CONCERNING THE SYSTEM OF THE WORLD.

BOOK III.

In the preceding books I have treated the principles of [natural] philosophy, yet not in a philosophical manner, but only mathematically, from which clearly it may be possible to argue over philosophical matters. These principles are the laws and conditions of forces and motions, which are considered especially according to philosophy. Yet the same, lest they may appear barren, I have illustrated by certain philosophical scholias, treating those which are general, and in which philosophy especially is considered to lay the foundations, such as the density and resistance of bodies, spaces devoid of bodies, and the motion of light and sound. It remains that we may give some instruction about the constitution of the systems of the world from the same principles. With this in mind I had composed a third book so that it might be read by many. But in which the principles put in place were not understood well enough, they [the readers] barely understood the strength of the consequences, nor were the prejudices cast aside to which they had become accustomed over many years: and therefore lest the matter be drawn into disputes, I have transferred the bulk of this book into propositions, in the mathematical manner, so that from these alone the principles may be read by those who have worked through the earlier principles. Nevertheless since there many propositions occur, which may add excessive delays even for readers versed in mathematics, I do not wish to be the authority that the reader should pursue all of these ; it would suffice for whoever that the definitions, laws of motion and the first three sections of Book I be read with care, then he may transfer to this book on the systems of the world, and he may consult the remaining propositions of the first book here put in place as it pleases.

[Introductory Note : The English title of the work being : *The Mathematical Principles of Natural Philosophy* ; it took a considerable time for Newton's assertion about the predominance of mathematics in describing natural phenomena to be accepted generally, rather than the hand-waving exercises of philosophers, which were then the order of the day. In Newton's view, ideas analogous to those of ancient Greek geometry could be applied to natural phenomena, so that from a carefully chosen set of axioms and laws, successive propositions could be established as in the ancient geometric texts, governing the natural world from simple collisions to the workings of the solar system, which he called ' The World'. Planetary astronomy provided an almost ideal platform on which to establish a coherent theory of dynamics from simple principles, applied to the motions of the planets, where the complications of unwelcome factors such as friction due to air resistance, etc., were absent. The main thrust of such a work must include derivations of Kepler's empirical Laws from the Law of Universal Gravitation. This then was Newton's ambitious project, and the use of geometry was central to the work and given dominance, though the new and revolutionary calculus – not without its detractors, played a strong supporting role, and actually underpinned the whole edifice that Newton had erected: indeed many of the propositions could not have been established without the use of calculus. Thus, in Newton's scheme, a geometrical point became a particle with a mass or reluctance to move initially or change its state of motion if already moving, its inertia ; interactions of particles that changed the states of rest or of uniform motion of these were ascribed to forces, that acted on the masses either at a distance or by direct contact, obeying the three laws of motion, and the trajectories followed by the particles were represented by lines familiar from geometry ; as were the velocities, accelerations, and the forces themselves, that might be added as the sides of parallelograms; Newton's world was thus one of motion, for which these new ideas of fluxions and fluents were admirably suited for the detailed analysis. A consequence of this time dependent dynamical approach was a disregard for quantities such as energy, that remained conserved with time; and perhaps more surprising, a disregard for the angular form of the laws governing motion, and the extension to bodies of differing shapes, rather than points, though some progress is made in this final book in that regard. Thus Newton's particle mechanics was correct, but incomplete in a number of fundamentals. Leibniz tried to introduce such a time-independent principle via his *vis viva* notion, which was a close relative of kinetic energy, without the half ; a source of much confusion to workers in the 18th century; in this regard it may be said, and perhaps it is an undue criticism, that Leibnitz was trying to jump onto someone else's bandwagon, so to speak ; supreme logician that he was, Leibnitz was no match for Newton as a natural philosopher.

In addition, one needs to bear in mind the cause and effect nature of Newton's arguments, thus, in geometry, the converse of a proposition may be as important and useful as the original proposition; this is not the case in physics : the converse of a proposition may not be physically significant, or the situation may arise from a different cause. In general, the irreversible increase in entropy, the arrow of time as it were, indicates that processes in the macroscopic world generally go in one direction rather than to be reversible, which in turn is the exception rather than the rule. Thus Newton does not necessarily dwell on the converses of his propositions, as one might do in geometry. Finally, in this introductory note, it should be made clear that this work is not an 'elementary' treatment of astronomical calculations, but a rather sophisticated one, in which unfortunately for us, most of the detailed calculations have been omitted by Newton, such as his method of finding the shape of the earth ; instead he plays with the reader at times, telling him at length about relatively trivial matters, such as adventurers sailing to remote places to measure the force of gravity there, which could be briefly encapsulated in a table. Thus, it is advisable to have an understandable text on classical celestial mechanics available to check results, such as that by W. M. Smart (Longmans; 1953) ; occasionally, results derived by Leseur & Janquier are inserted, and references made to the latter part of the book by Chandrasekhar are of interest.]

Required Rules of PHILOSOPHY.

Rule I.

No more causes of natural phenomena must be admitted, than which are both true and needed for the explanation of the phenomena.

Certainly philosophers say : Nature does nothing in vain, and more will be in vain from several things in excess than arises from a few. For nature is simple and does not indulge in superfluous causes.

Rule II.

Therefore the same causes are to be assigned to natural effects of the same kind, as far as it can come about.

Such as breathing in man and beast; the descent of stones in *Europe* and *America;* light in a cooking fire and in the sun; reflection of light on the earth and on the planets.

Rule III.

The qualities of bodies cannot be extended or remitted, and it is allowed that they be put in place for all bodies, agreed upon by experiment, for they are the universal qualities of bodies requiring to be present.

For the qualities of bodies become known only through experiments, and thus such qualities are required to be in general agreement whenever they square up generally with experiments ; and because the qualities cannot be diminished, they cannot be removed. Certainly qualities are not to be rashly fashioned out of dreams contrary to the steady course of experiments, nor is it required to depart from the analogy of nature, since that is simple and always accustomed to be in harmony within itself.

The extension of bodies can only become known from the senses, nor can the extension of all bodies be perceived: but because it is agreed upon by all the senses, from that the property of extension can be agreed upon for all bodies. We find out too that many bodies are hard. But the hardness arises from the hardness of the parts, and thence not only of these bodies which are perceived, but we may conclude also of all the others that we cannot perceive, that the indivisible particles are deservedly hard. Again, we gather that all bodies are impenetrable, not from reason but from the senses. The bodies we treat are found to be impenetrable, and thence we conclude that impenetrability is a universal property of bodies. All bodies are moveable, and by certain forces (we call which the forces of inertia) persevere in their motion or rest, and from these we may deduce the properties of observed bodies. The whole extension, hardness, impenetrability, mobility, and the force of inertia arise from the extension, hardness, impenetrability, mobility, and forces of inertia of the parts: and thence we conclude that the smallest parts of all bodies are to be extended, and to be hard and impenetrable, and to have mobility provided by the forces of inertia. And this is the foundation of all philosophy. Again the divided parts of bodies mutually touching each other can be separated in turn from each other, by phenomena that we know, and it is evident that the indivisible parts may be separated into smaller parts in a ratio from mathematics. Truly it is uncertain whether these distinct parts not yet divided are able to be divided and separated in turn by natural forces. But if in turn it may be agreed by a single experiment that some particles somehow undivided, with a hard and solid body being broken, that a division may be apparent: we would conclude on the strength of this rule, that not only may the separate parts be divided, but also that they may be continued to be divided indefinitely.

Finally, if all bodies revolving around with the earth are to exert a weight on the earth, [occasionally we will use the term *attract by gravity* for Newton's expression] and that for every individual quantity of matter, and the moon by its quantity of matter to attract the earth, and in turn our sea to have an attraction on the moon, and all the planets to mutually attract each other, and in a like manner the attraction of comets to the sun by gravity may be agreed upon generally from astronomical observations : it will be required to be said that by this rule all bodies mutually attract each other by gravitation. For there will be an even stronger argument for the phenomena of universal gravitation, than that for the impenetrability of bodies : concerning which certainly we have no experimental evidence, nor any observations. Yet I am not at all confirming that the force of gravity is essential for bodies. By the force *insita* I understand only the force of inertia. This is unchangeable; but the force of gravity is diminished on receding from the earth.

Rule IV.

In experimental philosophy, propositions are deduced from phenomena by induction, not from contrary hypothesis, for they must be considered as true hypotheses, either accurately or approximately true, from which either more accurate hypotheses may be made available, or the hypothesis be made free from exceptions.

This must come about lest the argument of induction be removed by [contrary] hypothesis.

PHENOMENA.

PHENOMENON I.

[Note : In Latin & Greek, the word *phaenomena* related to an appearance in the sky.]

The circumjovial planets, with radii drawn from the centre of Jupiter, describe areas proportional to the times, and the periodic times of these, with respect to the fixed stars at rest, are in the three halves power of their distances from its centre.

This is agreed from astronomic observations. The orbits of these planets are not different sensibly from concentric circles around Jupiter, and the motions of these may be taken from uniform circles. Truly astronomers agree that the periods are in the three halves ratio of the radii of their orbits ; and the same is evident from the following table.

The periodic time of the Jovian satellites.

$1^d.18^h.27'.34''$; $3^d.13^h.13'.42''$; $7^d.3^h.42'.36''$; $16^d.16^h.32'.9''$.

The distances of the satellites from the centre of Jupiter.

From observations	1	2	3	4	
Borelli .	$5\frac{2}{3}$	$8\frac{2}{3}$	14	$24\frac{2}{3}$	Jovian radii.
Townley *by micrometer*	5,52	8,78	13,47	24,72	
Cassini *by telescope*	5	8	13	23	
Cassini *by satellite eclipse.*	$5\frac{2}{3}$	9	$14\frac{23}{66}$	$25\frac{3}{16}$	
From the periodic times.	5,667	9,017	14,384	25,299	

With the best micrometers Master *Pound* has determined the elongations of the Jovian satellites and the diameter of Jupiter as follows. The maximum elongation of the fourth Jovian satellite from the centre of the sun was taken by a micrometer in a telescope tube 15 feet long, and produced around $8'.16''$ from the mean distance of Jupiter from the earth. That of the third satellite was taken in a telescope 23 feet long, and produced$4'.42''$ at the same earth to Jupiter distance. The maximum elongations of the remaining satellites at the same distance of the earth from Jupiter $2'.56''.47'''$, & $1'.51''.6'''$ arising from the periodic times.

The diameter of Jupiter was often taken from the micrometer in the 123 feet telescope, and reduced according to the mean distance of Jupiter from the sun or the earth, always produced an image less than $40''$, at no time less than $38''$, more often $39''$. In shorter telescopes this diameter is $40'$ or $41'$. For the light of Jupiter by unequal refraction is dilated to some extent, and this dilation has a smaller ratio to the diameter of Jupiter in the longer and more perfect telescopes than in the shorter and less perfect ones.

The times in which two satellites, the first and the third, will pass across the body of Jupiter, from the beginning of the ingression to the start of the exit, and from the complete ingression to the complete exit, have been observed with the aid of the same longer telescopes. And the diameter of Jupiter at its mean distance from the earth produced $37\frac{1}{8}''$ by the transition of the first satellite and $37\frac{3}{8}''$ from the transition of the third. Also the time in which the shadow of the first satellite passed through the body of Jupiter was observed, and thence the diameter of Jupiter at its mean distance from the earth produced around $37''$. We may assume the diameter of this to be approximately $37\frac{1}{4}''$; and the maximum elongations of the first, second, third and fourth satellites were respectively 5,965, 9,494, 15,141, & 26,63 radii of Jupiter.

PHENOMENON II.

The planets around Saturn, with radii drawn to Saturn, describe areas proportional to the times, and the periodic times of these, with the fixed stars, are in the three on two ratio of the distances from the centre of Saturn.

Certainly *Cassini* from his observations has established the distance of these from the centre of Saturn and the periodic times to be of this kind.

The periodic times of the satellites of Saturn.

$1^{d}.21^{h}.18'.27''$; $2^{d}.17^{h}.41'.22''$; $4^{d}.12^{h}.25'.12''$; $15^{d}.22^{h}.41'.14''$; $79^{d}.7^{h}.48'.00''$.

The distances of the satellites from the centre of Saturn in radii of the ring.

From observations.	$1\frac{19}{20}$.	$2\frac{1}{3}$.	$3\frac{1}{2}$.	8.	24.
From the periodic times.	1,93.	2,47.	3,45	8.	23,35.

The maximum elongation of the fourth satellite from the centre of Saturn gathered from observations is accustomed to be approximately eight radii. But the maximum elongation of this satellite from the centre of Saturn, taken from the best micrometer in the Huygens telescope 123 feet long, has produced a radius of $8\frac{7}{10}$ radii. And from this observation and from the periodic times, the distances of the satellite from the centre of Saturn in radii of the ring are 2,1. 2,69. 3,75. 8,7. and 25,35. The diameter of Saturn in the same telescope to the diameter of the ring was as 3 to 7, and the diameter of the ring produced on the 28th and 29th days of May of the year 1719 was $43''$. And thence the diameter of the ring at the mean position of Saturn from the earth is $42''$, and the diameter of Saturn $18''$. Thus these are in the longest and best telescopes, because hence the apparent magnitudes of celestial bodies in longer telescopes may have a greater proportion to the dilation of the [rays of] light at the ends of these bodies than in shorter telescopes. If all the stray light may be removed, the diameter of Saturn will remain not greater than $16''$.

PHENOMENON III.

The first five planets Mercury, Venus, Mars, Jupiter, and Saturn, surround the sun with their orbits.

Mercury and Venus may be shown to revolve around the sun from their lunar phases. With the full shape shining when they are situated beyond the sun ; half when away from the direction of the sun; sickle shaped when this side of the sun, and by its disc in the manner of a spot occasionally passing in front of the sun. From Mars also made full near to the sun in conjunction, & gibbous at right angles, it is evident that it goes around the sun. Concerning Jupiter and Saturn the same also may be demonstrated from their full phases : indeed it is evident that these shine with light borrowed from the sun from the shadows of the satellites projected onto themselves.

PHENOMENON IV.

The periodic times of the five primary planets, and either of the sun around the earth or the earth around the sun, with the fixed stars at rest, are in the three on two ratio of the mean distances from the sun.

That this ratio was found by *Kepler* has been acknowledged by everyone. Certainly the periodic times are the same, the same dimensions of the orbits, whether the sun goes around the earth, or the earth may be revolving around the sun. And indeed it is agreed from measurements of the periodic times amongst astronomers. Moreover the magnitudes of all the orbits have been determined most carefully by *Kepler* and *Boulliau* from observations: and the mean distances, which correspond to the periodic times, are not sensibly different from the distances which they found, and these among themselves are intermediate mainly, as can be seen from the following table.

♁ *The periodic times of the planets and of the earth about the sun, with respect to the fixed stars, in days and decimal parts of days.*

♄	♃	♂	♁	♀	☿
10759,275.	4332, 514	686,9785.	365,2565.	224,6176.	87,9692.

The mean distances of the planets and of the earth from the sun.

	♄	♃	♂	♁	♀	☿
Following *Kepler*	951000.	519650.	152350.	100000.	72400.	38806.
Following *Boulliau*	954198.	522520.	152350.	100000.	72398.	38585.
Following the periodic times	954006.	520096.	152369.	100000.	72333	38710.

There is no dispute concerning the distances of Mercury and Venus from the sun, since these may be determined from their elongations from the sun. Also concerning the distances of the outer planets from the sun all the disputation is removed by the eclipses of the satellites of Jupiter. And indeed the position of the shadow that Jupiter casts may be determined by those eclipses, and with that nominated the heliocentric longitude may be obtained. Moreover from the heliocentric and geocentric longitudes between these taken together the distance of Jupiter may be determined.

PHENOMENON V.

The primary planets, with radii drawn to the earth, describe areas not at all proportional to the times; but with radii drawn to the sun, the areas traversed are proportional to the times.

For with respect to the earth they are now progressing, now they are stationary, and now also they are regressing : But with respect to the sun they are always progressing, and that with just about a uniform motion, but yet a little faster at the perihelions and a little slower at the aphelions, thus so that there shall be described equal areas. The proposition is the most noteworthy of astronomy, and may be demonstrated to a high degree by the satellites of Jupiter, from which eclipses we have said that the heliocentric longitudes of this planet and the distances from the sun are determined.

PHENOMENON VI.

With a radius drawn to the centre of the earth, the moon describes areas proportional to the times.

It is evident from the apparent motion of the moon deduced from the apparent diameter. Moreover the motion of the moon is perturbed a little by the force of the sun, but I will ignore the insensible small errors in these phenomena.

PROPOSITIONS.

PROPOSITION I. THEOREM I.

The forces, by which the circumjovial planets are drawn perpetually from rectilinear motion and are retained in their orbits, with respect to the centre of Jupiter, are inversely as the squares of their distances from the same centre.

The first part of the proposition is apparent from the first phenomenon, and the second or third proposition of the first book : and the latter part by the first phenomenon, and the sixth corollary of the fourth proposition of the same book.

You may understand the same concerning the planets which accompany the planet Saturn, by the second phenomenon.

PROPOSITION II. THEOREM II.

The forces, by which the primary planets are perpetually drawn from rectilinear motion, and retained in their orbits, with respect to the sun, and to be inversely as the square of the distances from its centre.

The first part of the preposition is apparent from the fifth proposition, and the second proposition of the first book : and the latter part by the fourth phenomenon, and the fourth proposition of the same book. Moreover this first part of the proposition may be shown most accurately by the resting nature of the aphelions. For the smallest aberration from the square ratio must bring about a noteworthy motion in the individual revolutions (by Corol. I. Prop. XLV. Book. I.) and in more a large motion.

PROPOSITION III. THEOREM III.

The force, by which the moon is retained in its orbit, with respect of the earth, is inversely as the square of the distances of the places from the centre of this.

The first part of the assertion is apparent from the sixth phenomenon, and the second or third proposition of the first book : and the latter part by the slowest motion of the moon's apogee. For that motion, which in the individual revolutions is only of three degrees and three minutes, can be disregarded. For it is apparent (by Corol. I. Prop. XLV. Book. I.) that if the distance of the moon from the centre of the earth shall be to the radius of the earth as D to 1; the force by which such a motion may arise shall be reciprocally as $D^{2\frac{4}{243}}$, that is, reciprocally as that power of D of which the index is $2\frac{4}{243}$, that is, in a ratio of the distance a little greater than the inverse square, but which approaches by $59\frac{3}{4}$ times closer to the square than to the cube. Truly it arises from the action of the sun (as will be mentioned later) and therefore here can be ignored. In as much as the action of the sun draws the moon from the earth, is as the distance of the moon from the earth approximately ; and thus (by what were discussed in Corol. 2. Prop. XLV. Book. I.) the action is to the centripetal force of the moon as around 2 to 357,45, or1 to $178\frac{29}{40}$. And with so small a force of the sun, the remaining force by which the moon is retained in its orbit will be reciprocally as D^2. That which also will be agreed upon more fully by comparing this with the force of gravity, as shall be made in the following proposition.

Corol. If the mean centripetal force by which the moon may be retained in orbit may be increased first in the ratio $177\frac{29}{40}$ to $178\frac{29}{40}$, then also in the square ratio of the radius of the earth to the mean distance of the moon from the centre of the earth : the centripetal force will be found of the moon at the surface of the earth, because that force in place descending to the surface of the earth always may be increased in the inverse ratio of the square of the height.

PROPOSITION IV. THEOREM IV.

The moon it attracted towards the earth by gravity, and by the force of gravity is drawn always from rectilinear motion, and retained in its orbit.

The mean distance of the moon from the earth at syzygies is 59 times the earth radius, according to *Ptolemy* and most of the astronomers, 60 according to *Vendelin* and *Huygens*, $60\frac{1}{3}$ according to *Copernicus*, following *Street* $60\frac{2}{5}$ and according to *Tycho* $56\frac{1}{2}$. But *Tycho*, and all who follow his tables of refraction, by putting in place refractions of the sun and moon (generally contrary to the nature of light) greater than of the fixed stars, and that by as many as four or five minutes of arc, increased the parallax of the moon by just as many minutes, that is, as if between a 12th or 15th part of the whole parallax. This error may be corrected, and a distance around $60\frac{1}{2}$ of the earth's radius emerges, almost as that which has be designated by the others. We may assume the mean distance to be 60 of the earth's radius at sygyies ; and the lunar period with respect to the fixed stars to be made up from 27 days, 7 hours, and 43 minutes, as has been established from astronomy ; and the circumference of the earth to be 123249600 Parisian feet, as defined from French measurements : and if the moon may be supposed to be deprived of all its motion and released, so that by being urged by all that force, by which (by the Corol. of Prop. III.) it may be retained in its orbit, it may fall to the earth ; this distance of $15\frac{1}{12}$ Parisian feet it describes in a time of one minute. This is deduced by calculation accomplished either by Prop. XXXVI of the First Book, or (that returns the same) by the Corollary of the fourth proposition of the same book. For that arc which the moon in a time of one minute, by its mean motion, may describe at a distance of 60 earth radii, is around the versed sine of $15\frac{1}{12}$ Parisian feet, or more accurately 15 feet, 1 inch and $1\frac{4}{9}$ twelve parts of an inch. From which since that force by acceleration to the earth may be increased in the inverse square ratio, and thus at the surface of the earth shall be 60×60 times greater than at the surface of the moon ; a body by falling with that force in our region , must describe $60\times 60\times 15\frac{1}{12}$ i Parisian feet n an interval of one minute , and in an interval of one second $15\frac{1}{12}$ feet, or more accurately 15 feet. 1 inch. and $1\frac{4}{9}$ twelfth parts of an inch. And by the same force weights actually fall on the earth. For the length of a pendulum, in the latitude of Paris for a single second of oscillation, is of three feet and $8\frac{1}{2}$ lines [i.e. twelfth parts of an inch]; as *Huygens* observed. And the height, that a weight describes by falling in a time of one second, is to the half length of this pendulum in the square ratio of the circumference of the circle to its diameter (as *Huygens* also indicated), [indeed, in modern terms, $s = \frac{1}{2}gt^2$ and $t^2 = 4\pi^2 \frac{l}{g} \therefore s = \pi^2 \times \frac{l}{2}$.] and thus is 15 Parisian feet, 1 inch, $1\frac{7}{9}$. lines. And therefore the force by which the moon may be retained in its orbit, if it were falling onto the surface of the earth, will emerge equal to our force of gravity, and thus (by rules I and II.) is that force itself that we are accustomed to call gravity. For if gravity were different from that, bodies with the two forces taken jointly would be attracted towards the earth with twice the velocity, and would describe by falling a distance of $30\frac{1}{6}$ Parisian feet: entirely contrary to experience.

Here the calculation is founded on the hypothesis that the earth is at rest. For if the earth and the moon may be moving around the sun, and meanwhile also commonly revolving about the common centre of gravity : with the law of gravity remaining, the distance between the centres of the earth and moon in turn will be around $60\frac{1}{2}$ earth radii ; as will be apparent from beginning the computation. Moreover the computation can be entered into by Prop. LX. Book I.

Scholium.

The demonstration of the proposition can be explained more fully thus. If several moons were revolving around the earth, in the same way as it shall be in the system of Jupiter or Saturn : the periodic times (by an inductive argument) obey the laws of the planets found by *Kepler*, and therefore the centripetal forces of these shall be inversely as the squares of the distances from the centre of the earth, by Prop. I. of this Book. And if the smallest of these shall be infinitely small, and it may nearly touch the peaks of the highest mountains: the centripetal force of this by which it may be held in orbit, will be almost equal to the weights of bodies on the peaks of those mountains (by the preceding computation), and it may be effected that the same small moon, if it may be deprived of all the motion by which it goes on in orbit, with the deficiency of the centrifugal force by which it may remain in orbit, may fall to the earth, and that with the same velocity by which weights fall from the peaks of those mountains, on account of the equality of the forces by which they fall. And if that force by which that smallest moonlet fell, were different from gravity, and if that moonlet should have the customary force of gravity on the earth of bodies on the peaks of mountains : the same moonlet from each taken together would fall with twice the speed. Whereby since each of the forces, both these of the weights of bodies, and these of the moonlets, with regard to the centre of the earth, and between themselves shall be similar and equal, the same (by rules I. and II.) will have the same cause. And therefore that force, by which the moon may be kept in its orbit, that itself will be as we are accustomed to call the weight : and especially that the moonlet at the peak of a mountain may neither be without weight, nor that it will fall with twice the velocity that heavy bodies are accustomed to fall.

PROPOSITION V. THEOREM V.

The circumjovial planets gravitate towards Jupiter, the circumsaturn planets towards Saturn, and the circumsolar planets towards the sun, and by the force of their gravity they are always drawn from rectilinear motion, and held in curved orbits.

For the revolutions of the circumjovial planets around Jupiter, the circumsaturnal planets around Saturn, and of Mercury and Venus and of the remainder around the sun are phenomena of the same kind as with the revolution of moon around the earth ; and therefore (by rule II.) depend on causes of the same kind : especially since it may be shown that the forces, on which these revolutions depend, may be with respect to the centre of Jupiter, Saturn and the sun, and by receding from Jupiter, Saturn or the Sun they may decrease in the same ratio and by the law, by which the force of gravity decreases in receding from the earth.

Corol. 1. The force of gravity therefore is given towards all the planets. For no one can doubt that Venus, Mercury and the others are bodies of the same kind as Jupiter and Saturn. And since the attraction of all shall be mutual by the third law of motion, Jupiter towards all its satellites, Saturn towards its satellites, and the earth towards the moon, and the sun will gravitate towards all the primary planets.

Corol. 2. The gravity, with regard to any planet, is inversely as the square of the distance of the places from its centre.

Corol. 3. All the planets have a weight between each other, by Corol.1. and 2. And hence Jupiter and Saturn by attracting each other near conjunction, noticeably perturb each others motion, the sun disturbs lunar motions, the sun and the moon disturb our sea, as will be explained in the following.

Scholium.

Until now we have called that force, by which celestial bodies may be retained in their orbits, the centripetal force. Now the same is agreed to be the force of gravity, and therefore we will now hereafter call this the force of gravity. For the cause of that centripetal force, by which the moon may be held in orbit, must be extended to all the planets by rules I, II. & IV.

PROPOSITION VI. THEOREM VI.

All bodies are attracted by gravity to the individual planets, and the weights of these for whatever planet, for equal distances from the centre of the planet, are proportional to the quantity of matter in the individual planets.

Others have observed for a long time now that [equal] descent of all weights on the earth (at any rate with the very small unequal retardation removed which arises from the air) happen in equal times ; and certainly one can note the most accurate equality of the times in pendulums. I have tested the matter with gold, silver, lead, glass, common salt, wood, water, and with wheat. I have prepared two equal small round wooden boxes. One I filled with wood, and likewise I suspended a weight of gold at the other centre of oscillation (as nearly as I could). The boxes constituted the pendulums, hanging from equal eleven feet strings, in order that the weight, the shape and the air resistance generally were equal; and with equal oscillations, placed next to each other, they were going and coming together for a long time. Hence the amount of matter in the gold (by Corol. I. and 6. Prop. XXIV. Book II.) was to the amount of matter in the wood, as the action of the motive force in all the gold to the same action in all the wood ; that is, as the weight to the weight. And thus for the others. From these experiments clearly in bodies of the same weight the difference of the materials to be taken would have [an effect] to be less than a thousandth part of the whole matter. Truly now there is no doubt that the nature of gravity on the planets is the same as on the earth. For these terrestrial bodies raised as far as the orbit of the moon, and released together with the moon deprived of all motion, may fall likewise as on the earth; and now by what has been shown before it is certain that they describe equal spaces with the moon, and thus so that they [*i.e*. the quantities of matter in the bodies] shall be to the quantity of matter in the moon, as their weights shall be to the weight of the moon itself [at the radius of the moon's orbit]. Again because the satellites of Jupiter are revolving in times which are in the three on two ratio of the distances from the centre of Jupiter, the accelerative weights of these will be towards Jupiter inversely as the square of the distances from the centre of Jupiter ; and therefore at equal distances from Jupiter, the accelerative weights of these become equal. Hence by falling in equal times from the equal heights they describe equal distances ; thence so that it shall be as with weights here on our earth. And the circumsolar planets by the same argument, released from equal distances from the sun, describe equal distances in their descent towards the sun in equal times. Moreover the forces, by which unequal bodies may be equally accelerated, are as the bodies; that is, the weights are as the quantities of matter in the bodies. Again is apparent from the especially regular motion of the satellites that the proportional weights of Jupiter and of its satellites towards the sun are as the quantities of matter of these ; by Corol 3. Prop. LXV. Book I. For, if some of these should be attracted more towards the sun than the rest, for their quantity of matter : the motions of satellites (by Corol. 2. Prop. LXV. Book I.) would be perturbed from the inequality of the attraction. If, with equal distances from the sun, some satellite should be heavier towards the sun by its quantity of matter, than Jupiter by its quantity of matter, in some given ratio, for example *d* to *e:* the distances between the centre of the sun and the centre of the orbit of the satellite, always should be greater than the distance of the centre of the sun and the centre of Jupiter approximately in the three on two ratio ; as found in a certain initial calculation. And if a satellite should weigh less towards the sun in that ratio *d* to *e*, the distance of the centre of the orbit of the satellite from the sun would be less than the distance of the centre of Jupiter from the sun in the square root ratio approximately. And thus if in the distances from the sun equalities, the accelerative gravity of some satellite towards the sun should be greater or less than the accelerative weight of Jupiter towards the sun, by only a one thousandth part of the whole force of gravity ; the distance of the centre of the orbit of the satellite from the sun would become greater or less than the distance from Jupiter by a $\frac{1}{2000}$ th part of the whole distance, that is, by a fifth part of the extreme distance of the satellite from the centre of Jupiter : which indeed would be quite detectable for an exocentric orbit. But the orbits of the satellites of Jupiter are concentric, and therefore the gravitating acceleration of Jupiter and of the satellites towards the sun are equal to each other. And by the same argument the weight of Saturn and of its attendants towards the sun, at equal distances from the sun, are as the quantities of matter within themselves : and the weights of the moon and of the earth towards the sun are either zero, or accurately proportional to these masses. But they dos have some masses by Corol. 1. and 3. Prop. V.

For indeed the weights of parts of each of the individual planets to some other planet are between themselves as the matter in the individual parts. For if some parts should exert a greater force of gravity, others less, than for the quantity of the matter; the whole planet, for the kind of part with which it should abound the most, would gravitate more or less than for the quantity of the whole matter. But it does not matter if the parts refer to external or internal parts. If for example earthly bodies, which are now about us, were supposed raised to the orbit of the moon, and brought together with the body of the moon : if the weights of these were to the weights of the external parts of the moon as the quantities of matter in the same, truly in a greater or smaller ratio to the weights of the internal parts, the same would be in a greater or smaller ratio to the weight of the whole moon: contrary than what has been shown above.

*Corol.*1. Hence the weights of bodies do not depend on the forms and textures of these. For if they were able to be varied with the shape, they could be greater or less, according to the kind of shape, in an equal amount of matter, completely contrary to experience.

Corol. 2. Generally bodies, which are near the earth, are heavy on the earth ; and all weights, which are equally distant from the centre of the earth, are as the quantities of matter in the same. This is a characteristic of all that it is possible to establish by experiment, and therefore has been confirmed generally by rule III. If the ether or some other either may be freed from gravity, or for a quantity of its matter that may experience gravity less : because that (by the reasoning of *Aristotle, de Cartes,* and of others) may not differ from other bodies except in the shape of the matter, likewise it may be possible by a change of shape gradually to be transformed into a body of the same condition as these, but which may gravitate as greatly as possible, and in turn especially heavy bodies, by gradually adopting the form of these, gradually would be able to lose their weight. And hence the weights depend on the shapes of bodies, and could be varied according to their form, contrary to what has been proven in the above corollary.

Corol. 3. All spaces are not equally occupied. For if all spaces were to be equally filled, the specific gravity of the fluid by which the region of the air would be filled up, on account of the maximum density of matter, would concede nothing to the specific gravity of mercury, gold, or to any other body of the greatest density ; and therefore neither would it be possible for a body of gold nor of any other kind of body to descend in air. For bodies do not descent in fluids, unless they shall be of greater specific gravity. Because if the quantity of matter in a given space may be diminished by rarefaction, why may it not be possible to diminish it indefinitely?
[This is an attempt to explain the differentiation of matter by density in a body held together by gravity; the densest in the centre, and the least dense in the outer regions.]

Corol. 4. If all the particles of all solid bodies should be of the same density, nor able to be rarefied without pores, a vacuum is given. I say that bodies are to be of the same density, of which the forces of inertia are as their magnitudes [*i.e.* masses].

[In this case, if no differentiation is possible as the density is constant, then the body ends abruptly on the outside in a vacuum.]

Corol. 5. The force of gravity is of a different kind from the magnetic force. For magnetic attraction is not as matter attraction. Some bodies are attracted more, others less, most are not attracted. And the magnetic force in one and the same body is able to be retained and to be lost , and sometimes greater by far than the force of gravity, and in receding from a magnet it does not decrease in the ratio of the inverse square, but almost as the inverse, as far as can be ascertained from certain gross observations.

PROPOSITION VII. THEORM VII.

Gravitation happens in bodies universally, and that is to be proportional to the quantity of matter in the individual bodies.

All planets attract each other mutually as we have approved previously, and so that the gravity in each one separately can be considered to be reciprocally as the square of the distance of the places from the centre of the planet. And hence it is a consequence (by Prop. LXIX. Book 1. and its corollaries) that gravity be in the same proportion in all materials.

Again since the gravitational forces of all the parts of some planet *A* shall be present on some planet *B,* and the gravity of its parts shall be to the whole gravity, as the matter of its parts to the whole, and for every action there shall be an equal reaction (by the third law of motion); planet *B* in turn will attract planet *A* by gravity in all its parts, and by its gravity acting on each and every part to its total gravity, as the matter of the parts to the matter of the whole. *Q. E. D.*

Corol. 1. Therefore the whole gravity on a planet arises and is prepared from the gravity of all its individual parts. We have an example of this affair from magnetic and electric attractions. For the whole attraction arises from the attractions of the individual parts. This matter is understood in gravitation, by considering many smaller planets to run together into one sphere and to compose a larger planet. For the whole force must arise from the forces of the composing parts. If this may be objected to because all bodies, which are around us, must mutually attract each other by gravity, since yet gravity of this kind may by no means be experienced : I respond that the gravity in these bodies, since it shall be to the gravity of the whole earth as these bodies are to the whole earth [*i.e.* in terms of mass], is by far smaller than that which it may be possible to experience.

*Corol.*2. Gravitation in the individual particles of bodies is reciprocally as the square of the distance of the places between the particles. This is apparent from Corol 3. Prop. LXXIV. Book I.

PROPOSITION VIII. THEOREM VIII.

If the matter of two spheres mutually attracting each other shall be homogeneous on all sides in all directions, which are equally distant from the centres: the weight of the spheres of the one to the other shall be reciprocally as the square of the distance between the centres.

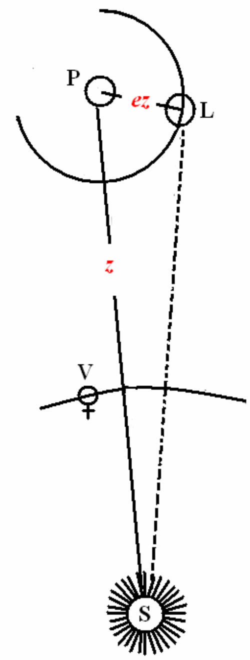
After I had found that the gravitation towards a whole planet arose from and was composed from the gravitation of the parts ; and the gravitation between the parts to be reciprocally proportional to the distances between the parts : I was doubting whether that reciprocal square proportion was obtained accurately with the whole force composed from the many forces, or whether indeed it was as an approximation. For it may come about that the proportion, which may be obtained well enough at greater distances, might be perceptibly in error near the surface of a planet on account of the unequal distances of the particles and on account of dissimilar situations. Yet truly, by Prop. LXXV and Prop. LXXVI. of Book I and their corollaries, I understood the truth of the proposition upon which the proposition here is acting.

Corol. 1. Hence the weights of the bodies in different planets are able to be found and to be compared. For the weights of equal bodies revolving in circles around planets are (by Corol. 2. Prop. IV. Book I.) as the diameters of the circles directly and inversely as the squares of the periodic times ; and the weights at the surfaces of the planets, or at any other distances from the centre, are greater or less (by this proposition) inversely in the square ratio of the distances. Thus from the periodic times of Venus around the sun of 224 and $16\frac{3}{4}$ hours, of the outermost circumjovial satellite around Jupiter of 16 days and $16\frac{8}{25}$ hours, of the Huygens satellite around Saturn of 15 days and $22\frac{2}{3}$ hours, and of the moon around the earth of 27 days, 7 hours and 43 min., by gathering together the mean distance of Venus from the sun and with the maximum heliocentric elongation of the outer circumjovial satellite from the centre of Jupiter 8′. 16″; with the mean distance of the Huygens satellite from the centre of Saturn 3′. 4″; and of the moon from the centre of the earth 10′. 33″ ; by entering upon a calculation [see *L* & *J* note following below] I found that of equal bodies and at equal distances from the centre of the sun, Jupiter, Saturn, and the earth , that their attractions towards the centre of the sun, Jupiter, Saturn, and the earth shall be as $1, \frac{1}{1067}, \frac{1}{3021}$, and $\frac{1}{169282}$ respectively [Cohen in his text unfortunately includes a comma, or equivalent to a decimal point in Newton's work, in his translation at this point, which is evidently incorrect.], and with the distances increased or diminished in the square ratio: the weights of equal bodies towards the sun, Jupiter, Saturn and the earth at the distances 10000, 997, 791, and 109 from the centres of these, and thus at the surfaces of these, will be as 10000, 943, 529, and 435 respectively. However great the weights of bodies shall be on the surface of the moon will be talked about in the following.

[Abridged note 68, *L* & *J*, p. 35, Book 3.
All these may be returned by algebraic proofs.

Let *S* be the centre of the sun, *V* the centre of Venus, *P* the centre of a primary planet, *L* a satellite at its maximum heliocentric elongation that the angle *LSP* measures, for which the angle *SLP* is right. The periodic time of Venus may be called *t* ; the periodic time of the satellite *L* about the primary planet P may be called θ. The distance *SP*, however great it may be, shall be called *z*; the ratio $\frac{SP}{SV}$ which is given by the Phenomena IV may be expressed by the ratio *a* to *b*, and hence there will be

$\frac{(SP=z)}{SV}=\frac{a}{b}$ and $SV=\frac{bz}{a}$; and with the radius present the sine of the maximum heliocentric elongation of the satellite *L*, or the sine of the angle *LSP* may be called *e*; Hence in the right-angled triangle *SLP*, $\frac{z}{\sin\frac{\pi}{2}}=\frac{PL}{(\sin LSP=e)}$; hence the side $PL=ez$.



Because the force of the sun on Venus and the force of the sun on the satellite are by Cor. 2 of Prop. IV, Book I, as the distance of Venus and the distance of the satellite from the centre of the sun and of the primary planet divided by the squares of the periodic times, either as $\frac{bz}{at^2}$ to $\frac{ez}{\theta^2}$, or if the force of the sun on Venus is taken as 1, the force of the primary planet on the satellite will be as $\frac{aet^2}{b\theta^2}$.

But the force of the primary planet on the satellite at the distance *PL*, is to the force by which it may itself act if it were at just as great a distance from the sun as Venus, inversely as the square of the distances, therefore there becomes $\frac{1}{e^2z^2}$ to $\frac{a^2}{b^2z^2}$, $\left(\text{or 1 to } \frac{a^2e^2}{b^2}\right)$ as $\frac{aet^2}{b\theta^2}$ to $\frac{a^3e^3}{b^3}\times\frac{t^2}{\theta^2}$; and again it will be found that the force of the sun on Venus is to the force of the primary *P* planet on the satellite, if it may be at just as great a distance by that amount that Venus is distant from the sun as 1 to $\frac{a^3e^3}{b^3}\times\frac{t^2}{\theta^2}$. This result follows for any positions of the circular orbits of the bodies, and hence is true for all positions; the quantities in the ratios are known from observation, and hence the force of the sun will be to the force of the planet as 1 to $\frac{a^3e^3}{b^3}\times\frac{t^2}{\theta^2}$. Thus, for example, let *P* be the centre of the planet Jupiter, and *L* the outer satellite, then *b* is to *a* as 72333 to 520096, while $e=\sin(8'16'')$, on inserting the ratio of the periodic times in seconds, and in those days using logarithms, the force of the attraction towards the centre of Jupiter is $\frac{1}{1067}$ of the attraction towards the sun.

Subsequent to Newton, astronomers have found more accurate values for the various distances and angles. Cohen gives a table comparing the present accepted values and Newton's values, in his Introduction. p.225.]

Corol. 2. Also the quantities of matter in the individual planets becomes known. For the quantities of matter in the planets are as the forces of these at equal distances from their centres; that is, on the sun, on Jupiter, Saturn, and the earth, they are as $1, \frac{1}{1067}, \frac{1}{3021}, \& \frac{1}{169282}$ respectively. If the parallax of the sun may be taken as greater than $10''$, and less than $30'''$, the quantity of matter on the earth must be increased or diminished in the cubed ratio.

Corol. 3. The densities of the planets become known also. For the weights of equal and homogeneous bodies on homogeneous spheres are at the surfaces of the spheres as the diameters of the spheres, by Prop. LXXII. Book I, and thus the densities of heterogeneous spheres are as these weights applied to the diameters of the spheres. But indeed the diameters of the sun, Jupiter, Saturn and of the earth in turn will be as 10000, 997, 791, and 109, and the weights at the surfaces of the same will be as 10000, 943, 529 and 435 respectively, and therefore the densities are as 100, $94\frac{1}{2}$, 67 and 400. The density of the earth which arises from this calculation does not depend on the parallax of the sun, but is determined from the parallax of the moon, and this therefore is defined correctly. Therefore the sun is a little denser than Jupiter, and Jupiter than Saturn, and the earth is four times denser than the sun. For the sun is rarefied by its remarkable heat. Truly the moon is denser than the earth, as will become apparent in the following.

Corol. 4. Therefore the smaller planets are denser, with all other things being equal. Thus the force of gravity approaches more to equality on the surfaces of these. And also the planets are denser, with all else being equal, which are closer to the sun ; as Jupiter with Saturn, and the earth with Jupiter. Certainly planets were to be gathered together at different distances from the sun so that as it were by a change in the density they might enjoy the heat from the sun more or less. Our water, if the earth were located in the orbit of Saturn, would be frozen, if in the orbit of Mercury it would depart at once into vapours. For the light of the sun, to which the heat is proportional, is seven times denser in the orbit of Mercury than with us : and with a thermometer I have found that with a seven-fold increase in the heat of the summer sun, water boils off. Truly there is no doubt why the matter of Mercury should not be accustomed to the heat, and therefore shall be denser than ours ; since all denser matter requires greater heat to take part in natural processes.

PROPOSITION IX. THEOREM IX.

Gravity on going downwards from the surfaces of planets decreased approximately as the distance from the centre.

If the matter of a planet were uniform as far as density is concerned, this proposition would be obtained accurately: by Prop. LXXIII. Book I. Therefore there is as great an error, as can arise from the inequality of the density.

PROPOSITION X. THEOREM X.

The motions of the planets in the heavens is able to be conserved for a very long time.

In the Scholium of Proposition XL, Book II, it has been shown that a sphere of frozen water, by moving freely in our air and by describing the length of its radius, may lose a $\frac{1}{4586}$th part of its motion by the resistance of the air. Moreover the same proportion will be obtained approximately in spheres with any size and speed. Now truly I deduce thus that sphere of our earth is denser than if the whole were made from water. If this sphere were entirely aqueous, in which some parts were rarer than water, on account of the smaller specific gravity these would rise and float on top. And by that reason the earthly sphere would arise covered by water on all sides, if some parts were rarer than water, and thus all the water flowing away might meet together in the opposite direction. And the account of our earth surrounded by great seas is the same. This, if it were not denser, would emerge from the seas, and by its part for the degree of lightness would stand out from the water, with seas flowing together in the opposite region. By the same argument the sunspots are lighter than the luminous matter of the sun on which they float. And in the formation of some kinds of planets, in which time all the heavier matter would seek the centre from the mass of water. From which, since with common ground uppermost, it shall be as if twice as dense as water, and a little further down in mines it may be found to be as much as three or four or even five times heavier: it is plausible that the abundance of all the matter in the earth shall be as if five times or greater than six times greater than if the whole were made from water; especially since it was shown before that the earth is to be as if four times denser than Jupiter. Whereby if Jupiter shall be a little denser than water, here in an interval of thirty days, in which it will describe a length of 459 of its radii, it may lose almost a tenth part of its motion in a medium of the same density as the density of our air. Truly since the resistance of the mediums may be reduced in the ratio of the weight and density, thus as water, which is $13\frac{3}{5}$ lighter than quicksilver, resists less in the same ratio ; and the air, which is in 860 parts lighter than water, resists less in the same ratio : if it [*i.e.* a body of the constitution of Jupiter] should ascend to the heavens where the weight of the medium, in which the planets are moving, is diminished indefinitely, the resistance will almost cease. At any rate we have shown in the scholium to Prop. XXII. Book II, that if it were to ascend to a height of 200 miles above the earth, the air there would be rarer than at the surface of the earth in the ratio 30 to 0,0000000000003998, or around 75000000000000 to 1. And hence the star of Jove by revolving in a medium of the same density with that upper air, in a time of 1000000 years, would not lose a ten hundred thousandth part of its motion to the resistance of the medium. Certainly in spaces closer to the earth, nothing is found that creates resistance other than exhalations and vapours. With these from a hollow glass cylinder with the air must diligently exhausted weight within the glass fall freely and without any sensible resistance ; a weight of gold itself and the finest feather dropped together fall with the same velocity, and in their case describing a height of four, six or eight feet reach the bottom at the same time, as has been ascertained by experiment. And therefore if it may ascend into the heavens devoid of air and exhalations, the planets and comets will be moving without any sensible resistance through that space for an exceedingly long time.

HYPOTHESIS I.

The centre of the system of the world is at rest.

This has been conceded by everyone, while some may contend that the earth, others the sun to be the centre of the system at rest. We may see which thence may follow.

PROPOSITION XI. THEOREM XI.

The common centre of gravity of the earth, the sun, and of all the planets is at rest.

For that centre (by Corol. IV of the laws) either may be at rest or may be progressing uniformly in a straight direction. But with that centre always progressing, the centre of the world also will be moving, contrary to the hypothesis.

PROPOSITION XII. THEOREM XII.

The sun is disturbed by the continual motion, but at no time does it recede far from the common centre of gravity of all the planets.

For since (by Corol. 2. Prop. VIII.) the matter in the sun shall be to the matter in Jupiter as 1067 to 1, and the distance of Jupiter from the sun shall be to the radius of the sun in a little greater ratio ; the common centre of gravity of Jupiter and the sun falls on a point a little above the surface of the sun. By the same argument since the matter in the sun shall be to the matter in Saturn as 3021 as 1, and the distance of Saturn from the sun shall be as the radius of the sun in a slightly smaller ratio : the common centre of gravity of Saturn and the sun lies a little below the surface of the sun. And by following in the footsteps of the same calculation, if the earth and all the planets were put in place at one side of the sun, the common centre of gravity of all scarcely would be different by the diameter of the whole sun from the centre of the sun. In all the other cases the distance of the centres is always less. And therefore since that centre of gravity will always be at rest, the sun will constantly be moving by the variation of the position of the planets in all directions, but at no time will it recede far from the centre.

[Thus, in this proposition, Newton gives us essentially a criterion for investigating the possibility of planetary systems circulating around nearby stars ; and in the same manner, gives a sign to other intelligent beings on planets elsewhere, if they exist, that the sun itself has such a planetary system.]

Corol. Hence the common centre of gravity of the earth, the sun and of all the planets is obtained for the centre of the world. For since the earth, the sun and the planets all gravitate amongst themselves mutually, and therefore, by its force of gravity, following the laws of motion will always be disturbed : it is evident that from the mobility of these, for the centre of the world to be at rest, they are unable to be consider at rest. If that body should be located in the centre in which all the bodies are attracted especially (as is the common opinion) that must be conceded to be the privilege of the sun. But since the sun may be moving, a point at rest must be selected, from which the centre of the sun departs minimally, and from which likewise besides it will depart less, only if the sun were denser and greater, so that it would be moving less.

PROPOSITION XIII. THEOREM XIII.

The planets are moving in ellipses having the centre of the sun as one focus, and with the radii drawn to that centre describing areas proportional to the times.

[See, e.g. Smart, *Celestial Mechanics*, the first few chapters, for a clear modern exposition of Kepler's Laws in terms of the law of gravitation.]

Above we have argued about these motions from phenomenae. Now with the principles of the motion known, we may deduced the motions from basic principles. Because the weights of the planets towards the sun are inversely as the squares of the distances from the centre of the sun ; if the sun were at rest and the remaining planets did not act on each other mutually, the orbits of these would be ellipses, having the sun at a common focal point, and the areas would be described in proportional times (by Prop. I. and Xl. and Corol. I. Prop. XIII. Book I.) ; but the actions of the planets among themselves are very small (so that they may be disregarded) and the motion of the planets in ellipses around the moving sun is less perturbing (by Prop. LXVI. Book I.) than if the motion were being undertaken as around the sun at rest.

Certainly the action of Jupiter on Saturn cannot be dismissed entirely. For the gravity towards Jupiter is to the gravity towards the sun (with equal distances) as 1 to 1067; [*i.e.* $\frac{M_{Sat.}\times M_{Jup.}}{M_{Sat.}\times M_{sun}}=\frac{1}{1067}$] and thus with Jupiter and Saturn in conjunction, because the distance of Saturn from Jupiter is as the distance of Saturn from the sun almost as 4 to 9 the gravity of Saturn towards Jupiter to the gravity of Saturn towards the sun will be as 81 to 16×1067 or as 1 to 211 approximately. [*i.e.* $\frac{acc_{SatJup}}{acc_{Satsun}}=\frac{\left(M_{Sat.}\times M_{Jup.}\right)/16}{\left(M_{Sat.}\times M_{sun}\right)/81}=\frac{1}{1067}\times\frac{81}{16}\approx\frac{1}{211}$]
And hence the perturbation of the orbit of Saturn arises from the individual conjunctions of this planet with Jupiter thus are perceptible so that astronomers are in difficulties with the same. On account of the variation in position of the planets in these conjunctions, the eccentricity of this now may be increased, now decreased, now the aphelion may be moved forwards and then perhaps it is drawn backwards, and the mean motion may be accelerated by the forces and then retarded. Yet the error in all the motion of this around the sun arising from such a force (except in the mean motion) can be almost avoided by putting in place the lower focus of its orbit at the common centre of gravity of Jupiter and the sun (by Prop. LXVII, Book I.) and therefore when it is a maximum, the error scarcely exceeds two minutes. And the maximum error in the mean motion scarcely exceeds two minutes per annum. But in the conjunction of Jupiter & Saturn the accelerating gravity of the sun towards Saturn, of Jupiter towards Saturn, and of Jupiter towards the sun are almost as 16, 81 and $\frac{16\times 81\times 3021}{25}$ or156069, and therefore the difference of the forces of gravity of the sun towards Saturn and of Jupiter towards Saturn is to the force of gravity of Jupiter towards the sun as 65 to 156609 or 1 to 2409.
[Because at the conjunction of Jupiter and Saturn, the distance of Saturn from the sun, Saturn from Jupiter, and Jupiter from the sun, are to each other as 9, 4, and 5,

approximately; we already have that $\frac{F_{Sat/Jup}}{F_{Sat/sun}} = \frac{(M_{Sat} \times M_{Jup.})/16}{(M_{Sat} \times M_{sun})/81} = \frac{1/16}{1067/81} = \frac{81}{1067 \times 16}$; while $\frac{F_{Jup/sun}}{F_{Sat/sun}} = \frac{(M_{sun} \times M_{Jup.})/25}{(M_{Sat} \times M_{sun})/81} = \frac{3021 \times 81}{1067 \times 25}$, and $\frac{F_{Jup/sun}}{F_{Sat/sun}} \times \frac{F_{Sat/sun}}{F_{Sat/Jup}} = \frac{F_{Jup/sun}}{F_{Sat/Jup}} = \frac{3021 \times 81}{1067 \times 25} \times \frac{1067 \times 16}{81} = \frac{3021 \times 16}{25}$.

Or,

$$F_{Sat/sun} : F_{Sat/Jup.} : F_{Jup/sun} :: \frac{1}{3021 \times 81} : \frac{1}{1067 \times 3021 \times 16} : \frac{1}{1067 \times 25}$$

or

$$acc_{sun/Sat} : acc_{Jup/Sat} : acc_{Jup/sun} :: \frac{1}{81 \times 3021} : \frac{1}{3021 \times 16} : \frac{1}{25}$$

i.e. $\frac{1}{81}, \frac{1}{16}$, and $\frac{3021}{25}$, by Cor. 1, Prop. VIII, that is, as 16, 81 and $\frac{16 \times 81 \times 3021}{25}$ or 156609 . Again, the difference of the accelerations of the sun on Saturn and of Jupiter on Saturn to the acceleration of Jupiter on the sun is as $\frac{a_{sun/Sat} - a_{Jup/Sat}}{a_{Jup/sun}} = \frac{\frac{1}{3021 \times 81} - \frac{1}{3021 \times 16}}{\frac{1}{25}} = \frac{65}{\frac{3021 \times 81 \times 16}{25}} = \frac{1}{2409}$; thus the effect of Saturn on Jupiter is much less than the effect of the sun on Jupiter (from an *L* & *J*. note).]

But the difference of this proportionality has the maximum effectiveness of Saturn in perturbing the motion of Jupiter, and therefore the perturbation of the orbit of Jupiter is far less than that of Saturn. The perturbations of the rest of the orbits are besides a great deal smaller except that the orbit of the earth is perceptibly disturbed by the moon. The common centre of gravity of the earth and the moon, travels around the sun – situated at the focus, in an ellipse, and a radius drawn to the sun will describe areas proportional to the time, truly the earth is revolving around this common centre in its monthly motion.

PROPOSITION XIV. THEOREM XIV.
The aphelions and nodes of orbits are at rest.

The aphelions are at rest by Prop. XI. Book I, and so that both the planes of the orbits, by Prop. I of the same book, and the nodes are at rest in the planes at rest. But yet from the revolution of the planets and from the actions of comets between themselves other inequalities may arise, but which here they are to be ignored on account of their insignificance.

Corol. 1. Also the fixed stars are at rest, therefore so that they maintain the given positions at the nodes and aphelions.

Corol. 2. And thus since nothing shall be evident of the parallax of these, arising from the annual motion of the earth, the forces of these, on account of the immense distance of the bodies, will give rise to no noticeable effect in the region of our system. Perhaps the fixed stars dispersed equally in all the parts of the heavens mutually destroy the forces from mutual attractions, by Prop. LXX. Book I.

Scholium.

Since the nearer planets of the sun (evidently Mercury, Venus, the earth, and Mars) on account of the smallness of the pairs of bodies acting in turn between themselves : the aphelions and nodes of these are at rest, unless perhaps they may be disturbed by the forces of Jupiter, Saturn and further bodies. And thence it can be deduced from the theory of gravitation, that the aphelia of these are moving a little in consequence with respect of the fixed stars, and that in the three on two proportion of the distances of the planets from the sun. So that if the aphelion of Mars should make 33′ , 20″ in a hundred years with respect of the fixed stars as a consequence; the aphelions of the earth, of Venus, and of Mercury in a hundred years would make 17′, 40″, 10′, 53″, and 4′, 16″ respectively. And these motions, on account of the smallness, are ignored in this proposition.

PROPOSITION XV. PROBLEM I.
To find the principal diameters of the orbits.

These are to be taken in the two on three ratio of the periodic times,
by Prop. XV, Book I, then one by one increased in the ratio of the sum of the masses of the sun and of each planet revolving to the first of the two mean proportionals between that sum and the sun, by Prop. LX, Book I. [See Smart, *Celestial Mechanics*, p.15]

PROPOSITIO XVI. PROBLEMA II.
To find the eccentricities and aphelions of the orbits.

The problem has been done in Prop. XVIII. Book I.

PROPOSITION XVII. THEOREM XV.
The daily motions of the planets is uniform, and the libration of the moon arises from its daily motion.

It is apparent by the first law of motion and Corol. 22. Prop. LXVI. Book I that Jupiter certainly is revolving with respect to the fixed stars in 9 hours and 56 minutes, Mars in 24 hours and 39 minutes, Venus in around 23 hours, the earth in 23 hours 56 minutes, the sun in $25\frac{1}{2}$ and the moon in 27 days 7 hours 43 minutes. It is evident that these are found from the phenomena [*i.e.* experimental data in modern jargon]. Spots in the body of the sun return at the same place on the solar disc in around $27\frac{1}{2}$ days, with respect to the earth ; and thus with respect to the fixed stars the sun is rotating in around $25\frac{1}{2}$ days. Truly because there is the monthly revolution of the moon about its axis : the same face of this will always look at the more distant focus of its orbit, as nearly as possible, and therefore according to the situation of that focus will hence deviate thence from the earth. This is the libration of the moon in longitude: For the libration in latitude has arisen from the latitude of the moon and the inclination of its axis to the plane of the ecliptic. *N. Mercator* has explained this theory of the libration of the moon more fully in letters from me, published in his *Astronomy* at the start of the year 1676.

[*Institutionum Astronomicarum libri duo*, p.286-7; see note on p.16 of Book I of *Newton's Letters*; these letters have been lost, but Newton's help had been fully acknowledged in Mercator's work.]

The outer satellite of Saturn may be seen to be revolving in a similar manner about its axis, with its same face always looking towards Saturn. For by revolving around Saturn, as often as it arrives at the eastern part of its orbit, it appears most decayed [in brightness] and the fullness seen to cease: as which can arise through certain spots on that part of the body which then is turned towards the earth, as *Cassini* noted. Also, the outer satellite of Jupiter may be seen to be to be revolving about its axis in a like motion, therefore so that it may have a spot on the part of its body turned away from Jupiter as in the body of Jupiter is discerned whenever the satellite passes between Jupiter and our eyes.

PROPOSITION XVIII. THEOREM XVI.

The axes of the diameters of the planets which are drawn normally to the same axes are the minor axes.

Planets must conform to a spherical shape with all the daily circular motion removed, on account of the equal weight of the parts on all sides. Through that circular motion it comes about that parts next to the equatorial axis will try to rise, receding from the axis. And thus the matter, if it shall be fluid, rising by itself will increase the diameter of the equator, truly the axis at the poles will be diminished by its falling. Thus the axis of Jupiter (in common agreement from observations) it taken to be shorter between the poles than from East to West. By the same argument, unless our earth were a little higher at the equator than at the poles, the seas would subside at the poles, and by ascending adjoining the equator, there they would flood everything.

[Again, need it be said, we have a present instance of Newton's insight : the melting of glaciers and polar ice caps due to global warming is and will continue to increase the equatorial bulge of the earth, leading to permanent flooding of equatorial regions, increased tidal activity, etc. ; in the same manner, the sea levels in the northern polar regions may remain unchanged or actually fall.]

PROPOSITION XIX. PROBLEM III.

To find the proportion of the axis of a planet to the same perpendicular diameter.

Our countryman *Norwood* around the year 1635 by measuring the distance between London and York as 905751 London feet, and by observing the difference of the latitude to be $2^0, 28'$ deduced the measure of one degree to be 367196 London feet, that is, 57300 Paris toises.

Picart by measuring the arc of $1^0, 22', 55''$ in the meridian between *Amiens* and *Malvoisine,* found the arc of 1^0 to be 57060 Paris hexafeet [or toises]. *Cassini* the elder measured the distance along the meridian from the town of *Colliore* in *Roussilon* to the observatory in Paris; and his son had added the distance from the observatory to the tower of the town Dunkirk. The total distance was $486156\frac{1}{2}$ toises and the difference of the latitudes of the towns Collioure and Dunkirk was $8^0, 31', 11\frac{1}{6}''$. From which the arc of 1^0

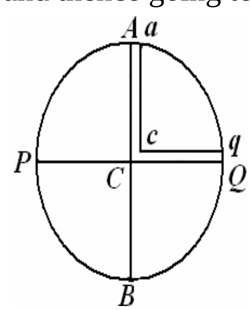
produced 57061 toises. And from these measurements a circuit around the earth is deduced to be 123249600 Paris feet, and its radius 19615800 Paris feet, from the hypothesis that the earth shall be a sphere.

In the latitude of Paris a heavy body by falling in a time of one second will describe 15 Paris feet, 1 inch and $1\frac{7}{9}$ lines as above, that is, $2173\frac{7}{9}$ lines. The weight of the body is diminished by the weight of the surrounding air. We may suppose the weight lost to be one eleven thousandth part of the total weight, and that heavy body by falling in a vacuum describes a height of 2174 lines in one second. [recall that 1 line is $\frac{1}{12}$ part of an inch.]

A body revolving uniformly in a circle at a distance of 19615800 feet from the centre, in single sidereal days of 23 hours, 56′ minutes, and 4″ seconds will describe in a time of one second an arc of 1433,46 feet, the versed sine of which is 0,052656, or of 7,54064 lines. And thus the force, by which weights descend at the latitude of *Paris*, is to the centrifugal force of bodies at the equator arising from the daily motion of the earth, as 2174 to 7,54064.

The centrifugal force of bodies at the equator of the earth is to the centrifugal force, by which bodies tend to move directly from the earth at the latitude of Paris of 48^0, 50′, 10″, in the square ratio of the radius to the sine of the complement of its latitude, that is, as 7,54064 to 3,267. This force may be added to the force by which the bodies descend at that latitude of Paris, and a body at that latitude by falling with that total force, in a time of one second describes 2177,267 lines, or 15 feet 1 inch and 5,267 lines. And the total force of gravity at that latitude will be to the centrifugal force of bodies at the equator of the earth as 2177,167 to 7,54064 or 289 to 1.

From which if *APBQ* may designate the figure of the earth now no longer spherical but generated by the revolution of an ellipse around the minor axis *PQ*, and *ACQqca* shall be a channel full of water, from the pole *Qq* to the centre *Cc*, and thence going to the equator *Aa* : the weight of water in the legs of the channel *ACca*, is to the weight of water in the other leg *QCcq* as 289 to 288, because the centrifugal force arising from the circular motion will sustain one part of the 289 parts and may be taken from the weight of the parts, and the weight 288 in the other leg will sustain the rest. [The alternate diagram for a prolate spheroid rather than the true oblate spheroid has been drawn in all three editions of the *Principia*, with *PQ* taken as the bulge; I have changed this to the oblate shape, as Chandrasekhar has done, but the orientation is now unusual, as the polar axis *PQ* is now horizontal ; this is the usual view of someone standing at the equator and looking North along *CQ*.] Again (from Proposition XCI. Corol. 2. Book I.) by entering upon a computation, [which is quite long and is presented by Chandrasakher in Ch. 20, p. 281 of his work on Newton,] I find that if the earth should be constructed from uniform matter, and it may be deprived of all motion, and its axis *PQ* to be to the diameter *AB* as 100 to 101; the gravity at the location *Q* on the earth becomes to the gravity at the same location *Q* in a sphere with centre *C* with the radius *PC* or *QC* described, as 126 to 125. And by the same argument the weight at the position *A* on the spheroid, by convoluting the ellipse *APBQ* described about the axis *AB*, is to the weight at the same place *A* on the sphere described with centre *C* and



with the radius *AC*, as 125 to 126. But the weight at the position *A* on the earth is the mean proportional between the weights on the said spheroid and sphere : so that therefore the sphere, by diminishing the diameter *PQ* in the ratio 101 to 100, is turned into the figure of the earth ; and this figure with a third diameter which is perpendicular to the two diameters *AB* and *PQ*, being diminished in the same ratio, is turned into the said spheroid ; and the gravity at *A*, in each case, is diminished in approximately the same ratio. Therefore the gravity at *A* on the sphere with centre *C* and described with the radius *AC* is to the gravity at *A* on the earth as 126 to $125\frac{1}{2}$, and the gravity at the position *Q* on the sphere with centre *C* and described with the radius *QC*, is to the gravity at the location *A* on the sphere described with centre *C* and with radius *QC*, in the ratio of the diameters (by Prop. LXXII. Book I.) that is, as 100 to101. Now these three ratios may be taken together, 126 to 125, 126 to $125\frac{1}{2}$, and 100 to 101: and the gravity at the location *Q* on the earth becomes to the gravity at the position *A* on the earth, as $126\times116\times100$ to $125\times125\frac{1}{2}\times101$, or as 501 to 500.

Now since (by Corol. 3, Prop. XCI, Book I.) the weight in either leg of the canal *ACca* or *QCcq* shall be as the distances of the places from the centre of the earth ; if those legs may be separated into transverse and equidistant surfaces proportional to the total parts, the weights of the individual parts in the leg *ACca* will be to the weights of just as many parts in the other leg, as the magnitudes and accelerating weights jointly ; that is, as 101 to 100 and 500 to 501, that is, as 505 to 501. And thus if the centrifugal force of each part in the leg *ACca* arising from the daily motion were to the weight of each part as 4 to 505, so that therefore from the weight of each part, by dividing into 505 parts, four parts may be subtracted; the weights will remain equal in each leg, and therefore the fluid might remain in equilibrium. Truly the centrifugal force of each part is to the weight of the same as 1 to 289, that is, the centrifugal force must be equal to the $\frac{4}{505}$ part of the weight is only the $\frac{1}{289}$. part. And therefore I say, following the golden rule of proportions, that if the centrifugal force were made $\frac{4}{505}$ as the height of the water in the leg *ACca* may exceed the height of water in the leg *QCcq* by the hundredth part of the total height: the centrifugal force $\frac{1}{289}$ may be made as the excess of the height in the leg *ACca* shall be of the height in the other leg *QCcq* by only the $\frac{2}{229}$ part. Therefore the diameter of the earth along the equator to the diameter through the poles is as 230 to 229. And thus since the mean radius of the earth, close to the measurement of *Picart*, shall be 19615800 Paris feet, or of 3923,16 miles (on putting that a mile shall be a measure of 5000 feet), the earth will be higher at the equator than at the poles by an excess of 85472 feet, or of $17\frac{1}{10}$ miles. And its altitude at the equator will be around 19658600 feet, and at the poles 19573000 feet.

If a planet shall be greater or smaller than the earth with its density remaining and the daily periodic time of revolution, the proportion of the centrifugal force to gravity will remain, and therefore the proportion of the diameter between the poles to the diameter along the equator will remain also. Or if the daily motion may be accelerated or retarded in some ratio, the centrifugal force will be augmented or decreased in that ratio squared, and therefore the difference of the diameters will be increased or diminished in the same ratio approximately. And if the density of the planet may be increased or decreased in some ratio, the drawing force of gravity also will be increased or decreased in the same ratio, and the difference of the diameters in turn will be diminished in the ratio of the of the increased gravity or diminished in the ratio of the diminished gravity. From which since the earth with respect to the fixed stars may be revolving in 23hours and 56 minutes, but Jupiter in 9 hours and 56 minutes and the squares of the times shall be as 29 to 5, and the densities of the revolving bodies as 400 to $94\frac{1}{2}$: the difference of the diameters of Jupiter will be to the smaller diameter as $\frac{29}{5}\times\frac{400}{94\frac{1}{2}}\times\frac{1}{229}$ to 1, or 1 to $9\frac{1}{3}$ approximately. Therefore the diameter of Jupiter drawn from the east to the west , to the diameter between the poles is as $10\frac{1}{3}$ to $9\frac{1}{3}$ approximately. From which since the major diameter of this shall be 37″, the minor diameter lying between the poles will be 33″, 25‴. From the erratic [refraction of the] light there may be added around 3", and the apparent diameters of this planet emerge 40″ and 36″ : which are to each other approximately as $11\frac{1}{6}$ to $10\frac{1}{6}$. This thus may be itself had from the hypothesis that the body of Jupiter shall be of uniform density. But if its body shall be denser towards the plane of the equator rather than the poles, it diameters may be in turn as 12 to 11, 13 to 12, or perhaps as 14 to 13. And indeed *Cassini* observed in the year 1691, that the diameter of Jupiter stretched out from east to west surpassed the other by about a fifteenth part. Moreover our countryman *Pound* with a telescope of 123 feet long and with the best micrometer, measured the diameters of Jupiter in the year 1719, as follows.

Times.		*Diam. max.*	*Diam. min.*	*Diameters ratio.*	
days	*hours.*	*part.*	*part.*		
Jan. 28	6	13,40	12,28	*as* 12	*to* 11
Mar. 6	7	13,12	12,20	$13\frac{3}{4}$	$12\frac{3}{4}$
Mar. 9	7	13,12	12,08	$12\frac{2}{3}$	$11\frac{2}{3}$
Apr. 9	9	12,32	11,48	$12\frac{1}{2}$	$11\frac{1}{2}$

Therefore the theory has agreed with the phenomena. For planets are warmed more in the light of the sun towards their equators, and therefore they are heated there a little more than towards the poles.

Even though the force of gravity be diminished at the equator by the daily rotation of our earth, and therefore there the earth rises higher than at the poles (if its matter shall be of uniform density), it will be apparent from experiments with pendulums which are reviewed in the following proposition.

[See Chandrasekhar Ch. 20 for a modern exposition of this proposition and related work.]

PROPOS1TION XX. PROBLEM IV.

To find and to compare amongst themselves, the weights of bodies in different regions of the earth.

Because the weights of the unequal legs of the channels of water *ACQqca* are equal; and the weights of the parts proportional to the whole legs, and similarly situated to the whole, are in turn as the weights of the whole, thus they are also equal to each other ; the weights of the equal and similarly situated parts are inversely as the legs, that is, inversely as 230 to 229. And likewise the ratio of any homogeneous and similarly situated bodies in the legs of the channels. The weights of these are reciprocally as the legs, that is, reciprocally as the distances of the bodies from the centre of the earth. Thus if bodies may be considered in the upper parts of the channels, or on the surface of the earth ; the weights of these in turn will be inversely as the distances of these from the centre. And the weights by the same argument, in any other regions over the whole surface of the earth, are inversely as the distances of the places from the centre; and therefore are given in proportion, from the hypothesis that the earth shall be a spheroid.

From such thence a theorem arises, that the increment of the weight in going from the equator to the pole, shall be nearly as the versed sine of twice the latitude, or which is the same thing, as the sine square of the right latitude. And the arc of the degrees of latitude may be increased in around the same ratio in the meridian. Therefore since the latitude of Paris shall be $48^0, 50'$, that of places on the equator $00^0, 00'$, and that of places at the poles 90^0, and the squares of the versed sines shall be as 11334, 00000 and 20000, with the radius present 10000, and the weight at the pole shall be to the weight at the equator as 230 to 229, and the excess of the weight at the pole to the weight at the equator shall be as 1 to 229: therefore the excess of the weight at the latitude of Paris to the weight at the equator shall be as $1\times\frac{11334}{20000}$ to 229, or 5667 to 2290000 . And therefore the total weighs in these places will be in turn as 2295667 to 2290000. Whereby since the lengths of isochronous pendulums shall be as the forces of gravity, and at the latitude of Paris the length of a pendulum oscillating in individual seconds shall be three Paris feet and $8\frac{1}{2}$ lines, or rather on account of the weight of the air $8\frac{5}{9}$: the length of the pendulum at the equator will be overcome by the length of the Parisian pendulum by an excess of 1,087 parts of a line. And by a similar calculation the following table can be constructed.

General latitude of the place.	*Length of the pendulum.*	*Measure of 1^0 along a meridian.*
Degrees	*Ft. Lin.*	*Toises.*
0	3 7,468	56637
5	3 7,482	56642
10	3 7,526	56659
15	3 7,596	56687
20	3 7,692	56724
25	3 7,812	56769
30	3 7,948	56823
35	3 8,099	56882
40	3 8,261	56945
1	3 8,294	56958

2	3 8,327	56971
3	3 8,361	56984
4	3 8,394	56997
45	3 8,428	57010
6	3 8,461	57022
7	3 8,494	57035
8	3 8,528	57048
9	3 8,561	57061
50	3 8,594	57074
55	3 8,756	57137
60	3 8,907	57196
65	3 9,044	57250
70	3 9,162	57295
75	3 9,258	57332
80	3 9,329	57360
85	3 9,371	57377
90	3 9,387	57382

Moreover it is agreed by this table that the inequalities of degrees shall be so small that in it may be taken as a sphere in the geographical shape of the earth: especially if the earth were a little denser towards the equatorial plane than towards the poles.

Now truly some astronomers sent to far away regions to make astronomical observations, observe that oscillating clocks may move slower near the equator than in our regions. And indeed *Richer* observed this in the year 1672 on the island of Cayenne. For while he observed the transit of fixed stars through the meridian in the month of August, he found his clock to be moving slower than for the mean motion of the sun, with a difference present of $2'.28''$ for individual days. Thereupon by arranging so that a simple pendulum might oscillate in single seconds as measured by a reliable clock, he noted the length of the simple pendulum, and this he did often each day for ten months. Then on returning to France he compared the length of this pendulum with the length of the Parisian pendulum (which was of three Parisian feet, and $8\frac{3}{5}$) and he found it to be shorter, with the difference being $1\frac{1}{4}$ lines.

Afterwards our countryman *Halley* sailing to the island of St. Helens in the year 1677, found the oscillations of his clock there to be moving slower than in London, but he did not measure the difference. Truly he rendered the pendulum shorter by $\frac{1}{8}$ part of an inch, or by $1\frac{1}{2}$ lines. And in order to do this, since the length of the screw at the end of the pendulum was insufficient, he interposed a wooden ring between the sheath of the screw and the pendulum bob.

Then in the year 1682, *Varin* and *Des Hayes* found the length of the pendulum oscillating in a time of one second at the royal observatory in Paris to be 3ft.,$8\frac{5}{9}$ lin. And on the island of Gorée by the same method they found the length of the isochronous pendulum to be 3 ft.,$6\frac{5}{9}$ lin., with a difference of the lengths present of 2 lin. And in the same year sailing to the islands of Guadeloupe and Martinique, the found the length of the isochronous pendulum on these islands to be 3ft., $6\frac{1}{2}$ lin.

After this *Couplet* the younger in the month of July of the year 1697, thus setting his pendulum clock to the mean motion of the sun at the royal observatory in Paris, so that in a long enough time the clock was in agreement with the motion of the sun. Then sailing to Lisbon he found that late in the month of November the clock would go slower than before, with a time difference of 2′, 13″ in 24 hours. And in the month of March following sailing to Paraiba he found there his clock to go slower than in Paris, with a difference present of 4′, 12″ in 24 hours. And it confirms that the pendulum swinging in one second should be shorter in Lisbon by $2\frac{1}{2}$ lin. and in Paraiba by $3\frac{2}{3}$ lines than in Paris. He would have corrected the differences better to be $1\frac{1}{3}$ and $2\frac{5}{9}$. For these differences [of the lengths] correspond to the differences of the times 2′, 13″, and 4′, 12″. There is less trust in gross observations of this kind.

In the next years (1699 and 1700) *des Hayes* sailing again to America, determined that on the islands of Cayenne and Granada the length of a pendulum swinging in a time of one second, should be a little less than 3 ft., $6\frac{1}{2}$ lin., and that on the island of St. Christophers that length should be 3 ft., $6\frac{3}{4}$ lin., and that on the island of St. Dominica the same should be 3 ft., 7 lin.

In the year 1704 *Father Feuillée* found at Portobello in America that the length of a pendulum oscillating in seconds was 3 Parisian ft., and only $5\frac{7}{12}$ lin, that is, almost a line shorter than at Paris, but from an erring observation. For then sailing to the island of Martinique, he found the length of the isochronous pendulum to be 3 Parisian ft., and $5\frac{10}{12}$ lin.

But the latitude of Paraiba is 6^0, 38′ to the South, and that of Portobello 9^0, 33′ to the North, and the latitudes of the islands of Cayenne, Gorée, Guadeloupe, Martinique, Granada, & St. Christophors [St. Kitts] and St. Dominica are respectively 4^0, 55′, 14^0, 40', 14^0, 00′, 14^0, 44′, 12^0, 6′, 17^0, 19', and 19^0, 48′ to the North. And the excess of the longitude of the Parisian pendulum over the lengths of the isochronous pendulums at these latitudes are a little greater than for the table of lengths of pendulums calculated above. And therefore the earth is a little higher at the equator than for the above calculation, and denser at the centre than in the mines near the surface, except perhaps the heat in the torrid zones may have increased the lengths of the pendulums a little.

Certainly *Picart* observed that iron rods, which in winter time were one foot in length frozen solid, heated in the fire emerged one foot with the quarter of a line. Then *de la Hire* observed that an iron rod which in the same manner in winter time was six foot long, when it was exposed to the summer sun emerged six feet in length with the two thirds part of a line. In the former case the heat was greater than in the latter case, truly in this latter case it was greater than the heat of the external parts of the human body. For metals become exceedingly hot in the summer sun. But the rod of a pendulum in the pendulum clock at no time is accustomed to be exposed to the heat of the summer sun, nor at any time to take in heat equal to the heat of the external surface of the human body. And therefore the rod of the pendulum in the clock three feet in length, indeed will be a little longer in summer time than in winter time, but scarcely by exceeding a quarter of a line. Therefore the total difference of the lengths of the pendulums which are isochronous in diverse regions, cannot be attributed to different amounts of heat. But nor is this difference to be attributed to errors of the astronomers sent from France. For whatever of their observations were not in perfect agreement among themselves, yet the errors are small and thus may be ignored. [One may note that at this time the science of statistics was virtually unknown.] And they all agree, that isochronous pendulums are shorter at the equator than at the royal observatory in Paris, with a difference not less than $1\frac{1}{4}$, not greater than $2\frac{2}{3}$. Through the observations made by *Richer* in Cayenne, the difference was $1\frac{1}{4}$. From these made by *des Hayes* that corrected difference produced $1\frac{1}{2}$ lines or $1\frac{3}{4}$ lines. From these measurements of others made with less accuracy the same result was produced as around 2 lines. And this discrepancy was able to arise partially from errors in observations, partially from the differences of the internal parts of the earth and the height of mountains, and partially from the different heats [that we would now call temperatures] in the air.

An iron rod three feet long, in winter time in England, is shorter than in summer time, I believe, by the sixth part of a line. On account of the heat at the equator this quantity [$\frac{1}{6}$th of a line] may be taken away from the difference of the lines $1\frac{1}{4}$ observed by *Richer*, and $1\frac{1}{12}$ line will remain which now agrees properly with that deduced before by theory $1\frac{87}{1000}$. Moreover the observations made by *Richer* in Cayenne, that he repeated in the individual weeks through ten months, and the lengths of the pendulum in the iron rod noted with the length of that similarly noted when he returned to France. Which diligence and caution seems to be lacking in the observations of others. If it is required to put trust in the observations of this, the earth is higher at the equator than at the poles in excess of around 17 miles as predicted by the above theory.

PROPOSITION XXI. THEOREM XVII.

The equinoctial points regress, and the axis of the earth by nutating in the individual yearly revolution, inclines twice towards the ecliptic and twice to the first position.

It is apparent by Corol. 20. Prop. LXVI. Book I. But that motion of nutation must be very small, and scarcely or not perceptible at all.

DE MUNDI SYSTEMATE.

LIBER TERTIUS III.

In libris praecedentibus principia philosophiae tradidi, non tamen philosophica sed mathematica tantum, ex quibus videlicet in rebus philosophicis disputari possit. Haec sunt motuum & virium leges & conditiones, quae ad philosophiam maxime spectant. Eadem tamen, ne sterilia videantur, illustravi scholiis quibusdam philosophicis, ea tractans quae generalia sunt, & in quibus philosophia maxime fundari videtur, uti corporum densitatem & restistentiam, spatia corporibus vacua, motumque lucis & sonorum. Superest ut ex iisdem principiis doceamus constitutionem systematis mundani. De hoc argumento composueram librum tertium methodo populari, ut a pluribus legeretur. Sed quibus principia posita satis intellecta non fuerint, ii vim consequentiarum minime percipient, neque praejudicia deponent, quibus a multis retro annis insueverunt: & propterea ne res in disputationes trahatur, summam libri illius transtuli, in propositiones, more mathematico, ut ab iis solis legantur qui principia prius evolverint. Veruntamen quoniam propositiones ibi quam plurimae occurrant, quae lectoribus etiam mathematice doctis moram nimiam injicere possint, auctor esse nolo ut quisquam eas omnes evolvat; suffecerit siquis definitiones, leges motuum & sectiones tres priores libri primi sedulo legat, dein transeat ad hunc librum de mundi systemate, & reliquas librorum priorum propositiones hic citatas pro lubitu consulat.

REGULAE PHILOSOPHANDI.

REGULA I.

Causas rerum naturalium non plures admitti debere, quam quae *& verae sint & earum phaenomenis explicandis sufficiant.*

Dicunt utique philosophi: Natura nihil agit frustra, & frustra fit per plura quod fieri potest per pauciora. Natura enim simplex est & rerum causis superfluis non luxuriat.

REGULA II.

ldeoque effectuum naturalium eiusdem generis eaedem assignandae sunt causae, quatenus fieri potest.

Uti respirationis in homine & in bestia; descensus lapidum in *Europa* & in *America;* lucis in igne culinari & in sole ; reflexionis lucis in terra & in planetis.

REGULA III.

Qualitates corporum quae intendi & remitti nequeunt, quaeque corporibus omnibus competunt in quibus experimenta instituere licet, pro qualitatibus corporum universorum habendae sunt.

Nam qualitates corporum non nisi per experimenta innotescunt, ideoque generales statuendae sunt quotquot cum experimentis generaliter quadrant ; & quae minui non possunt, non possunt auferri. Certe contra experimentorum tenorem somnia temere confingenda non sunt, nec a naturae analogia recedendum est, cum ea simplex esse soleat & sibi semper consona. Extensio corporum non nisi per sensus innotescit, nec in omnibus sentitur: sed quia sensibilibus omnibus competit, de universis affirmatur. Corpora plura dura esse experimur. Oritur autem durities totius a duritie partium, & inde non horum tantum corporum quae sentiuntur, sed aliarum etiam omnium particulas indivisas esse duras merito concludimus. Corpora omnia impenetrabilia esse non ratione sed sensu colligimus. Quae tractamus, impenetrabilitia inveniuntur, & inde concludimus impenetrabilitatem esse proprietatem corporum universorum. Corpora omnia mobilia esse, & viribus quibusdam (quas vires inertiae vocamus) perseverare in motu vel quiete, ex hisce corporum visorum proprietatibus colligimus. Extensio, durities, impenetrabilitas, mobilitas & vis inertiae totius oritur ab extensione, duritie, impenetrabilitate, mobilitate & viribus inertiae partium: & inde concludimus omnes omnium corporum partes minimas extendi & duras esse & impenetrabiles & mobiles & viribus inertiae praeditas. Et hoc est fundamentum philosophiae totius. Porro corporum partes divisas & sibi mutua contiguas ab invicem separari posse, ex phaenomenis novimus, & partes indivisas in partes minores ratione distingui posse ex mathematica certum est. Utrum vera partes illae distinctae & nondum divisae per vires naturae dividi & ab invicem separari possint, incertum est. At si vel unico constaret experimento quod particula aliqua indivisa, frangendo corpus durum & solidum, divisionem pateretur: concluderemus vi huius regulae, quod non solum partes divisae separabiles essent, sed etiam quod indivisae in infinitum dividi possent.

Denique si corpora omnia in circuitu terrae gravia esse in terram, idque pro quantitate materiae in singulis, & lunam gravem esse in terram pro quantitate materiae suae, & vicissim mare nostrum grave esse in lunam, & planetas omnes graves esse in se mutuo, & cometarum similem esse gravitatem in solem, per experimenta & observationes astronomicas universaliter constet: dicendum erit per hanc regulam quod corpora omnia in se mutua gravitant. Nam & fortius erit argumentum ex phaenomenis de gravitate universali, quam de corporum impenetrabilitate : de qua utique in corporibus coelestibus nullum experimentum, nullam prorsus observationem habemus. Attamen gravitatem corporibus essentialem esse minime affirmo. Per vim insitam intelligo solam vim inertiae. Haec immutabilis est. Gravitas recedendo a terra, diminuitur.

REGULA IV.

In philosophia experimentali, propositiones ex phaenomenis per inductionem collectae, non obstantibus contrariis hypothesibus, pro veris aut accurate aut quamproxime haberi debent, donec alia occurrerint phaenomena, per quae aut accuratiores reddantur aut exceptionibus obnoxiae.

Hoc fieri debet ne argumentum inductionis tollatur per hypotheses.

PHAENOMENA.

PHAENOMENON I.

Planetas circumjoviales, radiis ad centrum jovis ductis, areas describere temporibus proportionales, eorumque tempora periodica, stellis fixis quiescentibus, esse in ratione sesquiplicata distantiarum ab ipsius centro.

Constat ex observationibus astronomicis. Orbes horum planetarum non differunt sensibiliter a circulis jovi concenticis, & motus eorum in his circulis uniformes deprehenduntur. Tempora vero periodica esse in sesquiplicata ratione semidiametrorum orbium consentiunt astronomi ; & idem ex tabula sequente manifestum est.

Satellitum jovialium tempora periodica.

$1^d.18^h.27'.34''$; $3^d.13^h.13'.42''$; $7^d.3^h.42'.36''$;$16^d.16^h.32'.9''$.

Distantiae satellitum a centro jovis.

Ex observationibus	1	2	3	4	
Borelli .	$5\frac{2}{3}$	$8\frac{2}{3}$	14	$24\frac{2}{3}$	
Townlei *per microm.*	5,52	8,78	13,47	24,72	Semidiam. Jovis
Cassini *per telescop.*	5	8	13	23	
Cassini *per eclips.satell.*	$5\frac{2}{3}$	9	$14\frac{23}{66}$	$25\frac{3}{16}$	
Ex tcmporibus periodicis.	5,667	9,017	14,384	25,299	

Elongationes satellitum jovis & diametrum eius D. *Pound* micrometris optimis determinavit ut sequitur. Elongatio maxima heliocentrica satellitis quasii a centro jovis micrometro in tubo quindecim pedes longo capta fuit, & prodiit in mediocri jovis a terra distantia $8'.16''$ circiter. Ea satellitis tertii micrometro in telescopio pedes 123 longo capta fuit, & prodiit in eadem jovis a terra distantia $4'.42''$. Elongationes maximae reliquorum satellitum in eadem jovis a terra distantia ex temporibus periodicis prodeunt $2'.56''.47'''$, & $1'.51''.6'''$.

Diameter jovis micrometro in telescopio 123 pedes longo saepius capta fuit, & ad mediocrem jovis a sole vel terra distantiam reducta, semper minor prodiit quam $40''$, nunquam minor quam $38''$, saepius $39''$. In telescopiis brevioribus haec diameter est $40'$ vel $41'$. Nam lux jovis per inaequalem refrangibilitatem nonnihil dilatatur, & haec dilatatio minorem habet rationem ad diametrum jovis in longioribus & perfectoribus telescopiis quam in brevioribus & minus perfectis.

Tempora quibus satellites duo, primus ae tertius. transibant per corpus jovis, ab initio ingressus ad initium exitus, & ab ingressu completo ad exitum completum, observata sunt ope telescopii eiusdem longioris. Et diameter jovis in mediocri eius a terra distantia prodiit per transitum primi satellitis $37\frac{1}{8}''$ & per. transitum tertii $37\frac{3}{8}''$. Tempus etiam quo umbra primi satellitis transiit per corpus jovis observatum fuit, & inde diameter jovis in mediocri ejus a terra distantia prodiit $37''$ circiter. Assumamus diametrum ejus esse $37\frac{1}{4}''$ quamproxime; & elongationes maximae satellitis primi, secundi, tertii, & quasii aequales erunt, semidiametris jovis 5,965, 9,494, 15,141, & 26,63 respective.

PHAENOMENON II.

Planetas circumsaturnum, radiis ad saturnum ductis, areas describere temporibus proportionales, & *eorum tempora periodica, stellis quiescentibus, esse in ratione sesquiplicata distantiarum ab ipsius centro.*

Cassinus utique ex observationibus suis distantias eorum a centro saturni & periodica tempora huiusmodi esse statuit.

Satellitum saturniorum tempora periodica.

$1^d.21^h.18'.27''$; $2^d.17^h.41'.22''$; $4^d.12^h.25'.12''$; $15^d.22^h.41'.14''$; $79^d.7^h.48'.00''$.

Distantiae satellitum a centro saturni in simidiametris annuli.

Ex obscrvationibus.	$1\frac{19}{20}$.	$2\frac{1}{3}$.	$3\frac{1}{2}$.	8.	24.
Ex temporibus periodicis.	1,93.	2,47.	3,45	8.	23,35.

Quarti satellitis elongatio maxima a centro saturni ex observationibus colligi solet esse semidiametrorum octo quamproxime. At elongatio maxima satellitis huius a centro saturni, micrometro optima in telescopio Hugeniano pedes 123 longo capta, prodiit semidiametrorum octo cum septem decimis partibus semidiametri. Et ex hac observatione & temporibus periodicis, distantiae satellitum a centro saturni in semidiametris annuli sunt 2,1. 2,69. 3,75. 8,7. & 25,35. Saturni diameter in eodem telescopio erat ad diametrum annuli ut 3 ad 7, & diameter annuli diebus Maii 28 & 29 anni 1719. prodiit $43''$. Et inde diameter annuli in mediocri saturni a terra distantia est $42''$, & diameter saturni $18''$. Haec ita sunt in telescopiis longissimis & optimis, propterea quod magnitudines apparentes corporum coelestium in longioribus telescopiis majorem habeant proportionem ad dilatationem lucis in terminis illorum corporum quam in brevioribus. Si reiiciatur lux omnis erratica, manebit diameter saturni haud major quam $16''$.

PHAENOMENON III.

Planetas quinque primarios mercutium, venerem, martem, jovem & saturnum orbibus suis solem cingere.

Mercurium & venerem circa solem revolvi ex eorum phasibus lunaribus demonstratur. Plena facie lucentes ultra solem siti sunt; dimidiata e regione solis; falcata cis solem, per discum eius ad modum macularum nonnunquam transeuntes. Ex martis quoque plena facie prope solis conjunctionem, & gibbosa in quadraturis, certum est, quod is solem ambit. De jove etiam & saturno idem ex eorum phasibus semper plenis demonstratur: hos enim luce a sole mutuata splendere ex umbris satellitum in ipsos projectis manifestum est.

PHAENOMENON IV.

Planetarum quinque primariorum, & *vel solis circa terram vel terrae circa solem tempora periodica, stellis fixis quiescentibus, esse in ratione sesquiplicata mediocrium distantia a sole.*

Haec a *Keplero* inventa ratio in confesso est apud omnes. Eadem utique sunt tempora periodica, eaedemque orbium dimensiones, sive sol circa terram, sive terra circa solem revolvatur. Ac de mensura quidem temporum periodicorum convenit inter astronomos universos. Magnitudines autem orbium *Kepleriis* & *Bullialdus* omnium diligentissime ex observationibus determinaverunt: & distantiae mediocres, quae temporibus periodicis respondent, non differunt sensibiliter a distantiis quas illi invenerunt, suntque inter ipsas ut plurimum intermediae; uti in tabula sequente videre licet.

Planetarum ac telluris tempora periodica circa solem respectu
♁ *fixarum, in diebus* & *partibus decimalibus diei.*

♄	♃	♂	♁	♀	☿
10759,275.	4332, 524	686,9785.	365,2565.	224,6176.	87,969.

Planetarum ac tellutis distantiae mediocres a sole.

	♄	♃	♂	♁	♀	☿
Secundum *Keplerum*	951000.	519650.	152350.	100000.	72400.	38806.
Secundum *Bullialdum*	954198.	522520.	152350.	100000.	72398.	38585.
Secundum tempora periodica	954006.	520096.	152369.	100000.	72333	38710.

De distantiis mercurii & veneris a sole disputandi non est locus, cum hae per eorum elongationes a sole determinentur. De distantiis etiam superiorum planetarum a sole tollitur omnis disputatio per eclipses satellitum jovis. Etenim per eclipses illas determinatur positio umbrae quam jupiter proiicit, & eo nomine habetur jovis longitudo heliocentrica. Ex longitudinibus autem heliocentrica & geocentrica inter se collatis determinatur distantia jovis.

PHAENOMENON V.

Planetas primarios, radiis ad terram ductis, areas describere temporibus minime proportionales; at radiis solem ductis, areas temporibus proportionales percurrere.

Nam respectu terrae nunc progrediuntur, nunc stationarii sunt, nunc etiam regrediuntur: At solis respectu semper progrediuntur, idque propemodum uniformi cum motu, sed paulo celerius tamen in periheliis ac tardius in apheliis, sic ut arearum aequabilis sit descriptio. Propositio est astronomis notissima, & in jove apprime demonstratur per eclipses satellitum, quibus eclipsibus heliocentricas planetae huius longitudines & distantias a sole determinari diximus.

PHAENOMENON VI.

Lunam radio ad centrum terrae ducto, aream tempori proportionalem describere.

Patet ex lunae motu apparente cum ipsius diametro apparente collato. Perturbatur autem motus lunaris aliquantuium a vi solis, sed errorum insensibiles minutias in hisce phaenomenis negligo.

PROPOSITIONES.

PROPOSITIO I. THEOREMA I.

Vires, quibus planetae circumjoviales perpetuo retrahuntur a motibus rectilineis & in orbibus suis retinentur, respicere centrum jovis, & esse reciproce ut quadrata distantiarum locorum ab eodem centro.

Patet pars prior propositionis per phaenomenon primum, & propositionem secundam vel tertiam libri primi: & pars posterior per phaenomenon primum, & corollarium sextum propositionis quartae eiusdem libri.
Idem intellige de planetis qui saturnum comitantur, per phaenomenan secundum.

PROPOSITIO II. THEOREMA II.

Vires, quibus planetae primari; perpetuo retrahuntur a motibus rectilineis, & *in orbibus suis retinentur, respicere solem, & esse reciproce ut quadrata distantiarum ab ipsius centro.*

Patet pars prior propositionis per phaenomenon quintum, & propositionem secundam libri primi: & pars posterior per phaenomenon quasium, & propositionem quartam eiusdem libri. Accuratissime autem demonstratur haec pars propositionis per quietem apheliorum. Nam aberratio quam minima a ratione duplicata (per Corol. I. Prop. XLV. Lib. I.) motum apsidum in singulis revolutionibus notabilem, in pluribus enormem efficere deberet.

PROPOSITIO III. THEOREMA III.

Vim, qua luna retinetur in orbe suo, respicere terram, & *esse reciproce ut quadratum distantiae locorum ab ipsius centro.*

Patet assertionis pars prior per phaenomenen sextum, & propositionem secundam vel tertiam libri primi: & pars posterior per motum tardissimum lunaris apogaei. Nam motus ille, qui singulis revolutionibus est graduum tantum trium & minutorum trium in consequentia, contemni potest. Patet enim (per Corol. I. Prop. XLV. Lib. I.) quod si distantia lunae a centro terrae sit ad semidiametrum terrae ut *D* ad *I*; vis a qua motus talis oriatur sit reciproce ut $D^{2\frac{4}{243}}$, id est, reciproce ut ea ipsius *D* dignitas cuius index est $2\frac{4}{243}$, hoc est, in ratione distantiae paulo majore quam duplicata inverse, sed quae partibus $59\frac{3}{4}$ propius ad duplicatam quam ad triplicatam accedit. Oritur vero ab actione solis (uti posthac dicetur) & propterea hic negligendus est. Actio solis quatenus lunam distrahit a terra, est ut distantia lunae a terra quamproxime; ideoque (per ea quae dicuntur in Corol. 2. Prop. XLV. Lib. I.) est ad lunae vim centripetam ut 2 ad 357,45 circiter, seu 1 ad $178\frac{29}{40}$. Et neglecta solis vi tantilla, vis reliqua qua luna retinetur in orbe erit reciproce ut D^2. Id quod etiam plenius constabit conferendo hanc vim cum vi gravitatis, ut fit in propositione sequente.

Corol. Si vis centripeta mediocris qua luna retinetur in orbe augeatur primo in ratione $177\frac{29}{40}$ ad $178\frac{29}{40}$, deinde etiam in ratione duplicata semidiametri terrae ad mediocrem distantiam centri Lunae a centro terrae: habebitur vis centripeta lunaris ad superficiem terrae, posito quod vis illa descendendo ad superficiem terrae perpetuo augeatur in reciproca altitudinis ratione duplicata.

PROPOSITIO IV. THEOREMA IV.
Lunam gravitare in terram, & *vi gravitatis retrahi semper a motu rectilineo,* & *in orbe suo retineri.*

Lunae distantia mediocris a terra in syzygiis est semidiametrorum terrestrium, secundum *Ptolemaeum* & plerosque astronomotum 59, secundum *Vendelinum,* & *Hugeniu* 60, secundum *Copernicus* $60\frac{1}{3}$, secundumo Streetum $60\frac{2}{5}$ & secundum *Tychonem* $56\frac{1}{2}$. Ast *Tycho,* & quotquot eius tabulas refractionum sequuntur, constituendo refractiones solis & lunae (omnino contra naturam lucis) majores quam fixarum, idque scrupulis quasi quatuor vel quinque, auxerunt parallaxin lunae scrupulis totidem, hoc est, quasi duodecima vel decima quinta parte totius parallaxeos. Corrigatur iste error, & distantia evadet quasi $60\frac{1}{2}$ semidiametrorum terrestrium, fere ut ab aliis assignatum est. Assumamus distantiam mediocrem sexaginta semidiametrorum in syzygiis; & lunarem periodum respectu fixarum compleri diebus 27, horis 7, minutis primis 43, ut ab astronomis statuitur; atque ambitum terrae esse pedum Parisiensium 123249600, uti a *Galiis* mensurantibus definitum est: & si luna motu omni privari fingatur ac dimitti, ut urgente vi illa omni, qua (per corol. Prop.III.) in orbe suo retinetur, descendat in terram; Haec spatio minuti unius primi cadendo describet pedes Parisienses $15\frac{1}{12}$. Colligitur hoc excalculo vel per Propositionem XXXVI Libri Primi, vel (quod eodem reddit) per corollarium nonum propositionis quartae eiusdem libri, confecto. Nam arcus illus quem luna tempore minuti unius primi, medio suo motu, ad distantiam sexaginta semidiametrorum terrestrium describat, sinus versus est pedum Parisiensium $15\frac{1}{12}$ circiter, vel magis accurate pedum 15 dig.1. & lin.$1\frac{4}{9}$. Unde cum vis illa accedendo ad terram augeatur in duplicata distantiae ratione inversa, ideoque ad superficiem terrae major sit partibus 60×60 quam ad lunam; corpus vi illa in regionibus nostris cadendo, describere deberet spatio minuti unius primi pedes Parisienses $60\times60\times15\frac{1}{12}$, & spatio minuti unius secundi pedes $15\frac{1}{12}$, vel magis accurate pedes 15. dig. 1. & lin. $1\frac{4}{9}$. Et eadem vi gravia revera descendunt in terram. Nam penduli, in latitudine Lutetiae Parisiorum ad singula minuta secunda oscillantis, longitudo esse pedum trium Parisiensium & linearum $8\frac{1}{2}$; ut observavit *Hugenius.* Et altitudo, quam grave tempore minuti unius secundi cadendo describit, est ad dimidiam longitudinem penduli huius in duplicata ratione circumferentiae circuli ad diametrum eius (ut indicavit etiam *Hugenius)* ideoque est pedum Parisiensium 15. dig. 1. lin. $1\frac{7}{9}$. Et propterea vis qua luna in orbe suo rctinetur, si descendatur in superficiem terrae, aequalis evadit vi gravitatis apud nos, ideoque (per reg. I, & II.) est illa ipsa vis quam nos gravitatem dicere solemus. Nam si gravitas ab ea diversa esset, corpora viribus utrisque coniunctis terram petendo duplo velocius descenderent, & spatio minuti unius secundi cadendo describerent pedes Parisienses $30\frac{1}{6}$: omnino contra experientiam.

Calculus hic fundatur in hypothesi quod terra quiescit. Nam si terra & luna moveantur circum solem, & interea quoque circum commune gravitatis centrum revolvantur : manente lege gravitatis distantia centrorum lunae ac terrae ab invicem erit $60\frac{1}{2}$ semidiametrorum terrestrium circiter; uti computationem ineunti patebit.

Computatio autem iniri potest per Prop. LX. Lib. I.

Scholium.

Demonstratio propositionis sic fusius explicari potest. Si lunae plures circum terram revolverentur, perinde ut sit in systemate saturni vel jovis : harum tempora periodica (per argumentum inductionis) observarent legem planetarum a *Keplero* detectam, & propterea harum vires centripetae forent reciproce ut quadrata distantiarum a centro terrae, per prop. I. huius. Et si earum infima esset parva, & vertices altissimotum montium prope tangeret: huius vis centripeta qua retineretur in orbe, gravitates corporum in verticibus illorum montium (per computationem praecedentem) aequaret quamproxime, efficeretque ut eadem lunula, si motu omni quo pergit in orbe suo privaretur, defectu vis centrifugae qua in orbe permanserat, descenderet in terram, idque eadem cum velocitate qua gravia cadunt in illorum montium verticibus, propter aequalitatem virium quibus descendunt. Et si vis illa qua lunula illa infima descendit, diversa esset a gravitate, & lunula illa etiam gravis esset in terram more corporum in verticibus montium: eadem lunula vi utraque conjuncta duplo velocius descenderet. Quare cum vires utraeque, & hae corporum gravium, & illae lunarum, centrum terrae respiciant, & sint inter se similes & aequales, eaedem (per reg. I. & II.) eandem habebunt causam. Et propterea vis illa, qua luna retinetur in orbe suo, ea ipsa erit quam nos gravitatem dicere solemus: idque maxime ne lunula in vertice montis vel gravitate careat, vel duplo velocius cadat quam corpora gravia solent cadere.

PROPOSITIO V. THEOREMA V.

Planetas circumjoviales gravitare in jovem, circumsaturnum in saturnum, & *circumsolares in solem,* & *vi gravitatis suae retrahi semper a motibus rectilineis, & in orbibus curvilineis retineri.*

Nam revolutiones planetarum circumjovialium circa jovem, circumsaturniorum circa saturnum, & mercurii ac veneris reliquorumque circumsolarium circa solem sunt phaenomena eiusdem generis cum revolutione lunae circa terram ; & propterea (per reg. II.) a causis eiusdem generis dependent: praesertim cum demonstratum sit quod vires, a quibus revolutiones illae dependent, respiciant centra jovis, saturni ac solis, & recedendo a jove, saturno & sole decrescant eadem ratione ac lege, qua vis gravitatis decrescit in recessu a terra.

Corol. 1. Gravitas igitur datur in planetas universos. Nam venerem, mercurium, caeterosque esse corpora eiusdem generis cum jove & saturno, nemo dubitat. Et cum attactio omnis per motus legem tertiam mutua sit, jupiter in satellites suos omnes, saturnus in suos, terraque in lunam, & sol in planetas omnes primarios gravitabit.

Corol. 2. Gravitatem, quae planetam unumquemque respicit, esse reciproce ut quadratum distantiae locorum ab ipsius centro.

Corol. 3. Graves sunt planetae omnes in se mutuo per corol. 1. & 2. Et hinc jupiter & saturnus prope conjunctionem se invicem attrahendo, sensibiliter perturbant motus mutuos, sol perturbat motus lunares, sol & luna perturbant mare nostrum, ut in sequentibus explicabitur.

Scholium.

Hactenus vim illam, qua corpora coelestia in orbibus suis retinentur centripetam appellavimus. Eandem jam gravitatem esse constat, & propterea gravitatem in posterum vocabimus. Nam causa vis illus centripetae, qua luna retinetur in orbe, extendi debet ad omnes planetas per reg. I, II. & IV.

PROPOSITIO VI. THEOREMA VI.

Corpora omnia in planetas singulos gravitare, & pondera eorum in eundem quemvis planetam, paribus distantiis a centro planetae, proportionalia esse quantitati materiae in singulis.

Descensus gravium omnium in terram (dempta saltem inaequali retardatione quae ex aeris perexigua resistentia oritur) aequalibus temporibus fieri, jamdudum observarunt alii ; & accuratissime quidem notare licet aequalitatem temporum in pendulis. Rem tentavi in auro, argento, plumbo, vitro, arena, sale communi, ligna, aqua, tritico. Comparabam pyxides duas ligneas rotundas & aequates. Unam implebam ligno, & idem auti pondus suspendebam (quam potui exacte) in alterius centro oscillatonis. Pyxides ab aequalibus pedum undecim filis pendentes, constituebant pendula, quoad pondus, figuram, & aeris resistentiam omnino paria; & paribus oscillationibus, juxta positae, ibant una & redibant diutissime. Proinde copia materiae in auro (per Corol. I. & 6. Prop. XXIV. Lib. II.) erat ad copiam materiae in ligno, ut vis motricis actio in totum aurum ad eiusdem actionem in totum lignum; hoc est, ut pondus ad pondas. Et sic in caeteris. In corporibus eiusdem ponderis differentia materiae, quae vel minor esset quam pars millesima materiae totius, his experimentis manifesto deprehendi potuit. Jam vero naturam gravitatis in planetas eandem esse atque in terram, non est dubium. Elevari enim fingantur corpora Haec terrestria ad usque orbem lunae, & una cum luna motu omni privata demitti, ut in terram simul cadant; & per jam ante ostensa certum est quod temporibus aequalibus describent aequalia spatia cum luna, ideoque quod sunt ad quantitatem materiae in luna, ut pondera sua ad ipsius pondus. Porro quoniam satellites jovis temporibus revolvuntur quae sunt in ratione sesquiplicata distantiarum a centro jovis, erunt eorum gravitates acceleratrices in jovem reciproce ut quadrata distantiarum a centro jovis; & propterea in aequalibus a jove distantiis, eorum gravitates acceleratrices evaderent aequales. Proinde temporibus aequalibus ab aequalibus altitudinibus cadendo describerent aequalia spatia; perinde ut sit in gravibus in hac terra nostra. Et eodem argumento planetae m circumsolares, ab aequalibus a sole distantiis demissi, descensu suo in solem aequalibus temporibus aequalia spatia describerent. Vires autem, quibus corpora inaequalia aequaliter accelerantur, funt ut corpora; hoc est, pondera ut quantitates materiae in planetis. Porro jovis & eius satellitum pondera in solem proportionalia esse quantitatibus materiae eorum patet ex motu satellitum quam maxime regulari; per Corol 3. Prop. LXV. Lib. I. Nam si horum aliqui magis traherentur in solem, pro quantitate materiae suae, quam caeteri: motus satellitum (per Corol. 2. Prop. LXV. Lib. I.) ex inaequalitate attractionis perturbarentur. Si, paribus a sole distantiis, satelles aliquis gravior esset in solem pro quantitate materiae suae, quam jupiter pro quantitate materiae suae, in ratione quacunque data, puta *d* ad *e:* distantia inter centrum solis & centrum orbis satellitis, major semper foret quam distantia inter centrum solis & centrum jovis in ratione subduplicata quam proxime; uti calculo quodam inito inveni. Et si satelles minus gravis esset in solem in ratione illa *d* ad *e*, distantia centri orbis satellitis a sole minor foret quam distantia centri jovis a sole in ratione illa subduplicata. Ideoque si in aequalibus a sole distantiis, gravitas acceleratrix satellitis cujus is in solem major esset vel minor quam gravitas acceleratrix jovis in solem, parte tantum millesima gravitatis totius; foret distantia centri orbis satellitis a sole major vel minor quam distantia jovis a sole parte $\frac{1}{2000}$ distantiae totius, id est, parte quinta distantiae satellitis extimi a centro jovis: quae quidem orbis eccentricitas foret valde sensibilis. Sed orbes satellitum sunt jovi concentrici, & propterea gravitates acceleratrices jovis & satellitum in solem aequantur inter se. Et eodem argumento pondera saturni & comitum eius in solem, in aequalibus a sole distantiis, sunt ut quantitates materiae in ipsis: & pondera lunae ac terrae in solem vel nulla sunt, vel earum massis accurate proportionalia. Aliqua autem sunt per corol. 1. & 3. Prop. V.

Quinetiam pondera partium singularum planetae cujusque in alium quemcunque sunt inter se ut materia in partibus singulis. Nam si partes aliquae plus gravitarent; aliae minus, quam pro quantitate materiae: planeta totus, pro genere partium quibus maximae abundet, gravitaret magis vel minus quam pro quantitate materiae totius. Sed nec refert utrum partes illae externae sint vel internae. Nam si verbi gratia corpora terrestria, quae apud nos sunt, in orbem luna elevari fingantur, & conferantur cum corpore lunae : si horum pondera essent ad pondera partium externarum lunae ut quantitates materiae in iisdem, ad pondera vero partium internarum in majori vel minori ratione, forent eadem ad pondus lunae totius in majori vel minori ratione: contra quam supra ostensum est.

*Corol.*1. Hinc pondera corporum non pendent ab eorum formis & texturis.Nam si cum formis variari possent; forent majora vel minora, pro varietate formarum; in aequali materia: omnino contra experientiam.

Corol. 2. Corpora universa, quae circa terram sunt, gravia sunt in terram; & pondera omnium, quae aequaliter a centro terrae distant, sunt ut quantitates materiae in iisdem. Haec est qualitas omnium in quibus experimenta instituere licet, & propterea per reg. III. de universis affirmanda est. Si aether aut corpus aliud quodcunque vel gravitate omnino destitueretur, vel pro quantitate materiae suae minus gravitaret: quoniam id (ex mente *Aristotelis*, *Cartesii* & aliorum) non differt ab aliis corporibus nisi in forma materiae, posset idem per mutationem formae gradatim transmutari in corpus eiusdem conditionis cum iis, quae pro quantitate materiae quam maxime gravitant, & vicissim corpora maxime gravia, formam illius gradatim induendo, possent gravitatem suam gradatim amittere. Ac proinde pondera penderent a formis corporum, possentque cum formis variari, contra quam probatum est in corollaria superiore.

Corol. 3. Spatia omnia non sunt aequaliter plena. Nam si spatia omnia aequaliter plena essent, gravitas specifica fluidi quo regio aeris impleretur, ob summam densitatem materiae, nil cederet gravitati specificae argenti vivi, vel auri, vel corporis alterius cujuscunque densissimi; & propterea nec aurum neque aliud quodcunque corpus in aere descendere posset. Nam corpora in fluidis, nisi specifice graviora sint, minime descendunt. Quod si quantitas materiae in spatio dato per rarefactionem quamcunque diminui possit, quidni diminui possit in infinitum?

Corol. 4. Si omnes omnium corporum particulae solidae sint eiusdem densitatis, neque sine poris rarefieri possint, vacuum datur. Eiusdem densitatis esse dico,quarum vires inertiae sunt ut magnitudines.

Corol. 5. Vis gravitatis diversi est generis a vi magnetica. Nam attractio magnetica non en ut materia attracta. Corpora aliqua magis trahuntur, alia minus, plurima non trahuntur. Et vis magnetica in uno & eadem corpore intendi potest & remitti, estque nonnunquam longe major pro quantitate materiae quam vis gravitatis, & in recessu a magnete descrescit in ratione distantiae non duplicata, sed fere triplicata, quantum ex crassis quibusdam observationibus animadvertere potui.

PROPOSITIO VII. THEORMA VII.

Gravitatem in corpora universa fieri, eamque proportionalem esse quantitati materiae in singulis.

Planetas omnes in se mutua graves esse jam ante probavimus, ut & gravitatem in unumquemque seorsim spectatum esse reciproce ut quadratum distantiae locorum a centro planetae. Et inde consequens est (per Prop. LXIX. Lib. 1. & eius corollaria) gravitatem in omnes proportionalem esse materiae in iisdem.

Porro cum planetae cuiusvis *A* partes omnes graves sint in planetam quemvis *B,* & gravitas partis cujusque sit ad gravitatem totius, ut materia partis ad materiam totius, & actioni omni reactio (per motus legem tertiam) aequalis sit; planeta *B* in partes omnes planetae *A* vicissim gravitabit, & erit gravitas sua in partem unamquamque ad gravitatem suam in totum, ut materia partis ad materiam totius. *Q. E. D.*

Corol. 1. Oritur igitur & componitur gravitas in planetam totum ex gravitate in partes singulas. Cuius rei exempla habemus in attractionibus magneticis & electricis. Oritur enim attractio omnis in totum ex attractionibus in partes singulas. Res intelligetur in gravitate, concipiendo planetas plures minores in unum globum coire & planetam majorem componere. Nam vis totius ex viribus partium componentium oriri debebit. Siquis obiiciat quod corpora omnia, quae apud nos sunt, hac lege gravitare deberent in se mutua, cum tamen eiusmodi gravitas neutiquam sentiatur: respondeo quod gravitas in haec corpora, cum sit ad gravitatem in terram totam ut sunt haec corpora ad terram totam, longe minor ea quam quae sentiri possit.

*Corol.*2. Gravitatio in singulas corporis particulas aequales est reciproce ut quadratum distantiae locorum a particulis. Patet per Corol 3. Prop. LXXIV. Lib. I.

PROPOSITIO VIII. THEOREMA VIII.

Si globorum duorum in se mutuo gravitantium materia undique in regionibus, quae a centris aequaliter distant, homogenea sit : erit pondus globi alterutrius in alterum reciproce ut quadratum distantiae inter centra.

Postquam invenissem gravitatem in planetam totum oriri & componi ex gravitatibus in partes; & esse in partes singulas reciproce proportionalem quadratis distantiarum a partibus: dubitabam an reciproca illa proportio duplicata obtineret accurate in vi tota ex viribus pluribus composita, an vero quam proxime. Nam fieri posset ut proportio, quae in majoribus distantiis satis accurate obtinenet prope superficiem planetae ob inaequales particularum distantias & situs dissimiles, notabiliter erraret. Tandem vera, per Prop.

LXXV & LXXVI. Libri primi & ipsarum corollaria, intellexi veritatem propositionis de qua hic agitur.

Corol. 1. Hinc inveniri & inter se comparari possunt pondera corporum in diversos planetas. Nam pondera corporum aequallum circum planetas in circulis revolventium sunt (per Corol. 2. Prop. IV. Lib. I.) ut diametri circulorum directe & quadrata temporum periodicorum inverse ; & pondera ad supersicies planetarum, aliasve quasvis a centro distantias, majora sunt vel minora (per hanc propositionem) in duplicata ratione distantiarum inversa. Sic ex temporibus periodicis veneris circum solem dierum 224 & horarum $16\frac{3}{4}$, satellitis extimi circumjovialis circum jovem dierum 16 & horarum $16\frac{8}{25}$ satellitis Hugeniani circum saturnum dierum 15 & horarum $22\frac{2}{3}$ & lunae circum terram dierum 27. hor.7. min. 43, collatis cum distantia mediocri veneris a sole & cum elongationibus maximis heliocentricis satellitis cxtimi circumjovialis a centro jovis $8'$. $16''$. satellitis Hugeniani a centro saturni $3'$. $4''$. & lunae a centro terrae $10'$. $33''$. computum ineundo inveni quod corporum aequalium & a centro solis, jovis, saturni ac terrae aequaliter distantium pondera sint in solem, jovem, saturnum ac terram ut $1, \frac{1}{1067}, \frac{1}{3021}, \& \frac{1}{169282}$ respective, & auctis vel diminutis distantiis, pondera diminuuntur vel augentur in duplicata ratione: pondera aequalium corporum in solem, jovem, saturnum ac terram in distantiis 10000, 997, 791, & 109 ab eorum centris, atque ideo in eorum superficiebus, erunt ut 10000, 943, 529, & 435 respective. Quanta sint pondera corporum in superficie lunae dicetur in sequentibus.

Corol. 2. Innotescit etiam quantitas materiae in planetis singulis. Nam quantitates materiae in planetis sunt ut eorum vires in aequalibus distantiis ab eorum centris; id est, in sole, jove, saturno ac terra sunt ut $1, \frac{1}{1067}, \frac{1}{3021}, \& \frac{1}{169282}$ respective. Si parallaxis solis statuatur major vel minor quam $10''$, $30'''$, debebit quantitas materiae in terra augeri vel diminui in triplicata ratione.

Corol. 3. Innotescunt etiam densitates planetarum. Nam pondera corporum aequalium & homogenearum in sphaeras homogeneas sunt in superficiebus sphaerarum ut sphaerarum diametri, per Prop. LXXII. Lib. I. ideoque sphaerarum heterogenearum densitates sunt ut pondera illa applicata ad sphaerarum diametros. Erant autem verae solis, jovis, saturni ac terrae diametri ad invicem ut 10000, 997, 791, & 109, & pondera in eosdem ut 10000, 943, 529 & 435 respective, & propterea densitates sunt ut 100, $94\frac{1}{2}$, 67 & 400. Densitas terrae quae prodit ex hoc computo non pendet a parallaxi solis, sed determinatur per parallaxin lunae, & propterea hic recte definitur. Est igitur sol paulo densior quam jupiter, & jupiter quam saturnus, & terra quadrupo densior quam sol. Nam per ingentem suum calorem sol rarescit. Luna vero densior est quam terra, ut in sequentibus patebit.

Corol. 4. Densiores igitur sunt planetae qui sunt minores, caeteris paribus. Sic enim vis gravitatis in eorum superficiebus ad aequalitatem magis accedit. Sed & densiores sunt planetae, caeteris paribus, qui sunt soli propiores ; ut jupiter saturno, & terra jove. In diversis utique distantiis a sole collocandi erant planetae ut quilibet pro gradu densitatis calore solis majore vel minore frueretur. Aqua nostra, si terra locaretur in orbe saturni, rigesceret, si in orbe mercurii in vapores statim abiret. Nam lux solis, cui calor proportionalis est, septuplo densior est in orbe mercurii quam apud nos: & thermometro expertus sum quod septuplo solis aestivi calore aqua ebullit. Dubium vero non est quin materia mercurii ad calorem accommodetur, & propterea densior sit hac nostra; cum materia omnis densior ad operationes naturales obeundas majorem calorem requirat.

PROPOSITIO IX. THEOREMA IX.

Gravitatem pergendo a superficiebus planetarum deorsum decrescere in ratione distantiarum a centro quam proxime.

Si materia planetae quoad densitatem uniformis esset, obtineret haec propositio accurate: per Prop. LXXIII. Lib. I. Error igitur tantus est, quantus ab inaequabili densitate oriri possit.

PROPOSITIO X. THEOREMA X.

Motus planetarum in coelis diutissime conservari posse.

In scholio Propositionis XL. Lib. II. ostensum est quod globus aquae congelatae, in aere nostro libere movendo & longitudinem semidiametri suae describendo, ex resistentia aeris amitteret motus sui partem $\frac{1}{4586}$. Obtinet autem eadem proportio quam proxime in globis utcunque magnis & velocibus. Iam vera globum terrae nostrae densiorem esse, quam si totus ex aqua constaret, sic colligo. Si globus hicce totus esset aqueus, quaecunque rariora essent quam aqua, ob minorem specificam gravitatem emergerent & supernatarent. Eaque de causa globus terreus aquis undique coopertus, si rarior esset quam aqua, emergeret alicubi, & aqua omnis inde defluens congregaretur in regione opposita. Et par est ratio terrae nostrae maribus magna ex parte circumdatae. Haec si densior non esset, emergeret ex maribus, & parte sui pro gradu levitatis extaret ex aqua, maribus omnibus in regionem oppositam confluentibus. Eodem argumento maculae solares leviores sunt quam materia lucida solaris cui supernatant. Et in formatione qualicunque planetarum, ex aqua materia omnis gravior, quo tempore massa fluida erat, centrum petebat. Unde cum terra communis suprema quasi duplo gravior sit quam aqua, & paulo inferius in fodinis quali triplo vel quadruplo aut etiam quintuplo gravior reperiatur: verisimile est quod copia materiae totius in terra quasi quintuplo vel sextuplo major sit quam si tota ex aqua constaret; praesertim cum terram quasi quadruplo densiorem esse quam jovem jam ante ostensum sit. Quare si jupiter paulo densior sit quam aqua, hic spatio dierum triginta, quibus longitudinem 459 semidiametrorum suarum describit, amitteret in medio eiusdem densitatis cum aere nostro motus sui partem fere decimam. Verum cum resistentia mediorum minuatur in ratione ponderis ac densitatis, sic ut aqua, quae partibus $13\frac{3}{5}$ levior est quam argentum vivum, minus resistat in eadem ratione; & aer, qui partibus 860 levior est quam aqua, minus resistat in eadem ratione : si ascendatur coelos ubi pondus medii, in quo planetae moventur, diminuitur immensum, resistentia prope cessabit. Ostendimus utique in scholio ad Prop. XXII. Lib. II. quod si ascenderetur ad altitudinem milliarium ducentorum supra terram, aer ibi rarior foret quam ad superficiem terrae in ratione 30 ad 0,0000000000003998, seu 75000000000000 ad 1 circiter. Et hinc stella jovis in medio eiusdem densitatis cum aere illo superiore revolvendo, tempore annorum

1000000, ex resistentia medii non amitteret motus sui partem decimam centesimam millesimam. In spatiis utique terrae proximis, nihil invenitur quod resistentiam creet praeter aerem exhalationes & vapores. His ex vitro cavo cylindrico diligentissime exhaustis gravia intra vitrum liberrime & sine omni resistentia sensibili cadunt; ipsum aurum & pluma tenuissima simul demissa aequali cum velocitate cadunt, & casu suo describendo altitudinem pedum quatuor sex vel octo simul incidunt in fundum, ut experientia compertum est.
Et propterea si in coelos ascendatur aere & exhalationibus vacuos planetae & cometae sine omni resistentia sensibili per spatia illa diutissime movebuntur.

HYPOTHESIS I.
Centrum systematis mundani quiescere.

Hoc ab omnibus concessum est, dum aliqui terram, alii solem in centro systematis quiescere contendant. Videamus quid inde sequatur,

PROPOSITIO XI. THEOREMA XI.
Commune centrum gravitatis terrae, *solis* & *planetarum omnium quiescere.*

Nam centrum illud (per legum Corol. IV.) vel quiescet vel progredietur uniformiter in directum. Sed centro illo semper progrediente centrum mundi quoque movebitur contra hypothesin.

PROPOSITIO XII. THEOREMA XII.
Solem motu perpetuo agitari, sed nunquam longe recedere a communi gravitatis centro planetarum omnium.

Nam cum (per Corol. 2. Prop. VIII.) materia in sole sit ad materiam in jove ut 1067 ad I, & distantia jovis a sole sit ad semidiametrum solis in ratione paulo majore; incidet commune centrum gravitatis jovis & solis in punctum paulo supra superficiem solis. Eadem argumento cum materia in sole sit ad materiam in saturno ut 3021 ad 1, & distantia saturni a sole sit ad semidiametrum solis in ratione paulo minore: incidet commune centrum gravitatis saturni & solis in punctum paulo infra superficiem solis. Et eiusdem calculi vestigiis insistendo si terra & planetae omnes ex una solis parte consisterent, commune omnium centrum gravitatis vix integra solis diametro a centro solis distaret. Aliis in casibus distantia centrorum semper minor est. Et propterea cum centrum illud gravitatis perpetuo quiescit, sol pro vario planetarum sim in omnes partes movebitur, sed a centro illo nunquam longe recedet.

Corol. Hinc commune gravitatis centrum terrae, solis & planetarum omnium pro centro mundi habendum est. Nam cum terra, sol & planetae omnes gravitent in se mutuo, & propterea, pro vi gravitatis suae, secundum leges motus perpetuo agitentur: perspicuum est quod horum centra mobilia pro mundi centro quiescente haberi nequeunt. Si corpus illud in centro locandum esset in quod corpora omnia maxime gravitant (uti vulgi est opinio) privilegium istud concedendum esset soli. Cum autem sol moveatur, eligendum erit punctum quiescens, a quo centrum solis quam minime discedit, & a quo idem ad huc minus discederet, si modo sol densior esset & major, ut minus moveretur.

PROPOSITIO XIII. THEOREMA XIII.

Planetae moventur in ellipsibus umbilicum habentibus in centro solis, & *radiis ad centrum illud ductis areas describunt temporibus proportionales.*

Disputavimus supra de his motibus ex phaenomenis. Jam cognitis motuum principlis, ex his colligimus motus coelestes a priori. Quoniam pondera planetarum in solem sunt reciproce ut quadrata distantiarum a centro solis; si sol quiesceret & planetae reliqui non agerent in se mutuo, forent orbes eorum elliptici, solem in umbilico communi habentes, & areae describerentur temporibus proportionales (per Prop. I. & Xl. & Corol. I. Prop. XIII. Lib. I.) actiones autem planetarum in se mutuo perexiguae sunt (ut possint contemni) & motus planetarum in elliptibus circa solem mobilem minus perturbant (per Prop. LXVI. Lib. I.) quam si motus uti circa solem quiescentem peragerentur.

Actio quidem jovis in saturnum non est omnino contemnenda. Nam gravitas in jovem est ad gravitatem in solem (paribus distantiis) ut 1 ad 1067; ideoque in conjunctione jovis & saturni, quoniam distantia saturni a jove est ad distantiam saturni a sole fere ut 4 ad 9 erit gravitas saturni in jovem ad gravitatem saturni in solem ut 81 ad 16×1067 seu 1 ad 211 circiter. Et hinc oritur perturbatio orbis saturni in singulis planetae huius cum jove conjunctionibus adeo sensibilis ut ad eandem astronomi haereant. Pro vario situ planetae in his conjunctionibus, eccentricitas eius nunc augetur nunc diminuitur, aphelium nunc promovetur nunc forte retractitur, & medius motus per vices acceleratur & retardatur. Error tamen omnis in motu eius circum solem a tanta vi oriundus (praeterquam in motu medio) evitari fere potest constituendo umbilicum inferiorem orbis eius in communi centro gravitatis, jovis & solis (per Prop. LXVII. Lib. I.) & propterea ubi maximus est, vix superat minuta duo prima. Et error maximus in motu medio vix superat minuta duo prima annuatim. In conjunctione autem jovis & saturni gravitates acceleratrices solis ,in saturnum, jovis in saturnum & jovis in solem sunt sere ut 16, 81 & $\frac{16\times 81\times 3021}{25}$ seu 156069, ideoque differentia gravitatum solis in saturnum & jovis in saturnum est ad gravitatem jovis in solem ut 65 ad 156609 seu 1 ad 2409. Huic autem differentiae proportionalis est maxima saturni efficacia ad perturbandum motum jovis, & propterea perturbatio orbis jovialis longe minor est quam ea saturnii. Reliquorum orbium perturbationes sunt adhuc longe minores praeterquam quod orbis terrae sensibiliter perturbatur a luna. Commune centrum gravitatis terrae & lunae, ellipsin circum solem in umbilico positum percurrit, & radio ad solem ducto areas in eadem temporibus proportionales describit, terra vero circum hoc centrum commune motu menstruo revolvitur.

PROPOSITIO XIV. THEOREMA XIV.

Orbium aphelia & *nodi quiescunt.*

Aphelia quiescunt per Prop. XI. Lib. I. ut & orbium plana, per eiusdem libri Prop. I. & quiescentibus planis quiescunt nodi. Attamen a planetarum revolventium & cometarum actionibus in se invicem orientur inaequalitates aliquae, sed quae ob parvitatem hic contemni possunt.

Corol. I. Quiescunt etiam stellae fixae, propterea quod datas ad aphelia nodosque positiones servant.

Corol. 2. Ideoque cum nulla sit earum parallaxis sensibilis ex terrae motu annuo oriunda, vires earum ob immensam corporum distantiam nullos edent sensibiles effectus in regione systematis nostri. Quinimo fixae in omnes coeli partes aequaliter dispersae contrariis attractionibus vires mutuas destruunt, per Prop. LXX. Lib. I.

Scholium.

Cum planetae soli propiores (nempe mercurius, venus, terra, & mars) ob corporum parvitatem parum agant in se invicem: horum aphelia & nodi quiescent, nisi quatenus a viribus jovis, saturni & corporum superiorum turbentur. Et inde colligi potest per theoriam gravitatis, quod horum aphelia moventur aliquantulum in consequentia respectu fixarum, idque in proportione sesquiplicata distantiarum horum planetarum a sole. Ut si aphellum martis in annis centum conficiat 33′. 20″ in consequentia respectu fixarum; aphelia terrae, veneris, & mercurii in annis centum conficient 17′. 40″, 10′.53″, & 4′. 16″ respective. Et hi motus, ob parvitatem, negliguntur in hac propositione.

PROPOSITIO XV. PROBLEMA I.

Invenire orbium principales diametros.

Capiendae sunt hae in ratione subsesquiplicata temporum periodicorum, per Prop. xv. Lib. I. deinde sigillatim augendae in ratione summae massarum solis & planetae cujusque revolventis ad primam duarum medie proportionalium inter summam illam & solem, per Prop. LX. Lib. I.

PROPOSITIO XVI. PROBLEMA II.

Invenire orbium excentricitates & *aphelia.*

Problema confit per Prop. XVIII. Lib. I.

PROPOSITIO XVII. THEOREMA XV.

Planetarum motus diurnos uniformes esse, & *librationem lunae ex ipsius motu diurno oriri.*

Patet per motus legem 1. & Corol. 22. Prop. LXVI. Lib. I. Jupiter utique respectu fixarum revolvitur horis 9. 56′, mars horis 24. 39′, venus horis 23 circiter, terra horis 23. 56′, sol diebus $25\frac{1}{2}$ & luna diebus 27. 7 hor. 43′. Haec ita se habere ex phaenomenis manifestum est. Maculae in corpore solis ad eundem situm in disco solis redeunt diebus $27\frac{1}{2}$ circiter, respectu terrae ; ideoque respectu fixarum sol revolvitur diebus $25\frac{1}{2}$ circiter. Quoniam vera lunae circa axem suum uniformiter revolventis dies menstruus est: huius facies eadem ulteriorem umbilicum orbis eius semper respiciet quamproxime & propterea pro situ umbilici illius deviabit hinc inde a terra. Haec est libratio lunae in longitudinem: Nam libratio in latitudinem orta est ex latitudine lunae & inclinatione axis eius ad planum eclipticae. Hanc librationis lunaris theoriam D. *N. Mercator* in astronomia sua initio anni 1676 edita, ex literis meis plenius expofuit. Simili motu extimus saturni satelles circa axem suum revolvi videtur, eadem sui facie saturnum perpetuo respiciens. Nam circum saturnum revolvendo, quoties ad orbis sui partem orientalem accedit, aegerrime videtur, & plerumque videri cessat: id quod evenire potest per maculas quasdam in ea corporis parte quae terrae tunc obvertitur, ut *Cassinus* notavit. Simili etiam motu satelles extimus jovialis circa axem suum revolvi videtur, propterea quod in parte corporis jovi aversa maculam habeat quae tanquam in corpore jovis cernitur ubicunque satelles inter jovem & oculos nostros transit.

PROPOSITIO XVIII. THEOREMA XVI.

Axes planetarum diametris quae ad eosdem axes normaliter ducuntur minores esse.

Planetae sublato omni motu circulari diurno figuram sphaericam, ob aequalem undique partium gravitatem, affectare deberent. Per motum illum circularem fit ut partes ab axe recedentes iuxta aequatorem ascendere conentur. Ideoque materia si fluida sit ascensu suo ad aequatorem diametros adaugebit, axem vero descensu suo ad polos diminuet. Sic jovis diameter (consentientibus observationibus) brevior deprehenditur inter polos quam ab oriente in occidentem. Eodem argumento, nisi terra nostra paulo altior esset sub aequatore quam ad polos, maria ad polos subsiderent, & iuxta aequatorem ascendendo) ibi omnia inundarent.

PROPOSITIO XIX. PROBLEMA III.

Invenire Proportionem axis planetae ad diametros eidem perpendiculares.

Norwoodus noster circa annum 1635 mensurando distantiam pedum Londinensium 905751 inter *Londinum* &. *Eboracum,* & observando differentiam latitudinum 2. *gr.* 28′ collegit mensuram gradus unius esse pedum Londinensium 367196, id est, hexapedarum Parisiensium 57300.

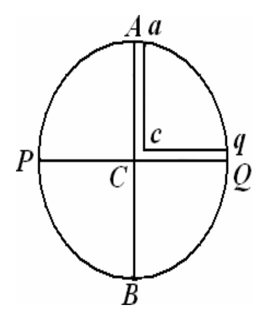
Picartus mensurando arcum gradus unius & 22′. 55″ in meridiano inter *Ambianum* & *Malvoisinam,* invenit arcum gradus unius esse hexapedarum Parisiensium 57060. *Cassinus* senior mensuravit distantiam in meridiana a villa *Collioure* in *Roussilion* ad observatorium Parisiense; & fillus eius addidit distantiam ab observatotio ad turrem urbis *Dunkirk.* Distantia tota erat hexapedarum $486156\frac{1}{2}$ & differentia latitudinum villae *Collioure* & urbis *Dunkirk* erat graduum octo & 31′. $11\frac{1}{6}$″. Unde arcus gradus unius prodit hexapedarum Parisiensium 57061. Et ex his mensuris colligitur ambitus terrae pedum Parisiensium 123249600, & semidiameter eiuis pedum 19615800, ex hypothesi quod terra sit sphaerica.

In latitudine *Lutetitae Parisiorum* corpus grave tempore minuti unius secundi cadendo describit pedes Parisienses 15 dig. 1 lin. $1\frac{7}{9}$ ut supra, id est, lineas $2173\frac{7}{9}$. Pondus corporis diminuitur per pondus aeris ambientis. Ponamus pondus amissum esse partem undecimam millesimam ponderis totius, & corpus illud grave cadendo in vacuo describet altitudinem linearum 2174 tempore minuti unius secundi.

Corpus in circulo ad distantiam pedum 19615800 a centro, singulis diebus sidereis horarum 23. 56′. 4″ uniformiter revolvens tempore minuti unius secundi describet arcum pedum 1433,46, cuius sinus versus est pedum 0,052656, seu linearum 7,54064. Ideoque vis, qua gravia descendunt in latitudine *Lutetitae,* est ad vim centrifugam corporum in aequatore a terrae motu diurno oriundam, ut 2174 ad 7,54064.

Vis centrifuga corporum in aequatore terrae est ad vim centrifugam,qua corpora directe tendunt a terra in latitudine *Lutetiae* graduum 48. 50′. 10″, in duplicata ratione radii ad sinum complementi latitudinis illius, id est, ut 7,54064 ad 3,267. Addatur haec vis ad vim qua gravia descendunt in latitudine illa *Lutetiae*, & corpus in latitudine illa vi tota gravitatis cadendo, tempore minuti unius secundi describet lineas 2177,267, seu pedes Parisienses 15 dig.1 & lin. 5,267. Et vis tota gravitatis in latitudine illa erit ad vim centrifugam corporum in aequatore terrae ut 2177,167 ad 7,54064 seu 289 ad 1.

Unde si *APBQ* figuram terrae designet jam non amplius sphaericam sed revolutione ellipseos circum axem minorem *PQ* genitam, sitque *ACQqca* canalis aquae plena, a polo *Qq* ad centrum *Cc*, & inde ad aequatorem *Aa* pergens: debebit pondus aquae in canalis crure *ACca*, esse ad pondus aquae in crure altero *QCcq* ut 289 ad 288, eo quod vis centrifuga ex circulari motu orta partem unam e ponderis partibus 289 sustinebit ac detrahet, & pondus 288 in altero crure sustinebit reliquas. Porro (ex Propositionis XCI. Corol. 2. Lib. I.) computationem ineundo, invenio quod si terra constaret ex uniformi materia, motuque omni privaretur, & esset eius axis *PQ* ad diametrum *AB* ut 100 ad 101; gravitas in loco *Q* in terram foret ad gravitatem in eodem loco *Q* in sphaeram centro *C* radio *PC* vel *QC* descriptam, ut 126 ad 125. Et eodem argumento gravitas in loco *A* in sphaeroidem, convolutione ellipseos *APBQ* circa axem *AB* descriptam, est ad gravitatem in eodem loco *A* in sphaeram centro *C* radio *AC* descriptam, ut 125 ad 126. Est autem gravitas in loco *A* in terram media proportionalis inter gravitates in dictam sphaeroidem & sphaeram : propterea quod sphaera, diminuendo diametrum *PQ* in ratione 101 ad 100, vertitur in figuram terrae ; & haec figura diminuendo in eadem ratione diametrum tertiam, quae diametris duabus *AB*, *PQ* perpendicularis est, vertitur in dictam sphaeroidem; & gravitas in *A*, in casu utroque, diminuitur in eadem ratione quam proxim.e. Est igitur gravitas in *A* in
sphaeram centro *C* radio *A C* descriptam, ad gravitatem in *A* in terram ut 126 ad $125\frac{1}{2}$, & gravitas in loco *Q* in sphaeram centro *C* radio *QC* descriptam, est ad gravitatem in loco *A* in sphaeram centro *C* radio *QC* descriptam, in ratione diametrorum (per Prop. LXXII. Lib. I.) id est, ut 100 ad 101. Conjungantur jam hae tres rationes, 126 ad 125, 126 ad $125\frac{1}{2}$, & 100 ad 101: & fiet gravitas in loco *Q* in terram ad gravitatem in loco *A* in terram, ut $126\times116\times100$ ad $125\times125\frac{1}{2}\times101$, seu ut 501 ad 500.



Jam cum (per Corol. 3. Prop. XCI. Lib. I.) gravitas in canalis crure utrovis *ACca* vel *QCcq* sit ut distantia locorum a centro terrae; si crura illa superficiebus transversis & aequidistantibus distinguantur in partes totis proportionales, erunt pondera partium singularum in crure *ACca* ad pondera partium totidem in crure altero, ut magnitudines & gravitates acceleratrices coniunctim ; id est, ut 101 ad 100 & 500 ad 501, hoc est, ut 505 ad 501. Ac proinde si vis centrifuga partis cuiusque in crure *ACca* ex motu diurno oriunda fuisset ad pondus partis eiusdem ut 4 ad 505, eo ut de pondere partis cuiusque, in partes 505 diviso, partes quatuor detracteret; manerent pondera in utroque crure aequalia, & propterea fluidum consisteret in aequilibrio. Verum vis centrifuga partis cuiusque est ad pondus eiusdem ut 1 ad 289, hoc est, vis centrifuga quae deberet esse ponderis pars $\frac{4}{505}$ est tantum pars $\frac{1}{289}$. Et propterea dico, secundum regulam auream, quod si vis centrifuga $\frac{4}{505}$ faciat ut altitudo aquae in crure *ACca* superet altitudinem aquae in crure *QCcq* parte centesima totius altitudinis: vis centrifuga $\frac{1}{289}$ faciet ut excessus altitudinis in crure *ACca* sit altitudinis in crure altero *QCcq* pars tantum $\frac{2}{229}$. Est igitur diameter terrae secundum aequatorem ad ipsius diametrum per polos ut 230 ad 229. Ideoque cum terrae semidiameter mediocris, iuxta mensuram *Picarti*, sit pedum Parisiensium 19615800, seu miliarium 3923,16 (posito quod milliare sit mensura pedum 5000) terra altior erit ad aequatorem quam ad polos excessu pedum 85472, seu miliiarum $17\frac{1}{10}$. Et altitudo eius ad aequatorem erit 19658600 pedum circiter, & ad polos 19573000 pedum.

Si planeta major sit vel minor quam terra manente eius densitae ac tempore periodico revolutionis diurnae, manebit proportio vis centrifugae ad gravitatem, & propterea manebit etiam proportio diametri inter polos ad diametrum secundum aequatorem. At si motus diurnus in ratione quacunque acceleretur vel retardetur, augebitur vel minuetur vis centrifuga in duplitata illa ratione, & propterea differentia diametrorum augebitur vel minuetur in eadem duplicata ratione quamproxime. Et si densitas planetae augeatur vel minuatur in ratione quavis, gravitas etiam in ipsum tendens augebitur vel minuetur in eadem ratione, & differentia diametrorum vicissim minuetur in ratione gravitatis auctae vel augebitur in ratione gravitatis diminutae. Unde cum terra respectu fixarum revolvatur horis 23. 56′, jupiter autem horis 9.56′, sintque temporum quadrata ut 29 ad 5, & revolventium densitates ut 400 ad $94\frac{1}{2}$: differentia diametrorum jovis erit ad ipsius diametrum minorem ut $\frac{29}{5}\times\frac{400}{94\frac{1}{2}}\times\frac{1}{229}$ ad 1, seu 1 ad $9\frac{1}{3}$ quamproxime. Est igitur diameter jovis ab oriente in occidentem ducta, ad eius diametrum inter polos ut $10\frac{1}{3}$ ad $9\frac{1}{3}$ quamproxime. Unde cum eius diameter major sit 37″, eius diameter minor quae polis interjacet, erit 33″. 25‴. Pro luce erratica addantur 3" circiter, & huius planetae diametri apparentes evadens 40″ & 36″ : quae sunt ad invicem ut $11\frac{1}{6}$ ad $10\frac{1}{6}$ quamproxime. Hoc ita se habet ex hypothesi quod corpus jovis sit uniformiter densum. At si corpus eius sit densius versus planum aequatoris quam versus polos, diametri eius possint esse ad invicem ut 12 ad 11, vel 13 ad 12, vel forte 14 ad 13. Et *Cassinus* quidem anno 1691 observavit, quod jovis diameter ab oriente in occidentem porrecta diametrum alteram superaret parte sui circiter decima quinta. *Poundus* autem noster telescopio pedum 123 longitudinis & optimo micrometro, diametros jovis anna 1719, mensuravit ut sequitur.

Tempora		*Diam.max.*	*Diam.min.*	*Diametri ad invicem.*	
dies	*hor.*	*part.*	*part.*		
Jan. 28	6	13,40	12,28	*ut* 12	*ad* 11
Mar. 6	7	13,12	12,20	$13\frac{3}{4}$	$12\frac{3}{4}$
Mar. 9	7	13,12	12,08	$12\frac{2}{3}$	$11\frac{2}{3}$
Apr. 9	9	12,32	11,48	$12\frac{1}{2}$	$11\frac{1}{2}$

Congruit igitur theoria cum phaenomenis. Nam planetae magis incalescunt ad lucem solis versus aequatores suos, & propterea paulo magis ibi decoquuntur quam versus polos.

Quinetiam gravitatem per rotationem diurnam terrae nostrae minui sub aequatore, atque ideo terram ibi altius surgere quam ad polos (si materia eius uniformiter densa sit) patebit per experimenta pendulorum quae recensentur in propositione sequente.

PROPOS1TIO XX. PROBLEMA IV.

lnvenire & *inter se comparare pondera corporum in terrae huius regionibus diversis.*

Quoniam pondera inaequalium crurum canalis aqueae *ACQqca* aequalia sunt; & pondera partium, cruribus totis proportionalium & similiter in totis sitarum, sunt ad invicem ut pondera totorum, ideoque etiam aequantur inter se; erunt pondera aequalium & in cruribus similiter sitarum partium reciproce ut crura, id est, reciproce ut 230 ad 229. Et par est ratio homogeneorum & aequalium quorumvis & in canalis cruribus similiter sitorum corporum. Horum pondera sunt reciproce ut crura, id est, reciproce ut distantia corporum a centro terrae. Proinde si corpora in supremis canalium partibus, sive in superficie terrae consistant; erunt pondera eorum ad invicem reciproce ut distantiae eorum a centro. Et eodem argumenta pondera, in aliis quibuscunque per totam terrae superficiem regionibus, sunt reciproce ut distantiae locorum a centro; & propterea, ex hypothesi quod terra sphaerois sit, dantur proportione.

Unde tale confit theorema, quod incrementum ponderis pergendo ab aequatore ad polos, sit quam proxime ut sinus versus latitudinis duplicatae, vel quod perinde est, ut quadratum sinus recti latitudinis. Et in eadem circiter ratione augentur arcus graduum latitudinis in meridiano. Ideoque cum latitudo *Lutetiae Parisiorum* sit 48 gr. 50′, ea locorum sub aequatore 00 gr. 00′, & ea locorum ad polos 90 gr. & duplorum sinus versi sint 11334, 00000 & 20000, existente radio 10000, & gravitas ad polum sit ad gravitatem sub aequatore ut 230 ad 229, & excessus gravitatis ad polum ad gravitatem sub aequatore ut 1 ad 229: erit excessus gravitatis in latitudine *Lutetiae* ad gravitatem sub aequatore, ut $1\times\frac{11334}{20000}$ ad 229, seu 5667 ad 2290000. Et propterea gravitates totae in his locis erunt ad invicem ut 2295667 ad 2290000. Quare cum longitudines pendulorum aequalibus temporibus oscillantium sint ut gravitates, & in latitudine *Lutetiae Parisiorum* longitudo penduli singulis minutis secundis oscillantis sit pedum trium Parisiensium & linearum $8\frac{1}{2}$, vel potius ob pondus aeris $8\frac{5}{9}$: longitudo penduli sub aequatore superabitur a longitudine synchroni penduli Parisiensis, excessu lineae unius & 87 partium millesimarum lineae. Et simili computo confit tabula sequens.

Latitudo universus loci.	*longitudo penduli.*	*Mensura gradus in meridiano.*
grad.	*ped. lin.*	*hexapedae.*
0	3 7,468	56637
5	3 7,482	56642
10	3 7,526	56659
15	3 7,596	56687
20	3 7,692	56724
25	3 7,812	56769
30	3 7,948	56823

35	3 8,099	56882
40	3 8,261	56945
1	3 8,294	56958
2	3 8,327	56971
3	3 8,361	56984
4	3 8,394	56997
45	3 8,428	57010
6	3 8,461	57022
7	3 8,494	57035
8	3 8,528	57048
9	3 8,561	57061
50	3 8,594	57074
55	3 8,756	57137
60	3 8,907	57196
65	3 9,044	57250
70	3 9,162	57295
75	3 9,258	57332
80	3 9,329	57360
85	3 9,371	57377
90	3 9,387	573 82

Constat autem per hanc tabulam quod graduum inaequalitas tam parva sit, ut in rebus geographicis figura terrae pro sphaerica haberi possit: praesertim si terra paulo densior sit versus planum aequatoris quam versus polos.

Jam vero astronomi aliqui in longinquas regiones ad observationes astronomicas faciendas missi, observarunt quod horologia oscillatoria tardius moverentur prope aequatorem quam in regionibus nostris. Et primo quidem D. *Richer* hoc observavit anna 1672 in insula *Cayennae.* Nam dum observaret transitum fixarum per meridianum mense *Augusto,* reperit horologium suum tardius moveri quam pro medio motu solis, existente differentia $2'.28''$ singulis diebus. Deinde faciendo ut pendulum simplex ad minuta singula secunda per horologium optimum mensurata oscillaret, notavit longitudinem penduli simplicis, & hoc fecit saepius singulis septimanis per mentes decem. Tum in *Galiiam* redux contulit longitudinem huius penduli cum longitudine penduli Parisiensis (quae erat trium pedum Parisinsium, & octo linearum cum tribus quintis partibus linere) & reperit breviorem esse, existente differentia lineae unius cum quadrante.

Postea *Halleius* noster circa annum 1677 ad insulam *Sanctae Hellenae* navigans, reperit horologium suum oscillatorium ibi tardius moveri quam *Londini,* sed differentiam non notavit. Pendulum vero brevius reddidit plusquam octava parte digiti, seu linea una cum semisse. Et ad hoc efficiendum, cum longitudo cochleae in ima parte penduli non sufficeret, annulum ligneum thecae cochleae & ponderi pendulo interpofuit.

Deinde anna 1682. D. *Varin* & D. *Des Hayes* invenerunt longitudinem penduli singulis minutis secundis oscillantis in observatorio regio Parisiensi esse pede 3. lin. $8\frac{5}{9}$. Et in insula *Gorea* eadem methodo longitudinem penduli synchroni invenerunt esse pede 3.

lin. $6\frac{5}{9}$, existente longitudinum differentia lin. 2. Et eodem anno ad insulas *Guadaloupam* & *Martinicam* navigantes, invenerunt longitudinem penduli synchroni in his insolis esse ped. 3. lin. $6\frac{1}{2}$.

Posthac D. *Couplet* filius anna 1697 mense *Julio*, horologium suum oscillatorium ad motum solis medium in observatorio regio Parisiensi sic aptavit, ut tempore satis longo horologium cum motu solis congrueret. Deinde *Ulyssipponem* navigans invenit quod mense *Novembri* proximo horologium tardius iret quam prius, existente differentia 2′. 13″ in horis 24. Et mense *Martio* sequente *Paraibam* navigans invenit ibi horologium suum tardius ire quam *Parisiis*, existente differentia 4′. 12″ in horis 24. Et affirmat pendulum ad minuta secunda oscillans brevius suisse *Ulyssipponi* lineis $2\frac{1}{2}$ & *Paraibae* lineis $3\frac{2}{3}$ quam *Parisiis*. Rectius posuisset differentias esse $1\frac{1}{3}$ & $2\frac{5}{9}$. Nam hae differentiae differentiis temporum 2′. 13″, & 4′. 12″ respondens. Crassoribus huius observationibus minus fidendum est.

Annis proximis (1699 & 1700) D. *Des Hayes* ad *Americam* denuo navigans, determinavit quod in insulis *Cayennae* & *Granadae* longitudo penduli ad minuta secunda oscillantis, esset paulo minor quam ped.3. lin. $6\frac{1}{2}$, quodque in insula *S. Christophori* longitudo illa esset ped.3. lin. $6\frac{3}{4}$, & quod in insula *S. Dominici* eadem esset ped. 3. lin. 7.

Annoque 1704 *P. Feuelleus* invenit in *Porto belo* in *America* longitudinem penduli ad minuta secunda oscillantis, esse pedum trium Parisiensium & linearum tantum $5\frac{7}{12}$, id est, tribus fere lineis breviorem quam *Lutetiae Parisiorum*, sed errante observatione. Nam deinde ad insulam *Martinicam* navigans, invenit longitudinem penduli isochroni esse pedum tantum tritim Parisiensium & linearum $5\frac{10}{12}$.

Latitudo autem *Paraibae* est 6 gr. 38′ ad austrum, & ea *Porto beli* 9 gr. 33′ ad boream, & latitudines insularum *Cayennae, Goreae, Guadiloupae, Martinicae, Granadae,* & *Sancti Christophori* & *Sancti Dominici* sunt respective 4 gr. 55′, 14 gr. 40', 14 gr. 00′, 14 gr. 44′, 12 gr. 6′, 17 gr. 19', & 19 gr. 48′ ad boream. Et excessus longitudinis penduli Parisiensis supra longitudines pendulorum isochronorum in his latitudinibus observatas sunt paulo majores quam pro tabula longitudinum penduli superius computata. Et propterea terra aliquanto altior est sub aequatore quam pro superiore calculo, & densior ad centrum quam in fodinis prope superficiem, nisi forte calores in zona torrida longitudinem pendulorum aliquantulum auxerint.

Observavit utique D. *Picartus* quod virga ferrea, quae tempore hyberno ubi gelabant frigora erat unius longitudine, ad ignem calefacta evasit pedis unius cum quarta parte lineae. Deinde D. *de la Hire* observavit quod virga ferrea quae tempore consimili hyberno sex erat pedum longitudinis, ubi soli aestivo exponebatur evasit sex pedum longitudinis cum duabus tertiis partibus lineae. In priore casu calor major fuit quam in posteriore, in hoc vero major fuit quam calor externarum partium corporis humani. Nam metalia ad solem aestivum valde incalescunt. At virga penduli in horologio oscillatorio nunquam exponi solet calori solis aestivi, nunquam calorem concipit calori externae superficiei corporis humani aequalem. Et propterea virga penduli in horologio tres pedes longa, paulo quidem longior erit tempore aestivo quam hyberno, sed excessu quartam partem lineae unius vix superante. Proinde differentia tota longitudinis pendulorum quae in diverus regionibus isochrona sunt, diverso calori attribui non potest. Sed neque erroribus astronomorum e *Gallia* missorum tribuenda est haec differentia. Nam quamvis eorum observationes non perfecte congruant inter se, tamen errores sunt adeo parvi ut contemni possint. Et in hoc concordant omnes, quod isochrona pendula sunt breviora sub aequatore quam in observatorio regio Parisiensi, existente differentia non minore quam lineae unius cum quadrante, non majore quam linearum $2\frac{2}{3}$. Per observationes D. *Richeri* in *Cayenna* factas differentia fuit lineae unius cum quadrante. Per eas D *Des Hayes* differentia illa correcta prodiit lineae unius cum semisse vel unius cum tribus quasiis partibus lineae. Per eas aliorum minus accuratas prodiit eadem quasi duarum linearum. Et haec discrepantia partim ab erroribus observationum, partim a dissimilitudine partium internarum terrae & altitudine montium, & partim a diversis aeris caloribus, oriri potuit.

Virga ferrea pedes tres longa, tempore hyberno in *Anglia,* brevior est quam tempore aestivo, sexta parte lineae unius, quantum sentio. Ob calores sub aequatore auferatur haec quantitas de differentia lineae unius cum quadrante a *Richero* observata, & manebit linea $1\frac{1}{12}$ quae cum linea $1\frac{87}{1000}$ per theoriam jam ante collecta probe congruit. *Richerus* autem observationes in *Cayenna* factas, singulis septimanis per menses decem iteravit, & longitudines penduli in virga ferrea ibi notatas cum longitudinibus eius in *Galiia* similiter notatis contulit. Quae diligentia & cautela in aliis observatoribus defuisse videtur. Si huius observationibus sidendum est, terra altior erit ad aequatorem quam ad polos excessu miliiarium septendecim circiter ut supra per theoriam prodiit.

PROPOSITIO XXI. THEOREMA XVII.

Punctae a*equinoctialia regredi, & axem terrae singulis revolutionibus annuis nutando his inclinari in ecllpticam* & *bis redire ad positionem priorem.*

Patet per corol. 20. Prop. LXVI. Lib. I. Motus tamen iste nutandi perexiguus esset debet, & vix aut ne vix quidem sensisibilis.