarius mediocris est semissis motus horarii in syzygiis lunae, ideoque est ad $16''. 35'''. 16^{iv}. 36^{v}$. ut quadratum sinus distantiae nodorum a syzygiis ad quadratum radii, sive ut *AZ qu.* ad *AT qu.* Nam si luna uniformi cum motu perambulet semicirculum *QAq,* summa omnium arearum *PDdM,* quo tempore luna pergit a *Q* ad *M,* erit area *QMdE* quae ad circuli tangentem *QE* terminatur; & quo tempore luna attingit punctum *n,* summa illa erit area tota *EQAn* quam linea *PD* describit, dein luna pergente ab *n* ad *q,* linea *PD* cadet extra circulum, & aream *nqe* ad circuli tangentem *qe* terminatam describet; quae, quoniam nodi prius regrediebantur, jam vero progrediuntur subduci debet de area priore, & cum aequalis sit areae *QEN*, relinquet semicirculum *NQAn.* Igitur summa omnium arearum *PDdM,* quo tempore luna semicirculum describit, est area semicirculi; & summa omnium quo tempore luna circulum describit est area circuli totius. At area *PDdM,* ubi luna versatur in syzygiis, est rectangulum sub arcu *PM* & radio *PT;* & summa omnium huic aequalium arearum, quo tempore luna circulum describit, est rectangulum sub circumferentia tota & radio circuli; & hoc rectangulum, cum sit aequale duobus circulis, duplo majus est quam rectangulum prius. Proinde nodi, ea cum velocitate uniformiter continuata quam habent in syzygiis lunaribus, spatium duplo majus describerent quam revera describunt; & propterea motus mediocris quocum, si uniformiter continuaretur, spatium a se inaequabili cum motu revera confectum describere possent, est semissis motus quem habent in syzygiis lunae. Unde cum motus horarius maximus, si nodi in quadraturis versantur, sit $33''. 10'''. 33^{iv}. 12^{v}$, motus mediocris horarius in hoc casu erit $16''.35'''.16^{iv}. 36^{v}$. Et cum motus horarius nodorum semper sit ut *AZ qu,* & area *PDdM* conjunctim, & propterea motus horarius nodorum in syzygiis lunae ut *AZqu.*

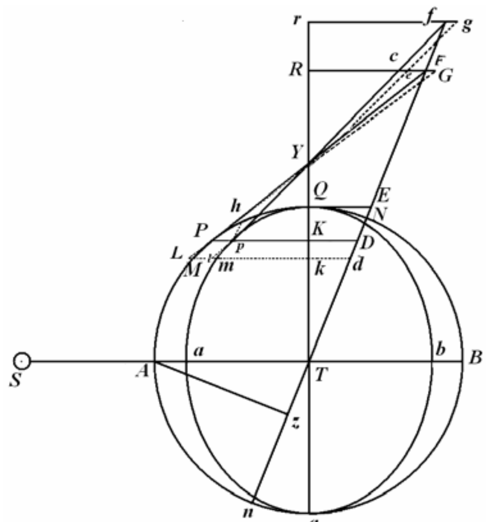
& area *PDdM* conjunctim, id est (ob datam aream *PDdM* in syzygiis descriptam) ut *AZ qu.* erit etiam motus mediocris ut *AZqu.* atque ideo hic motus, ubi nodi extra quadraturas versantur, erit ad $16''. 35'''.16^{iv}. 36^{v}$. Et cum motus horarius nodorum semper sit ut *AZ qu,* & area *PDdM* eonjunctim, & propterea motus horarius nodorum in syzygiis lunae ut *AZqu.* & area *PDdM* conjunctim, id est (ob datam aream *PDdM* in syzygiis descriptam) ut *AZ qu.* erit etiam motus mediocris ut *AZqu,* atque ideo hic motus, ubi nodi extra quadraturas versantur, erit ad $16''. 35'''.16^{iv}. 36^{v}$ ut *AZ qu.* ad *ATqu.*
Q. E. D.

PROPOSITIO XXXI. PROBLEMA XII.

Invenire motum horarium nodorum lunae in orbe elliptico.

Designet *Qpmaq* ellipsin, axe majore *Qq*, minore *ab* descriptam, *QAqB* circulum circumscriptum, *T* terram in utriusque centro communi, *S* solem, *p* lunam in ellipsi motam, & *pm* arcum quem data temporis particula quam minima describit, *N* & *n* nodos

linea *Nn* junctos, *pK* & *mk* perpendicula in axem *Qq* demissa & hinc inde producta, donec occurrant circulo in *P* & *M,* & lineae nodorum in *D* & *d.* Et si luna, radio ad terram ducto, aream describat tempori proportionalem, erit motus horarius nodi in ellipsi ut area *pDdm* & *AZq* conjunctim.



Nam si *PF* tangat circulum in *P,* & producta occurrat *TN* in *F,*& *pf* tangat elliptin in *p* & producta occurrat eidem *TN* in *f,* conveniant autem hae tangentes in axe *TQ* ad *Y;* & si *ML* designet spatium quod luna in circulo revolvens, interea dum describit arcum *PM,* urgente & impellente vi praedicta 3*IT,* seu 3 *PK* motu transverso describere posset, & *ml* designet spatium quod luna in ellipsi revolvens eodem tempore, urgente etiam vi 3*IT* seu 3*PK,* describere posset; & producantur *LP* & *lp* donec occurrant plano eclipticae in *G* & *g;* & jungantur *FG* & *fg,* quarum *FG* producta secet *pf*, *pg* & *TQ* in *c, e* & *R* respective, & *fg* producta secet *TQ* in *r.* Quoniam vis *31T* seu *3PK* in circulo est ad vim 3*IT* seu *3pK* in ellipsi, ut *PK* ad *pK,* seu *AT* ad *aT;* erit spatium *ML* vi priore genitum ad spatium *ml* vi posteriore genitum, ut *PK* ad *pK,* id est, ob similes figuras *PYKp* & *FYRc,* ut *FR* ad *cR.* Est autem *ML* ad *FG* (ob similia triangula *PLM, PGF)* ut *PL* ad *PG,* hoc est (ob parallelas *Lk, PK, GR)* ut *pl* ad *pe,* id est (ob similia triangula *plm*, *cpe*) ut *lm* ad *ce* ; & inverse ut *LM* est ad *1m,* seu *FR* ad *cR.* ita est *FG* ad *ce.* Er propterea si *fg* esset ad *ce* ur *fY* ad *cY,* id est, ut *fr* ad *cR* (hoc est, ut *fr* ad *FR* & *FR* ad *cR* coniunctim id est, ut *fT* ad *FI* & *FG* ad *ce* coniunctim) quoniam ratio *FG* ad *ce* utrinque ablata relinquit rationes *fg* ad *FG* & *fT* ad *FT,* foret *fg* ad *FG* ut *fT* ad *FT;* atque ideo anguli, quos *FG* & *fg* subtenderent ad terram *T,* aequarentur inter se. Sed anguli illi (per ea quae in praecedente propositione exposuimus) sunt motus nodorum, quo tempore luna in circulo arcum *PM,* in ellipsi arcum *pm* percurrit: & propterea motus nodorum in circulo & ellipsi aequarentur inter se. Haec ita se haberent, si modo *fg* esset ad *ce* ut *fT* ad *cT*, id est, si *fg* aequalis esset $\frac{ce\times fY}{cY}$. Verum ob similia triangula *fgp, cep,* est *fg* ad *ce* ut *fp* ad *cp;* ideoque *fg* aequalis est $\frac{ce\times fp}{cp}$; & propterea angulus, quem *fg* revera subtendit, est ad angulum priorem, quem *FG* subtendit, hoc est, motus nodorum in ellipsi ad motum nodorum in circulo, ut haec *fg* seu $\frac{ce\times fp}{cp}$ ad priorem *fg* seu $\frac{ce\times fY}{cY}$,id est, ut $fp\times cY$ ad $fY\times cp$, seu *fp* ad *fY & cY* ad *cp,* hoc est, si *ph* ipsi *TN* parallela occurrat *FP* in *h,* ut *Fh* ad *FY* & *FY* ad *FP;* hoc est, ut *Fh* ad *FP* seu *Dp* ad *DP,* ideoque ut area *Dpmd* ad aream *DPMd.* Et propterea, cum (per Corol. I. Prop. XXX.) area posterior & *AZq* conjunctim proportionalia sint motui horario nodorum in circulo, erunt area prior & *AZq* conjunctim proportionalia motui horario nodorum in ellipsi. *Q.E.D.*

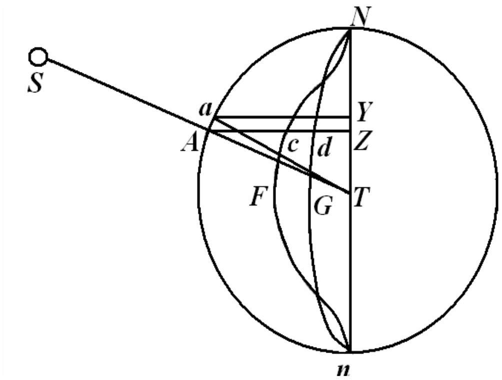
Corol. Quare cum; in data nodorum positione, summa omnium arearum *pDdm,* quo tempore luna pergit a quadratura ad locum quemvis *m,* sit area *mpqQEd,* quae ad ellipseos tangentem *QE* terminatur; & summa omnium arearum illarum, in revolutione integra, sit area ellipseos totius: motus mediocris nodorum in ellipsi erit ad motum mediocrem nodorum in circulo, ut ellipsis ad circulum; id est, ut *Ta* ad *TA,* seu 69 ad 70. Et propterea, cum (per. Corol. 2. Prop. XXX.) motus mediocris horarius nodorum in circulo sit ad $16''.\ 35'''.\ 16^{\text{iv}}.\ 36^{\text{v}}.$ ut *AZ qu.* ad *AT qu.* si capiatur angulus $16''.21'''.3^{\text{iv}}.\ 30^{\text{v}}.$ ad angulum $16''.\ 35'''.\ 16^{\text{iv}}.36^{\text{v}}.$ut 69 ad 70, erit motus mediocris horarius nodorum in ellipsi ad $16''.\ 21'''.\ 3^{\text{iv}}.\ 30^{\text{v}}.$ ut *AZq* ad *ATq;* hoc est, ut quadratum sinus distantiae nodi a sole ad quadratum radii.

Caeterum luna, radio ad terram ductio, aream velocius describit in syzygiis quam in quadraturis, & eo nomine tempus in syzygiis contrahitur, in quadraturis producitur; & una cum tempore motus nodorum augetur ac diminuitur. Erat autem momentum areae in quadraturis lunae ad eius momentum in syzygiis ut 10973 ad 11073, & propterea momentum mediocre in octantibus est ad excessum in syzygiis, defectumque in quadraturis, ut numerorum semisumma 11023 ad eorundem semidifferentiam 50. Unde cum tempus lunae in singulis orbis particulis aequalibus sit reciproce ut ipsius velocitas, erit tempus mediocre in octantibus ad excessum temporis in quadraturis, ac defectum in syzygiis, ab hac causa oriundum, ut 11023 ad 50 quam proxime. Pergendo antem a quadraturis ad syzygias, invenio quod excessus momentorum areae in locis singulis, supra momentum minimum, in quadraturis, sit ut quadratum sinus distantiae lunae a quadraturis quam proxime; & propterea differentia inter momentum in loco quocunque & momentum mediocre in octantibus, est ut differentia inter quadratum sinus distantiae lunae a quadraturis & quadratum sinus graduum 45, seu semissem quadrati radii; & incrementum temporis in locis singulis inter octantes & quadraturas, & decrementum eius inter octantes & syzygias, est in eadem ratione. Motus autem nodorum, quo tempore luna percurrit singulas orbis particulas aequales, acceleratur vel retardatur in duplicata ratione temporis. Est enim motus iste, dum luna percurrit *PM* (caeteris paribus) ut *ML,* & *ML* est in duplicata ratione temporis. Quare motus nodorum in syzygiis, eo tempore confectus quo luna datas orbis particulas percurrit, diminuitur in duplicata ratione numeri 11073 ad numerum 11023 ; estque decrementum ad motum reliquum ut 100 ad 10973, ad motus vera totum ut 100 ad 11073 quam proxime. Decrementum autem in locis inter octantes & syzygias, & incrementum in locis inter octantes & quadraturas, est quam proxime ad hoc decrementum, ut motus totus in locis illis ad motum totum in syzygiis & differentia inter quadratum sinus distantiae lunae a quadratura & semissem quadrati radii ad semissem quadrati radii conjunctim. Unde si nodi in quadraturis versentur, & capiantur loca duo aequaliter ab octante hinc inde distantia, & alia duo a syzygia & quadratura iisdem intervaliis distantia, deque decrementis motuum in locis duobus inter syzygiam & octantem, subducantur incrementa motuum in locis reliquis duobus, quae sunt inter octantem & quadraturam; decrementum reliquum aequale erit decremento in syzygia: uti rationem ineunti facile constabit. Proindeque decrementum mediocre, quod de nodorum motu mediocri subduci debet, est pars quarta decrementi in syzygia. Motus totus horarius nodorum in syzygiis, ubi luna radio ad terram ducto aream tempori proportionalem describere supponebatur, erat $32''.42'''.7^{iv}$. Et decrementum motus nodarum, quo tempore luna jam velocior describit idem spatium, diximus esse ad hunc motum ut 100 ad 11073 ; ideoque decrementum illud est $17'''43^{iv}.\ 11^{v}$, cuius pars quarta $4'''.25^{iv}.\ 48^{v}$. motui horario mediocri superius invento $16''.21'''.3^{iv}.\ 30^{v}$. subducta, relinquit $16''.16'''.37^{iv}.\ 42.^{v}$ motum mediocrem horarium correctum.

Si nodi versantur extra quadraturas, & spectentur loca bina a syzygiis hinc inde aequaliter distantia; summa motuum nodorum, ubi luna versatur in his locis, erit ad summam motuum, ubi luna in iisdem locis & nodi in quadraturis versantur, ut *AZqu.* ad *ATqu.*Et decrementa motuum, a causis jam expositis oriunda, erunt ad invicem ut ipsi motus, ideoque motus reliqui erunt ad invicem ut *AZqu.* ad *ATqu.* & motus mediocres ut motus reliqui. Est itaque motus mediocris horarius correctus, in dato quocunque nodorum situ, ad $16''.16'''.37^{iv}.42^{v}$. ut *AZqu.* ad *ATqu.*; id est, ut quadratum sinus distantiae nodorum a syzygiis ad quadratum radii.

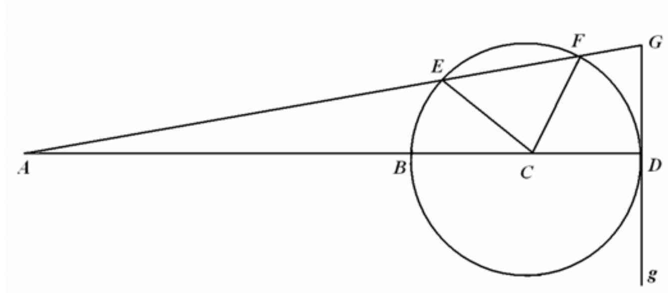
PROPOSITIO XXXII. PROBLEMA XIII.
Invenire motum medium nodorum lunae

Motus medius annuus est summa motuum omnium horariorum mediocrium in anno. Concipe nodum versari in *N,* & singulis horis completis retracti in locum suum priorem, ut non obstante motu suo proprio, datum semper servet situm ad stellas fixas. Interea vero solem *S,* per motum terrae, progredi a nodo, & cursum annuum apparentem uniformiter complere. Sit autem *Aa* arcus datus quam minimus, quem recta *TS* ad solem semper ducta, intersectione sui & circuli *NAn,* dato tempore quam minimo describit: & motus horarius mediocris (per jam ostensa) erit ut *AZq,* id est (ob proportionales *AZ, ZY*) ut



rectangulum sub *AZ* & *ZY* hoc est, ut area *AZYa.* Et summa omnium horariorum motuum mediocrium ab initio, ut summa omnium arearum *aTZA,* id est, ut area *NAZ.* Est autem maxima *AZYa* aequalis rectangulo sub arcu *Aa* & radio circuli ; & propterea summa circuli omnium rectangulorum in circulo toto ad summam totidem maximorum, ut area circuli totius ad rectangulum sub circumferentia tota & radio, id est, ut 1 ad 2. Motus autem horarius, rectangulo maximo respondens, erat $16''.16'''.\,37^{\text{iv}}.42^{\text{v}}.$ Et hic motus, anno toto sidereo dierum 365. hor. 6. min. 9. sit $39^{\text{gr}}.\,38'.\,7''.\,50'''.$ Ideoque huius dimidium $19^{\text{gr}}.49'.\,3''.55'''.$ est motus medius nodorum circulo toti respondens. Et motus nodorum, quo tempore sol pergit ab *N* ad *A,* est ad $19^{\text{gr}}.\,49'.\,3''.\,55'''.$ ut area *NAZ* ad circulum totum.

Haec ita se habent ex hypothesi, quod nodus horis singulis in locum priorem retractitur, sic ut sol anno toto completo ad nodum eundem redeat a quo sub initio digressus fuerat. Verum per motum, nodi fit ut sol citius ad nodum revertatur, & computanda iam est abbreviatio temporis. Cum sol anno toto conficiat 360 gradus & nodus motu maximo eodem tempore conficeret $39^{\text{gr}}.\,38'.\,7''.\,50'''$, seu 39,6355 gradus ; & motus mediocris nodi in loco quovis *N* sit ad ipsius motum mediocrem in quadraturis suis, ut *AZq* ad *ATq:* erit motus solis ad motum nodi in *N,* ut 360 *ATq* ad 39,6355 *AZq* ; id est, ut 9,0827646 *ATq* ad *AZq.* Unde si circuli totius circumferentia *NAn* dividatur in particulas aequales *Aa,* tempus quo sol percurrat particulam *Aa,* si circulus quiesceret, erit ad tempus quo percurrit eandem particulam, si circulus una cum nodis circa centrum *T* revolvatur, reciproce ut 9,0827646 *ATq* ad 9,0877646*ATq* +*AZq.* Nam tempus est reciprocc ut velocitas qua particula percurritur, & haec velocitas est summa velocitatum solis & nodi. Igitur si tempus, quo sol sine motu nodi percurreret arcum *NA,* exponatur per sectorem *NTA,* & particula temporis quo percurreret arcum quam minimum *Aa,* exponatur per sectoris particulam *ATa;* & (perpendiculo *aY* in *Nn* demisso) si in *AZ* capiatur *dZ,* eius longitudinis ut sit rectangulum *dZ* in *ZY* ad sectoris particulam *ATa* ut *AZq* ad 9,0827646*ATq* +*AZq,* id est, ut sit *dZ* ad $\frac{1}{2}AZ$ ut *ATq* ad 9,0827646*ATq* +*AZq,* rectangulum *dZ* in *ZY* designabit decrementum temporis ex motu nodi oriundum, tempore toto quo arcus *Aa* percurritur. Et si punctum *d* tangit curvam *NdGn,* area curvilinea *NdZ* erit decrementum totum, quo tempore arcus totus *NA* percurritur; & propterea excessus sectoris *NAT* supra aream *NdZ* erit tempus illud totum. Et quoniam motus nodi tempore minore minor est in ratione temporis, debebit etiam area *AaTZ* diminui in eadem ratione.



Id quod fiet si capiatur in *AZ* longitudo *eZ,* quae sit ad longitudinem *AZ* ut *AZq* ad $9{,}0827646ATq + AZq$. Sic enim rectangulum *eZ* in *ZY* erit ad aream *AZYa* ut decrementum temporis, quo arcus *Aa* percurritur, ad tempus totum quo percurreretur, si nodus quiesceret: & propterea rectangulum illud respondebit decremento motus nodi. Et si punctum *e* tangat curvam *NeFn,* area tota *NeZ,* quae summa est omnium decrementorum, respondebit decremento toti, quo tempore arcus *AN* percurritur ; & area reliqua *NAe* respondebit motui reliquo, qui verus est nodi motus, quo tempore arcus totus *NA* per solis & nodi conjunctos motus percurritur. Iam vero area semicirculi est ad aream figurae *NeFn,* per methodum serierum infinitarum quaesitam, ut 793 ad 60 quamproxime. Motus autem qui respondet circulo toti erat $19^{gr}.49'.\ 3''.55'''$. & propterea motus, qui figurae *NeFn* duplicatae respondet, est $1^{gr}.\ 29'.58''.2'''$. Qui de motu priore subductus relinquit $18^{gr}.19'.\ 5''.53'''$. motum totum nodi respectu fixarum inter sui ipsius coniunctiones cum sole; & hic motus de solis motu annuo graduum 360 subductus, relinquit $341^{gr}.\ 40'.\ 54'.\ 7'''$. motum solis inter easdem conjunctiones. Iste autem motus est ad motum annuum 360^{gr}. Ut nodi motus jam inventus $18^{gr}.19'.\ 5''.53'''$. ad ipsius motum annuum, qui propterea erit $19^{gr}.18'.\ 1''.23'''$. Hic est motus medius nodorum in anno sidereo. Idem per tabulas astronomicas est $19^{gr}.21'.21''.50'''$. Differentia minor est parte trecentesima motus totius, & ab orbis lunaris eccentricitate & inclinatione ad planum eclipticae oriri videtur. Per eccentricitatem orbis motus nodorum nimis acceleratur, & per eius inclinationem vicissim retardatur aliquantulum, & ad iustam velocitatem reducitur.

PROPOSITIO XXXIII. PROBLEMA XIV.
Invenire motum verum nodorum lunae.

In tempore quod est ut area *NTA-NdZ,* (*in fig. preced.*) motus iste est ut area *NAe,* & inde datur. Verum ob nimiam calculi difficultatem, praestat sequentem problematis constructionem adhibere. Centro *C,* intervallo quovis *CD,* describatur circulus *BEFD.* Producatur *DC* ad *A,* ut sit *AB* ad *AC* ut motus medius ad semissem motus veri mediocris, ubi nodi sunt in quadraturis, id est, ut $19^{gr}.18'.1''.23'''$. ad $19^{gr}.49'.3''.55'''$, atque ideo

BC ad *AC* ut motuum differentia $0^{gr}.31'.2''.32.'''$, ad motum posteriorem $19^{gr}.49'.3''.55'''.$ hoc est, ut 1 ad $38\frac{3}{10}$; dein per punctum *D* ducatur infinita *Gg,* quae tangat circulum in *D;* & si capiatur angulus *BCE* vel *BCF* aequalis duplae distantiae solis a loco nodi, per motum medium invento; & agatur *AE* vel *AF* secans perpendiculum *DG* in *G;* & capiatur angulus qui sit ad motum totum nodi inter ipsius syzygias (id est, ad $9^{gr}.11'.3''.$) ut tangens *DG* ad circuli *BED* circumferentiam totam; atque angulus iste (pro quo angulus *DAG* usurpari potest) ad motum motum nodorum addatur ubi nodi transeunt a quadraturis ad syzygias, & ab eodem motu medio subducatur ubi transeunt a syzygiis ad quadraturas; habebitur eorum motus verus. Nam motus verus sic inventus congruet quam proxime cum motu vero qui prodit exponendo tempus per aream *NTA-NdZ,* & motum nodi per aream *NAe;* ut rem perpendenti & computationes instituenti constabit. Haec est aequatio semestris motus nodorum. Est & aequatio menstrua, sed quae ad inventionem latitudinis lunae minime necessaria est. Nam cum variatio inclinationis orbis lunaris ad planum eclipticae duplici inaequalitati obnoxia sit, alteri semestri, alteri autem menstruae; huius menstrua inaequalitas & aequatio menstrua nodorum ita se mutuo contemperant & corrigunt, ut ambae in determinanda latitudine lunae negligi possint.

Corol. Ex hac & praecedente propositione liquet quod nodi in syzygiis suis quiescunt, in quadraturis autem regrediuntur motu horario $16''.19'''.\ 26^{iv}.$ Et quod aequatio motus nodorum in octantibus sit $1^{gr}.30'$. Quae omnia cum phaenomenis coelestibus probe quadrant.

Scholium.

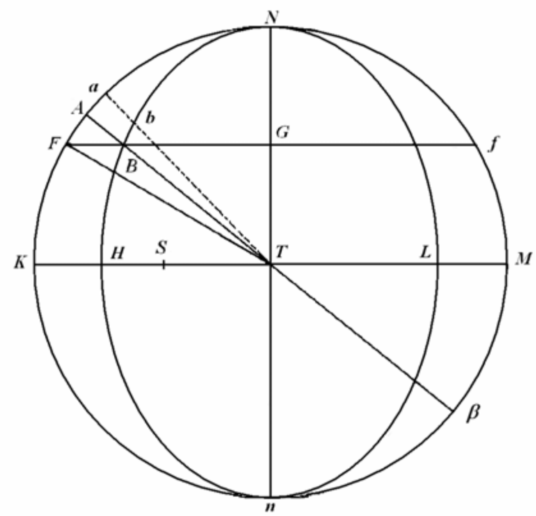
Alia ratione motum nodorum *J. Machin Astrom. Prof. Gresham. & Hen. Pemberton* M. D. seorsum invenerunt. Huius methodi mentio quaedam alibi facta est. Et utriusque chartae, quas vidi, duas propositiones continebant, & inter se in utrisque congruebant. Chartam vera D. *Machin,* cum prior in manus meas venerit, hic adjungam,

DE MOTU NODORUM LUNAE.

PROPOSITIO I.

" *Motus solis medius a nodo, definitur per medium proportionale geometricum, inter*
" *motum ipsius solis medium, & motum illum mediocrem quo sol celerrime recedit a*
"*nodoin quadraturis.*
" Sit *T* locus ubi terra, *Nn* linea nodorum lunae ad tempus quodvis datum, *KTM* huic ad
" rectos angulos ducta, *TA* recta circum centrum revolvens ea cum velocitate angulari qua
" sol & nodus a se invicem recedunt, ita ut angulus inter rectam quiescentem *Nn*
" & revolventem *TA,* semper fiat aequalis distantiae locorum solis & nodi. Jam si recta
" quaevis *TK* dividatur in partes *TS* & *SK* quae sint ut motus solis horarius medius ad
" motum horarium mediocrem nodi in quadraturis, & ponatur recta *TH* media
" proportionalis inter partem *TS* & totam *TK,* haec recta inter reliquas proportionalis erit
" motui medio solis a nodo.

" Describatur enim circulus *NKn M* centro *T* & radio *TK,* eodemque centro & semiaxibus
" *TH* & *TN* describatur ellipsis *NHn,* & in tempore quo sol a nodo recedit per arcum *Na,* si
" ducatur recta *Tba,* area sectoris *NTa* exponet summam motuum nodi & solis in eodem
" tempore. Sit igitur arcus *a A* quam minimus quem recta *Tba* praefata lege revolvens in
" data temporis particula uniformiter describit, & sector quam minimus *TAa* erit ut
" summa velocitatum qua sol & nodus tum temporis seorsim feruntur. Solis autem
" velocitas fere uniformis est, utpote cuius parva inaequalitas vix ullam inducit in medio
" nodorum motu varietatem.
" Altera pars huius summae nempe velocitas nodi in mediocri sua quantitate, augetur in
" recessu a syzygiis in duplicata ratione sinus distantiae eius a sole; per Coroll. Prop. 31.
" Lib. 3tii Princip. & cum maxima est in quadraturis ad solem in *K,* eandem rationem
" obtinet ad solis velocitatem ac ea quam habet *SK* ad *TS* hoc est ut (differentia



" quadratorum ex *TK* & *TH* vel) rectangulum *KHM* ad *TH* quadratum. Sed ellipsis *NBH*
" dividit sectorem *ATa* summae harum duarum velocitatum exponentem, in duas partes
" *ABba* & *BTb* ipsis velocitatibus proportionales.
" Producatur enim *BT* ad circulum in β, & a puncto *B* demittatur ad axem majorem
" perpendicularis *BG,* quae utrinque producta occurrat circulo in punctis *F* & *f,* &
" quoniam spatium *ABba* est ad sectorem *TBb* ut rectangulum $AB\beta$ ad *BT* quadratum
" (rectangulum enim illud aequatur differentiae quadratorum ex *TA*& *TB* ob rectam $A\beta$
" aequaliter & inaequaliter sectam in *T* & *B.)* Haec igitur ratio ubi spatium *ABba*
" maximum est in *K,* eadem erit ac ratio rectanguli *KHM* ad *HT* quadratum,
" sed maxima nodi mediocris velocitas erat ad solis velocitatem in hac ratione.
" Igitur in quadraturis sector *ATa* dividitur in partes velocitatibus
" proportionales. Et quoniam rectang. *KHM* est ad *HT* quadr. ut *FBI* ad *BG* quad.
" & rectangulum $AB\beta$ aequatur rectangulo *FBf.* Erit igitur areola *ABba* ubi maxima
" est ad reliquum sectorem *TBb,* ut rectang. $AB\beta$ ad *BG* quadr. Sed ratio harum
" areolarum semper erat ut $AB\beta$ rectang. ad *BT* quadratum; & propterea areola A*Bba*

" in loco *A* minor est simili areola in quadraturis, in duplicata ratione *BG* ad *BT* hoc est in " duplicata ratione sinus distantiae solis a nodo. Et proinde summa omnium areolarum "*ABba* nempe spatium A*BN* erit ut motus nodi in tempore quo sol digreditur a nodo per " arcum *NA.* Et spatium reliquum nempe sector est ellipticus *NTB* erit ut motus solis " medius in eadem tempore. Et propterea quoniam annuus motus nodi medius, is est qui " sit in tempore quo sol periodum suam absolverit, motus nodi medius a sole erit ad " motum ipsius solis medium, ut area circuli ad aream ellipseos, hoc est ut recta *TK* ad " rectam *TH* mediam scilicet proportionalem inter *TK* & *TS* ; vel quod eadem redit ut " media proportionalis *TH* ad rectam *TS*.

PROPOSITIO II.

" *Data motu medio nodorum lunae invenire motum verum.*

" Sit angulus *A* distantia solis a loco nodi media, sive motus medius solis a nodo. Tum " si capiatur angulus *B* cuius tangens sit ad tangentem anguli *A* ut *TH* ad *TK,* hoc est " in subduplicata ratione motus mediocris horarii solis ad motum mediocrem horarium " solis a nodo in quadraturis versante; erit idem angulus *B* distantia solis a loco nodi vero. " Nam jungatur *FT* & ex demonstratione ratione propositionis superioris erit angulus *FTN* " distantia solis a loco nodi medio, angulus autem *ATN* distantia a loco vero, & tangentes "horum angulorum sunt inter se ut *TK* ad *TH.*

" *Coroll.* Hinc angulus *FTA* est aequatio nodorum lunae, sinusque huius anguli ubi "maximus est in octantibus, est ad radium ut *KH* ad $TK + TH$. Sinus autem huius " aequationis in loco quovis alio *A* est ad sinum maximum, ut sinus summae angulorum " $FTN + ATN$ ad radium: hoc est fere ut sinus duplae distantiae solis a loco nodi medio (nempe 2. *ETN)* ad radium.

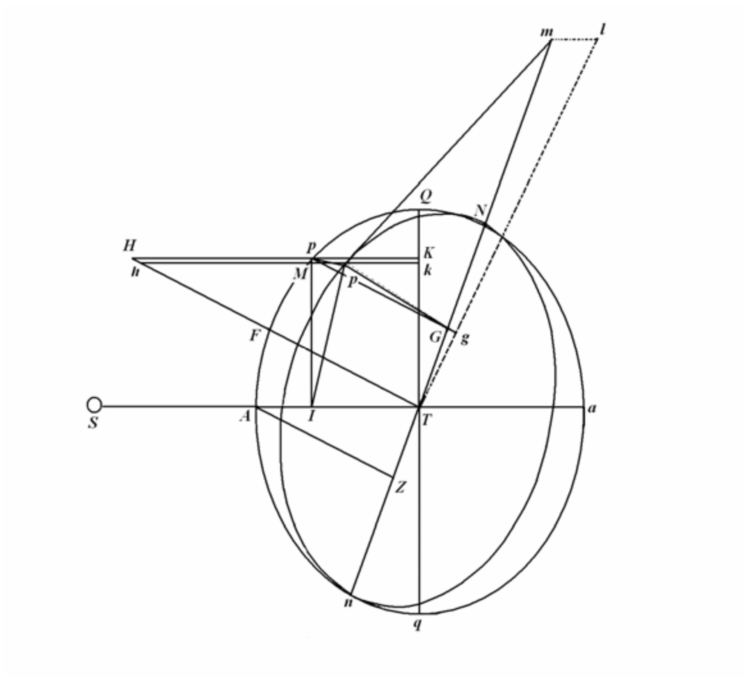
Scholium.

" Si motus nodorum mediocris horarius in quadraturis sit $16''.16'''.3^{iv}.42^{v}$. hoc est in anno " toto sidereo $39°.38'.7''.50'''$. erit *TH* ad *TK* in subduplicata ratione numeri 9,0827646 " ad numerum 10,0827646, hoc est ut 18,6524761 ad 19,6524761. Et propterea *TH* ad " *HK* ut 18,6514761 ad 1. hoc est ut motus solis in 1 anno sidereo ad motum nodi medium "$19^0.18'.1''.23'''\frac{2}{3}$.

" At si motus medius nodorum lunae in 20 annis Julianis sit $386^0.50'.15''$. sicut ex " observationibus in theoria lunae adhibitis deducitur: motus medius nodorum in anna " sidereo erit $19^0.20'.31''.58'''$. Et *TH* erit ad *HK* sit 360^{gr} ad $19^0.20'.31''.58'''$. " hoc est ut 18,61214 ad 1. unde motus mediocris horarius nodorum in quadraturis " evadet $16''.18'''.48^{iv}$. Et aequatio nodorum maxima in octantibus $1^0.29'.57''$.

PROPOSITIO XXXIV. PROBLEMA XV.
Invenire variationem horariam inclinationis orbis lunaris ad planum eclipticae.

Designent *A* & *a* syzygias; *Q* & *q* quadraturas; *N* & *n* nodos; *P* locum lunae in orbe suo ; *p* vestigium loci illius in plano eclipticae, & *mTl* motum momentaneum nodorum ut supra. Et si ad lineam *Tm* demittatur perpendiculum *PG*, jungatur pG, & producatur ea donec occurrat *Tl* in g, & jungatur etiam *PG*: erit angulus *PGp* inclinatio orbis, lunaris ad planum eclipticae, ubi luna versatur in *P*; & angulus *Pgp* inclinatio eiusdem post momentum temporis completum; ideoque angulus *GPg* variatio momentanea inclinationis. Est autem hic angulus *GPg* ad angulum *GTg* ut *TG* ad *PG* & *Pp* ad *PG* conjunctim. Et propterea si pro momento temporis substituatur hora; cum angulus *GTg* (per Prop. xxx.) sit ad angulum $33''.10'''.33^{iv}$.



ut $IT\times PG\times AZ$ ad $ATcub.$ erit angulus *GPg* (seu inclinationis horaria variatio) ad angulum $33''.10'''.33^{iv}$. ut $IT\times AZ\times TG\times\frac{Pp}{PG}$ ad *ATcub. Q. E.I.*

Haec ita se habent ex hypothesi quod luna in orbe circulari uniformitcr gyratur. Quod si orbis ille ellipticus sit, motus mediocris nodorum minuetur in ratione axis minoris ad axem majorem; uti supra expositum est. Et in eadem ratione minuetur etiam inclinationis variatio.

Corol. I. Si ad *Nn* erigatur perpendiculum *TF*, sitque *pM* motus horarius lunae in plano ellipticae; & perpendicula *pK, Mk* in *QT* demissa & utrinque producta occurrant *TF* in *H* & *h*: erit *IT* ad *AT* ut *Kk* ad *Mp*, & *TG* ad *Hp* ut *TZ* ad *AT*, ideoque $IT\times TG$

aequale $\frac{Kk\times Hp\times TZ}{Mp}$, hoc est, aequale areae *HpMh* ductae in rationem $\frac{TZ}{Mp}$: & propterea inclinationis variatio horaria $33''.10'''.33^{iv}$. ut *HpMh* ducta in $AZ\times\frac{TZ}{Mp}\times\frac{Pp}{PG}$ ad $ATcub.$

Corol. 2. Ideoque si terra & nodi singulis horis completis retraherentur a locis suis novis, & in loca priora in instanti semper reducerentur, ut situs eorum, per mensem integrum periodicum, datus maneret; tota inclinationis variatio tempore mensis illius foret ad $33''.10'''.33^{iv}$. ut aggregatum omnium arearum *HpMh*, in revolutione puncti *p* genitarum, & sub signis propriis + & – conjunctarum, ductum in $AZ\times TZ\times\frac{Pp}{PG}$ ad $Mp\times ATcub.$ id est, ut circulus totus *QAqa* ducta in $AZ\times TZ\times\frac{Pp}{PG}$ ad $Mp\times ATcub.$ hoc est, ut circumferentia *QAqa* ducta in $AZ\times TZ\times\frac{Pp}{PG}$ ad $2Mp\times ATquad.$

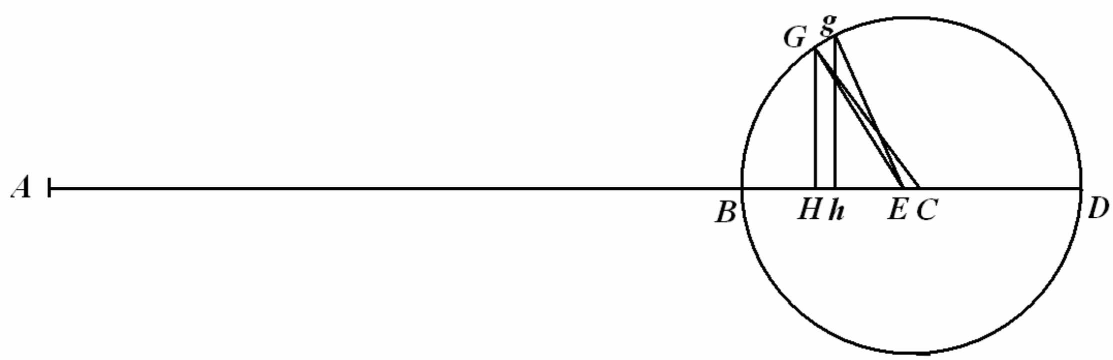
Corol. 3. Proinde in dato nodorum situ, variatio mediocris horaria, ex qua per mensem uniformiter continuata variatio illa menstrua generari posset, est ad $33''.10'''.33^{iv}$. ut $AZ\times TZ\times\frac{Pp}{PG}$ ad $2ATq.$, sive ut $Pp\times\frac{AZ\times TZ}{\frac{1}{2}AT}$ ad $PG\times 4AT$, id est (cum *Pp* sit ad *PG* ut sinus inclinationis praedictae ad radium, & $\frac{AZ\times TZ}{\frac{1}{2}AT}$ sit ad $4AT$ ut finus duplicati anguli *ATn* ad radium quadruplicatum) ut inclinationis eiusdem sinus ductus in sinum duplicatae distantiae nodorum a sole, ad quadruplum quadratum radii.

Corol. 4- Quonium inclinationis horaria variatio, ubi nodi in quadraturis versantur, est (per hanc propositionem) ad angulum $33''.10'''.33^{iv}$. ut $IT\times AZ\times TG\times\frac{Pp}{PG}$ ad $ATcub.$ id est, ut $\frac{IT\times TG}{\frac{1}{2}AT}\times\frac{Pp}{PG}$ ad $2AT$; hoc est, ut sinus duplicatae distantiae lunae a quadraturis ductus in $\frac{Pp}{PG}$ ad radium duplicatum: summa omnium variationum horariarum, quo tempore luna in hoc situ nodorum transit a quadratura ad syzygiam (id est, spatio horarum $177\frac{1}{6}$,) erit ad summam totidem angulorum $33''.10'''.33^{iv}$., seu 5878", ut summa omnium sinuum duplicatae distantiae lunae a quadraturis ducta in $\frac{Pp}{PG}$ ad summam totidem diametrorum; hoc est, ut diameter ducta in $\frac{Pp}{PG}$ ad circumferentiam; id est, si inclinatio sit $5^{gr}.1'$, ut $7\times\frac{874}{10000}$ ad 22, seu 278 ad 10000. Proindeque variatio tota, ex summa omnium horariarum variationum tempore praedicto conflata, est 163", seu 2'. 43".

PROPOSITIO XXXV. PROBLEMA XVI.

Dato tempore invenire inclinationem oritis lunaris ad planum eclipticae.

Sit *AD* sinus inclinationis maximae, & AB sinus inclinationis minimae. Bisecetur *BD* in *C*, & centro *C*, intervallo *BC* describatur circulus *BGD*. In *AC* capiatur *CE* in ea ratione ad *EB* quam *EB* habet ad 2*BA*: et si dato tempore constituatur angulus *AEG*



aequalis duplicatae distantiae nodorum a quadraturis, & ad *AD* demittatur perpendiculum GH: erit *AH* sinus inclinationis quaesitae.

Nam *GEq* aequale est

$$GHq + HEq = BHD + HEq = HBD +$$

$$HEq - BHq = HBD \ + BEq \ - 2BH \times BE$$

$$= BEq + 2EC \times BH = 2EC \times AB + 2EC \times BH = 2EC \times AH.$$

Ideoque cum 2*EC* detur, est *GEq* ut *AH*. Designet jam *AEg* duplicatam distantiam nodorum a quadraturis post datum aliquod momentum temporis completum, & arcus *Gg* ob datum angulum *GEg* erit ut distantia *GE*. Est autem *Hh* ad *Gg* ut *GH* ad *GC* & propterea *Hh* est ut contentum $GH \times Gg$, seu $GH \times GE$; id est, ut $\frac{GH}{GE} \times GEq$ seu $\frac{GH}{GE} \times AH$, id est, ut *AH* & sinus anguli *AEG* conjunctim. Igitur si *AH* in casu aliquo sit sinus inclinationis, augebitur ea iisdem incrementis cum sinu inclinationis, per Corol. 3. Propositionis superioris, & propterea sinui illi aequalis semper manebit. Sed *AH*, ubi punctum *G* incidit in punctum alterutrum *B* vel *D*, huic sinui aequalis est, & propterea eidem semper aequalis manet. *Q. E. D.*

In hac demonstratione supposui angulum *BEG*, qui est duplicata distantia nodorum a quadraturis, uniformiter augeri. Nam omnes inaequalitatum minutias expendere non vacat. Concipe jam angulum *BEG* rectum esse, & in hoc casu *Gg* esse augmentum horarium duplae distantiae nodorum & solis ab invicem; & inclinationis variatio horaria in eodem casu (per Corol. 3. Prop. novissimae) erit ad $33''.10'''.33^{iv}.$ ut contentum sub inclinationis sinu *AH* & sinu anguli recti *BEG*, qui est duplicata distantia nodorum a sole, ad quadruplum quadratum radii; id est, ut mediocris inclinationis sinus *AH* ad radium quadruplicatum; hoc est (cum inclinatio illa mediocris sit quasi $5^{gr}.\ 8'\frac{1}{2}$ ut eius sinus 896 ad radium quadruplicatum 40000, sive ut 224 ad 10000. Est autem variatio tota, sinuum differentiae *BD* respondens, ad variationem illam horariam ut diameter *BD* ad arcum *Gg*; id est, ut diameter BD ad semicircumferentiam *BGD* & tempus horarum $2079\frac{7}{10}$, quo nodus pergit a quadraturis ad syzygias, ad horam unam conjunctim; hoc est, ut 7 ad 11 & $2079\frac{7}{10}$ ad 1. Quare si rationes omnes conjungantur, fiet variatio tota BD ad $33''.10'''.33^{iv}.$ ut $224\times7\times2079\frac{7}{10}$. ad 110000, id est, ut 29645 ad 1000, & inde variatio illa BD prodibit $16',\ 23''\frac{1}{2}$.

Haec est inclinationis variatio maxima quatenus locus lunae in orbe suo non consideratur. Nam inclinatio, si nodi in syzygiis versantur, nil mutatur ex vario situ lunae. At si nodi in quadraturis consistunt, inclinatio minor est ubi luna versatur in syzygiis, quam ubi ea versatur in quadraturis, excessu 2'. 43"; uti in propositionis superiaris Corollario quarto indicavimus. Et huius excessus dimidio $1'.21''\frac{1}{2}$ variatio tota mediocris *BD* in quadraturis lunaribus diminuta fit 15'. 2", in ipsius autem syzygiis aucta fit 17'. 45". Si luna igitur in syzygiis constituatur, variatio tota in transitu nodorum a quadraturis ad syzygias erit 17'. 45": ideoque si inclinatio, ubi nodi in syzygiis versantur, sit $5^{gr}.17'.\ 20''$; eadem, ubi nodi sunt in quadraturis, & luna in syzygiis, erit $4^{gr}.\ 59'.\ 35''$. Atque haec ita se habere confirmatur ex observationibus.

Si jam desideretur orbis inclinatio illa, ubi luna in syzygiis & nodi ubivis versantur; fiat *AB* ad *AD* ut sinus graduum 4. 59'. 35" ad sinum graduum 5. 17'. 20", & capiatur angulus *AEG* aequalis duplicatae distantiae nodorum a quadraturis; & erit *AH* sinus inclinationis quaesitae. Huic orbis inclinationi aequalis est eiusdem inclinatio, ubi luna distat $90^{gr.}$ a nodis. In aliis lunae locis inaequalitas menstrua, quam inclinationis variatio admittit, in calculo latitudinis lunae compensatur, & quodammodo tollitur per inaequalitatem menstruam motus nodorum (ut supra diximus) ideoque in calculo latitudinis illius negligi potest.

Scholium.

Hisce motuum lunarium computationibus ostendere volui, quod motus lunares per theoriam gravitatis a causis suis computari possint. Per eandem theoriam inveni praeterea quod aequatio annua medii motus lunae oriatur a varia dilatatione orbis lunae per vim solis, iuxta Corol. 6. Prop. LXVI. Lib.1. Haec vis in perigaeo solis major est, & orbem lunae dilatat ; in apogaeo eius minor est, & orbem illum contracti permittit. In orbe dilatato luna tardius revolvitur, in contracto citius ; & aequatio annua, per quam haec inaequalitas compensatur, in apogaeo & perigaeo solis nulla est, in mediocri solis a terra distantia ad 11'. 50" circiter ascendit, in aliis locis aequationi centri solis proportionalis est ; & additur medio motui lunae ubi terra pergit ab aphelio suo ad perihelium, & in opposita orbis parte subducitur. Assumendo radium orbis magni 1000 & eccentricitatem terrae $16\frac{7}{8}$, haec aequatio, ubi maxima est, per theoriam gravitatis prodiit 11'. 49"'. Sed eccentricitas terrae paulo major esse videtur, & aucta eccentricitate haec aequatio augeri debet in eadem ratione. Sit eccentricitas $16\frac{11}{12}$, & aequatio maxima erit 11'. 51".

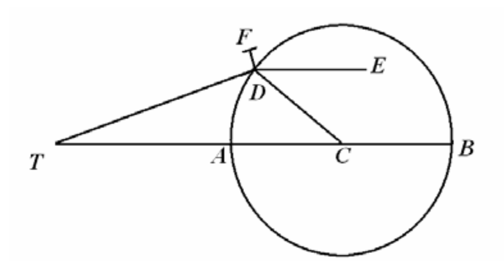
Inveni etiam quod in perihelio terrae, propter majorem vim solis, apogaeum & nodi lunae velocius moventur quam in aphelio eius, idque in triplicata ratione distantiae terrae a sole inverse. Et inde oriuntur aequationes annuae horum motuum aequationi centri solis proportionales. Motus autem solis est in duplicata ratione distantiae terrae a sole inverse, & maxima centri aequatio, quam haec inaequalitas generat, est $1^{gr.}\ 56'.\ 20''$ praedictae solis eccentricitati $16\frac{11}{12}$ congruens. Quod si motus solis esset in triplicata ratione distantiae inverse, haec inaequalitas generaret aequationem maximam $2^{gr.}\ 54'\ 30''$. Et propterea aequationes maximae, quas inaequalitates motuum apogaei & nodorum lunae generant, sunt ad $2^{gr.}\ 54'.\ 30''$ ut motus medius diurnus apogaei, & motus medius diurnas nodorum lunae sunt ad motum medium diurnum solis. Unde prodit aequatio maxima medii motus apogaei 19'.43", & aequatio maxima medii motus nodorum 9'. 24". Additur vero aequatio prior & subducitur posterior, ubi terra pergit a perihelio suo ad aphestum : & contrarium sit in opposita orbis parte.

Per theoriam gravitatis constitit etiam quod actio solis in lunam paulo major sit, ubi transversa diameter orbis lunaris transit per solem, quam ubi eadem ad rectos est angulos cum linea terram & solem jungente: & propterea orbis lunaris paulo major est in priore casu quam in posteriore. Et hinc oritur alia aequatio motus medii lunaris. pendens a situ apogaei lunae ad solem, quae quidem maxima est cum apogaeum lunae versatur in octante cum sole; & nulla cum illud ad quadraturas vel syzygias pervenit: & motui medio additur in transitu apogaei lunae a solis quadratura ad syzygiam, & subducitur in transitu apogaei a syzygia ad quadraturam. Haec aequatio, quam semestrem vocabo, in octantibus apogaei, quando maxima est, ascendit ad 3'. 45" circiter, quantum ex phaenomenis colligere potui. Haec est eius quantitas in mediocri solis distantia a terra. Augetur vero ac diminuitur in triplicata ratione distantiae solis inverse, ideoque in maxima solis distantia est 3'. 34"' & in minima 3'.56" quamproxime: ubi vero apogaeum lunae situm est extra octantes, evadit minor; estque ad aequationem maximam, ut sinus duplae distantiae apogaei lunae a proxima syzygia vel quadratura ad radium.

Per eandem gravitatis theoriam actio solis in lunam paulo major est ubi linea recta per nodos lunae ducta transit per solem, quam ubi linea illa ad rectos est angulos cum recta solem ac terram jungente. Et inde oritur alia medii motus lunaris aequatio, quam semestrem secundam vocabo, quaeque maxima est ubi nodi in solis octantibus versantur, & evanescit ubi sunt in syzygiis vel quadraturis, & in aliis nodorum positionibus proportionalis est sinui duplae distantiae nodi alterutrius a proxima syzygia aut quadratura: additur vero media motui lunae, si sol distat a nodo sibi proximo in anticedentia, sub ducitur si in consequentia, & in octantibus, ubi maxima est, ascendit ad 47" in mediocri solis distantia a terra, uti ex theoria gravitatis colligo. In aliis solis distantiis haec aequatio maxima in octantibus nodorum est reciproce ut cubus distantiae solis a terra, ideoque in perigaeo solis ad 49" in apogaeo eius ad 45" circiter ascendit.

Per eandem gravitatis theoriam apogaeum lunae progreditur quam maxime ubi vel cum sole conjungitur vel eidem opponitur, & regreditur ubi cum sole quadraturam facit. Et eccentricitas sit maxima in priore casu & minima in posteriore, per Corol. 7, 8 & 9. Prop.

LXVI. Lib. I. Et hae inaequalitates per eadem Corollaria permagnae sunt, & aequationem principalem apogaei generant, quam semestrem vocabo. Et aequatio maxima semestris est $12^{gr.}18'$ circiter, quantum ex observationibus colligere potui. *Horroxius* noster lunam in ellipsi circum terram, in eius umbilico inferiore constitutam, revolvi primus statuit. *Halleius* centrum ellipseos in epicyclo locavit, cuius centrum uniformiter revolvitur circum terram. Et ex motu in epicyclo oriuntur inaequalitates jam dictae in progressu & regressu apogaei & quantitate eccentricitatis. Dividi intelligatur distantia mediocris lunae a terra in partes 100000, & referat *T* terram & *TC* eccentricitatem mediocrem lunae partium 5505. Producatur *TC* ad *B*, ut sit *CB* sinus aequationis maximae semestris $12^{gr.}\ 18'$ ad radium *TC*, & circulus *BDA* centro *C*, intervallo *CB* descriptus erit epicyclus il1e in quo centrum orbis lunaris locatur & secundum ordinem literarum *BDA* revolvitur. Capiatur angulus *BCD* aequalis duplo argumento annuo, seu duplae distantiae veri loci solis ab apogaeo lunae semel aequato, & erit *CTD* aequatio semestris apogaei lunae & *TD*



eccentricitas orbis eius in apogaeum secundo aequatum tendens. Habitis autem lunae motu medio & apogaeo & eccentricitate, ut & orbis axe majore partium 200000; ex his eruetur verus lunae locus in orbe & distantia eius a terra, idque per methodos notissimas.

In perihelio terrae, propter majorem vim solis, centrum orbis lunae velocius movetur circum centrum *C* quam in aphelio, idque in triplicata ratione distantiae terrae a sole inverse. Ob aequationem centri solis in argumento annuo comprehensam, centrum orbis lunae velocius movetur in epicyclo *BDA* in duplicata ratione distantiae terrae a sole inverse. Ut idem adhuc velocius moveatur in ratione simplici distantiae inverse; ab orbis centro *D* agatur recta *DE* versus apogaeum lunae, seu rectae *TC* parallela, & capiatur angulus *EDF* aequalis excessui argumenti annui praedicti supra distantiam apogaei lunae a perigaeo solis in consequentia; vel quod perinde est, capiatur angulus *CDF* aequalis complemento anomaliae verae solis ad gradus 360. Et sit *D*F ad *DC* ut dupla eccentricitas orbis magni ad distantiam mediocrem solis a terra, & motus medius diurnus solis ab apogaeo lunae ad motum medium diurnum solis ab apogaeo Proprio conjunctim, id est, ut $33\frac{7}{8}$ ad 1000 & 52'. 27''. 16''' ad 59'. 8''. 10''' conjunctim, sive at 3 ad 100. Et concipe centrum orbis lunae locari in puncto *F*, & in epicyclo, cuius centrum, est *D*, & radius *DF*; interea revolvi dum punctum *D* progreditur in circumferentia circuli *DABD*. Hac enim ratione velocitas, qua centrum orbis lunae in linea quadam curva circum centrum *C* descripta movebitur, erit reciproce ut cubus distantiae solis a terra quamproxime, ut oportet.

Computatio motus huius difficilis est, sed facilior reddetur per approximationem sequentem. Si distantia mediocris lunae a terra sit partium 100000, & eccentricitas *TC* sit partium 550; ut supra: recta *CB* vel *CD* invenietur partium $1172\frac{3}{4}$, & recta *DF* partium

$35\frac{1}{5}$. Et Haec recta ad distantiam *TC* subtendit angulum ad terram quem translatio centri orbis a loco *D* ad locum *F* generat in motu centri huius: & eadem recta duplicata in situ parallelo ad distantiam superioris umbilici orbis lunae a terra, subdendit eundem angulum, quem utique translatio illa generat in motu umbilici, & ad distantiam lunae a terra subdendit angulum quem eadem translatio generat in motu lunae, quique propterea aequatio centri secunda dici potest. Et haec aequatio, in mediocri lunae distantia a terra, est ut sinus anguli, quem recta illa *DF* cum recta a puncto *F* ad lunam ducta continet quamproxime, & ubi maxima est evadit 2'. 25". Angulus autem quem recta *DF* & recta a puncto *F* ad lunam ducta comprehendunt, invenitur vel subducendo angulum *EDF* ab anomalia media lunae, vel addendo distantiam lunae a sole ad distantiam apogaei lunae ab apogaeo solis. Et ut radius est ad sinum anguli sic inventi, ita 2'. 25". sunt ad aequationem centri secundam, addendam, si summa illa sit minor semicirculo, subducendam si major. Sic habebitur eius longitudo in ipsis lluminarium syzygiis.

Cum atmosphaera terrae ad usque altitudinem milliarium 35 vel 40 refringat lucem solis, & refringendo spargat eandem in umbram terrae, & spargendo lucem in confinio umbrae dilatet umbram: ad diametrum umbrae, quae per parallaxim prodit, addo minutum unum primum in eclipsibus lunae, vel minutum unum cum triente.

Theoria vero lunae primo in syzygiis, deinde in quadraturis, & ultimo in octantibus per phaenomena examinari & stabiliri debet. Et opus hocce aggressurus motus medios solis & lunae ad templus meridianum in observatorio regio *Grenovicensi*, dic ultimo mensis Decembris anni 1700. st. vet. non incommode sequentes adhibebit: nempe motum medium solis ♑ $20^{gr.}\ 43'.\ 40'$, & apogaei eius ♋ $7^{gr.}\ 44'.\ 30''$, & motum medium lunae $15^{gr.}\ 21'.\ 00''$, & apogaei eius ♓ $8^{gr.}\ 20'.\ 00''$, & nodi ascendensis ♌ $27^{gr.}\ 24'.\ 20''$; & differentiam meridianorum observatorii huius & observatorii regii *Parisiensis* $0^{hor.}\ 9^{min.}\ 20^{sec.}$ motus autem medii luna: & apogaei eius nondum satis accurate habentur.