HOROLOGII OSCILLATORII

SECOND PART. [p. 21]

Concerning the descent of moving bodies and the motion of these on a Cycloid.

HYPOTHESIS.

I.

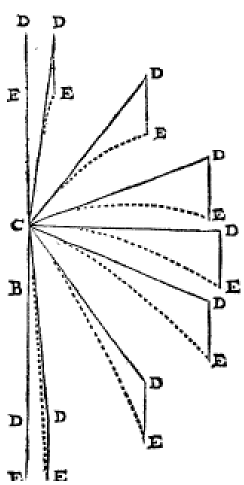
If there is no gravity, and the air offers no resistance to the motion of bodies, then any one of these bodies admits of a single motion to be continued with an equal velocity along a straight line.

II.

Now truly this motion becomes, under the action of gravity and for whatever the direction of the uniform motion, a motion composed from that constant motion that a body now has or had previously, together with the motion due gravity downwards.

III.

Also, either of these motions can be considered separately, neither one to be impeded by the other.



A body placed at C is released from rest, to travel a distance CB under the force of gravity for a certain time that we call F. And again it may be understood that the same body, if there were no gravity, may be given a motion in some direction by which in an equal time F, it will cross a distance CD with a uniform motion. Hence by adding the force of gravity the body will not reach the point D from C in the said time F, but some point E, placed under D, so that the line DE is always equal to the length CB; thus indeed, both the uniform motion, and that which arises from gravity, carry out their own motions, without the one hindering the other. For indeed the curved line traced out by this motion of the body will be defined in what follows, since it is not that of a uniform up or down motion but rather points sideways. Truly, when a line of constant motion CD [p. 22] tending downwards is affected by the perpendicular motion, it is apparent that the line CD is augmented by the line DE, by the added motion of gravity. Likewise, when the uniform motion CD points upwards in some direction, the same line CD is to be diminished by the line DE, as indeed, by acting for the time F, the body always arrives at the point E. Because if in both circumstances, by that reason stated, the two motions can be considered separately, then we are to consider that the one motion does not impede the other motion in any way, hence it is now appropriate to find the laws and the cause of the acceleration of the falling body. And indeed we will show those two at the same time.

PROPOSITIO I.

For a falling body, there are equal increases in speed in equal intervals of time, and the distances measured out from the start of the descent for equal time intervals continually increase by equal amounts.

Some body is placed at A and released from rest, falling in the first time interval a distance AB, and when it arrives at B, it has acquired a speed which thereafter, it would traverse some distance BD during the second time interval, with this steady motion. Hence we know that the distance gone through in the second time interval is greater than the distance BD, since even with the action of gravity suspended the distance traversed is BD. Truly the distance is found from the two parts of the motion, from the uniform component acquired by traveling through the distance BD, and from the motion due to the body falling, which by necessity is augmented by an amount equal to AB. [p. 23] Whereby we know that in the time for the second interval, the body arrives at E, which is equal to the sum of the distances BD and DE (equal to AB).

Truly concerning the velocity that the body shall have at E, at the end of the second time interval, we find that should be twice the velocity that it had at B at the end of the first time interval. Indeed we have said that the body is to be moved by a motion composed of two parts, from the constant motion acquired at B, and from the change in the motion produced by gravity, which change at the end of the second time interval shall clearly be the same as at the end of the first time interval, with the idea that in the course of the second time interval, the speed of the body has been increased by the same amount as in the first time interval. Whereby, with the speed acquired at the end of the first time interval kept intact, it appears that the speed at the end of the second time interval to be none other than twice the speed acquired at the end of the first time interval, or double that speed.

Whereas now, after the body arrives at E, if henceforth it should continue to move with a constant speed of the amount that it had acquired, it appears that in the third time interval, equal in length to those before, that it will traverse a distance equal to EF, which will be double the distance BD; because we have said that BD is to be run through with half of this speed, with a constant motion, and for an equal length of time. But again with the action of gravity added, for the body to travel through the third time interval, besides the distance EF, there is also the distance FG, itself equal to AB or DE. Thus at the end of the third time interval the body arrives at G. Truly the velocity here will now be the triple of that which the body had at B, at the end of the first time interval at B: since besides the speed acquired at E, that as we said was double the speed at B, the force of gravity for the descent of the third time interval, again contributes an increase in speed equal to that at the end of the first time interval. On account of which and the other speeds acquired, at the end of the third interval of time, the speed is agreed upon to be three times that which it was at the end of the first time interval.

In the same way it can be shown that by the time the fourth time interval has finished, both the distance GH should be three times the distance BD, and the distance HK should be equal to AB itself : and the velocity at K, at the end of the fourth time, to be four times that which it had at B, at the end of the first time interval. Thus for any distances considered in succession, which are travelled through in equal time intervals, each has been shown to increase in turn by an excess equal to the distance BD ; and likewise also the velocities are to be increased equally with equal time intervals.

[p. 24]

PROPOSITION II.

The distance completed by a falling body in a certain time interval, starting from rest, is half that distance that the body goes in the same time with a constant speed equal to the final speed acquired by the falling body.

These lengths may be placed in the previous proposition, where, for a body falling from A, some distance AB is traversed in a certain time, and BD truly is the distance gone in the same length of time where it is understood that the speed is constant and equal to that amount acquired at the end of the first time interval, or at the end of the distance AB. I say that the distance BD is twice AB.

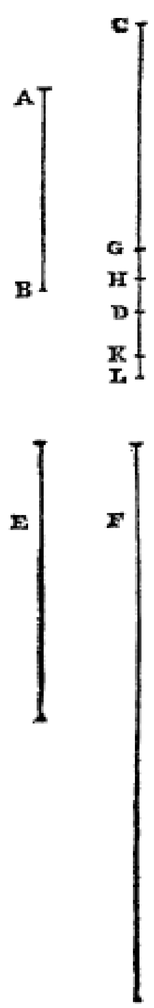
Indeed when the distances fallen through for the four first equal time intervals are AB, BE, EG, GH, there is a certain proportion between these : if we take twice as long time intervals for these times; as truly, for the first time, we now take these two for the distances AB and BE to have been traversed; for the second, truly the two times remaining for the distances EG and GK to be traversed, it is now required to show that the distances AE and EK, which are passed through from rest in equal time intervals, are thus to be as the distances AB and BE between themselves, which likewise are traversed in equal time intervals from rest.

Therefore, when AB is to BE as AE is to EK; then on inverting the ratio, it becomes, EB or DA to AB, thus as KE to EA: and also, on division, DB is to BA as the excess of KE over EA to EA.
[$DA/AB - 1 = KE/EA - 1$, giving $DB/BA = (KE - EA)/EA$; this is accepted as a true statement]
When moreover it shall be, as shown from the previous propositon, that interval KE is then the sum of twice AB, [for the sections FG and HK] as well as five times BD [for the section EF is twice BD, and the section GH is three times BD, from twice and three times the velocity over the same time intervals, giving $KE = 5 \times BD + 2 \times AB$] ; EA is truly equal to twice AB [for the sections AB and DE], as well as a single BD ; it is apparent that the excess of KE over EA to be equal to four times BD [for $KE - EA = 5 \times BD + 2 \times AB - BD - 2 \times AB = 4 \times BD$]. Therefore in like manner, [in the above equality to be proven], DB to BA will thus be four times BD to EA : hence four times BA will be equal to EA [as $4BD/EA = BD/BA$]: truly EA itself, as we have said, is equal to twice AB plus a single BD. Hence twice AB is equal to BD ; qed.

[p. 25]
PROPOSITIO III.

The two distances traversed, for a body falling for any two lengths of time from rest initially, are in the same ratio between themselves as the squares of the times of the falls, or also as the squares of the final velocities of each.

Indeed by the preceding propositon it will be shown the the distances AB, BE, EG, GK, and however many there might be, for a descent carried out and measured in equal time intervals, increase by equal amounts, each of which shall be equal to BD itself : Now, it is apparent, since BD is twice AB, the distance BE to be the threefold; EG the five fold of the same; GK the sevenfold, and the others following to be increasing following the progression of odd numbers from one, 1, 3, 5, 7, 9, &c. and for any quantity of these numbers, themselves of the following terms in the sequence, the sum of these makes a square; the side of the square itself taken from the multitude of numbers (as you wish, if the first three numbers are added, they make nine; if four, then sixteen; etc.) hence it follows that the distances, through which a falling body passes, each from rest, are in the same ratio between each other and of the squares of the times for which the descents lasted, if indeed commensurable times are taken.



The demonstration can easily be extended moreover for times that are not commensurable.
[i. e. time divisions that are not the same for the two motions; thus, the first motion might be recorded at one second intervals, and the next at half second or two second intervals, and so on].
Indeed let there be times of this kind used, which are in the ratio of the lines AB to CD, and the distances traversed in these time are E and F, and both obviously assumed to start form the instant the body is released. I say that the square ot the time AB is to the square of the time CD thus, as the distance E is to F.
[Thus, in this diagram, AB and the various lengths marked off in CL represent times; Huygens' proofs so far have relied on the same marking out of time in the descents; he now sets out to show by a *reductio ad absurdum* type argument that the same thing happens even if different time markings are used in the two descents, and thus the truth of the theorem is independent of the units in which time is measured.]

If indeed this were not the case; in the first place we would have the possibility that the ratio of the distance E to F is greater than the square AB to the square CD, and is surely equal to that instead, in which the square AB is to the square CG, [p. 26] with CG taken smaller than CD; from CD is taken the part DH, less than DG (the difference between CD and CG), in order that the remaining part CH is commensurable with AB; indeed it is agreed that this can be the case. Therefore CH is greater than CG. But as the square of the time AB is to the square of the time CH, thus the distance E descended in the time AB is to the distance descended in the time CH, as shown above [in the first units for time]. But truly this greater distance F is traversed in the time CD; Hence the ratio E to F is less than the square of AB to the square CH. Moreover, since the distance E to F is thus put to be as the square of AB to the square of CG by hypothesis; hence the ratio of the square of AB to the square of CG will be smaller too, than the square of AB to the square of CH, and hence the square of CG is greater than the square of CH; which is absurd, when CH is said to be greater than CG. Hence, we do not have the ratio of the distance E to F greater than the ratio of the square AB to the square CD.

[Let us use modern symbols to extract this part of the theorem, which can of course be shown quite easily by other means:

1. AB and CD are time intervals of different lengths which are not multiples of the same basic unit of time; and for each of these incommensurate time intervals there corresponds the distances E and F, for bodies dropped from rest.

2. In spite of this, the theorem contends that the ratio of the distances fallen E to F is still equal to the ratio of the time AB squared to the time CD squared.

Symbolically $(t_{AB}/t_{CD})^2 = E/F$. We are to prove the validity of this statement.

3. Let us assume that this is not the case, and that in fact for the given times, the ratio E to F is larger. Thus, t_{AB} is the time to fall $E = d_{AB}$; and t_{CD} is the time to fall $F = d_{CD}$, but we assume that $E/F = d_{AB}/d_{CD} > (t_{AB}/t_{CD})^2$ or $d_{AB}/(t_{AB})^2 > d_{CD}/(t_{CD})^2$.

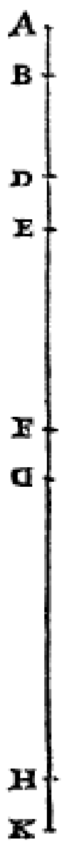
Hence, there will be a time for which the ratios are equal, and the equality can be achieved by choosing a shorter time CG for which $d_{AB}/d_{CG} = (t_{AB}/t_{CG})^2$, in an obvious notation.

4. There will be a time CH commensurate with the time AB somewhere in the interval CD for which CH > CG, for which the first part of the analysis can be applied : in this case, E/(distance corr. to time CH) $= d_{AB}/d_{CH} = (t_{AB}/t_{CH})^2$; but the greater distance F is travelled in the time CD, hence $E/F = d_{AB}/d_{CD} < (t_{AB}/t_{CH})^2$; and from the hypothesis, $E/F = (t_{AB}/t_{CG})^2$, it follows that $(t_{AB}/t_{CG})^2 < (t_{AB}/t_{CH})^2$, and hence $t_{CG} > t_{CH}$, which contradicts the assumption. Thus, CG cannot be less than CD; and likewise, it is shown next that CG cannot be greate than CD, and this can be shown in the same manner in modern terms; hence the equality remains.]

Now we may have, if possible, a smaller ratio; and the ratio of the distance E to F shall be the same as the ratio of the square AB to the square CL, with CL taken greater than CD, and from CL is taken LK from the small excess LD, by which CD is greater than CL; and thus as the remainder KC shall be commensurable with AB. Hence, as the square of the time AB to the square of the time CK, thus is the distance traversed E, passed in the time AB, to the distance run through in the time CK. Truly this second distance is smaller than the space traversed in the time CD, surely the distance F. Hence the ratio of the distance E to F is greater than the square of AB to the square of CK. Moreover, since the distance E to F is thus put to be as the square AB to the square CL; hence the ratio of the square of AB to the square of CL will be greater than that of the same square AB to the square CK, and likewise the square CL will be less than the square CK; which is absurd, when the square of CL is assumed be greater than the square CK. Hence, it does not have a smaller ratio of the distance E ad F than the square AB to the square CD. Whereby it is necessary that it has the same ratio. Again, with the speeds at the end of the times AB and CD which they have acquired between themselves, as with their own times; it is apparent that the ratio of the distances E to F also to be as the squares of the times AB and CD, for which they were sent. Thus the propositon is agreed upon.

PROPOSITON IV.

If a falling body is taken and aimed upwards, with the speed acquired at the end of the descent, so as it rises with the same time intervals as descending, then the same intervals of distance are traversed, and thus the body rises to the same height from which it descended. Likewise, as with equal time intervals, equal amounts of speed are taken from the motion

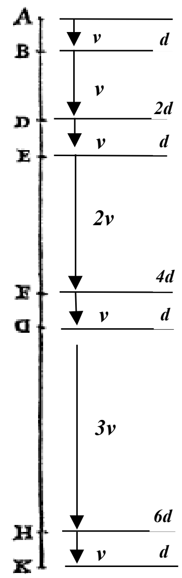


Indeed as in proposition 2 there shall be some distances, [p. 27]for equal small time intervals, passed through by the body falling from rest, of which the first is AB; the second is composed from BD, for which the speed is equal to that acquired in the passage through AB, and from DE itself equal to AB; the third is made up of EF, twice BD, and from FG, equal to AB ; the fourth is composed from GH, from three times BD, and from HK likewise itself equal to AB, and by the same reasoning again to increase, if there were more intervals. I say that for the same equal time intervals, the same individual distances in turn KG, GE, EB, BA, are traversed by the rising body, and the body starts with the speed acquired at the end of the descent K.

Moreover for the sake of brevity, we will consider each speed to be designated henceforth by a length of interval equivalent to that which a body with that uniform motion and time interval passed through in the descent we have considered.

[In what follows, let d be the distance fallen from rest AB, and v the speed at the end of this fall at the point B. In which case the distance BD is $2d$, while the distances DE, FG, HK are all of length d. The reader can refer to the modified diagram on the following page for ease in following the proof, though it must be appreciated that the added symbols are not part of the original geometrical presentation by Huygens.]

And thus from what has been shown in the said proposition, when the body arrives at K, it has the speed GH [$3v$] increased by the speed BD [v], i. e. it equals the speed KF [this is clearly wrong, as it gives a terminal speed of $5v$, rather than $4v$; hence the speed at K corresponds to GK or $4v$], since the distance KF [$8d$] is equal to HG [$6d$] and BD; for the individual sections HK and FG are each equal to AB; and hence likewise BD itself is twice the interval AB, as we have shown in proposition 2. And thus the speed acquired at the end of the descent K is to be converted on going up, if the body is to be brought up with a constant motion, and KF can be made the distance for one part of the time. And, by the action of gravity in the ascent, the ascent distance KF is diminished by the interval FG equal to AB, as is apparent from the said hypothesis originally assumed. Hence for the first part of the time the body ascends by as much as KG [$7d$] , by which in the same interval it last descended. Likewise truly the decrease necessary to the speed is the same as that acquired in the fall for the same part of time, that is the constant speed BD [The speed decreases from $4v$ to $3v$ in going up from K to G]. Thus the body, when it has ascended to G, has the speed remaining GH, as from the beginning of the ascent it had the speed HG together with the speed BD. Moreover the distance HG [6*d*] is equal to GD ; as this distance is equal to FE [4*d*], together with DB [2*d*] or with twice AB, or together with twice FG [*d*] and ED [*d*] ; Hence if from G, with a speed equal to that which it had [on descent], the body goes through another rise, and would make it through the interval GD [6*d*] in the next interval of time. But from the added action of gravity, the ascent is diminished by the interval DE [*d*] , equal to AB. Hence, in this second time interval, the body ascends through the interval GE [5*d*], which for a similar length of time the body also crossed in falling. Likewise moreover the speed must be diminished anew for as much as for one part of time, surely the speed BD [*v*]. Thus when the body ascends as far as E, it only has the speed FE [2*v*], which obviously is the speed left when the speed BD is taken from the speed GD. [p. 28] For BD, as we have now said, is equal to the sum of the two lengths DE and FG.



Huygens' diagram modified.

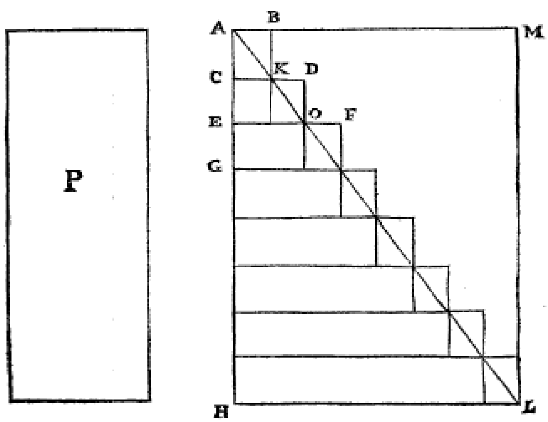
Moreover the length FE is equal to EA [4*d*], when FE is equal to twice BD, or BD together with twice AB, or twice AB and twice DE. Hence if from E with a constant speed, the body ascends for the duration of one time interval, it should give that amount of a distance EA. But with the effect of gravity taken into account, the ascent is diminished by the interval AB. Hence for this part of the time, only the distance EB has been risen, which for a like part of the time was passed through too in the descent. Here again truly as much of the speed is lost as is necessary for that acquired for a fall though one interval of time, that is, the speed BD. Thus the body, when it has ascended as far as B, has the speed BD itself left, since at E it had the speed FE [2*v*] the double of BD itself. If hence from B with a constant speed equal to that which it has initially, it executes a rise, to be taken in one part of the time, through a distance of DB, or twice AB. But with the action of gravity taken into account, the ascent is dimininshed to that interval which is equal to AB itself. Therefore from this part of the time the body only ascends through the interval BA, which it also passed through in the first descent. And in the end indeed at the limit of the time by necessity the body thus has returned back to A. But it would be absurd, as it is not possible, for a motion under gravity, that the body could ascend higher than from that which it fell. Also, for the amoung BD to be lost again for the speed that the body had at B, it is now apparent for the body placed at A that there is no speed remaining, and hence there is no higher extent of the motion. Thus it has been shown that a body will return to the same altitude from which it fell from rest, and the individual intervals, which the body passed through in equal time intervals, there are the same number of times for the times of return of the ascent : and also for the equal times it is apparent that the changes in the speeds of the motion are equal also. Hence the proposition is agreed upon.

Since truly in the demonstration of this following proposition, which depends on the preceeding one, it is assumed that there is a certain proportion between the distance intervals which the body passed through in falling successively through equal intervals of time, and this proportionality shall be the same, whatever equal time intervals may be taken ; since indeed from the nature of the investigation this is seen to be a necessary factor, and if it is denied, then it is admitting that the investigation of your distance intervals by proportion is in vain. Nevertheless, as the proposition can also be demonstrated without this, by the following method of Galileo, [p. 29] it is worthwhile to show this proportionality from this less perfect account than that which we have more carefully composed, and thus we will demonstrate this proposition anew.

PROPOSITIO V.

The interval traversed in a given length of time by a body falling from rest is half of the interval that the body would traverse at a constant rate in an equal time with the speed that it had acquired finally in the falling motion.

Let the whole time of the descent be AH, by which time in moving it will have traversed a certain interval, the size of which can be designated by the plane area P. With HL drawn perpendicular to AH of whatever length that will refer to that speed acquires at the end of the fall. Hence, on completing the rectangle AHLM, it is to be noted that the magnitude of the distance traversed in the time AH, with the final speed HL is understood. Therefore it is required to be shown that the rectangle of area P is half of the rectangle MH, that is, with the drawing of the diagonal AL to be equal to the triangle AHL.



If the area P is not equal to the area of the triangle AHL, it will hence be either smaller or greater than this. [There now follows a *reductio ad absurdum proof*]. In the first place, if it were possible, the plane area P could be made less than the area of the triangle AHL. Moreover, the whole of AH is divided into equal parts AC, CE, EG, &c., in order that the figure circumscribed to triangle AHL [p. 30] is equal to the sum of the rectangles BC, DE, FG, etc ; and as are the rectangles KE, OG, &c., for the other figure inscribed in the same triangle from the heights of the individual parts of the division of AH itself, is equal to the sum of the rectangles of the same height. ***In order that, as I say, the difference of this [larger] figure over that [smaller figure], may be less than the difference of the triangle AHL over the plane figure P.*** [Translator's bold italics] It is seen that it is possible to do this, with the total excess of the circumscribed figure over the inscribed figure equal to the smallest rectangle, having the base HL [made by adding the contributions AB, KD, OF, etc., which can thus be made as small as we please, by increasing the number of time intervals; Huygens does not actually say this explicitely, though it seems to be implied]. Hence the excess of the triangle AHL over the inscribed figure is entirely less than the excess over the plane figure P, and hence the figure inscribed in the triangle is greater than the plane figure P.

Moreover again, when the parts of the line AH of the time of the total descent are brought together. And when the speeds of the falling motion increase in the same proportion with the time of the descent (Prop. I); and the speed acquired at the end of the whole motion shall be HL; and CK shall be that speed which has been acquired at the end of the first part of the time AC; since as AH to AC, thus HL to CK. Similarly, EO will be the distance acquired at the end of the second part of the time CE, and thus henceforth. Moreover, it is apparent, with the time AC for the first, a certain space is to be passed through from the movement, which is greater than zero ; with CE truly the time for the second passage, then the interval traversed would be greater than KE, since the interval KE would be crossed in the time CE with a constant motion and with the speed CK. Indeed the distances have, with a constant speed of crossing, a ratio composed from the ratio of the times, and from the ratio of the velocities, and thus with the time AH, and with the constant speed equal to HL we can put the distance to traverse MH; with the second time CE, and with the speed CK then the distance traversed is KE, as the ratio of the rectangle MH to the rectangle KE is composed from the ratios AH to CE, & HL to EO.

Hence when, as I have said, the distance KE shall be that which is traversed in the time CE, with a speed equal to CK, moreover the falling body can be taken by the accelerated motion in the time CE, which now from the beginning of this time has the speed CK; it is clear that for this accelerated motion for the time CE, the body will cross a greater distance than KE. For the same reason, for the time of the third interval EG, a greater distance than OG will be put together, since clearly this distance has been put together from the same time EG, with a constant speed EO. And thus henceforth, for the invividual fractions of the time AH, greater distances will be tranversed by the moving body than rectangles of the inscribed figure, by the adjacent parts. Whereby the whole distance traversed with the accelerated motion will be greater than the inscribed figure. Truly that distance has been made equal to the plane area P. And thus the inscribed figure will be less than the distance P. The assumption made is hence absurd; indeed the same distance has been shown to be greater [as P includes the extra distances gone due to the acceleration]. Therefore the plane P is not less than the triangle AHL [which the minor sum approaches as the number of time divisions increases]. But it is required to show that P is not greater than the area of the triangle.

Indeed, let this be the case, [p. 31] if possible [*i. e.* area P > ΔAHL]; and AH is divided into equal parts, and to the same height of these, and circumscribed again to triangle AHL, as before, shall be the figure from the rectangles, thus in order that the one shall exceed the other of the smaller areas as the plane P shall exceed the triangle AHL, therefore it will be by necessity that the circumscribed figure is less than the plane P [for the new proposition to be true]. Now it is agreed, for the first part of the time AC, the distance to be transversed from the motion shall be less than BC, which is covered here in the same time AC with a speed equal to CK, which the body has gained at last at the end of the time of the motion. Similarly for the second part of the time CE, the distance travelled with the accelerated motion shall be less than DE, which is run through here in the same time CE, with the speed equal to EO, as finally at the end of the time CE is gained by the motion. And thus henceforth, for the individual parts of the time AH, smaller distances are passed through by the motion than are represented by the rectangles of the circumscribed figure, from the adjacent parts themselves. Whereby the whole distance executed by the accelerated motion, will be less than the distance represented by the circumscribed figure. Truly the distance is equal to that put for the plane P ; hence the plane P will also be less than the circumscribed figure. The new assumption is hence absurd, since this figure has been shown to be less than the figure P. Hence the plane P in not greater than the the triangle AHL, but it has been shown now not to be smaller. Hence it must be equal by necessity. QED.

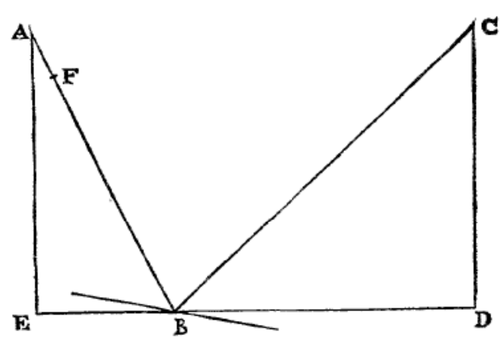
And these propositions indeed have established everything up to the present, for bodies equally ascending and descending inclined planes, and with the motion to the perpendicular known in agreement : since, with that which has been put in place concerning gravity, both cases are to be admitted by the same reasoning.

Thus truly it will no more more difficult now to demonstrate the following proposition than to concede the same, as Galileo has postulated and himself shown in a certain way. for he has tried to demonstrate that proposition produced afterwards, and which is present in the the last edition of his works, and which appears to be lacking in proof when judged by me. But here is the proposition.

PROPOSITION VI.

The speeds of bodies, acquired by falling down planes of different inclinations are equal, if the heights of the planes are equal.

We call the elevation of the plane the height along the perpendicular.
Thus let there be inclined planes, the sections of which make raised planes to the horizontal AB and CD ; and the heights of which AE and CD are equal. [p. 31] A body may fall down the plane AB from A; or again down the plane CB from C. I say that with the fall in both places the same change in the velocity is acquired.



If indeed by falling through CD a smaller velocity were acquired than by falling through AB, then it may have a velocity, by falling along CB only that which it could acquire by falling along FB, with the position of FB obviously less than AB. But it will acquire falling along CB that velocity by which again it will be able to rise the whole length of BC (see Prop. 4). Hence it will acquire along FB that velocity by which it will be able to rise the whole length of BC. And likewise falling from F to B, if again the motion should continue along BC, and since that can be made possible from the rebound at the oblique surface at B, it can ascend as far as C, that is, higher than it fell, hence this proposition is absurd.

In the same manner it can be shown that nor dropping the body along the plane AB will a smaller velocity be acquired than along CB. Therefore the same velocity is acquired by both planes, qed.

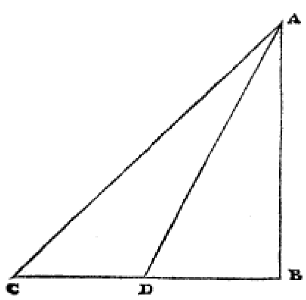
Because if truly for either plane, a perpendicular is taken equal to the height of that plane itself, through which the moveable object is placed to release, thus also it is agreed that it is to acquire the same velocity as by the inclined plane; in so much as the same has been shown.

Hence now directly the demonstration can proceed of the other Gailiean theorem, upon which everything else depends that he has related concerning the fall of bodies on inclined planes. Truly :

PROPOSITIO VII.

The times of the descent of bodies on planes with different inclinations, but which all have the same height, are to each other as the lengths of the planes.

Let AC and AD be inclined planes with the same height AB. I say that the time of descent down the plane AC to the time of descent down AD is as the length AC to AD.

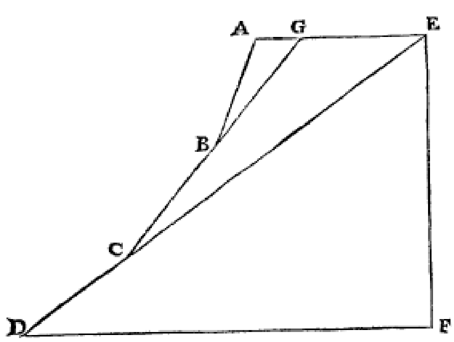


The time to pass along AC is indeed equal to the time of the motion along the same AC with a constant motion, [p. 33] with a speed equal to the half of that final speed which is acquired by falling through AC (Prop. II). Similarly the time to pass through AD equal to the time for a constant motion through the same AD, with half the final speed which is acquired by falling through AD. Moreover this half final speed is equal to that half final speed (Prop. V), and thus the said time of the equal motion through AC, to the time of the equal motion through AD, will be as AC to AD. Therefore the individual times themselves are equal, obviously to the time of descent along AC, to the time of descent along AD, and truly have the same ratio as AC to AD. qed.

It can be shown in the same way, that the time of descent along AC to the time of descent for the fall along the perpendicular AB, are as the lengths AC to AB.

PROPOSITIO VIII.

If from the same height by a continuous motion a body shall descent down some plane and then down another adjoining plane, at whatever inclination; the same velocity will always be acquired by falling vertically from the same height.



Let AB, BC and CD be contiguous planes, of which the final is A, above the horizontal line DF drawn through the end of the lowest plane D. The amount of the height is the vertical EF and with the body falling down that plane from A as far as D. I say that at D it will be having that velocity which, falling from E it will have at F.

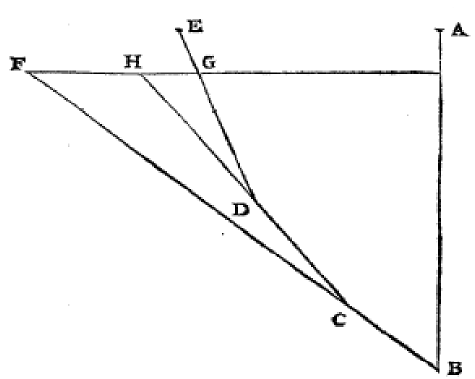
The line CB produced runs to meet the line AE in G. Likewise DC predicted [p. 34] runs to meet the same line AE in E. Since therefore falling down AB it acquires the same velocity at the end of the plane B, and as it does by falling along GB (Prop. VI) ; it is evident that, with the bend at B nothing is able to stand in the way of the motion, it will have just as much velocity when it has reached at C, as the amount if it should have descended down the GC; that is, as much as it would have from the descent down EC. Whereby it will cross the rest of the plane CD in the same manner and as if it had come from EC. Hence it has the same velocity at D, as if it had descended via the plane ED, that is, the same as by falling vertically through EF. Q.E.D.

Hence it is apparent also for the circumference of a circle, or for any curve with the moving body descending along it (for here it is allowed to consider such curves as composed from an infinite number of straight lines) always acquiring the same velocity as if the body had fallen from an equal altitude : and the velocity is of such a size as it might gain from falling vertically from the same height.

PROPOSITION IX.

If a descending body can reverse its motion, then it will rise to the same height from which it came, by advancing along any adjoining plane surfaces set at whatever inclinations.

Let the body fall from the height AB, and from the point B the inclined planes shall rise BC, CD, DE, the furthest of which E shall be at the same height as the point A. I say that if the moving body, upon falling through AB, can change direction so that it can pass through the said inclined planes, then it will reach as far as E. [p. 35]

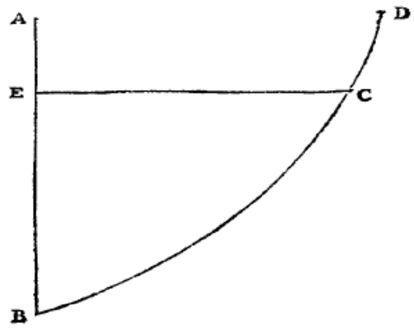


Indeed it could be said, if it were possible, that it could just get as far as G. [On this assumption] BC and CD are produced, while running to meet the line to the horizontal GF in F & H. Since indeed from the motion, with the planes BC and CD surmounted, the body may have as much velocity that it can ascend along DG, or along DH; for both these it is agreed that the same velocity is required to reach the horizontal GF, from proposition 6; Therefore, with the plane BC overcome, the moving body only will have what is necessary to ascend along CH, or CF. Therefore only as far as that by which it is able to ascend along BF, that is, the same velocity that it can acquire by descending along FB. But at B it has the velocity by which it can ascend as far as A. Therefore by that velocity which the body can acquire by descending along FB, is it able to ascend along BA, that is, higher than from where it had departed, which is not possible to be done [according to our assumption].

Moreover, in short it is the same demonstration for however many planes there are along which the moving body may rise. From which, if there was an infinite number of planes, that is, if the surface is put in place for some curve, along this surface too the moving body will rise to the height from which it came.

PROPOSITION X.

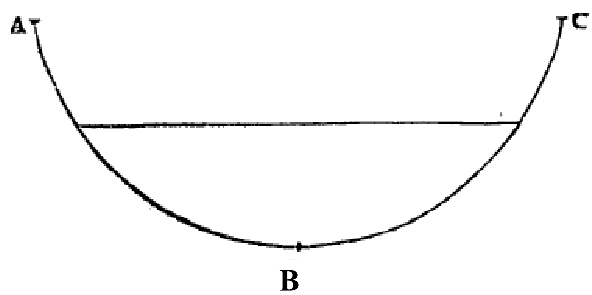
If the moving body falls vertically, or descends along some surface, and is considered to be carried up again by the impetus along some other curve, it will always have the same velocity at points with the same height when ascending as descending.



For if the moving body is falling through the height AB, from which the motion can continue along the surface BCD, in which the point C shall have the same height as the point E in AB. I say that the velocity of the moving body is not the same at C and at E. [Huygens' absurdity statement is always the first part of the proof] [p. 36]

When indeed at C that velocity may remain with the body by which it can ascend again as far as the point D, and equally the height A (Prop. IX) : and whenever from the descent through AE it shall acquire that velocity by which, from the converse motion, the ascent shall be through CD; It is apparent when it reaches C by ascending, it will have the same velocity which it had at E on descending. Q.e.d. [Thus, the last theorem is extended to curves.]

PROPOSITION XI.

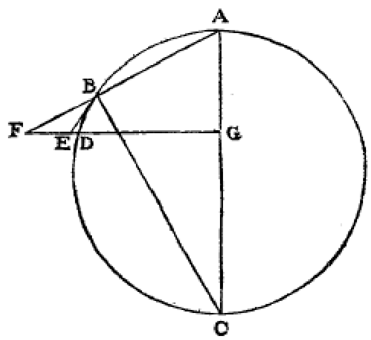


If a body is directed to fall along some surface, and then by the converse motion to rise along the same surface or along another like surface, and similarly be brought to the same position, then it will ascend and descend the same distance in equal times.

Just as the moving body can descend along the surface AB, and, when it arrives at B, by the returned motion to rise along the same surface AB, or similarly it can ascend along that similar curve BC with respect to the horizontal plane, and it is agreed from the previous demonstrations, to be returning to the same altitude from which it came. Moreover, since always, at the points of these with the same altitude, it has the same velocity from ascending and descending (Prop. X); it is apparent that the same line is to have its individual parts traversed twice with the same velocity: from which the times of each motion by necessity are equal. Q.e.d. [p. 37]

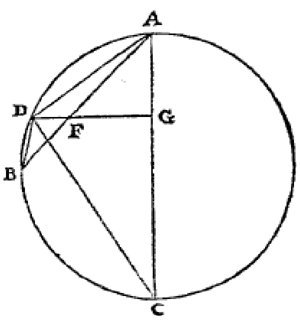
PROPOSITION XII.

ABC shall be a circle, with diameter AC, and to which FG shall be at right angles ; truly a line AF drawn from the end of the diameter A runs to meet this beyond the circle, which indeed by necessity will cut the circumference, considered at B. I say that the arc BD, intersected by the lines GF and AF, is less than the line DF.



For BC can be joined, and the tangent to the circumference can be drawn from the point B, which by necessity will cross the line FG between F and D. Therefore the angle BAC in the circle is equal to the angle EBC. Whereby the angle FBE, which together with the angle EBC make the right angle FBC, will be equal to BCA. Moreover, since the triangles ABC and AGF are similar, and the angle F is equal to the angle ACB; therefore the same angle F is equal to the angle FBE. Thus the triangle FEB is isosceles, having the equal legs FE and EB. Therefore the line ED added to each makes the line FD equal to the two lines BE and ED. Truly these two lines are agreed to be greater than the arc BD, with the same ends intercepted, and in the same part of the circle. Therefore FD will be greater than the same BD : whereby the proposition is agreed upon.

PROPOSITION XIII.

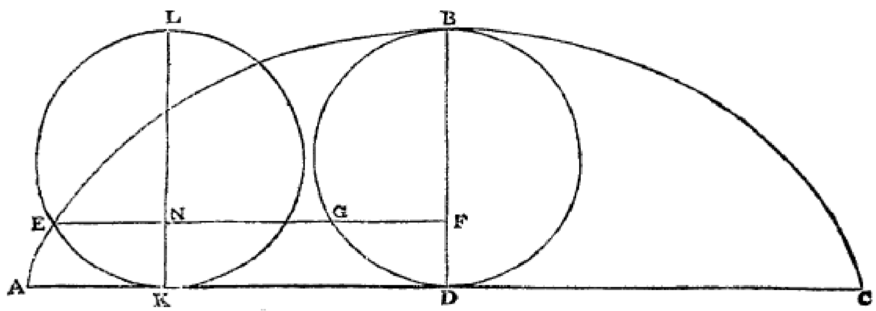


With the same points in place, if the line AB crosses DG within the circle ; I say that the arc BD, from the intersection of the circle with the lines GD and AB, is greater than the line DF.

Indeed the line DC can be joined, and the subtended chord drawn to the arc DB. Therefore, since the angle ABD is equal to the angle ACD, that is, to the angle ADG; and moreover since the angle DFB is greater than the angle ADF or ADG; and it follows that the same angle DFB will be greater than [p. 38] the angle DBF. Therefore in triangle DFB, the side DB will be greater than the side DF; from which the arc DB will be much greater than the same DF. Whereby the proposition is agreed on.

PROPOSITION XIV.

ABC is a cycloid the base of which is AC and the axis BD. Moreover, I have considered it to be generated according to the definition and mechanical description set out above in a satisfactory manner. And about the axis, BCE shall be described by the circle, and from any point E taken on the cycloid, EF is drawn parallel to the base AC, which crosses the axis BD at F, and shall cut the circumference in G. I say that the straight line GE is equal to the arc GB.



Indeed the circle LEK is drawn through the point E equal to BGD, and which touches the base of the cycloid in K, and the diameter KL is drawn. Therefore the straight line AK is equal to the arc EK ; but the whole length AD is equal to the semi-circumference KEL; therefore KD is equal to the arc EL or GB. Moreover, KD or NF is equal to EG, since EN is equal to GF, and NG is common to each. Therefore it is agreed that GE is equal to the arc GB. [p. 39.]
[p. 21]

HOROLOGII OSCILLATORII

PARS SECUNDA.

De descensu Gravium & motu eorum in Cycloide.

HYPOTHESIS.

I.

Si gravitas non esset, neque aër motui corporum officeret, unum quodque eorum, acceptum semel motum continuaturum velocitate aequabili, secundum lineam rectam.

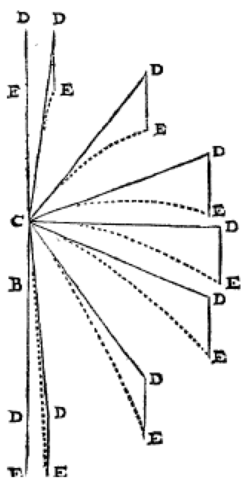
II.

Nunc vero fieri gravitatis actione, undecunque illa oriatur, ut moveantur motu composito, ex aequabili quem habent in hanc vel illam partem, & ex motu deorsum à gravitate profecto.

III.

Et horum utrumque seorsum considerari posse, neque alterum ab altero impediri.

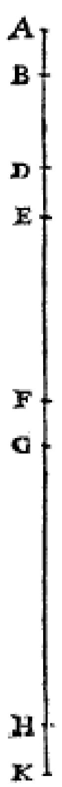
Ponatur grave C è quiete dimissum, certo tempore, quod dicatur F, vi gravitatis transire spatium CB. Ac rursus intelligatur idem grave accepisse alicunde motum quo, si nulla esset gravitas, transiret pari tempore F motu aequabili lineam rectam CD. Accedente ergo vi gravitatis non perveniet grave ex C in D, dico tempore F, sed ad punctum aliquod E, recta sub D situm, ita ut spatium DE semper aequetur spatio CB, ita



enim, & motus aequabilis, & is qui à gravitate oritur suas partes peragent, altero alterum non impediente. Quamnam vero lineam, composito illo motu, grave percurrat, cum motus aequabilis non recta sursum aut deorsum sed in obliquum tendit, è sequentibus definiri poterit. Cum vero deorsum in perpendiculari contingit motus aequabilis CD, [p. 22] apparet lineam CD, accedente motu ex gravitate, augeri recta DE. Item, cum sursum tendit motus aequabilis CD, ipsam CD diminui recta DE, ut nempe, peracto tempore F, grave inveniatur semper in puncto E. Quod si utroque hoc casu, seorsum, uti diximus, duos motus consideremus, alterumque ab altero nullo modo impediri cogitemus, hinc jam accelerationis gravium cadentium causam legesque reperire licebit. Et primum quidem duo ista simul ostendemus,

PROPOSITIO I.

Aequalibus temporibus aequales celeritatis partes gravi cadenti accrescere, & spatia aequalibus temporibus ab initio descensus emensa, augeri continue aequali excessu.



Ponatur grave aliquod, ex quiete in A, primo tempore lapsum esse per spatium AB, atque ubi pervenit in B, acquisivisse celeritatem qua deinceps, tempore secundo, motu aequabili, percurrere posset spatium quoddam BD. Scimus ergo spatium secundo tempore peragendum majus fore spatio BD, quia vel cessante in B omni gravitatis actione spatium BD percurreretur. Feretur vero motu composito ex aequabili quo percursurum esset spatium BD, & ex motu gravium cadentium, quo deprimi necesse est per spatium [p. 23] ipsi AB aequale. Quare ad BD addita DE, aequali AB, scimus tempore secundo grave perventurum ad E.

Quod si vero inquiramus quam velocitatem habeat in E, in fine secundi temporis, eam inveniemus duplam esse debere velocitatis quam habebat in B fine temporis primi. Diximus enim moveri composito motu ex aequabili cum celeritate acquisita in B, & ex motu à gravitate producto, qui cum tempore secundo idem plane sit ac primo, idea decursu temporis secundi aequalem celeritatem gravi contulisse debet atque in fine primi. Quare cum acquisitam in fine primi temporis celeritatem conservaverit integram, apparet in fine secundi temporis bis eam celeritatem inesse quam acquisiverat in fine temporis primi, sive duplam.

Quod si jam, postquam pervenit in E, pergeret deinceps tantum moveri celeritate aequabili, quantum illic acquisivit, apparet tempore tertio, prioribus aequali, percursurum spatium EF, quod duplum futurum sit spatii

BD; quia hoc percurri diximus dimidia hujus celeritatis, motu aequabili, & temporis parte aequali. Accendente autem rursus gravitatis actione, percurret tempore tertio, praeter spatium EF, etiam spatium FG, ipse AB ver DE aequale. Itaque in fine tertii temporis grave invenietur in G. Velocitatem vero hic habebit triplam jam ejus quam habebat in B, in fine primi temporis: quia praeter celeritatem acquisitam in E, quam diximus duplam esse acquisitae in B, vis gravitatis, temporis tertii decursu, aequalem rursus atque in fine primi celeritatem contulit. Quamobrem utraque celeritas, in fine temporis tertii, triplam celeritatem constituet ejus quae fuerat in fine temporis primi.

Eodem modo ostendetur tempore quarto peragi debere & spatium GH triplum soatii BD, & spatium HK ipsi AB aequale : velocitatemque in K, infine quarti temporis, fore quadruplam ejus quae fuerat in B, in fine temporis primi. Atque ita spatia quotlibet deinceps considerata , quae aequalibus temporibus peracta fuerint, aequali excessu, qui ipsi BD aequalis sit, crescere manifestum est; simulque etiam velocitates per aequalia tempora aequalite augeri.

[p. 24]

PROPOSITIO II.

Spatium peractum certo tempore à gravi, è quiete casum inchoante, dimidium estt ejus spatii quod pari tempore transiret motu aequabili, cum velocitate quam acquisivit ultimo casus momento.

Ponantur quae in propositione praecedenti, ubi quidem AB erat spatium certo tempore, à gravi cadente ex A, peractum BD vero spatium quod pari tempore transiri intelligebatur celeritate aequabili, quanta acquisita erat in fine primi temporis, seu in fine spatii AB. Dico itaque spatium BD duplum esse ad AB.

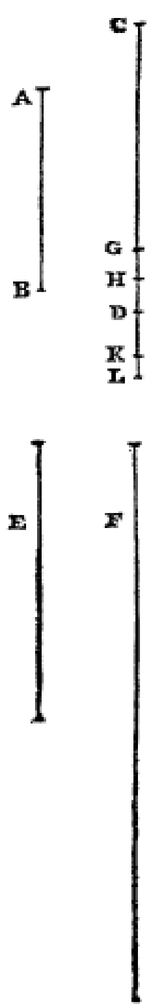
Quum enim spatia primis quatuor aequalibus temporibus à cadente transmissa sint AB, BE, EG, GH, quorum inter se certa quaedam est proportio : si eorum temporum dupla tempora sumamus, ut nempe pro primo tempore jam accipiantur duo illa quibus spatia AB, BE. peracta fuere; pro secundo vero tempore duo reliqua quibus peracta fuere spatia EG, GK, oportet jam spatia AE, EK, quae sunt aequalibus temporibus à quiete peracta, inter se esse sicut spatia AB, BE, quae aequalibus item temporibus à quiete percurrebantur.

Quum igitur sit ut AB ad BE, ita AE ad EK; & convertendo, ut EB sive DA ad AB ita KE ad EA: erit quoque, dividendo, DB ad BA ut excessus KE supra EA ad EA. Quum sit autem, ex ostensis propositione praecedenti, KE aequalis tum duplae AB, tum quintuplae BD : EA vero aequalis tum duplae AB, tum simplici BD; apparet dictum excessum KE supra EA aequari quadruplae BD. Sicut igitur DB ad BA ita erit quadrupla DB ad EA : unde EA quadrupla erit ipsius BA : eadem vero EA aequatur, uti diximus, & duplae AB & simplici BD. ergo BD duplae AB aequalis erit ; quod erat demonstratum.

[p. 25]

PROPOSITIO III.

Spatia duo, à gravi cadente quibuslibet temporibus transmissa, quorum utrumque ab initio descensus accipiatur, sunt inter se in ratione duplicata eorundem temporum, sive ut temporum quadrata, sive etiam ut quadrata celeritatum in fine cuiusque temporis acquisitarum.



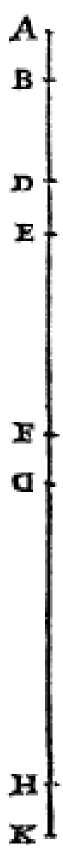
Quum enim ostensum sit propositione antecedenti spatia AB, BE, EG, GK, quotcunque fuerint, aequalibus temporibus à cadente peracta, crescere aequali excessu, qui sit ipsi BD aequalis : Patet nunc, quoniam BD est dupla AB, spatium BE fore triplum AB; EG quintuplum ejusdem AB; GK septuplum; aliaque deinceps auctum iri secundum progressionem numerorum imparium ab unitate, 1, 3, 5, 7, 9, &c. cumque quotlibet horum numerorum, sese consequentium, summa faciat quadratum, cujus latus est ipsa adsumptorum numerorum multitudo (velut si tres primi addantur, facient novem, si quatuor sexdecim) sequitur hinc spatia, à gravi cadente transmissa, quorum utrumque à principio casus inchoetur, esse inter se in ratione duplicata temporum quibus casus duravit, si nempe tempora commensurabilia sumantur.

Facile autem & ad tempora incomensurabilia demonstratio extendetur. Sint enim tempora hujusmodi, quorum inter se ratio ea quae linearum AB, CD. spatiaque temporibus his transmissa sint E & F, utraque nimirum ab initio descensus adsumpta. Dico esse, ut quadratum AB ad quadratum CD, ita spatium E ad F.

Si enim negetur; habeat primo, si potest, spatium E ad F majorem rationem quam quadratum AB ad quadratum CD, nempe eam quam quadratum AB ad quadratum CG, sumpta CG minore quam CD & à CD auferatur pars DH, minor quam DG excessus CD supra CG, atque ita ut reliqua HC commensurabilis sit ipse AB; hoc enim fieri posse constat. Erit ergo CH major quam CG. Atque ut quadratum temporis AB ad quadratum temporis CH, ita spatium E, quod tempore AB peractum AB peractum est ad spatium peractum tempore CH, per superiùs ostensa. Hoc vero spatio majus est illud quod tempore CD percurritur, nempe spatium F. Ergo spatii E ad spatium F minor est ratio quam quadrati AB ad quadratum CH. Sicut autem spatium E ad F, ita ponebatur esse quadratum AB ad quadratum CG; ergo minor quoque erit ratio quadrati AB ad quadratum CG, quam quadrati AB [p. 26] ad quadratum CH, ac proinde quadratum CG majus quadratum CH; quod est absurdum, quum CH major dicta sit quam CG. Non habet igitur spatium E ad F majorem rationem quam quadratum AB ad quadratum CD.

Habeat jam, si potest, minorem; sitque ratio spatii E ad F eadem quae quadrati AB ad quadratum CL, sumpta CL majore quam CD, & à CL auferatur LK minor excessu LD, quo CD superatur à CL; atque ita ut reliqua KC sit commensurabilis AB. Quia ergo ut quadratum temporis AB ad quadratum temporis CK, ita est spatium E, peractum tempore AB, ad spatium peractum tempore CK. Hoc vero spatio minus est spatium peractum tempore CD, nempe spatium F. erit proinde spatii E ad F major ratio quam quadrati AB ad quadratum CK. Sicut autem spatium E ad F, ita ponebatur esse quadratum AB ad quadratum CL. Ergo major erit ratio quadrati AB ad quadratum CL, quam ejusdem quadrati AB ad quadratum CK, ideoque quadratum CL minus erit quam qu. CK. quod est absurdum, quum CL major sit quam qu. CK. Ergo neque minorem rationem habet spatium E ad F quam quadratam AB ad quadratum CD. quare necesse est ut eandem habeat. Porro cum celeritates in fine temporum AB, CD acquisitae sint inter se sicut ipsamet tempora; apparet rationem spatiorum E ad F eandem quoque esse quae quadratorum temporum AB, CD, quibus transmissa sunt. Itaque constat propositum.

PROPOSITO IV.



Si grave celerate ea quam in fine descensus acquisivit sursum tendere caeperit fiet ut paribus temporis partibus, spatia quae prius sursum, eadem deorsum transeat, adeoque ad eandem unde descenderat altitudinem ascendat. Item ut aqualibus temporis partibus aqualia amittat celeritatis momenta.

Sunto enim ut in propositione 2, spatia quotlibet, aequalibus temporis [p. 27] partibus cadendo è quiete peracta, quorum primum AB; secundeun compositum ex BD, quod celeritate aequabili acquisita per AB transeundum erat, & ex DE ipse AB aequali; tertium compositum, ex EF, duplo ipsius BD, & ex FG, eidem AB aequali; quartum compositum ex GH, triplo ipsius BD, & ex HK ipsi itidem AB aequali, atque eadem ratione porro crescentia, si plura fuerit. Dico totidem aequalibus temporibus eadem spatia KG, GE, EB, BA, singula singulis peragenda esse à gravi sursum tendente, atque incipiente cum celeritate in fine descensus K acquisita.

Brevitatis autem gratia celeritas quaeque designetur deinceps longitudine spatii quod grave motu aequabili, cum celeritate illa, atque temporis parte una, quales in descensu considerabimus, transmissurum esset.

Itaque ex ostensis dicta propositione, cum in K grave pervenerit, habet celeritatem GH auctam celeritate BD, hoc est celeritatem KF, quia KF aequatur ipsis HG, BD, sunt enim partes singulae HK, FG, aequales ipsi AB, ac proinde utraque simul ipsi BD, quam esse duplam AB ostendimus propositione 2. Itaque celeritatem in fine descensus K acquisitam sursum convertendo, si grave aequabili motu ferretur, conficeret una temporis parte spatium KF. Atque, gravitatis actione accendente, deiminuetur ascensus KF spatio FG ipse AB aequali, ut patet ex dictis ad hypothesin initio sumptam. Ergo parte prima temporis ascendet grave tantum per KG, quo eodem spatio parte temporis novissima descenderat. Simul vero & celeritati tantum decessisse necesse est, quantum acquiritur temporis parte una deorsum cadendo, hoc est celeritatem BD. Itaque grave, ubi ad G ascenderit, habet celeritatem reliquam GH, cum initio ascensus habuerit celeritatem HG una cum celeritate BD. Est autem ipse HG aequalis GD; quum aequetur ipse FE una cum DB, hoc est una cum dupla AB, hoc est una cum duabus FG & ED; Ergo si ex G, cum celeritate aequabili, quantam illic habet, sursum pergeret, conficeret una parte temporis spatium GD. Accedente autem gravitatis actione, deminuetur ascensus iste spatio DE, ipsi AB aequali. Ergo, hac secunda parte temporis, ascendet per spatium GE, quod simili temporis parte etiam cadendo tranfierat. Simul autem celeritati tantum decessisse denuo debet quantum temporis parte una ex casu acquiritur, nempe celeritas BD. Itaque ubi usque ad E ascenderit, habet duntaxat celeritatem FE, quae nimirum relinquite [p. 28] quum à celeritate GD aufertur celeritas BD. Nam BD, ut jam diximus, aequalis est duabus DE, FG.

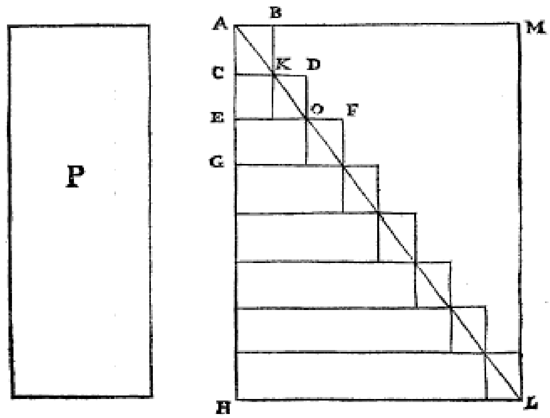
Est autem ipsi FE aequalis EA, quum FE aequetur ipsi BD bis sumptae, hoc est ipsi BD una cum dupla AB, hoc est una cum duabus AB, DE. Ergo si ex E cum celeritate aequabili, quantam illic dabet, sursum pergeret, confecturum esset una temporis parte spatium EA. Sed accidente actione gravitatis, diminuetur ascensus iste ipso spatio AB. Proinde hac parte temporis per spatium EB tantum ascendet, quod simili parte temporis descendendo quoque transierat. Hic vero rursus celeritati tandum decessisse necesse est quantum una temporis parte cadendo deorsum acquiritur, hoc est celeritate BD. Itaque grave, ubi usque ad B ascenderit, habet celeritatem ipsam BD reliquam, cum in E habuerit celeritatem FE ipsius BD duplam. Si ergo ex B cum celeritate aequabili, quantam illic habet, sursum pergeret, confecturum esset parte una temporis spatium aequale ipse DB, hoc est duplum AB. Sed accente gravitatis actione, diminuitur ascensus iste spatio quod ipsi AB aequale sit. Igitur hac parte temporis ascendet tantummodo per spatium BA, quod etiam primo descensus tempore tranfierat. Atque in fine quidem extremi temporis hujus necessario quam ad A, atque inde eo relapsum esse. At hoc absurdum esset, cum non possit, motu à gravitate profecto, altius quam unde decidit ascendere. Porro quum celeritati quam in B habebat rursus decesserit celeritas BD, patet jam gravi in A constituto nullam celeritatem superesse, ac proinde non altius excursurum. Itaque ostensum est ad eadem unde decidit altitudinem pervenisse, & singula spatia, quae aequalibus descensus temporibus transmiserat, eadem totidem ascensus temporibus remensum esse : sed & aequalibus temporibus aequalia ipsi decessisse celeritatis momenta apparuit. Ergo constat propositum.

Quia vero in demonstratione propositionis secundae, ex qua pendet praecedens, adsumptum fuit certam quandam esse proportionem spatiorum quae continuis aequalibus temporibus à gravi cadente transeuntur, quaeque eadem sit, quaecunque aequalia tempora accipiantur; quod quidem & ex rei natura ita se habere necesse est, & si negetur, fatendum frustra proportionem istorum spatiorum investigari. Tamen, quia propositum etiam absque hoc demonstrari potest, Galilei methodum sequendo, [p. 29] operae pretium erit demonstrationem, ab illo minus perfecte traditam, hic accuratius conscribere, itaque rursum hic demonstrabimus.

PROPOSITIO V.

Spatium peractum certo tempore, à gravi è casum inchoante, dimidium esse ejus spatii quod pari tempore transiret motu aequabili, cum celeritate quam acquisivit ultimo casus momento.

Sit tempus descensus totius AH, quo tempore mobile peregerit spatium quoddam cujus quantitas designetur plano P. ductaque HL perpendiculari ad AH, longitudinis cujuslibet, referat illa celeritatem in fine casus acquisitam. Deinde completo rectangulo AHLM, intelligatur eo notari quantitas spatii quod percurreretur tempore AH, cum celeritate HL. Ostendendum est igitur planum P dimidium esse rectanguli MH, hoc est, ducta diagonali AL, aequale triangulo AHL.



Si planum P non est aequale triangulo AHL, ergo aut minus eo erit, aut majus. Sit primo, si fieri potest, planum P minus triangulo AHL. dividatur autem AH in tot partes aequales AC, CE, EG, &c. ut, circumscripta triangulo AHL [p. 30] figura è rectangulis quorum altitudo singulis divisionum ipsius AH partibus aequetur, ut sunt rectangula BC, DE, FG, alteraque eidem triangulo inscripta, ex rectangulis ejusdem altitudinis, ut sunt KE, OG, &c. ut, inquam, excessus illius figura supra hanc, minor sit excessu trianguli AHL supra planum P. hoc enim fieri posse perspicuum est, cum totus excussus figurae circumscriptae super inscriptam aequetur rectangulo infimo, basin habenti HL. Erit itaque omnino excessus ipsius triangule AHL supra figuram inscriptam minor quam supra planum P, ac proinde figura triangulo inscripta major plano P. Porro autem, quum recta AH tempus totius descensus repartes referent. Cumque celeritates mobilis cadentis crescant eadem proportione qua tempora descensus (Prop. I); sitque celeritas in fine totius temporis acquisita HL; erit ea, quae in fine primae partis temporis AC acquiretur, CK; quia ut AH ad AC, ita HL ad CK. similiter quae in fine partis temporis secundae CE acquiritur, erit EO, atque ita deinceps. Patet autem, tempore primo AC, spatium aliquod à mobili transmissum esse, quod majus sit nihilo; tempore vero secundo CE transmissum esse spatium quod majus sit quam KE, quia spatium KE transmissum fuisset tempore CE, motu aequabili, cum celeritate CK. habent enim spatia, motu aequabili transacta, rationem compositam ex ratione temporum, & ratione velocitatum, ideoque cum tempore AH, celeritate aequabili HL percurri posuerimus spatium MH, sequitur tempore CE, cum celeritate CK, percurri spatium KE, quum ratio rectanguli MH ad rectangulum KE componatur ex rationibus AH ad CE, & HL ad EO.

Quum ergo, ut dixi, spatium KE sit illud quod transmitteretur tempore CE, cum celeritate aequabili CK, mobile autem feratur tempore CE motu accelerato, qui jam principio hujus temporis habet celeritatem CK; manifestum est isto accelerato motu, tempore CE, majus spatium quam KE confecturum. Eadem ratione, tempore tertio EG, majus spatium conficiet quam OG, quia nempe hoc confecturum esset tempore eodem

EG, cum celeritate aequabili EO. Atque ita deinceps, singulis temporis AH partibus, à mobili majora spatia quam sunt rectangula figurae inscriptae, ipsis partibus adjacentia, peragentur. Quare totum spatium motu accelerato peractum majus erit ipsa figura inscripta. Spatium vero illud aequale positum fuit plano P. Itaque figura inscripta minor erit spatio P. quod est absurdum; eodem enim spatio major ostensa fuit. Non est igitur planum P minus triangulo AHL. At neque majus esse ostendetur.

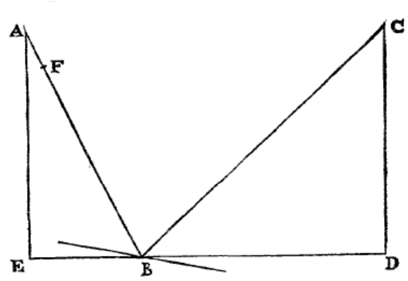
Sit enim, si potest; & dividatur AH in partes aequales, atque ad earum altitudinem, inscripta circunscriptaque rursus, [p. 31] ut ante, sit triangulo AHL figura ex rectangulis, ita ut altera alteram excedat minori excessu quam quo planum P superat triangulum AHL, erit igitur necessario figura circumscripta minor plano P. Constat jam, prima temporis parte AC, minus spatium à mobili transmitti quam sit BC, qui hoc percurreretur eodem tempore AC cum celeritate aequabili CK,quam demum in fine temporis AC mobile adeptum est. Similiter secunda parte temporis CE, minus spatium motu accelerato transmittetur quam sit DE, quia hoc percurreretur eodem tempore CE, cum celeritate aequabili EO, quam demum in fine temporis CE mobile assequitur. Atque ita deinceps, singulis partibus temporis AH, minora spatia à mobili trajicientur quam sunt rectangula figurae circumscriptae, ipsis partibus adjacentia. Quare totum spatium motu accelerato peractum, minus erit ipsa figura circumscripta. Spatium vero illud aequale positum fuit plano P; ergo planum P minus quoque erit figura circumscripta. quod est absurdum , cum figura haec plano P minor ostensa fuerit. Ergo planum P non majus est triangulo AHL, sed nec minus esse jam ostensum. Ergo aequale sit necesse est; quod erat demonstrandum.

Et haec quidem omnia quae hactenus demonstrata sunt, gravibus per plana inclinata descendentibus atque ascendentibus aeque ac perpendiculariter motis convenire sciendum est : cum, quae de effectu gravitatis posita fuerent, eadem ratione utrobique sint admittenda.

Hinc vero non difficile jam erit demonstrare propositionem sequentem quam concedi sibi, ut quodammodo per se manifestam, Galileus postulavit. nam demonstratio illa quam postea adferre conatus est, quaeque in posteriori operum ejus editione extat, parum firma meo quidem judicio videtur. Est autem propositio hujusmodi.

PROPOSITIO VI.

Celeritates gravium, super diversis planorum inclinationibus descendendo acquisitae, aequales sunt, si planorum elevationes fuerint aequales.



Elevationem plani vocamus altitudinem ejus secundum perpendiculum.

Sunto itaque plana inclinata, quorum sectiones factae plano ad horizontem erecto, AB, CD; quorumque sint aequales; [p. 31]& cadat grave ex A per planum AB. & rursus ex C per planum CB. dico utroque casu eundem gradum velocitatis in puncto B acquisiturum.

Si enim per CD cadens minorem velocitatem acquirere dicatur quam cadens per AB, habeat ergo. per CB cadens, eam duntaxat quam per FB acquireret, posita nimirum FB minore quam AB. Acquiret autem per CB cadens eam velocitatem qua rursus per totam BC possit ascendere (Prop. 4 huj.). Ergo & per FB acquiret eam velocitatem qua possit ascendere per totam BC. Ideoque cadens ex F in B, si continuet porro motum per BC; quod repercussu ad superficirem obliquam fieri potest; ascendet usque in C, hoc est, altius quam unde decidit, quod est absurdum.

Eodem modo ostendetur neque per planum AB decidenti minorem velocitatem acquiri quam per CB. Ergo per utraque plana eadem velocitas acquiritur, quod erat demonstrandum.

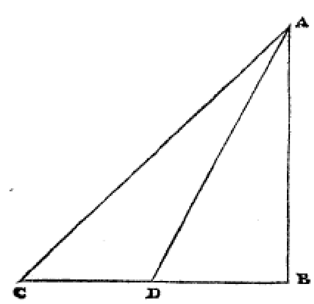
Quod si vero, pro plane alterutro, sumatur perpendiculum ipsum planorum elevationi aequale, per quod decidere mobile ponatur, sic quoque eandem quam per plane inclinata velocitatem ei acquiri constat; eadem namque est demonstrato.

Porro hinc jam recte quoque procedet demonstratio alterius theorematis Galilenai, cui reliqua omnia, quae de decensu super planis inclinatis tradidit, superstruuntur. Nempe

PROPOSITIO VII.

Tempora descensuum super planis diversimode inclinatis, sed quorum eadem est elevatio, esse inter se ut planorum longitudines.

Sint plana inclinata AC, AD quorum eadem elevatio AB. dico tempus descensus per planum AC ad tempus descensus per AD esse ut longitudo AC ad AD. Est enim tempus per AC aequale tempori motus aequabilis per eandem AC, cum celeritate dimidia ejus [p. 33] quae acquiritur casu per AC (Prop. II). Similiter tempus per AD est aequale tempori motus aequabilis per ipsam AD, cum demidia celeritate ejus quae acquiritur casu per AD. Est autem haec dimidia celeritas illi dimidiae celeritati aequalis (Prop. V), ideoque dictum tempus motus aequabilis per AC, ad tempus motus aequabilis per AD, erit ut AC ad AD. Ergo & tempora singulis istis aequalia, nimirum tempus descensus pe AC, ad tempus descensus per AD, eandem rationem habebunt, nempe quam AC ad AD. quod erat demonstrandum.

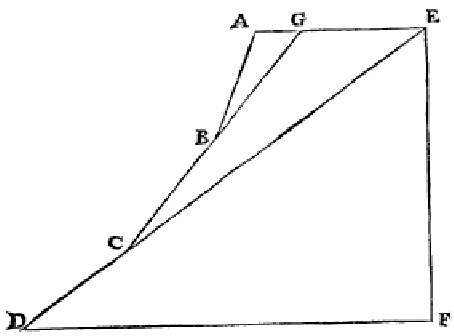


Eodem modo ostendetur & tempus descensus per AC, ad tempus casus par AB perpendicularem, esse ut AC ad AB longitudine.

PROPOSITIO VIII.

Si ex altitudine eadem descendat mobile continuato motu per quotlibet ac quodlibet plana contigua, utcunque inclinata; semper eandem in fine velocitatem acquireret cadendo perpendiculariter ex pari altitudine.

Sint plana contigua AB, BC, CD, quorum terminus A, supra horizontalem lineam DF per infimum terminum D ductam, altitudinem habeat quanta est perpendicularis EF



descendatque mobile per plana illa ab A usque in D. Dico in D eam velocitatem habiturum quam, ex E cadens, haberet in F.

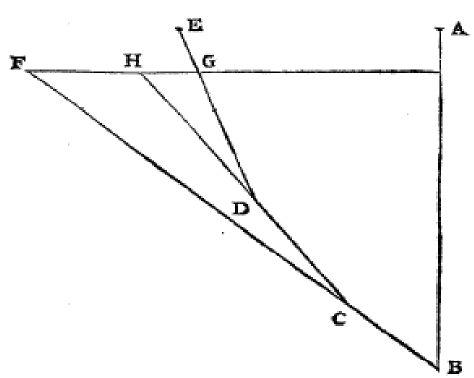
Producta enim CB occurrat rectae AE in G. Itemque DC producta [p. 34] occurrat eidem AE in E. Quoniam itaque per AB descendens enadem acquirit velocitatem in termino B, atque descendens per GB (Prop. VI) ; manifestum est, cum flexus ad B nihil obstare motui ponatur, tantam velocitatem habiturum ubi in C pervenerit, quantam si per GC planum descendisset; hoc est, quantam haberet ex descensu per EC. Quare & reliquum planum CD eodem modo transibit ac si per EC advenisset. ac proinde in D denique parem velocitatem habebit, ac si descendisset pe planum ED, hoc est, eandem quam ex casu perpendiculari per EF. quod erat demonstrandum.

Hinc liquet etiam per circuli circumferentiam, vel per curvam quamlibet lineam descendente mobili (nam curvas tanquam ex infinitis rectis compositae essent hic considerare licet) semper eandem illi velocitatem acquiri si ab aequali altitudine descenderit: tantamque eam esse velocitatem, quantam casu perpendiculari ex eadem altitudine adipisceretur.

PROPOSITIO IX.

Si grave, à descensu, sursum convertat motum suum, ascendet ad eandem unde venit altitudinem, per quascunque planas superficies contiguas, & quomodocunque inclinatas, incesserit.

Cadat grave ex altitudine AB, & ex puncto B inclinata sint sursum plana BC, CD, DE, quorum extremitas E sit eadem altitudine cum puncto A. Dico si mobile, post casum per AB, convertat motum ut pergat moveri per dicta plana inclinata, perventutum usque in E. [p. 35]

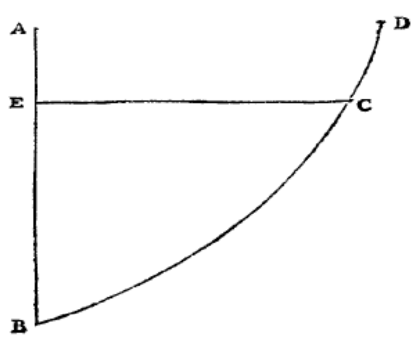


Dicatur enim, si fieri potest, tantum ad G perventurum. Producantur BG & CD, donec occurant horizontali GF in F & H. Cum igitur mobile, superatis planis BC, CD, habeat tantum eam velocitatem qua possit ascendere per DG, vel per DH; nam ad haec utraque eadem velocitate opus esse constat ex propositione 6; Ergo, superato plano BC, eam duntaxat habebat qua potuisset ascendere vel per CH, quam acquireret descendendo per FB. Atque in B habet velocitatem qua potest ascendere usque in A. Ergo illa velocitate quam acquirit grave descendendo per FB, posset ascendere per BA, hoc est, altius quam unde discesserat, quod fieri non potest.

Est autem eadem prorsus demonstratio quotcunque plana fuerint per quae mobile ascendat. Vnde & si infinita fuerit planorum multitudo, hoc est, si superficies aliqua curva ponatur, per hanc quoque ad eam ex qua venit altitudinem mobile assurget.

PROPOSITIO X.

Si mobile cadat perpendiculariter, vel per quamlibet superficiem descendat, ac rursus impetu concepto per quamlibet alium feratur sursum, habebit ascendendo ac descendendo in punctis aeque altis eandem semper velocitatem.

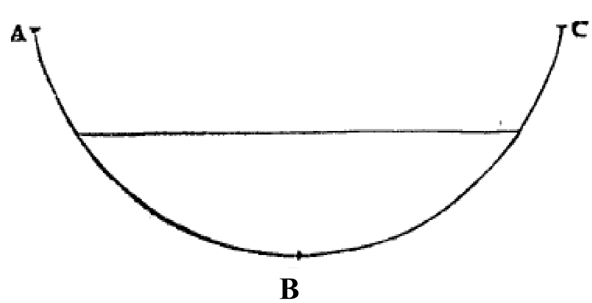


Ut si mobile ex altitudine AB decidens, motum deinde continuet per superficiem BCD, in qua punctum C sit pari altitudine atque in AB est punctum E. Dico in C eandem velocitatem inesse mobili atque in E fuerat. [p. 36]

Quum enim in C ea velocitas supersit mobili qua porro ascendat usque ad D punctum, aeque altum ac A (Prop. IX) : cumque & ex descensu per AE velocitatem eam acquirat qua, converso motu, ascensurum sit per CD; Patet cum pervenit ad C ascendendo, eandem ipsum habere velocitatem, quam habebat in E descendendo; qed.

PROPOSITIO XI.

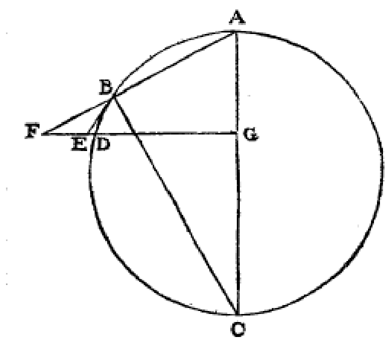
Si mobile per superficiem aliquam deorsum tendat, ac deinde converso motu sursum per eandem superficiem vel aliam similem similiterque positam feratur, aequalibus temporibus per idem spatium descendet atque ascendet.



Velut si per superficiem AB descendat mobile, atque, ubi ad B pervenerit, converso motu sursum per eandem AB, vel ei similem & respectu plani horizontalis similiter positam BC, ascendat, constat ex ante demonstratis, perventurum ad eandem ex qua venit altitudine. Cum autem perpetuo, in punctis quorum eadem altitudo, eandem velocitatem habeat ascendendo ac descendendo (Prop. X); apparet eadem lineam bis eadem velocitate singulis sui partibus percurri: unde & tempora utruisque motus aequalia esse necesse est; qed. [p. 37]

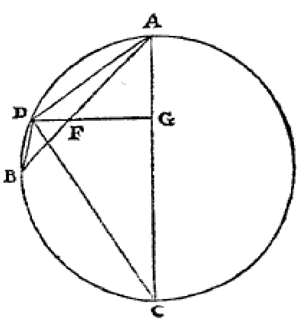
PROPOSITIO XII.

Esto circulus ABC, diametro AC, cui ad angulos rectos sit FG; huic vero occurrat à termino diametri A educta AF extra circulum, quae quidem necessario secabit circumferentiam, puta in B. Dico arcum BD, lineis GF, AF interceptum, minorem esse recta DF.



Iungatur enim BC, & ducatur ex B puncto tangens circumferentiam recta BE, quae necessario occurret rectae FG inter F & D. Est igitur angulus BAC in circulo aequalis angulo EBC. quare & angulus FBE, qui una cum EBC constituit angulum rectum FBC, erit aequalis BCA. Quia autem similia sunt triangula ABC, AGF, erit & angulus F aequalis angulo ACB. Ergo idem angulus F aequalis angulo FBE. Itaque isosceles est triangulus FEB, habens crura aequalia FE, EB. Addita ergo utrique eorum recta ED, fiet FD, aequalis duabus BE, ED. Hasce vero duas majores esse constat arcu BD, iisdem terminis intercepto, & in eandem partem cavo. Ergo & FD eodem arcu BD major erit : quare constat propositum.

PROPOSITIO XIII.

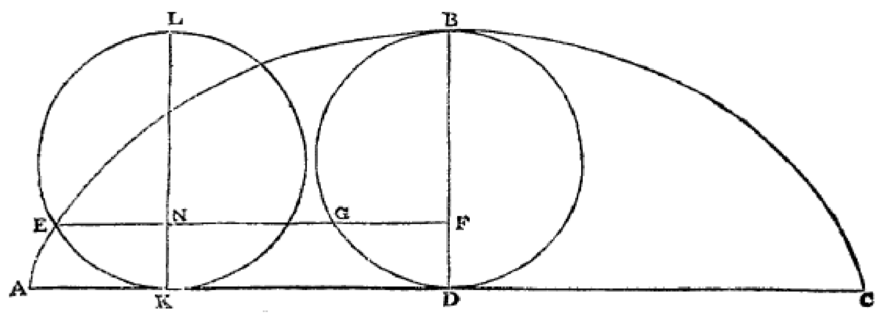


Idem positis, si recta AB, occurrat ipsi DG intra circulum; Dico arcum BD, rectis GD, AB interceptum majorem esse recta DF.

Iungatur enim DC & decatur arcui DB subtensa DB. Quoniam ergo angulus ABD aequalis ACD, hoc est, angulo ADG; angulus autem DFB major angulo ADF, sive ADG; erit [p. 38] idem DFB eiam major DBF. Ergo in triangulo DFB latus DB major latere DF; unde multo magis arcus DB superabit eandem DF. Quare constat propositum.

PROPOSITIO XIV.

Sit cyclois ABC cujus basis AC axis BD. Quomodo autem generetur ex definitione & descriptione mechanica superius traditis satis manifestum arbitror. Et circa axem BD circulus descriptus sit BCD, & à quolibet puncto E in cycloide sumpto agatur EF basi AC parallela, quae occurrat axi BD in F, secetque circumferentiam BGD in G. Dico rectam GE arcui GB aequalem esse.



Describatur enim per E punctum circulus LEK ipsi BGD aequalis, quique tangat basin cycloidis in K, & ducatur diameter KL. Est igitur recta AK arcui EK aequalis; sed tota AD aequalis semicircumferentiae KEL; ergo KD aequalis arcui EL sive GB. Est autem KD sive NF aequalis EG, quoniam EN aequalis GF, & communis utrique NG. Ergo constat & GE aequalem esse arcui GB. [p. 39.]