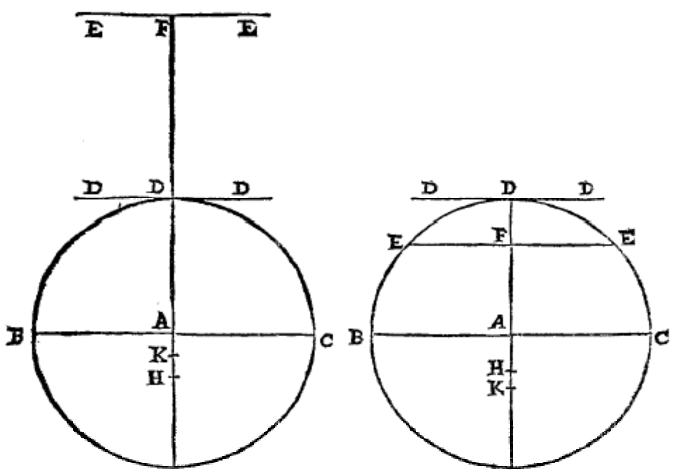
HOROLOGII OSCILLATORII

PART FOUR b. [p. 126]

PROPOSITION XXI.

How the centre of oscillation can be found for plane figures.

By understanding what has been demonstrated up to this point, it is now easy to define the centre of oscillation for a large number of figures, which are accustomed to be considered in geometry. And in the first place, concerning the plane figures that we have discussed above, we have defined two kinds of motion for these oscillations; that is, either about an axis placed in the same plane as the figure, or about an axis set up at right angles to the plane of the figure. The first of these we call motion into the plane, and the second lateral motion, or to the side.



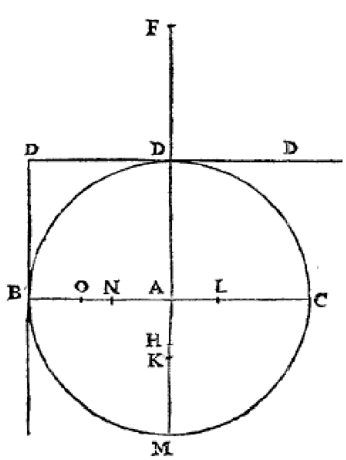
For the first method of disturbance, truly where the motion is about an axis placed in the same plane as the figure BCD about the axis EF, as shown in the diagram : here, a wedge is considered drawn on the figure, and cut by a plane which passes through the plane of the figure, which is in this case through DD, in order that the line of intersection is parallel to the axis of the oscillation ; and the distance of the centre of gravity from this intersection is given, as here AD; and likewise the subcentre of the said wedge upon the same intersection, which here is given as DH. The centre of oscillation K of the figure BDC is found by dividing the rectangle DAH by the distance FA; since from that division the distance AK is generated, which is the distance the centre of oscillation lies below the centre of gravity. For the rectangle DAH, multiplied by the number of elemental areas of the figure BDC, is equal to the sum of the squares of the distances of these elements from the line BAC, which passes through the centre of gravity [p. 127] A, drawn parallel to the axis of oscillation EE [by Prop.10, of this section]. Whereby, on dividing the same rectangle by the distance FA, the distance AK is found, by which the centre of oscillation lies below the centre of gravity A. [by Prop. 18, of this section.]

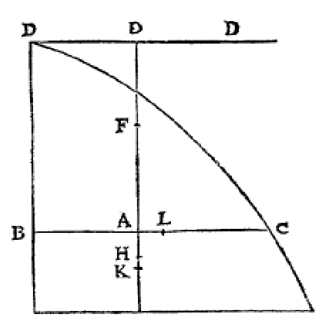
Thus it has been shown that for the axis of oscillation DD, the point H corresponding to the centre of oscillation is the same as the subcentre of the wedge cut by the plane through DD ; and thus DH is the length of the simple pendulum isochronous to the figure BCD. One of these results has been noted by others before, yet not demonstrated. Moreover, it is not our intention to pursue just how the centre of gravity of the wedge constructed on a plane figures is to be found, as these are known in many cases now. Just as, if the figure BCD is a circle, then DH is equal to $\frac{5}{8}$ th of the diameter. If the figure is a rectangle, then DH = $\frac{2}{3}$ of the diameter. Thus the same reasoning also applies for a rod, or for a weighted line, as previously discussed, which is suspended from either end, which is isochronous with a pendulum two thirds of its length. For indeed a line of this kind can be considered to be a rectangle of the smallest width.

Now, if the figure is a triangle, with a vertex pointing up, then DH shall be $\frac{1}{4}$ of the diameter. If a vertex is pointing downwards, then it is $\frac{1}{2}$ of the diameter.

Moreover, as by proposition 16, it was shown that the motion of any kind of plane figure could thus be found. Truly, we can give the figure BCD any one of a number of positions, by inverting it around the axis BAC, or placing it parallel to the horizontal, or being inclined at an angle, as long as the same axis FE is kept in place, then the length of the isochronous pendulum FK also stays the same. This has been shown by that proposition.

Again, when a plane figure is set in motion about an axis erected at right angles to to the plane of the figure ; which we have defined as lateral motion; as if the figure BCD itself is moving about the axis, which is understood to be erected through the point F in the plane DBC; now in this case, as well as the wedge upon the figure which is cut by the plane drawn through the line DD, and tangent to the figure at the highest point, another wedge is also to be considered, which is cut by the plane through the line BD which is a tangent to the figures at the side, and which is at right angles to the tangent DD. Thus it is necessary to be given, besides the centre of gravity A of the figure and the subcentre HD of the first wedge, also the subcentre of the second wedge LB. Thus indeed the rectangles DAH and BAL are known, which added together here gives the area to be divided, which





then can also be called the rectangle of oscillation. This rectangle, divided by the distance

FA, gives the distance AK, by which the centre of oscillation lies below the centre of gravity A.

If indeed FA is the axis of the figure BCD for the wedge cut by the line [p. 128] BD upon the whole figure, then it is possible for the wedge upon the half figure DBM to be taken cut by the plane through DM. For if the subcentre of this wedge upon DM is OA, then the distance of the centre of gravity of the plane figure DBM from the same line DM shall be NA, and it is agrees that the rectangle OAN is equal to the rectangle BAL [by Prop. 12, of this section]. Thus the rectangle OAN added to the rectangle DAH constitutes the area to be divided by the distance FA, in order to give the distance AK.

Certainly the demonstration of these has been shown in the preceding, obviously as when the rectangles DAH and BAL, or DAH and OAN, are multiplied by the number of elemental parts of the figure, then they are equal to the sum of the squares of the distances from the centre of gravity A; or, which is the same here, from the axis of gravity to the parallel axis of oscillation; and hence the said rectangle divided by the distance FA gives the length of the interval AK [by Prop. 18 of this section; this completes the summary of Huygens' method for plane shapes].

The Centre of Oscillation of the Circle.

For with a circle, certain rectangles DAH and BAL are shown to be equal to each other and these together give rise to half the square of the radius. Hence, if FA is taken equal to the radius AB, thus as one ratio to the other, from which AK is half the radius, the distance from the centre of gravity to the centre of oscillation. If therefore the circle is set in motion from an axis through D taken in the circumference, then DK is equal to three quarters of the diameter DM.

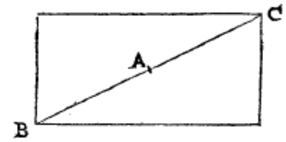
[From symmetry, DAH = BAL, and with AH = $\frac{1}{4}r$, then 2 × rect.DAH = $\frac{1}{2}r^2$; hence, from the general result : 2 × DA.AH = FA.AK, giving AK = $\frac{1}{2}r^2/r = r/2$;
or DK = $3r/2$ = 3/4 × diameter. This result is in agreement with modern theory. We will not verify the rest that follow, but there is no reason to expect them to be incorrect, as the theory developed is consistent with a modern analysis.]

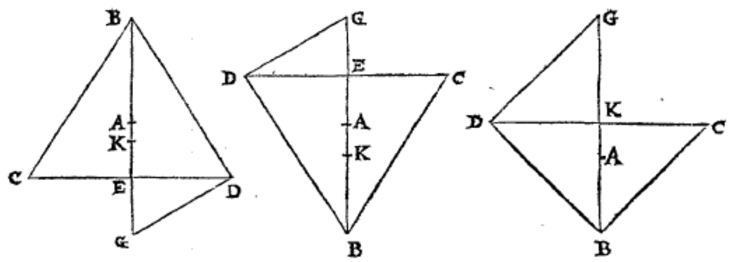
In this manner we have enquired about the centres of oscillation of the following plane figures, which can be simply written down. As surely, [p. 129]

The Centre of Oscillation of a Rectangle.

For any rectangle, such as CB, the area to be divided, or the area of the rectangle of oscillation, is found to be equal to the third part of the square of the semidiagonal AC. Hence it follows, if the rectangle is suspended from some angle, and the rectangle is set in motion laterally, then the simple pendulum isochronous to that is equal to $\frac{2}{3}$ of the length of the whole diagonal [See diagram on next page].

The Centre of Oscillation of an isosceles Triangle.





In an isosceles triangle CBD of this kind with the vertex pointing up, the area to be divided is equal to the sum of the 18^{th} part of the square of the diameter BE and the 24^{th} part of the square of the base CD. Hence, if DG is drawn from the base angle, perpendicular to the side DB, which crosses the diameter BE produced in G; and A is the centre of gravity of the triangle ; with the interval GA divided into four equal parts, and one of these AK can be placed next to BA; then BK is the length of the isochronous pendulum, if the triangle is suspended from the vertex B. Moreover, when the triangle is suspended from the mid-point of the base E, the length of the isochronous pendulum EK is equal to half the length of BG.

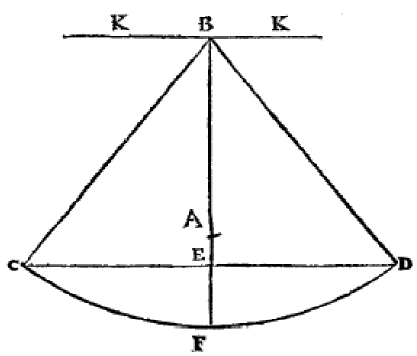
Hence it is evident that when a right-angled isosceles triangle is suspended from the mis-point of the base, the length of the isochronous pendulum is equal to its own diameter. And similarly, if it is suspended from its own right angle, the isochronous pendulum is the same length.

The Centre of Oscillation of a Parabola.

In the case of the section of a parabola bounded by a line at right angles to the axis, the area to be divided is equal to the sum of the $\frac{12}{127}^{th}$ part of the square of the axis, together with the fifth part of the square of half the base. When [p. 130] the parabola is suspended from the vertex point, the length of the isochronus pendulum is found to be the sum of $\frac{5}{7}$ of the axis and $\frac{1}{3}$ of the above side. Truly when it is suspended from the mid-point of the base, the length of the isochronous pendulum is $\frac{4}{7}$ of the axis and $\frac{1}{2}$ the length of the above side.

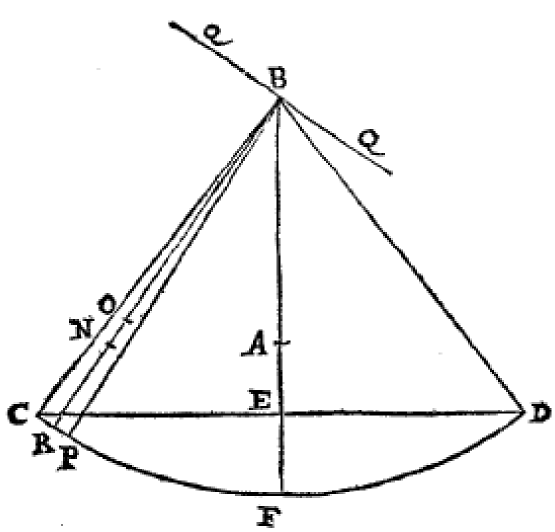
The Centre of Oscillation of the Sector of a circle.

In the sector or a circle BCD, if the radius BC is called r : the semi-arc CF, p : and the semi-chord CE, b : then the area to be divided is equal to $\frac{1}{2}rr - \frac{4bbrr}{ppp}$, that is, half the square of BC, less the square BA; by putting A equal to the centre of gravity of the sector, and indeed BA $= \frac{2br}{3p}$.



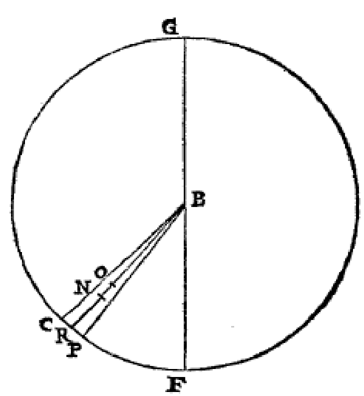
Moreover,if the sector is suspended from B, from the centre of its own circle, the length of the isochronous pendulum is $\frac{3pr}{4b}$, that is, three quarters of the line, which is to the radius BF as the arc CFD to the chord CD. Moreover this is found from the known subcentre of the wedge; as that cut from above the whole sector, by the plane drawn through BK parallel to the semichord CD, the subcentre of this wedge upon BK we find to be : $\frac{1}{8}r - \frac{3}{8}a + \frac{3pr}{8b}$, and called the versed sine BF; as of that upon the half sector BFC that is cut by the plane through BF, the subcentre of the wegde upon BF we find to be $\frac{3}{8}b - \frac{3br}{8a} + \frac{3pr}{8a}$.

But also by another way, the centre of oscillation of the sector is more easliy found, which is in this manner. It is understood that the sector BCP is the smallest part of the sector BCD, which can have triangles in place of segments. Moreover the squares of the distances of these elemental parts from the point B, are equal to the squares of the distances from the line BR, by dividing the sector in two, added together with the squares of the distances from the line BQ, which is at right angles to BR. But, the ratio of these squares to those given is greater, since the angle CBP is very small; and thus these have been taken as zero. [p. 131]



The position of BO is two thirds of BR, that is, the position O of the centre of gravity of the triangle BCP; and BN is a third of a quarter of BR; as surely N is the centre of gravity of the wedge, drawn upon the triangle BCP cut by the plane through BQ. From these put in place, the square is constructed, from the distances of the elemental triangles BCP to the line BQ, equal to the rectangle NBO multiplied by the number of the same elemental triangles. Thus the rectangle NBO, thus muliplied, is agreed to be equal to the squares of distances from the point B of the elemental triangle BCP. Moreover the squares of these distances, to the squares of the sum of all the distances of the whole sector BCD, as the sector BCP to the sector BCD, that is, as the number of elemental sectors BCP, to the number of elemental BCD; this indeed is easily understood to be the number of equal sectors BCP into which the sector BCD is divided. Hence the rectangle NBO, multiplied by the number of elemental sectors BCD, is equal to the sum of the squares of the distances of the elemental areas of these from the point B. Thus the rectangle NBO, divided by BA, the distance between the point of suspension and the centre of gravity of the sector, gives the length of the isochronous pendulum, when the sector is suspended from B, [by Prop. 17 of this section]. Moreover the rectangle NBO = $\frac{1}{2}rr$: and the distance BA, as we have said before now, is equal to $\frac{2br}{3p}$. Hence, with the division performed, the length $\frac{3pr}{4b}$ for the length of the isochronous pendulum is found, as was likewise found before.

The Centre of Oscillation of the Circle, in another way from above.

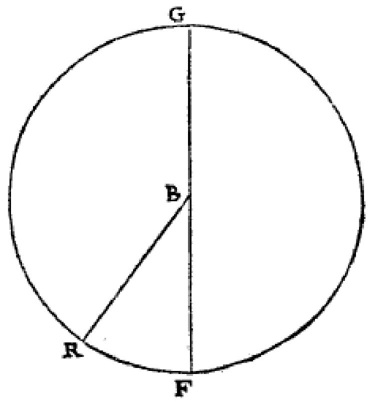


It is also possible to find the centre of oscillation of the circle in the same most simple way. For the circle shall be GCF, the centre of which is B; and an elementas sector in the circle is understood to be BCP, thus as in the sector BCD. [p. 132]

Therefore when, according to the method of exposition, the sum of the squares from the distances of the elemental sectors BCP to the centre B, is equal to the rectangle NBO, that is , to half the square of the radius, multiplied by the number of elemental sectors; moreover the circle is composed from this number of sectors ; hence the squares are from the distances of the elemental areas of the whole circle to the centre B, equal to half the square of the radius, multiplied by the number of these elemental parts of the circle.

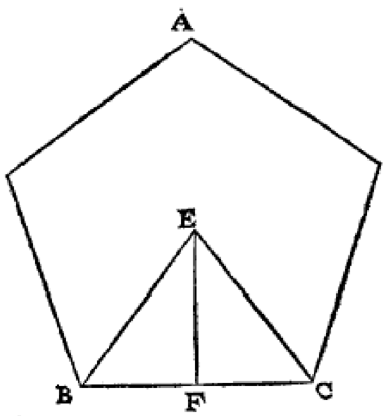
Moreover B is the centre of the gravity of the circle. Hence the said half of the square of the radius, here will be the area to be divided by the distance between the point of suspension and the centre B, in order to find the interval between the centre of oscillation and the centre of the circle B [by Prop. 18 of this section]. Which thus demonstrates the equivalence of this approach with that earlier.

The Centre of Oscillation of the circumference of a circle.



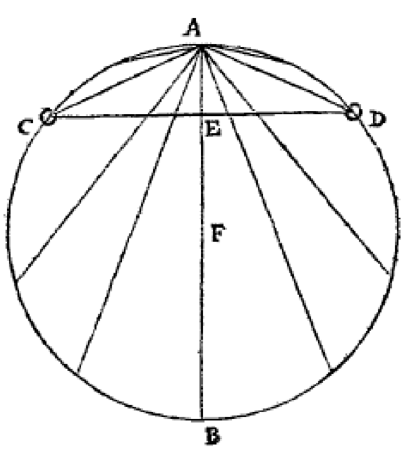
The centre of oscillation of the circumference of a circle is easily found according to this arrangment. [p. 133] Indeed the circumference of the circle is to be described with centre B and radius BF. Therefore the square BR, multipled by the number of elemental lengths into which the circumference is understood to be divided, is equal to the sum of the squares of the distances of the elemental lengths from the centre B. Whereby the square BR is this area to be divided [by Prop. 18 of this section]. Hence it is apparent that if the circle is supended from a point on the circumference G, then the length of the isochronous pendulum is equal to the diameter GF.

The Centre of Oscillation of Regular Polygons.



In the same manner, the isochronous pendulum for some regular polygon such as ABC can be found. For the area to be divided can be constructed, equal to half of the square of the perpendicular from the centre of the polygon to the side of the polygon, together with the 24th part of the square of the side. But, if the pendulum isochronous to the perimeter of the polygon is sought, the area to be divided is equal to the square of the perpendicular from the centre to the side, together with the12th part of the square of the side.

The use of other [equivalent] rectangles and volumes in this theory.



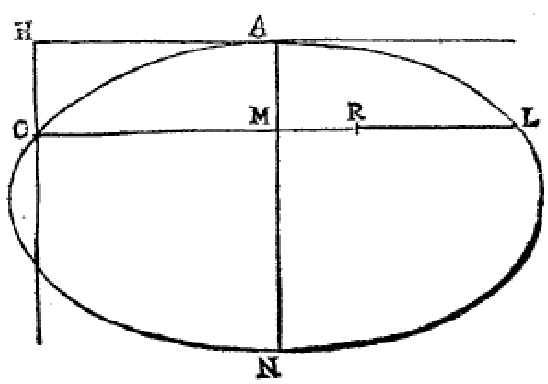
It is pleasing besides to consider others bodies in place of the original ones that constitute the same pendulum. As if for example the proposition may be, for some given point of suspension A, and for a length AB, to find the position of two equal weights C and D, equally placed from A and from the perpendicular distance AB, which set in motion about the axis through A, perpendicular to the plane through ACD, are isochronous to the given simple pendulum of length AB.

Putting AB = a, and with CD drawn which cuts AB at right angles in E, the indeterminate AE = x : EC or ED = y. Hence the square AC = $xx + yy$. This indeed multiplied by the number of elemental parts of the weights C and D, which are here understood to be minimal in size, is equal to the squares of the distances of the same elemental parts from the axis [p. 134] of suspension A. Hence the square AC, or $xx + yy$, divided by the distance E, which truly is between the axis of suspension and the centre of gravity of the weighes C and D, gives $\frac{xx+yy}{x}$, the length of the isochronous pendulum [by Prop. 17 of this section]; that therefore is necessary to be equal AB or a. Thus $\frac{xx+yy}{x} = a$. And $yy = ax - xx$. Thus it is apparent that the position of the points C and D is on the circumference of the circle, the centre of which is F, where AB is divided in two, with the radius $\frac{1}{2}a$, or FA. Hence, whenever two equal weights are placed on the circumference ABCD, equidistant from A, and that are set in motion about A, they are isochronous to a pendulum having a length equal to the diameter AB.

Thus it is also observed, if the circumference ACBD is divided into two equal parts by the points A and B, and a weight is divided between A and B in some ratio, and the circle is set in motion about an axis through A, then it will be isochronous to the same simple pendulum AB.

Truly let there be an exemple of this kind for solids. AN is a weightless rigid line. It is proposed that to some acceptable point on the line, such as M, another line or rod OML with a given weight, is joined at right angles, and this line is bisected at M. From the point of suspension A, this pendulum is set in motion from the side, and the oscillations will be isochronous with that of a simple pendulum of length AN.

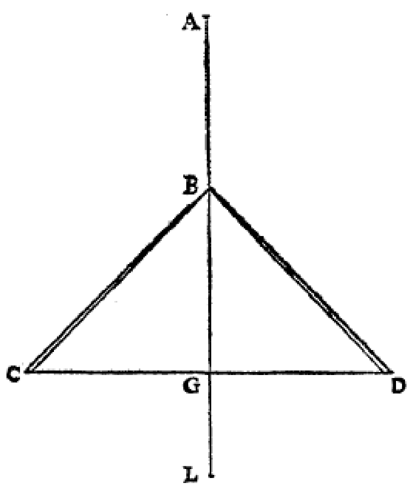
OH is drawn parallel to AN, and AH is parallel to OM, and OR is equal to $\frac{2}{5}$ OL. Thus the subcentre of the wedge constructed upon the line OL, and cut by the plane drawn through OH, will be OR. But the subcentre of the other wedge upon the same OL, cut by the plane through the line AH, is itself AM (for the wedge here is no more than a rectangle). Whereby that rectangle, that above we have called the rectangle of oscillation, is only the rectangle OMR; since truly, on division by the length AM, the distance OL of the centre of oscillation is given from the point of suspendion A, below the point M. [p. 135]



In now AN = a ; AM = x; MO or ML = y. Hence the rectangle OMR = $\frac{1}{3}yy$, from which by division by AM, there is given $\frac{1}{3}\frac{yy}{x}$, which thus ought to be equal to the length of MN, when we want the centre of oscillation of the rod to be in N. Hence the equation is formed : $\frac{1}{3}\frac{yy}{x} + x = a$. Thus $y = \sqrt{3ax - 3xx}$. Which indicates that the points O & L lie on an ellipse, of which the line

AN is the minor axis; and indeed the latus rectum is three times AN, according to the division of the ordinates to the axes. [Note that *x* and *y* are defined contrary to modern convention, and the concept of eccentricity was not known as such at this time.]

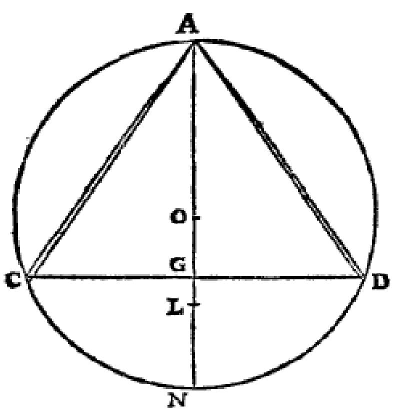
Hence it can be shown, when all the rods are parallel to OL and terminate on the ellipse, that the oscillations are isochronous with the simple pendulum AN, also the whole area uf the ellipse, suspended from the point A and set in motion laterally, are isochronous with the simple pendulum AN. Also, it is the same for any part of the ellipse cut by one or two line perpendicular to the axis AN.



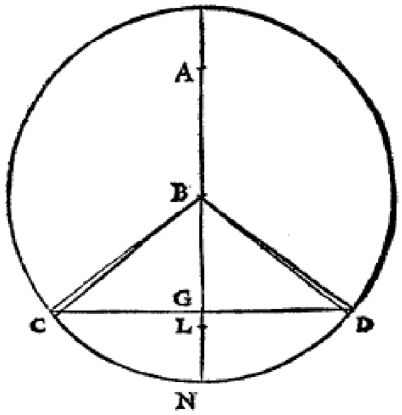
We will consider another example of weights in place of area, in which some noteworthy features spring to mind.

IF AB is a weightless rod, suspended from A; and it is necessary, to that given point B [p.136], to affix a triangle with two equal parts, and which recede from the axis AB at equal angles, for which the angles at B are minimum, or considered to be infinitely small, and for which, thus suspensed from A, isochronous oscillations give rise to a simple pendulum of given length AL.

Here, draw CG perpendicular to BG, and by placing AB = a; AL = b; BG = x; CG = y: the equaqtion is found:
$y = \sqrt{(2ab - 2aa - \frac{8}{3}ax + \frac{4}{3}bx - xx)}$. From which it is apparent, the bases of the triangles C and D, which bases here are considered as points which lie on the circumference of a circle; which surely has the simple term -*xx*.



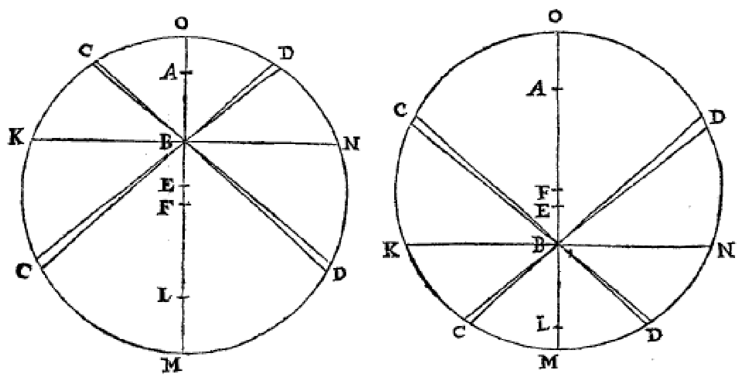
Moreover it is allowed to consider here, if *a* is equal to zero, that is, if the point where the triangles BC and BD are joined is the same as the point A; then the equation will become $y = \sqrt{\frac{4}{3}bx - xx}$. And hence, in that case, if we take AO = $\frac{2}{3}b$, that is, equal to $\frac{2}{3}$ AL, and with the centre O the circle A DN is described; the bases of the triangles AC and AD, are on the circumference of that circle. Therefore when whatever two of the most acute triangles, which are constructed from A to the circumference ACND, with corresponding magnitude and position, they have the point L as the centre of oscillation, with AL put equal to $\frac{3}{4}$ of the diameter AN; and when the whole circle is composed from these triangles of the same size, and that from any portion of this, such as ACND, the sides AC and AD are equal ; it has been shown, as for the whole circle, so for the size of any part we have considered, the centre of oscillation lies at L.



Again, if in the equation we have found we put $\frac{8}{3}a = \frac{4}{3}b$, or $2a = b$; that is, if the triangles are considered to be joined at B, that bisects the length AL, then $y = \sqrt{2aa - xx}$, and this equation shows that if with centre B, and radius put equal to twice BA, a circumference is described that is the locus of the bases of the most acute triangles BC and BD, of which truly [p. 137], the centre of oscillation is L, when suspended from the point A. Also, when the whole circle and any sector, having the axis in the line AL, is composed from like triangles of this kind, it has been shown that the centre of oscillation of these, suspended from the point A, is the point L.

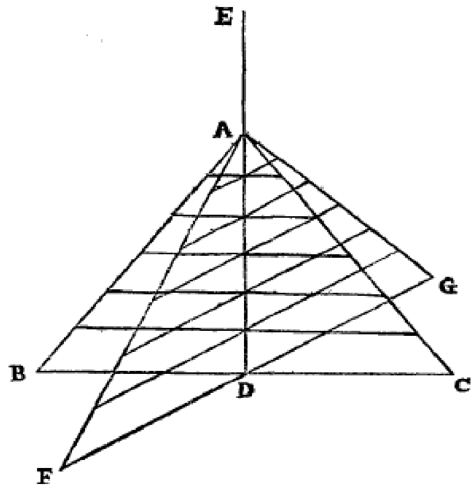
Thus the sector of any circle, suspended from a point that is distant from the centre by an amount equal to half the length of the side of the square inscribed in the circle, has an isochronous pendulum of length equal to the whole length of the side of the same square. And thus for this one case, with only the length of the arc given, the pendulum isochronous to the sector is found.

Again, according to the general construction of the first equation, $y = \sqrt{(2ab - 2aa - \frac{8}{3}ax + \frac{4}{3}bx - xx)}$, AL is divided in half at E, and the third part of EF is added to BE; and F is the centre of the circle to be described ; moreover the radius FO is taken equal to that amount which is twice the difference of the squares AE and EF.



Thus if, from the point B, two of the most acute angled equal triangles are set up on the circumference described, as BC and BD; then the centre of oscillation of these is L, suspended from the point A [p. 138]. Whereby for any part of the described circle, the vertex of this portion is B, and the axis truly the line AL, such as both CBD are described; and with the suspension from A, for such the centre of oscillation is agreed to be the same point. And thus also for the segments of the circle KON and KMN, which make the line KBN perpendicular to AB.

And this indeed should be sufficient to be noteworthy concerning the lateral motions of plane and line shapes. To which we may still add : from the centres of oscillation found for rectilinear figures, or for figures which are symmetric about the axis, as isosceles triangles or sections of a parabola by a straight line ; we can also find the centres of oscillation of slanting figures, composed from disjointed forms of these above, such as for scalene triangles and parabolas without right lines. As for example , the isosceles triangle BAC, the axis of which is AD, and which is understood to be suspended from the point E; for if there is another scalene triangle FAG, having the same axis AD, and the base FG is equal in length to the base BC; then I say that this triangle, suspended from E, is isochronous to the first triangle BAC.

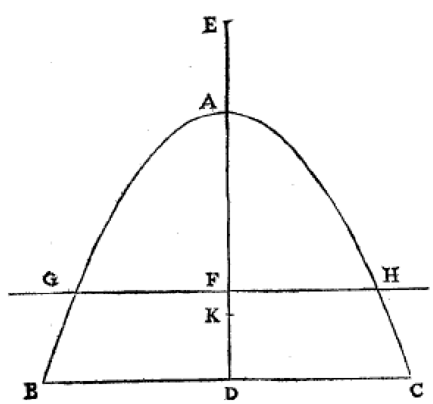


Since indeed the rod, or the heavy line, FG, joined to the weightless line ED at D, placed at an angle, and suspended from E, is isochronous to the line BC, similarly joined at D [by Prop. 16 of this section; thus, the centre of gravity of both lines are the same, have the same moment of inertia essentially about their c. of g. and hence about any other common axis; and and so have equal periods] ; and it comes about in a likewise manner for the rest of the rods for both triangles, which cut the axis AD in the same points, and which are all equal to each other : hence by necessity the whole triangles, which are undertood to be able to be composed from lines or rods , are isochronous. This can be shown for other figures in the same way. [p.139]

PROPOSITION XII.

How the Centre of Oscillation is found for Solid Figures.

We are able also to readily find the centre of oscillation in solid figures, as shown previously. If indeed ABC is a solid figure, suspended from an axis through the point E which is understood to be at right angles to the plane of the page; and moreover, the centre of gravity is F : now with the planes EFD and GFH drawn through F, the second of which is parallel to the horizontal, and the former is traversed by the axis E; then as by proposition 14, with the sum of the squares of the distances of the elemental weights [or volume elements] of the solid ABC found from the planes GFH and EFD; *i. e.* , by finding the sum of both the rectangles, which multiplied by the said number of elemental parts, equals the said sum of the squares of the distances; the



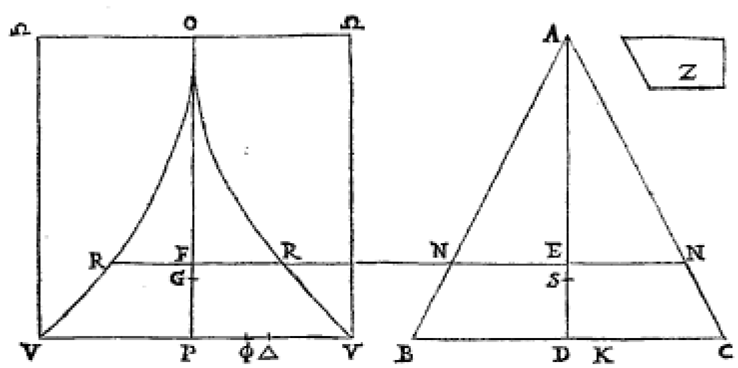
sum of these rectangles divided by the distance EF, which is the distance from the centre of the centre of gravity to the point of suspension, gives the interval FK by which the centre of motion is below the centre of gravity F. This is indeed apparent from proposition 18. Moreover we will now give some examples of using this proposition.

The Centre of oscillation of a Pyramid.

In the first place, ABC is the pyramid, having vertex A, axes AD, and the base truly a square of which the side is BC. It is put in place to be set in motion about the axis which passes through the vertex A, which is at right angles to the plane of the page.

Here a plane proportional figure OVV is to be put in place from the side, following proposition 14, which agrees with the parabola left when the semiparabola OVΩ with vertex O is taken from the rectilinear figure ΩP. [p. 140]

For thus between the sections BC and NN of the pyramid, there are corresponding lines VV and RR in the plane figure. Thus, as the centre of gravity E is at a distance from the vertex of the pyramid equal to three quarters of the axis AD, thus too the centre of gravity F, of the figure OVV, is at a distance of three quarters of the diameter OP from



the vertex O.

Again by considering the horizontal plane NE, passing through the centre of gravity of the pyramid ABC, that cuts the same figure OVV along RF; and the subcentre of the wedge is found, upon the figure OVV cut by the plane through the line OΩ, and OG is the subcentre, (for it can be shown to be $\frac{4}{5}$ of the diameter OP) ; the rectangle OFG multiplied by the number of elemental areas of the figure OVV is equal to the sum of the squares of the distances from the line RF (by Prop. 10 of this section), and hence also to the sum of the squares of the distances from the plane NE, of the elemental volumes of the solid ABC. Moreover the rectangle OFG becomes equal to $\frac{3}{80}$ of the square of OP, or of the square of AD.

Consequently, in order to find the sum of the squares of the distances from the plane AD, the subcentre of this wedge must first be known, cut upon the square base of the pyramid BC, by a plane passing through the line through B, understood to be parallel to the axis A [i. e. this wedge has its edge along that side of the base which goes into the plane of the page at B; thus, each elemental square formed from cuts of the pyramid out of the plane of the page forms a wedge with the same subcentre placed 2/3 along the side]; and this subcentre is BK; and this is $\frac{2}{3}$ BC. Likewise it is required to know the distance between the centre of gravity of half the figure OPV from OP; which is called ΦP; and this is equal to $\frac{3}{10}$ PV. From this, PV is bisected in Δ, if the ratio ΔP to PΦ is made thus as 5 to 3, thus the rectangle BDK, which is $\frac{1}{12}$ of the square of BC, to another area Z; this will be, multiplied by the number of elemental volumes of the solid ABC, equal to the squares of the distances from the plane AD (by Prop. 15 of this section). Moreover it is apparent that the area Z becomes equal to $\frac{1}{20}$ of the square of BC.

Thus, the whole area to be divided, is equal here to the sum of $\frac{3}{80}$ of the square of AD and $\frac{1}{20}$ of the square BC. Hence as here ,if the suspension is from A, the vertex of the pyramid, and thus the distance for which the division is to be made [p. 141] AE, is equal to $\frac{3}{4}$ AD; hence ES becomes equal to $\frac{3}{20}$ AD, the distance by which the centre of agitation is below the centre of gravity, to which is added $\frac{1}{15}$ of the third proportion between AD and BC; or the whole length AS is equal to $\frac{4}{5}$ AD plus the said $\frac{1}{15}$ of the third proportion.

The Centre of Oscillation of a Cone.

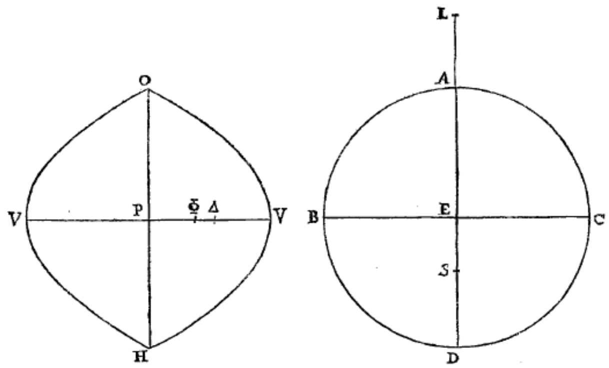
Concerning the case that ABC is a cone, everything is gone about in the same manner, except that here the area Z is equal to the rectangle ΔPΦ (by Prop. 15 of this section), that is, it is $\frac{1}{20}^{\text{th}}$ of the square of PV or BD, or $\frac{3}{80}^{\text{th}}$ of the square of BC. Whereby, the whole area to be divided in the cone will be $\frac{3}{80}^{\text{th}}$ of the square of AD, together with $\frac{3}{80}^{\text{th}}$ of the swuare of BC. And hence, with the suspension put in place from the vertex A, ES becomes, for the distance by which the centre of agitation lies below the centre of gravity, equal to $\frac{1}{20}$ AD and $\frac{1}{20}^{\text{th}}$ of the third proportion between AD and BC; or the whole length AS is equal to $\frac{4}{5}^{\text{th}}$ of AD together with $\frac{1}{5}$ of the third proportion between AD and DB. And thus it is shown, if AD and DB are equal, that is, if the cone ABC is right, then AS becomes equal to the axis AD.

Again, it also follows from proposition 20, that this right cone suspended from the centre of the base D, is isochronous with itself suspended from the vertex A, as was shown from the above right-angles triangle.

Centre of Oscillation of a Sphere.

If ABC is a sphere, the proportional plane figure OVH placed at the side, is composed from parabolas with the common base OH, equal to the diameter of the sphere AD. Truly the sphere is cut by planes through the centre E, of which BC is parallel to the horizontal, and AD truly to the vertical : in order that the sum of the squares of the distances of the elemental volumes from the plane AD can be foun, the distance of the centre of gravity of the parabola OVH from OH is to be noted, which is ΦP, and that is $\frac{2}{5}$ VP. Hence, with PV bisected in Δ, it is agreed that rectangle ΔPΦ, multiplied by the number of elemental volumes of the sphere ABC, is equal to the sum of the squares of the distances from the plane AD (by Prop. 15 of this section). Moreover the rectangle ΔPΦ is equal to $\frac{1}{5}$ th of the square of PV, or of the square BE.

But it is evident that the sum of the squares of the distances from the plane BC, is equal to the sum of the squares of the distances from the plane AD, and hence to the same rectangle ΔPΦ, multiplied by the said numbe of elements. Hence the area to be divided, for the sphere ABC is twice the area of the rectangle ΔPΦ ; likewise equal to $\frac{2}{5}$ of the square of the radius EB.



Thus, if the sphere is suspended from the point A on its surface, [p. 142] then ES, the distance from the centre of the sphere E to the centre of agitation S, will be equal to $\frac{2}{5}$ th of the radius AE. The whole length AS is equal to $\frac{7}{10}$ of the diametrer AD. If truly the sphere is suspended from some other point, such as L, then ES is equal to $\frac{2}{5}$ of the third proportion between LE and ED.

Centre of Oscillaton of a Cylinder.

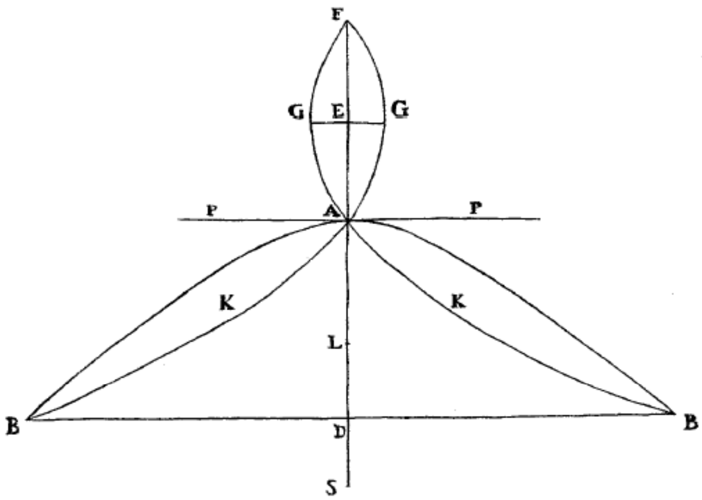
For a cylinder, we find that the area to be divided is equal to $\frac{1}{12}$ th of the square of the height, together with $\frac{1}{4}$ of the square of the radius of the base. Hence if the cylinder is suspended from the centre of the upper base, the length of the isochronous pendulum is equal to $\frac{2}{3}$ of the height, together with half of that height, which is in the same ratio to the radius as the radius is to the height.

Centre of Oscillation of a Parabolic Conoid.

For a parabolic conoid, the rectangle of oscillation is $\frac{1}{18}$ th of the square of the height, with $\frac{1}{6}$ th of the square of the radius of the base. Hence, if it is suspended from the vertex, the length of the isochronous pendulum is $\frac{3}{4}$ of the axis, together with $\frac{1}{4}$ of that number which is in the same ratio to the radius as the radius has to the axis, that is, together with $\frac{1}{4}$ of the latus rectum of the generating parabola.

Centre of Oscillation of a Hyperbolic Conoid.

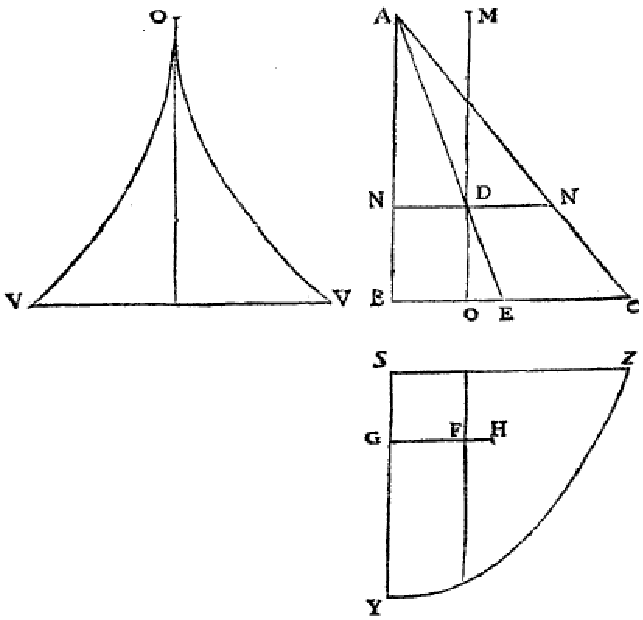
The centre of oscillation of a hyperbolic section of a cone can also be found. If indeed, for example, the hyperbola BAB is the section of this cone through the axis ; having the axis AD, and the transverse line AF : the proportional plane figure is BKAKB, within the base BB, [p. 143] and with similar portions AKB of parabolic curves passing through each other at the vertex, and having the axes GE bisected by the transverse line AF, and parallel to the base BB. And indeed the centre of gravity L of this figure BKAKB is as far from the vertex A, as the centre of gravity of the cone ABB; and the ratio of the axis AD to AL, is thus as (3 × FA + 2 × AD), to (2 × FA + 3 × AD/2). Hence the distance of the centre of gravity of the half-figure ADBK to AD can be found, and also I say that the subcentre of the wedge on the figure BKAKB, cut by the plane through the line AP and parallel to BB, can also be found; and from these consequently the centre of oscillaton of the cone, from wherever suspended; provided the axis, around which it can move, is parallel to the base of the cone. And indeed the area can be found to be divided, if the axis AD is put equal to the length of the transverse side AF, to be equal to $\frac{1}{20}^{\text{th}}$ of the square of AD added to $\frac{11}{200}^{\text{th}}$ of the square of DB. Moreover , the line AL is then $\frac{7}{10}$ AD.



Hence, if the cone of this kind is suspended from the vertex A, the length of the isochronous pendulum AS can be found, equal to $\frac{27}{35}$ AD plus $\frac{31}{140}$ of the third proporton between the two proportions AD and DB.

The Centre of oscillation of a half Cone.

Subsequently for the halves of certain solids, which happen to be cut through the axis, it is possible to find the center of oscillation. As if half of the cone ABC, having the vertex A, and the base is the diameter of the semicircle BC [p. 144] : indeed the centre of gravity D of this is known, since AD is $\frac{1}{4}$ of the line AE, thus cutting BC in E, thus as the quarter of the circumference of the circle is to the radius, thus as $\frac{2}{3}$ CB is to BE. Then indeed E is the centre of gravity of the semicircular base, and likewise the centres of gravity of all the parallel segments to the base of the semicon ABD are on the line AE.

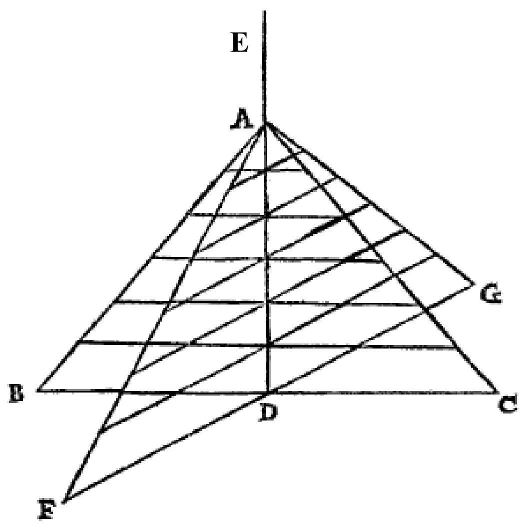


And again by putting a certain proportional figure in place laterally, OVV, which is the same as that used in the description of the whole cone : by which truly the sum of the squares can be found of the distances of the elemental parts of the semi-cone from the horizontal plane ND drawn through the centre of gravity.

Truly in order that the squares of the distances from the plane MDO to the vertical can be gathered together, by having another proportionalal figure SYZ too, thus by prop. 14 above, the vertical sections of which show proportonal lines in a correspondence to the lines of the semi-cone ABC itself , and the distance of the centre of gravity F of this figure from SY is known, as it is agreed to be equal to the distance DN, of the centre of gravity of the semi-cone from the plane of the triangle AB; and with the subcentre HG of the wedge cut upon the figure SZY put in place, from the plane drawn through SY, the rectangle GFH becomes known. Truly the product of this by the number of elemental parts of the semi-cone ABC is equal to the sum of the squares of the distances of the parts of the semi-cone, from the plane MDO. It is also possible to find the rectangle GFH, even if the subcentre HG is not known, in the following manner.

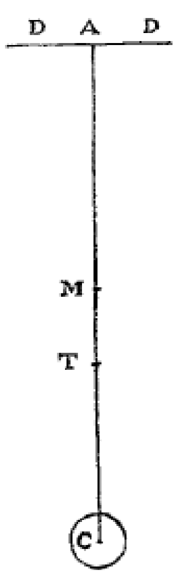
Above we have said, when we were working on the cone, that the sum of the squares of the distances from the plane through the axis [p. 145] of this, is equal to $\frac{5}{80}$ th of the square of the diameter of the base, or $\frac{5}{20}$ of the square of the radius, multiplied by the number of elements of the whole cone. Thus in this case, for the semi-cone ABC, the sum of the squares of the distances from the plane AB is equal to $\frac{3}{20}$ of the square BC, multiplied by the number of elemental parts of the semi-cone. Also, the rectangle HGF, multiplied by the number of elemental parts of the semi-cone ABC, is equal to the square of the distances from the plane AB, as in apparent from proposition 9. Hence rectangle HGF is equal to $\frac{3}{20}$ of the square BC. Moreover, putting AB = a; BC = b; and the square of the circumference, described by the radius BC = q; then EB = $\frac{2bb}{5q}$. When ND becomes equal to three quarters , then ND or GF = $\frac{1bb}{2q}$. By taking the square of this the rectangul HGF, that was $\frac{3}{20}$ of the square BC, then the rectangle GFH = $\frac{5}{20}bb-\frac{3bb}{4qq}$. Moreover, this rectangle, multiplied by the number of elemental parts of the semi-cone ABC, is equal to the sum of the squarees of the distances from the plane MDO. But the sum of the squares of the distances from the plane MD is equal to, as for the cone, $\frac{3}{80}aa$, multiplied by the number of elemental parts of the semi-cone ABC. Thus, the whole area to be divided, is here equal to $\frac{3}{80}aa+\frac{3}{20}bb-\frac{3bb}{4qq}$.

Hence indeed the centre of oscillation is found for the suspension of any semi-cone, as long as it is from an axis which is parallel to the base of the triangle from the section AB. Indeed it is to be noted, when in short the kind of figure SZY is unknown, that nevertheless the subcentre GH of the wedge cut on the plane through SY thus can be found. For, since the rectangle HGF is equal to $\frac{3}{20}bb$, or to the square of BC, and GF is equal to $\frac{1bb}{2q}$, then GH = $\frac{3}{20}q$.



Again, for the semi-cylinder and also for the parabolic semi-cone, the centres of oscillation can be found, and of other semi-solids; which we leave to be discovered by others.

Moreover, as for plane figures, and thus here for solid figures as well, there is a place for the centres of oscillation for the oblique figures that we have discussed, which can be set up just as for plane figures from disjointed lines, and for which the centres of oscillation are no different for solid figures than they are for plane figures. Thus, if there are two cones ABC and AFG, the one right and the other scalene; and of which both the diameters and the bases are equal; these are isochronous, either suspended from the vertex or from some other axis, at the same distance from their centres of gravity; provided that the axis of oscillation is at right angles to the plane of the triangle through the diameter, and that the axis from which the scalene cone has been suspended is at at right angles to the plane of the base. [p. 146]

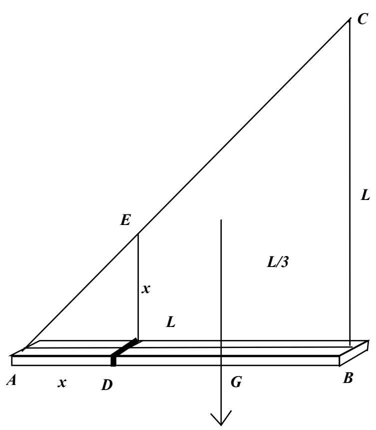


PROPOSITON XIII.

To control the movement of the clock by the addition of a small secondary weight that can move up or down the pendulum rod which is divided in a certain ratio into parts.

So that we can be expedient here, we consider that, in the first place the centre of oscillation of the pendulum itself has been found, constructed from a given rod with weight, and with an heavy weight attached to the lower part. AC is the rod, with the weight attached, and the length of this is called *a*. Moreover, it is understood that the weight of the rod itself, as well as the attached weight C, are considered to be divided into equal small parts : the rod consists of a certain number *b* of these equal small weights, while the weight C has the number *c*; then, by putting these in the ratio *b* to *c*, thus is the ratio of the weight of the rod to the total weight attached found. Therefore, the length of the isochronous simple pendulum can be found if the sum of the squares of the distances of all the elemental weights measured from the point of suspension A, is divided by the sum of the distances of the same weights (by Prop. 6 of this section). AC is first bisected at M, and then cut at T, in order that AT is twice TC. Hence, M is the centre of gravity of the rod AC, and AT is the subcentre of the [hypothetical] wedge placed above and cut by the plane through AD, perpendicular to AC; here the wedge is actually a triangle; the sum of the squares, from the distances of the individual elements of the rod from the point A, [p. 147] is equal to the rectangle AMT, which together with the square AM, is equal to the rectangle TAM, which is to be multiplied by the allowed number of elemental weights *b*; *i. e.* $\frac{1}{3}aab$; since MA is $\frac{1}{2}a$, & TA $\frac{2}{3}a$, and hence the rectangle TAM = $\frac{1}{3}aa$. Indeed the sum of the squares, from the distances of the elemental weights C from the same point A, is equal to the square of AC, multiplied by the number of elemental weights of the weight itself ; that is, *aac*. Hence the sum of all the squares of the elemental distances along the rod, added to the contribution of weigh C is $\frac{1}{3}aab + aac$.

[The reader may permit us to take a short pause to look at Huygens' method of computing moments of inertia, without the use of modern integration. The use of the wedge technique is particualarly easy to understand when applied to a uniform rectilinear rod of mass M, and where the length L of the rod is much greater than its width and thickness, as shown in the diagram. The sum of the squares of the elemental masses is $\sum x_i^2.\Delta m_i = \frac{M}{L}\sum x_i^2.\Delta x_i$. If the width is taken as 1 for a uniform thins sheet, then the sum of the squares of the lengths *x* is equivalent to the sum of the moments of the volume elements $x\Delta x.1$. Now, for a rectilinear volume such as the wedge ABC, the sum of the moments of the elemental volumes is equal to the volume of the whole wedge × the distance of the centre of gravity or subcentre from A, where we have taken a density of 1 also, so that mass or weight can be exchanged for volume. In the present circumstance, the volume is $\frac{1}{2}L^2.1$; and the distance AG is 2*L*/3. Hence, $\frac{M}{L}\sum x_i^2.\Delta x_i = \frac{M}{L}\times\frac{L^2}{2}\times\frac{2L}{3} = \frac{1}{3}ML^2$. Thus, we have Huygens' result, and we have shown in an elementary way how it was found. It is harder to evaluate the centre of mass location for objects with curved shapes. Wedges based on circles and conic sections had been investigated by Gregorius a few years earlier, and expounded in his *Opus geometricum*...... This work has not been directly quoted by Huygens, as Gregorius had erroneously included a proof of the squaring of the circle, which Huygens had shown to be flawed. The work does contain many interesting developments, and includes a method of integration based on summing geometric progression; Leibnitz is said to have found it of great help in formulating his calculus;

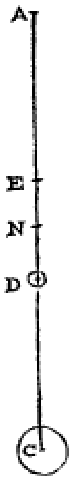


and of course it contains the rudiments of the natural logarithm as the area under the section of a hyperbola. At present, a small part of this work is included on this website.]

Again, the sum of all the distances of the elemental parts of the rod AC from the point A, is equal to $\frac{1}{2}ba$; which is *a*, the length of the rod itself, multiplied by half the number of elemental weights contained in the rod. And all the distances of the elemental weights of the weight C are *ac*, at the same distance *a* from A. Thus so that the sum of both distances is $\frac{1}{2}ab + ac$. The sum of the squares first found $\frac{1}{2}aab + aac$ is to be divided by this amount, and the length of the isochronous pendulum is $\frac{\frac{1}{3}aab+aac}{\frac{1}{2}ab+ac}$ or $\frac{\frac{1}{3}ab+ac}{\frac{1}{2}b+c}$.

Thus the ratio that can be formed, for the length AC to the length of the isochronous pendulum, is that equal to half the weight of the rod added to the attached weight, to a third of the weight of the rod added to the weight of the attached weight, and multiplied by the length of the rod. Moreover it is necessary to measure the length AC, from the point of suspension to the centre of gravity of the weight C; since a ratio of this size may not be obtained otherwise, or just as it be considered too small.
[From our point of view, a rigid compound pendulum with moment of inertia *I* about the vertex A performs SHM, at least for small oscillations (with the angle of the rod to the vertical θ, mass M, and distance of the c. of g. from the A equal to *l*), obeys an equation of form : $I\ddot{\theta} = -Mgl\theta$; for which the period is $T_c = 2\pi\sqrt{\frac{I}{Mgl}}$. If $I = Mk^2$, where *k* is the radius of gyration, then $T_c = 2\pi\sqrt{\frac{k^2}{gl}}$. The period of the equivalent simple pendulum of the same mass, where *k* = *l*, is given by $T_s = 2\pi\sqrt{\frac{l}{g}}$. In the present case, the moment of inertia is that of a rod of length *a* and mass *b* about the end of the rod A, which is $\frac{1}{3}a^2b$, to which is added the moment of inertia of a point mass *c* at distance *a*, or ca^2; hence $I_C = \frac{1}{3}a^2b + ca^2$, and this is acted on by the sum of moments $bga/2 + cga$ in an equation of the form: $I_C\ddot{\theta} = -(bga/2 + cga)\theta$, giving a period $T_c = 2\pi\sqrt{\frac{\frac{1}{3}a^2b+ca^2}{bga/2+cga}}$. Thus the length of the equivalent isochronous pendulum $l = \frac{\frac{1}{3}ab+ca}{b/2+c}$, as Huygens has shown. The addition of the sliding weight is now obvious, as now follows.]

Because if now, besides the weight C, another weight D is underestood to be attached to the rod above the weight C. The size of this weight, or the number of elemental parts is *d* : and the distance AD is *f*. In order that a simple pendulum can be found isochronous to this composite pendulum, the square of the distance of this particular weight D from the point A must be added to the above sum of the squares, which is seen to be the term *dff*. Thus, in order that the sum of all the squares now becomes $\frac{1}{3}aab + aac + ffd$. Likewise, to the sum of the distances is to be added the distance of the particular weight D, which makes *df*. Hence the sum of all the distances is $\frac{1}{2}ba + ca + df$; the sum of all the squares is to be divided by this sum, and gives $\frac{\frac{1}{3}aab+aac+ffd}{\frac{1}{2}ab+ac+fd}$ as the length of the equivalent isochronous pendulum.



For if indeed the length of this isochronous simple pendulum is considered to be given equal to p, and the rest of all the other distances have been given, except [p.148] the distance AD or f, which determines the positon of the weight D : then this distane can be found in the following manner: by setting $\frac{\frac{1}{3}aab+aac+ffd}{\frac{1}{2}ab+ac+fd}$ equal to p, from this equation one obtains $ff = pf + \frac{\frac{1}{2}abp+cap-\frac{1}{3}aab-aac}{d}$. And hence $f = \frac{p}{2} \pm \sqrt{\frac{1}{4}pp + \frac{\frac{1}{2}abp+cap-\frac{1}{3}aab-aac}{d}}$. Where it is to be understood that there are two [positive] roots, if $\frac{1}{2}abp+cap$ is less than $\frac{1}{3}aab+aac$; that is, if the length p is less than $\frac{\frac{1}{3}ab+ac}{\frac{1}{2}b+c}$, which was found before to be the length of the isochronous pendulum, or the distance from the centre of oscillation from the point of suspension for the composite pendulum composed from the rod and the weight C.

Thus it is apparent, if we wish to put this into effect, so that by the application of the weight D, the motion of the pendulum can be changed ; then this can be done from two different points between A and C, from which both can bring about the same change in the speed : such as at the points D or E. Which locations are put at equal distances from the point N, which is half the length of p from A, that is, by half the length of the equivalent simple isochronous pendulum here considered equal to the length of the composite isochronous pendulum. Moreover it is apparent, that when this length p is put a little less than AC, then the point N is a little more than the mid-point of the rod AC.

Again, the length p of the equivalent simple isochronous pendulum can be determined from the above equation: $f = \frac{p}{2} \pm \sqrt{\frac{1}{4}pp + \frac{\frac{1}{2}abp+cap-\frac{1}{3}aab-aac}{d}}$. For it appears that $\frac{1}{4}pp + \frac{1}{2}.\frac{abp+2acp}{d}$ cannot be less than $\frac{\frac{1}{3}.abb+acc}{d}$ [for real roots]. Hence p cannot be less than $-\frac{(ab+2ac)}{d} + \frac{a}{d}\sqrt{(\frac{4}{3}bd+4cd+bb+4bc+4cc)}$. Because if p is put equal to this quantity, that is, if $\frac{1}{4}pp + \frac{\frac{1}{2}abp+acp}{d} = \frac{\frac{1}{3}abb+acc}{d}$, then in the same above equation, $f = \frac{1}{2}p$, [as the discrimiant is zero] that is, $f = -\frac{(ab+2ac)}{2d} + \frac{a}{2d}\sqrt{(\frac{4}{3}bd+4cd+bb+4bc+4cc)}$. From which the distance of the weight D from the point A is determined for which the acceleration of the pendulum will always be a maximum.

The use of this mechanism for clocks can again be shown as follows. For example, the individual oscillations of the pendulum of a clock may be noted in seconds. Moreover, the weight of the rod is $\frac{1}{50}$ th of the weight attached to the end of the pendulum : and besides this, there is another small weight that can be moved along the length of the rod, which has the same weight as the rod, [p.149]. Now it is enquired, where on the rod should the weight be placed, in order that the clock advances by one minute in a space of 24 hours. Likewise, where should it be placed for a gain in time of 2 minutes; likewise, 3 minutes, and so on.

Sixty times twenty four hours gives1440, truly the number of minutes contained in a day. From these one is taken away, when an increase of one minute is required : 1439 remain. Moreover, the ratio of 1440 to 1439 squared, is nearly that which 1440 has to 1438. Hence, if the seconds of the simple pendulum are noted, the length can be understood to be divided into 1440 equal parts, and to be divided into 1438 parts for the other pendulum that advances on the other pendulum by one minute in a time of 24 hours. Thus p prevails here with 1438 parts. [For $\frac{s}{p} = \frac{T^2}{T'^2} = \frac{1440^2}{1439^2} = \frac{1440^2}{1440^2(1-1/1440)^2} \simeq \frac{1440}{1438}$, where p from above is the length of the simple pendulum isochronous to the inaccurate compound pendulum, and s is the length of the similar accurate simple pendulum that keeps the correct time. The problem is then to find the location of the sliding mass on the defective compound pendulum to make it the same as the ideal simple pendulum.]

Moreover since the pendulum of the clock, made from a metal rod and suspended with a weight, is placed isochronous with a simple pendulum of 1440 parts ; in the first place the length of this rod has to be determined, from the above equation.
[That is, we must find the length of the composite pendulum, that keeps perfect time, a for these parameters from the length of the simple isochronous pendulum s, for which $s = \frac{\frac{1}{3}ab+ca}{b/2+c} = \frac{a(\frac{1}{3}b+c)}{b/2+c}$, giving $a = \frac{sb/2+sc}{\frac{1}{3}b+c}$. The pendulum that gains each day has the equivalent simple isochronous pendulum of length p.]
For $\frac{\frac{1}{3}ab+ac}{\frac{1}{2}b+c}$ is equal to the length of the simple pendulum, where the isochronous pendulum is made from a rod having a length a, weight b, and with an affixed mass of which the weight is c. Hence, if the length of the simple isochronous pendulum is called s, then $\frac{\frac{1}{2}bs+cs}{\frac{1}{3}b+c} = a$. With these values put in place here: $c = 50$; $b = 1$; $s = 1440$; then the length of the rod becomes $a = 1444\frac{4}{5}$ for the exact composite pendulum. Now, since $f = \frac{p}{2} \pm \sqrt{\frac{1}{4}pp + \frac{\frac{1}{2}abp+cap-\frac{1}{3}aab-aac}{d}}$, this becomes $f = \frac{p}{2} \pm \sqrt{\frac{1}{4}pp + 72962p - 105061210}$.
Hence again, if p is 1438 parts, as we have said ; then it is found that $f = 1331\frac{1}{2}$, and truly the size of s, or of the simple pendulum, that gives the correct number of oscillations is 1440. The length of the pendulum, if set up as three feet, which we have called the horary, has f as 33 inches plus an extra 3 twelfth parts called lines. Or, by taking this length f from the total length of three feet, there are left two inches and 9 lines that can be taken up from the centre of oscillation of the composite pendulum, in order that the place of the weight D can be found, for an advance of one minute in the time of 24 hours. [Thus, simple proportion is used to give the distance of D from the vertex A : 36 inches is to 1440 parts as f in inches is to $1331\frac{1}{2}$ parts, giving f in inches = 33.2875, and the distance from the centre of oscillation is 2.7125 inches] In the same way we can find from calculaton the rest of the distances into which the rod is to be divided, the length p to be put in place one after another : and we show these here in the table [p.150], indeed it is according to the numbers in this table that the rod has been divided up in the description of the clock that was shown above in figure IV of section I. But the increases proceed on a daily basis, as now we have shown this, by 15 seconds or a quarter of a

minute. For example, if the sliding weight D is moved to the division 73, 4, it is found that the clock is just beginning to be slow by 15 seconds in 24 hours; a difference of 15 seconds; it will be necessary to move the weight up to the number 85, 6, in order to correct it. The centre of oscillation is higher than the centre of gravity C by 1, 4 divisions. [Thus, the numbers in the right-hand column represent the numerber of lines, which divided by 12 corresponds to the number of inches the weight D must be raised; *e. g*. 32, 6 = 32.6/12 = 2.72 inches, etc.]

The gain in time of the clock in 24 hours. *min.*	*sec.*		The no. of divisions the sliding weight is taken up from the centre of oscillation. *Lines and tenth parts of lines of the scale .*
0,	15	————	7, 0
0,	30	————	15, 2
0,	45	————	23, 3
1,	0	————	32, 6
1,	15	————	41, 9
1,	30	————	51, 7
1,	45	————	62, 2
2,	0	————	73, 4
2,	15	————	85, 6
2,	30	————	99, 0
2,	45	————	114, 1
3,	0	————	131, 8
3,	15	————	154, 3
3,	30	————	192, 6

PROPOSITON XXIV.

It is not possible to be given the ratio of the centre of oscillation for pendulums suspended between cycloids, and how this difficulty that has arisen can be removed.

If the propositions presented above concerning a pendulum suspended between cycloidal parts are subjected to a subtle examination with these propositions which are concerned with the centre of oscillation ; it can be seen that these lack somewhat in perfection regarding the equality of the oscillations we prefer. And the first point to be in doubt, for finding the generating circle of the cycloid, whether the length of the pendulum is taken from the point of suspension to the centre of gravity [p. 151] of the attached weight, or should it be taken to the centre of oscillation; which is often seen to be at some distance apart from the other distance, and to be larger when the sphere or lead lens is larger. How much indeed is the difference, if the diameter of the sphere should equal a quarter or third part of the length of the pendulum ? Because, if we accept the length to the centre of oscillation as the said distance; then it is not yet sufficiently clear that what has been agreed upon for the centre of oscillation a pendulum that continues its motion with its length unchanged actually applies when it is used to compare the motion of a pendulum that continually changes its length as it moves along the cycloidal path. Indeed it can be seen that the centre of oscillation is changing its distance according to the different lengths ; which however is not understood from this method. The thing is most difficult to explain from reason, if we are to follow a perfect exactness from the previous propositions : just as in the demonstration of equal times in the cycloid we have considered the motion of a point weight for the motion taken by the pendulum. [Thus, some propositions have followed from the use of the centre of oscillation, for a constant length of pendulum, and others have followed from the motion of the centre of mass.] But if we examine this effect, then we find that these difficulties do not cause much trouble, when it is not so much the magnitudes of the weights that are required (though that which is bigger is better) from which the pendulum is made, but the difference of the heights of the centres of gravity and of oscillation that is the source of the trouble here. For if we still want to avoid these difficulties, then as a consequence, we can make the sphere or lens of the pendulum move around its own axis to the horizontal, with the distance to the axis as large as possible ; and with the rod of the pendulum necessarily split into two forks at the lower end. [Thus, the sphere does not rotate with the rod, but stays as a point mass, and its own moment of inertia does not come into play, as it does not rotate about its own axis] Indeed it can be made in this manner, and from the nature of the motion, in order that the sphere of the pendulum always keeps the same position with respect to the horizontal plane; and thus any points on the sphere, and equally the centre of the sphere itself traverses the same cycloid. Thus now we stop considering the centre of oscillation; it follows that such a pendulum is no less perfect in keeping time if all the weight is considered to act at one point.

PROPOSITION XXV.

Concerning a universal and constant measure, and how it can be established.

Certainly, keeping a constant length measure, which is not a nuisance by being spoiled or destroyed either by the change of seasons or by the passage of long periods of time, is a most useful thing, and sought by many long ago. Which if it had been found in the olden times, then there might not now be so much disputes arising from confusion over Roman, Greek, and Hebrew feet. [Such as Biblical references to the length of Noah' Ark and the like.] Truly a measure of this kind is easily understood with the help of our clock ; for without clocks it would scarcely be possible to have such a measure. [p. 152] Although indeed, from certain trials with the swing of a simple pendulum, the rotaton of the whole heavens can be counted in terms of the swings of such a pendulum, or from a known part of this rotation from the distances of the fixed stars, following the right ascension; neither can this be done in this way with certainty, which can be addressed by putting clocks to use, on account of the long hours of work of a most tedious and irksome nature, associated with the anxiety of counting regularly. Moreover, besides clocks, anyone who investigates the exactness of this kind of measurement, must also bring to his attention centres of oscillation ; thus here at last, after discussing these things, we present this determination of length.

The most suitable clocks for this sort of application are those for which the number of swings are recorded, either of whole or half second duration, and which have been constructed with pointers to show the number of these swings. Indeed after the clock was built, from observations of the fixed stars , by the method that we have shown in the section on the construction of clocks : another simple pendulum, that is, a lead sphere , or made from some other dense material, attached by a thin thread, has been suspended nearby, and set in motion with a small swing ; and by extending or shortening the length of the thread, then resetting this after a quarter of an hour or half of this time, the swings of the pendulum can be adapted to the to and fro motions of the clock. Moreover I have said that the pendulum has to be started with a small motion, since small oscillations put in place of 5 or 6 degrees, have satisfactory equal times, which is not the case for large swings. Then, with an acceptable measure of the distance, from the point of suspension to the centre of oscillation of the simple pendulum, if single seconds values are repeated, divide the length of the pendulum into three parts, and this division is the length of a single foot, that we have called a HORARY above [for which a three feet length is equivalent to the second] : and which, from this agreement, not only can be set up everywhere for the nations, but also is regularly renewed in time. Thus the provision of this little measure surely makes it possible for all the others to be known too, one they have been expressed as a proportion of this amount. Such as now above, the Paris foot according to our horary is in the ratio 864 to 881; which is the same as if, for the Parisian foot considered first, we have said that the simple pendulum is composed from three of these feet, with eight and a half lines, that correspond to an oscillation of one second. Moreover the Paris foot is in the ratio to the Rhenish foot, which is used in our country, of 144 to 139 ; that is, with five lines less than of the other. And thus this kind of foot, and any others, can be assigned values that last indefinitely.

Moreover just as the centre of oscillation can be found for a sphere suspended by any length [p. 153], as has been shown above. Truly, if the distance between the point of suspension *A* to the centre of the sphere *C* is in the same ratio to the radius of the sphere as two fifths of the radius is to the other length taken to end in the sought centre of oscillation *D*, at a distance *CD* lower than the centre by this amount.
[Thus, if l is the distance *AC*, and r is the radius, and the other length *CD* is l', then $\frac{l}{r} = \frac{2r/5}{l'}$, or $l' = 2r^2/5l$.].
Moreover, it is readily apparent why it is necessary to consider the centre of oscillation, in the setting up of accurate feet for the horary. For, if the distance from the point of suspension to the centre of the sphere is taken, the size of the sphere is not defined in proportion to the length of the thread, and the length of the pendulum will not be a reliable means by which the times of one second are measured by the returns; but where the sphere is larger, then here that distance measured between the centre of the sphere and the point of suspension is found to be less [in comparison]. Since in isochronous pendulums, the centres of oscillation are certainly equidistant from the points of suspension; moreover the centre of oscillation descends more below the centre for a large sphere than for a small one. [These comments relate to the general rule that $ll' = k_{cm}^2$.]

Hence it was necessary by these who, before this determination of the centre of oscillation, undertook to set up a method for measuring a general length, to designate the diameter of the suspended sphere and indeed the proportion of the diameter to the length of the suspending string, which was set equal to a third or a fourth part, and to be measured in some known way, either in finger or thumb widths.Since now with the invention of our first clock, that noble institution, the Royal Society of the English has taken the business in hand itself, and recently under the auspices of the most learned London astronomer, Gabriel Mouton. For by this more recent method, now put in place with some certainty, that which is found is that which is sought : and though I scarcely know how big the error may be, provided they do not exceed the size of the sphere that I have just mentioned. Moreover it may be possible beforehand, to come to some agreement about how the experiments are to unfold, thus, in order that the labour of counting the oscillations can get under way, and also by performing useful calculations. On account of which, with the position of the centre of oscillation agreed upon mainly, a certain ratio follows, with nothing binding except the necessity of agreeing with the laws; and here now by using larger spheres rather than small ones, since these are less impeded by moving through the air.

Other figures, as well as spheres suspended from strings, have been suitable for measurements, such as cones, cylinders, and other solid figures and plane shapes, the centres of oscillation of which we have presented above ; since, from the point of suspension to the centre of oscillation, it is certain that the interval is the same as that of the isochronous pendulum. Also, not only can we use these clocks which indicate the recurrance of the pendulum in seconds or half seconds ; but for any other pendulum proposed made in the correct length that can be obtained, as long the number of oscillations in a given time can be carried out from the known proportions of the wheels, or the number of teeth. Indeed from the simple pendulum the number of swings can be found that are gone through in the space of an hour, which are agreed upon for individual swings, or by taking two or three at a time. The number of which if squared will be, as the square 3600, the number of seconds that bring about the passage of one hour, to the square of that number, thus as the length of the simple pendulum found, (the length of which is always taken from the point of suspension to the centre of oscillation) to the length of the three foot horary pendulum that we have discussed. This indeed is hence agreed upon, since for any two pendulums the ratio of the lengths are thus as the squares of the times arising from the numbers in which the individual swings are carried out ; and therefore the contrary ratio that may be had from the squares of the numbers gives the the number of oscillations of each carried out. For, as there was so much agreement with the trials up to this point, that Theorem about the lengths of pendulums, these truly are in the square ration of the times, with which the individual swings are carried out; now the demonstration of this from the above exposition has been shown. For indeed we will show, for the single swings of the pendulum, suspended between the cycloids, to the distance fallen, to have a certain ratio to half the length of the pendulum ; that certainly is as the circumference of the circle to its diameter; this is easily gathered, for the time of oscillation of two pendulums are between each other, thus as with the vertical times of descent from half the heights of these. Which half heights, or also the total, are in the square ratio of the time, by which they have traversed in their vertical descent (by Prop. 3, Part. 2); the same square ratio also is had of the times, which the individual oscillations measure out. Moreover from the smallest oscillations of a pendulum, suspended between the cycloids, there is no distinguishable difference from the smallest oscillations of a simple pendulum, with the same length as this. And thus the lengths of both of the simple pendulums have the same square ratio of the times, with which the small oscillations are carried out.

Since in the counting of any oscillations, which pass in an hour or in half an hour of time, the work is not tedious, as the clock assists by counting the seconds, shown by the seconds hand; a simple pendulum of any length may be taken, of which the number of oscillations in a period of one hour can be found in this way ; for from the length of the three foot pendulum, to the second, this number will emerge by calculation as before. [p. 155]

PROPOSITION XXV.

To find the distances that weights traverse falling vertically in a given time.

Up to the present, anyone who has investigated this measurement, has found it necessary to consult experimental evidence; from these experiments set up it has not been easy to give an exact determination of the distances, on account of the speed acquired by falling towards the end of the motion. Moreover, from our Prop. 25, concerning the fall of weights, and from knowing the length of the pendulum to within the second, we are able as a consequence to determine this time reliably without doing an experiment. In the first place, we examine the distance that the weight slips past in a time of one second; from which all the other distances can then be deduced. Since as we have said that the length of the pendulum that measures time to the second is that of the 3 foot horarum [*i. e*. Huygens' standard reference pendulum]: and moreover, the time for a small oscillation is to the time to descend vertically from half the height of the pendulum, as the circumference of the circle to its diameter, that is, as 355 to 113
[this is π to 7 sig. figs. ; also, a weight freely falling a distance equal to $l/2$ from rest, or half the length of the pendulum l, from our perspective, does this in a time $t = \sqrt{l/g}$, while the time for the swing of the pendulum from one side to the other is $T = \pi\sqrt{l/g}$, from which the above ratio $T/t = \pi$ follows; that is, the time for a weight to fall a distance h is 1/π.× the time for a swing of a pendulum of length $2h$.]
If the ratio is taken, as the number of seconds of the former to the latter, thus the time of one second, or expressed as sixty sixtieth parts of a second to the other, then the latter make $19'''\frac{1}{10}$, the time to fall from rest through a distance of half the length of the pendulum, which surely is 18 inches.
[the symbol ''' represents 1/60 of a second; thus, a time of 1 second corresponds ot a time of falling 60/π = 19.098.. sixtieths of a second, or ~ 19.1/60 seconds.]
Moreover, the squares of these times are thus to the distances gone through in these times, as was shown in the above proposition. Hence, if the ratio is formed as the square from $19'''\frac{1}{10}$ [corresponding to 18 inches] to the square from $60'''$ [corresponding to the unknown distance in inches fallen in one second], that is, as 36481 to 360000, thus 18 inches to other, gives 14 feet, 9 and a half inches, to be the distance fallen vertically in a time of one second. [For $19.1^2/60^2 = 18/h$.] Moreover, since the horarium foot is in the ratio to Parisien foot as 881 to 864; this will be the same height, reduced to that measure, as nearly 15 feet and one inch. And these in short agree with the most accurate of our experiments, for which that point of time, when the fall has finised, cannot be distinguished from judgement by the eye or ear ; but the distance fallen through, that we have tried to expound here by other means, is known to be free from any error.

The half oscillation of a pendulum suspended from a wall or an upright table can be taken to show the change in time for a weight to fall. In order that at the same moment as the small spherical bob of the pendulum is released, a lead weight is dropped, and with each held in place by a narrow thread, which is severed by an applied fire. But first, for the weight to be released, another small length of string is attached, of which the length, as when the whole length is drawn out by the falling weight, the pendulum has not yet pushed against the wall. [p. 156]. The other end of this string is stuck to ruled paper or to prepared thin parchment ; this is applied to the wall or table, in order that it can easily follow the pulled string, and to fall along its length in a straight line; crossing in that place, where the sphere of the pendulum strikes the table. Therefore by taking the whole string, the part of the ruled paper above can be drawn down, by falling with the weight, before the pendulum strikes the table. When a certain amount of the paper has slid past, the sphere, which has been lightly dusted with soot, leaves a mark. Moreover, here from the added length of string, a definite measurement of the distance fallen can be obtained.

Moreover, for the passage of the pendulum through the air, there nothing about this that we understand, except that a predetermined measurement with falling bodies agrees exactly with experiments [Thus, air resistance is negligibe]. Neither within reason is it so large, as with the heights which are given for these to ascend, that it is possible to see any difference; as with solid bodies made of metal or made from some lighter material, taken with a little more volume. Indeed light materials, which cut through the air by falling, thus they may be weighed according to the size of the body, as the wooden sphere, or even made of cork, can be compared with lead : obviously when the diameters of these to the diameter of lead have that ratio, that the specific gravity of lead has to specific gravity of wood or cork, then the weights of the spheres [given by specific gravity × volume]are thus to each other as the surface areas of these. Nevertheless, as the bodies fall with equal speeds, to the extent that it is not possible to distinguish between them, and which have different intrinsic weights, as there is no benefit served by having the diameters in that ratio. Indeed they can be made equal to each other, provided both are large enough : or they are not dropped from an excessive height. And indeed that too has been paid attention to here, for either the height is taken to be so great, or in mediums as with the height, so much is to be thrown away from the lightness of the body; as on account of the air resistance, the acceleration movement is so different from that which we have demonstrated above, with the proportion departing the most from the calculated values. And if in general, for any body considered, that has been set up to be slipping through air or any other liquid, with a certain speed, according to the ratio of its weight to its surface area ; then it is not possible to exceed that speed, or rather to reach htat speed. Which surely is that speed the body will have, that if in air, or in that liquid forcing the body up, holding the body, the same as if it possible to sustain it floating [Hints of terminal velocity]. Truly, concerning these, perhaps at another time, with many things to be done, the occasion will arise.

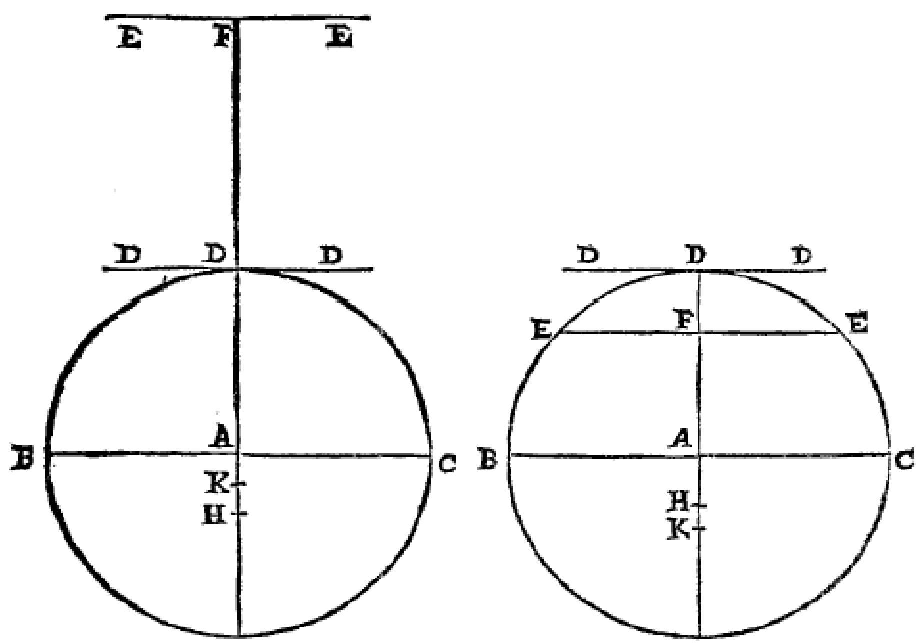
HOROLOGII OSCILLATORII

[p. 126]

PROPOSITIO XXI.

Quomodo in figuris planis centra oscillationis inveniantur.

Intellectis quae hactenus demonstrata sunt, facile jam erit in plerisque figuris, quae in Geometria considerari consueverunt, definire oscillationis centra. Atque ut de planis primum dicamus; duplicem in iis oscillationis motum supra definivimus; nempe, vel circa axem in eodem cum figura plano jacentem, vel circa eum qui ad figurae planum erectus sit. Quorum priorem vocavimus agitationem in planum, alterum agitationem in latus.



Quod si priore modo agitetur, nempe circa axem in eodem plano jacentem, sicut figura BCD circa axem EF; hic, si cuneus super figura intelligatur abscissus, plano quod ita secet planum figurae, ut intersectio, quae hic est DD, sit parallela oscillationis axi; deturque distantia centri gravitatis figurae ab hac intersectione, ut hic AD; itemque subcentrica cunei dicti super eadem intersectione, quae hic sit DH. Habebitur centrum oscillationis K, figurae BDC, applicando rectangulum DAH ad distantiam FA; quoniam ex applicatione hac orietur distantia AK, qua centrum oscillationis inferius est centro gravitatis. Est enim rectangulum DAH, multiplex secundum numerum particularum figurae BDC, aequale quadratis distantiarum ab recta BAC, quae per centrum gravitatis [p. 127] A parallela ducitur axi oscillationis EE [Prop. 10, huius]. Quare, applicando idem rectangulum ad distantiam FA, orietur distantia AK, qua centrum oscillationis inferius est centro gravitatis A. [Prop. 18, huius]

Hinc manifestum est, si axis oscillationis sit DD, fieri centrum oscillationis H punctum; adeoque longitudinem DH, penduli simplicis isochroni figurae BCD, esse tunc ipsam subcentram cunei, abscissi plano per DD, super ipsam DD. Quod unum ab aliis ante animadversum fuit, non tamen demonstratum.

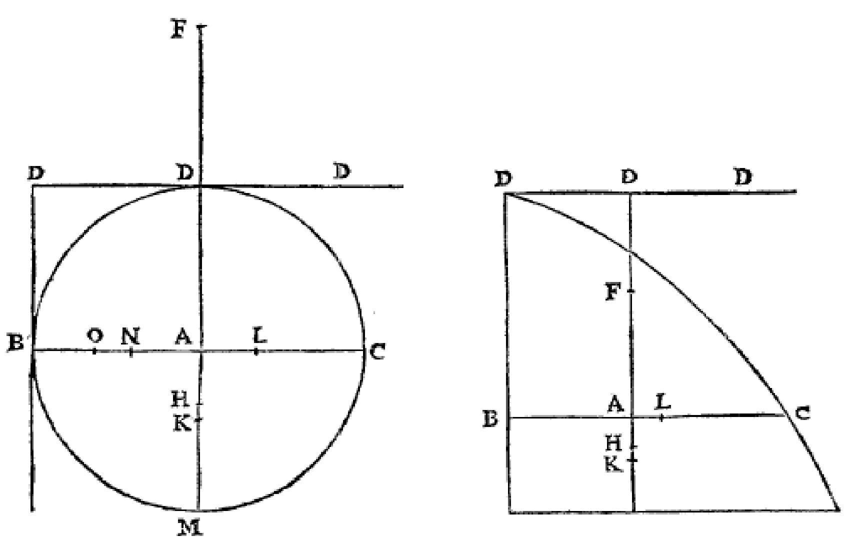
Quomodo autem centra gravitatis cuneorum super figuris planis inveniantur, persequi non est instituti nostri, & jam in multis nota sunt. Velut, quod si figura BCD sit circulus, erit DH aequalis $\frac{5}{8}$ diametri. Si rectangulum, erit DH = $\frac{2}{3}$ diametri. Unde & ratio apparet cur virga, seu linea gravitate praedita, altero capite suspensa, isochrona sit pendulo longitudinis subsesquialterae. Considerando nempe lineam ejusmodi, ac si esset rectangulum minimae latitudinis.

Quod si figura triangulum fuerit, vertice sursum converso, sit DH $\frac{1}{4}$ diametri. Si deorsum, $\frac{1}{2}$ diametri.

Quod autem propositione 16 demonstratum fuit, id ad hujusmodi figurae planae motum ita pertinere sciendum. Nempe se aliam atque aliam positionem demus figurae BCD, invertendo eam circa axem BAC, ut vel horixonti parallela jaceat, vel oblique inclinetur, manente eodem agitationis axe FE, etiam longitudo penduli isochroni FK eadem manebit. Hoc enim ex propositione illa manifestum est.

Porro quando figura plana, circa axem ad planum figurae erectum, agitatur; quam vocavimus agitationem in latus; velut se figura BCD moveatur circa axem, qui per punctum F intelligitur ad planum DBC erectus; hic jam praeter cuneum super figura, qui abscinditur plano ducto per DD, tangentem figuram in puncto summo, alter quoque considerandus cuneus, qui abscinditur plano per BD, tangentem figuram in latere, quaeque tangenti DD sit as rectos angulos. Oportetque dari, praeter figurae centrum gravitatis A, subcentricamque HD cunei prioris, etiam subcentricam LB cunei posterioris. Ita enim nota erunt rectangula DAH, BAL, quae simul sumpta faciunt hic spatium applicandum, quod deinceps etiam Rectangulum oscillationis vocabitur. Quod nempe, applicatum ad distantiam FA, dabit distantiam AK, qua centrum oscillationis K inferius est centro gravitatis A.

Si vero FA sit axis figurae BCD, potest, pro cuneo abscisso per [p. 128] BD super figura tota, adhiberi cuneus super figura dimidia DBM abscissus plano per DM. Nam, si



cunei hujus subcentrica super DM sit OA, distantia vero centri gr. figurae planae DBM ab eadem DM sit NA, aequale esse constat rectangulum OAN rectangulo BAL [Prop. 12, hujus.]. Itaque rectangulum OAN, additum rectangulo DAH, constituet quoque planum applicandum ad distantiam FA, ut fiat distantia AK.

Et horum quidem manifesta est demonstratio ex praecedentibus, quippe cum rectangula DAH, BAL, vel DAH, OAN, multiplicia secundum numerum particularum figurae, aequalis sint quadratis distantiarum a centro gravitatis A; sive, quod idem hic est, ab axe gravitatis axe oscillationis parallelo; ac proinde rectangula dicta, ad distantiam FA applicata, efficiant longitudinem intervalli AK [Prop. 18, huj.].

Centrum Oscillationis Circuli.

Et in circulo quidem rectangula DAH, BAL, inter se aequalia esse liquet, simulque efficere semissem quadrati a semidiametro. Unde, si fiat ut FA ad semidiametrum AB, ita haec ad aliam, ejus dimidium erit distantia AK, a centro gravitatis ad centrum oscillationis. Si igitur circulus ab axe D, in circumferentia sumpto, agitetur, erit DK aequalis tribus quartis diametri DM.

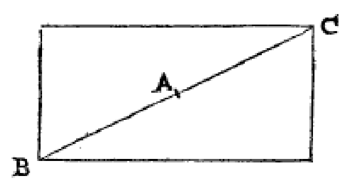
Ad hunc modum & in sequentibus figuris planis centra oscillationis quaesivimus, quae simpliciter adscripsisse sufficiet. Nempe, [p. 129]

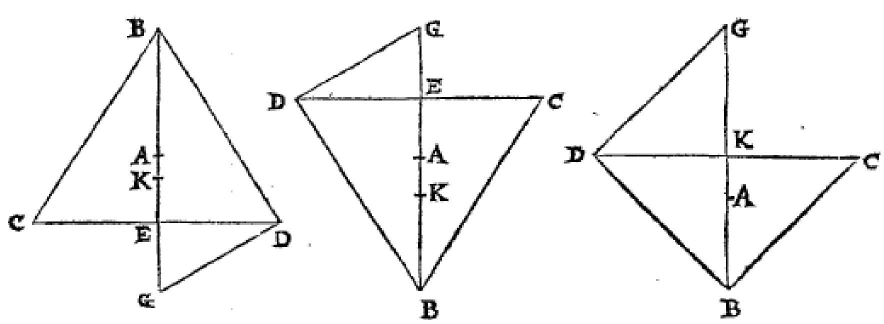
Centrum Oscillationis Rectanguli.

In rectangulo omni, ut CB, spatium applicandum, sive rectangulum oscilationis, invenitur aequal tertia parti quadrati a semidiagono AC. Unde sequitur, si rectangulum ab aliquo angulorum suspendatur, motuque hoc laterali agitur, pendulum illi isochronum esse $\frac{2}{3}$ diagonii totius.

Centrum Oscillationis Trianguli isoscelis.

In triangulo isoscele, cujusmodi CBD, spatium applicandum aequatur parti decimae





osctavae quadrati a diametro BE, & vegesimae quartae quadrati baseos CD. Unde, si ab angulo baseos ducatur DG, perpendicularis super latus DB, quae occurrat productae diametro BE in G; sitque A centrum gravitatis triangulis ; divisoque intervallo GA in quatuor partes aequales, una earum AK apponatur ipsi BA; erit BK longitudo penduli isochroni, si triangulum suspendatur ex vertice B. Cum autem ex puncto mediae basis E suspenditur, longitudo penduli isochroni EK aequabitur dimidiae BG.

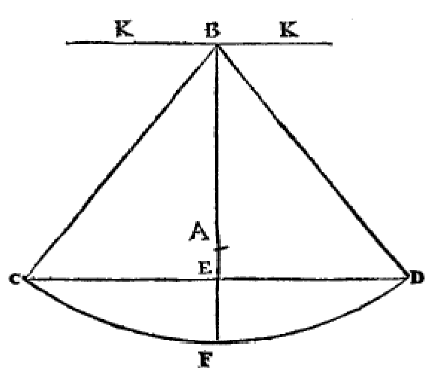
Atque hinc liquet, triangulum isosceles rectangulum, si ex puncto mediae basis suspendatur, isochronum esse pendulo longitudinem diametro suae aequalem habenti. Similiterque, si suspendatur ab angulo suo recto, eidem pendulo isochronum esse.

Centrum Oscillationis Parabolae.

In parabolae portione recta, spatium applicandum aequatur $\frac{12}{127}$ quadrati axis, una cum quinta parte quadrati dimidiae basis. Cumque [p. 130] parabola ex verticis puncto suspensa est, invenitur penduli isochroni longitudino $\frac{5}{7}$ axis, atque insuper $\frac{1}{3}$ lateris recti. Cum vero ex puncto mediae basis suspenditur, erit ea longitudo $\frac{4}{7}$ axis, & insuper $\frac{1}{2}$ lateris recti.

Centrum Oscillationis Sectoris circuli.

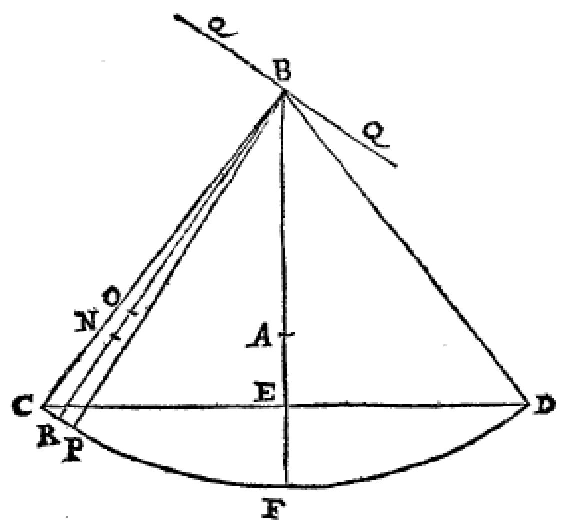
In circuli sectore BCD, si radius BC vocetur r : semi arcus CF, p : semisubtensa CE, b : sit spatium applicandum aequale $\frac{1}{2}rr - \frac{4bbrr}{ppp}$, hoc est, dimidio quadrati BC, minus quadrato BA; ponendo A esse centrum gravitatis sectoris. Tunc enim BA $= \frac{2br}{3p}$.



Si autem suspendatur sector ex B, centro circuli sui, sit pendulum ipsi ioschronum $\frac{3pr}{4b}$, hoc est, trium quartum rectae, quae sit ad radium BF ut arcus CFD ad subtensam CD. Haec autem inveniuntur cognitis subcentricis cuneorum; tum illius qui super sectore toto abscinditur, plano ducto per BK parallalem subtensae CD, cujus cunei subcentricam super BK invenimus esse $\frac{1}{8}r - \frac{3}{8}a + \frac{3pr}{8b}$, vocando a sinum versum BF; tum illius super dimidio sectore BFC abscinditur plano per BF, cujus nempe cunei subcentricam super BF invenimus $\frac{3}{8}b - \frac{3br}{8a} + \frac{3pr}{8a}$.

Sed & alia via, sectoris centrum oscillationis, facilius invenitur, quae est hujusmodi. Intelligatur sectoris BCD pars minima sector BCP, qui trianguli loco haberi potest. Quadrata autem, a distantiis particularum ejus a pucto B, aequalia sunt quadratis distantiarum ab recta BR, bifariam sectorem dividente, una cum quadratis distantiarum ab recta BQ, quae ipsi BR est ad angulos rectos. Sed, horum quadratorum ad illa, ratio quavis data est major, quoniam angulus CBP minimus; ideoque illa pro nullis habenda sunt. [p. 131]

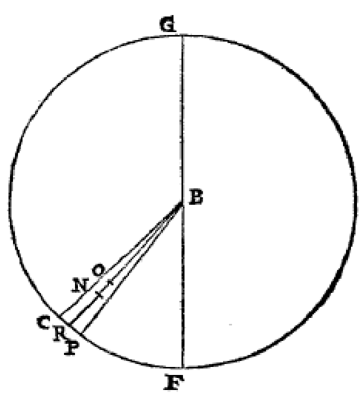
Posita vero BO duarum tertiarum BR, hoc est, posito O centro gravitatis trianguli BCP; & BN trium quartarum BR; ut nempe N sit centrum gravitatis cunei, super triangulo BCP abscissi plano per BQ. His positis, constat quadrata, a distantiis particularum trianguli BCP ab recta BQ, aequari rectangulo NBO multiplici secundum particularum ejusdem trianguli numerum. Itaque rectangulum NBO, ita multiplex, aequale censendum quadratis distantiarum a puncto B particularum trianguli BCP. Sunt autem quadrata distantiarum harum, ad quadrata distantiarum totius sectoris BCD, sicut sector BCP ad sectorem BCD, hoc est, sicut numerus particularum sectoris BCP, ad numerum particularum sectoris BCD; hoc enim facile intelligitur, eo quod sector BCD dividatur in sectores qualis BCP. Ergo rectangulum NBO, multiplex secundum numerum particularum sectoris BCD, aequale erit quadratis differentiarum particularum ejus a puncto B. Ideoque rectangulum NBO, applicatum ad BA, distantiam inter suspensionem & centrum gravitatis sectoris, dabit longitudinem penduli isochroni, cum sector ex B suspenditur [Prop. 17, huj.]. Est autem rectangulum NBO = $\frac{1}{2}rr$: distantia autem BA, ut jam ante diximus , equal to $\frac{2br}{3p}$. Unde, facta applicatione, oritur $\frac{3pr}{4b}$, longitudo penduli isochroni, ut ante quoque inventa fuit.



Centrum Oscillationis Circuli, aliter quam supra.

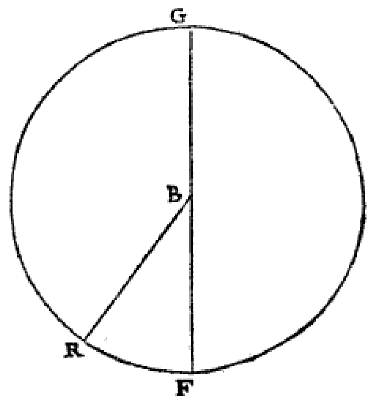
Eodem modo etiam simplicissime, in circulo, centrum oscillationis invenire licet. Sit enim circulus GCF, cujus centrum B; sectorque in eo minimus intelligatur BCP, sicut in sectore BCD. [p. 132]

Cum igitur, secundum modo exposita, quadrata, a distantiis particularum sectoris BCP ad centrum B, aequentur rectangulo NBO, hoc est, dimidio quadrato radii, multiplici secundum sectoris ipsius particularum numerum; circulus autem ex ejusmodi sectoribus componatur; erunt proinde quadrata, a distantiis particularum circuli



totius ad centrum B, aequalia dimidio quadrato radii, multiplici secundum numerum earundem circuli particularum.

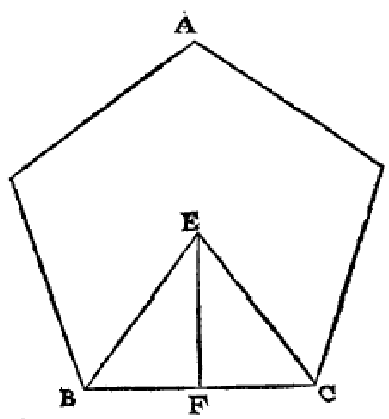
Est autem B centrum gravitatis circuli. Ergo dictum dimidium quadratum radii, hic erit spatium applicandum distantiae inter suspensionem & centrum B, ut habeatur intervallum, quo centrum oscillationis inferius est ipso centro B [Prop. 18, huj.]. Quod & supra ita se habere ostendimus.



Centrum oscillationis Peripheriae circuli.

Facilius etiam, centrum oscillationis circumferentiae circuli, hoc [p. 133] pacto reperitur. Esto enim circumferentia descripta centro B, radio BF. Quadratum igitur BR, multiplex secundum numerum particularum in quas circumferentia divisa intelligitur, aequatur quadratis a distantis omnium earum particularum ad centrum B. Quare quadratum BR erit hic spatium applicandum [Prop. 18, huj.]. Patetque hinc, si suspensio sit ex G, puncto circumferentiae, penduli isochroni longitudinem aequari diametro GF.

Centrum oscillationis Polygonorum ordinatorum.

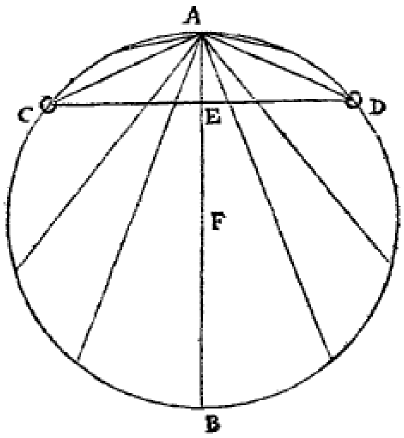


Haud absimiliter & polygono cuivis ordinato, ut ABC, pendulum isochronum invenitur. Fit enim, spatium applicandum, aequale semissi quadrati perpendicularis ex centro in latus polygoni, una cum vigesima quarta parte quadrati lateris. At, si perimetro poligoni pendulum isochronum quaeratur,sit spatium applicandum aequale quadrato perpendicularis a centro in latus, cum duodecima parte quadrati lateris.

Loci plani & solidi usus in hac Theoria.

Est praeterea & Locorum contemplato in his non injucunda. Ut si propositum sit, dato puncto suspensionis A, & longitudine AB, invenire locum duorum ponderum aequalium C, D, aequaliter ab A & a perpendiculari AB distantium, quae agitata circa axem in A, perpendicularem plano per ACD, isochrona sint pendulo simplici longitudinis AB.

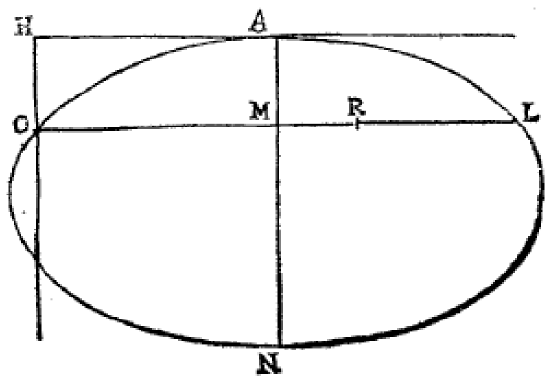
Ponatur AB = a, ductaque CD, quae secet AB ad angulos rectos in E, sit AE indeterminata = x : EC vel ED = y. Ergo quadratum AC = $xx + yy$. Hoc vero multiplex secundum numerum particularum ponderum C, D, quae hic minima intelliguntur, aequatur quadratis distantiarum earundem particularum ab axe [p. 134] suspensionis A. Ergo quadratum AC, sive $xx + yy$, applicatum ad distantiam AE, quae nempe est inter axem suspensionis & centrum gravitatis ponderum C, D, efficiet $\frac{xx+yy}{x}$, longitudinem penduli isochroni [Prop. 17, huj.]; quam propterea oportet aequalem esse AB sive a. Itaque $\frac{xx+yy}{x} = a$. Et $yy = ax - xx$. Unde patet, locum punctorum C & D, esse circumferentiam circuli, cujus centrum F, ubi AB difariam dividitur, radius autem $\frac{1}{2}a$, sive FA. Ergo, ubicunque in circumferentia ABCD duo pondera aequalia, aequaliter ab A distantia, ponitur, ea, ex A agitata, isochrona erunt pendulo longitudinem habenti aequalem diametro AB.



Atque hinc manifestum quoque, & circumferentiam ACBD, si gravitas ei tribuatur, & quamlibet ejus portionem, aequaliter in A vel B divisam, & ab axe per A suspensam, eidem pendulo AB isochronam esse.

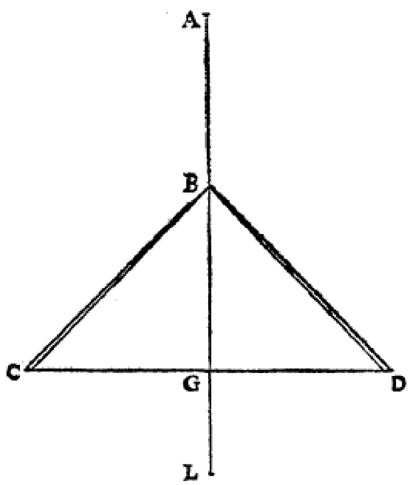
Loci vero solidi exemplum esto hujusmodi. Sit AN linea inflexilis sine pondere. Propositumque sit, ad punctum in ea acceptum, ut M, affigere ipsi ad angulos rectos lineam, seu virgam, pondere praedictam OML, ad M bifariam divisam, cujus in latus agitatae oscillationes, ex suspensione A, isochronae sint pendulo simplici longitudinis AN.

Ducatur OH parallela AN, & AH parallela OM, & sit OR aequalis $\frac{2}{5}$ OL. Itaque cunei super recta OL, abscissi plano per OH ducto, subcentrica erit OR. Sed cunei alterius super eadem OL, abscissi plano per rectam AH, (est autem cuneus hic nihil aliud quam rectangulum) subcentrica erit ipsa AM. Quare rectangulum illud, quod supra Oscillationis vocavimus, erit solum rectangulum OMR; quod nempe, applicatum ad longetudinem AM, dabit distantiam centri oscillationis linea OL, ex A suspendae, infra punctum M. [p. 135]



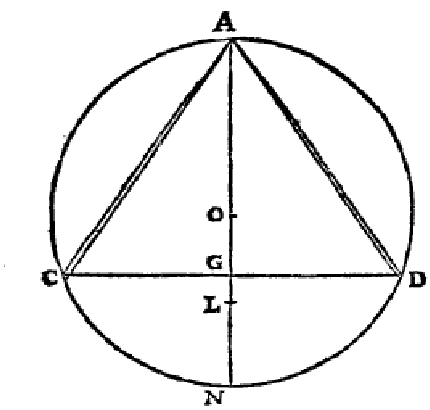
Sit jam AN = a ; AM = x; MO vel ML = y. Est ergo rectangulum OMR = $\frac{1}{3}yy$, quo applicato ad AM, sit $\frac{1}{3}\frac{yy}{x}$, quae longitudo itaque ipsi MN aequalis esse debebit, cum velimus centrum oscillationis virgae esse in N. Fit ergo aequatio $\frac{1}{3}\frac{yy}{x} + x = a$. Unde $y = \sqrt{3ax - 3xx}$. Quod significat puncta O & L esse ad Ellipsin, cujus axis minor AN; latus rectum vero, secundum quod possunt ordinatim ad axem hunc applicatae, ipsius AN triplum.

Hinc vero manifestum sit, cum omnis virga ipsi OL parallela, & ad Ellipsin hanc terminata, oscillationes isochronas habeat pendulo simplici AN, etiam totum Ellipseos planum, ex A suspensum & in latus agitatum, ipsi AN pendulo isochronum fore. Sed & partem Ellipseos quamlibet, quae lineis una vel duabus, ad AN perpendicularibus, abscindetur.



Caeterum adscribemus & aliud loci plani exemplum, in quo nonnulla notatu digna occurrunt.

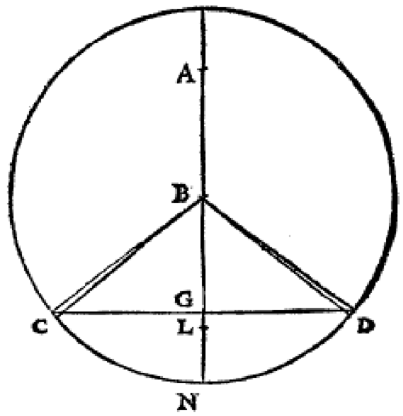
Si virga AB ponderis expers, suspensa ex A; oporteatque, ad datum [p.136] in ea punctum B, affigere triangula duo paria, & paribus angulis ab axe AB recedentia, quorum anguli ad B minimi, sive infinite parvi existimandi, quaeque, ita suspensa ab A, oscillationes isochronas faciant pendulo simplici datae longitudinis AL.



Hic, ducta CG perpendiculari in BG, & ponendo AB = a; AL = b; BG = x; CG = y: invenitur aequatio $y=\sqrt{(2ab-2aa-\frac{8}{3}ax+\frac{4}{3}bx-xx)}.$ Ex qua patet, bases triangulorum C, & D, quae bases hic ut puncta considerantur, esse ad circuli circumferentiam; quia nempe habetur terminus simplex $-xx$.

Licet autem hic animadvertere, quod si a sit nihilo aequalis, hoc est, si punctum, ubi affiguntur trianguli BC, BD, sit idem cum puncto A; tum futura sit aequatio $y=\sqrt{\frac{4}{3}bx-xx}$. Ac proinde, hoc casu, si sumatur AO = $\frac{2}{3}b$, hoc est, = $\frac{2}{3}$ AL, centroque O per A circulus describatur A DN; erunt bases triangulorum AC, AD , ad illius circumferentiam. Cum igitur quaelibet duo triangula acutissima, quae ex A ad circumferentiam ACND constituuntur, magnitudine & situ sibi respondentia, centrum oscillationis habeant punctum L, posita AL = $\frac{3}{4}$ diametri AN; cumque circulus totus ex ejusmodi triangulorum paribus componatur; uti & portio ejus quaelibet, ut ACND, latera AC, AD aequalia habens; manifestum est, tum circuli totius, tum portionis qualem diximus, centrum oscillationis esse in L.

Rursus, si in aequatione inventa ponatur $\frac{8}{3}a=\frac{4}{3}b,$ seu $2a = b$; hoc est, si triangula affigi intelligantur in B, quod longitudinem AL secet bifariam, erit $y=\sqrt{2aa-xx}$, quae aequatio docet, quod si centro B, radio qui possit duplum BA, circumferentia describatur,

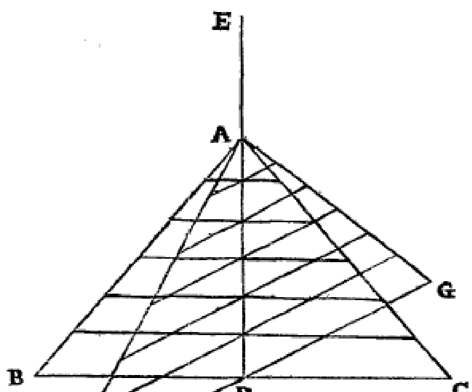


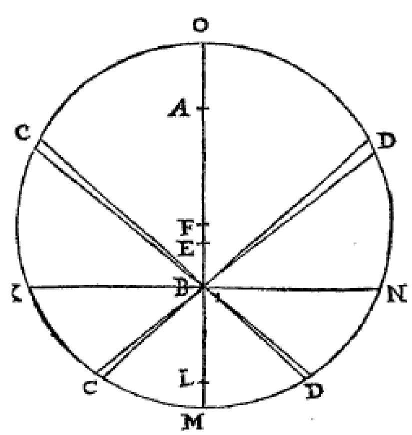
ea erit locus basium triangulorum acutissimorum BC, BD, quorum [p. 137] nempe, ex A suspensorum, centrum oscillationis erit L punctum. Cumque & circulus totus, & sector ejus quilibet, axem habens in recta AL, ex hujudmodi triangulorum paribus componatur, manifestum est & horum, ex A suspensorum, centrum oscillationis esse punctum L.

Adeoque quilibet circuli sector, suspensus a puncto quod distet, a centro circuli sui, semisse lateris quadrati circulo inscripti, pendulum isochronum habebit toti eidem lateri aequale. Atque ita, hoc uno casu, absque posita dimensione arcus, pendulum sectori isochronum invenitur.

Porro, ad universalem constructionem aequationis primae, $y=\sqrt{(2ab-2aa-\frac{8}{3}ax+\frac{4}{3}bx-xx)}$, dividatur AL bifariam in E, & adponatur ad BE pars sui tertia EF; eritque F centrum describendi circuli; radius autem FO aequalis sumendus ei, quae potest duplum differentiae quadratorum AE, EF.

Si itaque, ex puncto B, ad descriptam circumferentiam triangula duo paria acutissima constituantur, ut BC, BD; illorum, ex A suspensorum [p. 138], centrum oscillationis erit L. Quare & portionis cujuslibet descripti circuli, cujus portionis vertex sit in B, axis vero in recta AL, quales sunt utraque CBD; posita suspensione ex A; centrum oscillationis idem punctum esse constat. Atque adeo etiam circuli segmentorum KON, KMN, quae facit recta KBN perpendicularis ad AB.





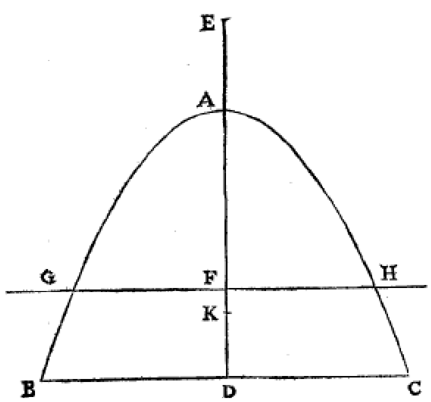
Et haec quidem de motu laterali planorum, ac linearum, animadvertisse sufficiat. Quibus hoc tantum addimus; inventis centris oscillationis figurarum rectarum, seu quae aequaliter ad axem utrinque constitutae sunt; ut trianguli isoscelis, vel parabolicae sectionis rectae; etiam obliquarum, quae velut luxatione illarum efficiuntur, ut trianguli scaleni, & parabolae non rectae, centra oscillationis haberi. Ut si, exempli, triangulum BAC isosceles, cujus axis AD, a puncto E suspensum intelligatur; sit vero & aliud triangulum scalenum FAG, axem eundem habens AD, & basin FG aequalem basi BC; etiam hoc triangulum, ex E suspensum, priori BAC isochronum esse dico.

Quia enim virga , seu linea gravis, FG, affixa virgae sine pondere ED in D, situ obliquo, suspensaque ex E, isochrona est virgae BC, similiter in D addixae [Prop. 16, huj.] ; idemque evenit in virgis caeteris trianguli utriusque, quae axem AD secant in iisdem punctis, atque inter se aequales sunt : necesse est tota triangula, quae ex lineis, seu virgis iisdem composita intelligi possunt, isochrona esse. In aliis figuris similis est demonstratio. [p.139]

PROPOSITIO XII.

Quomodo, in solidis figuris, oscillationis centra inveniantur.

In solidis porro figuris facile quoque, per ante demonstrata, centrum oscillationis invenire licebit. Si enim sit solidum ABC, suspensum ab axe, qui, per punctum E, intelligatur hujus paginae plano ad rectos angulos; centrum autem gravitatis sit F : ductis jam per F planis EFD, GFH, quorum posterius sit horizontali parallelum, alterum vero per axem E transeat; inventisque, per propositionem 14, summis quadratorum a distantiis particularum solidi ABC a plano GFH, itemque a plano EFD; hoc est, inventris rectangulis utrisque, quae, multiplicia secundum numerum dictarum particularum, aequalia sint dictis quadratorum summis; rectangula haec applicata ad distantiam EF, qua nempe axis suspensionis distat a centro gravitatis, dabunt intervallum FK, quo centrum agitationis K inferius est centro gravitatis F.



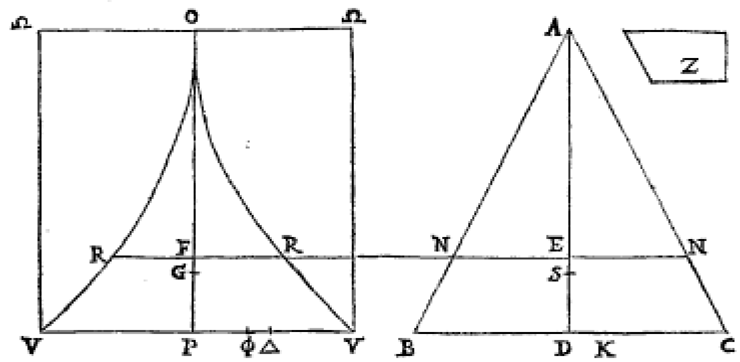
Hoc enim patet ex propositione 18. Dabimus autem & horum exempla aliquot.

Centrum oscillationis in Pyramide.

Sit primum AB pyramis, verticem habens A, axem AD, basin vero quadratum, cujus latus BC. Ponaturque agitari circa axem qui, per verticem A, sit hujus paginae plano ad angulos rectos.

Hic figura plana proportionalis OVV, a latere adponenda, secundum propositionem 14, constabit ex residuis parabolicis OPV, quae nempe supersunt, cum, a rectangulis ΩP, auferentur semiparabolae OVΩ, verticem habentes O. [p. 140]

Sicut enim inter se sectiones pyramidis BC, NN, ita quoque rectae VV, RR, ipsius in figura respondes. & sicut centrum gravitatis E distat, a vertice pyramidis, tribus quartis axis AD, ita quoque centrum gravitatis F, figurae OVV, distabit tribus quartis diametri OP a vertice O.



Intellecto porro horizontali plano NE, per centrum gravitatis pyramidis ABC, quod idem figuram OVV secet secundum RF; inventaque subcentrica cunei, super figura OVV abscissi plano per OΩ, quae subcentrica sit OG, (est autem $\frac{4}{5}$ diametri OP) erit rectangulum OFG, multiplex per numerum particularum figurae OVV, aequae quadratis distantiarum ab recta RF (Prop. 10, huj.), ac proinde quoque quadratis distantiarum a plano NE, particularum solidi ABC. Fit autem rectangulum OFG aequale $\frac{3}{80}$ quadrati OP, vel quadrati AD.

Deinde, ad inveniendam summam quadratorum a distanciis a plano AD, noscenda primo subcentrica cunei, super quadrata basi pyramidis BC abscissi, plano per rectam quae in B intelligatur axi A parallela; quae subcentrica sit BK; estque $\frac{2}{3}$ BC. Noscenda item distantia centr. gr. dimidiae figurae OPV ab OP; quae sit ΦP; estque $\frac{3}{10}$ PV. Inde, divisa bifariam PV in Δ, si fiat ut ΔP ad PΦ, hoc est, ut 5 ad 3, ita rectangulum BDK, quod est $\frac{1}{12}$ quadrati BC, ad aliud spatium Z; erit hoc, multiplex secundum numerum particularum solidi ABC, aequale quadratis distantiarum a plano AD (Prop. 15, huj.). Apparet autem, fieri spatium Z aequale $\frac{1}{20}$ quadrati BC.

Itaque, totum spatium applicandum, aequatur hic $\frac{3}{80}$ quadrati AD, cum $\frac{1}{20}$ quadrati BC. Unde, si suspensio, ut hic, posita fuerit in A, vertice pyramidis, ideoque distantia, ad quam applicatio facienda, [p. 141] AE aequalis $\frac{3}{4}$ AD; fiet hinc ES, intervallum quo centrum agitationis inferius est centro gravitatis, aequae $\frac{1}{20}$ AD, atque insuper $\frac{1}{25}$ tertiae proportionalis duabus AD, BC; sive tota AS aequalis $\frac{4}{5}$ AD, praeter dictam $\frac{1}{15}$ tertiae proportionalis.

Centrum oscillatonis Coni.

Quod si ABC conus fuerit, omnia eodem modo se habebunt, nisi quod spatium Z hic sit aequae rectangulo ΔPΦ (Prop. 15, huj.), hoc est $\frac{1}{20}$ quadrati PV vel BD, sive

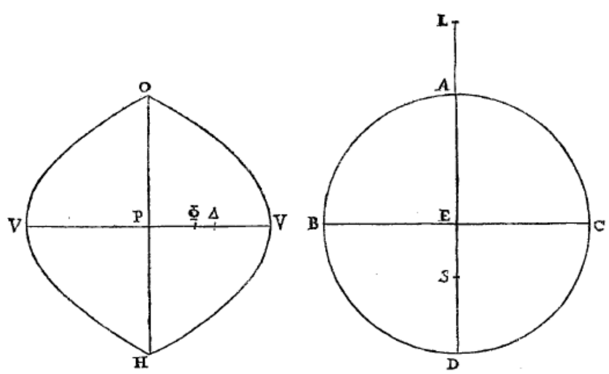
$\frac{3}{80}$ quadrati BC. Quare, totum spatium applicandum, in cono erit $\frac{3}{80}$ quadrati AD, una cum $\frac{3}{80}$ quadrati BC. Ac proinde, posita suspensione ex vertice A, fiet ES, qua centrum agitationis inferius est centro gravitatis, aequalis $\frac{3}{20}$ AD, & $\frac{1}{20}$ tertiae proportionalis duabus AD, BC; sive tota AS aequalis $\frac{4}{5}$ AD, una cum $\frac{1}{5}$ tertiae proportionalis duabus AD, DB. Atque hinc manifestum est, si AD, DB aequales sunt, hoc est, si conus ABC sit rectangulis, fieri AS aequalem axi AD.

Sequitur quoque porro, ex propositione 20. conum hunc rectangulum, si ex D centro basios suspendatur, isochronum fore sibi ex vertice A suspenso, quamadmodum & de triangulo rectangulo supra ostensum fuit.

Centrum oscillatonis in Sphaerae.

Si ABC sit sphaera, erit figura plana proportionalis, a latere adponenda, OVH, ex parabolis composita, quarum basis communis OH, aequalis sphaerae diametro AD. Secta vero sphera planis per centrum E, quorum BC sit horizontali parallelum, AD vero verticale : ut inveniatur summa quadratorum a distantiis a plano AD, noscenda est distantia centri gr. parabolae OVH ab OH, quae sit ΦP, estque $\frac{2}{5}$ VP. Deinde, divisa PV bifariam in Δ, constat rectangulam ΔPΦ, multiplex per numerum particularum spherae ABC, aequari quadratis distantiarum a plano AD (Prop. 15, huj.). Est autem rectangulum ΔPΦ aequale $\frac{1}{5}$ quadrati PV, vel quadrati BE.

Atqui, quadrata distantiarum a plano BC, aequalia esse liquet quadratis distantiarum a plano AD, ac proinde eidem rectangulo ΔPΦ, multiplici per dictum particularum numerum. Ergo spatium applicandum, in sphera ABC, erit duplum rectanguli ΔPΦ ; ideoque aequale $\frac{2}{5}$ quadrati a radio EB.



Itaque, si sphera suspensa sit ex puncto in superficie sua A, erit [p. 142] ES, a centro spherae E ad centrum agitationis S, aequalis $\frac{2}{5}$ semidiametri AE. Totaque AS aequalis $\frac{7}{10}$ diametri AD. Si vero ex puncto alio, ut L, sphera suspensa sit; erit ES aequalis $\frac{2}{5}$ tertiae proportionalis duabus LE, EB.

Centrum oscillationis Cylindri.

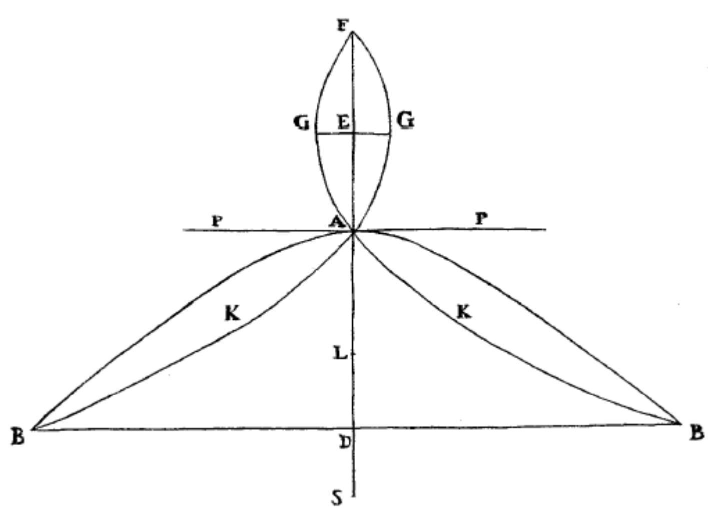
In cylindro, invenimus spatium applicandum aequari $\frac{1}{12}$ quadrati altitudinis, una cum $\frac{1}{4}$ quadrati a semidiametro basis. Unde si cyclindrus a centro basis superioris suspendatur, sit longitudo penduli isochroni aequalis $\frac{2}{3}$ altitudinis, una cum semisse ejus, quae sit ad semidiametrum basis ut haec ad altitudinem.

Centrum oscillationis Conoidis Parabolici.

In conoide parabolico, rectangulum oscillationis est $\frac{1}{18}$ quadrti altitudinis, cum $\frac{1}{6}$ quadrati a semidiametro basis. Unde, si a puncto verticis fuerit suspensum, sit longitudo penduli isochroni $\frac{3}{4}$ axis, cum $\frac{1}{4}$ ejus quae sit ad semidiametrum basis, sicut haec ad axem, id est, una cum $\frac{1}{4}$ lateris recti parabolae genitricis.

Centrum oscillatonis Conoidis Hyperbolici.

In conoide quoque hyperbolico centrum oscillationis inveniri potest. Si enim, exempli gratia, sit conoides cujus sectio per axem, hyperbola BAB; axem habens AD, latus transversum AF : erit figura plana ipsi proportionalis BKAKB, contenta basi BB, [p. 143] & parabolicae lineae portionibus similibus AKB, quae parabolae per verticem A transeunt, axemque habent GE, dividentem bifariam latus transversum AF, ac parallelum basi BB. Et hujus quidem figurae BKADB, centrum gravitatis L, tantum distat a vertice A, quantum centrum gravitatis conoidis ABB; estque axis AD ad AL, sicut tripla FA cum dupla AD, ad duplam FA cum sesquialtera AD. Deinde & distantia centri gr. figurae dimidiae ADBK, ab AD, invenire potest, atque etiam subcentrica cunei super figura BKAKB, abscissi plano per AP, parallelam BB; hujus inquam cunei subcentrica, super ipsa AP, inveniri quoque potest; atque ex his consequenter centrum agitationis conoidis, in quavis suspensione; dummodo axis, circa quem movetur, sit basi conoidis parallelus. Atque invenio quidem,

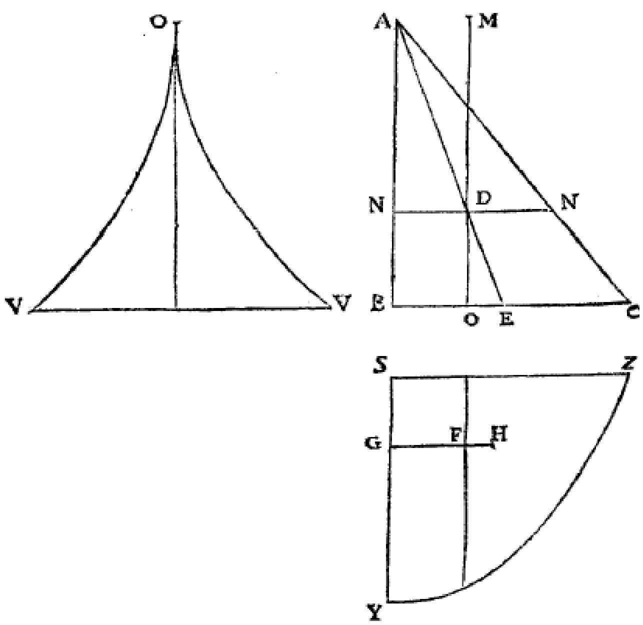


si axis AD lateri transverso AF aequali ponatur, spatium applicandum aequari $\frac{1}{20}$ quadrati AD, cum $\frac{11}{200}$ quadrati DB. Tunc autem AL est $\frac{7}{10}$ AD.

Unde, si conoides hujusmodi ex vertice A suspendatur, invenitur longitudino pendulo isochroni, AS, aequalis $\frac{27}{35}$ AD, cum $\frac{31}{140}$ tertiae proportionalis duabus AD, DB.

Centrum oscillatonis dimidii Coni.

Denique & in solidis dimidiatis quibusdam, quae fiunt sectione per axem, centrum agitationis invenire licebit. Ut si sit conus dimidiatus ABC, verticem habens A, diametrum semicirculi baseos BC [p. 144] : ejus quidem centrum gravitatis D notum est, quoniam AD sunt $\frac{1}{4}$ rectae AE, ita dividentis BC in E, ut, sicut quadrans circumferentiae circuli ad radium, ita sint $\frac{2}{3}$ CB ad BE. Tunc enim E est centrum gravitatis semicirculi baseos, ideoque in AE centra gravitatis omnium segmentorum semiconi ABD, basi parallelorum.

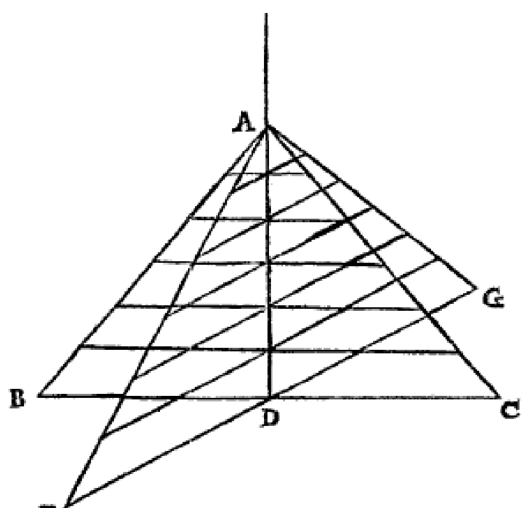


Et figura quidem porro proportionalis a latere pondenda, OVV, eadem est quae in cono toto supra descripta fuit : per quam nempe invenietur summa quadratorum, a distantiis particularum semiconi a plano horizontali ND, per centrum gravitatis ducto. Verum quadrata distantiarum, a plano verticali MDO, ut colligantur, altera quoque figura proportionalis SYZ, sicut supra prop. 14 adhibenda est, cujus nempe sectiones verticales, exhibeant lineas proportionales sectionibus sibi respondentibus in semicono ABC, & hujus figurae cognoscenda est distantia centri gr. F ab SY, quam aequalem esse constat distantiae DN, centri gr. semiconi a plano trianguli AB; positaque HG subcentrica cunei abscissi super figura SZY, ducto plano per SY, noscendum est rectangulum GFH, cujus nempe multiplex, secundum numerum particularum semiconi ABC, aequabitur quadratis distantiarum semiconi in planum MDO. Licebit vero cognoscere rectangulum illud GFH, etiamsi subcentricae HG longitudo ignoretur, hoc modo.

Diximus supra, cum de cono ageremus, quadrata distantiarum a plano [p. 145] per axem ejus, aequari $\frac{1}{80}$ quadrati a diametro basis, sive $\frac{5}{20}$ quadrati a semidiametro, multiplicis per numerum particularum coni totius. Unde & hic, in semicono ABC, quadrata distantiarum a plano AB aequalia erunt $\frac{3}{20}$ quadrati BC, multiplicis per numerum particularum ipsius semiconi. Sed & rectangulum HGF, multiplex per numerum particularum semiconi ABC, aequatur quadratis distantiarum a plano AB, ut patet ex propositione 9. Ergo rectangulum HGF aequale $\frac{3}{20}$ quadrati BC. Ponendo autem AB = a; BC = b; & quadrantem circumferentiae, radio BC descriptae = q; sit EB = $\frac{2bb}{5q}$. Cujus cum ND tribus quartis aequetur, fiet proinde ND, sive GF = $\frac{1bb}{2q}$. Cujus quadratum auferendo a rectangulo HGF, quod erat $\frac{3}{20}$ quadrati BC, fiet rectangulum GFH = $\frac{5}{20}bb - \frac{3bb}{4qq}$. Hoc autem rectangulum, multiplex per numerum particularum semiconi ABC, aequatur quadratis distantiarum a plano MDO. At quadratis distantiarum a plano MD aequantur, ut in cono, $\frac{3}{80}aa$, multiplices per numerum particularum semiconi ABC. Itaque, totum spatium applicandum, aequabitur hic $\frac{3}{80}aa + \frac{3}{20}bb - \frac{3bb}{4qq}$.

Unde quidem centrum agitationis invenitur in omni suspensione semiconi, dummodo ab axe qui sit parallelus basi trianguli a sectione AB. Notandum vero, cum figura SZY sit ignorae prorsus naturae, subcentricam tamen GH, cunei super ipsa abscissi plano per SY, hinc inveniri. Nam, quia rectangulum HGF aequale erat $\frac{3}{20}bb$, sive quadrati BC, & GF aequalis $\frac{1bb}{2q}$, sit inde GH aequalis $\frac{3}{20}q$.

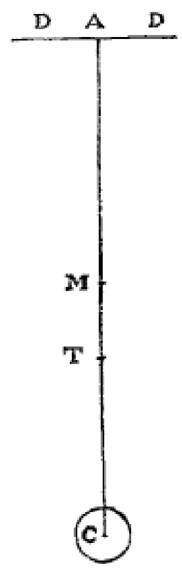
Porro, etiam semicylindri, & semiconoidis parabolici, centra agitationis inveniri possunt, atque aliorum insuper semisolidorum; quae aliis investiganda relinquimus.



Quaemadmodum autem in figuris planis, ita & hic in solidis figuris locum habet, quod de obliquarum centris agitationis illic diximus, quae veluti luxatione rectarum constituuntur, quarum centra oscillationis non different a centris oscillationis rectarum. Sic, si coni duo fuerit ABC, AFG, alter rectus, alter scalenus; quorum & diametri & bases aequales; hi ex vertice suspensi, vel a quibuscunque axibus, aequaliter a centris eorum gravitatis distantibus, isochroni erunt; dummodo axis, unde conus scalenus suspensus est, rectus sit ad planum trianguli per diametrum, quod planum base est ad angulos rectos.
[p. 146]

PROPOSITO XIII.

Horologiorum motum temperare, addito pondere exiguo secundario, quod super virga penduli, certa ratione divisa, sursum deorsumque moveri posset.



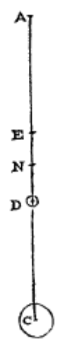
Ut hoc expediamus, primo penduli ipsius, ex virga gravitate praedita, & appenso parte ima pondere, compositi, centrum oscillationis inveniendum est.

Sit virga, cum appenso pondere, AC, cujus longitudo dictatur a. Intelligantur autem, tum virga ipsa, tum pondus appensum C, in particulas minimas aequales divisa, earumque particularum virga habeat numerum *b*, pondus vero C numerum *c*, ponendo nempe *b* ad *c*, sicut gravitas virgae ad gravitatem appensi ponderis. Longitudo igitur penduli simplicis, dato isochroni, habebitur, si summa quadratorum a distantiis particularum omnium a puncto suspendionis A, dividatur per summam earundem distantiarum (Prop. 6, huj.). Secetur AC bifariam in M; tum vero in T, ut AT sit dupla TC. Quia ergo M est centrum gravitatis lineae AC, & AT subcentrica cunei super ipsa abscissi plano per AD, perpendicularem ad AC; qui cuneus hic revera triangulum est; erit summa quadratorum, a distantiis particularum virgae a puncto A, [p. 147] aequalis rectangulo AMT, una cu quadrato AM; hoc est, rectagulo TAM, multiplici secundum numerum particularum *b*; hoc est, $\frac{1}{3}aab$; quia MA est $\frac{1}{2}a$, & TA $\frac{2}{3}a$, ac proinde rectanguluram TAM = $\frac{2}{3}aa$. Summa vero quadratorum, a distantiis particularum ponderis C ab eodem puncto A, aequabitur quadrato AC, multiplici secundum numerum particularum ipsius ponderis; hoc est, *aac*. Adeoque summa quadratorum omnium, tam a distantiis particularum virgae, quam ponderis C, erit $\frac{1}{3}aab + aac$.

Porro, distantiae omnes particularum virgae AC a puncto A, aequantur $\frac{1}{2}ba$; longitudini scilicet virgae ipsius, quae est *a*, multiplici secundum semissem numeri particularum quas continet. Et distantiae omnes particularum ponderis C, ab eodem puncto A, sunt *ac*. Ita ut summa utrarumque distantiarum sit $\frac{1}{2}ab + ac$. Per quam dividendo summam quadratorum prius inventam, $\frac{1}{2}aab + aac$, sit $\frac{\frac{1}{3}aab+aac}{\frac{1}{2}ab+ac}$ sive $\frac{\frac{1}{3}ab+ac}{\frac{1}{2}b+c}$, longitudo penduli isochroni.

Quae itaque habebitur, si fiat, ut dimidia gravitas virgae, una cum gravitate appensi ponderis, ad trientem gravitatis virgae, una cum gravitate ejusdem appendsi ponderis, ita longitudo AC ad aliam. Oportet autem sumere longitudinem AC, a puncto suspensionis A ad centrum gravitatis ponderis C; cum magnitudinis ejus ratio hic non habeatur, ac veluti minimum consideretur.

Quod si jam, praeter pondus C, alterum insuper D virgae inhaerere intelligatur, cujus gravitas, seu particularum numerus sit *d* : distantia vero AD sit *f*. Ut pendulum simplex huic ita composito isochronum inveniatur, addenda sunt ad summam superiorem quadratorum, quadrata distantiarum particularum ponderis D a puncto A, quae quadrata



apparet esse *dff*. Adeo ut summa omnium jam sit futura $\frac{1}{3}aab + aac + ffd$. Item, ad summam distantiarum, addendae distantiae particularum ponderis D, quae faciunt *df*. Ac summa proinde distantiarum omnium erit $\frac{1}{2}ba + ca + df$; per quam dividenda est ista quadratorum summa, & sit $\frac{\frac{1}{3}aab+aac+ffd}{\frac{1}{2}ab+ac+fd}$, longitudo penduli isochroni.

Quod si vero, haec longitudo penduli isochroni, datae aequalis postuletur, quae sit *p*, & reliqua omnia quae prius data sint, praeter [p.148] distantiam AD seu f, quae determinat locum ponderis D : sitque invenienda haec distantia, id fiet hoc modo. Nempe, cum postuletur $\frac{\frac{1}{3}aab+aac+ffd}{\frac{1}{2}ab+ac+fd}$ aequale *p*, orietur ex hac aequatione $ff = pf + \frac{\frac{1}{2}abp + cap - \frac{1}{3}aab - aac}{d}$. Et $f = \frac{p}{2} \pm \sqrt{\frac{1}{4}pp + \frac{\frac{1}{2}abp + cap - \frac{1}{3}aab - aac}{d}}$. Ubi animadvertendum, duas esse veras radices, si $\frac{1}{2}abp + cap$ minus sit quam $\frac{1}{2}aab + aac$; hoc est, si longitudino punduli isochroni, sive distantia centri oscillationis a suspensione, in pendule composito ex virga AC & pondere C.

Unde patet, si velimus efficere, ut, applicato pondere D, acceleretur penduli motus; posse duobus locis, inter A & C, illud disponi, quorum utrolibet eadem celeratis pendulo concilietur :velut in D vel E. Quae loca aequaliter distantibus a puncto N, quod abest ab A, semisse longitudinis p, hoc est, semisse penduli simplicis, cui compositum hoc isochronum postulabatur. Apparet autem, quando haec longitudo *p* tantum exiguo minor ponitur quam AC, etiam punctum N exiguo superius esse puncto medio virgae AC.

Porro, ex aequatione superiori, $f = \frac{p}{2} \pm \sqrt{\frac{1}{4}pp + \frac{\frac{1}{2}abp + cap - \frac{1}{3}aab - aac}{d}}$ habetur determinatio longitudinis *p*. Patet enim, $\frac{1}{4}pp + \frac{\frac{1}{2}abp + acp}{d}$ non minus esse debere quam $\frac{\frac{1}{3}.abb + acc}{d}$. Unde non debebit esse minor quam $\frac{a}{d}\sqrt{(\frac{4}{3}bd + 4cd + bb + 4bc + 4cc)} - \frac{ab-2ac}{d}$. Quod si p aequatur huic quantitati, hoc est, si $\frac{1}{4}pp + \frac{\frac{1}{2}abp + acp}{d}$ fuerit aequale $\frac{\frac{1}{3}abb + acc}{d}$, erit jam, in eadem superiori aequatione, $f = \frac{1}{2}p$, hoc est, $\frac{a}{2d}\sqrt{(\frac{4}{3}bd + 4cd + bb + 4bc + 4cc)} - \frac{ab-2ac}{2d}$. Quo determinatur distantia ponderis D a puncto A, ex qua maxime omnium acceleret motum penduli.

Atque haec ad horologiorum usum sic porro adhibentur. Sit, exempli gratia, pendulum horologii, quod singulis oscillationibus scrupula secunda notet. Virgae autem gravitas sit $\frac{1}{50}$ gravitatis appensi ponderis in imo pendulo : & praeter hoc, sit aliud exiguum pondus mobile secundum virgae longitudinem, cujus gravitatis easdem [p.149] ponatur quae ipsius virgae. Quaeritur jam, quo loco hoc virgae imponendum, ut uno scrupulo primo acceleretur horologii motus, spatio 24 horarum. Item, ubi collocandum, ut duorum scrupulorum primorum sit acceleratio; item, ut trium, quatuor, atque ita porro.

Ductis viginti quatuor horis sexagies, fiunt 1440, quot nempe scrupula prima una die continentur. Ex his unum aufer, quando unius scrupuli acceleratio quaeritur : supersunt 1439. Ratio autem 1440 ad 1439 duplicata, proxime est ea quae 1440 ad 1438. Ergo, si penduli simplicis, secunda scrupula notantis, longitudo divisa intellegatur in partes aequales 1440, earumque 1438 alii pendulo tribuantur, hoc praecedet alterum illud, in 24 horis, uno scrupulo primo. Adeo ut hic p valeat partes 1438.

Quia autem pendulum horologii, ex virga metallica & pondere appenso compositum, isochronum ponitur pendulo simplici partium 1440; invenienda primum est virgae illis longitudo, ex aequatione superius posita. Erat nempe $\frac{\frac{1}{3}ab+ac}{\frac{1}{2}b+c}$ aequale longitudini penduli simplicis, quod isochronum composito ex virga habente longitudinem a, gravitatem b, & pondere affixo cujus gravitats c. Ergo si longitudino penduli simplicis isochroni dicatur s. Erit $\frac{\frac{1}{2}as+cs}{\frac{1}{3}b+c}=a.$ Positoque, ut hic, $c = 50$; $b = 1$; $s = 1440$; fiet $a = 1444\frac{4}{5}$, longitudo virgae.

Iam, quia erat $f=\frac{p}{2}\pm\sqrt{\frac{1}{4}pp+\frac{\frac{1}{2}abp+cap-\frac{1}{3}aab-aac}{d}}$, fiet $f=\frac{p}{2}+\text{vel}-\sqrt{\frac{1}{4}pp+72962p-105061210}$. Unde porro, si p sit, uti diximus, partium 1438; invenietur $f = 1331\frac{1}{2}$, qualium nempe s, seu pendulum simplex, secunda scrupual oscillationibus designans, continet 1440. Cujus longitudo si pedum trium statuatur, quos horarios vocavimus, habebit f uncias 33, & 3 unciarum uncias, quas lineas vocant. Vel, auferendo hanc longitudinem f a tota trium pedum longitudine, supererunt duae, lineae 9, a centro oscillationis penduli compositi sursum sumendae, ut habeatur locus ponderis D, unius scrupuli primi accelerationem praetans tempore 24 horarum. Eodem modo reliquas distantias, quibus virga dividenda est, calculo investigavimus, aliam atque aliam ponendo longitudinem p : easque subjecta tabella exhibemus [p.150], secundum cujus numeros etiam virga penduli divisa est, quae superius in descriptione horologii fuit exhibitae. Procedunt autem accelerationes diurnae, ut jam illic advertimus, per 15 scrupula secunda, seu primorum scrupulorum quarantes. Ex. gr. si, pondere mobili D haerente in parte 73, 4, inveniatur horologium tardius justo incedere, in 24 horis, differentia 15 secundorum scrupulorum; oportebit sursum adducere pondus D, usque ad numerum 85, 6, ut corrigatur.

Acceleratio horologii spatio 24 horarum.			Partes, a centro osc. sursum accipiendae
scrup.	*pr.*	*sec.*	*Linea & decima linearum pedis horarii.*
0,	15		7, 0
0,	30		15, 2
0,	45		23, 3
1,	0		32, 6
1,	15		41, 9
1,	30		51, 7
1,	45		62, 2
2,	0		73, 4
2,	15		85, 6
2,	30		99, 0
2,	45		114, 1
3,	0		131, 8
3,	15		154, 3
3.	30		192. 6

Centrum oscillationis altius est centro gravitatis C partibus 1, 4.

PROPOSITO XXIV.

Centri oscillationis rationem habere non posse, in pendulis inter Cycloides suspensis; & quomodo hinc orta difficultas tollatur.

Si quis, subtili examine, contulerit ea quae in superioribus, de pendulo inter cycloides suspenso, demonstravimus, cum his quae ad centrum oscillationis pertinent; videbitur ei deesse aliquid ad perfectam illam, quam praeferimus, oscillationum aequalitatem. Ac primo dubitabit, an, ad inveniendum circulum cycloidis genitorem, penduli longitudo accipienda sit a puncto suspensionis ad [p. 151] centrum gravitatis appensi plumbi, an vero centrum oscillationis; quod, ab altero illo, saepe sensibili intervallo distat, atque eo majore, quo major fuerit sphaera aut lens plumbea. Quid enim, si spaerae diameter quartam, aut tertiam partem, penduli longitudinis aequit ? Quod si ad centrum oscillationis illam longitudinem accipiendam dicamus, non tamen expediet quo pacto ea, quae de centro oscillationis ostensa sunt, conveniant pendulo continue longitudinem suam immutanti, quale illud quod inter cycloides movetur. Posset enim videri, etiam centrum oscillationis mutari, ad singulas diversas longitudines; quod tamen hoc modo intelligendum non est. Res sane explicatu difficillima, si omnimodam ακριβειαν sectemur. Nam in demonstratione temperorum aequalium in cycloide, mobile, per eam delaturm, veluti punctum gravitate praeditum consideravimus. Sed, si ad effectum spectemus, non magni facienda est difficultas haec; cum ponderis, quo pendulum constat, magnitudo in horologiis tanta non requiratur (et si quo majus eo melius) ut differentia centrorum gravitatis, & oscillationis, aliquid hic turbare possit. Quod si tamen effugere prorsus has tricas velimus, id ita consequemur, si sphaeram lentemve penduli, circa axem suum horizontalem, mobilem efficiamus : axis extrema utrinque, virgae penduli imae, inserendo : quae idcirco ut bifida hac parte sit necesse est. Fit enim hoc modo, ex motus natura, ut eandem perpetuo positionem, respectu horizontalis plani, sphaera penduli servet, atque ita puncta ejus quavis, aeque ac centrum ipsum, cycloides easdem percurrant. Unde cessat hic jam centrorum oscillationis consideratio; nec minus perfectam temporum aequalitatem tale pendulum consequitur, quam si puncto unico omnis ejus gravitatis contineretur.

PROPOSITIO XXV.

De mensura universalis, & perpetuae, constituendae ratione.

Certa, ac permanens magnitudinum mensura, quae nullis casibus obnoxia sit, nec temporum injuriis, aut longinquitate aboleri aut corrumpi possit, res est & utilissima, & a multis pridem quaesita. Quae sit priscis temporibus reperta fuisset, non tam perplexae nunc forent, de pedis Romani, Graeci, Hebraeique veteris modulo, disceptationes. Haec vero mensura, Horologii nostri opera, facile constituitur; cum sine illo nequaquam, aut aegre admodum, haberi possit. [p. 152] Etsi enim, simplici pendulorum oscillatione, hoc a quibusdam tentatum fuerit, numerando recursus que tota caeli conversione continentur, vel parte ejus cognita, per fixarum stellarum distantias, secundum ascensionem rectam; nec certitudo eadem hoc modo, quae adhibitis horologiis, contingit, & labor longe est molestissimus ac taediosissimus, propter numerandi solicitudinem. Quia autem, praeter horologia, aliquid, ad exactissimam hujus mensurae inquisitionem, etiam centrorum oscillationis notitia confert; ideo hic demum, post eorum tractationem, hanc determinationem subjicimus.

Aptissima huic rei sunt horologia, quorum oscillationes singulae secunda scrupula, vel eorum semisses, notant, quaeque indicibus etiam, ad ea demonstrando, instructa sunt. Postquam enim, fixarum stellarum observationibus, compositum fuerit, methodo illa quam in horologii descriptione ostendimus : aliud pendulum simplex, hoc est, sphaera plumbea, aut alia materia gravi constans, ex tenui filo religata, juxta suspendenda est, motuque exiguo impellanda; ac tantisper producenda, aut contrahenda fili longitudo, donec recursus ejus, per quadrantem horae, aut semissem, una ferantur cum reciprocationibus penduli horologio aptati. Dixi autem exiguo motu impellendum pendulum, quia oscillationes exiguae, puta 5 vel 6 partium, satis aequalia tempora habent, magnae vero non item. Tunc, accepta mensura distantiae, a puncto suspensionis ad centrum oscillationis penduli simplicis; eaque , si recursus singuli scrupula secunda valeant, in tres partes divisa; facient hae singulae longitudinem pedis, quem HORARIUM in superioribus vocavimus : quique, hoc pacto, non solum ubique gentium consitiui possit, sed & venturo aevo redintegrari. Adeo ut & moduli penum omnium aliorum, semel ad hunc proportionibus suis expressi, certo quoque in posterum cognosi possint. Sicut jam supra, pedem Parisiensem ad hunc horarium esse diximus, ut 864 ad 881; quod idem est ad si, posito prius pede Parisiensi, dicimus tribus huiusmodi pedibus, cum octo lineis & dimidia, constitui pendulum simplex, cujus oscillationis scrupulis secundis horariis responsurae sint. Pes autem Parisiensis ad Rhenanum, quo in patra nostra utuntur, se habet ut 144 ad 139; hoc est, quinque lineis suis diminutus, alterum illum relinquit. Atque ita & hic pes, & alii quilibet, perpetuo duraturas mensuras accipiunt.

Quomodo autem centrum oscillationis in sphaera, ex qualibet longitudine [p. 153] suspensa, inveniatur, in superioribus demonstratum est. Nempe, si fiat ut distantia inter punctum suspensionis & sphaerae centrum, ad semidiametrum ejus, ita haec ad aliam; ejus duas quintas, a centro deorsum acceptas, terminari in quaesito oscillationis centro. Facile autem apparet cur necessaria sit hujus centri consideratio, ad accuratam pedis Horari constitutionem. Nam, si a puncto suspensionis ad sphaerae centrum distantia accipiatur, sphaerae autem magnitudo non definiatur proportione ad fili longitudinem, non erit certa mensura penduli cujus recursus secunda scrupula metiantur; sed quo major erit sphaerae, hoc minor invenietur mensura illa, inter centrum sphaerae & punctum suspensionis intercepta. Quia in isochronis pendulis, centra quidem oscillationis a punctis suspensionum aequaliter distant; amplius autem descendit centrum oscillationis infra centrum sphaerae majoris, quam minoris.

Hinc necesse fuit illis, qui, ante hanc centri oscillatorii determinationem, mensurae universalis constituendae rationem inierunt; quod, jam inde a prima Horologii nostri inventione, noblis illa Societas Regia Anglicana sibi negotium sumpsit, & recentius doctissimus Astronomus Lugdunensis, Gabriel Moutonus; his, inquam, necesse fuit designare globuli suspensi diametrum, vel proportione certa ad fili longitudinem, cujus nempe tricesimam vel aliam partem aequaret; vel mensura quadam cognita, ut digiti vel pollicis. Sed hoc posteriore modo, ponitur jam certi aliquid, quod id ipsum est quod quaerendum est : etsi scio vix sensibilem errorem fore, dummodo sphaerae istam, quam jam dixi, magnitudinem non multum excedant. Priore autem posset quidem aliquo pacto res explicari; sed ita, ut numerandarum oscillationum labor subeundus sit, calculoque etiam utendum. Quamobrem praesit, centra oscillationis adhibendo, certam rationem sequi, nullisque praeter necessitatem legibus obligari; atque hic jam majoribus sphaeris quam exiguis potius utendum, quod illae occursu aeris minus impediantur.

Caeterum, non sphaerae tantum ex filio suspensae, sed & coni, cylindri, aliaque omnia solida, planaque, quorum centra oscillationis superius exhibuimus, ad hanc mensuram investigandem apta sunt; quoniam, a puncto suspensionis ad centrum oscillationis, certum idemque omnibus isochronis pendulis est intervallum. Neque etiam illa duntaxat horologia, quae secunda scrupula aut eorum semisses singulis penduli recuribus indicant, ad haec usurpare possumus; sed & alia quacunque penduli longitudine instructis propositum obtinebitur, dummodo ex rotarum proportionum certo,seu dentium numero,c cognoscatur numerus oscillationum certo tempore peragendarum. Invento enim pendulo simplici, cujus libratones singulae conveniant vel singulis, vel binis ternisve recursibus horologii, constabit jam hinc, quot penduli illius vices horae spatio transfigantur. Quarum numerus si quadretur, erit ut quadratum e 3600, numero scrupulorum secundorum horam unam efficientium, ad quadratum illius numeri, ita longitudino penduli simplicis inventi, (quae longitudino semper a puncto suspensionis ad centrum oscillationis accipienda est) ad longitudinem penduli illius horarii tripadalis, quod diximus. Hoc enim inde constat, quod duorum quorumvis pendulorum longitudines sunt inter se, sicut quadrata quorumvis pendulorum longitudines sunt inter se, licut quadrata temporum quibus singulae oscillationes transeunt; ideoque contrariam rationem habent quadratorum a numeris, quos efficiunt oscillationes aequalibus temporum intervallis peractae. Nam, cum hactenus experientia tantum comprobatum fuerit Theorema illud, de pendulorum longitudinibus; eas nempe duplicatem habere rationem temporum, quibus oscillationes singulae peraguntur; nunc ejus demonstratio ex superius traditis manifesta est. Cum enim ostenderimus, singulos recursus penduli, inter cycloides suspensi, ad casum perpendicularem, e dimidia penduli longitudine, certam rationem habere ; eam scilicet quam circumferentia circuli ad diametrum suam; facile hinc colligitur, tempora oscillationum in duobus pendulis esse inter se, sicut tempora descensus perpendicularis ex dimidiis eorum altitudinibus. Quae altitudines dimidiae, sive etiam totae, cum habeant rationem duplicatam temporum, quibus ipsae descensu perpendiculari percurruntur (Prop. 3, Part. 2); eaedem quoque duplicatam rationem habebunt temporum, quae oscillationes singulas metiuntur. Ab oscillationibus autem minimis penduli, inter cycloides suspensi, non different sensibiliter oscillationes minimae penduli simplicis, cujus eadem sit longitudo. Itaque & pendulorum simplicium longitudines, duplicatam rationem habebunt temporum, quibus oscillationes minimae transfiguntur.

Quod siquis oscillationum numerandarum, quae horae aut semihorae tempore transeunt, laborem non defugiat; horologium que adsit, cujus indes secunsa scrupula demonstret; quaecunque accipiatur penduli simplicis longitudo, ejus numerus oscillationum, quae hora una continentur, hoc modo cognoscetur; atque inde longitudo pendulo tripedalis, ad secunda scrupula, ut antea, calculo prodibit. [p. 155]

PROPOSITIO XXV.

Spatium definire, quod gravia, perpendiculariter cadentia, dato tempore percurrunt.

Hanc mensuram quicunque hactenus investigarunt, experimenta consulere necesse habuerunt; quibus, prout hactenus instituta fuere, non facile ad exactam determinationem pervenire, propter velocitatem cadentium,sub finem motus acquisitam. Ex nostra autem prop. 25, de Descensu gravium, cognitaque longitududine penduli ad secunda scrupula, absque experimento, per certam consequentiam, rem expedire possumus. Ac primo quidem spatium illius inquiremus, quod unius scrupuli secundi tempore grave praeterlabitur ; ex quo quaelibet alia deinde colligere licebit. Quia igitur penduli, ad secunda scrupula, longitudinem diximus esse pedum Horariorum 3 :tempus autem unius oscillationis minimae, est ad tempus descensus perpendicularis ex dimidia penduli altitudine, ut circumferentia circuli ad diametrum, hoc est , ut 355 ad 113 : si fiat, ut numerus horum prior ad alterum, ita tempus unius secundi scrupuli,sive sexaginta tertiorum, ad aliud; fient $19'''\frac{1}{10}$, tempus descensus per dimidiam penduli altitudinem, quae nempe est pedis unciarum 18. Sicut autem quadrata temporum, ita sunt spatia illis temporibus peracta, quemadmodum superiori propositione fuit ostensum. Ergo, si fiat ut quadratum ex $19'''\frac{1}{10}$ ad quadratum ex $60'''$, hoc est,ut 36481 ad 360000, ita 18 unciae ad aliud, fient ped. 14. unc. 9. lin. 6, altitudo descensus perpendicularis , tempore unius secundi. Cum autem pes Horarius sit ad Parisiensem, ut 881 ad 864; erit eadem altitudo, ad hanc mensuram reducta, proxime pedum 15 & unciae unius. Atque haec cum accuratissimis experimentis nostris prorsus conveniunt, in quibus punctum illud temporis, quo casus finitur, non aurium aut oculi judicio discernitur; quorum neutrum hic satis tutum est; sed spatium descendendo peractum, alio modo, quem hic exponere tentabimus, absque ullo errore cognoscitur.

Penduli, ad parietem tabulumve erectam, suspensi dimidia oscillatio moram temporis, cadendo absumpti, indicat. Cujus sphaerula, ut eodem momento ac plumbum casui destinatum dimittatur, utraque filo tenui connexa tenentur, quod admoto igne inciditur. Sed prius, casuro plumbo, funiculus alius adnectitur, ejus longitudinis, ut cum totus exierit a plumbo tractus, nondum ad parietem illidatur pendulum [p. 156]. Funiculi ejus caput alterum, regulae chartaceae, aut ex tenui membrana patatae, cohaeret; ita ad parietem tabulamve applicatae, ut trahentem funem facile seque possit, rectaque secundum longitudinem suam descendere; eo loci transiens, quo penduli sphaera ad tabulam accidet. Absumpto igitur funiculo toto, pars insuper regulae deorsum trahitur a cadens plumbo, priusquam pendulum ad tabulam pertingat. Quae quanta sit pars, sphaera fuligine leviter infecta, regulamque praeterlabentem signans, indicat. Huc autem addita funiculi longitudine, spatium cadendo emensum certo definitum habetur.

Aeris autem occursum, quasi nullus esset in his intelligimus, ut mensura cadentibus corporibus praefixa cum experimentis exacte consentiat. Nec sane tantus est ille, ut in altitudinibus his, quo ascendere datur, sensibile discrimen inducere possit; dummodo solido corpora e metallo, aut, si levior materia constent, mole grandiuscula accipiantur.

Levitas enim materiae, in iis quae cadendo aerem secant, ita magnitudine corporis pensatur, ut sphaera lignea, vel etiam e subere formata, paria faciat cum plumbea : quando nimirum diameter harum ad plumbae diametrum eam rationem habuerit, quam gravitas plumbi propria ad ligni suberisve gravitatem. Tunc enim gravitates sphaerarum erunt inter se sicut earum superficies. Veruntamen, ut aequali celeritate, quantum sensu percipi potest, decidant corpora, quae multum intrinseca gravitate differunt, nequaquam opus est ut proportio illa diametrorum servetur. Possunt enim inter se aequalia esse, dummodo utraque satis magna sint : aut ex non nimia altitudine decidant. Etenim illud quoque hic animadvertendum est, tantam vel altitudinem esse posse ; vel, in mediocri etiam altitudine, tantam projecti corporis levitatem; ut ob aeris renitentiam, acceleratio motus tandem ab illa, quam in superioribus demonstravimus, proportione plurimum recessura sit. Namque in universum, corpori cuilibet, per aerem aliudve liquidum labente, certu celeritatis modus, pro ratione ponderis ac superficiei suae, constitutus est; quem excedere, aut potius ad quem pervenire nunquam possit. Quae nempe celeritas ea est, quam si aer, aut liquor ille sursum tendens, haberet, suspensum corpus idem sibi innatans sustinere posset. Verum de his, alias fortasse, pluribus agendi occasio erit.