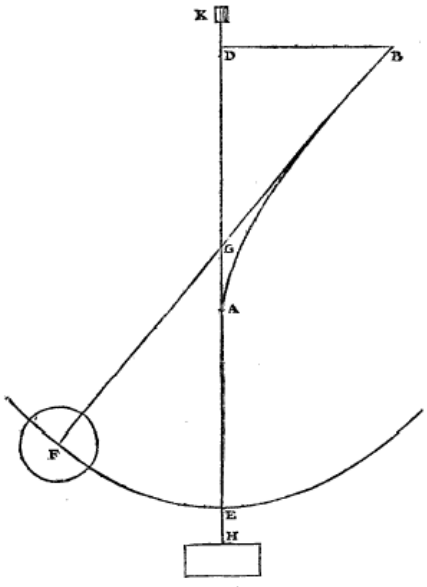
HOROLOGII OSCILLATORII

PART FIVE. [p. 157]

In which the construction of another kind of clock is considered from the motion of a circular pendulum; and Theorems on Centrifugal Force.

There is another kind of oscillatory motion, besides that which we have been using until now. Truly of that kind where the pendulum weight is carries round in a circle. Thus we have deduced the construction of another kind of clock, almost at the same time as the other; and likewise based on the same principles of equal times; but used less frequently on account of the ease and simplicity of construction of the other kind. Moreover, many clocks of this kind have been constructed successfully. For these, the final hand is seen to be carried around in a continued and equal motion, designating the seconds; while for our first clock, and all other clocks, the hand is carried around in small jumps. Likewise here too, there is an absence of all the din and noise arising from the working of the clock; though for astronomical observations, the sound of the single seconds ticking away is something useful that one would not be without. And indeed, I had put in place a description of these clocks with the these other matters to be published at last, which have been delayed, concerning circular motion and what can be called centrifugal force; for which it may be said I have many arguments, and these may follow this work if leisure time permits. But, in order that the eager may enjoy the more mature of these new labours without either useless speculation or by some unforeseen cause that may intercede and prevent further publication, I have added on also to this part, in addition to what has been resolved, a short explanation of the making of this second kind of clock, and likewise the theorems expounded pertaining to centrifugal force; and the demonstration of these are left for another time.



The Construction of the second kind of clock.

I am not lead by necessity to show here the arrangement of the wheels, from which the inside of this clock are constructed, since that can be easily put in place by clockmakers [p. 158], and it can be changed in a number of ways; for it is sufficient to explain that part by which the motion is regulated by a certain method. The figure shows this part of the clock.

DH is understood to be erected perpendicular to the horizontal, and to be capable of rotating about the two end poles. . To this line a plate is fixed at A, with the length some given amount, and with the curve following the line AB; which is that paraboloid we have shown in Prop. 8 of part

3; from the evolution of this, after a certain line was connected to it, describes a parabola. Here this line is AE; truly the parabola, described from the evolution of the whole length BAE, refers to the line EF. A string BGF is applied to the curve BA, the end of which describes the parabola. A weight is attached to that point F. Moreover, while the axis DH is turning about itself, the string BGF, extended in a straight line, thus describes a horizontal circule, by leading a small sphere around; which will be greater or less, according to the size of the force making it move from the axis DH, generated by the wheels of the clock acting on the drum K: but in such a way that all the motions are contained on the surface of the parabolic conoid. And from this circuits of the same time always emerge, [p. 159], as will be apparent from what we will say about the motion.

For if we wish to observe single orbits of half seconds, it is required that the latus rectum of the parabola EF is $4\frac{1}{2}$ inches of our horarium feet, that is half the length of the pendulum, to which the individual half second swings are near. But from the latus rectum, a length hangs from the side of the paraboloid AB; that indeed is $\frac{27}{16}$ of that length: and likewise of the length AE, which is half the length of the latus rectum of the parabola. If truly we want single seconds for the times of the orbits of the pendulum, four times the previous length of both the latus rectum and the line AE are to be taken.

Again, although we have described the string BGF up to now as just a single string, knowing the outstanding length, so that the string above can be made double, and the ends meet at F in an angle of some 20 or 30 degrees. The end of the plate AB at B must be wide enough, that it is sufficient for the spreading apart of the strings, or itself to be made in two parts. For with this agreed upon, the circular motion of the weight F is made continuous, free from any other support, and each attached string is extended in a straight line; which cannot be done, if the pendulum is only stretched by a single string. Where nevertheless that force supplied for the movement, either by the wheels of the clock or a weight, or by some other force, is required to be known for the continuation of this circular motion. Which force truly comes to the axis KH from the drum K, and after a little struggle, it is maintained once imparted to the sphere F.

Moreover, so that this can be easily done, it is necessary that the drum KH can rotate freely. Since by no better reason than to have been thoroughly verified, that the lower part should be made from hard steel, and by supposing that it has the plane surface of a diamond; of which the smallest particle is sufficient, placed in a small hole in the lower plate.

For the rest, in place of the string BGF, for the section that must be in contact with the curve AB, it is best to use a thin chain made from gold or from some other metal, by which the length is kept from changing. And this we have found to be the case with the first clock too, when the pendulum has been suspended between the cycloids. But there the continuous flexing of the chain, from the friction of the rings which were very small, impeded the free movement of the pendulum.

Theorems Concerning Centrifugal Force, Arising from Motion in a Circle.

1.

If two small equal moving objects travel round unequal circles in equal time intervals ; the ratio of thecentrifugal force on the circumference of the greater circle to that on the smaller [p. 160]***, is thus as the ratio of the circumferences or diameters of these circles.***

11.

If two small equal objects are carried around the circumferences of unequal circles with the same speed, then the centrifugal forces of these are in the inverse ratio of the diameters.

111.

If two small equal objects are carried around equal circumferences with unequal speeds, but each maintains a constant motion, such as we wish to be understood in these theorems, then the centrifugal force of the faster, to the force of the slower, is in the ratio of the squares of the speeds.

1V.

If two equal small bodies are carried around the circumferences of unequal circles with equal centrifugal forces, then the time to complete a revolution in the greater circumference, to the time to complete a revolution in the lesser circumference, is in the inverse square ratio of the diameters.

V.

If a small object is carried aroung in the circumference of a circle with that speed acquired by falling from a height, which is equal to a quarter of the diameter; then the object will have a centrifugal force equal to its own weight ; that is, the rope is stretched from the centre by the same force by which it is suspended.

V1.

On the hollow surface of a paraboloid of a cone, which has a vertical axis, all the circular orbits of small objects, travelling around circumferences parallel to the horizontal, however small or large thay may be, are carries out in equal times : which individual times are equal to two oscillations of the pendulum, the length of which is equal to half the latus rectum of the generating parabola.

V11.

If two small moving objects, suspended from strings of unequal lengths, are rotating thus so that they travel round the circumferences of circles parallel to the horixontal, and with the other ends of the strings kept fixed **[on the axis]**; ***then by this motion the strings describe a conoidal surface, moreover if the altitudes of the cones are of equal height then the times of completing the rotations are also equal.***

V111.

If two moving bodies, as previously, are rotating by moving in a cone, suspended by either equal or unequal strings; and if the altitudes of the cones are unequal then the times to complete rotations are in the inverse square ratio of the altitudes.

1X.

If a pendulum, by moving laterally on the surface of a cone, make circuits which are minimally different; then the individual times of these, to the time to fall vertically from twice the height of the pendulum, have the same ratio as the circumference of a circle to the diameter : and hence the periods of the two smallest lateral oscillations of the pendulum are equal.

X.

If a small object is carried round the circumference of a circle with the length of the pendulum equal to the radius, for which the time for the individual orbit is solved, then on solving for the motion on the cone for the smallest displacement, the centrifugal force is equal to the weight of the object, and there are two small lateral oscillations possible.

X1.

For any kind of pendulum, with lateral movements on the cone, the orbiting times are equal to the times to the time to fall vertically from a height equal to the length of the string, for which the angle of inclination of the string to the horizontal plane is 2 degrees and 54 minutes approximately. Truly being exact, if the ratio of the sine of the given angle to the radius, is as the square inscribed in the circle to the square of the circumference of this circle.

X11.

If two pendulums with equal weights, but with the lengths of the strings unequal, are rotating with conical motion, and the heights of the weights on the cones are equal; then the forces by which their strings are stretched are in the same ratio as the lengths of the strings.

X111.

If a simple pendulum is given the maximum swing from the side, that is, if the pendulum falls through the whole arc of the quadrant; when it arrives at the lowest point of the circumference, it pulls its string with a force more than three times the force present when it was simply suspended.

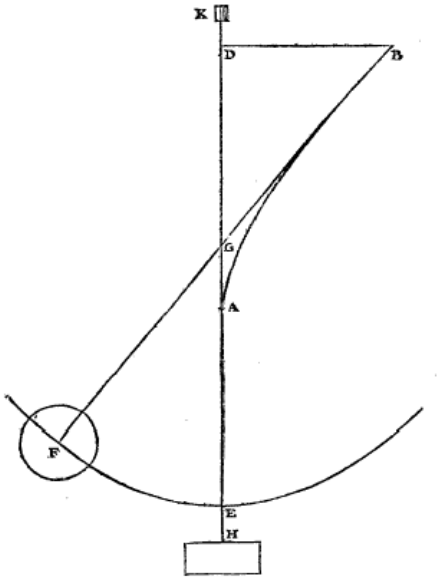
THE END.

HOROLOGII OSCILLATORII

PARS QUINTA.

Constructionem aliam, e circulari pendulorum motu deductam, continens; & Theoremata de Vi Centrifuga.

Est & aliud Oscillatorii motus genus, praeter id quod hactenus pertractavimus. Ejusmodi nempe, quo, per circuli ambitum, pendulum pondus circumfertur. Unde aliud quoque horologii commentum deduximus, eodem fere tempore quo prius illud;; sed cujus usus minus percrebuit, propter alterius illius constructionem, quodammodo simpliciorem facilioremque. Plura tamen hujus quoque generis de quo nunc loquimur, nec sine successu, constructa fuere : estque in his singulare illud, quod continuo atque aequabili motu circumferri cernitur index postremus, qui secunda scrupula designat; cum in priore nostro horologio, omnibusque aliis, subsultim quasi feratur. Item hoc quoque, quod absque strepitu, sonoque omni, moveantur hac ratione constructa automata. quanquam, ad observationes astronomicas, sonus ad singula secunda scrupula repetitus, utilatate non careat. Et constitueram quidem, descriptionem horum cum iis demum edere, quae ad motum circularem & Vim Centrifugam, ita enim eam vocare libet, attinent; de quo argumento plura dicenda habeo, quam quae hoc tempore exequi vacet. Sed, ut nova nec inutili speculatione maturius fruantur harum rerum studiosi, neve casu aliquo intercidat, hanc quoque partem, praeter destinatum, caeteris adjunxi, qua machinae hujus fabrica breviter exponitur, simulque Theoremata traduntur, ad vim centrifugam pertenentia; demonstratione ipsorum in aliud tempus dilata.



Horologii secundi constructio.

Non necessarium duxi, ut rotatum, quibus interiora horologii constant, dispositionem hic exhiberem; cum ea ab artificibus facile [p. 158] ordinari, variisque modis mutari possit; sed eam partem explicari satis esse, quae motum ejus certa ratione moderatur. Cujus partis hic figura expressa est.

Ad axis DH ad horizontalem erectus intelligendus est, ac super polis duobus mobilis. Huic ad A affixa est lamina, latitudine aliqua praedita, curvataque secundum lineam AB; quae est paraboloides illa de qua ostendimus, propos. 8. partis 3, evolutione ejus, postquam ipsi recta quaedam juncta fuerit, describi parabolam. Ea recta hic est AE; parabolam vero, ex evolutione totius BAE descriptam, refert linea EF. Filum curvae BA applicatum, cujus extremo puncto parabola describitur, est BGF. Pondus illi affixum F. Dum autem axis DH in sese vertitur, filum BGF, in rectam

lineam extensum, sphaerulum F una circumducit, ita ut circulos horizontali paralleros percurrat; qui majores minoresque erunt, prout majori aut minori vi axis DH, ab rotis horologii in tympanidium K agentibus, incitabitur : sed ita, ut omnes in superficie conoidis parabolici contineantur. Atque hoc ipso aequalia semper circuitus tempora evadent [p. 159], ut ex iis, quae de hoc motu postea dicemus, apparebit.

Quod si circuitus singulos, secundorum scrupulorum semisses notare vilimus, oportet latus rectum parabolae EF esse $4\frac{1}{2}$ unciarum pedis Horarii nostri, hoc est dimidium longitudinis penduli, cujus singulae oscillationes semiscrupulum secundum impenderent. Ex parabolae autem latere recto, pendet magnitudo lateris recti paraboloidis AB; quippe quod illius $\frac{27}{16}$ continet : atque item longitudinis AE, quae lateris recti parabolae dimidium est. Si vero secunda scrupula unoquoque circuitu expleri desideremus, quadrupla priorem accipienda sunt, tum latera recta, tum linea AE.

Porro, etsi filum BGF veluti unicum ac simplex hactenus designavimus, sciendum tam longe praestare ut parte superiori duplex sit, ac versus F in angulum coeat, 20 vel 30 partium. In quem finem & laminae AB latitudo ad B tanta esse debet, quanta isti filorum divarications sufficit, vel & ipsa bifida facienda. Hoc pacto enim motus circularis ponderis F, absque alio ullo adminiculo, continuatur, ac filum utrumque sibi annexum in rectum extendit; quod non faceret, si unico tantum filo teneretur. Ubi tamen vim illam ab horologii rotis, vel pondere vel alia potentia motis, ad continuationem hujus motus circularis requiri sciendum. Quae nempe vis per tympanidium K ad axem KH pervenit, ac minimo nisu, motum sphaerae F semel inditum, conservat.

Hoc autem quo facilius possit, liberrimam axis KH revolutionem esse oportet. Quod nulla ratione melius perfici compertum, quam si, parte sui ima, durato chalybo constet, suppositamque habeat adamantis superficiem planam; cujus minima quaevis particula hic sufficit, subter laminam perforatam collocanda.

Caeterum in locum fili BGF, qua parte curvae AB applicari debet, catenulum tenuem ex auro, aliove metallo, adhibere licebit, quo melius invariata servetur longitudo. Atque hoc in priore quoque horologio, ubi pendulum inter cycloides suspensum est, experti sumus. Sed ibi flexus catenulae continuus, attritu annulorum, perexiguo licet, non parum impedit liberam penduli agitationem.

DE VI CENTRIFUGA ex motu circuli, Theoremata.

1.

Si mobilia dua aequalia, aqualibus temporius circumferentias inaequales percurrant; erit vis centrifuga in majori circumferentia, [p. 160] ***ad eam quae in minori, sicut ipsae inter se circumferentiae, vel earum diametri.***

11.

Si duo mobilae aequalia, aequali celeritate ferantur, in circumferentiis inaequalibus; erunt eorum vires centrifugae in ratione contraria diametrorum.

111.

Si duo mobilae aequalia in circumferentiis aequalibus ferantur, celeritate inaequali, sed utraque motu aequabili, qualem in his omnibus intelligi volumus; erit vis centrifuga velocioris, ad vim tardiorism in ratio duplicata celeritatum.

1V.

Si mobila duo aequalia, in circumferentiis inaequalibus circumlata; vim centrifugam aequalem habuerint; erit tempus circuitus in majori circumferentia,ad tempus circuitus in minori, in subdupla ratione diametrorum.

V.

Si mobile in circumferentia circuli feratur ea celeritate, quam acquirit cadendo ex altitudine, quae sit quartae parti diameter aequalis ; habebit vim centrifugam suae gravitati aequalem ; hoc est, eadem vi funem quo in centro detinetur intendet, atque cum ex eo suspensum est.

V1.

In cava superficie conoidis parabolici, quod axem ad perpendicularum erectum habeat, circuitus omnes mobilis, circumferentias horizonti parallelas percurrentis, sive parvae sive magna fuerint, equalibus temporibus peraguntur : quae tempora singula aequantur binis oscillationibus penduli, cujus longitudo sit dimidium lateris recti parabolae genitricis.

V11.

Si mobila duo, ex filis inaequalibus suspensa, gyrentur ita ut circumferentias horizontali parallelas percurrant, capite altero fili immoto manente; fuerint autum conorum, quorum superficiem fila hoc motu describunt, altitudines aequarum; tempora quoque circulationum aeqalia erunt.

V111.

Si mobila duo, uti prius, motu conico gyrentur, filis aequalibus vel inaequalibus suspensa; fuerintque conorum altitudines inaequales; erunt tempora circulationum in subduplicata ratione ipsarum altitudinum.

1X.

Si pendulum, motu conico latum, circuitus minimos faciat; eorum singulorum tempora, ad tempus casus perpendicularis ex dupla penduli altitudine, eam rationem habent, quam circumferentia circuli ad diametrum : ac proinde aeqalia sunt tempori duarum oscillationum lateralium, ejusdem penduli, minimarum.

X.

Si mobile in circumferentiae feratur, circuitusque singulos absolvat eo tempore, quo pendulum, longitudinem semidiametri circumferentiae ejus habens, motu conico circuitum minimum absolveret, vel duplicem oscillationem minimam lateralem : habebit vim centrifugam suae gravitati aequalem.

X1.

Penduli cujuslibet, motu conico lati, tempora circuitus aequalia erunt tempori casus perpendicularis, ex altitudine penduli filo aequali; cum angulus inclinatinis fili, ad planum horizontalis, fuerit partium 2 scrup.54, proxime. Exacte vero, si anguli dicti sinus fuerit ad radium, ut quadratum circulo inscriptum ad quadratum a circumferentia ejus.

X11.

Si pendula duo, pondere aequalia, sed inaequali filorum longitudine, motu conico gyrentur, fuerintque conorum altitudines aequales ; erunt vires, quibus fila sua intendent, in eadem ratione quae est filorum longitudinis.

X111.

Si pendulum simplex oscillatione laterali maxima agitetur, hoc est, si per totum circuli quadrantem descendat; ubi ad punctum imum circumferentiae pervenerit, triplo majori vi filum suum trahet, quam si ex illo simpliciter suspensum foret.

FINIS.