[p. 21.]

ON THE AMAZING CANON OF LOGARITHMS with their outstanding use in trigonometry.

BOOK II a.

Chapter 1.

Since geometry is the art of measuring proposed quantities with complete accuracy, and as a figure (at least on the strength of geometrical considerations) can be decomposed into the sides of triangles, or by further triangulation into smaller triangles, then a figure is composed of triangles with some of its angles and sides measured, and all the other triangular parts are to be found, and from which the extent of the figure can be found. Therefore it is clear that the arithmetical solution of any geometrical question depends on the principles by which triangles are solved.

The triangle is to be either planar or spherical.

Concerning plane triangles : Prop. 1.

Prop. 1. *The three angles of a plane triangle add up to two right angles.*

Thus with two angles given, take the sum of these from 180 degrees, and the third angle is found. Likewise with one taken from 180, there remains the sum of the other two angles. [p. 21.]

A triangle either has a right angle or the angles are oblique [i. e. slanting, and so includes oblique and acute angles in modern terms].

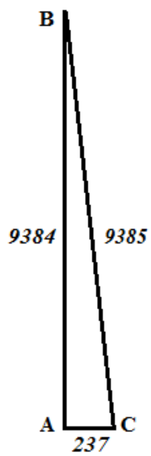
In right-angles triangles the sides that embrace the right-angle are called the legs, and the hypotenuse is the side that the right-angle subtends.

Prop. 2. *In a right-angles triangle, the logarithm of a leg is equal to the sum of the logarithms of [the sine of] the angle to that and the logarithm of the hypotenuse.*

Since it is apparent from the principles of trigonometry, each side is to the sine opposite to it, as the hypotenuse is to the total sine: and (by Prop.5 of Ch.2, Book 1) of these four numbers in proportion, the logarithms of the second and third is equal to the logarithms of the first and the fourth : but the logarithm of the fourth is equal to 0 nothing (by the coroll. 6 def., Ch.1, Book 1) Thus (from above) the logarithm of the side is equal to the sum of the logarithm of the [sine of the] angle that it subtends and the logarithm of the hypotenuse.

Corol. *Hence for any two given, of the hypotenuse and side and the angle that it subtends, the third side and hence all of the remaining parts of the right-angles triangle can be found.*

Since indeed these three, with the total sine make up four numbers in proportion, it is clear that the fourth of these, whatever it is, can be put in place and found according to problem 3, Ch. 5 of Book 1.



As in triangle ABC, with the right-angle at A, the hypotenuse BC 9385 is given, with the leg [or side] AB 9384. The sizes of the other angles C and B is sought. Therefore from the logarithm AB, 635870 – 000, take the logarithm BC, 634799–000. There remains 1071, the logarithm of [the sine of] the angle C, to which there corresponds the angle 89 deg. $9\frac{3}{4}$ min. in the table for the angle C, and from the opposite for the complement, obviously 0 deg. $50\frac{1}{4}$ min. for the angle B. [p. 23.]

[Thus, no interpolation between neighbouring values in the tables has been done for the sides AB and BC; however, these particular numbers have been chosen to have the smallest difference in the logarithms, for which a difference Δ can still be found in the table, and in which case an interpolation has been done between the values for 89 deg., 9 min, for which the log is 1101; and 89 deg.,10 min, for which the log is 1058 :

no.	Δ num.	log.	Δ log.
9383925 AB	-	635870	-
9384930 BC	1015	634799	-1001
9385934	1004	633729	-1071
		1071	

Hence, the differences in the values of these numbers and the logs are almost the same, and give accurate enough results.]

And vice versa if the angle C is given together with the leg of the right-angled triangle AB, and the hypotenuse BC is sought.

From the logarithm AB, 635870 – 0000 take the logarithm of the [sine of the] angle C. 1071, and there comes about 634799 – 0000, the Logarithm of the hypotenuse sought, BC, 9385.

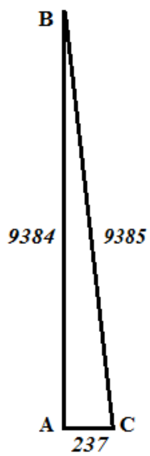
In the third case if with BC and the angle C given, AB is sought : then add the logarithm of BC, 634799 – 0000, to 1071, the logarithm of the angle C, and 635870 – 000 is produced, the Logarithm of the number 9384 that corresponds to the leg AB required. Neither is the remaining leg AC required to be known from the angle B (which is the complement of the angle C.) now known to be given. Thus all the parts of this right-angled triangle are now known.

Prop. 3. *In a right-angled triangle the logarithm of any leg is equal to the sum from the differential of the opposite angle and the logarithm of the remaining leg.*

Since from the common teachings of triangles it is agreed [*e.g.* Pitiscus, Book III], that each leg is in the same ratio to the tangent of its own opposite angle, as the remaining leg to the total sine : and since (by Prop.5 of Ch.2 of Book1) from these four proportionals , the logarithms of the means (that is, of the differential [or tangent] of the angle, and the logarithm of the embracing leg) are equal to the logarithms of the same subtending leg, and the logarithm of the total sine, (which is zero or 0) ; the logarithm of the leg is equal to the sum as above. [*i. e.* $a = b \tan A$, where C is the right-angle, from the sine rule.]

Coroll. *Thus from the legs of the right-angle, and from the angle opposite these, with any two given, the third (by this proportion) and hence the rest of all the parts of the right-angled triangle become known (from the preceding).*

Since these three with the total sine constitute four numbers in proportion, it is clear, that whatever the fourth one of these is, it can be put in place in the proportionality and found, by problem 3, Ch.5 of Book 1. [p. 24.]



As of the preceding triangle ABC, with A taken as the right-angle with the sides AB, 9384, and AC, 137. The angle B is sought [from $AC = AB \tan B$,] . From the logarithm AC, 42924534 – 000, take 635870 – 000, the logarithm of AB, and the differential [tangent] of the angle B is found 42288664 is found, 0 g. 50' which is requires. But if the leg AC, 137 is given : and the angle B 0 g. 50' , the leg AB is found by taking 42288664, the differential of the angle B from the logarithm AC, which is 42924534 – 000. Hence indeed 635870 – 000 is come upon, and this is the Logarithm of the number 9384, which is the leg sought of A B. In the third place, from the given leg AB, 9384, and from the angle B, 0 g. 50' : in order that the leg AC may be obtained : add 635870 – 000, the Logarithmus of the leg AB to 42288664, the differential of the angle B, and 42924534 – 00 is arrived at, the logarithm of 137 of AC sought. Moreover the hypotensue can be obtained from the preceding proposition. Also the angle C is apparent. since it is the complement of the angule B, now known. And thus in this way, and from any side given in advance, from any other part of the right-angle given all the other parts can thus become known.

You have now all the skills you need to solve right-angles triangles.

We now proceed to consider other oblique–angled triangles.

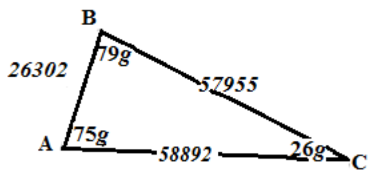
Chapter II.

Prop. 4. *In any triangle, the sum found from the logarithms of [the sine of] any angle you please and of a side including it, is equal to the sum of the logarithms of the side and of the [the sine of] angle opposite to those.*

Since the sine of every angle to the opposing side is in the same ratio for all the angles in a triangle: and thus the product formed from the right sine of any angle, and with any side including that, is equal to the product of the side subtending the first angle and the sine of the angle subtended. [p. 25.] Thus (by Prop.5 , Ch.2, Book 1) the sum from the logarithms , etc is equal, as above. [The familiar sine rule.]

Coroll. *Thus from any two angles of a given kind, and with their subtended lines, if three are given, any fourth, and all the remaining parts of the triangle will be known.*

Indeed of these four proportional numbers, any sought can be put in the fourth place, and found by Prob.3, Ch.5, Book 1.



Let any oblique-angled triangle ABC be given, with AB, 26302; BC, 57955; and the angle C, 26 degrees : The angle A is to be found, which can be obtained thus : Add 5454707 – 00 , the logarithmic of BC to 8246889, surely the logarithm of [the sine of] C, 26 degrees, and they make 13701596 – 00. Hence take the logarithm AB, which is 13354921 – 00, and there remains 346675, the logarithm of [the sine of] 75 degrees and a little more, obviously of the angle A sought, if A were acute : otherwise 105 degrees (by sections 1 and 2, Ch. 3, Book 1) if it were obtuse. On the other hand, if now the angle A of 75 degrees is given, and the angle C and the side BC as above : and AB is sought [for AB =BC sin C / sin A], then add 5454707 – 00 the logarithm of BC to 8246889 , the logarithm of the angle C, and the sum becomes, as above, 13701596 – 00, from which is taken 346675 the logarithm of the angle A, giving 13354921 – 00, the logarithm of the side AB, and of the number sought 26302. Now you have the angle B of 79 degrees, with the angles A , 75 degrees and C, 26 degrees, by Prop. 1 of this Book. From which now, in the same way the side AC opposite to the angle B is found, 58892, just as recently that side AB was known opposite the angle C. Now the side AB opposite the angle C can become known. Thus, all the angles and sides of the triangle become apparent.
[It is convenient for us to use a spreadsheet to reproduce the true Napier's Logs, as required; this is necessary to check the accuracy of the original calculations, and of course there *had* to be a small error in a generating table that limited the accuracy of the original tables !]

	Orig. No.'s	Orig. Log. (nearest)	Given as	True value
BC	5795(183)	5455577	5454707 – 00	5455003.339
sin C	4383712	+ 8246889	$C = 26^0$	8246892.393
AB	2630(312)	− 13354817	13354921– 00	-13355482.62
sin A		346683	346675 (found)	346413.112
		$A = 74^0$, 59'	$A = 75^0$	$A = 75^0$

[Translator's Digression.

Thus, Napier uses the sine rule to write sinA = BC sinC / AB : and he designed his tables to evaluate the fourth proportion of problems such as this. The approximate numbers and their logs can be found in the tables : In the first table below, we have used values from the log tables, and found a value for A short by 1'. Napier used more accurate values in his calculation : it appears that he either used a table with a closer mesh to find intermediate values, and it is of course possible that he used as he suggests the geometric/arithmetic means of existing values to determine further closer- spaced intermediate values, though this would be very tedious. However, it is more likely that he referred to the original table that he had originally constructed, and which is described in the *Constructio* : such a table of continued proportions was an approximation of a particular exponential decay that we describe below; it is evident that he has picked out the values of the sines and their logarithms at intervals of one minute in the argument from this table. It was a remarkable example of the economic management of paper by this wily Scot, that on adjacent pages of the tables, he was able to present the sine, the sine of the complement or cosine, the tangent, and the secant, and the logarithms, of any whole degree in minutes; and thus of the whole quadrant in 45 pages. This task occupied Napier for some 20 years from the early 1590's until 1614, when the tables were finally published to great acclaim. The further encumbrance of a table of continued proportions would seem to be excessive for the navigational requirements of master mariners bound perhaps for the East or West Indies!

We may make further mention here of the emergence of the technique known as prosthaphaeresis, which had been developed by a number of astronomers in the latter part of the 16^{th} century, as an aid for speeding up multiplication ; in many respects its use resembled that of logarithms, and utilised the identity

$$\sin a \sin b = \tfrac{1}{2}[\cos(a - b) - \cos(a + b)],$$

or one related to it, in conjunction with a table of sines, such as the gargantuan work of Pitiscus, which eventually in the third edition published at the same time as Napier's log tables, gave the sines of the angles in a right angle in steps of 10 seconds of arc for a radius of 1,00000,00000,00000.

There used to be some confusion in log tables between natural logs and Napierian logs : the natural log of N we call here ln N, and the Napierian log of N we call Nlog N , as there does not seem to be any agreed way of writing these logs, while the ordinary base 10 log of N is log N. It is not hard to establish that the function $N = 10^7 \exp(-n/10^7)$ establishes the relation between a number N and its Napierian logarithm n, though of course Napier did not go beyond the finite difference stage of working with such quantities, in which case the equation is replaced by $N = 10^7 \times (1 - 1/10^7)^n$. This notation was of course totally unknown at the time, but it aids in our understanding of how these logarithms behave. The inverse function is thus : $n = -10^7 \ln(N/10^7)$, which we can write as $\mathrm{Nlog}N = 10^7 \ln(10^7/N)$. The function Nlog$N$ that Napier based his tables on thus is infinite when the argument N is zero, and decreases to the value zero when $N = 1 \times 10^7$, and subsequently increases slowly towards negative infinity. The argument $N > 0$ always.

Now we can understand fully what is meant by the rounding process used for whole numbers with less than 7 significant figures : essentially, we always work with the full number of places, and if the need arises, we can divide or multiply by some power of ten a number of time as explained in Ch. IV, Section 9, of this translation; in the above example this has been possible without any trouble, as the powers of ten have cancelled in any case. In the above table, the second column corresponds to the logarithms of the whole numbers including the parts in brackets taken from the table, and which gives the value of A in the third column; as a check, we have included the true values of the whole 7 digit numbers in the last column. It appears that Napier used his more extensive tables for the proportional numbers and their logs to find a number answering to the correct number of digits to that required by the given number, and then added 23025842, the Napierean log of 1000000 or 1.0×10^6, the appropriate number of times to slide the decimal point along. Note that adding the log divides the original number by 10.

End of Digression.]

In oblique-angled triangles the legs [*b* and *c*] *are the sides which support any angle* [*A*], *and the base*[*a*] *is the line the angle* [*A*] *subtends.* [p. 26.]

Prop. 5. *In oblique-angled triangles, the logarithm of the sum of the legs taken from the sum formed from the logarithm of the difference of the legs, and of the differential* [tangent] *of half the sum of the opposite angles, leaves the differential* [tangent] *of half the difference of the same.*

[*i.e.* $-\log(b+c)+\log(b-c)+\log\tan(\mathrm{B}+\mathrm{C})/2=\log\tan(\mathrm{B}-\mathrm{C})/2$; or

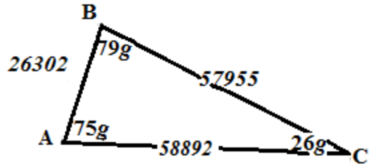
$$\frac{\tan(\frac{B-C}{2})}{\tan(\frac{B+C}{2})}=\frac{b-c}{b+c},$$

the well-known half-tangent formula, where the sides *a, b, c* and the angles A, B, and C can be permuted]

Because, as the the sum of the legs to the difference of the legs, thus the tangent of half the sum of their opposite angles itself has to the tangent of half the difference of the same. Thus the quantities are in proportion, and (by Prop.1 of Ch.2, Book1) and the differences or the excesses are in proportion. Therefore by necessity (by Prop.4 of Ch.2, Book1) we can conclude as above.

Coroll. *Thus from two legs or sides, and the included angle, the angles of the remaining opposite sides can become known (from this) : and hence (from before) the remaining side.*

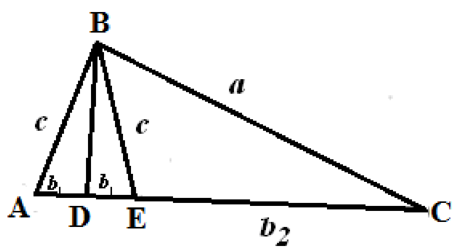
For by taking the logarithm of the sum of the sides, from the sum formed from the logarithm of the difference of the same, and the tangent of the sum of the half-angles opposite added together, there is found the tangent of the half-difference of the same angles : from which half difference added to the said half-sum, the greater angle comes about, and by subtraction the smaller.



As of the repeated above oblique-angled triangle ABC; the sides AB 26302, and BC 57955, are given, and the contained angle B of 79 degrees. Moreover, the remaining angles A and C are sought. The sum of the legs AB and BC is 84257, and the logarithm of this is 24738819 – 0, and the difference of the same AB and

BC is 31653, the logarithm of which is 34529210 – 0. Since the angle B is given as 79 degrees, the sum of the angles A and C (by Prop 1 of this section) is 101 degrees, and half the sum is indeed 50 degrees and 30 m., the tangent of which is – 1931766, which added to the logarithm 34529210 – 0, gives + 7858625 the logarithm of the tangent of 24 degrees 30', which is the half-difference of the angles A & C sought. [p. 27.] Hence therefore the half difference 24 degrees 30' added to the half sum 50 degrees 30' becomes equal to 75 degrees for the larger angle A sought, and by subtraction the same $24\frac{1}{2}$ degrees from $50\frac{1}{2}$ degrees, there remains 26 degrees for the smaller angle B.

Definition. *In oblique-angled triangles, the true base is either the sum of the cases : and then the difference of the cases is called the other base; or the true base is the difference of the cases, and then the sum of the cases is called the other base.*



As in the triangle ABC, [the small case letters have been added here; see the Latin version for the original diagram] the smaller case is the section AD, and the larger case is the section DC. The sum of the sections or cases AC is the true base. And in this triangle take the smaller section AD, or the section DE equal to that, from the larger DC, and there remains the difference of the sections E C, that we can call the other base. On the other hand truly in the triangle EBC, DE is the smaller section, (equal to DA). The larger section is DC, and the difference of the sections, EC is the true base. Moreover the sum of the sections, obviously AC, we call the other section.

Prop. 6. *In oblique-angled triangles, the sum of the logarithms of the sum and difference of the sides, is equal to the sum of the logarithms of the bases, the true and the other.*

Since the true base is to the sum of the sides, as the difference of the sides to the other base : Thus, (by Prop.5 of Ch.2, Book 1) we must by necessity conclude that the logarithms of the bases are equal to the sum of the logarithms of the sum and difference of the legs, as above.

[Thus we have two triangles, each with a common angle C and side *a* , and with equal sides *c* of the same length; in one case the angle A is acute, and in the other the corresponding angle E is obtuse. It follows that $a^2 = h^2 + b_2^2; h^2 = c^2 - b_1^2; h = BD.$ Hence, $a^2 = c^2 + b_2^2 - b_1^2;$ or : $a^2 - c^2 = b_2^2 - b_1^2$, the required result.]

Coroll. *Thus from an oblique-angles triangle of given sides, there can be made two right-angled triangles of known hypotenuses with one other leg of these, which (by Prop 2 of this) also all the remaining parts of the oblique-angled triangle can become known. .*

For by adding the logarithm of the sum of the sides to the logarithm of the difference of the sides, and hence by taking the logarithm of the true base, there arises the logarithm of the other base, by Prop.4, Ch. 2 and problem 3, Ch. 5, Book 1. Thus the half sum of

the bases is the greater section, and the half sum of the difference is the smaller section. [Thus, $\log(a-c)+\log(a+c)=\log(b_2-b_1)+\log(b_2+b_1)=\log \mathrm{EC}+\log \mathrm{AC}.$]
Thus the sides are given of the above triangle ABC [p. 28.], clearly AB is 26302, and BC is 57955, and the base AC is 58892, and the remaining sides and angles are to be found. The sum of the sides is 84257, the logarithm of which is 24738819 – 0. The difference of the sides is 31653, the logarithm of this is 34529210 – 0. Add these logarithms, and hence they make 59268029 – 00, from which is taken 5293461 – 00, the logarithm of the base AC, and there remains 53974568 the logarithm of the alternate base 45286: add that to the true base, and they make 104178, of which the half is 52089, the major section [b_2]. Take that from the same, and they make 13606, of which the half is 6803, AD the minor section. [Thus, $\mathrm{AC}+\mathrm{EC}=2.b_2$ and $\mathrm{AC}-\mathrm{EC}=2.b_1$.]

Thus with the hypotenuse AB and the other side AD of the right-angles triangle ADB obtained; and the hypotenuse BC and the side CD of the right-angled triangle BDC found, the angles of the right-angled triangle (by 2 of this section) at A, B, and C can be found, and as a consequence also all the parts of the oblique angled triangle can be found from the parts set out.

Nor should the problem be solved otherwise if the sides of the triangle EBC are given, and the rest of the parts of the triangle are sought. For from the sides and the true base EC, the other base AC can be obtain, and from these the other section, and the rest, as above.

CONCLUSION.

You now have a complete and perfect set of rules for solving all plane triangles, which if it should seem to be a little toilsome in finding the variable lines with logarithms : yet in the computation of the motions of the planets (in which obviously the eccentricities of orbits, the elongations of the perigees [Auges] *and apogees, in the diameters of the epicycles, and other lines, that remain the same), with the logarithms of these once known exactly, they can be used ever after without any change, surely a marvel for ease of use and certainty.*

Now spherical triangles follow, the most difficult of all, as commonly set out by others, yet by our logarithms the most easy of all.

[p. 29.]

Concerning Spherical Triangles.

CH. III.

Basics 1. *In spherical triangles, of all the angles that angle nearest in size to the quadrant, and the side subtending it, are in doubt* [i.e., as to whether or not the acute angle or its supplement should be used if the angle is almost right]. *For they are either of the same kind or of different kinds, depending on whether they have been produced by computation, or set up by hypothesis.*[Spherical triangle have three sides, which are measured as arcs or angles from the centre of the sphere, and three angles; two parts are of the same kind if they are both acute or obtuse angles.]
2. Truly either of any two given oblique [meaning non-right angles] *angles whatever is of the same kind as the side subtending it. Thus for a given part, the kind of the remaining part is apparent.*
3. If some angle of the triangle is nearer to being a quadrant than the side subtending it, then there are two sides of the same kind, and with the third side less than the quadrant.
4. Truly if some side of the triangle is nearer to being a quadrant, than the angle subtending it : then there are two angles of the same kind, and the third angle is greater than a quadrant.
5. A spherical triangle either is, or is not, quadrantal.
6. For a quadrantal triangle contains a quadrant or angle equal to a quadrant, the quadrant is either that of the angle or of the side.

Thus, we show that quadrantal triangles without a right-angle are to be found with equal ease, and to be compared with the right-angled case.
7. A quadrantal triangle has either a simple or several right-angles.
8. A quadrantal with several right angles has either two or three right angles.
9. The triquadrantal (or triangle) with three quadrants has each arc or angle equal to a quadrant.
10. Thus any triangle is a triquadrantal in which any three parts are equal, and not being given opposite to an individual quadrant.
11. A triangle is two right angled, when each of two angles and their individual sides are equal to quadrants.
12. In any triangle with two right angles, the oblique angle is equal to its own subtending side.
13. Every triangle in which some part is equal to a quadrant, and some oblique angle is equal to its own subtending arc is a triangle with two right- angles.
14. Every triangle having any two individual parts equal to quadrants, and the third unequal, is a two right- angled triangle.
15. The remaining quadrantal triangles are said to be simple.

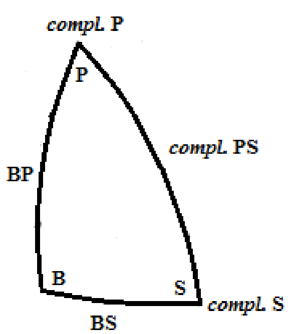
[We pause to remark that two coordinate systems are used in the following to locate an observer's position on the earth, in an intuitive manner. (Consult a text or web definition for more complete details): the first is now known as the *horizontal coordinate system*, in which the earth is at rest at the centre of an infinite celestial sphere on which heavenly bodies move, the Zenith is directly overhead, the observer's position is taken as the origin of coordinates, the angle a star makes with the horizontal is its *altitude*, while the *azimuth* is the angle measured in the horizontal plane from a fixed direction, such as North, given by the ZP or Zenith-Pole star meridian.
The second or *equatorial coordinate system* takes the centre of the earth as the origin, while bodies move on the celestial sphere, measured according to their *declination* or angle to the PP' axis (fixed in direction) or by their complementary latitude, where P and P' are the pole star and nadir, and by their *right ascension* as a measure of their celestial longitude about the polar axis. The right ascension is the angle or azimuth swept out by the hour circle of some celestial body referred to a fixed direction in space, taken to be first point in the constellation *Aries* or ♈, the location of the sun at sunrise at the Vernal Equinox ; this angle has a diurnal time dependence as well as a yearly variation, and is usually measured as a time.]

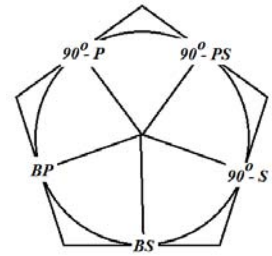
Concerning Simple Quadrantal Triangles. [p. 30.]

CAP. IV.

1. For a single quadrantal triangle is one in which a single part is equal to a quadrant, and moreover the remaining five parts are not equal to quadrants.
2. Of these five parts which are not quadrants, the three which are placed the furthest from the right angle, or on the quadrant side, we can convert into their complements, and all five retaining the original order are placed in a circle, or in a five angled arrangement, and which we call circular parts.

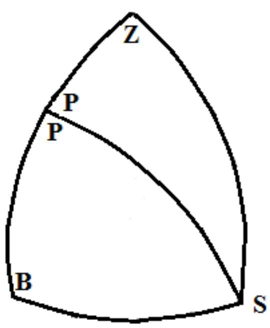
In the first case, let BPS be a spherical triangle with a right angle at B. The five oblique parts of this triangle which are not right angles are these : the side BP containing the right angle ; another the oblique angle P; PS the side subtending the right angle; S the remaining oblique angle; and SB, the remaining side containing the right angle. For which, for the sake of making the calculation easier, we take the side BP itself unchanged; we take the complement of the angle P; the complement of the side PS; and the complement of the angle S; and the side SB itself unchanged, and we place these five parts to be kept in their natural order, on the margin of a circle, and we call these the circular parts.





[This is Napier's introduction to his well- known mnemonic for finding the unknown arc or angle for a quadrantal spherical triangle. The arcs on either side of the right angle formed by two arcs or by a quadrant below remain unchanged, while the complements of the other two angles and the opposite arc are taken, as shown more conveniently in this extra diagram, to which a general formula is applied below.]

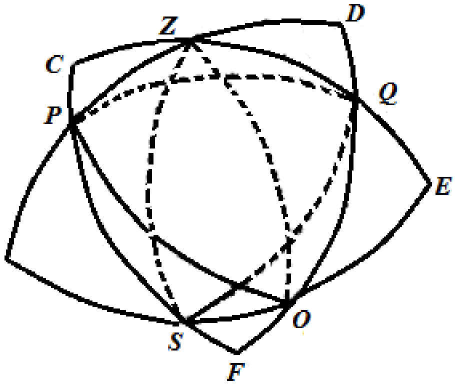
Similarly, the following triangle is a simple quadrantal triangle, but without a right-angled triangle SPZ (composed from the vertices of the Eastern sun S, the Pole [star] P, and the Zenith Z), with ZS the quadrantal side. The five unchanged non-quadrantal parts of this spherical triangle are : the other angle Z contained by a side of the quadrant; the side PZ for the [angular] distance of the pole from the zenith. The angle P subtended by the quadrant. The side PS for the [angular] distance of the pole from the sun, and then the final angle S which is embraced by the quadrant [ZS and the arc PS]. For which, to make the calculation easier, we take the angle Z [unchanged], or PZS, which is the arc of the position of the sun from the North pole . The complement of PZ, which is the elevation of the pole. The complement of the angle P, or the angle ZPS, [p. 31.] which is the difference of the ascensions, that is, the difference of the times of sunrise or sunset from the sixth hour. The complement of the side PS, which is the declination of the sun; and the angle of the sun S or PSZ, which we call the angle of the position of the sun (obviously with respect to the pole and the zenith). We set up these five parts on the margin around a circle or pentagon, and we call these circular. Neither shall there be made other circular parts from the above right-angles triangle BPS, if P is the pole, S is the sun, and B the North-facing point as you wish. The side BP gives the elevation of the pole; the complement of P gives the difference of the ascension; the complement PS the declination of the sun ; the complement S the angle of the position of the sun ; and hence BS the azimuth of the sun. Which are in short the same circular parts, but which are traversed the one clockwise and the other anticlockwise. Thus the same is true for all quadrantals with right angles as for those without right angles.



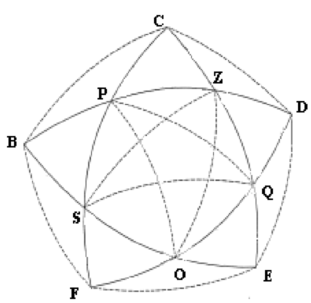
Corol. 3 *Hence it is the case that there are many triangles with different parts, that agree with these circles in a straight forward manner, and these can be resolved by our circular method.*

As it appears clear enough from the above two triangles BPS and PZS joined together. In which all the natural parts (besides PS and BS of the one, and PS and PZS of the other) are clearly different : yet truly the circular parts are all in agreement (as has been said above).

4. *The uniformity of the circular parts is most apparent with right-angles made on the surface of a sphere from five arcs of great circles, of which the first cuts the second, the second the third, the third the fourth, and finally the fourth cuts the fifth* [p. 32.] *at right angles : thus all the remaining angles are made oblique.*



An example : The meridian DB of a place cuts the horizon BE in the point B. The horizon BE cuts the arc of the great circle EC, which can be drawn with the sun S as its pole, in the point E . The circle EC, which goes around the sun, cuts the sun's meridian CF in the point C. The meridian of the sun CF cuts the equator FD in the point F: and finally the equator FD, cuts the meridian of the region DB in the point D. All these five sections cut orthogonally at the points B, E, C, F, and D, and make right angles: with the remaining sections cut at oblique angles in the points Z, P, S, O, and Q. From these sections, five right angles can be made :
PBS [between the local meridian at B and the local horizon];
SFO [between the sun's meridian and the celestial equator with the pole at P];
OEQ [between the horizontal and the arc with the sun as pole];
QDZ [between the celestial equator and the local meridian from B looking North],
and ZCP [between the sun as pole circle and the sun's meridian],
although the natural parts of these may differ, and in particular the triangles may be varied, nevertheless these five circular arcs set out above remain the same, without any distinction.



[This is a remarkable geometrical entity in its own right, that can be extracted from the natural elements from which it was composed by Napier. A small note in the AMS Journal for July,1898, p. 552, by Prof. E.D. Lovett, which is on the web, illustrates some or the charm of this construction, and which the present translator has taken the liberty to re-label and present here. The object is the spherical quadrantal pentagon PSOQZ , which is self-polar, with associated right-angled spherical triangles such as CPZ : if the arcs forming this triangle are extended into quadrants, and the polar quadrants of P and Z are constructed, then the outer quadrantal star pentagon can be formed BCDEF, the polars of the vertices of which is the inner star inner star pentagon, to which Napier's original diagram does not really do justice, considering the symmetry of the structure, but which he arrived at from his above diagrams. This theorem

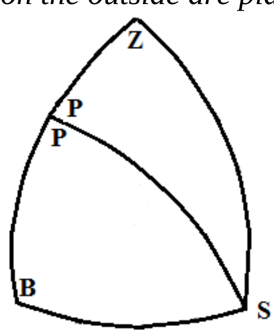
is also demonstrated and expanded on in the chapter by Sommerville in the Napier Tercentenary Memorial volume. Not to be outdone, nature has provided the South African carrion flower, showing some similarity to the above and growing in a pot in my own backyard, of the genus *Stapelia*.]

5. It is apparent that the uniform parts of the circle are present also in arcs that do not subtend right angles. These are made on the surface of a sphere from five points, the first of which from the second, the second from the third, the third from the fourth, and the fourth from the fifth, with the distances of the arcs equal to a quadrant; all the other distances are not equal to quadrants.

As in the preceding scheme, the points P from Q, Q from S, S from Z, Z from O, and O from P, are equally distant with the interval of a quadrant [*i.e.,* the diagonals of the pentagon SPZQO]: but indeed the distances from P to Z, Z to Q, Q to O, O to S, and S to P in turn, are not formed from quadrant arcs [*i.e.,* the sides of the regular pentagon]. And from these quadrantal arcs that do not support right angles, the five angles PZQ, ZQO, QOS, OSP, and SPZ can be made [*i.e.,* the angles of the regular pentagon]: and though some of the parts differ from natural parts, nevertheless here the same circular parts remain unchanged, as above. Obviously, these are : the elevation of the pole [BP] ; the complement of BPS or SPZ, the difference of the ascensions ; the complement of PS, which is SF, the declination of the sun ; the complement of PSB, or PSZ, the angle of the position of the sun; BS, the azimuth of the sun: which are equality in agreement with the above triangles, [p.33.], and not only from the sun, but indeed also from all the triangles which arise between the remaining sections of the whole number produced from these ten arcs: which are many and confusing, and that here we can dismiss. This short account is warning enough about the confusion of the natural parts, and their rules to be avoided and removed, and to be replaced with a single rule for these few circular parts.

6. Of the five circular parts, there are always three that are required to be known, of which two are given, and the third has to be found.

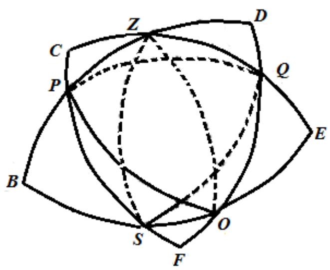
7. Of these three parts, one is central, and the other two are on the outside, and the two on the outside are placed one on either side or opposite.



For example, there are these three parts proposed in the question : the position or azimuth of the sun [BS], the elevation of the pole [BP], and the difference of the ascensions : of which, the elevation of the pole is said to be the middle or intermediate part, and the remaining two are neighbours to it on either side, or are said to be placed around this part. Again, if the three parts to be called into question were : the declination of the sun [the complement of PS], the elevation of the pole [BP] , and the angle of the position of the sun [PSZ], then as before, the elevation of the pole is called the intermediate or middle part, and with the declination of the sun and the angle of the position of the sun called the extremes, removed from the mean, or are said to be placed opposite. A like ratio can be found for the remaining five parts.

8. The logarithm of the sine of the middle part is equal to the logarithms of the tangents of the extremes [which is called the antilogarithm of the differentials in the original], *or to the logarithms of the cosines of the opposite extremes.*
[The identity is now usually written in a form without logs, or with sine and cosine interchanged : the cosine of an angle is equal to the product of the neighbouring cotangents and also to the product of the opposite sines. One of the angles is a right-angle which has been removed from the original 6 parts of the circle. The use of a right angle in Napier's theorem simplifies calculations using spherical triangles; see the note at the end of this section.]

This theorem is proved by induction [not the modern mathematical meaning of the term, but rather a setting out of the rules : Napier does not prove his rules in this book, but states them and shows how to use them; remember that this was a 'hands-on' type of publication.] of any three parts or triplicates, which can be set in place from the five circular of the first right-angled BPS, and which fall to be resolved in the question, but we omit triplicates from the following none right-angled triangle PZS, since all the circular parts of this (set out in sections 18, & 19, & 20) are for the same quantizes which preceded. Hence for the five circular parts of the right-angles triangle BPS, (which are BS, or the position of the rising sun : the complement BSP or the angle of the sun's position: the complement SP, or the declination of the sun : the complement SPB, or the difference of the ascensions : and PB, or the elevation of the pole). The three parts of these which fall into the category of being extremes in the question are:



(1) either BS, the complement of BSP, and the complement of Sp; or
(2) the complement of BSP, the complement of SP, and the complement of SPB : or
(3), the complement of SP, the complement of SPB, and PB ; [p. 34.]
(4), the complement of SPB, PB, and BS : or
(5), PB, BS, and the complement of BSP.

Indeed since in all these triplicates, the tangent of the one extreme is to the intermediate right sine, as the total sine is to the tangent of the other extreme, (such as is apparent from common trigonometry.) Thus (by our demonstrations, Prop. 5, Ch. 2, Book I) the logarithms of the means (which are of the intermediate by corollary 6, def., Ch.1.Book 1) equal to the logarithms of the tangents of the extremes , which are the differentials of the same (from sect. 22 and 25, Ch.3, Book 1). Therefore the logarithm of the single intermediate sine is equal to the differentials of the extremes on either side, as we have asserted in the first part of the theorem. The confirmation of the second part follows.

Therefore of the same five parts of the circle, these three that fall to being opposite the extremes, are either : (1), PB, the complement BSP, and the complement SPB , or (2), BS, the complement SP, and PB ; or (3), the complement BSP, the complement SPB, and

BS ; or (4), the complement SP, PB, and the complement BSP ; or (5), then the complement SPB, BS, and the complement SP.

But in all these triplicates or from the five cases, the right sine of the complement of the one extreme has the same ratio to the right sine of the intermediate as the total sine to the right sine of the other complement (that has been shown in a more detailed manner by Regiomontanus, Copernicus, Lansbergius, Pitiscus, and others, than can be repeated in this short account.) Thus from our demonstration (Prop. 5, Ch.2, Book I) the logarithms of the complements of these extremes are equal to the logarithms of the means, that is, (as has been said) to the logarithm of the sine of the only intermediate. But the logarithms of these complements of the opposite extremes are the antilogarithms of the same parts of the [*i.e.* cosines rather than sines] (by def., sections 13 and 16, Ch.2, Book I) therefore it follows in these cases, that the logarithm of single intermediate is equal to the antilogarithms of the same opposite extremes, as the second part of the theorem asserts. Thus the whole theorem is in agreement. [p. 35.]Besides this approval of all the cases that can occur, the theorem can be clearly seen from the preceding sections 4 and 5, in which figure the homologous constitution of the circular parts argues for a similar set of ratios : thus in order that from one intermediary and its extremes on either side, nor indeed can anything be denied for the other four intermediaries with their respective extremes placed opposite.

[Recall that at the time of Napier's writing, algebra was in its infancy as far as suitable notations were concerned; to aid our understanding, therefore, we append the following derivation. Thus, the basic formulae for the angles A, B, C and the sides *a, b, c* of a spherical triangle are the cosine rules for sides (I),:

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A \quad \text{(I)},$$

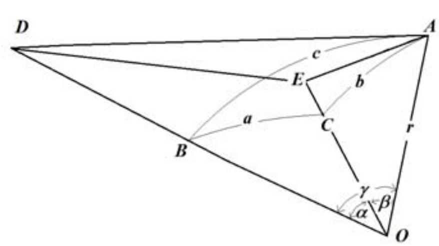
and similarly for cos *b* and cos *c* in a cyclic manner ; and for angles :

$$\cos A = -\cos B \cdot \cos C + \sin B \cdot \sin C \cdot \cos a \quad \text{(II)};$$

and similarly for cos *B* and cos *C* in a cyclic manner ; and the sine rule :

$$\sin a : \sin b : \sin c = \sin A : \sin B : \sin C \quad \text{(III)}.$$

The reader may refer to a text on spherical trigonometry, such as Todhunter's *Spherical Trigonometry*, for elementary derivations of these formulas. For example, the cosine rule for sides (I) may be shown in the following elementary manner :-



The spherical triangle *ABC* shown is drawn on the surface of a sphere with centre *O* and radius *r*, and subtends the arcs *BC, CA*, and *AB*, with respective arc lengths *a* , *b*, and *c* ; with the

corresponding angles *A*, *B*, and *C* defined between the tangents to the respective arcs on the surface. These arcs are viewed obliquely in 3 dimensions, and so appear as arcs of ellipses, and of course the tangent lines of the arcs are at right angles to the corresponding radius. If the arcs are measured in radians, or the radius is equal to one, then $\alpha = a$; $\beta = b$; & $\gamma = c$. The radii *OC* and *OB* are extended to cut the tangent plane formed by the arcs at *A*, at the points *E* and *D*. The tangent line segments *AD* and *AE* are drawn subsequently as shown, and the planar triangle *DAE* completed. We have at once from the planar cosine rule : $DE^2 = AE^2 + AD^2 - 2 \cdot AE \cdot AD \cdot \cos A$; and from the geometry of the tangents to the sphere lying on great circles, we have :
$EA = r \cdot \tan \beta$ and $DA = r \cdot \tan \gamma$;

Hence, $DE^2 = r^2 \cdot \tan^2 \beta + r^2 \cdot \tan^2 \gamma - 2 \cdot r^2 \cdot \tan \beta \cdot \tan \gamma \cdot \cos A$;

also, in triangle ODE :

$$DE^2 = OE^2 + OD^2 - 2 \cdot OE \cdot OD \cdot \cos \alpha ;$$

but, $OD \cdot \cos \gamma = r$ and $OE \cdot \cos \beta = r$; hence $OD = r \cdot \sec \gamma$ and $OE = r \cdot \sec \beta$, and it follows that

$$\therefore DE^2 = \sec^2 \gamma + \sec^2 \beta - 2 \cdot \sec \beta \cdot \sec \gamma \cdot \cos \alpha = \tan^2 \beta + \tan^2 \gamma - 2 \cdot \tan \beta \cdot \tan \gamma \cdot \cos A;$$

$$\therefore 1 - \sec \beta \cdot \sec \gamma \cdot \cos \alpha = -\tan \beta \cdot \tan \gamma \cdot \cos A;$$

$$\therefore \cos A = \cot \beta \cdot \cot \gamma \cdot \sec \beta \cdot \sec \gamma \cdot \cos \alpha - \cot \beta \cdot \cot \gamma = \frac{\cos \alpha - \cos \beta \cdot \cos \gamma}{\sin \beta \cdot \sin \gamma};$$

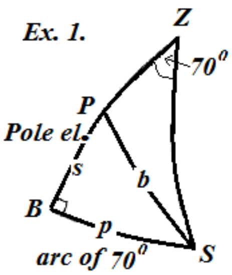
$$\text{hence}: \cos a = \cos \beta \cdot \cos \gamma + \cos A \cdot \sin \beta \cdot \sin \gamma = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A.$$

By setting one of the angles equal to 90^0 in these formulas, and setting the five other circular parts with the right angle ignored in order around a circle, Napier was able to derive his mnemonic, which incorporated the results of all these formulas ; this may be stated in the form :
In a right-angled spherical triangle, the cosine of any part is equal to the product of the co-tangens of the adjacent parts, and also to the product of the sines of the opposite parts; or in an equivalent manner, with the complements of the parts taken. See the example below for a diagram.]

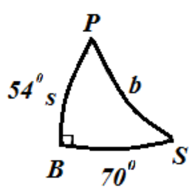
A General Deduction.

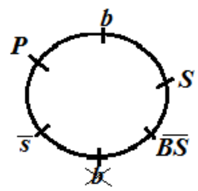
9. Hence it follows for single quadrantals, that from any two given parts some third can become known. For indeed, either the intermediate part is sought, and its logarithm is found by adding the differentials of the given extremes placed in position : or one of the extremes is sought, and the differential of this emerges from the subtraction of the differential of the known extreme from the known intermediate logarithm, as in the five triples of the preceding theorem with the right angled triangle, and the other angles not being right. Or again, the intermediate one is sought, and the logarithm of this comes about from the addition of the antilogarithms [not in the modern sense of course] of the opposite given extremes; or, finally one of the opposite extremes is sought, and the *antilogarithm of this is obtained from the subtraction of the other extreme logarithm from the known intermediate. Moreover two arcs of different kinds are now found to correspond to these logarithms, antilogarithms, and differences. Therefore from the kind of arc sought from the second, the third, or the fourth section of this chapter, or by hypothesis, the true arc can itself become known.*

Ex. 1.



As in the previous example in the seventh section, there are three circular parts in the question : the azimuth of the sun BS, the elevation of the pole BP, and the difference of the ascensions BPS, that is, in the right-angled triangle BPS, the parts BS, PB, and the complement of SPB : or in the quadrantal non right-angled triangle PZS, the parts PZS, the complement of PZ, and the complement of SPZ. Of any three parts that may be given, let the extremes placed on either side be given : the position of the rising sun BS, or PZS, 70 degrees : and the ascension difference, the complement of SPB, or the complement of SPZ, 16 gr. 24', 27'' : and the intermediate part [p. 36.] PB is sought, or the complement of PZ, which is the elevation of the pole. Therefore the differential of 70 degrees, viz, –10106827 is to be added to the differential 16 degrees 24', 27'', 12226180 and there comes about 2119353, the logarithm of 54 degrees for the elevation of the pole sought.





[Note that SP is the meridian arc, joining the position of the sunrise S to the pole, about which the celestial sphere rotates ; BS is part of the local horizontal, and the difference of the right ascensions corresponds to 16^0, 24', 27''. Napier's Rules are applied to this triangle, the parts of which are related according to : $\cos c = \cot A.\cot B = \sin \bar{a}.\sin \bar{b}$, etc. and these can be permuted in a cyclic fashion. Thus in this case,

$$\cos s = \cos \mathrm{BP} = \cot 70^0 \cot 16^0, 24', 27'';$$

and hence

$$\sin \mathrm{BP} = \tan 70^0 \tan 16^0, 24', 27'',$$

from which the sum of logarithms follows.]

A Reminder.

As well as the elevation of the pole found by in this method, also in the second place from the same example the azimuth of the sun can be obtained from the elevation of the pole, and with the angle of the sun's position given. Likewise in the third place, the angle of the sun's position can be found from the azimuth of the sun and from the same given declination of the pole. In the fourth place, the declination of the sun [PS] can be found from the given azimuth angle of the sun, and the difference of the ascensions. And in the fifth place, the difference of the ascensions can be found from the declination of the sun and from the elevation of the pole.

Second Example.

The azimuth of the rising sun can be given, either BS or PZS, as 70 degrees : and the elevation of the pole taken as 54 degrees, which is PB, or the complement PZ. Moreover, the difference of the ascensions is required, obviously the complement of SPB, or the complement of SPZ. And, since here similarly the extreme parts are placed on each side of the middle part, hence take the differential of the azimuths of the sun or 70 degrees, which is -10106827 from the logarithm of the elevation of the pole 2119353, and hence there comes out 12226180, which is thus nearly the arc of the differential of the ascensions sought 16 degrees, 24', 27".

[*i.e.*, in this case we have $\sin s = \tan 70^0 \tan \mathrm{P}$, from which the result follows.]

A Reminder.

This example can be imitated to find in the second place the declination of the sun from the difference of the ascensions, and from the given elevation of the pole. Likewise (3) : the angle of the position of the sun is found from the declination of the sun, and from the difference of the ascensions; (4) : the azimuth of the sun from the angle of the position of the sun, and from the declination of the same [p. 37.] ; (5) : the elevation of the pole is obtained from the azimuth of the sun, and from the angle of the position of the sun. Likewise in a contrary manner, the following can be found. (6) : the difference of the ascensions can be found from the declination of the sun and from the given angle of the position of the sun; (7): the declination of the sun can be found from the angle of the position of the sun, and from the azimuth of the sun. (8): the angle of the position of the sun is obtained from the azimuth of the sun and from the given elevation of the pole; (9): the azimuth of the sun is found from the elevation of the pole, and the difference of the ascensions. (10) : finally the elevation of the pole is found from the difference of the ascension and from the given declination of the sun.

Third Example.

In the last example of the seventh part of this chapter, these three circular parts were proposed : the declination of the sun [which is the quadrant angle of 90^0 – PS or *b*, in the above diagram, hence the complement of PS], the elevation of the pole [BP], and the angle of the position of the sun. In the right-angled triangle BPS, these are : the complement of PS, BP, and the complement of BSP, and in the other quadrantal triangle that is not right-angled, PZS, these are, the complement of PS, the complement of ZP, and ZSP. Of which three, the opposite extremes are given obviously, the declination of the sun, which is the complement of PS, 11 degrees, 35' 51", and the angle of the position of the sun, which is the complement of BSP, or ZSP, 34 degrees, 19' 21" almost. And the

intermediate part BP is sought, or the complement of ZP, which is the elevation of the pole. Therefore the logarithm of the complement of PS, cosine 11 degrees, 35' 51", which is 206271 to be added is to the logarithm of the cosine 34 degrees 19' 21" of BS, which is 1913082, and there arises 2119353, the logarithm of the sine of 54 degrees which is the elevation BP of the pole sought.
[Thus, $\cos s = \sin\left(90^0 - 11.6^0\right) \times \sin\left(90^0 - 34.3^0\right)$, giving $\cos s = \cos 36^0$, approximately.]

A Reminder.

Besides the elevation of the pole now found in this way, you can find : (2) the azimuth of the sun [i.e, the angle from North in the local horizontal plane you have to turn through in a clockwise sense to reach the current sun's meridian] by the same rule from the same declination and the difference of the ascensions given; (3) the angle of the sun's position from the difference of the ascensions and the deviation of the pole; (4) the declination of the sun from the elevation of the pole and the azimuth of the sun; and (5) you can find the difference of the ascensions from the azimuth of the sun and from the given angle of the sun's position. [p. 38.]

Fourth Example.

From the given declination of the sun SP, the complement of 11 degrees, 35', 51", and from the elevation of the pole BP, or the complement PZ of 54 degrees, let the angle of the position of the sun be sought, the complement of the angle BSP, or the angle PSZ [for the sun's meridian ZS in the local celestial sphere and the local horizon are perpendicular] : and, since here similarly the extreme parts of the middle value are put in position, therefore the logarithm of the cosine of 11 degrees, 35' 51", which is 206271, is taken from the logarithm of the sine of 54 degrees, which is 2119353, and there is in excess 1913082 , the logarithm of the cosine of 34 degrees 19', 21" nearly, which is the angle of the sun's position sought.

[In this case, BP is known as 36^0, PS is the complement of 11^0, 35' ,51" or ~ (90^0 -11.6^0), and we take $\cos \mathrm{BP} = \sin\left(\mathrm{compl.PS}\right) \cdot \sin\left(\mathrm{compl.BSP}\right)$ as the circular parts to use : hence, $\sin\left(\mathrm{compl.BSP}\right) = \cos \mathrm{ZSP} = \cos 36^0 / \cos 11.6^0$, giving ZSP = 34.2^0.]

Final advice.

Besides the angle of the position of the sun that is acquired from this first exercise, from the same exercise: (2), the angle of the sun's declination is found from the given difference of the ascensions and from the sun's azimuth; (3), the difference of the ascensions is found from the given elevation of the pole and from the angle of the sun's position; (4), the elevation of the pole is found from the sun's azimuth and from the same given declination; (5), the sun's azimuth is acquired from the angle of the sun's position and the difference of the ascensions; (6), (with the contrary order) the angle of the sun's position is found from the sun's azimuth and the given difference of the ascensions; (7) the declination of the sun is found from the angle of the position of the sun, and from the given elevation of the pole; (8) the difference of the ascensions is found from the declination of the pole, and from the same given azimuth; (9) the elevation of the pole is found from the given difference of the ascensions, and the angle of the position of the sun; finally(10) the azimuth of the sun is acquired from the elevation of the pole, and from the given declination of the sun.

And thus by the imitation of these four examples, thirty various questions can be solved in the case of the right-angled quadrantal, and as many in the non right-angle case can be solved by this porism, with the aid of only one addition or subtraction [p. 39]. For an understanding of other kinds of arcs, see the examples of the latter parts of this porism that follow (third, fourth, fifth, and sixth.)

De Canonis mirifici LOGARITHMORUM
præclaro usu in Trigonometria.

LIBER II.

Cap. 1.

Quum Geometia sit ars benè metiendi, Dimensio sit magnitudinú propositarum, magnitudines figuram (potentia saltem) constituant, figura sit triangulú, aut triangulatum : Triangulatum verò compositum sit ex triangulis, quibus suisque partibus mensuratis, mensurabitur & illud, illiusque partes omnes. Certum igitur est ex triangulorum doctrina omnis Geometricæ quæstionis Solutionem Logisticam pendere.

Triangulum aut rectilineum est, aut Sphæricum.

De rectilineis, prop. 1.

Prop. 1. *Rectilinei tres anguli æquantur duabus rectis.*

Unde duobus datis, aufer eorum aggregatum ex 180. gradibus, & proveniet tertius. Idem unico ex 180. gradibus ablato, restat reliquarum duorum aggregatum. [p. 21.]

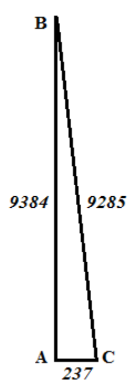
Rectilineum aut rectangulú est, aut obliquangulum.

In rectangulis crura vocamus, quæ rectum angulum ambiunt : hypotenusam, quæ subtendit.

Prop. 2. *In rectangulo Logarithmus cruris æquator aggregato ex Logarithmo anguli arci anguli oppositi, & Logarithmo hypotenusæ.*

Quum ex Trigonometriæ principiis pateat, alterutrumvis crus se habere ad sinú anguli ei oppositi, ut hypotenusa ad sinum totum : & (per prop. 5.cap.2.lib.1) horú quatuor proportionaliú logarithmi secundi & tertiii, æquetur logarithmis primi & quarti : quarti auté Logarithmus sit 0.seu nihil (per collarium 6 def.cap.1.lib.1) Ideo (ut supra) Logarithmus cruris æquatur aggregato ex Logarithmo anguli quem subtendit, & Logarithmo hypotenuæ.

Corol. *Unde hypotenusæ, cruris, & anguli quem subtendit, duobus quibuscunque datis, tertium, atque inde reliquæ omnes rectanguli partes innotescent.*



Quia enim hæc tria, cum sinu toto constituunt quatuor proportionalia, certum est eorum quodvis quarto loco posse constitui, & per 3.probl.cap.5.lib.1. acquiri.

Ut trianguli A.B.C. in A rectanguli, detur hypotenusa B.C 9385, cum crure AB 9384. Quæruntur anguli obliqui C. & B. Ex Logarithmo igitur A.B. 635870 – 0000, aufer Logarithmum BC.634799–000. Super sunt 1071 Logarithmus anguli C, cui in tabula respondent 89 g. $9\frac{3}{4}$ pro angulo C, & ex adverso 0 g. $50\frac{1}{4}$ pro ejus complemento, angulo scilicet B. [p. 23.]

Vice versa si detur angulus C, cum crure recti anguli A.B., & quæratur hypotenusa B.C.

Ex Logarithmo A.B. 635870 – 0000 aufer Logarithmum anguli C. 1071, & provenient 634799 – 0000 Logarithmus B.C.9385 hypotenusæ quæsitæ.

Tertio si datis B.C. , & angulo C, quæratur A B : adde Logarithmum B.C. 634799 – 0000 ad 1071 Logarithmum angulum C., & producentur 635870 – 000 Logarithmus numeri 9384 cruri A. B. quæsito respondentis. Nec secus ipsum crus reliquum A.C. ex angulo B. (qui est complementum anguli C.) jam cognito habetur. Atque ita omnes hujus rectanguli partes innotescunt.

Prop. 3. *In rectangulo Logarithmus cujusvis cruris, est æqualis aggregato ex differentiali oppositi anguli, & Logarithmo reliqui cruris.*

Quum ex vulgari doctrina triangulorum constet, quod alterutrum crus se habeat ad tangentem sibi oppositi anguli, ut reliquum crus ad sinum totum : & quum (per prop.5.cap.2.lib.1) ex his quatuor proportionalibus Logarithmi mediorum (id est, differentialis anguli, & Logarithmis cruris eum ambientis) æquentur Logarithmis cruris eundem subtendentis, & sinus totius (qui est nihil, seu 0) ideo Logarithmus cruris, est æqualis aggregato, &c. ut supra.

Corol. *Unde ex cruribus recti, & anguli alteri eorum opposito, duobus quibuscunque datis, tertium (per hanc) atque proinde ceteræ omnes rectanguli partes (per præced.) innotescent.*

Quandoquidem hæc tria cum sinu toto constuant quatuor proportionalia, certum est, eorum quodvis quarto loco posse collocari, & per 3.probl.cap.5.lib.1. acquiri. [p. 24.]

Ut præcedentis trianguli A B C, in A rectanguli datis cruribus A B, 9384. & A C, 137. Quæritur angulus B. Ex Logarithmo A C, 42924534 – 000. aufer 635870 – 000, Logarithmum A B. & provenient 42288664, differentialis anguli B, 0 g. 50' ii, quæsiti. Verum si dentur crus AC, 137 : & angulus B. 0 g. 50' ii, habebitur crus A B auferendo 42288664. differentialem anguli B. à Logarithmo A C. qui est 42924534 – 000. Inde enim proveniens 635870 – 000. est Logarithmus numeris 9384. qui crus est A B. quæsitum. Tertio datis crure A B, 9384. & angulo B, 0 g. 50' ii : ut habeatur crus AC. adde 635870 – 000, Logarithmum cruris AB. ad 42288664, differentialem anguli B, et provenient 42924534 – 00, Logarithmus 137, cruris A C, quæsiti. Hypotenusa autem B C per præced. prop. habetur. Angulus etiam C, patet, quum sit complementum anguli B, jam cogniti. Et ita per hanc, & præmissam, ex latere quovis, & parte alia quavis rectanguli datis reliquæ omnes ejus partes innotescent.

Completam erto habes rectangulorum rectilineorum scientiam : sequitur obliquangulorum.

De triangulis rectilineis præsertim obliquangulis.

Cap. II.

Prop. 4. ***In omni triangulo, aggregatum ex Logarithmis anguli cujusvis, & lateris eum ambientis, æquatur aggregato ex Logarithmis lateris, & anguli eis oppositorum.***

Quia omnium laterum ad oppositorum angulorum sinus eadem est ratio : & ita factum ex anguli cujusvis sinu recto, & latere quovis eum ambiente, æquatur facto ex latere subtendente priorem angulum, & sinu anguli subtensi à priore latere. [p. 25.] Ideo (per prop.5.cap.2.lib.1) aggragatum ex Logarithmis &c. æquatur. ut supra.

Corol. *Unde ex duobus angulis quibuscunque datæ speciei, & suis subtendibus, si tria dantur, quartum quodcunque, atque cætera omnes trianguli partes innotescent.*

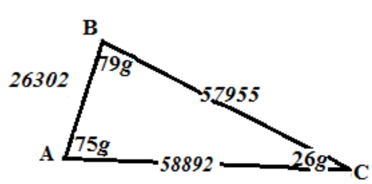
Horum enim quatuor proportionalium quodvis quæsitum potest quarto loco constitui, & per 3.probl.cap.5.lib.1. inveniri.
Ut obliquanguli A.B.C detur AB. 26302, & B C. 57955, & angulus C. 26 graduum : Quæraturque angulus A, qui sic habetur. Adde 5454707 – 00 Logarithmum BC. ad 8246889 Logarithmum scilicet C 26 graduum, & fient 13701596 – 00. Hinc aufer Logarithmum AB, qui est 13354921 – 00, restant 346675 Logarithmus 75 graduum, & paulò pluris, anguli scilicet, A quæsiti, si A prædicatur acutus : alioqui 105 g (per 1.& 2.sect.cap.3.lib.1) si pronuncietur obtusus.

Vice versa si detur angulus A jam 75 graduum, atque angulus C. & latus BC. ut supra : & quæratur A B. adde 5454707 – 00 Logarithmum B C. ad 8246889 Logarithum anguli C, fient, ut supra, 13701596 – 00, à quibus aufer 346675 logarithmus anguli A, provenient 13354921 – 00 Logarithmus lateris A B, & numeri ejus 26302 quæsiti. Habitis jam angulis A. 75 gr. & C. 26 gr., erit angulus B. 79 gr. per 1.hujus. Ex quo jam habito, non secus acquiritur latusei oppositum A C. 58892, quam nuperrimè ex angulo C. innotuit latus ei oppositum A B. Itaquae jam angulo C. innotuit latus ei oppositum A B. Itaque jam patent omnes hujus obliquanguli partes.

I*n obliquangulis crura vocamus, quæ angulum quemvis ambiunt : basim quæ subtendit.* [p. 26.]

Prop. 5. ***I****n obliquangulis, Logarithmus aggregati crurum subductus à summa facta ex Logarithmo differentiæ crurum, & differentiali semi-aggregati suorum oppositorum angulorum, relinquit differentialem semi-differentiæ eorundem.*

Quia, ut aggregatum crurum ad differentiam crurum, ita tangens semi-aggregati suorum oppositorum angulorum, se habet ad tangentem semi-differenciæ eorumdem. Unde analoga sunt, & (per prop.1.cap.2.lib.1) eorundem differentiæ seu excessus sunt æquales. Necessario igitur (per prop.4.cap.2.lib.1) concludimus ut supra.

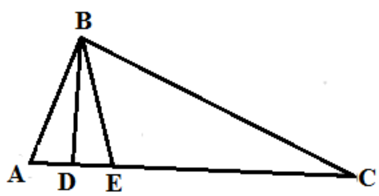


Corol. *Unde ex duobus cruribus, & angulo compræhenso, innotescunt (per hanc)anguli reliqui oppositi : atque inde (per præmissam)reliquum latus.*

Nam subducto Logarithmo aggretati crurum, à summa facta ex logarithmo differentiæ eorundem, & differentiali semi-aggregati oppositorum angulorum additis, proveniet differentialis semi-differentiæ eorundem angulorum : qua semi-differentia addita ad semi-aggregatum dictum, proveniet angulus major, & subtracta minor.

Ut repetiti superioris obliquanguli A B C. dentur crura, A B 26302, & B C. 57955, & angulus compæhensus B. 79 graduum. Quærantur autem reliqui anguli A & C. Aggregatum curum AB. & BC est 84257, ejusque Logarithmus est 24738819 – 0. differentia autem eorundem A B, & B C est 31653, ejusque Logarithm. est 34529210 – 0. Quumque B angulus detur 79 gr. erit (per 1.hujus) aggregatum angulorum A & C, graduum 101, semi-aggregatum vero 50 g. 30 m., cujus differentialis est – 1931766, quo ad 34529210 – 0, provenient + 7858625 differentialis graduum 24. 30', qui sunt semi-differentia angulorum A & C quæsitorum. [p. 27.] Hanc ergo semi differentiam 24. 30' adde ad semi-aggregatum 50. 30', fient 75 gradus, pro angulo A quæsitorum majore, & subtrahe eosdem $24\frac{1}{2}$ gradus ab eisdem $50\frac{1}{2}$ gradibus, & relinquentur 26 gradus pro angulo B quæsitorum minore.

Definitio. *In obliquangulis vera basis semper est vel aggregatum casuum : & tunc differentia casuum basis alterna vocatur ; vel vera basis est differentia casuum : & tum aggregatum casuum vocamus alternam.*



Ut trianguli A B C casus minor est AD : casus major est DC. Casuum aggregatum A C est basis vera. Et in hoc triangulo aufer casum minorem A D,seu ei æqualem D E à casu majore D C, relinquetur differentia casuum E C, quam basim alternam vocamus. Contrà vero in triangulo E B C casus minor est D E (cui æquatur DA). Casus major est DC, & casum differentiæ E C est basis vera. Casuum autem aggregatum, scilicet A C, basim alternam vocamus.

Prop. 6. ***I****n obliquangulis, summa Logarithmorum aggregati & differentiæ crurum, est æqualis summa Logarithmorum basium, veræ, & alternæ.*

Quia basis vera se habet ad aggregatum crurum, ut differentia crurum ad basim alternam : Ideo (per prop.5.cap.2.lib.1) necessatiò concludimus, basium Logarithmos æquari Logarithmis aggregati & differentiæ crurum, ut supra.

Corol. *Unde ex obliquangulo datorum laterum, fiunt duo rectangula notarum hypotenusarum cum altero cujusque crure, quæ (per2.hujus)reliquas etiam omnes obliquanguli partes notas reddunt.*

Nam addito Logarithmum aggregati crurum ad Logarithmum differentiæ crurum, & hinc ablato Logarithmo basis veræ, proveniet Logarithmus basis alternæ, per prop.4.cap.2. & probl.3.cap.5.lib.1. Harum itaque basium semi-aggretatum est casus major : semi-differentia vero casus minor. Ut superioris trianguli A B C dentur [p. 28.] latera, videlicet crus A B 26302, & crus B C 57955, & basis A C 58892, & quærantur cætera. Aggregatum crurum est 84257, ejusque Logarithmus est 24738819 – 0. Differentia crurum est 31653, ejusque Logarithmus est 34529210 – 0. Hos Logarithmos adde, fient inde 59268029 – 00. à quibus aufer 5293461 – 00 Logarithmum basis A C, restant 53974568 Logarithmus numeri 45286 basis alternæ : quam ad veram adde, fient inde 104178, quorum dimidium est 52089, DC, casus major. Eandem ab eadem aufer, fient inde 13606, quorum dimidium est 6803, A D casus minor.

Rectanguli itaque A D B. habitis jam, hypotenusa A B, & crure altero A D. atque rectanguli B D C habitis, hypotenusa B C, & crure D C, innotescunt (per 2.hujus) anguli rectangulorum apud A & B & C, & per consequens omnes etiam obliquanguli oblati partes ex præmissis propalantur.

Nec secus agendum foret si darentur latera trianguli E B C, & cætera partes quærentur. Ex cruribus enim & basi vera EC, innotescit basis alterna A C, atque ex his uterque casus, & cætera, ut supra.

CONCLUSIO.

Perfectam igitur & completam jam habes omnium triangulorum rectilineorum doctrinam, quæ si aliquantulum operosa in Logarithmis rectarum variabilium inveniendis videatur : In motibus tamen planetarum computandis (in quibus scilicet eccentricitates orbium, elongationes Augium & apogæorum, in epicyclorum diametri, & aliæ rectæ, eadem & invariabiles permanent) eorum logarithmis exactè semel notati, semper in posterum, sine ulla mutatione subservient, miranda certè fæcilitate, & certitudine.

Sequuntur jam Sphærica triangula, omnium difficillima, ut vulgò ab aliis traduntur, per Logarithmos tamen nostros omnium facillima.

[p. 29.]

De Triangulis Sphæricis.

CAP. III.

**Sent-
entia.**

1.In Triangulis Sphæricis angulus omnium quadranti quantitate proximus, & latus subtendens dubia sunt, An ejusdem, an diversa sint speciei, nisi id aut computus, aut hypothesis prodat.

2. Duorum vero obliquorum angulorum quilibet est ejusdem speciei, cujus est latus eum subtendens. Unde alterius datæ, reliqui patet species.

3. Si trianguli angulus aliquis propinquior sit quadranti, quam latus eum subtendens, erunt duo latera ejusdem speciei, & tertium quadrante minus.

4. Si vero trianguli latus aliquod propinquius sit quadranti, quam eo subtensus angulus : erunt duo ejus anguli ejusdem speciei, & tertius quadrante major.

5. Triangulum Spæricum aut est quadrantale, aut non.

6. Quadrantale est cujus aut latus, aut angulus æquatur quadranti.

Unde, non rectanguli quadrantalis scientiam æquè facilè, ac rectanguli comparari posse, docemus.

7. Quadrantale triangulum aut est multiplex, aut simplex.

8. Multiplex quadrantale aut est trirectangulum, aut birectangulum.

9.Trirectangulum est cujus singulæ partes quadrantes æquantur.

10. Unde omne triangulum, cujus trium partium non oppositarum singulæ quadranti æquantur, Trirectangulum est .

11. Birectangulum est, cujus duo tantum anguli, & sua subtendentia latera sigillatim quadranti æquantur.

12. In omni birectangulo angulus obliquus æquator, suo subtendenti lateri.

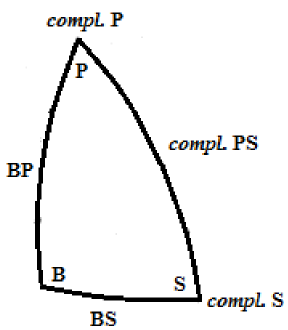
13. Omne Triangulum cujus pars aliqua æquatur quadranti, & angulus aliquis obliquus æquatur suo subtendenti, Birectangulum est.
14. Omne Triangulum habens duas quascunque partes sigillatim quadranti æquales, & tertiam inæqualem, Birectangulum est.
15. Cætera quadrantulia simplicia dicuntur.

[p. 30.]

De simplicibus Quadrantalibus.

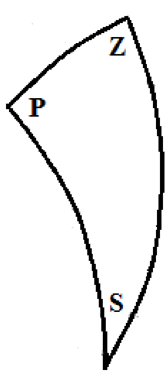
CAP. IV.

1.Quadrantale simplex est, cuius unica tantum pars quadranti æquatur, ceteræ autem quinque partes sunt non quadrantes.
2. Harum quinque partium non quadrantiom, Tresquæ à recto angulo, seu quadrante latere, situ remotiores sunt, in suo complementa convertimus, & retento pristino ordine omnes quinque in circularem, seu pentagonalem situm statuimus, & circulares vocamus.



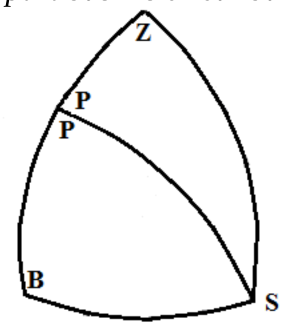
Sit primo triangulum BPS in B rectangulum. Ejus quinque partes obliquæ, seu non quadrantes, sunt hæ. BP latus ambiens rectum. P angulus obiquus alter. PS latus subtendens rectum. S angulus reliquus obliquus. SB reliquum latus ambiens rectú. Pro quibus nos facilioris calculi gratia assumimus latus BP ipsum; complementum anguli P : Complementum lateris PS; complementum anguli S; atque ipsum latus SB, & servato naturali situ has quinque partes ordine statuimus, ut à margine, & circulares vocamus.

Similiter sit secundò triangulum quadrantale simplex, non rectangulum (ex centris solis orientis, poli, & zenth factum) SPZ, in latere ZS quadrantale. Ejus quinque partes non quadrantes pristinæ sunt. Z angulus alter ambitus à latere quadrante. Latus PZdistantia poli à zenith. P angulus subtensus à quadrante. Latus PS distantia poli à Sole, & angulus denique S alter angulorum quos quadrans ambit. Pro



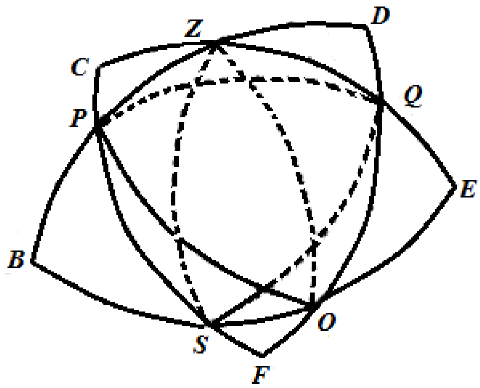
quibus nos ad faciliorem computum nostrum assumimus ipsum angulum Z , seu PZS, qui est arcus plagæ Solis à septentrione. Complenteum PZ, quod est ipsa elevatio poli: Complementum anguli P, seu angulum ZPS [p. 31.] quod est differentia ascensionalis, id est, differentia temporis ortus vel occasus Solis ab hora sexta. Complementú lateris PS quod est Solis declinatio : & angulum ipsum S seu PSZ, quem angulum positionis Solis (respectu scilicet poli & zenith) vocamus. Has quinque partes etiam circulari vel pentagono situ statuimus, ut à margine, & circulares vocamus. Nec aliæ fient circulares partes superioris trianguli rectanguli BPS, si P polum, S solum, & B cardinem borealem seu septentrionalem posueris. Fient enim latus BP elevatio poli, complementum P differentia ascensionalis, Complementum PS declinatio solis, complementum S angulus positionis solis : ac denique BS plaga solis. Quæ sunt eædem prorsus circulares partes, quæ supra, & eodem situ levorsum quo ille dextrorium dispositæ. Et ita in omnibus quadrantalibus tam rectangulis, quam non rectangulis.

Corol. 3 *Hinc sit quod plurima sint triangula in partibus suis naturalibus haud conformia, quæ in partibus his circulibus prorsus conveniunt, & hac nostra circularium methodo resolvuntur.*



Ut satis lucidè apparet in duobus superioribus triangulis BPS, & PZS conjunctis. In quibus omnes naturales partes (præter PS & BS hujus, & PS & PZS illius) prorsus differunt : circulares verò partes omnes (ut suprà dictum est) conveniunt.

4. *Hæc circularium partium uniformitas manifestissimè patet in rectangulis factis in superficie globi ex quinque circulis magnis, quorum primus secet secundum, secundus tertium, tertius quartum, quartus quintum,* [p. 32.] *quintus denique primum ad rectos angulos: reliquæ verò sectiones omnes ad angulos obliquos fient.*



Exempli gratia : Meridianus regionis DB, secat horizontem BE in puncto B. Horizon BE secat circulum EC, qui solem ambit (id est, qui circa solem tanquam polum ducitur) in puncto E. Circulus EC, qui solem ambit, secat meridianum solis CF in puncto C. Meridianus solis CF æquatorem FD in puncto F : & tandem æquator FD, secat meridianum regionis DB in puncto D. Et omnes hæ quinque sectiones in punctis B. E. C. F. D. orthogonaliter & ad rectos angulos fiunt : factis cæteris sectionibus in punctis Z. P. S. O. Q. ad angulos obliquos. Fientque ex his sectionibus rectangula quinque. PBS. SFO, OEQ. QDZ, & ZCP, quorum quamvis partes naturales differat, & in singulis triangulis varientur, circulares tamé quinque partes eædem sunt, quæ supra, absque ullo discrimine.

5.Eadem circularium partium uniformitas, patet etiam in quadrantalibus non rectangulis, factis in superficie globi ex quinque punctis, quorum primus distet a secundol, secundas à tertius, tertius à quarto, quartus à quinto, quintus à primo distantiis & arcubus æqualibus quadranti, aliæ vero punctorum distantiæ inæquales sint quadranti.

Ut in eodem præcedente schemate puncta, P à Q, Q ab S, S ab Z, Z ab O, atque O à P, dostant spatiis quadranti æqualib[9] : at verò P ab Z, Z à Q, Q ab O, O ab S, S à P, distant ab invicem arcubus non quadrantibus. Et fient ex his distantiis quadratantalia non rectangula quinque, PZQ, ZQO, QOS, OSP, & SPZ : quorum quamvis naturales partes differant : partes tamen circulares eædem & immutabiles hic permanent, quæ supra. Scilicet, elevatio poli, differentia ascensionalis, declinatio solis, angulus positionis solis, & plaga solis : quæ omnibus superioribus triangulis [p.33.] ex æquo conveniunt, nec his duntaxat solis, verum etiam omnibus triangulis quæ oriuntur ex inter sectionibus cæteris horum decem arcuum ad integros circulos productorum : quæ plurima & confusa sunt, missa hic facimus. Hac epitome satis est monuisse omnem confusionem naturalium partium, & suarum regularum, his paucis circularibus partibus & sua regula unica evitari, ac tolli.

6. Quinque circularium partium, tres semper in quœstionem cadunt, quarum duœ dantur, tertia quœritur.

7. Atque harum trium una est intermedia, & duœ sunt extremœ, quœ scilicet intermediœ aut circumponuntur, aut opponuntur.

Verbi gratia, Sint partes tres in quæstione propositæ hæ, plaga solis, elevatio poli, & differentia ascensionalis : quarum , elevatio poli pars intermedia dicitur, & reliquæ duæ extremæ ei vicinæ, aut circumpositæ vocantur, verum si tres partes in quæstionem cadétes forent, declinatio solis, elevatio poli, & angulus positionis solis, vocabitur (ut prius) elevatio poli intermedia, sed declinatio solis & angulus positionis solis, extremæ à media remotæ, seu ei oppositæ dicentur. Par ratio est in reliquis quinque.

8. Logarithmus intermediœ œquatur differentialibus circumpositurum extremarum, seu antilogarithmis oppositarum extremarum.

Hoc theorema probatur inductione omnium trium partium seu triplicitarum, quæ ex quinque circularibus partibus quadrantalis prioris BPS rectanguli, constitui possunt, & in quæstionem cadere, posterioris autem non rectanguli PZS triplicitates omittimus, quia ejus omnes partes circulares (ex 18, & 19, & 20 præmissis) eædé prorsus sunt quantitate quæ prioris. Quinque ergo partiu circularium rectanguli BPS, (quæ sunt BS, seu plaga solis orientis : complementum BSP seu angulus positionis solis: complementum SP, seu declinationem solis : complementum SPB, seu differentia ascensionalis : & PB, seu elevatio poli) tres illæ quæ in quæstionem extremarum circumpositarum cadunt, sunt aut primò BS, complem. BSp, & compl. Sp; aut secundò compl. BSP, compl. SP, & compl. SPB : aut tertio [p. 34.] comp. SP, compl. SPB, & PB : aut quartò compl. SPB, PB, & BS : aut quintò sunt PB, BS, & complem. BSP.

Verum quia in omnibus his triplicitatibus, Tangens alterius extremæ est ad sinum rectum intermediæ, ut sinus totus ad tangentem reliquæ extremæ (pro ut vulgaribus demonstrationibus Trigonometriæ patet.) Ideò (per nostras demonstrationibus prop. 5 cap. 2. lib.1) Logarithmi mediarum (qui sunt Logarithmus solius intermediatiæ per corol. 6.def.cap.1.lib.1) æquantur logarithmis tangentium harum extremarum sunt differentiales earumdem (ex sect. 22. & 25. cap.3.lib.1) Logarithmus igitur solius intermediæ æquatur differentialibus circumpositarum extremarum, ut priore parte Theorematis asservimus. Sequitur posterioris partis confirmatio.

Earundem ergo quinque partium circularium, tres illæ quæ in questionem extremarum intermediæ oppositarum cadunt, sunt aut primò PB, comp. BSP, & comp. SPB : aut secundò BS, comp.SP, & PB : aut tertiò compl. BSP, comp. SPB, & BS : aut quartò comp. SP, PB, & cop. BSP : aut quintò denique comp. SPB, BS, & comp. SP.

Sed in omnibus his triplicitatibus seu quinque casibus, sinus rectus complementi alterius extremæ se habet ad sinum rectum intermediæ, ut sinus totus ad sinum rectum complementi reliquæ (quod fusius à Regiomontano, Copernico, Lansbergio, Pitisco, & aliis demostratur, quam ut brevi hac epitome repetendum sit) Ideò per nostras demonstrationes (prop. 5, cap.2. lib.1) Logarithmi complementorum harum extremarum æquantur Logarithmis mediarum, id est, (ut dictum est) Logarithmo solius intermediæ. At Logarithmi complementorum harum extremarum oppositarum sunt earundem ipsarum partium antilogarithmi (ex def.sect.13.& 16.cap.2.lib.1) sequitur ergo in casibus, quod logarithmus solius intermediæ æquetur antilogarithmis suarum extremarum oppositaruum, ut asserit posterior theorematis pars. Totum ataque theorema constat. [p. 35.]Præter hanc probationem per inductionem omnium casuum, qui occurrere possunt, potest idem theorema lucidè perspici ex 19[a] & 20[a] præcedetibus, in quorum schemate, homologa circularium partium constitutio earundem analogiæ similitudinem arguit : ita ut quod de una intermedia & suis extremis circumpositis, aut oppositis verè enuntiatur, de cæteris quatuor intermediis & suis extremis respectivè circumpositis, aut oppositis negati non possit.

Porisma generale.

9.Hinc sequitur in quadrantalibus simplicebus, quod ex duabus partibus quibuscunque datis tertia quavis innotescet. Semper enim aut intermedia quœritur, & ejus logarithmus habetur addedo differentiales circumpositarum extremarum daturum : aut altera extremarum quœritur, & ejus differentialis emergit ex subtractione differentialis reliquœ extremœ datœ à Logarithmo intermediœ notœ : ut quinque prioribus triplicitatibus rectanguli prœcedentis theorematis, & totidem non rectanguli: aut intermedia quœritur, & ejus Logarithmus provenit addendo antilogarithmos oppositarum extremarum datarum : aut denique altera extremarum oppositarum quœritur : & ejus antilogarithmus ex subductione antlogarithmi reliquœ extremœ oppositœ datœ ex Logarithmo intermediœ notœ habetur. Ut in quinque posterioribus casibus rectanguli prœcedentis theoremat is & totidem non rectanguli. Horum autem Logarithmorum, antilogarithmorum, & differentialium jam inventorum cuilibet respondent duo arcus diversarum specierum . Ex specie igitur quœsiti arcus per secundam, tertiam, quartam hujus ,aut per hypothesim nota, ipse arcus verus innotescet.

Ut in priore exemplo septimæ, Tres quæstionis partes circulares sunt, plaga solis, elevatio poli, & differentia ascensionalis, id est, in rectangulo BPS, partes BS, PB, & complem. SP : vel in non rectangulo quadrantali PZS, partes PZS, comp. PZ & compl. SPZ : quarum trium dentur extremæ circumpositæ, scilicet plaga solis orientis BS, vel PZS, 70 gr : & differentia ascensionalis compl.SPB, vel compl. SPZ, 16 gr. 24', 27" : & quæratur intermedia [p. 36.] pars PB, vel compl. PZ, quæ est elevatio poli. Additur ergo differentialis 70. gr., viz, –10106827 ad differentialem 16 gr. 24', 27" & provenient 2119353, Logarithmus 54 graduum pro elevatione polo quæsita.

Admonitio.

Præter elevationem poli hoc modo inventam, habetur etiam secundò eadem praxi plaga solis ex elevatione poli, & angulò positionis solis. Item tertiò angulus positionis solis ex plaga solis, & ejusdem declinatione datis. Quartò declinatio solis ex angulo positionis solis, & differentia ascensionali. Quintò differentia ascensionalis ex declinatione solis, & elevatione poli.

Secundum exemplum.

Detur plaga solis orientis BS, seu PZS, 70. graduum : & elevatio poli 54. graduum, quæ est PB, aut compl. PZ. Quæratur autem differentia ascensionalis, scilicet compl. SPB, vel compl. SPZ, Et, quia hic similiter extremæ partes circumponuntur intermediæ, ergo aufer differentialem plagæ solis seu 70. graduum, qui est – 10406827. ex Logarithmo elevationis poli inde 12226180. differentialis graduum 16. 24' 27", arcus differentiæ ascensionalis quæsitæ.

Admonitio.

Ad hujus exempli imitationé habetur secundò declinatio solis ex differentia ascensionali, & elevatione poli datis. Item tertiò angulus positionis solis ex declinatione solis, & differentia ascensionali. Quartò plaga solis ex angulo positiois solis, & declinatione ejusdem [p. 37.] Quintò elevatio poli habetur ex plaga solis, & angulo positionis solis. Item contrà habetur. Sextò differentia ascensionalis ex declinatione solis, & angulo positionis solis datis. Septimò declinatio solis ex angulo positionis solis, & plaga ejus. Octavò angulus positionis solis habetur ex plaga solis & solis & elevatione poli datis. Nonò plaga solis ex elevatione poli, & differentia ascendionali. Decimò tantem elevatio poli habetur ex differentia ascensionali, & declinatione solis datis.

Tertium exemplum.

In posteriore exemplo ejusdem septimæ tres quæstionis partes circulares proponuntur hæ, declinatio solis, elevatio poli, & angulus positionis solis. Eæ sunt in rectangulo BPS compl. PS. BP & compl. BSP, & in non rectangulo quadrantali PZS, eæ sunt, compl. PS compl. ZP & ZSP. Quarum trium dentur extremæ oppositæ scilicet declinatio solis, quæ est compl. PS 11 gr. 35' 51", & angulus positionis solis, qui est compl. BSP, seu ZSP 34 gr. 19' 21" serè. Et quæratur intermedia pars BP, seu comp. ZP, quæ est elevatio poli. Additur ergo antilogarithmus 11 gr. 35' 51", qui est 206271 ad antilogarithmus 34 gr. 19' 21", qui est 1913082, provenient 2119353, Logarithmus 54 graduum pro elevatione poli quæsita.

Admonitio.

Præter elevationé poli hac jam modo inventa, poteris secundò per eandem praxim habere plaga solis ex ejusdem declinatione, & differentia ascensionali datis. Tertiò angulum positionis solis ex differentia ascendionali & devatione poli. Quartò declinationem solis ex elevatione poli & plaga solis. Et quintò invenies differentiam ascensionalem ex plaga solis & angulo positionis solis datis.

[p. 38.]

Quartum exemplum.

Detur declinatio solis compl. SP. 11 gr. 36' 51", & elevatio poli BP, seu compl. PZ graduum 54 . Quæratur autem angulus positionis solis compl. BSP, seu PSZ : & , quia hic similiter extremæ partes intermediæ opportunitur, igitur auferendus erit antilogarithmus 11 gr.35' 51", qui est 206271 ex logarithmo 54 graduum, qui est 2119353, & supererunt 1913082 antilogarithmus 34 graduum 19', 21" serè, qui sunt angulus positionis solis quæsitus.

Admonitio.

Præter angulum positionis solis hac prima praxi acquisitum, habetur secundò eadem praxi declinatio solis ex dais differentia ascendionali & plaga solis. Tertiò habetur differentia ascensionalis ex datis elevatione poli & angulo positionis solis. Quartò elevatio poli invenitur ex plaga solis & ejusdem declinatione datis. Quintò plaga solis acquiritur ex angulo positionis solis & differentia ascensionali. Sextò (cótrario ordine) angulus positionis solis invenitur ex plaga solis & differentia ascensionali datis. Septimò declinatio solis habetur ex angulo positionis solis, & elevatione poli datis. Octavò differentia ascensionalis ex declinatione poli, & ejusdem plaga invenitur. Nonò elevatio poli habetur ex data differentiæ ascensionali, & angulo positionis solis. Decimò tandem acquiritur plaga solis, ex elevatione poli, & declinatione solis datis.

Atque ita ad imitationem horum quatuor exemplorum, triginta variæ solvuntur quæstiones in quadrantali rectangulo, & totidem in non rectangulo solvuntur hoc porismate, beneficio unius tantummodo additionis vel [p. 39.] subtractionis. Cæterùm ad intelligentiam posterioris partis hujus porismatis, de arcuum speciebus, vide exempla, (tertium, quartum, quintum, & sextum) sequentia.