SECTION IX.

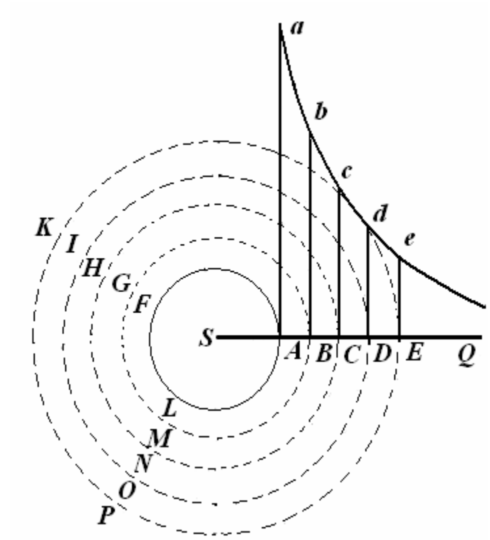
Concerning the circular motion of fluids.

HYPOTHESIS.

The resistance, which arises from the deficiency of lubricity of the parts, with all else being equal, is proportional to the velocity, by which the parts of the fluid may be separated from each other.

PROPOSITION LI. THEOREM XXXIX.

If an infinitely long solid cylinder may be rotating in an infinite uniform fluid with a given uniform motion about an axis in place , and the fluid may be disturbed into an orbit by the impulse of this alone, and each and every part of the fluid will persevere uniformly in its motion, I say that the periodic times of the parts of the fluid shall be as their distances from the axis of the cylinder.



Let *AFL* be the cylinder driven uniformly about the axis *S* acting on the orbit, the fluid may be separated by the concentric circles *BGM*, *CHN*, *DIO*, *EKP*, etc. into innumerable concentric solid orbits of this, with the same thickness. And because the fluid is homogeneous, the impressed forces of the contiguous orbits made between each other will be (by the hypothesis) as the translations of these between each other [*i.e*. their relative velocities], and as the contiguous surfaces on which impressed forces are made. If the impressed force on some orbit is greater or less from the concave side rather than from the convex side; the stronger impressed force will prevail, and the motion will be accelerated or decelerated, so that it is corrected into the direction of the motion itself, or into the contrary. Hence so that each and every orbit may persevere in its own regular motion, the impressed forces must be equal on each part in turn, and must be made on each in opposing directions. From which since the impressed forces are as the contiguous surfaces and the translations of these one to the other,
[Thus, the constant tangential frictional force on an orbit is assumed to vary as the relative velocity between neighbouring surfaces, and as the area of either surface; thus $rel.vel.\times area = const;$ or $r^2 \frac{d\omega}{dr}\delta r \ \propto \delta r$, as the area in this case is that of a cylindrical surface of unit length, and the relative velocity is the limiting difference in the angular speeds;]
the translations will be inversely as the surfaces, that is, inversely as the distance of the surfaces from the axis [*i.e.*, in this case, as the area in contact is proportional to the radius of the orbit]. Moreover, the applied differences of the angular motions about the axis are

as these translations to the distances, or as the translations directly and the distances inversely ; that is, as with the ratios taken together, inversely as the square of the distances. [*i.e.* the *y* and *x* coordinates in the adjoining diagram.]

[Thus, according to *B.* & *R.*, p. 296, we may consider the friction between two adjacent solid cylinders. Suppose ω and $\omega+\delta\omega$ are the angular velocities of two consecutive cylinders, and let *r* be the radius of their common surface, then the relative velocity of the two cylinders is $(\omega+\delta\omega)r-\omega r = r\frac{d\omega}{dr}\delta r$; the internal friction between the rotating fluid cylinders is proportional both to the difference (or differential) of the velocities, and to the common area in contact, which is proportional to the radius in this case, giving the frictional force proportional to $r^2\frac{d\omega}{dr}\delta r$. Now this quantity must remain constant, as we have steady state conditions, and hence $r^2\frac{d\omega}{dr}=-\alpha$, for some constant α . This equation may be integrated to give $\omega=\frac{\alpha}{r}+\beta$, where α and β are constants depending on the angular velocities of the inner and outer cylinders. Hence, from known values of ω and r , the constants α and β can be found. This is the integration performed next by Newton, where the outer limit is infinite and so β is zero. In which case $\omega=\frac{\alpha}{r}$, and Newton's result follows.]

Whereby if, to the parts of the indefinite right line *SABCDEQ*, the perpendiculars *Aa*, *Bb, cC, Dd, Ee*, &c. may be erected inversely proportional to the squares of the individual parts *SA, SB, SC, SD, SE*, etc., and through the ends of the perpendiculars a hyperbolic [like] curve may be considered to be drawn; the sum of these differences [of the angular speeds] will be, that is, the whole angular motion, as the corresponding sum of the lines *Aa, Bb, Cc, Dd, Ee*, that is, if for the fluid medium being uniformly put in place, the number of the orbits may be increased and the width may diminished indefinitely, as the analogous hyperbolic areas for these sums *AaQ, BbQ, CcQ, DdQ, EeQ*, etc. And the times are inversely proportional to the angular motions, also they will be inversely proportional to these areas. Therefore the periodic time of any particular curve *D* varies inversely as the area *DdQ*, that is (by the known quadratures of the curves), directly as the distance *SD. Q.E.D.*

Corol. I. Hence the angular motions of the particles of the fluid are inversely as the distances of these from the axis of the cylinder, and the absolute velocities are equal.

[That is, the whole mass rotates about the vertical axis with the same tangential velocity, whatever the radius; Newton later acknowledges in a Scholium that this state of affairs is unstable, unless some other condition applies. It is evident that Newton had constructed such a device, but he has not given any details here.]

Corol. 2. If the fluid may be contained in a cylindrical vessel of infinite length, and another cylinder may be contained within, but each cylinder may be revolving about their common axis, and the times of the revolutions shall be as the radii of these, and each part of the fluid may continue in its own motion : the period times of the individual parts will be as the distances of these from the axis of the cylinders.

[Thus, from $\omega = \frac{\alpha}{r}$ we have $\frac{T_1}{T_2} = \frac{r_1}{r_2}$.]

Corol. 3. If some common angular motion may be added to this kind of motion by the cylinder and the fluid, because by this new motion the friction of the mutual parts of the fluid does not change, the motions of the parts among themselves will not be changed. For the translations of the parts from one to the other depend on friction. Any part will persevere in that motion which, with the frictional forces acting in opposite directions on each side, may not be accelerated or retarded more.

Corol. 4. From which if all of the angular motion of the external cylinder may be removed from the whole system of the cylinders and fluid, the motion of the fluid will be had for a cylinder at rest.

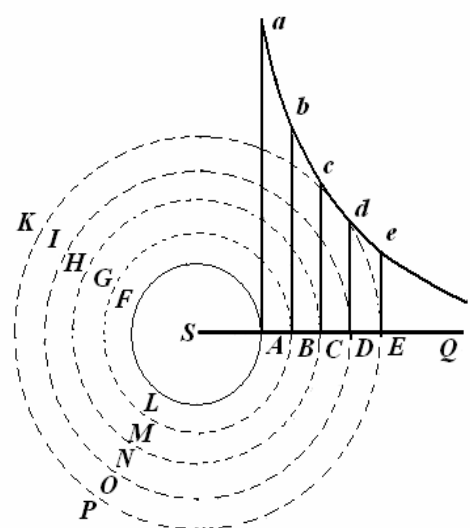
Corol. 5. Therefore if with both the fluid and the exterior cylinder at rest, the interior cylinder may be revolving uniformly; it will communicate a circular motion to the fluid, and gradually it will be propagated through the whole fluid ; nor will the first kind of motion cease to increase until the individual parts may acquire the motion defined by corollary four.

Corol.6. And because the fluid tries to propagate its motion more widely at this stage, it will carry the exterior cylinder around by its force around unless strongly retained ; and its motion will be accelerated until the periodic times of both cylinders are equal to each other. Because if the exterior cylinder may be strongly retained, it will try to retard the motion of the fluid ; and unless the interior cylinder be pressed on by some extrinsic force so that motion may be conserved, it will have the effect of gradually slowing that down.

All of which can be proven in deep water at rest.

PROPOSITION LII. THEOREM XL.

If a solid sphere, in an infinite uniform fluid, may be rotating with a uniform motion about a given axis in place, and the orbiting fluid may be agitated by the impulse of this alone, and moreover every part of the fluid may continue uniformly in its motion : I say that the periodic times of the parts of the fluid will be as the squares of the distances from the centre of the sphere.



Case 1. Let *AFL* be the sphere driven uniformly in obit about the axis *S*, and the fluid [in a plane normal to the axis] may be separated into innumerable concentric spheres of equal thickness by the concentric circles *BGM*, *CHN*, *DIO*, *EKP*, &c. Moreover suppose these orbits to be solid, and because the fluid is homogeneous, forces of contact of the orbits made between each other, will be (by hypothesis) as the translations of these from one to the other [*i.e*. the relative distance gone per unit time between the shells, or the velocity gradient, in modern terms], and as the surfaces in contact on which the forces act. If the force impressed on some orbit is either greater or less from the concave part as from the convex part; the stronger impressed force will prevail, and the velocity of the orbit will either accelerate or decelerate, as it is returned by its motion in the same or contrary direction. Hence so that each one of the orbits will persevere uniformly in its orbit, the forces impressed from each part must become equal between each other, and are made in opposite directions. Hence since the impressions shall be as the contiguous surfaces and as the translations of these relative to each other, that is, inversely as the square of the distances of the surfaces from the centre. But the differences of the angular motions about the axis are as these applied translations to the distances, or as the translations directly and the distances inversely, that is, with the ratios taken jointly, as the cube of the distances inversely. Whereby if the perpendiculars *Aa*, *Bb*, *Cc*, *Dd*, *Ee*, &c. may be erected, the perpendiculars of *SA*, *SB*, *SC*, *SD*, *SE*, &c. are themselves inversely proportional to the cubes of the innumerable individual parts of the right line *SABCDEQ*; the sum of the differences, that is, the whole angular motion will be as the corresponding sum of the lines *Aa*, *Bb*, *Cc*, *Dd*, *Ee* : that is (if the number of orbits may be increased and the width may be diminished indefinitely towards uniformly establishing a fluid medium) as the hyperbolic [like] area from these analogous sums *AaQ*, *BbQ*, *CcQ*, *DdQ*, *EeQ*, &c. And the periodic times inversely proportional to these angular motions also will be inversely proportional to these areas. Therefore the periodic time of any orbit *DIO* will be inversely as the area *DdQ*, that is, from the known quadratures of the curves, directly as the square of the distance *SD*. That which was wished to show in the first place.

[As in the previous proposition, with the surface area now proportional to r^2, we may consider the frictional force to become as $r^3 \frac{d\omega}{dr}\delta r$, which is constant, and hence $r^3 \frac{d\omega}{dr} = -\alpha$. This may be integrated to give $\omega = \frac{\alpha}{r^2} + \beta$, where we consider the angular integration to be treated as in case 2 below ; as the fluid is infinite, this integration becomes $\omega = \frac{\alpha}{r^2}$, as required.]

Case 2. As many right lines as you wish may be drawn from the centre of the sphere, which may contain a given angle with the axis, mutually greater than each other by a given angle, and from these right lines rotated about the axis, the orbits are to be cut into innumerable rings; and each and every ring will have four rings contiguous to itself, one interior, another exterior, and two lateral ones. There cannot be friction from any inner and outer rings in the motion, except by that acting in equal and opposite directions, just as I have treated in the first case. This is apparent from the demonstration of the first case. [Thus, any cross-section normal to the axis can be treated as in the first case.] And therefore any series of rings going from the sphere along straight lines to infinity, will be moved by the law of the first case, unless they may be impeded by friction from the sides. But in the motion made, according to this law, the friction of the rings on the sides is zero ; and thus no motion may be impeded, so that it may become less by this law. If annuli which are equally distant from the centre, either may be rotating swifter or slower around the poles than around the ecliptic; the slower ones may be accelerated, and the velocities

may be retarded from the friction of the motion, and thus the periodic times always approach towards equality, by the law of the first case. Therefore here the friction does not impede that motion less than following the law of the first case, and therefore that law will be obtained : that is, the times of the individual annuli will be as the squares of the distances of these from the centre of the sphere. Which was the second case to be demonstrated.
[Clearly there is a problem with a fluid behaving in this way: for the angular dependence of the motion has not been resolved. This is of course a rather difficult problem, and Newton proceeds to argue his way out of difficulties by considering reduced causes and reduced effects .]

Case 3. Now each and every annulus may be divided by transverse sections into innumerable particles, constituting the fluid substance absolutely and uniformly ; and because these sections may not regard the law of circular motion, but only bring about the circulation of the fluid, the circular motion will persevere as at first. All the annuli from these sections as the smallest roughness and the friction force are changed minimally, either will be changed equally, or not be changed equally. And with the proportion of the causes remaining, the proportion of the effect will remain, that is, the proportion of the motions and of the periodic times . *Q. E. D.*

Moreover, since the circular motion, and hence the centrifugal force arising, shall be greater at the ecliptic than at the poles ; some cause will have to be present by which the individual particles may be retained in their circles ; lest the matter, which is at the ecliptic, may not always recede from the centre and from the exterior vortices may move to the poles, and thence by the axis may be returned to the ecliptic by circulating perpetually.

Corol. 1. Hence the angular motion of the parts of the fluid around the axis of the sphere, are inversely as the squares of the distances from the centre of the sphere, and the absolute velocities inversely as the same applied squares to the distances from the axis.

Corol. 2. If a sphere may be revolving with a given uniform motion around an axis in position in a fluid to be similarly at rest, the motion will be communicated to the fluid in the manner of a vortex, and this motion will be propagated little by little to infinity ; and nor will the first motion cease in the individual parts of the fluid accelerated, as the periodic times of the individual parts shall become as the squares of the distances from the centre of the sphere.

Corol. 3. Because the interior parts of the vortices on account of their greater velocity rub against and exert a force on the exterior parts, and perpetually they communicate the motion to these by that action, and these exterior ones likewise transfer the quantity of motion by that action at this stage to other more exterior ones, and by this action they clearly maintain an invariant quantity of motion, it is apparent that the motion is perpetually transferred from the centre to the circumference of the vortex, and may be absorbed by the infinite circumference. The matter between any two concentric spherical surfaces will never be accelerated by the vortex, because there all the motion taken from the interior matter is always transferred to the outer matter.

Corol. 4. Hence towards the conservation of the vortex by moving constantly in the same state, some principle of action is required, by which the sphere may always accept the same quantity of motion, that will be impressed on the matter of the vortex. Without such a kind of principle, it is necessary that the sphere and the interior parts of the vortex, always propagating their motion into the exterior parts, and not receiving any new motion, are slowed down little by little and forced to stop in the orbit.

Corol. 5. If another sphere might drift towards this vortex at a certain distance from the centre of this, and meanwhile it may be rotating constantly at an inclination by some force about a given axis ; by its motion it may snatch some fluid from the vortex : and at first this new and very small vortex may be rotating together with the sphere around the centre of the other, and meanwhile its motion will spread out wider, and little by little it will be propagated to infinity, according to the manner of the first vortex. And by the same reasoning, by which the sphere may be seized by the motion of the other sphere, also by its motion it may seize the other sphere, thus so that the two spheres may be revolving about some point between the two, and on account of that mutual circular motion they may fly away from each other, unless they may be restrained by some force. Afterwards if the constantly impressed forces may be stopped, by which the spheres may persevere in their motions, and all may be permitted by the laws of mechanics, the motion of the spheres will become less (on account of the reason I have assigned in Corol. 3. and 4.) and the vortices at last will come to rest.

Corol. 6. If several spheres may be revolving constantly in given locations with certain velocities about given axes in position, just as many vortices may be made going off th infinity. For the individual spheres by the same reason, by which some one may propagate its motion to infinity, also they will propagate their motions to infinity, thus so that any part of the infinite fluid will be disturbed by that motion which may result from the actions of all the spheres. From which the vortices will not be defined within certain boundaries, but will extend out a little mutually between themselves, and the spheres by the actions of the vortices will always be moving mutually from their positions, as has been explained in the above corollary ; and nor may it be clear how they may keep their positions among each other, unless retained by some force. Moreover with these forces ceasing, which constantly impressed on the spheres preserve these motions, the matter on account of the reason assigned in the third and fourth corollaries, will relax little by little in the vortices and cease to act.

Corol. 7. If the fluid similarly may be enclosed in a spherical vessel, and may be disturbed into a vortex by a sphere present at the centre rotating uniformly, moreover the sphere and the vessel may be revolving around in the same direction, and the periodic times of these shall be as the squares of the radii : the parts of the fluid later will continue in their motions without acceleration or retardation, so that the periodic times of these shall be as the squares of the distances from the centre of the vortex. No other constitution of vortices shall be able to remain.

Corol. 8. If the vessel, inclosing the fluid, and the sphere may maintain this motion and in addition may be rotating with a common angular motion about some given axis ; because in this new motion the friction of the parts of the fluid shall not be changed from one to the other, nor will the motion of the parts among themselves be changed. For the translations of the parts amongst themselves depends on the friction. Any part will persevere in that motion, by which it shall not be retarded more by the friction of one side than accelerated more by the friction from the other.

Corol. 9. From which if the vessel may be at rest and the motion of the sphere may be given, the motion of the fluid will be given. For take a plane crossed by the axis of the sphere and to be revolving in a contrary motion ; and suppose the sum of the times of the revolution of this and of the revolutions of the sphere to be to the time of revolution of the sphere, as the square of the radius of the vessel to the square of the radius of the sphere : and the periodic times of the parts of the fluid with respect to its plane will be as the squares of their distances from the centre of the sphere.

Corol. 10. Thus if the vessel either may be moving around the same axis with the sphere, or around some different axis with some given velocity, the motion of the fluid will be given. For if the angular motion of the vessel may be taken away from the whole system, all the same motions will remain between themselves which were given at first by Corol. VIII. And this motion will be given by Corol. IX.

Corol. 11. If the vessel and the fluid are at rest and the sphere may be revolving with a uniform velocity, the motion will be propagated little by little through the whole fluid in the vessel, and the vessel may be driven around unless strongly detained, nor will the fluid and the vessel cease to accelerate as at first, until the periodic times of these shall be equal to the periodic times of the sphere. Because if the vessel may be detained a little by some force or may be revolving at some constant and uniform motion, the medium may arrive little by little at the state of the motion defined in Corollaries VIII, IX, and X, nor will it persevere in any other state. Then truly if, with these forces ceasing by which the vessel and the sphere will be rotating with a certain motion, the whole motion of the system may be allowed by the laws of mechanics; the vessel and the sphere will act in turn on each other by the fluid medium, nor will their first motion cease to propagate through the fluid between each other, so that the periodic times of these are equal to each other, and the whole system becomes the image of a single solid body revolving at the same time.

Scholium.

In all these I suppose the fluid consists of matter of uniform density and fluidity. For such a fluid it is in which the sphere likewise shall be able to propagate by the same motion, in the same time interval, similar and equal motions, always at equal distances to each other, wherever it may be placed in the fluid. Indeed the matter by its own circular motion may try to recede from the axis of the vortex, and therefore presses on all the further matter. From this compression there shall be stronger friction of the further parts and the separation from each other shall be more difficult ; and as a consequence the fluidity of the matter may be diminished. Again if the parts of the fluid are somewhere coarser or larger, the fluidity will be less there, on account of the fewer surfaces into which the parts may be separated from each other in turn. In cases of this kind the deficiency in fluidity either the lubricity or viscosity of the parts I suppose to be restored in some other way or by some other condition. Unless this may be done, the matter where it is less fluid will cohere more and will be slower, and thus the motion is undertaken more slowly and will propagate further than by the account given above. If the shape of the vessel may not be spherical, the particles will be moving along non-circular lines, but by conforming to the same shape as the vessel, and the periodic times will be as the squares of the average distances from the centre approximately. In the parts between the centre and the circumference, where the spaces are wider, the motions will be slower, and where faster where more restricted, nor yet will the faster particles wish reach the circumference. For the arcs describe smaller curves, and trying to recede from the centre will not be diminished by the decrease of its curvature, as it will be increased by the increase of the velocity. By going from the narrower spaces into the wider they will recede a little more from the centre, but they will be slowed down by this recession, and by approaching later from the wider to the narrower spaces they will be accelerated, and thus the individual particles will be retarded and accelerated in turn for ever. Thus these will be held in a rigid vessel. For the constitution of vortices in an infinite fluid is known by corollary six of this proposition.

Moreover I have tried to investigate the properties of vortices in this proposition, so that I might investigate if by some account or other celestial phenomena might be able to be explained by vortices. For the phenomenon is, that the periodic times of revolution of the planets around Jupiter are in the three on two ratio of their distances from the centre of Jupiter, and the same rule prevails with the planets which are revolving around the sun. Moreover these rules prevail as the most accurate on each of the planets, to the extent that astronomical observations can provide at this time. And thus if these planets may be carried by revolving around Jupiter and the sun in vortices, these vortices also will have to be revolving by the same law. Indeed the periodic times of the parts of the vortices will be emerging in the square ratio of the distances from the centre of motion : nor can that ratio be diminished and reduced to the three on two, unless either the matter of the vortex there shall be more fluid that is at a greater distance from the centre, or the resistance, which arises from the deficiency of the lubricity of the parts of the fluid, from the increase in the velocity by which the parts of the fluid may be separated in turn, may be increased in a greater ratio than that by which the velocity may be increased. Of which still neither the one nor the other can be considered to be in agreement. The thicker and less fluid parts will be trying to reach the circumference, except the heavy parts at the centre ; and it is very likely that, even if for the sake of demonstrating such a hypothesis I might propose at the beginning of this section, that the resistance should be proportional to the velocity, yet the resistance shall be in a smaller ratio than that is of the velocity. With which conceded, the periodic times of the parts of the vortex will be in a ratio greater than in the duplicate ratio of the distances from its centre. So that if the vortices (as it is the opinion of some) may be moving faster near the centre, then slower as far as to a certain boundary, then anew faster near the circumference ; certainly neither the three on two ratio [sesquiplicate] nor some other ratio can be obtained and determined with certainty. And thus philosophers will have to consider by what agreement that phenomenon of the three on two ratio may be explained by vortices.

PROPOSITION LIlI. THEOREM XLI.

Bodies, which carried by a vortex return in orbit, are of the same density as the vortex, and are moved by the same law with its parts as far as the velocity and the course determined.

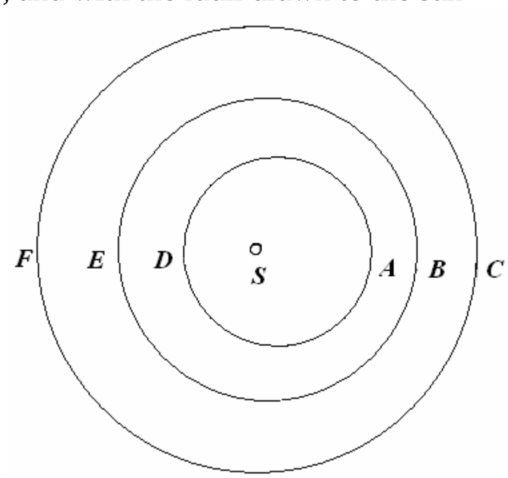
For if some very small part of the vortex may be supposed to be congealed, the particles of which or physical points maintain a given situation among themselves : this, since neither as far as its density, nor as far the insitu force or its figure may be changed, will move by the same law as before: and on the contrary, if a frozen and solid part of the vortex shall be of the same density with the rest of the vortex, and it may be dissolved into the fluid ; this will be moving by the same law as before, unless perhaps particles of this now made fluid may move amongst themselves. Therefore the motion of particles amongst themselves may be ignored, being considered as nothing to the whole progressive motion, and the whole motion will be as before. But the motion likewise will be as the motion of other parts of the vortex equally distant from the centre, therefore so that a solid dissolved in the fluid shall be a part of the vortex just the same as the other parts. Hence a solid, if it shall be of the same density as the matter of the vortex, will be moving with the same motion as the parts themselves, in matter nearby being relatively at rest. If it shall be denser, now it will try more than at first to recede from the centre of the vortex ; and thus that force of the vortex, by which at first it was being held in place in equilibrium in its orbit, now being overcome, it will recede from the centre and describe a spiral on revolving, no more returning in the same orbit. And by the same argument if it shall be rarer, it will approach towards the centre. Therefore it will not return in the same orbit unless it shall be of the same density as the fluid. But in that case it has been shown, that it would be revolving by the same law with the parts of the fluid equally distant from the centre of the vortex. *Q.E.D.*

Corol. 1. Hence a solid which is revolving in a vortex and always returns in the same orbit, is relatively at rest in the fluid in which it floats around.

Corol. 2. And if as long as the vortex shall be of a uniform density, the same body can be revolving at any distance from the centre of the vortex.

Scholium.

Hence it is clear that the planets cannot be carried by vortex bodies. For the planets following the Copernican [Newton means Kepler here] hypothesis are carried around the sun, revolving in ellipses with the sun at a focus, and with the radii drawn to the sun describing areas proportional to the times. But the parts of a vortex are unable to revolve in such a motion. *AD*, *BE*, and *CF* may designate three orbits described around the sun *S*, of which the outermost circle *CF* shall be concentric with the sun, and the aphelions of the two interior shall be *A*, *B* and the periheliums *D*, *E*. Therefore a body that is revolving in the orbit *CF*, with the radius drawn to the sun by describing areas proportional to the times, will be moving with a uniform motion. But a body, which is revolving in the orbit *BE*, will be moving slower at the *B* and faster at the perihelion *E*, following the astronomical laws ; yet since following the laws of mechanics the matter of the vortex in the narrower space between *A* and *C* ought to be moving faster than in the wider space between *D* and *F*; that is, faster at the aphelions than at the perihelion. Which



two disagree amongt themselves. Thus at the start of sign of Virgo, where the aphelion of Mars now turns, the distances between the orbits of Mars and of Venus is to the distance of the same orbits at the beginning of the sign Pisces as three to two approximately, and therefore the matter of the vortex between these orbits at the beginning of Virgo must be in the ratio of three to two. For as the space is narrower through which the same quantity of matter pases in one revolution of the time, it must cross with a greater velocity there. Therefore if the Earth may be carried relatively at rest by that in this celestial matter, and may be revolving in one circle around the sun with it, its velocity at the beginning of Pisces shall be to the velocity of the same at the beginning of Virgo in the ratio of three on two. From which the apparent diurnal motion of the sun at the beginning of Virgo should be as seventy minutes, and at the beginning of Pisces less than forty eight minutes : yet since (shown by experiment) this apparent motion of the motion of the sun shall be greater at the start of Pisces than at the start of Virgo, and therefore the Earth moves faster at the start of Virgo than at the start of Pisces. And thus the hypothesis of vortices disagrees generally with astronomical phenomina, and not only regarding explanations but also regarding the perturbations of the heavenly motions it brings about. On account of which, truly how these motions may be completed in free spaces without vortices, can be understood from book one, and in the system of the world now will be discussed more fully.

SECTIO IX.

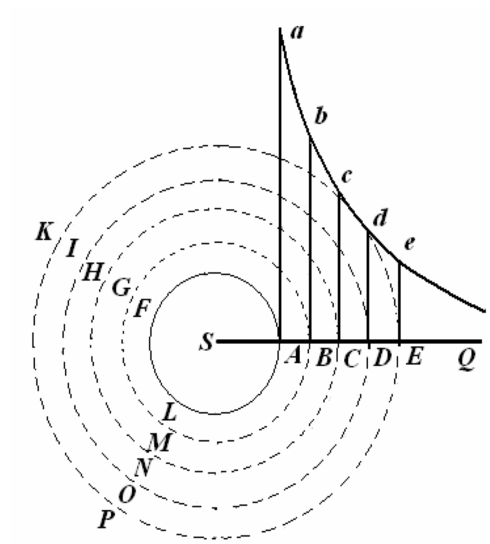
De motu circulari fluidorum.

HYPOTHESIS.

Resistentiam, quae oritur ex defectu lubricitatis partium fluidi, caeteris paribus, proportionalem esse velocitati, qua partes fluidi separantur ab invicem.

PROPOSITIO LI. THEOREMA XXXIX.

Si cylindrus solidus infinite longus in fluido uniformi & infinito circa axem positione datum uniformi cum motu revolvatur, & ab huius impulsu solo agatur fluidum in orbem, perseveret autem fluidi pars unaquaeque uniformiter in motu suo, dico quod tempora periodica partium fluidi sunt ut ipsarum distantiae ab axe cylindri.



Sit *AFL* cylindrus uniformiter circa axem *S* in orbem actus, & circulis concentricis *BGM, CHN, DIO, EKP,* &c. distinguatur fluidum in orbes cylindricos innumeros concentricos solidos eius, idem crassitudinis. Et quoniam homogeneum est fluidum, impressiones contiguorum orbium in se mutuo factae erunt (per hypothesin) ut eorum translationes ab invicem, & superficies contiguae in quibus impressions fiunt. Si impressio in orbem aliquem maior est vel minor ex parte concava quam ex parte convexa; praevalebit impressio fortior, & motum orbis vel accelerabit vel retardabit, prout in eandem regionem cum ipsius motu vel in contrariam dirigitur. Proinde ut orbis unusquisque in motu suo uniformiter perseveret, debent impressiones ex parte utraque sibi invicem aequari & fieri in regiones contrarias. Unde cum impressiones sunt ut contiguae superficies & harum translationes ab invicem, erunt translationes inverse ut superficies, hoc est, inverse ut superficierum distantiae ab axe. Sunt autem differentiae motuum angularium circa axem ut hae translationes applicatae ad distantias, sive ut translationes directe & distantiae inverse ; hoc est, coniunctis rationibus, ut quadrata distantiarum inverse. Quare si ad infinite rectae *SABCDEQ* partes singulas erigantur perpendicula *Aa, Bb, cC, Dd, Ee,* &c. ipsarum *SA, SB, SC, SD, SE,* &c. quadratis reciproce proportionalia, & per terminos perpendicularium duci intelligatur linea curva hyperbolica; erunt summae differentiarum, hoc est, motus toti angulares, ut respondentes summae linearum *Aa, Bb, Cc, Dd, Ee,* id est, si ad constituendum medium uniformiter fluidum, orbium numerus augeatur & latitudo minuatur in infinitum, ut area hyperbolicae his summis analogae *AaQ, BbQ, CcQ, DdQ, EeQ,* &c. Et tempora motibus angularibus reciproce proportionalia, erunt etiam his areis reciproce proportionalia. Est igitur tempus periodicum particulae cuiusvis *D* reciproce ut area *DdQ*, hoc est (per notas curvarum quadraturas) directe ut distantia *SD*. *Q.E.D*.

Corol. I. Hinc motus angulares particularum fluidi sunt reciprocae ut ipsarum distantia ab axe cylindri, & velocitates absolute sunt aequales.

Corol. 2. Si fluidum in vase cylindrico longitudinis infinite contineatur, & cylindrum alium interiorem continear, revolvatur autem cylindrus uterque circa axem communem, sintque revolutionum tempora ut ipsorum semidiametri, & perseveret fluidi pars unaquaeque in motu suo: erunt partium singularum tempora periodica ut ipsarum distantiae ab axe cylindrorum.

Corol. 3. Si cylindro & fluido ad hunc modum motis addatur vel auferatur communis quilibet motus angularis , quoniam hoc novo motu non mutatur attritus mutuus partium fluidi, non mutabuntur motus partium inter se. Nam translationes partium ab invicem pendent ab attritu. Pars quaelibet in eo perseverabit motu, qui, attritu utrinque in contrarias partes facto, non magis acceleratur quam retardatur.

Corol. 4. Unde si toti cylindrorum & fluidi systemati auferatur motus omnis angularis cylindri exterioris, habebitur motus fluidi in cylindro quiescente.

Corol. 5. Igitur si fluido & cylindro exteriore quiescentibus, revolvatur cylindrus interior uniformiter, communicabitur motus circularis fluido, & paulatim per totum fluidum propagabitur; nec prius desinet augeri quam fluidi partes singulae motum corollario quarto definitum acquirant.

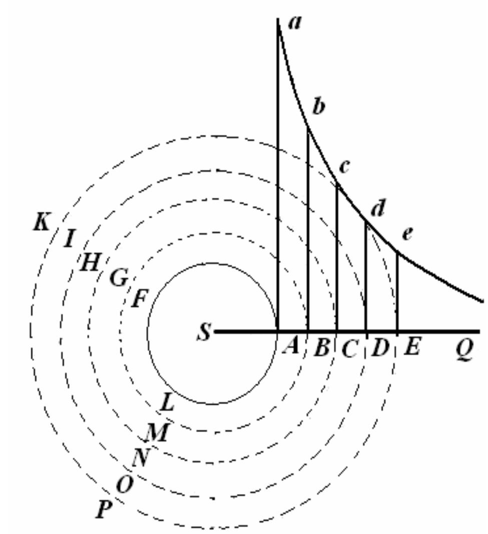
Corol.6. Et quoniam fluidum conatur motum suum adhuc latius propagare, huius impetu circumagetur etiam cylindrus exterior nisi violenter detentus ; & accelerabitur eius motus quoad usque tempora periodica cylindri utriusque aequentur inter se. Quod si cylindrus exterior violenter detineatur, conabitur is motum fluidi retardare; & nisi cylindrus interior vi aliqua extrinsecus impressa motum illum conservet, efficiet ut idem paulatim cesset.

Quae omnia in aqua profunda stagnante experiri licet.

PROPOSITIO LII. THEOREMA XL.

Si sphaera solida, in fluido uniformi & infinito,circa axem positione datum uniformi cum motu revolvatur, & ab huius impulsu solo agatur fluidum in orbem , perseveret autem fluidi pars unaquaeque uniformiter in motu suo: dico quod tempora periodica partium fluidi erunt ut quadrata distantiarum a centro sphaerae.

Cas: 1. Sit *AFL* sphaera uniformiter circa axem *S* in orbem acta, & circulis concentricis *BGM*, *CHN*, *DIO*, *EKP*, &c. distinguatur fluidum in orbes innumeros concentricos eiusdem crassitudinis. Finge autem orbes illos esse solidos , & quoniam homogeneum est fluidum, impressiones contiguorum orbium in se mutuo factae, erunt (per hypothesin) ut eorum translationes ab invicem & superficies contigua in quibus impressiones fiunt. Si impressio in orbem aliquem maior est vel minor ex parte concava quam ex parte convexa; praevalebit impressio fortior, & velocitatem orbis vel accelerabit vel retardabit, prout in eandem regionem cum ipsius motu vel in contrariam dirigitur. Proinde ut orbis unusquisque in motu suo perseveret uniformiter, debebunt impressiones ex parte utraque sibi invicem aequari, & fieri in regiones contrarias. Unde cum impressiones sint ut contiguae superficies & harum translationes ab invicem; erunt translationes inverse ut superficies, hoc est, inverse ut quadrata distantiarum superficierum a centro. Sunt autem differentiae motuum angularium circa axem ut hae translationes applicatae ad distantias, sive ut translationes directe & distantiae inverse , hoc est, coniunctis rationibus ut cubi distantiamm inverse. Quare si ad rectae infinitae *SABCDEQ* partes singulas erigantur perpendicula *Aa, Bb, Cc, Dd, Ee,* &c. ipsarum *SA, SB, SC, SD, SE,* &c. cubis reciproce proportionalia, erunt summae differentiarum, hoc est, motus toti angulares, ut respondentes summae linearum *Aa*, *Bb*, *Cc*, *Dd*, *Ee* : id est (si ad constituendum medium uniformiter fluidum, numerus orbium augeatur & latitudo minuatur in infinitum) ut area hyperbolicae his summis analogae *AaQ, BbQ, CcQ, DdQ, EeQ,* &c. Et tempora periodica motibus angularibus reciproce proportionalia erunt etiam his areis reciproce proportionalia. Est igitur tempus periodicum orbis cuiusvis *DIO* reciproce ut area *DdQ*, hoc est, per notas curvarum quadraturas, directe ut quadratum distantiae *SD*. Id quod volui primo demonstrare.



Cas. 2. A centro spherae ducantur infinitae rectae quam plurimae, quae cum axe datos contineant angulos, aequalibus differentiis se mutua superantes , & his rectis circa axem revolutis concipe orbes in annulos innumeros secari; & annulus unusquisquc habebit annulos quatuor sibi contiguos, unum interiorem, alterum exteriorem & duos laterales. Attritu interioris & exterioris non potest annulus unusquisque, nisi in motu iuxta legem casus primi facto, aequaliter & in partes contrarias urgeri. Patet hoc ex demonstratione casus primi. Et propterea annulorum series quaelibet a globo in infinitum recta pergens, movebitur pro lege casus primi, nisi quatenus impeditur ab attritu annulorum ad latera. At in motu hac lege facto attritus annulorum ad latera nullus est ; neque ideo motum, quo minus hac lege fiat, impediet. Si annuli, qui a centro aequaliter distant, vel citius revolverentur vel tardius iuxta polos quam iuxtas eclipticam ; tardiores accelerarentur, & velociores retardarentur ab attritu mutuo, & sic vergerent semper tempora periodica ad aequalitatem, pro lege casus primi. Non impedit igitur hic attritus quo minus motus fiat secundum legem casus primi, & propterea lex illa obtinebit : hoc est, annulorum singulorum tempora periodica erunt ut quadrata distantiarum ipsorum a centro globi, Quod volui secundo demonstrare.

Cas. 3. Dividatur iam annulus unusquisque sectionibus transversus in particulas innumera, constituentes substantiam absolute & uniformiter fluidam ; & quoniam hae sectiones non spectant ad legem motus circularis, sed ad constitutionem fluidi solummodo conducunt, perseverabit motus circularis ut prius. His sectionibus annuli omnes quam minimi asperitatem & vim attritus mutui aut non mutabunt, aut mutabunt aequaliter. Et manente causarum proportione manebit effectuum proportio, hoc est, proportio motuum & periodicorum temporum. *Q. E. D.* Caeterum cum motus circularis, & inde orta vis centrifuga, maior sit ad eclipticam quam ad polos ; debebit causa aliqua adesse qua particulae singulae in circulis suis retineantur ; ne materia, quae ad eclipticam est, recedat semper a centro & per exteriora vorticis migret ad polos, indeque per axem ad eclipticam circulatione perpetua revertatur.

Corol. I. Hinc motus angulares partium fluidi circa axem globi, sunt reciproce ut quadrata distantiarum a centro globi, & velocitates absolutae reciproce ut eadem quadrata applicata ad distantias ab axe.

Corol. 2. Si globus in fluido quiescente similari & infinite circa axem positione datum uniformi cum motu revolvatur, communicabitur motus fluido in morem vorticis, & motus iste paulatim propagabitur in infinitum; neque prius cessabit in singulis fluidi partibus accelerari, quam tempora periodica singularum partium sint ut quadrata distantiarum a centro globi.

Corol. 3. Quoniam vorticis partes interiores ob maiorem suam velocitatem atterunt & urgent exteriores, motumque ipsis ea actione perpetuo communicant, & exteriores illi eandem motus quantitatem in alios adhuc exteriores simul transferunt, eaque actione servant quantitatem motus sui plane invariatam , paret quod motus perpetuo transfertur a centro ad circumferentiam vorticis, & per infinitatem circumferentiae absorbetur. Materia inter sphaericas duas quasvis superficies vortici concentricas nunquam accelerabitur, eo quod motum omnem a materia interiore acceptum transfert semper in exteriorem.

Corol. 4. Proinde ad conservationem vorticis constanter in eodem movendi statu, requiritur principium aliquod activum, a quo globus eandem semper quantitatem motus accipiat, quam imprimit in materiam vorticis. Sine tali principio necesse est ut globus & vorticis partes interiores, propagantes semper motum suum in exteriores, neque novum aliquem motum recipientes, tardescant paulatim & in orbem agi desinant.

Corol. 5. Si globus alter huic vortici ad certam ab ipsius centro distantiam innataret, & interea circa axem inclinatione datum vi aliqua constanter revolveretur ; huius motu raperetur fluidum in vorticem: & primo revolveretur hic vortex novus & exiguus una cum globo circa centrum alterius, & interea latius serperet ipsius motus, & paulatim propagaretur in infinitum, ad modum vorticis primi. Et eadem ratione, qua huius globus raperetur motu vorticis alterius, raperetur etiam globus alterius motu huius, sic ut globi duo circa intermedium aliquod punctum revolverentur, seque mutuo ob motum illum circularem fugerent, nisi per vim aliquam cohibiti. Postea si vires constanter impresse, quibus globi in motibus suis perseverant, cessarent, & omnia legibus mechanicis permitterentur, languesceret paulatim motus globorum (ob rationem in Corol. 3. & 4. assignatam) & vortices tandem conquiescerent.

Corol. 6. Si globi plures datis in locis circum axes positione datos certis cum velocitatibus constanter revolverentur, fierent vortices totidem in infinitum pergentes. Nam globi singuli eadem ratione, qua unus aliquis motum suum propagat in infinitum, propagabunt etiam motus suos in infinitum, adeo ut fluidi infiniti pars unaquaeque eo agitetur motu qui ex omnium globorum actionibus resultat. Unde vortices non definientur certis limitibus, sed in se mutuo paulatim excurrent , globique per actiones vorticum in se mutuo perpetuo movebuntur de locis suis, uti in corollario superiore expositum est ; neque certam quamvis inter se positionem servabunt, nisi per vim aliquam retenti. Cessantibus autem viribus illis quae in globos constanter impressae conservant hosce motus, materia ob rationem in corollario tertio & quarto assignatam, paulatim requiescet & in vortices agi desinet.

Corol. 7. Si fluidum similare claudatur in vase sphaerico, ac globi in centro consistentis uniformi rotatione agatur in vorticem, globus autem & vas in eandem partem circa axem eundem revolvantur, sintque eorum tempora periodica ut quadrata semidiametrorum: partes fluidi non prius perseverabunt in motibus suis sine acceleratione & retardatione, quam sint eorum tempora periodica ut quadrata distantiarum a centro vorticis. Alia nulla vorticis constitutio potestesse permanens.

Corol. 8. Si vas, fluidum inclusum, & globus servent hunc motum & motu praeterea communi angulari circa axem quemvis datum revolvantur; quoniam hoc motu novo non mutatur attritus partium fluidi in se invicem, non mutabuntur motus partium inter se. Nam translationes partium inter se pendent ab attritu. Pars quaelibet in eo perseverabit motu, quo sit ut attritu ex uno latere non magis tardetur quam acceleretur attritu ex altero.

Corol. 9. Unde si vas quiescat ac detur motus globi, dabitur motus fluidi. Nam concipe planum transire per axem globi &, motu contrario revolvi; & pone summam temporis revolutionis huius & revolutionis globi esse ad tempus revolutionis globi, ut quadratum semidiametri vasis ad quadrarum semidiametri globi : & tempora periodica partium fluidi respectu plani huius erunt ut quadrata, diastantiarum suarum a centro globi.

Corol. 10. Proinde si vas, vel circa axem eundem cum globe, vel circa diversum aliquem data cum velocitate quacunque moveatur dabitur motus fluidi. Nam si systemati toti auferatur vasis motus angularis, manebunt motus omnes iidem inter se qui prius, per corol. VIII. Et motus isti per corol. IX. dabuntur.

Corol. 11. Si vas & fluidum quiescant & globus uniformi cum motu revolvatur, propagabitur motus paulatim per fluidum totum in vas, & circumagetur vas nisi violenter detentum, neque prius desinent fluidum & vas accelerari, quam sint eorum tempora periodica aequalia temporibus periodicis globi. Quod si vas vi aliqua detineatur vel revolvatur motu quovis constanti & uniformi, deveniet medium paulatim ad statum motus in corollariis VIII, IX, & X. definiti, nec in alio unquam statu quocunque perseverabit. Deindc vero si, viribus illis cessantibus quibus vas & globus certis motibus revolvebantur, permittatur systema totum legibus mechanicis; vas & globus in se invicem agent mediante fluido, neque motus suos in se mutuo per fluidum propagare prius cessabunt, quam eorum tempora periodica aequentur inter se, & systema totum ad instar corporis unius solidi simul revolvatur

Scholium.

In his omnibus suppono fluidum ex materia quoad densitatem & fluiditatem uniformi constare. Tale est in quo globus idem eodem cum motu, in eodem temporis intervallo, motus similes & equales, ad aequales semper a se distantias, ubivis in fluido constitutus, propagare possit. Conatur quidem materia per motum suum circularem recedere ab axe vorticis, &propterea premit materiam omnem ulteriorem. Ex hac pressione sit attritus partium fortior & separatio ab invicem difficilior; & per consequens diminuitur materia fluiditas. Rursus si partes fluidi sunt alicubi crassiores seu maiores, fluiditas ibi minor erit, ob pauciores superficies in quibus partes separentur ab invicem. In huiusmodi casibus deficientem fluiditatem vel lubricitate partium vel lentore aliave aliqua conditione restitui suppono. Hoc nisi fiat, materia ubi minus fluida est magis coherebit & segnior erit, ideoque motum tardius recipiet & longius propagabit quam pro ratione superius assignata. Si figura vasis non sit sphaerica, movebuntur particulae in lineis non circularibus sed conformibus eidem vasis figure, & tempora periodica erunt ut quadrata mediocrium distantiarum a centro quamproxime. In partibus inter centrum & circumferentiam, ubi latiora sunt spatia, tardiores erunt motus, ubi angustiora velociores, neque tamen particulae velociores petent circumferentiam. Arcus enim describent minus curves, & conatus recedendi a centro non minus diminuetur per decrementum huius curvaturae, quam augebitur per incrementum velocitatis. Pergendo a spatiis angustioribus in latiora recedent paulo longius a centro, sed isto recessu tardescent , & accedendo postea de latioribus ad angustiora accelerabuntur, & sic per vices tardescent & accelerabuntur particulae singulae in perpetuum. Haec ita se habebunt in vase rigido. Nam in fluido infinito constitutio vorticum innotescit per propositionis huius corollarium sextum.

Proprietates autem vorticum hac propositione investigare conatus sum, ut pertentarem siqua ratione phaenomena coelestia per vortices explicari possint. Nam phaenomenon est, quod planetarum circa iovem revolventium tempora periodica sunt in ratione sesquiplicata distantiarum a centro iovis , & eadem regula obtinet in planetis qui circa solem revolvuntur. Obtinent autem hae regulae in planetis utrisque quam accuratissime, quatenus observationes astronomicae hactenus prodidere. Ideoque si planetae illi a vorticibus circa ioven & solem revolventibus deferantur, debebunt etiam hi vortices eaden lege revolvi. Verum tempora periodica partium vorticis prodierunt in ratione duplicata distantiarum a centro motus: neque potest ratio illa diminui & ad rationem sesquiplicatam reduci, nisi vel materia vorticis eo fluidior sit quo longius distat a centro, vel resistentia, quae oritur ex defectu lubricitatis partium fluidi, ex aucta velocitate qua partes fluidi separantur ab invicem, augeatur in maiori ratione quam ea est in qua velocitas augetur. Quorum tamen neutrum rationi consentaneum videtur. Partes crassiores & minus fluide, nisi graves sint in centrum, circumferentiam petent ; & verisimile est quod, etiamsi demonstrationum gratia hypothesin talem initio sectionis huius proposuerim, ut resistentia velocitati proportionalis esset, tamen resistentia in minori sit ratione quam ea velocitatis est. Quo concesso, tempora periodica partium vorticis erunt in maiori quam duplicata ratione distantiarum ab ipsius centro. Quod si vortices (uti aliquorum est opinio) celerius moveantur prope centrum, dein tardius usque ad certum limitem, tum denuo celerius iuxta circumferentiam ; certe nec ratio sesquiplicata neque alia quaevis certa ac determinata obtinere potest. Viderint itaque philosophi quo pacto phaenomenon illud rationis sesquiplicatae per vortices explicari possit.

PROPOSITIO LIlI. THEOREMA XLI.

Corpora, quae in vortice delata in orbem, redeunt, eiusdem sunt densitatis cum vortice, & eadem lege cum ipsius partibus quoad velocitatem & cursus determinationem moventur.

Nam si vorticis pars aliqua exigua, cuius particulae seu puncta physica datum servant situm inter se, congelari supponatur : haec, quoniam neque quoad densitatem suam, neque quoad vim insitam aut figuram suam mutatur, movebitur eadem lege ac prius: & contra, si vorticis pars congelata & solida eiusdem sit densitatis cum reliquo vortice, & resolvatur in fluidum; movebitur haec eadem lege ac prius, nisi quatenus ipsius particulae iam fluidae factae moveantur inter se. Negligatur igitur motus particularum inter se, tanquam ad totius motum progressivum nil spectans, & motus totius idem erit ac prius. Motus autem idem erit cum motu aliarum vorticis partium a centro aequaliter distantium, propterea quod solidum in fluidum resolutum sit pars vorticis caeteris partibus consimilis. Ergo solidum, si sit eiusdem densitatis cum materia vorticis, eodem motu cum ipsius partibus movebitur, in materia proxime ambiente relative quiescens. Sin densius sit, iam magis conabitur recedere centro vorticis quam prius; ideoque vorticis vim illam, qua prius in orbita sua tanquam in aequilibrio constitutum retinebatur, iam superans, recedet a centro & revolvendo describet spiralem, non amplius in eundem orbem rediens. Et eodem argumento si rarius sit, accedet ad centrum. Igitur non redibit in eundem orbem nisi sit eiusdem densitatis cum fluido. Eo autem in casu ostensum est, quod revolveretur eadem lege cum partibus fluidi a centro vorticis aequaliter distantibus. *Q.E.D.*

Corol. 1. Ergo solidum quod in vortice revolvitur & in eundem orbem semper redit, relative quiescit in fluida cui innatat.

Corol. 2. Et si vortex sit quoad densitatem uniformis, corpus idem ad quamlibet a centro vorticis distantiam revolvi potest.

Scholium.

Hinc liquet planetas a vorticibus corporeis non deferri. Nam planetae secundum hypothesin *Copernicaem* circa solem delati revolvuntur in ellipsibus umbilicum habentibus in sole, & radiis ad solem ductis areas describunt temporibus proportionales. At partes vorticis tali motu revolvi nequeunt. Designent *AD, BE, CF*, orbes tres circa solem *S* descriptos, quorum extimus *CF* circulus sit soli concentricus, & interiorum duorum aphelia sint *A*, *B* & perihelia *D*, *E*. Ergo corpus quod revolvitur in orbe *CF*, radio ad solem ducto areas temporibus proportionales describendo, movebitur uniformi cum motu. Corpus autem, quod revolvitur in orbe *BE*, tardius movebitur in aphelio *B* & velocius in perihelio *E*, secundum leges astronomicas; cum tamen secundum leges mechanicas materia vorticis in spatio angustiore inter *A* & *C* velocius moveri debeat quam in spatio latiore inter *D* & *F*; id est, in aphelia velocius quam in perihelio, Quae duo repugnant inter se. Sic in principia signi virginis, ubi aphelium maris iam versatur, distantia inter orbes martis & veneris est ad distantiam eorundem orbium in principio signi piscium ut ternarius ad binarium circiter, & propterea materia vorticis inter orbes illos in principio piscium debet esse velocior quam in principio virginis in ratione ternarii ad binarium. Nam quo angustius est spatiam per quod eadem materiae quantitas eodem revolutionis unius tempore transit, eo maiori cum velocitate transire debet. Igitur si terra in hac materia coelesti relative quiescens ab ea deferretur, & una circa solem revolveretur, foret huius velocitas in principio piscium ad eiusdem velocitatem in principio virginis in ratione sesquialtera. Unde solis motus diurnus apparens in principio virginis maior esset quam minutorum primorum septuaginta, & in principia piscium minor quam minutorum quadraginta & octo : cum tamen (experientia teste) apparens iste solis motus maior sit in principia piscium quam in principio virginis, & propterea terra velocior in principio virginis quam in principio piscium. Itaque hypothesis vorticum cum phaenomenis astronomicis omnino pugnat, & non tam ad explicandos quam ad perturbandos motus coelestes conducit, Quomodo vera motus isti in spatiis liberis sine vorticibus peraguntur intelligi potest ex libro primo, & in mundi systemate jam plenius docebitur.