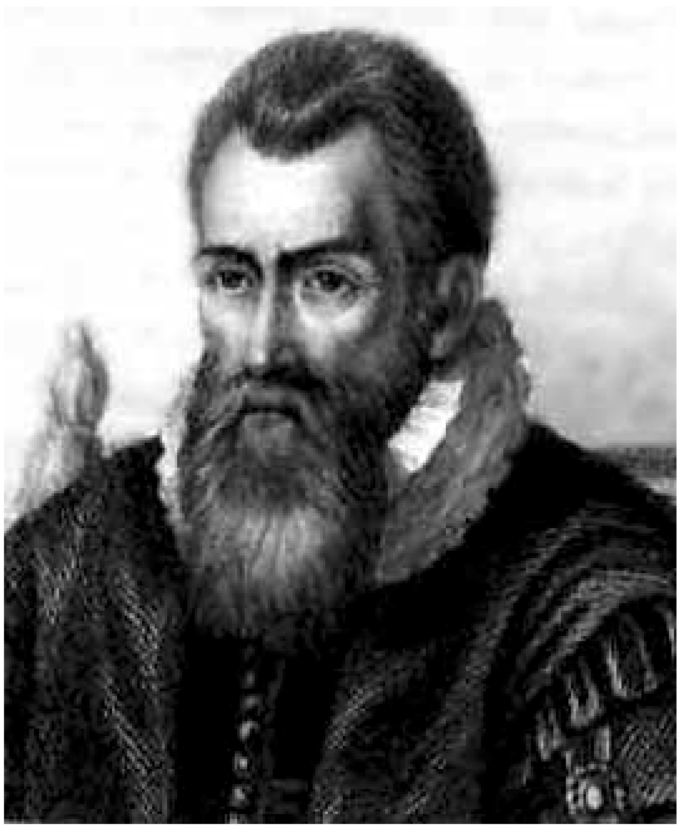
COMPSCI 790

History of Computing and Computers

Assignment 1

The Works of John Napier



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John Napier contributed to many fields of science and philosophy. His mathematical inventions and discoveries laid a considerable part of the foundation of modern computer science. Napier University in Scotland is named after him. The tower in which he was born - the Tower of Merchiston - lies at the centre of Napier University's Merchiston campus [Napier University 2001].

John Napier was born in Merchiston near Edinburgh in Scotland in 1550. He was the eldest son of Archibald Napier and Janet Bothwell. Napier studied at St Salvator's College, St Andrews. There he became interested in the mysteries of the Apocalypse [Macdonald 1966]. It is thought that Napier studied and travelled in Europe after leaving university, although there is no evidence of his attendance at any particular institutions.

The Napier family had accumulated their wealth and notoriety through the service of successive Scottish kings. Archibald became the seventh laird of Merchiston when he married Janet who was the daughter of the Bishop of Orkney [Gazetteer Scotland 2001].

In 1571 John Napier married Elizabeth, daughter of Sir James Stirling of Keir. She gave birth to his first son Archibald. Elizabeth died in 1579 and Napier married Agnes, daughter of Sir James Chisholm of Cromlix. She gave birth to five sons and five daughters.

One of John Napier's most important works was a religious one: "A Plaine discovery of the Whole Revelation of St John", completed in 1593 (or 1594 under the Gregorian calendar). Protestant Reformation had exploded in Europe in 1517 when a devout German Augustinian monk Martin Luther took exception to the sale of "Indulgences" by a wandering Dominican and printed a series of theses condemning such activities by the Catholic Church. "Indulgences" were certificates that absolved the purchaser from sin and guaranteed them a place in heaven – for a price. The essence of Protestantism was to focus on faith and the teachings of the bible, rather than on the papacy and on the ritual of the Catholic Church [Roberts 1996].

Napier's work is a study of the Apocalypse as described in St John's writings in the book of Revelations in the Bible. Napier used reason and mathematical methods to attempt to make sense of the writings. He took an active part in Church politics during the threatened invasion of the Spanish Armada in 1588. The Presbytery of Edinburgh appointed him as a commissioner to the General Assembly in that year. "A Plaine Discovery" is very critical of the Catholic Church, and is dedicated to King James VI, later also James I of England. Napier also speaks very plainly about the duty of rulers with respect to the Church; these were bold words at the time [Gibson 1914].

"A Plaine Discovery" was later translated into Dutch, French and German. Napier had intended to write the work in Latin – as was the scholarly tradition – but he was more interested in making the work public due to the political events and so gave priority to speed of completion. The work gave Napier a reputation as an erudite scholar in Britain and in continental Europe [Gibson 1914].

It is possible that the victory of James over the Presbyterian Party and his imposition of bishops on the Presbyterian Church in an attempt to Anglicise it encouraged Napier to cease his theological studies and devote himself to mathematics. Mathematical and scientific exploration was not looked on fondly in the latter part of the 16th century, and it is perhaps due to Napier's obvious piety and notoriety as a theologian that he was not persecuted for his brilliance. His descendant Mark Napier when writing about him tried to justify a rumour about Napier carrying a black cockerel with him by suggesting that it was an attempt to frighten servants into confessing crimes [Gibson 1914].

John Napier is credited with the invention of logarithms. In 1614 the *"Mirifici Logarithmorum Canonis Descriptio; Ejusque usus in utraque Trigonometria; ut etiam in omni Logitica Mathematica, Amplissimi, facillimi, and expeditissimi explicatio. Authore ac Inventore, Ioanne Nepero, Barone Merchistonii, &c Scoto."*, more commonly known as "The Descriptio". The book contained 90 pages of tables and a description of them [Gibson 1914]. Napier stated that he had begun to contemplate the idea of logarithms twenty years earlier, which would be around 1594. In 1594 Napier was told that the Danish[1] astronomer Tycho Brahe had heard of his efforts and was very much looking forward to the publication of a canon of logarithms. It is believed that a friend of Napier's - Dr John Craig – was part of the delegation going to Denmark to meet King James' fiancée Anne of Denmark. Craig probably met Brahe and related Brahe's excitement to Napier on his return to Scotland [Boyer 1968].

[1] Skane, the part of Denmark in which Brahe was born in is now part of Sweden [Van Helden 1995].

The *Descriptio* was translated into English by Edward Wright and published in 1616 by his son Samuel Wright. The English edition was prefaced by Napier, where he stated that:

> "But now some of our countrymen in this Island, well affected to these studies and the more public good, procured a most learned mathematician to translate the same into our vulgar English tongue, who, after he had finished it, sent the copy of it to me to be seen and considered on by myself. I having most willingly and gladly done the same, find it to be most exact and precisely conformable to my mind and the original." [Boyer 1968].

Napier was obviously impressed with the translation.

Napier's second work on his logarithms was published in 1619, after his death. "*Mirifici Logarithmorum Canonis Constructio"* was edited by his friend Henry Briggs, his son Robert Napier wrote the preface. "The Constructio" describes the method that Napier used to calculate the logarithms [Gibson 1914].

The invention of logarithms was hugely significant in that it made multiplication, division and root extraction far less laborious, particularly for very large numbers. Astronomers, navigators, mathematicians and other scientists were excited by the invention. Calculations were made using Napier's tables and the logarithm identities:

$$\log xy = \log x + \log y$$

$$\log \frac{x}{y} = \log x - \log y$$

In the process of developing his canon, Napier perfected the use of the decimal point and managed to define the logarithmic function as a differential equation [Tee 2001].

> "… though he did not introduce decimal fractions, he did introduce the decimal point, and showed, in the *Constructio*, the perfect simplicity and generality that attended its use. The decimal point is one of those simple devices that we take for granted but that needed a genius to invent; many years elapsed before its use became quite general." [Gibson 1914]

The astronomer Kepler who worked with Tycho Brahe, and who discovered that celestial bodies had elliptical orbits hailed logarithms as a great invention, as they eased his calculations considerably [Boyer 1968].

Napier's enthusiasm for simplifying the rigour of everyday mathematical calculations lead him to invent what he termed "Numbering Rods" and to describe their construction and use in his work "*Rabdologiae, seu Numerationis per Virgulas Libri*

Duo: Cum Appendice de expecitissimo Multiplicationis Promptuario. Quibus accessit et Arithmeticae Loaclis Liber Unus", commonly referred to as "The Rabdologaie".

The rods are based on the Arabian Lattice method of multiplication, whereby the numbers to be multiplied are lined up one across the top of a grid, and one down the right side. Each cell of the grid is divided into two diagonally, and the product of each two corresponding digits from the original numbers is written in each cell, with the tens in the upper part of the diagonal and the units in the lower.

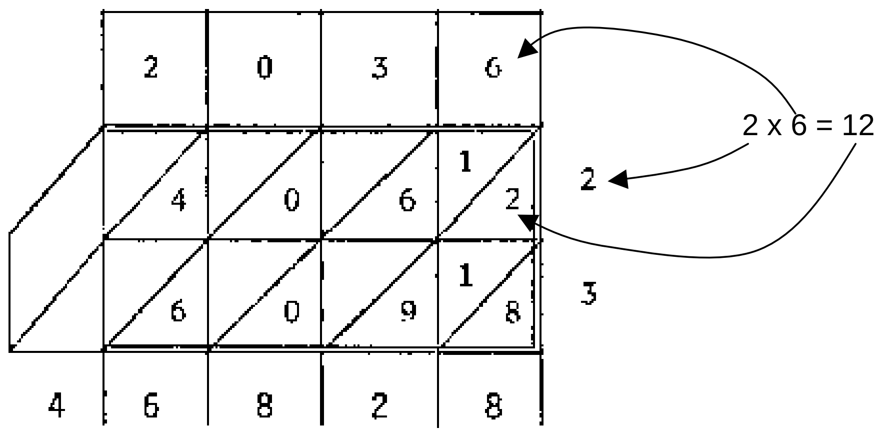

Figure 1 - Arabian Lattice multiplication [Wilson 1978]

The numbers in each parallelogram formed by the diagonals on each row are then added together to form the digits of one number for each row. The decimal point for the number in each row is shifted right one place for the corresponding place of the digit in the vertical number. The rows are added together to get the final result.

In the above example, the pairs added for the first row would be (a blank half-cell counts as 0):

$$0+4,\ 0+0,\ 1+6,\ 2\ =\ 4072$$

We then adjust for the fact that the 2 at the end of the row is in the tens column, so actually corresponds to 20, giving 40720.

The second row adds up as (note adding is from right to left so there is a carry from the 1 + 9 to the 0 + 0):

$$0+6,\ 0+0,\ 1+9,\ 8\ =\ 6108$$

There is no adjustment for the decimal point, as this row corresponds to the units. Finally, the two numbers are added together to produce the correct result:

$$40720 + 6108 = 46828$$

Napier's ingenuity was in creating a portable version of this system that was practical and convenient. The Rabdologiae begins with a description of the making of a set of rods. Napier suggests using "silver, ebony, boxwood or some strong material of a similar nature". He states that:

> "…ten oblong rods are required for numbers up to five figures; twenty are needed for numbers up to nine figures; or thirty of numbers up to 13 figures" [Hawkins 1978].

Each of the four faces of each rod is divided into nine squares. The top square is one of the 10 digits and the consequent squares vertically descending are the n^{th} multiple of that digit. The multiple is written in the same fashion as in the Arabian Lattice, with the digits split over a diagonal. Each opposite face of a rod contains the digit that is the complement of the other, e.g. 2 and 7 or 1 and 8. Below is what an "unfolded" rod would look like:

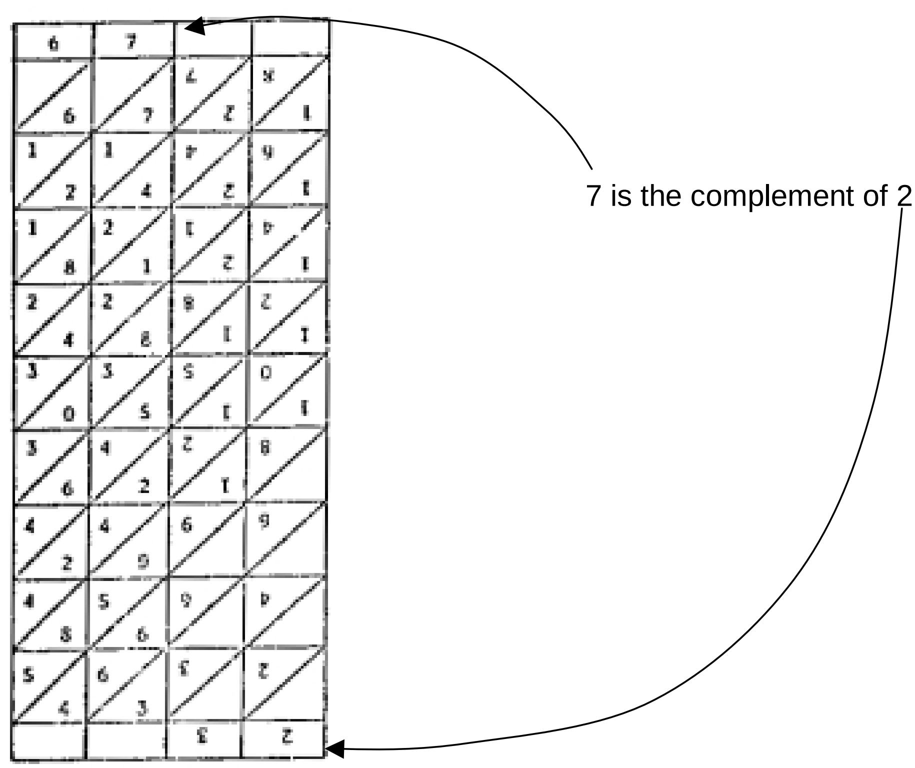

Figure 2 - An unfolded rod [Gibson 1914]

A set of ten rods made in this fashion will produce 4 copies of each digit, thereby making it possible to:

> "…produce not only every number less than the figures in 11 111, without exception, but every number below the 11 places of 10^{10}, except that number, in which, together with its counter-part there are five figures of the same value; or 8 figures of two kinds or 10 figures of three kinds." [Hawkins 1978]

To perform a multiplication, the user lines up rods with top squares corresponding to the digits one of the numbers being multiplied. Then for each digit of the other number the user takes the summed digits from the parallelograms in the corresponding row and makes the appropriate correction by shifting the decimal point right.

In the example below 2085 is being multiplied by another number; a guide rod showing all 10 digits is placed alongside.

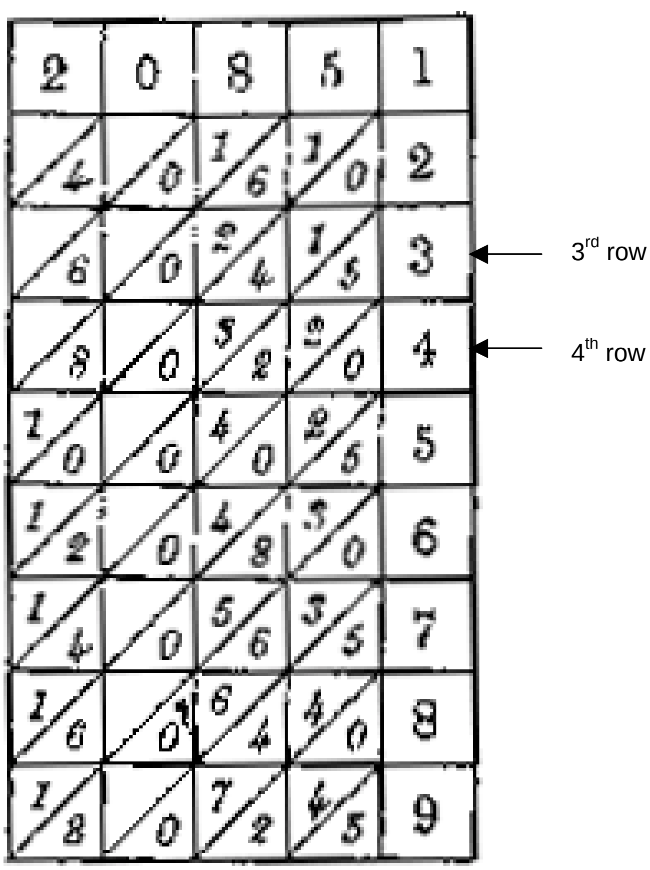

Figure 3 - A multiplication using rods [Gibson 1914]

If for example the other number was 34, the fourth row would produce the following pairs of numbers:

$8+0,\ 0+3,\ 2+2,\ 0\ = 8340$ with no adjustment because 4 is the units.

For the digit 3 in the tens column we get:

$6+0,\ 0+2,\ 4+1,\ 5\ = 6255$ with adjustment for then tens gives 62550

Adding these together:

$62550+8340 = 2085\times 34 = 70890$ …the correct answer.

Napier had created a convenient portable calculating machine. A certain amount of work still had to be performed by the user, but the rods became very popular. The term "Napier's bones" was coined. As late as the 1830s in Sweden a paper strip version of the rods was still being used [Tee 2001].

Napier wrote the Rabdologiae in Latin. It was later translated into Italian and Dutch, but was not translated into English until W.F. Hawkins did so in his doctoral thesis at the University of Auckland, completed in 1978.

Napier did not stop at his portable arithmetic tool. In an appendix to the Rabdologiae he describes the design of a machine to lighten the burden of multiplication even further. The appendix is titled "The Promptuary for Lightning Multiplication".

The Promptuary consists of a number of strips of a firm material (Napier suggests ivory) and a box. To multiply number less than 100 000 one hundred strips are required. Napier states that he made 200 strips for his own Promptuary. This would be an expensive exercise with ivory!

> "One hundred of the strips must be one quarter unit thick, and the other hundred half as thick or a little more, depending on the material used. Spread the 100 thick strips before you with their wide margins at the top and the narrow margins below and nearer to you. These are called direct strips. Lay out the thin ones with their wide margins on the right and their narrow margins on the left, namely, side on to the direct strips. These are called transverse strips." [Hawkins 1978].

Each strip is then divided into 10 squares. The wide margin that Napier refers to above is marked with one of the digits 0-9 and each of the ten squares is divided up into a matrix of nine smaller squares, each of which is bisected by a diagonal to form two triangles. In each of those triangles, a digit is marked or is left blank. Each of the 10 large squares containing the matrices is also bisected with a clearly marked diagonal. The digit numbering system is a clever adaptation of the Arabian Lattice method. Napier provided a description of the numbering by means of a diagram showing the positions of a pattern of variables and the arithmetical method to be used to find the values of those variables – basically just a splitting of the digits of the multiples for the strip's number over the matrix of triangles. The squares of the 0 strip are entirely blank. The following diagram, which was drawn by Hawkins and added in the notes to his translation of Napier's Rabdologiae, shows the numbers of the squares for all nine distinct strips:

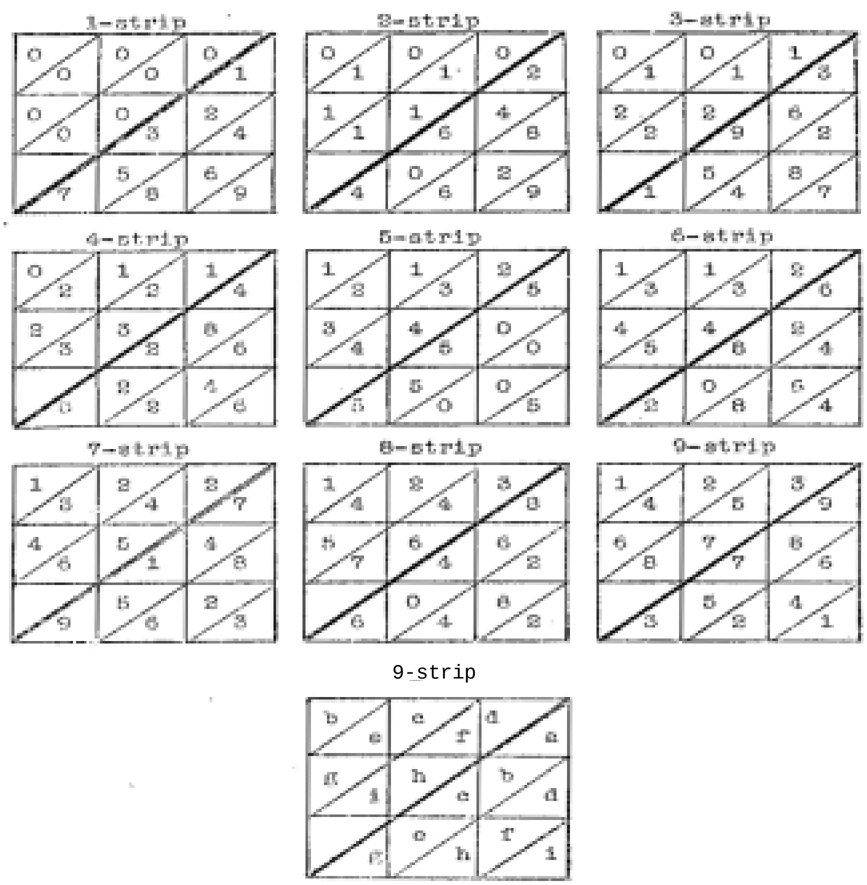

Figure 4 - The numbering on each type of strip [Hawkins 1978]

The numbering for each strip is repeated in each square of that strip, so the 1 strip will contain 10 repetitions of the above 1-strip numbering.

The other set of strips – the "transverse" strips are divided up in much the same way as the direct ones, but the main diagonal bisection is especially important. Each strip has a cutting pattern, whereby one or two of the small triangles are cut out from the squares on the strip. The cutting pattern – again in a diagram drawn by Hawkins is shown below:

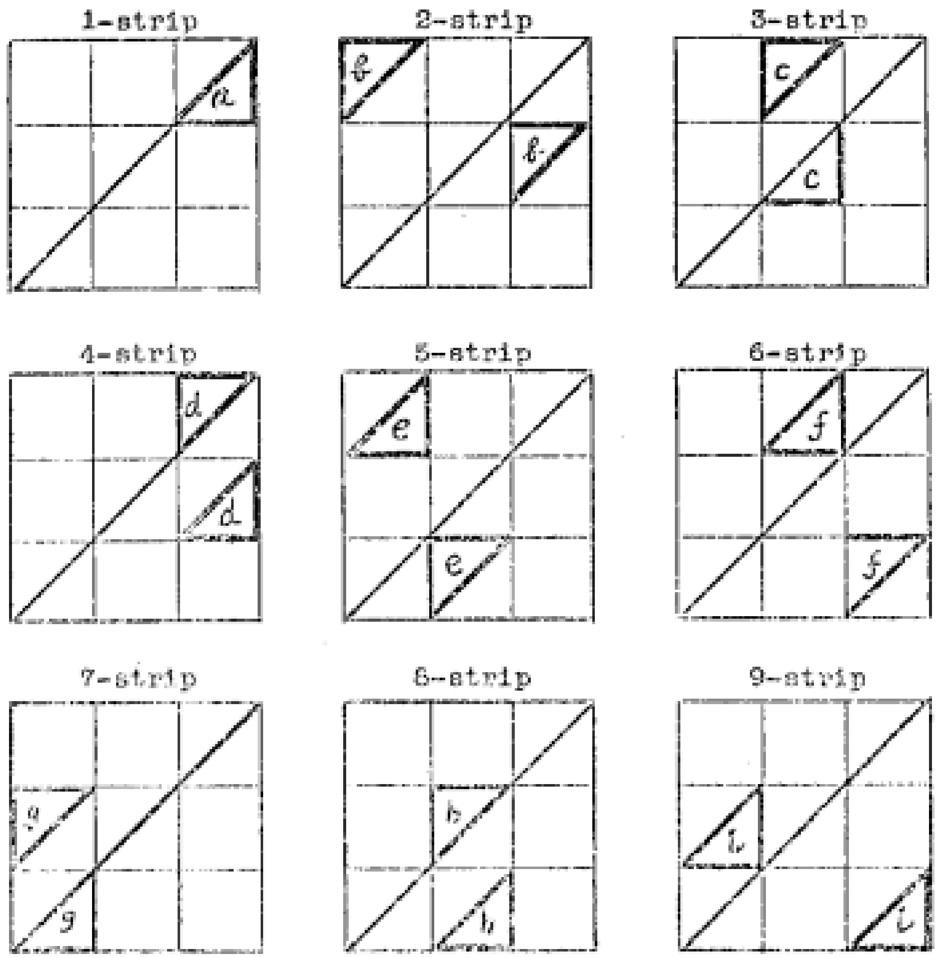

Figure 5 - Cuts in the transverse strips [Hawkins 1978]

These cuts form peepholes when the transverse strips are placed over the direct strips, showing corresponding digits on the direct strips below.

The basic idea of a multiplication is to lay down "direct" strips corresponding to the digits of one of the multiplicands and lay down "transverse" strips corresponding to the digits of the other multiplicand on top of these. The sums of the digits within the parallelograms formed by the main diagonals bisecting the ten squares on each strip form the digits of a number for each row. These numbers are added to form the final result in the same manner as with the numbering rods.

Napier goes on to describe the method of division with the Promptuary by transforming the divisor to its reciprocal using the "Tables of Lansbergius" or similar tables giving "extreme proportionals".

Napier also gives detailed plans of the design of a box on the top of which the multiplication was to be performed. The box holds the rods that are not in use in a series of slots around the side, marked with each digit. The box top has exactly the right size to provide a snug fit for the strips to be laid down in. Napier's diagram of the box is shown below:

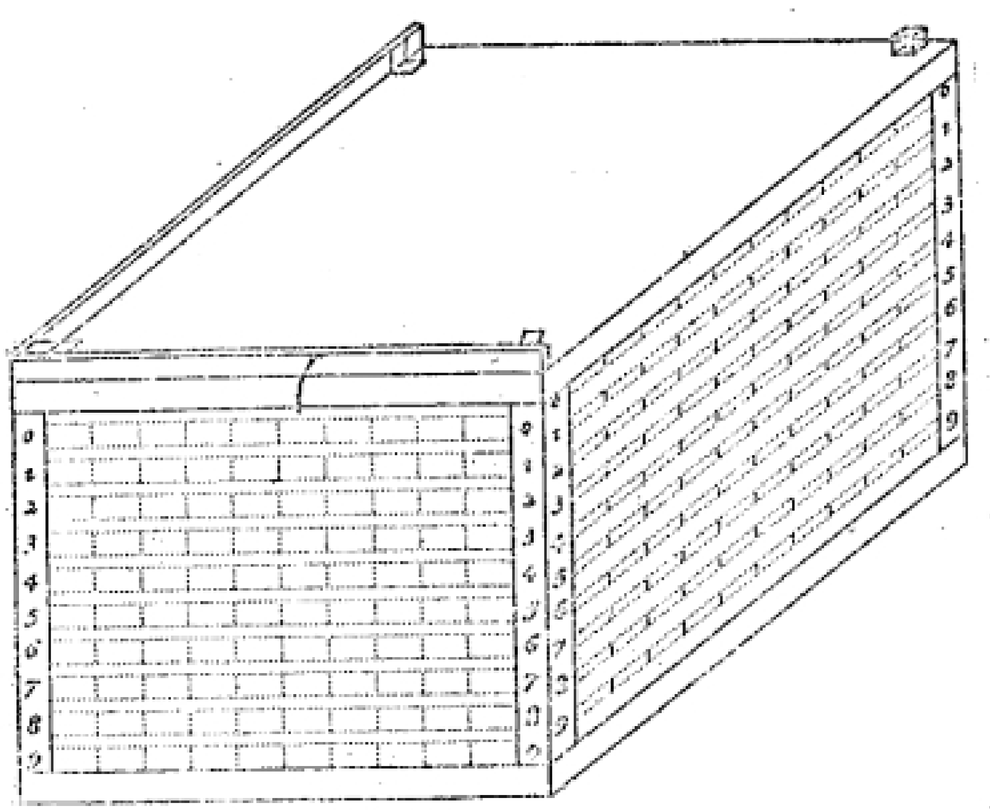

Figure 6 - The Promptuary's box

Thus through a combination of practicality and ingenuity Napier had created a very early calculating machine.

The conceptualisation of the binary system has been variously attributed to Gottfried Leibniz and the ancient Chinese, but it is now apparent that John Napier was the first to suggest it [Tee 2001].

In another appendix to the Rabdologiae Napier gives algorithms for conversion between decimal and binary and methods for binary arithmetic operations. He did not see any serious applications for binary or "local arithmetic" as he termed it:

> "I came upon this Arithmetical Table, which must justly be called a game rather than hard work." [Hawkins 1978]

Napier thought that the tedium of conversion would be off-putting [Tee 2001].

Napier's system uses counters in a row of cells labelled with both letters of the alphabet and successive powers of 2 starting from 1. The counters are used as we might use the 1 in modern binary notation to represent a "set bit". He goes on to propose a system of representation of binary numbers using the letters of the alphabet. Addition is carried out by assembling the two numbers – represented by the letters corresponding to the binary places – in alphabetical order. Wherever two of the same letter appears, the two letters are replaced by one instance of the letter in the place higher. For example $abc + c = abcc = abd$, this corresponds to $111 + 100 = 1011$ in modern notation. Napier describes how this can be calculated by placing two counters into a cell in the row of cells and then replacing them with one in the cell above.

Napier also proposes a "chess board" arrangement where arithmetic is performed by moving counters diagonally, horizontally or vertically on a board of squares indexed by the letters and powers of 2 written along the edges of the board.

Napier completes his Rabdologiae with a phrase that demonstrates his roots: "All praise to God!"

Napier made many other contributions to mathematics. He produced a vast amount of work on trigonometry, in particular spherical trigonometry. He also helped develop - along with other 16th century mathematicians - the foundations of the modern algebra system [Hawkins 1978].

There is no doubt that John Napier, Baron of Merchiston made a significant contribution to the development of computer science. We can be glad that the political events of the time turned his attention from his theological studies to mathematics. His early calculating machines were a demonstration to ordinary people that arithmetic could be made easier and faster through the use of mechanical devices. Modern computing relies almost entirely on the binary system, which Napier's sweeping vision first alighted on. Napier's creation of logarithms accelerated the development of all fields of science by greatly easing the burden of calculations involving large numbers.

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