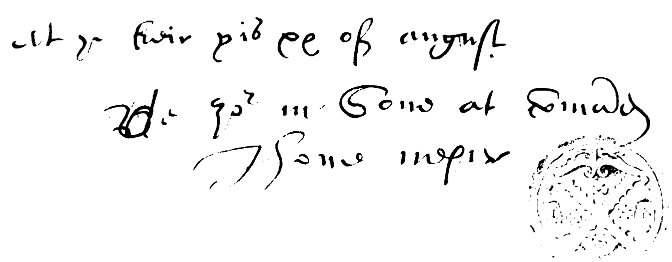
Engraved by R.M. Hodgetts. From the original in the possession of the Lord Napier



DE ARTE LOGISTICA.

DE ARTE LOGISTICA

JOANNIS NAPERI

MERCHISTONII BARONIS

LIBRI QUI SUPERSUNT.

IMPRESSUM EDINBURGI

M.DCCC.XXXIX.

At a General Meeting of the MAITLAND CLUB, held at Glasgow in the Hall of Hutcheson's Hospital, on Saturday the 26th of January, 1839,—

"RESOLVED, That 'THE BARON OF MERCHISTOUN HIS BOOKE OF ARITHMETICKE AND ALGEBRA' be printed for the Members, from the Original Manuscripts, in the possession of MARK NAPIER, ESQUIRE, Advocate."

Extracted from the Minutes of the Club.

JOHN SMITH, *Ygst, Secretary.*

THE MAITLAND CLUB.

NOVEMBER, M.DCCC.XXXIX.

THE RIGHT HONOURABLE

THE EARL OF GLASGOW,

PRESIDENT.

HIS ROYAL HIGHNESS THE DUKE OF SUSSEX.
HIS GRACE THE DUKE OF ARGYLL.
JOHN BAIN, ESQ.
ROBERT BELL, ESQ.
SIR DAVID HUNTER BLAIR, BART.
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HIS GRACE THE DUKE OF BUCCLEUCH & QUEENSBERRY.
10 JAMES BOGLE, JUN. ESQ.
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THE MOST NOBLE THE MARQUESS OF BUTE.
ALEXANDER CAMPBELL, ESQ.
THE HONOURABLE HENRY COCKBURN, LORD COCKBURN.
JAMES T. GIBSON CRAIG, ESQ.
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JAMES DENNISTOUN, ESQ.
JAMES DOBIE, ESQ.
RICHARD DUNCAN, ESQ. (TREASURER.)

THE MAITLAND CLUB.

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THE VERY REV. DUNCAN MACFARLAN, D.D.
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ANDREW RUTHERFURD, ESQ.
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WILLIAM SMITH, ESQ.

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80 DUNCAN STEWART, ESQ.
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WILLIAM STIRLING, ESQ.
WILLIAM STIRLING, ESQ. YR.
JOHN STRANG, ESQ.
THOMAS THOMSON, ESQ.
WILLIAM B. D. D. TURNBULL, ESQ.
DAWSON TURNER, ESQ.
PATRICK FRASER TYTLER, ESQ.
90 ADAM URQUHART, ESQ.

TO THE RIGHT HONOURABLE

FRANCIS LORD NAPIER OF MERCHISTON,

ETC. ETC. ETC.

My dear Lord Napier,

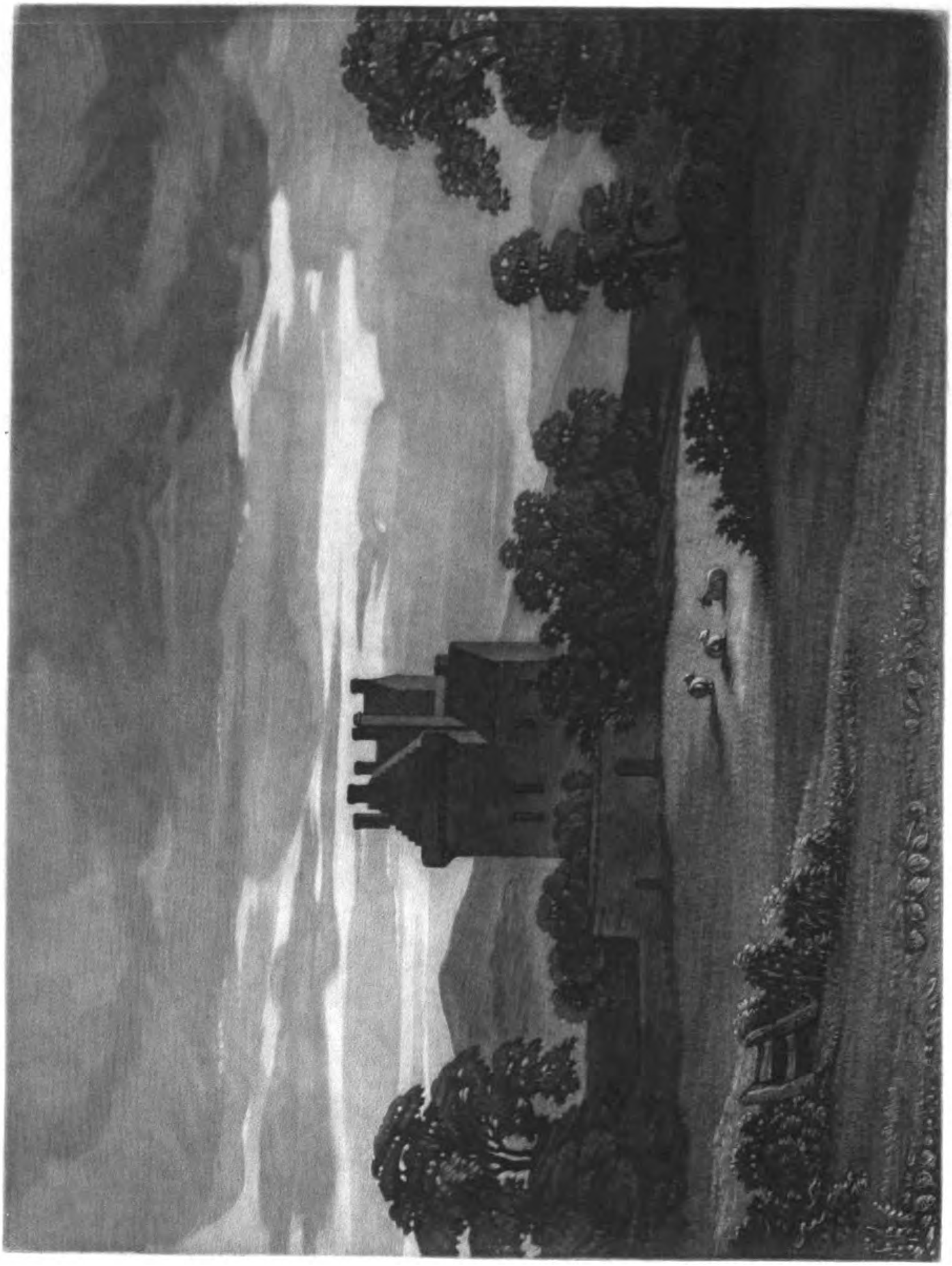
This Memorial of your great Ancestor, which it has been my ambition to present in a form worthy of the genius it records, I dedicate to you, in remembrance of the exemplary liberality with which the Archives of your noble family have been always open to illustrate the History and the Letters of Scotland.

Yours affectionately,

MARK NAPIER.

Edinburgh, *November* 1, 1839.

INTRODUCTION.



INTRODUCTION.

IN the Memoirs of NAPIER of MERCHISTON, published in 1834, some account was given of two manuscript treatises—one of Arithmetic, and the other of Algebra—composed in Latin by that celebrated mathematician, and which had remained inedited in the charter-chest of his family, and indeed unknown to the world, until the Memoirs were published. Upon that occasion, little more could be afforded than a very imperfect review of their contents. The idea subsequently occurred, that it might gratify the lovers of science if these mathematical studies of (to adopt the expressions of the historian Hume) "The celebrated Inventor of Logarithms, the person to whom the title of GREAT MAN is more justly due than to any other whom his country ever produced,"—were added as an appendix to a new edition of his Life. I have been induced, however, to publish the treatises in their present independent and more becoming form, by the spirited interposition of the Bannatyne and Maitland Clubs of Scotland; whose unanimous patronage of the work,—with their characteristic care for, and pride in, the ancient letters of Scotland,—has alone enabled me to render the volume so worthy a memorial of Scotish genius.

Napier's scientific manuscripts came into the possession, not of his eldest son Archibald, the first Lord Napier of Merchiston, but of a younger son Robert. From the congeniality of their pursuits, Robert became his father's literary executor, and edited his posthumous work, entitled "Logarithmorum Canonis Constructio," being his secret of the construction of that Canon of the Logarithms which he had published three years before his death. Robert Napier, designed of Bowhopple, Culcreugh, and Drumquhannie, was the second son of Napier's second marriage. The late Colonel Milliken Napier, Robert's lineal male representative, was still in possession of many private papers of the family at the close of last century. Upon one occasion, when the Colonel was called from home on the service of his country, these papers, along with a portrait of the great Napier, and a Bible with his autograph, were deposited for safety in a room of the house of Milliken, in

Renfrewshire. During the owner's absence the house was burned to the ground, and the precious relics perished.

As these manuscripts had not been arranged, nor properly examined, the extent of the loss is unknown; but it is more than probable that among them were the originals from which Napier's son Robert made the transcripts accidentally preserved, and now given to the public. The manner in which these transcripts had escaped the fate of the Culcreuch papers appears from the following note, written in the MS. volume itself, by Francis seventh Lord Napier, grandfather of the present Peer to whom it now belongs:—

"John Napier of Merchiston, inventor of the Logarithms, left his manuscripts to his son Robert, who appears to have caused the following pages to have been written out fair from his father's notes, for Mr Briggs, Professor of Geometry at Oxford. They were given to Francis, the fifth Lord Napier, by William Napier of Culcreuch, Esq., heir-male of the above-named Robert. Finding them, in a neglected state, amongst my family papers, I have bound them together, in order to preserve them entire.

"NAPIER."

"*7th March*, 1801."

The transcripts are entirely in the handwriting of Robert Napier himself, as is ascertained beyond doubt by a comparison with some of his letters in Lord Napier's charter-chest; and this title, written on the first leaf, is also in his handwriting:—

"The Baron of Merchiston his booke of Arithmeticke and Algebra. For Mr Henrie Briggs, Professor of Geometrie at Oxforde."

That they were taken directly from the author's own papers, is proved by the note which the transcriber adds at the conclusion of the fragment on Geometrical Logistic:—

"I could find no more of this Geometricall pairt amongst all his fragments."

And also by another note which immediately follows the abrupt termination of the Algebra:—

"There is no more of his Algebra orderlie sett doun."

These notes, of which fac-similes will be found in their respective places in this volume, seem to have been addressed to Henry Briggs; and they not only prove that the transcripts were made from Napier's private manuscripts, but that Napier himself had so far written out the treatises in the form now published, and that more remained to be digested and arranged.

The late Lord Napier, whose untimely death in China, a few years ago, attracted the mournful interest of the public, intrusted to myself, before his departure on that mission, the many curious and valuable historical relics of his family, with full permission to publish whatever might serve to illustrate the history and letters of Scotland. This task I have now accomplished, with at least industrious zeal, in the Memoirs of Merchiston, the History of the Lennox, the Memoirs of Montrose, and, finally, the editing of this beautiful, though unfinished treatise, De Arte Logistica.

The authenticity of the MS., as an unpublished work of the Inventor of Logarithms, being thus unquestionable, the date of its composition becomes an interesting enquiry, in reference to the history of science in Scotland. As Napier died in the year 1617, no later date can be assigned to these mathematical studies than the commencement of the seventeenth century—an epoch when the light of science is only said to have dawned, in our country, because of the publication of his own invention of Logarithms. From the internal evidence, however, there is every reason to suppose, that, when so much of his Arithmetic and Algebra was thus "orderlie sett doun" by Napier, the system of Logarithms was not only unknown to the world, but had not as yet been developed in the mind of the inventor himself. Consequently, a much earlier date must be assigned to the compositions now published. They are, in all probability, the first-fruits of that immortal genius which eventually drew from the mysterious depths of Numbers their pearl of greatest price. Napier's Arithmetic and Algebra will be regarded with reverential interest by all who can, in any degree, appreciate the analytical power, and the practical effects of the Logarithms; especially if, in those treatises, we can trace the preparatory labours of this secluded Scotish Baron, ere he so unexpectedly displayed himself on a pinnacle of science to which Kepler himself paid homage.

Napier published his great work, entitled "Mirifici Canonis Logarithmorum Descriptio," in the year 1614. By that time his mind was so deeply engrossed with the subject, and he was so constantly employed,—during the short interval between the date of that publication and his death in 1617,—with his plans for extricating science from the trammels of ordinary calculation, that his Arithmetic and Algebra can hardly be imagined to have been his latest compositions. But what appears decisive upon the point is, that there cannot be discovered throughout the whole of the work now printed an allusion even to the principle of the Logarithms. A complete exposition of the subject was not, indeed, to be expected in his treatise of Arithmetic; but neither is it to be supposed that he would have written, for instance, the fifth chapter of his second book of Arithmetic, "De Multiplicationis et Partitionis Compendiis Miscellaneis," after having published the Logarithms, without the slightest allusion to that means of substituting, for the real numbers, a set of artificial numbers rendering the whole art of calculation compendious beyond the most sanguine hopes of emancipated Astronomy.

It is even more improbable that the treatises in question were composed, or so far reduced to order, at some period in the long interval which must have elapsed between the author's conception of the Logarithms, and the construction of those elaborate works by which the glory of the invention was secured to himself, and the benefit bestowed upon mankind. In his book of Geometrical Logistic, Napier seems to allude to the Arithmetic of Surds as a discovery of his own, the full value of which he promises afterwards to disclose. Had he been conscious at this time of the far more important secret of the Logarithms,—which, after his first conception of the idea, was his daily labour to the end of his days,—there would, in all probability, have been some allusion to it. Indeed, his great invention must have formed a most interesting chapter of his work DE ARTE LOGISTICA, had he lived to complete it.

Not only is there no mention of his Canon of the Logarithms in these treatises, but when, to illustrate the relation between the powers of numbers and their indices, there is exhibited the following table,—

	I.	II.	III.	IIII.	V.	VI.	VII.
1.	2.	4.	8.	16.	32.	64.	128,—

no allusion occurs to the fact, that this table also affords an illustration of the logarithmic principle. The upper series are not only indices to the lower series, considered as powers, but they are Logarithms to those numbers, being an arithmetical adapted to a geometrical progression. The word Logarithms was compounded by Napier before he published his discovery, and this term itself embodies the principle illustrated (though on a very limited scale) by the above arrangement of figures. Αριθμοι signifies numbers; λογαριθμοι, the ratios of numbers, or rather the number of ratios, λογων αριθμος. Now, although Napier expounded his system of Logarithms through his geometrical idea of fluxions (a demonstration and a term afterwards adopted by Newton), and not through the doctrine of powers and exponents, yet he could not have composed the term Logarithms without a perfect knowledge of the connexion of his system with the arithmetic of proportions.

It can scarcely be doubted, then, that the composition of Napier's treatise of Numbers is prior in date even to his conception of the Logarithms; a conclusion that will seem very natural to any one who has examined his works on the subject, and who is at all capable of appreciating the command of the science of Numbers which those works evince. Undoubtedly, his great invention was not (as is alleged of the theory of gravitation) the result of a sudden thought or suggestion, in some happy moment of genius. It was laboriously elicited from the arcana of Numbers,—deliberately extracted, by a sort of Cæsarean operation, from the unripe womb of analytical science. Napier himself alludes to this predetermination of his genius, in a most interesting letter, the last he ever wrote. It is prefixed as a dedication, to the Chancellor of Scotland, of his RHABDOLOGIA, or the art of computing by means of figured rods, better known by the name of "Neper's bones." In the present attempt to trace the chronological order of his works, I may here take the liberty to translate this letter from the original Latin, the language in which he composed all his mathematical works:—

"To the most illustrious Alexander Seton, Earl of Dunfermline, Lord of Fyvy and Urquhart, High Chancellor of Scotland, &c.

"The difficulty and prolixity of calculation (most illustrious Sir), a toil which is apt to deter most people from the study of mathematics, I have, all my life, with what powers and little genius I possess, laboured to eradicate. And, with that end in view, I published of late years the Canon of Logarithms (for a long period elaborated by me), which, rejecting the natural numbers, and the more difficult operations performed by them, substitutes in their place others affording the same results by means of easy additions, subtractions, and extractions of roots. Of which Logarithms, indeed, I have now found out another species much superior to the former, and intend, if God shall grant me longer life, and the possession of health, to make known the method of constructing, as well as the manner of using them. But the actual computation of this new Canon, on account of the infirmity of my bodily health, I have left to some who are well versant in such studies; and especially to that most learned man, Henry Briggs, public professor of geometry in London, my most beloved friend. Meanwhile, however, for the sake of those who may prefer to work with the natural numbers as they stand, I have excogitated three other compendious modes of calculation. The first is by means of numerating rods, and this I have called RHABDOLOGIA. Another, by far the most expeditious of all for multiplication, and which, on that account, I have not inaptly termed the PROMPTUARY OF MULTIPLICATION, is by means of little plates of metal disposed in a box. And, lastly, a third method, namely, LOCAL ARITHMETIC performed upon a chess-board. But, to the publication of this little work, concerning the mechanism and use of the rods, I was specially impelled, not merely by the fact that they are so approved of as to be already almost in common use, and even carried to foreign countries; but because it also reached my ears that your kindness advised me so to do, lest they should be published in the name of another, and I be compelled to sing with Virgil,

> 'Hos ego versiculos feci,' &c.

This very friendly counsel from your Lordship ought to have the greatest weight with me; and most assuredly, but for that, this little treatise of the use of the rods (to which the other two compendious methods are added, by way of appendix), would never have seen the light. If, therefore, any thanks be due from the students of mathematics for these little books, they all belong to you as your just right, my noble Lord; to you, indeed, they must spontaneously fly, not only as patron, but a second parent: especially since I am assured that you have done these rods of mine such high honour as to have them framed, not of common materials, but of silver. Accept, then, my Lord, in good part, this small work, such as it is; and, though it be not worthy of so great a Mæcenas, take it under your patronage as a child of your own. And so I earnestly pray God to preserve you long, to us and the state, to preside over justice and equity.

"Your Lordship's very much obliged

"JOHN NAPIER,

"Baron of Merchiston."

The date of the volume to which this letter is prefixed is 1617, and Napier died upon the 4th of April in that same year. We learn from it the chronological order of the composition of all his published works. In the first place, he had long and laboriously wrought out—"à me longo tempore elaboratum"—his Canon of the Logarithms. Then he excogitated—"excogitavimus"—the mechanical system of calculating rods, for the sake of those who might be distrustful of his artificial system of Logarithms. His Promptuary of Multiplication, which immediately follows the Rhabdologia, he states to be the latest of all his inventions—"omnium ultimo à nobis inventum sit hoc Multiplicationis Promptuarium"—though he gives it precedence in the volume, according to his estimate of its importance. He had previously invented his mode of calculating with the abacus or chess-board, in the preface of which he again refers to the origin of all these inventions,—namely, that it was the constant labour of his life to rend the fetters with which science was yet subdued. " In the course," says he, " of devoting every moment of my leisure,"—doubtless, from what he considered his great calling, the exposition of the Revelations,—" to the invention of these compendious methods of calculation, and to the enquiry by what means the labour and toil of calculation might be removed, besides the Logarithms, Rhabdologia, the Promptuary of Multiplication, and other devices, I hit upon a certain arithmetical table, which, as it performs the more troublesome operations of common arithmetic upon an abacus or chess-board, may be considered an amusement rather than labour; for, by means of it, addition, subtraction, multiplication, division, and even the extraction of roots, are accomplished, simply by moving counters hither and thither upon the board. Unwilling either to consign it to oblivion, or to publish so small a matter by itself, I have prefixed it to my Rhabdologia, in addition to the Promptuary, for the benefit of the studious—and the criticism of the learned."

That Napier composed his Rhabdologia after the year 1614, when he published his Canon of the Logarithms, is also indicated by the fact, that in the minor work (Lib. I. cap. 2) he selects, as a numerical example, "Annus Domini 1615." Now, supposing the Rhabdologia an invention of earlier date than the Logarithms (which latter, as will presently appear, must be referred to some time in the previous century), the year of our Lord 1615, when thus selected for the example, was a year yet to come; and it is not likely that such an example would have occurred to him, in place of a year either present or past. Probably the year selected was that in which Napier was writing at the time, a supposition perfectly consistent with his statement, in the letter to the Chancellor, that this minor invention was for the benefit of those who might be disinclined to use the Logarithms, and that it had become well known, and even been carried abroad, before he was induced to publish it. For 1615 was the year after the publication of the Logarithms, and the Rhabdologia was not printed until 1617.

There is only one circumstance which seems to interfere with this chronology; but it is easily explained. In the fifth chapter of the second book of his Arithmetic, when expounding various compendious methods of multiplication and division, Napier adds, "Sive omnium facillime per ossa Rhabdologiæ nostræ." This might be supposed to afford decisive evidence that the Rhabdologia was composed before the book of Arithmetic. It must be observed, however, that the expressions quoted are written on the margin of Robert Napier's manuscript, and there is a mark in the text where the note should be inserted,—a direction complied with in printing the present volume (p. 42). Probably this note was an addition of the transcriber's; or perhaps Napier himself had subsequently added it to his MS., after the invention of the calculating rods had occurred. If the note be contemporary with the composition of the treatise, then undoubtedly the latter is of a subsequent date to his Rhabdologia. But there is another circumstance which seems to render this supposition totally out of the question. In Napier's Arithmetic occurs a genesis of decimal fractions, and perhaps the earliest on record. But the peculiar notation of decimals,—from which the system derives all its power as a reciprocation of the Arabic scale, and which Napier could not have failed to use and comment upon in any notice of a decimal fraction, when once aware of the expedient,—is not hinted at in the treatise on Arithmetic. Now, in the Rhabdologia, there is a section (of the fourth chapter) entitled "Admonitio pro Decimali Arithmeticâ,"—being his exposition of that very notation of decimals which is in use at the present day. Napier had hit upon this admirable expedient when constructing his Canon of the Logarithms,—the first mathematical work in which the arithmetic of decimal fractions was developed, and displayed in full operation. This circumstance of itself appears to determine that the composition of his treatise on Arithmetic is prior in date both to the invention of the Logarithms and of the Rhabdologia.

It is very probable that Napier's progress, in composing a complete digest of Numbers, might have been interrupted, and that work set aside for the time, in consequence of those very operations having suggested his more profound inventions; but that he composed his Arithmetic and Algebra subsequently to those inventions is most improbable, considering that the dates of his published works are 1614 and 1617, and that he died in the month of April of the latter year. Moreover, it appears that, at the time of his death, Napier was occupied in bringing to perfection his most elaborate and beautiful work, the "Constructio Logarithmorum;" which treatise, although it did not see the light until two years after his death, when it was edited by his son Robert, he had left in a state for publication. This volume, with some profound and original aids to the science of trigonometry added as a supplement to it, must have occupied all the time which he devoted to mathematics, (for he was still deeply engaged with the Apocalypse,) between the date of the publication of the Canon of Logarithms and the date of his death. The concluding sentences of his Canon afford so interesting a view of the last labours of Napier's life, that I may here quote them from the English translation of 1616, which he revised himself:—

"Now, therefore, it hath been sufficiently showed that there are Logarithms, what they are, and of what use they are; for with help of them, we have both demonstratively showed, and taught by examples of both kinds of trigonometry, that the arithmetical solution of any geometrical question may most readily be performed without trouble of multiplication, division, or extraction of roots. You have, therefore, the admirable table of Logarithms that was promised, together with the most plentiful use thereof, which, if (to you of the learned sort) I by your letters understand to be acceptable to you, I shall be encouraged to set forth also the way to make the table. In the mean time, make use of this short treatise, and give all praise and glory to God, the high Inventor and Guider of all good works."

Then follows an isolated sentence which he terms "Admonitio;" and, probably, he refers to his theological labours when, in that sentence, he speaks of "rerum graviorum cura" having interfered with the perfecting of his mathematical work:—

"Seeing that the calculation of this table, which ought to have been perfected by the labour and pains of many calculators, has been accomplished by that of one alone, it will not be surprising if many errors have crept into it. I beseech you, benevolent readers, pardon these, whether occasioned by the weariness of calculation, or an oversight of the press; for as for myself, the infirm state of my health, and weightier occupations, have prevented my adding the last finish. But if I shall understand that the use of this invention proves acceptable to the learned, I may, perhaps, in a short time (God willing) publish the philosophy of it, and the method either of amending this Canon, or of constructing a new one upon a more convenient plan; that thus, through the diligence of many calculators, a Canon, more highly finished and accurate than the labour of a single individual could accomplish, may at length see the light. Nothing is perfect at its birth."

Yet the work had been long and intensely laboured, and came from his hands nearly perfect. No sooner had KEPLER perused it, than he addressed his enthusiastic letter to this "most illustrious Baron," as he calls him,—the mysterious hermit of the sciences, whose startling appeal to the great continental astronomers, issuing from the terra incognita of

Scotland, had come upon the immortal German like a voice from another world. Ere Kepler's letter reached Scotland, Napier was no more. The response came not from "The Baron of Merchiston," who was no longer to be found in his old tower on "the Borough-muir," nor in his yet more romantic retreats in the Lennox and Menteith. Had he been spared to reply, and to express his delight at the homage of Kepler, it might have been said, of this eloquent and electric sympathy between the distant orbits of Kepler and Napier,—

> "Rude Scotia's mountains now have found a tongue;
> Benlomond answers, through his misty shroud,
> Back to the joyous Alps, who call to him aloud."

But, to return to a less poetical view of the matter, Robert Napier, the transcriber of the manuscripts now published, was he who responded to Kepler's ardent request, by publishing his father's "Logarithmorum Canonis Constructio;" and, to complete this review of the nature and chronological order of Napier's mathematical studies, I shall here translate the preface to his posthumous work, which was published in 1619:—

"Some years ago, my father, of ever venerated memory, published the use of the Wonderful Canon of Logarithms; but the construction and method of generating it, he, for certain reasons, was unwilling to commit to types, as he mentions upon the seventh and the last pages of the Logarithms, until he knew how it was judged of and criticised by those who are versed in this department of letters. But since his death I have been assured, from undoubted authority, that this new invention is much thought of by the most able mathematicians, and that nothing would delight them more than if the construction of his Wonderful Canon, or so much at least as might suffice to illustrate it, were published for the benefit of the world. Although, therefore, it is very manifest to me that the author had not put his last finish to this little work, I have done what in me lay to satisfy their laudable desires, as well as to afford some assistance, especially to those who are weak in such studies, and apt to stick at the very threshold. I doubt not, however, that this posthumous work would have seen the light in a far more perfect and finished state, if to the author himself, my dearest father,—who, according to the opinion of the best judges, possessed among other illustrious gifts this in particular, that he could explicate the most difficult matter by some sure and easy method, and in the fewest words,—God had granted a longer use of life. You have, then (benevolent reader), the doctrine of the construction of Logarithms, which here he calls artificial numbers,—for he had this treatise beside him composed for several years before he invented the word

Logarithms,—most copiously unfolded, and their nature, accidences, and various adaptations to their natural numbers, perspicuously demonstrated. I have also thought good to subjoin to the Construction itself a certain appendix, concerning the method of forming another and more excellent species of Logarithms, to which the inventor himself alludes in his epistle prefixed to the Rhabdologia, and in which the Logarithm of unity is 0. The treatise which comes last is that which, tending to the utmost perfection of his Logarithmic trigonometry, was the fruit of his latest toil,—namely, certain very remarkable propositions for resolving spherical triangles, without the necessity of dividing them into quadrantal or rectangular triangles, and which are absolutely general. These, indeed, he intended to have reduced to order, and to have successively demonstrated, had not death snatched him from us too soon. I have added some lucubrations upon these propositions, and also upon the new species of Logarithms, calculated by that most excellent mathematician, Henry Briggs, public professor in London, who undertook most willingly the very severe labour of this Canon, in consequence of the singular affection that existed between him and my father of illustrious memory,—the method of construction and explanation of its use being left to the inventor himself. But now, since he has been called from this life, the whole burden of the business rests upon the learned

Briggs, as if it were his peculiar destiny to adorn this Sparta. In the mean while, reader, enjoy these labours such as they are, and receive them in good part. Farewell.

"ROBERT NEPER."

From this history of Napier's studies, derived chiefly from himself and his son, the conclusion is inevitable, that the work now given to the public was composed before his conception of the Logarithms; and that that invention, as well as his subsequent minor inventions, arose out of the very attempt, to abridge the toils of calculation, which he indicates in the fifth chapter of the second book of his Arithmetic, "De Multiplicationis et Partitionis Compendiis Miscellaneis." In order to ascertain, then, the age of his Arithmetic and Algebra, we must determine, as nearly as possible, the period of time when he formed his conception of the Logarithms.

No one who has taken the trouble to examine Napier's original works, or who understands any thing of the nature and construction of Logarithms, can doubt that many years of labour must have elapsed, between the first idea of the system in the inventor's mind, and its publication, in 1614, accompanied with calculated tables of Logarithms. The work published after his death had been composed for several years, as we are informed by his son, ere Napier had compounded the term Logarithms, under which denomination, however, they were first published. He himself alludes to the long labour bestowed upon his Canon, and to the interesting fact that his operose calculations were entirely performed by himself. M. Biot, in his review and "extrait" of the Memoirs of Napier, published in the Journal des Savans for the year 1835, after passing the highest encomiums upon the genius of Napier's works, thus speaks of the mechanical labour of the calculations :—

"Besides the merit of the invention, Napier's tables are a prodigy of laborious patience. As we reflect upon the time and toil it must have cost him to calculate all those numbers, we shudder at the chances there were of his being cut off ere he had realized his idea, and of its having died with him. It has been said, and Delambre repeats the observation, that the last figures of his numbers are inaccurate. This is true; but it would have been a more valuable truth to have ascertained whether the inaccuracy resulted from his method, or from some error of calculation in applying it. This is what I have done; and I have discovered an error of the kind, a very slight error, in the last term of the second progression, which he forms preparatory to the calculation of his tables. All the successive steps being deduced from this one, the trifling error in question is thus carried on through the calculations. I corrected his error, and then, adopting his own method, but abridging the operations by our more rapid processes of developement, I calculated the logarithm of 5000000, which is the last in Napier's table, and consequently that upon which all the errors accumulate. I found for its value 6931471.808942; whereas, by the modern series, it ought to be 6931471.805599. Thus the difference commences at the tenth figure. I calculated, in like manner, the hyperbolic logarithm, with Napier's numbers corrected, and found for its value 2.3025850940346, while, by our present tables, it is 2.3025850929940; so the real difference only falls on the ninth decimal. But this is beyond the range of the tables of Callet, which are in daily use. If Napier had even commanded the services of a village schoolmaster, to calculate, by the inventor's own method of subtractions, a geometrical progression still slower than what he used, a desideratum to which he himself calls attention, the tables of Briggs, calculated to fourteen decimals, would have possessed no superiority over those of Napier. His minor inventions are scarcely worth mentioning after this immense invention of Logarithms. That was sufficient for the lifetime, as it is for the fame, of a single individual."

It is interesting to compare this judgment upon Napier's labours, pronounced in the nineteenth century from the highest tribunal of science, with that modest "Admonitio" of his own, already quoted, wherein he anticipates the chance, and deprecates the criticism, of casual errors in his unaided calculations. Biot tells us, that he could only discover "une petite faute de ce genre, une très petite faute." We may assume, then, that Napier's great invention had first occurred to him many years before its publication. Indeed the following details, which will serve to determine the chronology both of the invention of Logarithms and of the treatises now submitted to the public, afford a curious proof that he was in possession of the secret, and preparing his publication of the Logarithms, some time before the close of the sixteenth century.

It is a remarkable fact, not generally known, that Tycho Brahe obtained some hint of this boon to be conferred upon science, twenty years before Napier's other avocations, added to the tedious labour of the calculations and his own diffidence, permitted him to give his invention to the world. Sir Archibald Napier of Merchiston, the father of the great Napier, was Justice-Depute in the reign of Mary of Scotland. His colleague in that office was Sir Thomas Craig of Riccarton, celebrated for his work on Feudal law.

The third son of the Feudist was John Craig, afterwards Dr John Craig, physician to James VI.; and he was also the friend of John Napier, the son of his father's colleague in office. But there was another, and perhaps the stronger tie. Young Craig was much given to mathematical studies. There is one record, indeed, of his success in those pursuits, which of itself is sufficient to distinguish his memory in that respect,—a record rarely met with in his own country, and still seldomer perused. I allude to a small volume of Latin epistles, printed at Brunswick in the year 1737, and dedicated by their collector, "Rud. Aug. Noltenius," to the Duke of Brunswick. The three first letters in this collection are from Dr John Craig to Tycho Brahe, and they prove that he was upon the most friendly and confidential footing with the Danish astronomer. He addresses Tycho as his "honoured friend," and signs himself, "your most affectionate John Craig, Doctor of Philosophy and Medicine." The first of these letters (which are written in Latin) commences with the fact, that, "About the beginning of last winter that illustrious man, Sir William Stewart, delivered to me your letter, and the book which you sent me." The date of this letter is not given; but, in the Library of the University of Edinburgh, there is a mathematical work by Tycho Brahe, which bears, upon the first blank leaf, a manuscript sentence in Latin to the following effect: "To Doctor John Craig of Edinburgh, in Scotland, a most illustrious man, highly gifted with various and excellent learning, Professor of Medicine, and exceedingly skilled in the Mathematics, Tycho Brahe hath sent this gift, and with his own hand hath written this at URANIBURG, 2d November 1588."

It appears from contemporary records, that, in the month of August 1588, Sir William Stewart, commanding the royal guard of Scotland, was sent to Denmark to arrange the preliminaries of King James's marriage, and that he returned to Edinburgh upon the 15th of November 1588. There can be no doubt that the volume above-mentioned is that referred to in Craig's epistle to Tycho. Neither can it be doubted that this was Dr John Craig, third son of Sir Thomas Craig of Riccarton, and physician to James VI. He was raised to his highest post in the royal household by a gift from that monarch, recorded in the Fœdera, "dilecto nobis Johanni Craigio in medicinis doctori, officium et locum ordinarii, et primarii medici nostri," dated at Westminster, 20th June, 1603. This was the friend of Tycho Brahe and of Napier. The Danish astronomer transmitted his present to John Craig, from his palace of Uraniburg, with the air of the Monarch of Science. Napier, from his old tower of Merchiston, when writing, in the year 1608, to his own son, who was gentleman of the bed-chamber to King James in England, thus remembers the King's physician:—"Ye sall make my commendatiouns to Doctor Craig."

The facts now stated serve to throw light upon a sentence, respecting the invention of Logarithms, which occurs in one of Kepler's letters,—a sentence which has been little noticed, and never rightly understood. Kepler, when writing to his favourite correspondent Petrus Cugerus (a mathematician of Dantzic, and the master of Helvelius) upon the subject of the economy of the heavenly bodies, and after reveling in his deepest calculations, declares that, of all the methods of calculation in aid of astronomy, nothing excels the invention of Napier; and yet (he adds), even so early as the year 1594, a certain Scotchman, in a letter written to Tycho, held out some promise of that Wonderful Canon. The precise words of Kepler's letter are, "Nihil autem supra Neperianam rationem esse puto: etsi, quidem, Scotus quidam, literis ad Tychonem A. CIƆIƆXCIV. scriptis, jam spem fecit Canonis illius Mirifici." Kepler was the pupil of Tycho, and joined him as such after the reverse of fortune which expelled the illustrious Dane from his palace of Uraniburg. This accounts for Kepler's having obtained some partial knowledge of the letter to which he alludes. But who was the "certain Scotchman" from whom Tycho received this important communication before the year 1594,—a date so precisely given by Kepler, that we cannot doubt its accuracy?

John Craig had long intended to pay a visit to Tycho Brahe. This appears by his letter of the year 1589, already quoted, in which he states that five years before he had made an attempt to reach Uraniburg, but had been baffled by storms and the inhospitable rocks of Norway; and that ever since, being more and more attracted by the accounts, brought by ambassadors and others, of Tycho's fame, and the magnificence of his observatories, he had been longing to visit him. In the year 1590, James VI., the patron of Craig, spent some days at Uraniburg, before returning to Scotland from his matrimonial expedition. It cannot be doubted that James's physician, who was long about this monarch's person in a medical capacity, and eventually at the head of his medical staff, would seize the propitious opportunity, of the progress of his royal master, to visit his friend; and it is not unlikely that Craig himself had suggested or encouraged the visit of his royal master to Uraniburg. Dr Craig, it is most probable, was the "certain Scotchman" to whom Kepler alludes, and who, in the year 1594, had written a promise of the

Logarithms to Tycho Brahe. For after his visit to Tycho, in the year 1590, the person in Scotland, to whom Craig would most eagerly unfold the wonders of Uraniburg, was Napier of Merchiston.

Frederick II. of Denmark, the munificent patron of Tycho, had established that philosopher on the island of Huen, situated at the mouth of the Baltic, adding honours and revenues, and every aid and encouragement which the most ardent astronomer could desire. Upon the 8th of August, 1576, the first stone of the far-famed castle or palace of Uraniburg was laid in Tycho's principality. It was a vast quadrangle, the dimensions being sixty feet every way, and flanked with lofty towers thirty-two feet in diameter, the observatories of this palace of science. Tycho is also said to have fitted it up with certain mysterious tubes, and other telegraphic contrivances, which enabled him to communicate with his domestics as if by magic, and obtain secret knowledge of his many visitors long before their arrival.

In the year 1590, that in which King James visited Tycho, Napier's fertile genius, unaided by the encouragement of royal patronage, was teeming with various discoveries in mechanical science (besides his speculations in the science of Numbers), which, like Archimedes of old, he intended should be applied for the defence of the island against foreign invasion. A report made to Napier by his friend Dr Craig, of all he had seen and heard at Uraniburg, when there with his Majesty, would perfectly account for the following sentence of Napier's admonitory letter to King James (on the subject of his Majesty's supposed inclination to Popery), dated from Merchiston, 29th January, 1593:—" For let not your Majesty doubt but that there are within your realm, as well as in other countries, godly and good ingynes, versed and exercised in all manner of honest science, and godly discipline, who, by your Majesty's instigation, might yield forth works and fruits worthy of memory, which otherwise, lacking some mighty Mæcenas to incourage them, may perchance be buried with eternal silence."

His conversations with Dr Craig might suggest to Napier, not merely this hint to King James, that his Majesty should patronise science in Scotland, but the transmission of a hint to Tycho himself on the subject of astronomical calculation. If the King of Denmark, when he invested Tycho with something like eastern splendour, could have added the power that still lay hid in Arabic numbers, a false system of the world would not have been re-established at Huen. It was in the means of astronomical calculation that the science of Tycho Brahe was most deficient. Now Kepler tells us, that even so early as in the year 1594, (the year following that in which Napier's letter to King James is dated,) a certain Scotchman had written to Tycho some promise of the Logarithms. After the facts referred to, it can scarcely be doubted that this correspondent from Scotland was none other than Tycho's old friend and correspondent there, the learned physician of King James, and also the friend of Napier, with whom he could not fail to have often discussed the subject of the royal visit to Uraniburg. And this appears to be placed beyond all doubt, when, to what has been already stated, we add the following anecdote, somewhat imperfectly recorded by Anthony-à-Wood.

That indefatigable and amusing collector of literary gossip thus narrates it, in the Athenæ Oxonienses :—" It must now be known that one Dr Craig, a Scotchman, coming out of Denmark into his own country, called upon Joh. Neper, Baron of Marcheston, near Edinburgh, and told him, among other discourses, of a new invention in Denmark, by Longomontanus, as 'tis said, to save the tedious multiplication and division in astronomical calculations. Neper being solicitous to know further of him concerning this matter, he could give no other account of it than that it was by proportional numbers. Which hint Neper taking, he desired him at his return to call upon him again. Craig, after some weeks had passed, did so, and Neper then shewed him a rude draught of what he called Canon Mirabilis Logarithmorum. Which draught, with some alterations, he printing in 1614, it came forthwith into the hands of our author Briggs, and into those of Will. Oughtred, from whom the relation of this matter came."

Any one at all conversant with the history of science, and the nature of the invention of Logarithms, will at once perceive, that, whatever foundation in fact this anecdote may have, it is here inaccurately and ignorantly told. Longomontanus was the pupil and assistant of Tycho at Uraniburg, and a most distinguished mathematician. He, in common with many others, was well acquainted with a principle of numerical progression, the extraordinary generalization of which by Napier is that which constitutes the invention of Logarithms. Archimedes had first observed and speculated upon such progressions, but without discovering the Logarithms. It could not possibly be, that a hint from Longomontanus had suggested to Napier his great invention; for if a hint of the kind could have urged any human intellect thus rapidly upon the conception of the Logarithms themselves, that hint had arisen in the school of Alexandria, was submerged in the middle ages, and rose again with the letters of Greece,—a hint which Tycho had, which Stifellius, Byrgius, Longomontanus, and Kepler himself had; yet no more was made of it after the revival of letters, than had been by Archimedes before their fall. Kepler too, informs us, that in the year 1594, something regarding the generalization of this numerical principle had actually been reported in a letter to Tycho; yet the secret was still undiscovered until Napier published his Canon in 1614. In a letter, dated 11th March, 1618, to his friend Schikhart, Kepler, after descanting upon the various difficulties and resources of trigonometry, exclaims,—"A Scotish Baron has started up, his name I cannot remember, but he has put forth some wonderful mode by which all necessity of multiplications and divisions is commuted to mere additions and subtractions, nor does he make any use of a table of sines; still, however, he requires a canon of tangents, and the variety, frequency, and difficulty of additions and subtractions, in some cases exceed the labour of multiplication and division." This was the first crude and inaccurate idea formed by Kepler of the work which he had not yet studied; and already the Scotish Baron whose name he could not remember was in his grave! But of this fact Kepler was not aware even on the 28th July, 1619, when he thus addressed Napier himself:—

"But the chief cause that impeded my progress this year, in framing the Rudolphine Tables, was an entirely new, but happy calamity, which has befallen a part of the Tables I had long ago completed. I mean, most illustrious Baron, that book of thine, which, published at Edinburgh in Scotland five years ago, I first saw at Prague the year before last. At that time I had not leisure to study it; but last year, having met with a little book by Benjamin Ursin (long my familiar, and now astronomer to the Margrave), wherein he briefly gives the marrow of the matter, extracted from your own work, I was awakened to its merits. Scarcely, indeed, had I made trial of it, in a single example, when I became aware, to my great delight, that you had generalized a certain play of Numbers, which I myself, in a very minute degree, had practically adopted for many years, and had proposed to incorporate with my Tables; especially in the matter of parallaxes, and in the minutes of duration and delay in eclipses; of which method this very Ephemeris exhibits an example. I knew, indeed, that this method of mine was only applicable in the single case of an arc differing in no sensible degree from a straight line. But of this I was ignorant, that, from the excesses of the secants, LOGARITHMS could be generated, thus rendering the method universal throughout any extent of arc. Then indeed my mind, above all things, was eager to ascertain, whether, in this little book of Ursin's, these Logarithms had been accurately investigated. Calling to my aid, therefore, Janus Gringalletus Sabaudus, my familiar, I ordered him to subtract the thousandth part of the whole sine,—again, to subtract the thousandth part of that residue, and to repeat this operation more than two thousand times, until there remained about the tenth part of the whole sine; but of the sine from which a thousandth part had been subtracted, I computed the logarithm with the greatest care, beginning from the unit of that division which Pitiscus most frequently uses, namely, the duodecimal. The logarithm thus computed, I arranged uniformly with the remainders of all the subtractions. In this manner, I ascertained that these logarithms were as nearly perfect as possible, although a few errors had crept in, either of the press, or in that minute distribution of the greater logarithms about the beginning of the quadrant. I mention all this to you by the way, in order that you may understand how gratifying it would be, to me at least (and I should think to others), if you would put the world in possession of the methods by which you proceeded. Of these, I make no doubt, you have many, and most ingenious, at your command intuitively. And so the promise held out by you on the 57th page of your work," (to put the world in possession of his methods of constructing the Logarithms, should he understand that the invention was approved of by men of science,) "has fallen due to the public. And now let us grapple with your Tables."

This interesting letter, written after he to whom it was addressed had been dead for nearly two years, affords the most complete refutation of that part of the anecdote, in the Athenæ Oxonienses, which seems to impute Napier's invention to a sudden thought suggested by a hint from Kepler's intimate friend and companion Longomontanus. Kepler himself, in this letter, refers the invention to its true source, namely, Napier's surpassing command of the science of Numbers. Kepler, whose own immortal discoveries in Astronomy obtained for him the daring title of Legislator of the Heavens, bowed at the shrine of the Logarithms, and at once, and without reserve, acknowledged Napier as his master in Logistic. A quaint indication, of his estimate of Napier's invention, is also to be discovered in the frontispiece to those Rudolphine Tables of which he speaks in his letter. The telescope of Galileo, the elliptical orbit of a planet, the system of Copernicus, and a female figure with the Napierian Logarithm of half the radius of a circle arranged as a glory round her head, are there delineated as figurative of the mighty impulses which Astronomy had received in those days. But that the anecdote of Anthony-à-Wood has some foundation in fact is obvious, when we compare it with what has been previously stated. The Doctor Craig therein mentioned is undoubtedly Napier's friend, the physician of King James; and the "coming out of Denmark into his own country," refers to his return with his Majesty in 1590. Moreover, that Craig had some conversation with his friend the Baron of Merchiston on the subject of astronomical calculations, and that the result was some communication on the subject of the Logarithms, seems to be verified by the independent evidence afforded by Kepler's other letter, where he says that a certain Scotchman, in a letter to Tycho written in the year 1594, already held out some hopes of that wonderful Canon; a fact which Kepler had subsequently become aware of, probably from his inspection of Tycho's scientific papers and correspondence. Thus, by a very curious chain of evidence, it seems proved, that Napier had made some advance in his construction of the Logarithms so early as 1594, and that he had set himself deliberately to the task, after the difficulties which harassed the throne of Astronomy at Uraniburg (in consequence of the crudeness of numerical science) had been reported to him in 1590. But, as Kepler himself observes, the Inventor of Logarithms must have previously commanded many and most ingenious numerical resources. He must have been far in advance of his age in a profound knowledge of the mysterious play of Numbers. His mind must have been thoroughly and deeply imbued with the Logistic art, and analytical power, ere he proposed to himself the invention of that great lever of science. Now it is in the Latin treatises, which these observations preface, that we may trace Napier's command of Numbers, before he had formed a conception of the Logarithms. And thus the present work may be said to afford a most interesting chapter in the history of the progress of Science. A digest of the whole art of Logistic, equal, in so far as the then existing numerical notation admits of the comparison, to Euler's in modern times, composed prior to the year 1594, amid the storms of faction which then distracted Scotland, was an achievement worthy of being crowned by the invention of Logarithms, for the natural discovery of which, in the gradual development of algebraic resources, the analytic art was at the time an age too young.

Another circumstance may here be mentioned which also affords evidence, that, many years prior to the publication of the Logarithms, Napier's fame as a mathematician was established with the learned of his own countrymen, and that he was the person to whom Dr Craig would be most eager to converse on the subject of his visit to Tycho Brahe. The celebrated lawyer, and learned and accomplished author, Sir

John Skene of Currie Hill, when, in the course of preparing his treatise "De Verborum Significatione" (first published in the year 1597), he came to the word "particata vel perticata terræ," which he explains, "from the French word 'perche,' meikle used in the English lawes, ane ruid of land," adds this sentence:—"But it is necessare that the measurers of land called landimers, in Latin 'agrimensores,' observe and keep ane just relation betwixt the length and the breadth of the measures quhilk they use in measuring of lands, quhairanent I find na mention in the lawes and register of this realme, albeit ane ordinance thereanent be made by King Edward the First, King of England, the 33d yeir of his reigne; and because the knawledge of this matter is very necessare in measuring of lands dayly used in this realme, I thought good to propone certaine questions to John Naper, fear of Merchistoun,—ane gentleman of singular judgement and learning, especially in the mathematical sciences,—the tenour quhairof, and his answers made thereto, followis." Napier's answers are of considerable length, and given with his characteristic simplicity and power.

If the composition of the treatises now published is to be referred to a date prior to the year 1594, which seems to be placed beyond doubt by what has been stated, they are interesting not merely as the works of the Inventor of Logarithms, but as being the very first of the kind in Scotland, and among the first systematic works on Numbers after the revival of letters in Europe. The sixteenth century was the rudest period of algebraic science in Europe. Leonardo of Pisa, indeed, composed his work before the invention of printing, and early in the thirteenth century; but this had been lost sight of, and was not known for more than a century after Napier's death, when the manuscript was discovered at Florence. The first printed work on the subject was that of Lucas de Burgo, from whom is generally dated the decided dawn of Algebra in Europe. De Burgo's principal work was printed about the year 1494. The second printed work upon Arithmetic and Algebra appeared in 1539. This was a work of the great but eccentric Cardan, of whom it is affirmed by Scaliger that he was so devoted to astrology as actually to starve himself to death, that his own astrological prediction might be fulfilled,—a very equivocal illustration of his favourite science. Cardan died in the year 1575. Germany produced one or two mathematicians, who, at the same time that Cardan wrote, gave a more decided impulse to Numbers. Hitherto nothing had been added to that recondite science, since the introduction of the Arabic numerals, and the rude and imperfect symbols of Burgo's Algebra, except in the theory of equations, which had received a great extension from Tartalea and Cardan. The defect which materially impeded the system was in notation, the mainspring of numerical science. In the year 1544, Michael Stifellius, a Lutheran clergyman, published at Nuremberg his Arithmetica Integra, a very original Latin treatise on Arithmetic and Algebra, wherein he viewed numerical quantities, and their combinations, closely and ingeniously, and gave an impulse to Algebra by improving its notation. He was the first to introduce the signs + and — for plus and minus, and also the character $\surd$ (derived from the letter R), to denote the radix or root. Moreover, he entered systematically into the consideration of arithmetical and geometrical progressions, pointing out what may now be termed the logarithmic properties of a corresponding series of the powers of a given number, and the exponents of those powers, which latter term he uses. But in this speculation he had formed no conception of the possibility of changing the infinite series of natural numbers from an arithmetical to a geometrical progression, and then of generating a corresponding arithmetical progression.

In the year 1552 appeared the first treatise upon Arithmetic and Algebra in the English language. Its author was the unfortunate Robert Recorde, a mathematician of undoubted genius, but so little appreciated that he was allowed to linger out his last days in the Fleet, where he was imprisoned for debt, and where he died about the year 1558. His works are curious and original, but rude and puerile in their style. They are generally in the form of a dialogue between a Master and Scholar, and under such quaint titles, as "The Pathway to Knowledge"—"The Ground of Arts"—"The Castle of Knowledge"—"The Whetstone of Wit." Napier's style of communicating even his elementary rules is clear, simple, and philosophical. His son tells us, that, "according to the opinion of the best judges, my dearest father possessed, amongst other great endowments, this in particular, that he could explicate the most difficult matter by some sure and easy method, and in the fewest words." Accordingly, he never indulged in such facetiæ as the following, which occurs in Recorde's Arithmetic, a work, nevertheless, of original genius:—"Master. 'Exclude number, and answer this question—How many years old are you?'—Scholar. 'Mum.'—Master. 'So that, if number want, you answer all by mummes. How many miles to London?'—Scholar. 'A poak full of plums.'—Master. 'If number be lacking, it maketh men dumb; so that, to most questions, they must answer mum. What call you the science you desire so greatly?'—Scholar. 'Some call it Arsemetrick, and some Augrime.'—

Master. 'Both names are corruptly written; Arsemetrick for Arithmetic, as the Greeks call it; and Augrime for Algorisme, as the Arabians sound it.'"

Recorde was ingenious and inventive, and is remarkable, in the history of Algebra (which owes so much to successive inventions in the art of notation), for having added to its characters the sign of equality. Mr Babbage had overlooked the fact. That distinguished mathematician, in his history of notation, observes, "It is a curious circumstance that the symbol which now represents equality was first used to denote subtraction, in which sense it was employed by Albert Girard, and that a word signifying equality was always used instead, until the time of Harriot." The works of Girard and Harriot did not appear until the seventeenth century was far advanced, and long after Napier's death. Napier's book of Arithmetic, as already shown, must be referred to a date prior to the year 1594; and it will presently appear that his book of Algebra is of a date still earlier. If Mr Babbage's observation were historically accurate, we might claim for Napier the merit of having hit upon this ingenious device, in his unpublished work, a century before the time of Harriot. For, it will be observed, he adopts it in his chapter of equations, and defines it in these terms:—"Betwixt the parts of an equation that are equal to each other, a double line is interposed, which is the sign of equality; thus 1 ℞=7, which is pronounced, one thing equal to seven." The fact is, however, that this notation was already more or less in use; and it is to Recorde that the merit of the first idea is due; for, in his work, first published in 1552, he says, " And to avoid the tedious repetition of these woordes, ' is equal to,' I will sette, as I doe often in woorke use, a paire of paralleles, or gemowe lines of ane length, thus =, because noe two thynges can be moare equalle."

In France, the celebrated Ramus wrote an elementary treatise on Arithmetic and Algebra, about the year 1560; but he left the science as he found it. Raphael Bombelli, whose Algebra was published at Bologna in the year 1572, in Italian, wrote more elaborately and profoundly, but did not add any thing of consequence to the labours of his predecessors. The first that can be said to have done so, between the time of Stifellius and of Napier, was Simon Stevinus of Bruges, who published La Practique d'Arithmetique about the year 1582. He afterwards put forth other works upon Arithmetic and Algebra, along with a translation of some books of Diophantus; in all of which he evinced a remarkable genius for his subject. Algebraic notation received at his hands another of those impulses by which it has so gradually reached its present power; and he appears to have been the first who expressly mooted the doctrine of a decimal division of unit, which idea, however, remained to be developed and practically applied as a system by Napier.

A contemporary of Napier's was Vieta, whose name reflects lustre upon France. He generalized the language of Algebra, by employing letters to denote known as well as unknown quantities, and he extended the theory of equations. It is not at all likely that Napier ever saw any of his treatises, which were only first collected into one volume by Schooten, in 1646. All the other conspicuous treatises, illustrative of the progress of numerical science, are subsequent to the death of Napier. At some period of his life he had perused the work of Stevinus, as he mentions this author in the Rhabdologia; but considering how very few works of the kind existed at the time when Napier must have composed his numerical treatises, how slowly books were then spread abroad, and that literary communication between Scotland and the Continent was so slight as to leave Kepler in ignorance of the death of the Inventor of Logarithms two years after that event, it is not to be supposed that Napier had at his command even those scanty stores of information which other countries could afford, on the abstruse subjects to which he was devoted. The most accurate chronology of his treatises, so far as can be ascertained, would seem to be between the publications of Stevinus of Germany, in 1582, and those of Vieta of France, about the commencement of the next century. Of all his predecessors and contemporaries, however, Napier was the one destined to create the greatest revolution in Numbers. This fact alone must render the perusal of his early and preparatory labours very interesting to all who are capable of appreciating his great invention. Countries the most distinguished in Europe, for men of science, had produced in that recondite path the few remarkable men so briefly and imperfectly noticed above. But even the gigantic Kepler (on his own authority we state it) had struggled in vain against the spell that yet bound the Arabic system. From Archimedes to Kepler, not one of the victorious in the field of science had struck a blow sufficient to extricate the best wing of mathematics. Until Napier arose, the Arabic scale remained undeveloped. The name of Recorde is barely sufficient to give England a place in the previous history of the progress of science; and as for Scotland, when Napier was acquiring for his country the fame of one of the greatest impulses which human knowledge has received, it was distinguished by mists which science had not penetrated, and by the Douglas wars, whose baronial leaders knew little of the denary system beyond their mail-clad hands.

Napier, in the first chapter of his Arithmetic, refers to what he terms Geometrical Logistic, as forming the subject of his third book, and to Algebra, as being the subject of a fourth book. It would appear, however, that his Algebra, so far as " orderlie sett doun," is an earlier production than either his Arithmetic or the fragment of Geometrical Logistic. This is manifest from several circumstances :—1. It is entitled " The Algebra of John Naper, Baron of Merchistoun ;" but not Liber Quartus, in correspondence with the other books. 2. Arithmetic is referred to in it ; but there is no reference to his own book of Arithmetic, as there would have been, according to his practice throughout the rest of the manuscript, had that been previously composed. 3. The treatise of Algebra is itself divided into two books ; and, while there is a systematic reference to its component parts, there is no reference to any previous books. 4. Napier adopts in his Algebra the nomination and notation which had been introduced before his time ; whereas, in his Arithmetic and Geometrical Logistic, he adopts and expounds a peculiar nomination and notation of his own, applicable to the arithmetic of Surds, and whereby he proposes to supersede that with which he operates in his treatise of Algebra. There can be little doubt, therefore, that his Algebra is a work of still earlier date than the other books ; and these, as has been seen, were prior in date to his conception of the Logarithms, which was some time before the year 1594. Yet there is no appearance of crude or hasty composition in his earliest work. It is stamped with the same characteristics—clear and simple exposition, profound and original views, and symmetrical arrangement—which eminently distinguish all his compositions.

Having ascertained the most probable chronology of the treatises now printed, as being the occupations of a period embracing a course of years prior to 1594, we must compare the fact with Napier's time of life. He was born at Merchiston in the year 1550; so that he was only of age in 1571, the year of his marriage in Scotland. Even at this early period of his life, he must have been far advanced in his abstruse studies, in order to have obtained that complete command of Logistic,—a term he uses to denote the whole analytic art,—which had enabled him, prior to the year 1594, to digest every department of numerical science into a more comprehensive and perfect institute of the subject than had yet appeared. Napier himself has recorded an anecdote which illustrates the extraordinary precocity of his genius; and, while referring to it, I may add a few particulars relative to his early studies. It appears, from the original records of the University of St Andrews, that Napier was incorporated in the College of St Salvator in the year 1563. But there is an earlier and very curious notice, in reference to his education. His mother's brother was that celebrated character, Adam Bothwell, the first reformed Bishop of Orkney; the same who performed the marriage ceremony between Queen Mary and the Earl of Bothwell, and who afterwards anointed and crowned the infant James VI. This prelate concludes a letter (still preserved in the Napier charter-chest) on the subject of his own private affairs, addressed "To his Bruder the Laird off Merchistoun in Loudeanne," Napier's father, with the following remarkable and prophetic sentence:—

"I pray you, schir, to send your sone Jhone to the schuyllis; oyer to France or Flandaris; for he can leyr na guid at hame, nor get na proffeit in this maist perullous worlde,—that he may be savet in it,—that he may do frendis efter honour and proffeit,—as I dout not but he will: quhem, with you and the remanent of our successione, and my sister your pairte, God mot preserve eternalle. At the yairdis in Kirkwall, this v day of December, the yeir of God 1560, be

"Your Bruder at powair,

ADAME, Bischopp off Orknay."

Napier, at the date of this letter, (in which will be observed a promiscuous use of the Roman and the Arabic notation), was ten years of age, and his public education had not commenced. His father was only about sixteen years his senior, which probably induced the Bishop to tender his advice. But it was not until 1563 that young Napier matriculated at St Andrews; and how soon his genius there impelled him to the deepest speculations, we learn from himself. In his address "To the Godly and Christian reader," prefixed to his Commentaries on the Apocalypse, published in 1593, he tells us:—"In my tender yeares and barneage in Sanct Androis, at the schooles, having, on the one part, contracted a loving familiaritie with a certaine gentleman, a Papist,—and, on the other part, being attentive to the sermons of that worthy man of God, Maister Christopher Goodman, teaching upon the Apocalyps, I was so moved in admiration against the blindness of Papists, that could not most evidently see their seven-hilled citie Rome painted out there so lively by Saint John, as the mother of all spiritual whoredom, that not onely burstit I out in continual reasoning against my said familiar, but also, from thenceforth, I determined with myself (by the assistance of God's Spirit) to employ my studie and diligence to search out the remanent mysteries of that holy book,—as to this houre (praised be the Lorde!) I have bin doing at al such times as conveniently I might have occasion." It is a curious trait of the early power of his mind, that, when only fourteen years of age, he should have listened so intensely to an exposition of the Apocalypse from the pulpit, bursting forth afterwards in disputation with his papistical friend and companion, until he conceived the daring project of leaving not a mystery of prophecy undiscovered,—a project eventually realized, as he supposed, by those profound but fruitless speculations, in which he has been followed by Mede, Sir Isaac Newton, and a host of moderns, who have added nothing to his labours. The anecdote not only proves the precocity of his genius, but indicates the immediate cause of his very early attraction to that profound study of numerical science, which must have been co-extensive with the progress of his very learned and long laboured work, the "Plain Discovery of the whole Revelation of St John." His varied illustrations of such propositions as, for instance, that "the forty-two months, a thousand and two hundred and threescore prophetical days, three great days and a half, and a time, times, and half a time, mentioned in Daniel and the Revelation, are all one date," prove that he was deeply versed in ancient chronology and numbers. Moreover, in attempting to expound the mystery of the name and number 666, he evinces a knowledge of the numeral system of the Greeks, and that they worked arithmetically with the letters of their alphabet, instead of the Arabic or Indian notation now in use. It must be observed also, that the arrangement and structure of his theological work is mathematical.

There is reason to believe that Napier had made some researches abroad, relative to the history of the Arabic notation, when a very young man. In his Memoirs, I had ventured the hypothesis that he must have been abroad for a time after his studies at St Andrews. This was founded on the facts, that, as appears from the records of the University, he did not remain at St Salvator's long enough to take a degree, or to complete the usual course of studies there; and that, from the terms of a letter of the Bishop of Orkney to the Laird of Merchiston in the year 1668, young Napier seems to have been absent from home of that date. Besides, the almost invariable custom of Scotland then was, for all young men of family, having any pretensions to a learned education, to complete their studies on the Continent. Napier's Commentaries on the Apocalypse prove that he was a most accomplished scholar, that he possessed a general knowledge of the arts and sciences, and a great command of the learned and continental languages. Under these circumstances, it is more difficult to believe that he had never quitted his own rude country, than that he had obtained the usual advantage of spending some years of his youth in France, Italy, and the Low Countries; more especially, as this very plan of his education had been earnestly recommended by his uncle, the Bishop of Orkney. In coming to this natural and indeed inevitable conclusion, when writing his Memoirs, I was not aware of a very interesting fact, subsequently mentioned to the Royal Asiatic Society by the Right Honourable Sir Alexander Johnston, (a lineal descendant of the Inventor of Logarithms,) as chairman of the Committee of Correspondence. At the anniversary meeting of the Society held on the 9th of May, 1835, Sir Alexander, in the course of an address, for whose varied information he received the thanks of the Society, took occasion to communicate the following facts relative to Napier, and the curious result of a projected life of him by Lord Napier, which I have extracted from the proceedings of the learned body to whom they were addressed :—

" The province of Madura again became an object of literary interest in the eighteenth century, in consequence of my grandfather, the fifth Lord Napier of Merchiston, having determined to write the life of his ancestor, John Napier of Merchiston, and to prefix to it a history of the knowledge which the people of India had of mathematics. It appearing by John Napier's papers, that he had, from the information he obtained during his travels, adopted the opinion, that numerals had first been discovered by the college of Madura, and that they had been introduced from India by the Arabs into Spain, and into other parts of Europe, Lord Napier was anxious to examine the sources from whence John Napier had derived his information upon this subject; and, when he himself was abroad, he visited Venice and other places in Italy, in which he thought it was likely he should find an account of the information collected by the members of the Jesuit mission at Madura, upon this and other parts of Hindoo science. Having been successful in obtaining some interesting documents relative to the object of his researches, he returned to Scotland, and submitted them to the then Mr Mackenzie (afterwards Colonel Mackenzie), who had been recommended to him by Lord Seaforth, as a young man who had devoted himself to the study of mathematics. Lord Napier died before he had completed his life of John Napier; and Mr Mackenzie, whose mind had been turned to the subject of Hindoo science by Lord Napier, applied for, and obtained, through Lord Seaforth, a commission in the East India Company's Engineers on the Madras establishment, in order that he might have a favourable opportunity of prosecuting at Madura, the site of the ancient Hindoo college, his enquiries into the knowledge which the Hindoos possessed, in early days, of arithmetic, and the different branches of mathematics. On Mr Mackenzie's arrival at Madras, finding that my father and mother (the latter being the daughter of his patron, Lord Napier, and then engaged in completing the life which had been commenced by her father) were stationed at Madura, where my father held a political situation of high trust under his friend Lord Macartney, he obtained leave from Lord Macartney, the then Governor of Madras, to join them. As soon as Mr Mackenzie reached Madura, he began his enquiries relative to the ancient Hindoo college of that place; and, in conjunction with my father and mother, formed the plan of reviving, under the protection of the English government, the Hindoo college. In furtherance of this plan, my father having obtained from the Nabob of Arcot, the then sovereign of the country, some deserted ruins in the jungle, about a mile from the fort of Madura, which were supposed to have been connected in former days with the proceedings of the Hindoo college, built upon them, at considerable expense, the house which has ever since been known at that place by the name of Johnston House, and which is still my property, laying out its different compartments, under the direction of Mr Mackenzie, in such a manner as might best suit the adaptation of it as a building, in which the mathematical instruction that Mr Mackenzie wished to be circulated amongst all the natives of the country might be pursued. The pillars which supported this house were divided into six compartments, upon each of which all the diagrams were to be carved which were necessary to illustrate a course of arithmetic, geometry, mechanics, hydrostatics, optics, and astronomy, there being a building erected upon the roof, in which plane and spherical trigonometry were to be taught: two orreries were to be erected, the one illustrating the Ptolemaic, the other the Copernican, system of the universe; and lectures were to be given in Tamil, Telugu, Malayalam, and Canarese, pointing out the superior utility of the Copernican over the Ptolemaic system, and the great practical utility to which the sciences of Europe might be applied in every department of practical knowledge. Mr Mackenzie, shortly after he had finished this building for my father, was obliged to quit Madura on account of the public service, and the plan of the college was, owing to his absence, not then carried into effect."

It is to be lamented that the life of Napier, undertaken by the nobleman here mentioned, and subsequently by his accomplished daughter, never saw the light. In the preface to the Memoirs published in 1834, I had erroneously stated that Mrs Johnston's papers on the subject had perished by fire. I have since learned, from Sir Alexander himself, that the papers referred to in his address to the Asiatic Society, and which were bound together in the form of a quarto volume, were lost at sea, on board of the Jane, Duchess of Gordon, when that vessel perished on her voyage from Ceylon, in 1809. As Francis, fifth Lord Napier, was he to whom the then Napier of Culcreuch presented the valuable manuscripts which compose the present volume, it is not unlikely that the papers, from which it appeared that John Napier had prosecuted some enquiries abroad relative to the history of the Arabic or Indian notation, had also been obtained from the family of Robert Napier. The fact that Napier had made this investigation is the more interesting, that he himself was the first who fully developed the system. He added to it the Logarithms, and the reciprocation of the scale in the working of decimal fractions. "There are two improvements," says Wallis (the friend and contemporary of Newton), "which we have added to the Algorism of the Arabs, since we received it from them; to wit, that of Decimal Fractions and that of Logarithms."

In the manuscripts now printed, and which have so fortunately escaped the fate of nearly the whole of Napier's private papers illustrative of his studies, it was not to be expected that he would allude to his researches abroad, or enter into antiquarian details relative to the origin of the present system of arithmetical notation. But that he well understood the principle of its operation, and had also turned his mind to new inventions in notation, may be gathered from these manuscripts. In the first chapter of the second book of his Arithmetic, in which he discusses the nomination and notation of numerical quantities, he says that "every idiom supplies its own vocal nomination; but that the written names of integers are the nine significant figures,—1, 2, 3, 4, 5, 6, 7, 8, 9; and that these signify various numerical values, according to their change of place. This progress," he adds, "is from right to left; and the circle 0, which has no signification wherever it be placed, is merely used to indicate the progress of the significant figure, by representing the vacancy which that has occasioned in its progress. When the significant figure is occupying its first place, it is named," says Napier, "according to its own individual value; when it has progressed to the second place, it is named by its tenfold value; in the third place, a hundredfold; and so on by an infinite progress, each step attaching to the figure a value equal to the multiplication of the last step by ten." Such is Napier's explanation of that all-powerful though simple expedient; and none more accurate and lucid has been afforded during the two centuries and a half which have elapsed since he composed his digest of Numbers. The extreme ingenuity with which he adapted to this principle—of numerical values in geometrical progression, indicated by the progressive motion of the same digit in space—a new set of digits of his own invention, intended for the Arithmetic of Surds, is well worthy of attention.

Napier commences what he terms Geometrical Logistic with an important principle,—his profound consideration and command of which most probably paved the way to his great invention. He views quantity and number under two different aspects or conditions; namely, either as separated into distinct parts, capable of being exactly expressed by absolute numbers, or fractions of numbers, which he terms discrete quantity and number; or, as consisting of an infinite continuity of parts, not to be so expressed by numbers; and this he terms concrete quantity or numbers. "If," says he, "$3a$" (by which he means 3 of any given quantity) "refer to three digital lines, thus, ——— ——— ———, it is a discrete number; but if it refer to a concrete and continuous tri-digital line, in this form, ———+———+———, it is called a concrete number; though not a proper concrete number, but only in relation to the quantity to which it refers. A proper concrete number," Napier adds, "is the root of a number whose root cannot be measured; that is, exactly and finitely expressed by any number, integral or fractional." He had already explained that the numerical quantity, generally termed the Power of a number, but which he terms Radicatum, is that which can be reduced to unit, when divided one or more times by some other number, the number of the partitions being the index, and the dividing number the radix, or root. Euler, the great algebraical writer of the eighteenth century, had not a more profound knowledge of the properties of the relative numerical quantities, Root, Power, and Exponent, than Napier in the sixteenth century had of what he called Radix, Radicatum, and Index. Indeed, his opening of the subject of involution is less perplexing than Euler's, whose statement might leave the student at a loss to know why the square of a number is called the second power, and not the first. For Euler, after having said that a power of a number derives its rank, or particular denomination, from the number of times it is multiplied by itself, informs us, that a square is obtained by multiplying a number once by itself, and a cube, by multiplying a number twice by itself. Why, then, is a square called the second power, since the elevation is obtained by the first multiplication of the number by itself? Napier avoids all such perplexity, when he commences by saying, that the first step in the process of involution is to " multiply unit by the radix, which multiplication returns the radix itself; secondly, multiply that again by the radix, and the duplicatum (the expressive term he uses for square or second power) is produced; and so on, according to the quality of the index, which," he says, " is determined by the number of units composing the index." Thus, whether he explain the nature of a radicated quantity, as that which a certain number reduces to simple unity by one or more divisions, or as that which is raised to its particular denomination by one or more multiplications of the same number into itself, he so brings unit into his statement of the involution or evolution, as at once to make manifest the meaning of the algebraic law, that, although the powers of a number are the products of the successive multiplications of that number into itself, yet every number, when thus taken as the root of successive powers, is considered the first power of itself.

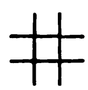
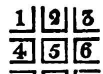
Moreover, Napier did not fail to observe, that there are certain numbers which neither are to be obtained by the multiplication of any number whatever into itself, nor can be resolved into simple unit by division by any number whatever. Thus, the number 16 is reduced to 1 by four times dividing by the number 2. This discovers 2 as what Napier called the quadrupartient root of 16, and 16 as the quadruplicate, or fourth power of 2. But there is no number, for instance, which by involution will produce the number 10. The bipartient or square root of 9 is 3, because 3 times 3 is 9; but what is the square root of 10? In other words, what is the number which, being multiplied by itself, produces 10? Not 3 times 3, that being only 9, nor yet the next number, 4, which gives too much, namely 16. The doctrine of fractions, indeed, enables us to express exactly numerical quantities between 3 and 4, nearer to each other than the relative quantities indicated by 3 and 4, and consequently a nearer approximation to the quantity sought. This approximation, however, will still be found to consist of terms, the one too much, and the other too little, exactly to express what is required; and a curious property it has, that these fractional terms may be brought closer and closer together, by an endless approximation, and still no number obtained which, by being multiplied into itself, will produce the precise number. Hence, numerically speaking, the number 10 (being a simple example of an infinity of numbers in the same predicament) has no root; that is, no root capable of finite expression in discrete or absolute number. "Now," says Napier, "a concrete number proper is the root of such irreduceable number, and these roots are commonly called surd and irrational."

Napier had too strong a hold of his subject to reject these latent and ineffable roots as no quantity at all, or as incapable of submission to the rules of Arithmetic, and the purposes of computation. He views them in their proper concrete character of quantity or magnitude, rather than as number or multitude; and calls them "nomina," because susceptible, he says, rather of being named than numbered. Moreover, he had considered these quantities so profoundly as to discover all their computative properties, and fully to illustrate them under the operation of all the rules of Arithmetic relative to discrete number and quantity.

Having digested this important chapter of his system of Logistic, Napier was struck with the necessity of a new nomination and notation wherewith to work, more easily and effectually, the peculiar quantities in question. The ancient geometricians had derived their nomination of the successive powers of a number, entirely from the three local dimensions of nature,—length, breadth, and thickness. Napier, in his Commentaries on the Revelation, took those co-existing qualities as an illustration of the Trinity; but he was not satisfied with their illustration of the geometrical progression of numbers. A geometrical progression, of which we hear so much in all statements of the Logarithmic principle, is so called, because the successive numerical powers which compose it were named from the geometrical extensions—represented by a line, a surface or square, and a solid or cube. The square contains two of the local dimensions,—length and breadth; in the cube are the whole three,—length, breadth, and thickness. In any progression of numbers, increasing from unit by a common ratio of multiplication, the second power contains two dimensions of the root, and was called by the ancient geometricians a square number, or square; so the third power, being composed of three dimensions of the root, obtained the name of a cubic number, or cube. But here the imperfect analogy became exhausted; for, while the powers of a number are infinite, extension in space admits of no other distinctions than the three above mentioned. Hence, by an expedient incongruous in idea as well as unwieldy in practice, the higher powers were designated as if repetitions of the geometrical dimensions. This will be seen from the following table, which is given by Napier in his earlier work on Algebra, p. 92.

√♀	Radix quadrata.
√ℭ	Radix cubica.
√♀♀	Radix quadrati quadrata.
√ß	Radix supersolida.
√♀ℭ	Radix quadrati cubica.
√ſß	Radix secunda supersolida.
√♀♀♀	Radix quadrati quadrati quadrata.
√ℭℭ	Radix cubici cubica. Et sic de cæteris in infinitum.

Here the signs are just an initial abbreviation of the words, and scarcely less unmanageable in practice. Napier had subsequently become sensible of the defect both of this notation and nomination. The numbers, or rather quantities, particularly dependent upon the nomenclature in question, are surds, which cannot be expressed in effable numbers, whether integers or fractions. Accordingly, in his Arithmetic (p. 11), after showing, for instance, that the surd quantity, called the cube root of 9, lurks between the numbers 2 and 3, Napier adds, "But geometricians, studious of greater accuracy, prefer the mode of prefixing the sign of the index to the radicate itself, instead of including the root between two terms. For example, they note the tripartient root of 9 thus, √ȼ9, which they pronounce the cube root of 9. I, however, note it thus, ∟9, and call it the tripartient root of 9; of which signs I shall more fully speak in their proper place. Hence arises geometrical or concrete numbers, commonly called irrational and surd." This was tantamount to naming the cube root of 9 (or whatever power be taken) the third root, and noting it by an equivalent numeral index; for we shall immediately see that the symbol ∟ was an invention of Napier's, to represent the numeral 3 used for a special purpose.

Along with a nomination expressive of the index, or quality of the root, it was Napier's object to establish a notation which, in like manner, would immediately suggest to the eye, not merely that a surd root was taken, but what number of root it was; and this by symbols entirely new and unappropriated. Since his day, when roots were to be expressed by radical signs, instead of by the number itself evolved, the expedient devised was to retain the contracted R, as the initial of the word root, and to place within it a small numeral, as the index of the quality of the root. Thus $\sqrt[3]{10}$ expresses the cube root, or, as Napier would have called it, the tripartient or third root, of the number ten considered as a radicatum or power. This notation,—which, it will be observed, gets rid of the unwieldy expedient of repeating the initials ℨ, 𝔠, and ß, for square, cube, and supersolid, and presents to the eye the number of evolutions requisite to obtain the particular root,—was not invented until many years after Napier's death. Now, the notation to which he refers, in the sentence quoted above, was so devised as at once to express that a root, of that number to which it was attached, was the quantity taken, the particular number or index of that root, and the fact that the root was a surd root, or concrete quantity. He took this simple combination of equal lines  intersecting each other at equal intervals. In the nine compartments, which this figure presents, he inserted the nine numerals,—for the sake, he says, of assisting the memory. From the natural arrangement of the figures within, it is easy to remember the number appertaining to each compartment. Separate the compartments thus, , and still the memory will retain the relation of each to the original form, even when the numerals are withdrawn. By this simple process there is actually obtained an equivalent for the nine significant digits of Arithmetic, susceptible of the same combinations, yet perfectly distinct in character. Napier's unit of this system is ⌋, and when the root which he wishes to represent is beyond the original value of the highest of his digits, which is ┌, corresponding to 9, he obtains the tenth value by moving his unit one step to the left, indicating the move by placing a circle on the right of the significant digit, thus, ⌋°. In like manner, his unit in its first place combined with the same advanced a step, thus ⌋⌋, indicates the root of the 11th division, or undecupartient root, as he named it. Thus having invented a new set of digits equivalent to 1, 2, 3, 4, 5, 6, 7, 8, 9, he submitted them to the same law of progression as in ordinary arithmetic.

Upon comparing the fragment of Geometrical Logistic with the first book of his Algebra, which Napier calls the nominate part of Algebra, it will be obvious, that the latter had been framed before the conception of the former. Having composed two books of Algebra, he had extended his plan so as to embrace the whole art of Logistic; and the nominate part of his Algebra would probably have merged in his third book called Geometrical Logistic, while the second book of Algebra, which he called the positive or cossic part, in which unknown quantities are treated of under fictitious representations, would have formed the fourth and last book of his great digest of Numbers. It is quite consistent with the nature of the subject to suppose, that the reason why Napier's book of the Logistic of concrete quantities by concrete numbers terminates abruptly, and why no other fragment of it could be found, was, that at this stage of his speculations, he had set himself to the invention of Logarithms, which may indeed be considered as the grand result of the Logistic of concrete quantities by concrete numbers. These, in all probability, were the very speculations which led him to that achievement, of which Playfair has said, that, "at a period when the nature of series, and when every other resource of which he could avail himself, were so little known, his success argues a depth and originality of thought, which, I am persuaded, have rarely been surpassed."

Napier's device for the notation of surds is of itself sufficient to prove that his invention of the Logarithms was subsequent to such speculations; for the mind which accomplished the Logarithms, and compounded that name for them, could scarcely fail very soon to perceive that the radical signs, expressive of a surd root, were to be entirely superseded by fractional exponents. Dr Hutton committed a strange anachronism, and one which does injustice to the genius of Napier, in the following passage:—"The notation of powers and roots, by the present mode of exponents, has introduced a new and general arithmetic of exponents or powers; for hence powers are multiplied by only adding their exponents, divided by subtracting the exponents, raised to other powers, or roots of them extracted, by multiplying or dividing the exponent by the index of the power or root. So $a^2 \times a^3 = a^5$, and $a^{\frac{1}{2}} \times a^{\frac{1}{4}} = a^{\frac{3}{4}}$; $a^5 \div a^3 = a^2$, and $a^{\frac{3}{4}} \div a^{\frac{1}{2}} = a^{\frac{1}{4}}$; the second power of a^3 is a^6, and the third root of a^6 is a^2. This algorithm of powers led the way to the invention of Logarithms, which are only the indices or exponents of powers; and hence the addition and subtraction of Logarithms answer to the multiplication and division of numbers, while the raising of powers and the extracting of roots is effected by multiplying the logarithm by the index of the power, or dividing the logarithm by the index of the root."—Math. Dict. If the algorithm of which Dr Hutton here speaks had belonged to science before Napier invented the Logarithms, Playfair's eulogy of him, quoted above, would not have been merited; for the arithmetic of powers and exponents might have disclosed the Logarithms even to an ordinary mathematician. The more accurate statement is, that the invention of Logarithms led the way to the algorithm of powers. Napier,—writing, be it remembered, some time in the sixteenth century, before Vieta, Harriot, Girard, and Oughtred, and when Algebra was not cultivated at all in this country,—had no assistance from the algebraic refinement of working known quantities by means of other symbols than the significant digits, or of expressing powers by means of small letters instead of numerals. He did not, for instance, consider $aaaa$ as the quadruplicatum (to use his own term) of any number a; still less did he consider the same quantity in this form, a^4; he had neither the literal notation of powers, nor the numeral notation of indices; for although, in explaining their genesis, he named the indices, one, two, three, &c., and also noted and arranged them above the powers by the numerals 1, 2, 3, &c., yet he did not systematically attach them to the root for the expression of the power. To have done so would have been to anticipate the notation of Descartes, whose epoch is 1637, twenty years after Napier's death. But, while he had not these advantages to lead him to the Logarithms, the arithmetic of exponents was so obviously to be deduced from that invention, that, having achieved the Logarithms, he could not fail immediately to perceive the application of fractional indices for the expression of roots,—and, consequently, that his proposed notation of surds, in his Geometrical Logistic, was comparatively crude and useless. This fact, which it would occupy too much space fully to illustrate here, may be offered as internal evidence, afforded by the manuscripts now printed, that they are studies which must have preceded the conception of his great invention.

That which may truly be said to have led Napier to the invention of Logarithms, is the profound view he took of the Logistic of concrete quantity, after having thoroughly mastered that of discrete quantity. The properties of what we may now term the Logarithms of discrete ratios, namely, the series of natural numbers taken to enumerate the steps or terms of any geometrical progression represented by integers, had been known since the days of Archimedes. But the desideratum was, to prove that the infinite series of natural numbers, 1, 2, 3, 4, 5, 6, &c., might be viewed as in a geometrical progression, and then to discover the indices of all these numbers, considered as powers of some given number,—which would disclose the corresponding arithmetical series, and the Logarithms par excellence. Less profoundly considered, the idea that the arithmetical series, or progression by equidifference of the natural numbers, could be viewed as a geometrical series, or progression by multiplication, seemed a contradiction in terms. Napier commences by demonstrating that the progressive increase, or decrease, of a concrete or continuous quantity may be conceived to be generated by a motion in space, so regulated, in respect of the velocity, as to generate a continuous geometrical progression, of infinitely small ratios of magnitude, various terms of which might be represented, or infinitely nearly so, by the series of natural numbers, which thus all become terms of a geometrical progression. In like manner, he demonstrated the genesis of a corresponding arithmetical progression by a simultaneous motion, the velocity of which, being equal throughout and not increasing or decreasing, generated magnitudes in an arithmetical progression, which, at any given point of the progress, might be represented by a numerical expression that would serve for the index, or number of the ratios (logarithm), of the corresponding point in the simultaneous geometrical motion. His idea of motion, thus taken to generate proportional magnitudes, was analogous to the law of the Arabic notation in discrete numbers, where the significant digit may be conceived to generate a decuple progression, by travelling in a line from right to left. There are various circumstances connected with the doctrine of Logarithms, unnecessary to be considered here, showing its natural affinity to the system of the Arabic notation, which in fact only became perfectly developed when the Logarithms were invented; and it is singular that, among the mathematical stores of those distant climes from which was derived the refined and powerful notation in question, and throughout the long ages of its operation in the hands of genius, not a trace can be discovered of a conception of that development of the scale which Napier accomplished in the Logarithms and the arithmetic of Decimal fractions. The Chinese are said to have laid claim to the invention; but the splendid copy of the Logarithms which issued from the imperial press of Pekin, contains certain errors which have been recently discovered in the European tables, previously published.

Napier's mode of demonstrating the Logarithms, by the motion of points (fluxu puncti) generating two lines—the one "when the point describing the same goeth forward equal spaces in equal times or moments," and the other (which, to facilitate his operations, he took in the decreasing ratio, having complete command of the arithmetic of negative quantities,) "when the point, describing the same in equal times, cutteth off parts continually of the same proportion to the lines from which they are cut off,"—was afterwards adopted by Sir Isaac Newton, in similar terms, to illustrate his own discovery of Fluxions. The accidental circumstance, or first idea, which may have led any great inventor into the path of his distinction, can rarely be discovered. I have endeavoured to illustrate Napier's progress to the Logarithms through his mathematical studies now published. Newton himself, in his treatise of the Quadrature of Curves, announces the method that led him to his great discovery of Fluxions. "I consider," he says, "mathematical quantities in this place not as consisting of very small parts, but as described by a continued motion. Lines are described, and therefore generated, not by the opposition of parts, but by the continued motion of points; superficies by the motion of lines; solids by the motion of superficies; angles by the rotation of the sides; portions of time by a continual flux; and so in other quantities. These geneses really take place in the nature of things, and are daily seen in the motion of bodies. And after this manner the ancients, by drawing moveable right lines along immoveable right lines, taught the genesis of rectangles. Therefore, considering that quantities which increase in equal times, and by increasing are generated, become greater or less according to the greater or less velocity with which they increase and are generated, I sought a method of determining quantities from the velocities of the motion or increments with which they are generated; and calling these velocities of the motions or increments Fluxions, and the generated quantities Fluents, I fell by degrees upon the method of Fluxions, which I have made use of here in the quadrature of curves, in the years 1665 and 1666." It is remarkable, that, instead of merely referring to the ancients in this passage, Newton had not rather said,—'And after this manner Napier, by drawing a moveable point along a right line, taught the genesis of Logarithms; and when I speak of quantities becoming greater or less according to the greater or less velocity with which the increase and decrease are generated, and of determining quantities from the velocities of the motions or increments with which they are generated,—and when I call these velocities of the motions or increments Fluxions,—I avail myself of Napier's demonstration, I adopt his language,—and even his very expressions, "fluxu," and "incrementi aut decrementi."' Newton may have closely studied Napier's published works, or he may never have seen them. The first idea is suggested by the above remarkable coincidences of thought and expression; and also by the fact that Newton's Commentaries on Scriptural prophecy is little else than a less elaborate repetition of Napier's. The other idea, however, is after all the more likely, from this circumstance, that, throughout the voluminous published works of Newton, mathematical and theological, no notice of Napier is to be found. Yet the great lever with which Newton worked was the Logarithms; and the Binomial Theorem, says Baron Maseres, "is so very closely connected with the subject of Logarithms, as to be the foundation of the best methods of computing them."

In Napier's Arithmetic, (p. 50,) which Newton certainly never saw, there is a figurate diagram, worthy of especial notice, as being an unknown anticipation of the Arithmetical Triangle of the celebrated Blaise Pascal; presented, however, in a far more beautiful form than that of the French mathematician. The properties of this triangle are so intimately connected with the Binomial Theorem, that Bernoulli, on that account, somewhat rashly claims for Pascal the merit of the invention. "Nous avons trouvé," he says, "ce merveilleux theorème aussi bien que M. Newton, d'une manière plus simple que la sienne. Feu M. Pascal a été le premier qui l'a inventée." Maseres, who republished Pascal's mathematical works, says of them,—"These works are so full of genius and invention, that I thought I should do a service to the mathematicians of Great Britain by republishing them in this collection. Some of them, and more especially his Arithmetical Triangle, have a considerable connexion with Logarithms, by affording a good demonstration of Sir Isaac Newton's

Binomial Theorem, in the case of integral and affirmative powers, which is of great use in the construction of Logarithms." But Napier had anticipated Pascal in this invention, which, however, only received its most valuable algebraic application, dependent as that is upon the modern exponential notation, at the hands of Sir Isaac Newton.

There are other indications, in the manuscripts now submitted to the public, of an inventive genius in Numbers, far before the times in which Napier wrote. Professor Playfair, in his Dissertation on the Progress of Mathematical and Physical Science, when speaking of Albert Girard,—a Flemish mathematician, whose principal work, Invention Nouvelle en Algèbre, was printed in 1669,—observes, "He appears to have been the first who understood the use of negative roots in the solution of geometrical problems, and is the author of the figurative expression which gives to negative quantities the name of 'quantities less than nothing,'—a phrase that has been severely censured by those who forget that there are correct ideas which correct language can hardly be made to express. The same mathematician conceived the notion of imaginary roots, and showed that the number of the roots of an equation could not exceed the exponent of the highest power of the unknown quantity." Sir John Leslie, who continued Playfair's Dissertation, also comments upon Girard's introduction of the phraseology, 'quantities greater and less than nothing,' and upon his discovery and nomenclature of impossible quantities. It is remarkable that Playfair and Leslie, who have expressed great admiration for the genius of Napier, had not looked at or studied his published works. Napier's command of the arithmetic of + and —, the signs of positive and negative quantities, (or as he more properly phrased it, abundant or abounding, and defective quantities,) was of great assistance to him in realising his conception of Logarithms; and, in the first chapter of his Canon Mirificus, he says, (to quote from the English translation of 1616, revised by himself,) "Therefore we call the logarithms of the sines abounding, because they are always greater than nothing, and set this mark + before them, or else none; but the logarithms which are less than nothing we call defective or wanting, setting this mark — before them." Dr Horsley had fallen into the same mistake of attributing the origin of this phraseology to Girard. But as for the quantity called impossible, these authors had not the means of knowing that Napier was the person who first discovered the use of it in mathematics, and that he had somewhat exultingly recorded the fact. Having laid the foundation, (p. 19,) by an exposition of the arithmetic of + and — not inferior to Euler's, he makes the announcement, in his Logistica Geometrica, which I shall here translate. "Seeing, therefore, that a surd uninome may be the root either of an abounding or of a defective number, and that its index may be either even or odd, from this fourfold cause it follows, that some surds are abounding, some defective, some both abounding and defective, which I term 'gemina;' some neither abounding nor defective, which I call 'nugacia.' The foundation of this great Algebraic secret I have already laid in the sixth chapter of the first book; and, though hitherto unrevealed by any one else, so far as I know, the value of it to this art, and to Mathematics in general, shall presently appear."

There can be no doubt that by "nugacia" Napier means the impossible quantity, and that he was the very first to conceive the idea, and to propose its use, in the arithmetic of surds and the theory of equations. He explains precisely its nature, and gives rules for its notation, and applies to it the startling designations of "a quantity absurd and impossible, nonsensical and signifying nothing." He shows that all roots, whether abounding or defective, (or, as he also terms it, greater or less than nothing,) when multiplied to an even index produce an abounding radicate. Thus, take for the root $+2$, and let it be multiplied to the index 4, (in other words, raised to the fourth power,) and the radicate will be $+16$; in like manner, raise —2 to the index 4, and the radicate will be +16; so —2 and +2 are equally the quadripartient root of 16. Hence, says he, every abounding radicate, having an even index, has two roots. These were what he called geminal. But, he adds, if roots both abounding and defective belong to the abounding radicate with an even index, then there is no root of any kind left for the same radicates having the defective sign prefixed. Hence, for example, ⊐—16 (in modern notation, $\sqrt[4]{-16}$) is an impossible quantity. The "great emolument" which Napier expected to bestow upon Mathematics by this ghost of a quantity, can only be well understood by profound mathematicians. Euler has devoted a chapter to impossible or imaginary quantities, which he concludes with these observations: "It remains for us to remove any doubt which may be entertained concerning the utility of the numbers of which we have been speaking; for those numbers being impossible, it would not be surprising if they were thought entirely useless, and the object only of an unfounded speculation. This, however, would be a mistake; for the calculation of imaginary quantities is of the greatest importance, as questions frequently arise of which we cannot immediately say whether they include any thing real and possible or not; but when the solution of such a question leads to imaginary numbers, we are certain that what is required is impossible."

But it appears, that, when Napier had broken ground upon the great field of equations, and with a vigour and command of the subject which obviously was about to anticipate the conquests of many of his illustrious successors, he was arrested in that progress by another idea. The stars were becoming too many for Tycho and Kepler; so he promised them the Logarithms. His mind had already penetrated the Arabic system in every direction. He had already orderly set down how the various operations of number and quantity gradually unfold,—how the vast fabric produces itself, growth after growth, every rule the parent of another, and the whole intimately related, in all its parts, in endless generations of Numbers. Having traced the genealogy of Numbers upwards from nothing, he had taken zero as the pivot for a reciprocal scale, and unfolded the logistic of "quantitates minores nihilo," and "quantitates impossibiles et nihil significantes." Then he had viewed unit as broken into another infinite scale, and had reduced to order all the operations of Arithmetic upon its fractions. It remained to condense the vast system into greater simplicity and power, and also to display unit as the pivot for a reciprocal play of the Arabic scale, as he had done by zero. This he accomplished by his invention and calculation of Logarithms. Moreover, in his brilliant performance of that promise to Tycho, he first applied—before either Newton or Leibnitz—

the great doctrine of Variable Quantity, and thus he not merely afforded materials for, but, to this extent, had actually anticipated, the algebraic conquests of Newton. Newton is well designed the Prince of Mathematicians. Napier is the King of Numbers.

It adds not a little to the interest with which these manuscripts must be viewed, that they had been written out for Henry Briggs, the satellite of Napier, and to whom Napier himself refers as " amico mihi longe carissimo." Briggs was ten years younger than Napier, and, about the year 1596, had been appointed Professor of Geometry, in the munificent establishment founded by Sir Thomas Gresham, where he devoted himself particularly to Astronomy, and became known to the most celebrated men of his day. In a letter to Archbishop Usher dated August 1610, Briggs says, " Concerning eclypses, you see by your own experience that good purposes may in two years be honestly crossed; and therefore, till you send me your tractate you promised the last year, do not look for much from me; for if any other business may excuse, it will serve me too. Yet I am not idle in that kind, for Kepler hath troubled all, and erected a new frame for the motions of all the seven upon a new foundation, making scarce any use of any former hypotheses; yet I dare not much blame him, save that he is tedious and obscure, and at length coming to the point, he hath left out the principal verb—I mean his tables both of middle motion and prosthaphæreseon—reserving all, as it seemeth, to his Tab. Rudolpheus, setting down only a lame pattern in Mars; but I think I shall scarce with patience expect his next books, unless he speed himself quickly." But ere those long promised Tables were published, the Logarithms appeared, and Kepler immediately remodeled his work upon this new chapter in science. Briggs, however, was the first to catch fire at the discovery. In another letter to Usher, dated Gresham House, 10th March 1615, after speaking of the Arabic versions of the Greek philosophers, he adds, "Napper, Lord of Markinston, hath set my head and hands a-work with his new and admirable Logarithms: I hope to see him this summer, if it please God; for I never saw book which pleased me better, or made me more wonder: I purpose to discourse with him concerning eclipses, for what is there which we may not hope for at his hands?" Dr Thomas Smith, the biographer of Usher and Briggs, has painted in vivid colours the state of excitement into which the latter was thrown by the Canon Mirificus. He says that Ursin, Kepler, Frobenius, Batschius, and others, received it with great honour, but none more so than Briggs. "He cherished it as the apple of his eye; it was ever in his bosom, or his hand, or pressed to his heart; with greedy eyes, and mind absorbed, he perused it again and again. In his study, or in his bed, his whole thoughts were bent upon illustrating it, and bringing it to the last stage of perfection; he considered that his study could not be more fruitfully, or beautifully, or gloriously bestowed than upon this most illustrious discipline; for he regarded all other work as idleness; it was the theme of his praise in familiar conversation with his friends, and ex cathedrâ he expounded it to his disciples."

Mr Hallam, in his Introduction to the Literature of Europe, recently published, has not failed to give a prominent place to Napier. But it is to be regretted that he had not derived his information, relative to Napier's works, from accurate sources; and I may take this opportunity of correcting some mistakes into which he has fallen. Speaking of the Logarithms, he says, "This Napier first published in 1614, with the title Logarithmorum Canonis Descriptio, seu Arithmeticarum Supputationum Mirabilis Abbreviatio. He died in 1618; and in a posthumous edition, entitled Mirifici Logarithmorum Canonis Descriptio, 1618, the method of construction, which had been at first withheld, is given; and the system itself, in consequence perhaps of the suggestion of his friend Briggs, underwent some change." But the real title of Napier's original publication is, "Mirifici Logarithmorum Canonis Descriptio; ejusque usus in utrâque Trigonometriâ, ut etiam in omni Logisticâ Mathematicâ, amplissimi, facillimi, et expeditissimi Explicatio." It is of importance to preserve the true title, because it bears evidence on the face of it, that, although (probably in reference to Tycho) he specially adapted his system to Trigonometry, he was perfectly aware of its value to the whole analytic art—"in omni Logisticâ Mathematicâ." Napier died in 1617. Nor am I aware of any such posthumous edition of the Canonis Descriptio, dated in 1618. Probably this is an inaccurate reference to the distinct work of Napier's, already mentioned, published in 1619, under the joint editorship of Robert Napier and Henry Briggs, and along with which appeared a reprint of the former work. The title of the Constructio, the most beautiful of Napier's works, is,—"Mirifici Logarithmorum Canonis Constructio; et eorum ad naturales ipsorum numeros Habitudines; una cum Appendice de aliâ eâque præstantiore Logarithmorum specie condendâ. Quibus accessere Propositiones ad triangula spherica faciliore calculo resolvenda. Una cum Annotationibus aliquot doctissimi D. Henrici Briggii in eas et memoratam appendicem." In the Memoirs of Napier I had called attention to the fact, that his works were more frequently referred to than examined; and that even Playfair, Leslie, and Dr Horsley were not acquainted with the original institute of the Logarithms. M. Biot, accordingly, when reviewing the Memoirs, had sought out the original editions of Napier's works, and he appears to have been much struck with their power. Referring to the development, and logarithmic application, of the algebraic calculus since Napier's time, he adds,—"I declare, however, to the honour of Napier, that these means produce nothing which cannot be very easily attained by his own method; and if, as it is natural to suppose, this assertion may seem to our analysts something more than rash, I hope presently to afford unanswerable proofs of its accuracy. But to form this just idea of Napier's operations, it is necessary to study his own works, especially his second work, in which he unfolds his method,—and not to rely upon extracts." In reference to the above, M. Biot prefixes to his review an investigation which he entitles, "Analyse et restitution de l'ouvrage original de Napier, intitulé, Mirifici Logarithmorum Canonis Constructio."

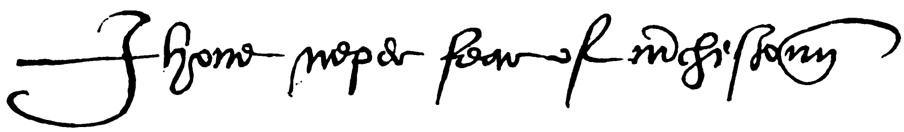
In the title-page of the Constructio, of which Briggs himself was one of the editors, will be found an allusion to Napier's improvement of his own system; and in the preface, the improvement is declared to have originated with Napier himself, nor is there any allusion to a suggestion from Briggs. In his letter to the Chancellor, Napier also mentions the improvement he intended, without any reference to Briggs. Indeed, the original edition of his Canon contains an allusion to the same, before Briggs knew of the Logarithms. Mr Hallam's doubt on the subject, which he thus repeats in another passage,—"It is uncertain from which of them the change in the form of Logarithms proceeded,"—would not have occurred had he examined these works. We have the assurance both of Napier and Briggs that there is no dubiety about the matter. Indeed, Briggs has recorded the exact state of the case in an interesting statement prefixed to his Arithmetica Logarithmica, published in London in 1624, and which I shall here translate:—"That these Logarithms differ from those which that illustrious man, the Baron of Merchiston, published in his Canon Mirificus, must not surprise you. For I myself, when expounding publicly in London their doctrine to my auditors in Gresham College, remarked that it would be much more convenient that 0 should stand for the logarithm of the whole sine, as in the Canon Mirificus; but that the logarithm of the tenth part of the same whole sine, that is to say, 5 degrees, 44 minutes, and 21 seconds, should be 10,000,000,000. Concerning that matter I wrote immediately to the author himself; and, as soon as the season of the year, and the vacation-time of my public duties of instruction permitted, I took a journey to Edinburgh, where, being most hospitably received by Napier, I stuck to him for a whole month. But, as we held discourse concerning this change in the system of Logarithms, he said that for a long time he had been sensible of the same thing, and had been anxious to accomplish it, but had published what he had already prepared, until he could construct tables more convenient, if other weighty matters, and the frail state of his health would suffer him so to do. But, he conceived, the change ought to be effected in this manner, that 0 should become the logarithm of unity, and 10,000,000,000 that of the whole sine. This I could not but admit was by far the fittest modification; so, casting aside what I had already prepared, I commenced under his advice to bend my mind to the calculation of these tables; and in the following summer I again took journey to Edinburgh, where I submitted to him the principal part of those tables which are here published; and I was about to do the same even the third summer, had it pleased God to have spared him to us so long."

It is not for the sake of asserting his right to what, at all events, is a simple derivative idea, that these proofs are referred to. It is because the dubiety expressed by Mr Hallam might infer that Napier had done injustice to Briggs, whose genius he appreciated, and whose friendship and admiration he cordially returned. "I will acquaint you," says Lilly, in his Life and Times, "with one memorable story related unto me by John

Marr, an excellent mathematician and geometrician, whom I conceive you remember. He was servant to James I. and Charles I. When Merchiston first published his Logarithms, Mr Briggs, then reader of the astronomy lectures at Gresham College in London, was so surprised with admiration of them, that he could have no quietness in himself until he had seen that noble person whose only invention they were. He acquaints John Marr therewith, who went into Scotland before Mr Briggs, purposely to be there when these two so learned persons should meet. Mr Briggs appoints a certain day when to meet at Edinburgh, but, failing thereof, Merchiston was fearful he would not come. It happened one day as John Marr and the Lord Napier were speaking of Mr Briggs—'Oh! John,' saith Merchiston, 'Mr Briggs will not come now.' At the very instant one knocks at the gate; John Marr hasted down, and it proved to be Mr Briggs, to his great contentment. He brings Mr Briggs into my Lord's chamber, where almost one quarter of an hour was spent, each beholding other with admiration, before one word was spoken. At last Mr Briggs began,—'My Lord, I have undertaken this long journey purposely to see your person, and to know by what engine of wit or ingenuity you came first to think of this most excellent help unto Astronomy, viz. the Logarithms; but, my lord, being by you found out, I wonder nobody else found it out before, when, now being known, it appears so easy.' He was nobly entertained by the Lord Napier; and every summer after that, during the Laird's being alive, this venerable man went purposely to Scotland to visit him."

An engraving of the old Castle of Merchiston, in its primitive state, where this symposium was held, illustrates the present volume. It is as nearly as possible a fac-simile of a sketch by "Grecian Williams." The portrait of Napier is from the original belonging to the Lord Napier, and which has always been in possession of the Napier family. It is engraved now for the first time. The fac-simile of the autograph is from a letter of Napier's to his father, dated from the residence of his father-in-law, Stirling of Keir. An odd idea has arisen that he sometimes designed himself "Peer of Merchiston." In the beautiful and ably illustrated pictorial edition of the works of Shakspere, now publishing, the following ingenious note is founded on the error:—"A remarkable illustration of our belief, that Peer and Fere were cognate terms, and that a Fere or Fear was one holding of the Crown in Fee, is furnished by the title which the famous John Napier attached to his name. At the end of the dedication to his 'Plain Discovery of the whole Revelation of St John,' in the edition of 1645, Napier signs himself ' Peer of Merchiston.' Mr Mark Napier, in the Life of his great ancestor (1834), says, that the true signature is Fear of Merchiston, and that Fear means that he was invested with the Fee of his paternal barony. Peer might have been a printer's or transcriber's substitution for Fear, or Fear might have been rejected by Napier for the more common word Peer."—(Illust. Henry IV., Act I.)

But there can be no doubt, that, in the instance relied upon, Peer was simply a misprint for Feer. That Napier had so written his signature for the edition of 1645, the only instance of its occurrence, is put out of the question by the fact, that, before that date, he had been dead for eight-and-twenty years. The following is a perfect fac-simile of his signature to a lease, dated at his place of Gartnes, 23d April, 1584:—



MARK NAPIER.

11, Stafford Street,
November 1, 1839.

DE ARTE LOGISTICA.

THE

BARON OF MERCHISTON HIS BOOKE OF ARITHMETICKE AND ALGEBRA.

FOR MR HENRIE BRIGGS

PROFESSOR OF GEOMETRIE

AT OXFORDE.