THE FINAL SECTION, V.

Miscellaneous items concerning the circular motions of fluids, motion of the air in producing sounds, and the internal motion of fluids.

CHAPTER XXII.

Concerning the circular motions of fluids.

Having considered a particular chapter for the rectilinear motion of fluids, the circular motions of the same fluids remains to be considered, and concerning which, attempts have been made with a great deal of care, by several of the most outstanding geometers to derive the nature of the phenomenon.

PROPOSITION LXXIX. THEOREM.

630. *If an earthlike globe of water were turning in a uniform motion about its axis with liquid matter enveloping its whole surface ; the surface of the liquid, or the surface of the whole globe of water, cannot maintain a spherical surface, but rather the figure of a certain spheroid will be induced, the axis of which shall become less than the equatorial diameter.*

*PEp*A shall be the globe of fluid revolving about its axis P*p* , of which the poles shall be P, *p*, and with the right line AE normal to the axis P*p* passing through the centre C of equatorial diameter. It must be shown, that the surface of the liquid body shall be a spheroid being maintained by the act of rotating in a circle about the axis P*p*, the axis of which P*p* shall be less than the equatorial diameter AE.

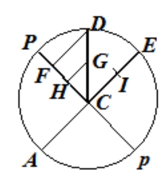


Fig. 144

Demonstration : The channels CP, CE & CD shall emerge from the centre of the body of fluid C, communicating between each other by the centre C, and full of fluid or liquid, the first of which PC shall coincide with P*p*, and CE shall be the perpendicular to the axis of the same, truly the third CD inclined to both at some angle. If the fluid mass were to be at rest, then it would be agreed (§.251.) that its surface would be spherical, and thus $\mathrm{CE} = \mathrm{CD} = \mathrm{CP}$. But now we will consider that fluid to be in a steady motion of rotating about P*p*, while the individual parts of the fluid E, I, G, D, &c. shall be completing their revolutions in the same time, and thus the speeds HG, FD, &c. of these will be in proportion to the radii CE, CI.

Truly, when the centrifugal force of all the circulating bodies is taken together [*i.e.* integrated], and because the centrifugal forces of circulating bodies of this kind are proportional to the radii, which the fluids will describe in lateral motions in equal times, the centrifugal forces of all the parts of the channel DC will be smaller than all the forces of the parts of the channel EC; and as these centrifugal forces are opposite to the forces of gravity ; thus the pressure of each column of fluid is equivalent to the excess, by which the gravitational force, or the absolute weight of a column, exceeds its centrifugal force, and if, with DC and EC equal, the excess of the weight EC above its centrifugal force is less than from the excess of the weight DC above its centrifugal force, the pressure of the column EC will be less than the pressure of the column DC, from which, since the pressures shall be connected together, they will not be able to remain in equilibrium, but the lesser pressure of the column EC goes to increase DC, and thus all the water in the column EC will be led away, thus so that EC shall be made greater than DC. By a similar argument it is arranged, that the column DC shall be greater than PC, since for this column PC it shall be agreed, namely for the axis P*p*, that it does not participate in the revolution of the motion of the mass, thus nor is it influenced by any centrifugal force, for its weight to equate to the difference, by which the weight of the column DC exceeds its centrifugal force. Therefore CE is greater than CD, and this is greater than CP or C*p*, and therefore the spheroidal figure *p*EPA is produced by the rotation acting around P*p*, of which P*p* will be the minor axis, and the mass of fluid adopts the figure of a spheroid. Q.E.D.

COROLLARY I.

631. Hence weights around a spheroid of this kind do not tend towards its centre, therefore neither do they tend towards the centre of the earth, if at the beginning of things the earth material consisted of heavy and of fluid parts, and that itself began to rotate. Indeed in the following figure, PEA shall be the shape of a fluid earth, likewise a string NM with a small weight attached M [*i.e.* a plumb bob or plummet], and this string attached at N with the aid of the plummet M will compose itself normal to the surface PDE. For the plummet M will descend as far as possible, but only to the extent that the surface PDE of the same immediate neighborhood would be perpendicular to its direction NM. And NM, or for that line RDQ parallel to the perpendicular of the curve at D, the angle DNM will be deflected from the right line DC drawn from the point of suspension N to the centre C, thus so that hence it would appear that the plummet M does not tend towards the centre of the spheroid EPA.

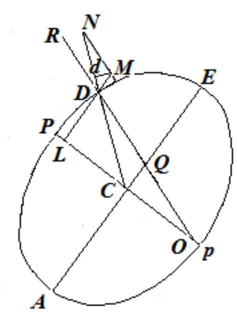


Fig. 145

COROLLARY II.

632. Therefore ND will be to the short line DM, parallel to the diameter of the equator CE, as the absolute weight of the plummet M to its centrifugal force at D. For if ND expresses the weight of the body, the short line DM expresses the force, by which it is withdrawn from the direction ND and kept in the direction NM. For the sides NM and DM are equivalent to the force ND, and with the rotational motion ceasing about the axis P*p,* the pendulum NM places itself in the position ND, from which, so that it may be detained in the position NM, there is a need for another force acting laterally along DM ; truly this lateral force is the centrifugal force arising from the circular motion of the spheroid, which exerts itself on the moving body, following the direction LDM of the radius of that circle, which the point D of the spheroid will describe in a single rotation

about the axis P*p*; and, as a consequence, the weight of the plummet M itself shall be to the strength of the centrifugal force at D parallel to the equator, as ND to DM.

COROLLARY III.

633. If the weight of the plummet M shall be to its centrifugal effect under the equator at E, as P to N, and with the right line DQO perpendicular to the curve PDE at D, there will be $LO:CO=P.EC:N.DC$. For the weight is to the centrifugal force at E, as P to N, and the centrifugal effect at E, to the centrifugal effect at D, shall be as EC to DL, therefore from the ratio of the weight to the centrifugal effect at D, that is (§. 632.) ND:DM, or $DC:CQ=[(P:N)\times CE:DL]=P.EC:N.DL$, [for, as we move around the circumference from E to P via D, at present the weight P is considered constant, while the centrifugal force has diminishes to the ratio LD:CE at D, with the initial value EC at E and final value zero at P.] and thus also $N.DC:N.CQ=P.EC:N.DL,$ or, on interchanging, $N.DC:P.EC=N.CQ:N.DL=CQ:DL$ (or on account of the parallel lines EC, DL) $=CO:LO,$ therefore on inverting there shall be $LO:CO=P.EC:N.DC$.

PROPOSITION LXXX. PROBLEM.

634. *With the property of the curve* PDE *elicited from the preceding corollary* (§.633.*), to construct and to define a curve of this kind.*

I. PDE shall be the curve sought, & CB the rectangle circumscribed to that, and the sides of this rectangle are produced to A and M, thus so that $BA=BP=EC$, and

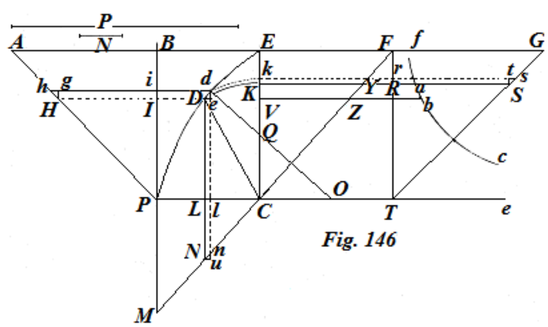


Fig. 146

$PM=PC$, and both PA and CM are connected. Next, on AE produced there is taken EF to EC in the given ratio P to N, and with the right line FT drawn through F parallel to EC, EF may be extended to G, thus so that $FG=ST=EC$. Thereafter any element of the curve may be taken D*d*, and with centre C, with the circular arcs DK, *dk*, described through the ends of the element D*d* , the lines KS and *ks* through K, *k* are acting parallel to PT. With which prepared and with TG drawn, there will be :

II. Because the squares of DL and LC shall be equal to the square of DC or TR, also the triangles, which are the halves of the squares, clearly PIH and CIN likewise shall be equal to the triangle TRS, and the triangles *Pih, Cln* likewise shall be equal to the triangle T*rs*; therefore, with the subtraction of the smaller from the larger, there will be : trapez.hI $-$ trapez. Nl $=$ trapez.Rs, or also the rect. gI $-$ rect. Nl $=$ rect. Sr, because with the points D, *d* connected, the trapeziums considered and the corresponding rectangles vanish in equal ratios.

[Note from the geometry that the increments of the areas of the contributing triangles are opposite, as one increases so the other decreases. Hermann thus continues to differentiate from first principles in a geometric manner.]

III. Because (following the hypothesis) DO is normal to the curve, the elementary triangles *Dde* and DLO will be similar, and thus
$\mathrm{LO}:\mathrm{LD}\left(=de:\mathrm{D}e\right)=\mathrm{I}i:\mathrm{L}l,\ \ \&\,\mathrm{LD}:\mathrm{LC}=\mathrm{HI}:\mathrm{LN},$ therefore from the equality,
$\mathrm{LO}:\mathrm{LC}=\mathrm{HI.I}i:\mathrm{LN.L}l$, or by inverting $\mathrm{LO}:\mathrm{CO}=\mathrm{HI.\ I}i:\mathrm{HI.\ I}i-\mathrm{LN.L}l$ (or from no. II of this) $=\mathrm{HI.I}i:\mathrm{RS.R}r$. And (§.633.) $\mathrm{LO}:\mathrm{CO}=\mathrm{P.EC}:\mathrm{N.DC}$, therefore
$\mathrm{P.EC}:\mathrm{N.DC}=\text{rect. H}i:\text{rect. R}t$, but because EF: FG (following the hypothesis) $=\mathrm{P}:\mathrm{N}$, and thus $\mathrm{P.FG}=\mathrm{P.EC}=\mathrm{N.EF}$, there will be
$\mathrm{N.EF}:\mathrm{N.DC}=\mathrm{EF}:\mathrm{DC}\left(\mathrm{RT}\right)=\text{rect K}r:\text{rect. R}t$: and consequently also
$\mathrm{H}i:\mathrm{R}t=\mathrm{K}r:\mathrm{R}t,$ hence the rect. HJ. I*i*, or H*i* is equal to the rect. KR.R*r* or K*r* everywhere; therefore the sum of all the H*i*, which are contained in the trapezium AHIB, are equal to the sum of all the *Kr*, which are present in the rect. EKRF, that is, the trapezium $\mathrm{ABIH}=\text{rect. EKRF.}$ Hence with any trapezium assumed AHJB in the triangle APB, and the rect. EKRF made equal to that, both the arc KD described with centre C, and the interval CK of the ordinate described HI produced towards I, will cross on the curve sought at the point D; hence thus it may be agreed that the curve PDE is algebraic, since all its points are able to be found geometrically. Which was required to be shown.

PROPOSlTION LXXXI. PROBLEM.

635. *To deduce from the previous proposition the construction of the curve* PDE *shown from the principle of the equilibrium of the channels, or of the columns of fluid* DC, PC *according to the explanation treated in proposition* LXXXIX.

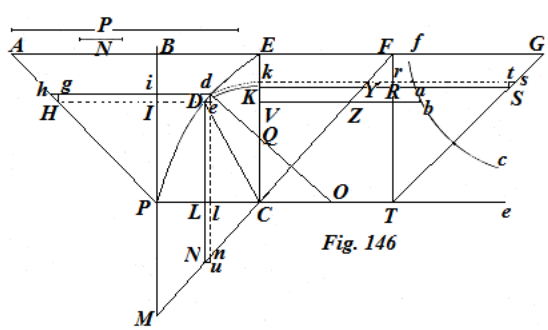


Fig. 146

With the construction of the previous proposition in place, because EF to FG or AB is as P to N, or as the weight to the centrifugal effect at E [It is evident even at this stage in the evolution of physics, that Hermann does not consider the centrifugal force as 'real' in the same sense as the gravitational force, but is an apparent force produced by the rotational motion, whereby he terms it *conatus centrifugus*, meaning literally, the attempt to flee from the centre], truly the centrifugal effect at E to the effect at D, shall be just as

EC to DL, or BP to IP, that is, as AB to HI; from the equal weight, to the centrifugal effort at D, just as EF to HI, thus so that HI always will indicate the centrifugal effort of a particle D in the channel DL or DC, and thus all the ordinates HI, which will be contained in the triangle HIP, or this triangle itself, expresses the centrifugal effort of the whole channel DL or DC. And the excess of the weight of the column DC over the weight of the column PC will be equivalent to the centrifugal effort of the column DC or DL (for of these two the centrifugal effect at the individual points equidistant from the axis PC are equal, and thus the centrifugal of the columns DC and DL will be equal), therefore the rectangle KVXR is equal to the triangle HIP, since EF or KR expresses the action of the weight, and by making $\mathrm{CV}=\mathrm{CP}$ the right line KV, the difference, by which the column DC exceeds the other PC and (§. 31.) the volume KV drawn at the weight EF or KR, expresses its weight, that is, the excess of the weight DC & PC. Therefore the whole triangle PAB also will be equal to the rect.VF ; and as a consequence with the equal distances PHI and VX taken from these, there will remain the trapezium $\mathrm{AHIB}=\text{rect. EKRF}$, as it was found in the previous proposition.

COROLLARY I.

636. Therefore because the triangle PBA = EKRF, there will be $\mathrm{EF}:\frac{1}{2}\mathrm{AB}\ (=\mathrm{P}:\frac{3}{2}\mathrm{N})=\mathrm{EC}:\mathrm{EV}=\mathrm{CE}:\mathrm{CE}-\mathrm{CP}$, there will be on re-arranging : $\mathrm{CE}:\mathrm{CP}=\mathrm{P}:\mathrm{P}-\frac{1}{2}\mathrm{N}.$

COROLLARY II.

637. If on calling CE, *a*; PC, *b*; CL, *x*; ID; *y*, & EF there will be *ap*:*n*, which we will call f with Huygens , and because (§. 634 and §.635.) $\frac{1}{2}\mathrm{AB}^2=\mathrm{EV.EF}$, there will be analytically, $af-f\sqrt{(xx+yy)}=\frac{1}{2}aa-\frac{1}{2}yy,$ which freed from asymmetry becomes : $y^4=4ffyy-4afyy+2aayy-4aaff+4a^3f-a^4+4ffxx$. Which is the very same equation that Huygens falls upon, establishing his calculation into the equilibrium of the channels DC and EC, not indeed, as we have done in the present proposition, into the equilibrium of the columns PC and DC, from which calculation the formula may have emerged more simply. See. *Discours de la Cause de la Pesanteur par Mr.Huygens* pag.157.

SCHOLION.

638. Truly we have supposed in these propositions, along with the praiseworthy Huygens, the weight of bodies to be uniform or equal everywhere. Because if indeed the weights may be put in the direct ratio of the distances from the centre C, the curve PDE will be different from that, which we have shown in the two previous propositions. For,

PROPOSITION LXXXIL THEOREM

639. *If the weight of bodies is proportional to the distances of the positions* D *from the centre* C , *the curve* PDE *will be a conic ellipse, of which the semiaxis* PC *will be to radius of the equator* EC *in the square root ratio* $\mathrm{P}-\mathrm{N}$ *to* P , *where* P *to* N *is now, as the weight below the equator at* E *to the centrifugal force at the same point.*

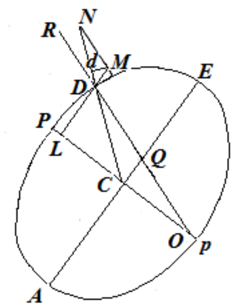


Fig. 145

Following (§.632.) ND is to DM, or DC to CQ, as the weight at D to the central force at D, while (following the hypothesis) $\text{wt. at D} : \text{wt. at E} = \mathrm{DC} : \mathrm{CE}$, and wt. at E to centrifugal effort at E (following the hypothesis) $= \mathrm{P} : \mathrm{N}$, therefore from the equation of the weight at D to the centrifugal effort at $\mathrm{E} = \mathrm{P.DC} : \mathrm{N.CE}$, and the centrifugal effort at E to the centrifugal effort at D $= \mathrm{EC} : \mathrm{DL} = \mathrm{N.EC} : \mathrm{N.DL}$, therefore anew from the equation of the weight at D to the centrifugal effort at $\mathrm{D} = \mathrm{DC} : \mathrm{CQ}$ or P.DC to $\mathrm{P.CQ} = \mathrm{P.DC} : \mathrm{N.DL}$, therefore $\mathrm{P.CQ} = \mathrm{N.DL}$; and therefore $\mathrm{DL} : \mathrm{CQ} = \mathrm{LO} : \mathrm{CO} = \mathrm{P} : \mathrm{N}$, and thus the ratio LO to CO is the same or given. Therefore from the construction in place, §.634 no. I. and (no. III. §. 634) there is $\mathrm{LO} : \mathrm{CO} = \mathrm{H}i : Rt$ (that is the no. before this) $= \mathrm{P} : \mathrm{N}$, therefore the sum of all the H*i*, or AHIB to the sum of all the R*t* or

$\mathrm{RFGS} = \mathrm{P} : \mathrm{N}$, or $2.\mathrm{AHlB} : 2.\mathrm{RFGS} = \mathrm{EC}^2 - \mathrm{DL}^2 : \mathrm{EC}^2 - \mathrm{DC}^2 = \mathrm{P} : \mathrm{N}$, and by interchanging $\mathrm{EC}^2 - \mathrm{DL}^2 : \mathrm{CL}^2 = \mathrm{P} : \mathrm{P} - \mathrm{N}$. From which it is agreed, this curve PDE to be a conic ellipse, of which the conjugate semiaxis PC, CE shall be between themselves in the square root proportion $\mathrm{P}-\mathrm{N}$ to P. Q.E.D.

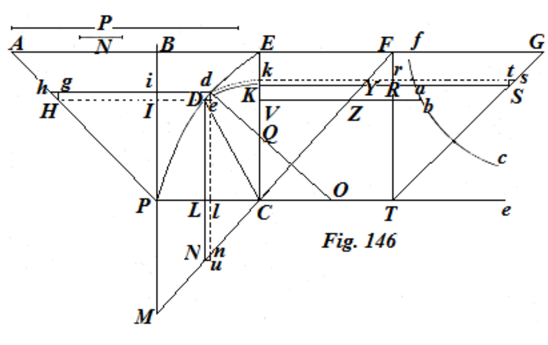


Fig. 146

Otherwise from the principle of equilibrium of the channels DC *and* PC.

640. CF shall be drawn, and because EF: FG (constr.) $= \mathrm{P} : \mathrm{N}$; the right line EF can express the weight at E, and because the weight is proportional to the distance from the centre, KY will express the weight at D, and the remaining ordinates in the triangle CKY will express the weight at the remaining locations of the channel DC, therefore the weight of the whole channel DC will be expressed by the triangle CKY. By an equal argument, if

CV were $=\mathrm{CP}$, the triangle CVZ will express the weight or the gravity of the channel PC; truly the difference of the weights DC and PC, which is expressed by the trapezium VKYZ , is equivalent to the centrifugal effort of the whole channel DL or DC (§. 635.) being expressed by the triangle PHI, therefore $\mathrm{KYZV}=$ triang. PIH, and thus $2.\mathrm{KYZV}=\mathrm{DL}^2$. And 2.KYZV is to $\mathrm{KC}^2-\mathrm{VC}^2$ or to $\mathrm{DC}^2-\mathrm{PC}^2$, or what is the same, to $\mathrm{LC}^2+\mathrm{DL}^2-\mathrm{PC}^2$, as EF to FG, or P to N . Therefore $\mathrm{DL}^2:\mathrm{DL}^2+\mathrm{LC}^2-\mathrm{PC}^2=\mathrm{P}:\mathrm{N}$, and by rearranging the ratio, $\mathrm{DL}^2:\mathrm{PC}^2-\mathrm{LC}^2=\mathrm{P}:\mathrm{P}-\mathrm{N}$. From which now again that will be clear, what was shown by the preceding paragraph, evidently the curve PDE to be a conic ellipse, of which the semiaxis PC shall be to EC in the square root ratio of $\mathrm{P}-\mathrm{N}$ to P. Q.E.D.

SCHOLION

641. It remains still, that we shall have to express the ratio of the absolute force of gravity to the centrifugal effect under the equator , that is the ratio P to N, by numbers, which will be shown easily with the benefit of logarithms. A shall be the height, which a weight beginning to fall naturally from rest will pass through in a vacuum in a time of one second on a vertical line, which Huygens found to be 15 ft. and 1 line; and R shall indicate the radius of the equator of the earth, and with P and N retained for the names of gravity and the centrifugal effect under the equator. By (§.151.) $\sqrt{(2\mathrm{A}:\mathrm{P})}$ expresses the time, in which the height A is completed by the naturally accelerated motion, which time, (following the hypothesis) is of one second; and, because with one diurnal revolution of the earth, with respect to the fixed stars, is 23 hours and 56 min. or of 86160 seconds, this will be called the number *n*, and $n\sqrt{(2\mathrm{A}:\mathrm{P})}$ will be the expression of one diurnal revolution in seconds, and because this same time (§.183.) also is expressed by $p\sqrt{(\mathrm{R}:\mathrm{N})}$, as *p* is the exponent of the ratio of the circumference to the radius, we will have the equation $n\sqrt{(2\mathrm{A}:\mathrm{P})}=p\sqrt{(R:\mathrm{N})}$ and thus, $\mathrm{P}:\mathrm{N}=2nn\mathrm{A}:pp\mathrm{R}$. Now, because *n* indicates 86160"; A, 15. ft. 1 lin.; $p=2.355:113=710:113$, and because, following the measurement of the earth by Piccart one degree in the meridian is 57060 hexapods, the logarithm around the earth will be had easily form the logarithm of 57060 by adding the logarithm of the number 360; and hence with hardly any further trouble the logarithm of the equatorial radius R can be found also. Therefore with the logarithms of these numbers substituted, the logarithm from $2nnA=10.5710475$, and the $\log.pp\mathrm{R}=8.1108142$; therefore the difference of these logarithms, that is, the logarithm of the ratio $2nn\mathrm{A}:pp\mathrm{R}=2.4612333$, to which the logarithm in the tables becomes approximately equal to the number 289. Therefore the ratio is $2nn\mathrm{A}:pp\mathrm{R}$, or $\mathrm{P}:\mathrm{N}=289:1$. Which was required to be found.

642. This ratio $\mathrm{P}:\mathrm{N}=289:1$, is the same as that, which first Huygens in his tract *De Caussa Gravitatis*, [*The Cause of Weight*] pag. 146. and later the illustrious Newton has shown in the latest Cambridge edition of his *Princ. Phil. Nat. Math*. pag. 379. Therefore

since in the case of propositions LXXX. & LXXXI, CE (§.636.) is to PC, as P to $\mathrm{P}-\frac{1}{2}\mathrm{N}$, there will be $\mathrm{CE}:\mathrm{CP}=578:577$ just as Huygens found on p.156 of the book mentioned. Truly in the case of proposition LXXXII, there was $\mathrm{PC}:\mathrm{EC}=\sqrt{\mathrm{P}-\mathrm{N}}:\sqrt{\mathrm{P}}$; or $=\sqrt{288}:\sqrt{289}$, which also is equal to the ratio $577:578$. For the ratio

$\sqrt{P-N}:\sqrt{\mathrm{P}}=\sqrt{(\mathrm{PP}-\mathrm{PN})}:\mathrm{P}$, and

$\sqrt{(\mathrm{PP}-\mathrm{PN})}=\mathrm{P}-\frac{\mathrm{PN}}{2P}+\&=\mathrm{P}-\frac{1}{2}\mathrm{N}$ approximately, therefore

$\sqrt{(\mathrm{P}-\mathrm{N})}:\sqrt{\mathrm{P}}=\mathrm{P}-\frac{1}{2}\mathrm{N}:\mathrm{P}=577:578.$ Truly Newton, with the nature of his curve PAQB not determined, or of the section of the earth through the axis PQ, and with much calculation used in the first edition *Princ. Phil. Nat*. p. 424. found the diameter of the earth along the equator to the diameter through the poles, to be as 692 to 689; but in the latest edition he found the one to the other to be as 230 to 229. With each differing greatly from the determinations of Huygens and from our own ; no wonder, since his calculation is complex, based on principles different from ours, with those attractions used, the laws of which were set out by the most elegant theorems in the first book. In Prop. XX. Book III it is skillfully established, the weights of equal bodies arranged on the surface of the earth, with the distances of the same to be present inversely proportional from the centre of the earth, and from that he deduced, how the channels EC and PC shall be of equal weights, and thus whatever particles of these similar and similarly placed also shall be of the same weight. For, because the weights of bodies are as the masses and the actions of gravity jointly, the actions of gravity everywhere shall be in the reciprocal ratio of the masses; from which, since the masses shall be proportional to EC and PC themselves , the acceleration acting at E to the acceleration acting at P, shall be as PC to EC, and thus for the remaining masses.

The most distinguished. David Gregory also alluded to the same property on p.269 of his *Elem. Astr. Phys.* Prop. LII. fol. 269, but did not attend to defining the figure of the section of the earth through the axis.

643. Indeed with this property established, clearly how the accelerating actions of the weights of the figure EP*p* revolving about the axis P*p* at E and D, are inversely proportional to the distances EC and DC from the centre, who would believe the absolute weights of bodies at the same points E and D of these to be directly proportional to the distances EC and DC ? Yet that is thus the case ; for these accelerative forces are inversely proportional to their distances from the centre, with the centrifugal effects of the same bodies removed, as far as these efforts are exerted on bodies along these directions, according to which the weight acts, but in the opposite sense. That is, if a weight at E, with the centrifugal force at that point removed, were to a weight at D, equally with the centrifugal force at this point removed D, but that, which is exerted on the body along the same direction CD, along which the weight acts on the body, shall be as DC to EC, the absolute force at E will be to the absolute force at D, as EC to DC. Towards

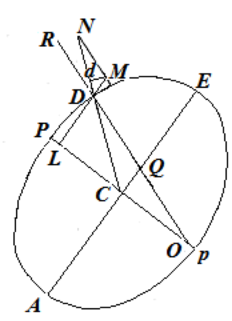


Fig. 145

demonstrating this it will required first to define the kind of curve PDE, [Fig.145, 146,] the analysis of which is required to be entered into. The absolute weights at E and D may be indicated by *g*E, *g*D, and the effect of the centrifugal force along the directions CE,

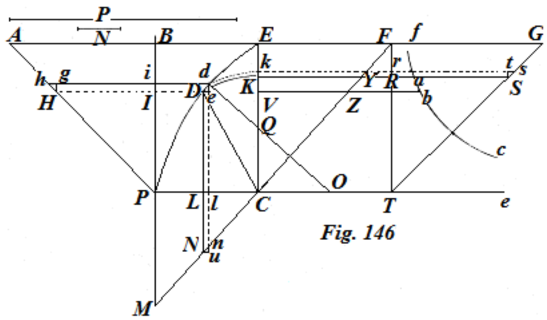


Fig. 146

CD at these same points by *c*E, *c*D. And from the hypothesis there will be $g\mathrm{E} - c\mathrm{E} : g\mathrm{D} - c\mathrm{D} = \mathrm{DC} : \mathrm{EC}$, and $g\mathrm{E} : g\mathrm{E} - c\mathrm{E} = \mathrm{P} : \mathrm{P} - \mathrm{N}$, and $c\mathrm{E} : g\mathrm{E} = \mathrm{N} : \mathrm{P}$, likewise the centrifugal effect at D along the direction OM is to the centrifugal effect at E, as DL to EC, and finally the centrifugal effect at D along DN is to the centrifugal force along DM, as DL to DC*;* therefore from the equation, that is, with everything in the foregoing multiplied into the foregoing, and everything in the subsequent into the subsequent, and with everything elicited requiring to be elicited, there will be had

$c\mathrm{D} : g\mathrm{D} - c\mathrm{D} = \mathrm{N.DL}^2 : (\mathrm{P} - \mathrm{N})\mathrm{EC}^2$,

or by inverting and adding, $g\mathrm{D} : c\mathrm{D} = (\mathrm{P} - \mathrm{N}).\mathrm{EC}^2 + \mathrm{N.DL}^2 : \mathrm{N.DL}^2$, but

$c\mathrm{D} : g\mathrm{E} = \mathrm{N.DL}^2 : \mathrm{P.DC.EC}$, therefore there becomes anew from the equation

$g\mathrm{D} : \mathrm{gE} = (\mathrm{P} - \mathrm{N.})\mathrm{EC}^2 + \mathrm{N.DL}^2 : \mathrm{P.DC.EC}$. Therefore the weight at D is to the weight at E as this fraction $(\mathrm{P} - \mathrm{N}).\mathrm{EC}^2 + \mathrm{N.DL}^2 : \mathrm{P.DC.EC}$, and thus following the method set out above §.633, the weight at D to the centrifugal effect at D is along DN [rather than the DM in the text], that is,

$\mathrm{DC} : \mathrm{CQ} = (\mathrm{P} - \mathrm{N}).\mathrm{EC}^2 + \mathrm{N.DL}^2 : \mathrm{N.DC.DL} = (\mathrm{EF} - \mathrm{EC}).\mathrm{EC} + \mathrm{DL}^2 : \mathrm{DC.DL}$, because (by constr.) in fig.146. $\mathrm{EF} : \mathrm{EC} = \mathrm{P} : \mathrm{N}$, or in the ratio of the weight under the equator at E to the centrifugal effect. Hence $\mathrm{DL} : \mathrm{CQ}$ or $\mathrm{LO} : \mathrm{CO} = (\mathrm{EF} - \mathrm{EC}).\mathrm{EC} + \mathrm{DL}^2 : \mathrm{DC}^2$ (§.634, no. III) = rect. $\mathrm{H}i$: rect. $\mathrm{R}t$. and on interchanging and inverting

$\mathrm{PI.I}i : (\mathrm{EF} - \mathrm{EC}).\mathrm{EC} + \mathrm{DL}^2 = \mathrm{CK.K}k : \mathrm{CK}^2 \left(\text{or } \mathrm{CD}^2\right)$. Hence it follows further

$\mathrm{EF.EC} : (\mathrm{EF} - \mathrm{EC}).\mathrm{EC}, + \mathrm{DL}^2 = \mathrm{EC}^2 : \mathrm{DC}^2$, and thence it is elicited $\overline{\mathrm{EF} - \mathrm{EC}}$ by

$\overline{\mathrm{EC}^2 - \mathrm{DL}^2} = \mathrm{EF.\ CL}^2$ (or, because $\mathrm{EF} - \mathrm{EC} : \mathrm{EF} = \mathrm{P} - \mathrm{N} : \mathrm{P}$) the equation

$\overline{\mathrm{P} \ \ - \mathrm{N}}.\overline{\mathrm{EC}^2 - \mathrm{DL}^2} = \mathrm{P.CL}^2$. Which is the same as that, I say, we have found above (§.639.) ; from which, since in the investigation of the curve PDE, as we have seen to be a conic ellipse in the location mentioned, we have presumed the absolute weights at E, D, &c., to be proportional to the distances EC, DC, now that itself, which was the case according to that supposition or hypothesis; just as the conclusion was derived from the principle, that the accelerations acting at E and D would be [inversely, in text] proportional with the distances EC, DC of these points from the centre C . But there is no need for an indirect deduction, since the matter can be proven directly: For, since we have found $(\mathrm{P}-\mathrm{N}).\mathrm{EC}^2-\mathrm{DL}^2.(\mathrm{P}-\mathrm{N})=\mathrm{P.CL}^2$, there will be

$(\mathrm{P}-\mathrm{N}).\ \mathrm{EC}^2,+\mathrm{N.DL}^2=\mathrm{P.DL}^2+\mathrm{P.CL}^2=\mathrm{P.DC}^2$; & , because above we have found

$g\mathrm{D}:g\mathrm{E}=(\mathrm{P}-\mathrm{N}).\mathrm{EC}^2,+\mathrm{N.DL}^2=\mathrm{P.DC.EC}$, generally there will be

$g\mathrm{D}:g\mathrm{E}=\mathrm{P.DC}^2:\mathrm{P.DC.EC}=\mathrm{DC}:\mathrm{EC}$. Q.E.D.

644. Therefore from the principles of the illustrious Newton established, and of the cel. Gregory, the section of the earth through the axis will be a conical ellipse, which we have determined in Prop. LXXXII, and thus the diameter of the earth through the equator to the diameter through the poles will be in the square root ratio P to $\mathrm{P}-\mathrm{N}$, that is (§.642.) $=\mathrm{P}:\mathrm{P}-\frac{1}{2}\mathrm{N}$, that is, as $=578:577$. Which ellipse differs very little from Huygens curve, which we have given shown in propositions LXXX and LXXXI for two different forces.

645. Again on making $\mathrm{CV}=\mathrm{CP}$, and with V*b* drawn equal to the length of a pendulum at the pole P noting the time in seconds, and with a hyperbola *bc* described through the point *b* between the asymptotes CE, CT, of which the ordinates K*a*, EF provide the lengths of isochronous pendulums, and with its oscillations measuring seconds at the locations D and E. For (§.178.) the central forces, by which isochronous pendulums are driven, are of pendulums with directly proportional lengths, from which since (following the hypothesis) the accelerations acting at P, D and E shall be inversely proportional to the distances of these points from the centre *c*, or from the nature of the hyperbola F*ab*, directly, with the respective ordinates V*b*, K*a*, E; and since the ordinate V*b* now may represent the length of the pendulum at the pole P with its oscillations being noted in seconds, the remaining K*a*, E*f*, &c. generally will indicate the lengths of all the isochronous pendulums at the locations D, E, &c. Therefore from this principle it will be allowed to construct a table without any difficulty, from which the time in seconds of the pendulum may be defined from the oscillations of its known length at individual degrees of latitude, but only if enough leisure time were available for us to undertake this calculation, and except for the great Newton who has lifted us up by his labour, and who has shown such in prop. XX. Book III. of the latest edition *Princ. Phil. Nat.*

646. Indeed in Huygens' hypothesis of uniform gravity, such as we have followed in propositions LXXX and LXXXI, the length of the pendulum at the pole P will itself be had to the length of an isochronous pendulum at any point D, as $\mathrm{EF.PC}+\frac{1}{2}\mathrm{DL}^2$ to $\mathrm{EF.PC}-\frac{1}{2}\mathrm{DL}^2$. The demonstration of which follows easily from the preceding. For at any location the length of the pendulum is proportional to the magnitude of the acceleration acting on the pendulum at that location, and this action of the acceleration is always the excess of absolute gravity over the centrifugal effect at the same location, taken in the direction of gravity.

PROPOSITION LXXXIII. THEOREM.

647. *If the cylinder ABDE filled with water as far as to H is considered to be rotating in a uniform motion about the axis FP, the water by this circular motion will be raised to the sides of the tube* BA, DE , *truly in the middle to be depressed to* M ; *thus so that its surface, which in the tube at rest was sensibly flat, may be changed into the concave surface* LMN , *which it will retain as long as the motion of the water persisted in the same course.*

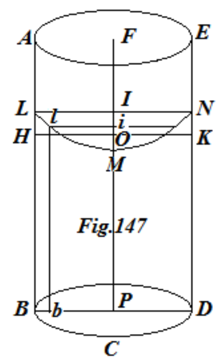


Fig.147

Now, because the circulating parts of the water make an attempt to recede from the centre, and with that a greater centrifugal force is had, where the speeds of circulation were greater, moreover the orbits with greater speeds than the mean velocity acting on an orbit are close to the sides of the vessel, and therefore the greater the effect of the centrifugal force, which, because they are impeded by the sides themselves and the fluid is water, are exerted along directions perpendicular to the plane of the base, as consequently, since the centrifugal forces on lines perpendicular to the base near the sides the centrifugal forces shall be maximum, it follows this water must be higher than in the middle, in order that the excess pressure from the weight of water above may be able to maintain a water pressure equal to the pressure at the centre of the cylinder, otherwise the water would not be able to remain in a steady state; therefore it is apparent, as the lines *bl* closer to the sides of the vessel BL, shall be greater than at the middle MP, the surface of the water LMN shall become concave.

648. Thus with the ordinates LI, *li* drawn to the axis PM, the abscissas MI, M*i* will express the centrifugal forces of the filaments of water BL, *bl* revolving about the axis PM, from which, if the velocities of the circulating fluids were, as some power *m* of the radii PB, P*b* &c., then there would be $\mathrm{MI{:}M}i = \mathrm{PB}^{2m-1} : \mathrm{P}b^{2m-1}$, and thus the curve MIL will be one from an infinite number of parabolas. For generally the square of the [speed of] the circulation is in a ratio composed from the centrifugal force and of the radius of the circulation by the first formula [*i.e.* $\mathrm{V}^2 = \mathrm{R.C}$], §.183 and thus the centrifugal forces MI will be as the square of the speed, or (following the hypothesis) as PB^{2m} applied to the radii PB of the circulation, and thus they are directly, as of the respective magnitudes PB^{2m-1} . Hence, if $m = \frac{3}{2}$, that is, the speeds shall be on the three on two ratio of the radii, the curve will be a conic parabola MIL.

SCHOLIUM I.

649. It is possible also to test the present proposition easily by experiment. For if a bucket may be hanging form a long rope and half full of water, with that rope always turning in a circle, while it may be held very rigid; then the bucket, itself suddenly released or given a dexterous impulse in the opposite sense, will follow a circular motion with great speed, and such also will press on the water; hence the water in the bucket pressed upon by this whirlpool motion, at once will be observed to be held together, so that the water is drawn up the walls of the bucket and it may subside in the middle, and thus its surface may lead to a concave figure in place of a plane one: to the surface of such a kind as was mentioned in the first place.

SCHOLIUM II.

650. Why indeed may not that requiring to be explained be deduced in an easy manner from our proposition, why in any whirlpool should solid bodies as well as vortices in other parts be driven away from the centre. For, because in the cylindrical vessel AD the water HBDK driven in a circle is itself always raised towards the walls, thus so that the surface adopts the cavity LMN; therefore it is evidently the case for each, because if the water were to be covered with a rigid lid KH, the individual points of the rigid cover with the middle O excepted will themselves be raised by the circulating water and therefore the different pressures being exerted are to be endured, clearly with these greater where the points are further from the centre of the cover O. And with that force, by which the points of the cover are trying to raise themselves, likewise with the reaction force itself acting on the water, which also without is understood to be without the cavity ; that is, by which force this cylindrical surface is being pressed from the water HD driven in a circle and thus fleeing from the axis MP, also by such a force is its reaction, of which the water is repelled by the force towards the axis MP ; thus so that, if some body shall be in water, as *b*, that shall be driven towards P in the direction BP perpendicular to the surface of the cylinder.

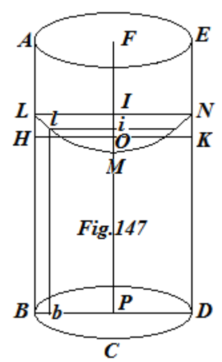


Fig.147

Thus also, with vortices acting in circles with great swiftness, the fluid having tried to slip away is repelled by the surfaces of the vortex, and solid bodies with the repelled fluid which are in the vortex, follow directions normal to the surface of the vortex, and a weight [*i.e.* such as a planet] is described in this manner according to some hypothetical vortex. Truly if only the remaining phenomena of gravity were allowed to be explained with the same simplicity as in this vortex system.

CHAPTER XXIII.

Concerning the movement of air in the production of sound.

651. Before the illustrious Newton no one had dared to treat the theory of sound geometrically, not surprising, since an enquiry of this kind shall be surrounded by these difficulties, which were only able to be considered by the most sagacious man and by other like-minded geometers. Nevertheless he has shown us the most elegant, the most outstanding theorem from geometry, concerning the accelerations in air of elastic pulses, in Prop. XLVIII, Book II, *Princ. Phil. Nat. Math*. in the old edition and in Prop. XLVII. in the latest edition, after he had set out his hypothesis concerning the production of sound in Prop. XLIII. To which the theory, least I am mistaken, corresponds to this.

652. Clearly it is understood from the vibrations [Hermann uses the word *trembling* rather than *vibrating*] of the parts of a noise producing body with the elastic air around alternately to be propelled and thus to be a little denser, afterwards to be relaxed and to go back in the opposite direction. For with the parts of the trembling body going forwards, the contiguous parts of the air driven forwards are denser ; truly on returning, and thus without the motion from the compressing force, the density of the air itself in all directions, as much as it is able, will expand out sideways by itself, thus so that in the outward direction, the vibrations of the body will be condensing the air, and by going back again the air will be rarefied. Because the parts of the vibrating body produce vibrations in the air itself, likewise again the parts of the air driven forwards maintain vibrations in the contiguous air, and this into others nearby and thus henceforth, but not everywhere with the same harmony, thus so that, if certain parts of the air shall be driven forward, they may be propelled forwards as one and all the remaining also are driven forwards, but in order that they may be propelled and become denser, others shall return and will become denser, again others shall return and become denser later, again they may be rarefied and return, and those interchanging in turn. With regard to the propulsion of parts of the air of this kind and again, their requiring to be returned from whence they came, or intervals of progressing and regressing, will be called the pulses of the air, which intervals on account of equal intervals of time are present equally everywhere. And with these in place the following problem occurs requiring to be solved.

PROPOSITION LXXXIV. PROBLEM.

653. *If the elasticities of the air are proportional with respect to the density of the air, to determine the speed of the elasticity of the air in one of its pulses through the air from the vibration of a trembling body or by some other method of excitation.*

I. EF shall be the width of one pulse, evidently it may be understood from one outward tremble of the vibrating body in contact with the air which is going to propel the air, and to be returned within the interval AB, of which the perpendicular EP shall be erected from the mid-point E, and within that the right line ME expresses the elasticity [*i.e.* the pressure] of the condensed air AB. With the part of the vibrating body returning, or rather with the compressing force moved back, which leads to a density of the air AB greater than normal, the air AB according to its own nature, will try to diffuse out equally in each direction (§.652.) ; but by considering its increase only which shall be towards D, the air before BC itself will be propelled, and that by condensing will be reduced to a volume CD approximately equal to AB. The interval EF, between the mid-points of the parts of the air of densities AB, CD, is the interval of one pulse, because it will be equivalent to the journey, which the vibrating body completes by going out and returning in. So that the speeds of the air at the individual points of the interval EF may be understood, again may be found by proceeding as follows.

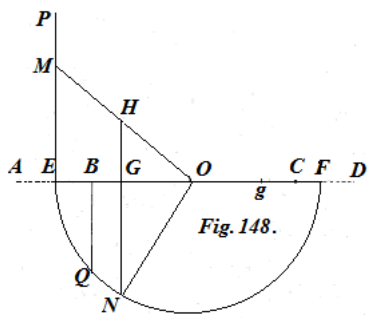


Fig. 148.

II. EF may be divided into two equal parts at O, OM may be joined, and with the right line GH drawn through any point G of the interval EF parallel to the line EP crossing the line OM at H, finally there becomes $Fg = EG$. The [original atmospheric] air EB is considered to have extended itself to EG: the accelerating force of the progressing point G is required ; truly this force is due to the elasticity of the air EG with the elasticity of the air FG taken away, which resists the progression. Now the rarity of the progressing air EG, is as EG, and of the rarified resistance [*i.e.* for pressures less than atmospheric pressure] , as FG, and because the densities shall be inversely proportional to the rarefactions, and the elasticities directly as the densities [*i.e.* for pressures greater than atmospheric pressure] ; the elasticities of this air will be inversely proportional to their rarefactions, from which, since (following the hypothesis) the elasticity of the air AB shall be EM, the elasticity of the expanded air EG will be $\mathrm{EM.EB:EG}$, and the elasticity of the expanded air FG shall be $\mathrm{EM.EB:FG}$. Therefore the accelerating force of the point G, or the difference of the elasticities $= \mathrm{EM.EB:EG}, -\mathrm{EM.EB:FG}$ (or because *g*F and EG are equal) $= \mathrm{EM.EB.G}g : \mathrm{EG.FG}$. Now, if on account of the narrowness of the pulses, and of the parts of these, the rectangle EGF can be assumed equal to the given rectangle BEF, the accelerating force of the point progressing
$\mathrm{G = EM.EB.\ G}g : \mathrm{EB.\ EF = EM.G}g : \mathrm{EF = EM.GO : EO}$, or (on account of the similar

triangles OEM and OGH) = GH. Therefore the force of the acceleration of the point G shall be approximately as the ordinate GH of the right angled triangle OEM, that is, as its distance GO from the centre of the vibration, as the celebrated Newton first found, in the place mentioned above. [*i.e.* the air in the starting pulse executes s.h.m. about O so that we may set $x = A\cos\omega t; \dot{x} = -A\omega\sin\omega t; \ddot{x} = -\omega^2 x$, where $A = \mathrm{EO}; x = \mathrm{GO} = \mathrm{EO}\cos\omega t; \dot{x} = v = -\omega\mathrm{EO}\sin\omega t; \max.\mathrm{acc}.\ddot{x} = -\omega^2\mathrm{EO} = \mathrm{EM}$] From which if with the centre O and radius OE, the semi-circle ENF shall be described, and EP shall be the mean proportional between EM and EO [*i.e.* $\mathrm{EP}^2 = \mathrm{EM.EO} = \omega^2 A^2 \text{or } \mathrm{EP} = \omega A$]. The speed of the point G (§.148) will be, as $\mathrm{GN.EP} : \mathrm{EO}\ [= A\omega\sin\omega t]$. Truly the time, in which the air EB extends itself into the distance EG (§.149) will be expressed by the arc EN:EP. Which were all required to be found.

COROLLARY I.

654. Hence the speeds of the pulses will be in a ratio composed from the square root of the distances between the pulses and from the square root of the densities. Or more briefly, the speeds are as EP. For the speeds of the points G put in place similarly at different EF, are as the homologous EP, because from the similitude of the circles, all the ratios are made equal to GN:NO or EO.

COROLLARY II.

655. Truly the times of the pulses are in the reciprocal ratio of the speed, or as EO:EP.

COROLLARY III.

656. The pendulum, whose length shall be the third proportional to EM and EO, will complete one oscillation composed from the time of one going and returning in the same time, by which one pulse resolves in going and returning.

SCHOLIUM.

657. The preceding determinations concerning the accelerating force, the velocity, and of the time bearing on the progression of the point G, cannot be true mathematically unless they must have physical accuracy as well. For because the accelerating force of the point G mentioned has been found above to be proportional to the quantity $\mathrm{EM.EB.G}g : \mathrm{EG.FG} = 2\mathrm{EM.EB.GO} : \mathrm{GN}^2$, the moment of the acceleration acting will be as $2\mathrm{EM.EB.GO}.-d\mathrm{GO} : \mathrm{GN}^2 = 2\mathrm{EM.BE.GN}.d\mathrm{GN} : \mathrm{GN}^2 = 2\mathrm{EM.EB}.d\mathrm{GN} : \mathrm{GN}$ shall be proportional to the moment of the speed acquired, or VdV, by calling this speed V, and dV its element, and $-d\mathrm{GO}$ indicates the decreasing element of the line GO, truly

the former *d*GN will be the element of the ordinate GN, hence $\text{EM.log.}\left(\text{GN}^2 : \text{BQ}^2\right)$ is, as the square of the velocity of the progressing point G.

CHAPTER XXIV.

Concerning the internal motion of fluids.

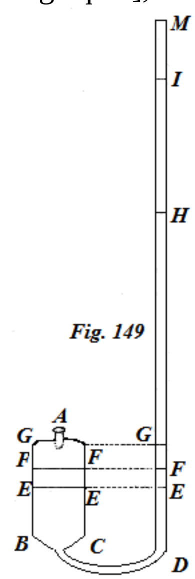
658. By this title here it is not understood to mean the internal motion of the molecules of any fluid at rest in its natural state, but rather that motion of the particles which is accustomed to be excited by external or accidental causes, where heat especially is being referred to, which beyond doubt is produced from the excitation of particles in motion in a hot body from external causes. Indeed however the internal motion of this kind may be disturbed exceedingly, yet it can be treated accurately enough by the laws of physics for its average measurement.

PROPOSITION LXXXV. THEOREM.

659. *Heat, with all else being equal, is in a ratio composed from the density of the hot body, and from the square of the ratio of the disturbances of the particles of the same.*

The movement of the particles is the mean speed among the speeds of the particles by which the particles of a hot body are moving around. This mean speed may be called V, and the density of the body D. Now, because heat consists of a faster motion of the particles, heat will be, as the forces of the particles of a hot body on some body nearby receiving heat, but these forces are in a ratio composed from the square of the speeds and from the simple density, or, as D.V^2. Therefore the heat also is as D.V^2. Q.E.D.

It was said in the proposition *cæteris paribus* [*i.e*. with everything being equal], that is, in bodies of similar kinds.



SCHOLIUM.

660. It will now be permitted to elicit from this proposition a way of measuring the movements of the particles of the air. The tube ADM may be considered in the form of an inverted syphon, of which the shorter leg AD shall be constructed from the glass container ABC and the longer leg DM hermetically sealed at M will exceed the length somewhat of the tubes accustomed to be used in ordinary barometers. The tube may be filled with mercury in winter time with the opening A uncovered considered downwards, and with the branch DM kept in a vertical situation, with the part D placed upwards, which with the help of another bent tube inserted duly can be put firmly in place into the open orifice A. [At this stage M is the lowest point, and mercury is poured into the combined tubes via another bend tube rising higher than D, which is then removed and inverted

again.] The whole tube full of mercury must be inverted, so that it may be put in place anew, as the diagram shows ; it may be restored, and the mercury in the branch MD as is the custom in ordinary barometers will itself drop to a height DH of 27 or 28 inches, with the superfluous quicksilver flowing out through the open aperture A. But, because the container ABC at this stage will be full of mercury, only by sucking will so much of the excess be drawn out, as far as provided that the surface of the remaining FF shall be about the middle of the container, or a little higher, thus so that BF or CF shall be $\frac{2}{3}$ of the total height of the container BG. Afterwards with the opening A closed, and thus with the communication of the ampoule with the exterior air cut off, the instrument will be properly prepared for use. Indeed we may consider the air AFF to be extended into the interval AEE by heat, thus so the quicksilver in the other tube may rise from H to I, the pressure of the column IE will be equivalent to the pressure of the air AEE at its base EE, and this is the force (§. 659), as made from the square of the mean speed of the particles of the air by the density of the air. It will be called, as set out in the proposition in the place mentioned, the mean velocity V, and the pressure of the air on the surface EE, as $\mathrm{V}^2:\mathrm{GE}$, for the density of the air AEE is as I: AE or I:GE; therefore $\mathrm{IE}=\mathrm{V}^2:\mathrm{GE},$ or $\mathrm{IE.GE}=\mathrm{V}^2=\mathrm{IG.GE}+\mathrm{GE}^2=\mathrm{V}^2.$ Hence $\mathrm{V}=\sqrt{(\mathrm{IE.GE})}$ or also $=\sqrt{(\mathrm{IG.GE}+\mathrm{GE}^2)}$. Therefore, if there may be called $\mathrm{FH}=a, \mathrm{EE}=b$. The diameter of the tube $\mathrm{MD}=c,$ and $\mathrm{GF}=e$, and finally with the variable $\mathrm{HI}=x$, V will be found, or the mean velocity of the particles of the air, as $\sqrt{\left(ab^4+b^4cx+bbccx+abbccx+bbccxx+c^4xx\right)}:bb.$ Hence with x in some observation, V will itself further be found. Q.E.D.

THE END .

SECTIO V ET ULTIMA,

Continens Miscellanea de motu circulari fluidorum, de motu aëris in producendo sono, & de motu interno fluidorum.

CAPUT XXII.
De motu circulari fluidorum.

Posteaquam præcipua capita ad motum fluidorum rectilineum attigimus, contemplandus superest motus circularis eorundem fluidorum, idque tanto magis quanto majori cum cura nonnulli ex præstantissimis philosophis Geometris difficiliora naturæ phænomena inde derivare conati sunt.

PROPOSITIO LXXIX. THEOREMA.

630. *Si globus Terraqueus æquabili motu circa axem suum conversus materia fluida in tota sua superficie circundatus esset; fluidi vel potius totius globi aquei superficies sphaërica manere non posset, sed figuram indueret sphæroidis cujusdam, cujus axis minor futurus esset diametro æquatoris.*

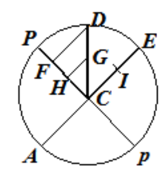


Fig. 144

Sit *PEpA* globus fluidus revolvendus circa axem P*p* , cujus poli sint P, *p*, & recta AE axi P*p* normalis per centrum C transiens diameter æquatoris. Probari debet fore, ut manens corporis fluidi & circa P*p* in gyrum acti superficies sit sphæroidea, cujus P*p* axis minor sit diametro æquatoris AE.

Demonstr. Ex centro corporis fluidi C exeant tubi CP, CE & CD ad centrum C inter se communicantes & pleni liquoris seu fluidi, quorum primus PC axi P*p* congruat, & CE eidem axi perpendicularis sit, tertius vero CD ad ambos ut libet inclinatus. Si moles fluida in quiete stet, constat (§.251.) ejus superficiem sphæricam fore, atque adeo $CE = CD = CP$. Sed cogitemus jam eam æquabili motu revolvi circa P*p*, singulæ fluidi partes E, I, G, D, &c. suas revolutiones simul absolvent, atque adeo earum celeritates radiis CE, CI, HG, FD, &c. proportionales erunt.

Verum, quia cum omni motu circulari conatus centrifugus mobilis circulantis conjunctus est & ejusmodi conatus centrifugi radiis proportionales sunt circulorum, quos mobilia in orbem lata pari tempore describunt, conatus centrifugi omnium partium tubi DC minores erunt omnibus conatibus partium tubi EC; unde ii conatus centrifugi conatibus gravitatis contrarii sunt ; atque adeo pressio cujusque columnæ fluidæ æquivalet excessui, quo conatus gravitatis, seu pondus absolutum columnæ, excedit conatum ejus centrifugum, & si, existentibus DC & EC æqualibus, excessus ponderis EC supra ejus conatum centrifugum minor est excessu ponderis DC supra ejus conatum centrifugum, erit pressio columnæ EC minor pressione columnæs DC, unde, cum communicantes sint, non poterunt in æquilibrio consistere, sed debitior pressio columnæ EC cedet fortiori DC, atque adeo aqua in columna EG attolletur, adeo ut EC major fiat quam DC. Pari argumento conficitur, columnam DC majorem esse quam PC, quandoquidem hæc columna PC utpote axi P*p* congruens, motum revolutionis massæ fluidæ non participat, neque adeo ullo conatu centrifugo pollet, pondere suo æquat differentiam, qua pondus columnæ DC excedit ejus conatum centrifugum. Sunt ergo CE major quam CD, & hæc major quam CP vel C*p*, ac proinde figura *p*EPA circa P*p* in gyrum acta producit sphæroidem, cujus P*p* erit axis minor, & massa fluida hujus sphæroidis figuram induet. Quod erat demonstrandum.

COROLLARIUM I.

631. Hinc gravia circa ejusmodi sphæroidem non tendunt ad ejus centrum, ergo nec ad telluris centrum tendunt, si in primordiis rerum terra materia fluida & gravi constiterit, & in se ipsam converti coeperit. Sit enim in altera figura PEA, figura telluris fluidæ, item filum NM cum annexo pondusculo M, & hoc filum in N fixum ope pondusculi M in situm superficiei PDE normalem sese componet. Nam corpusculum M descendet quantum potest, tantum autem descendere potest, usque dum superficie PDE vicinissimum ejusque directio NM eidem superficiei perpendicularis fuerit. Atque NM, vel ei parallela RDQ curvæ perpendicularis in D, angulo DNM a recta DC ex puncto suspensionis N ad centrum C ducta, deflectit, ut adeo hinc appareat pondusculum M non tendere ad centrum sphæroidis EPA.

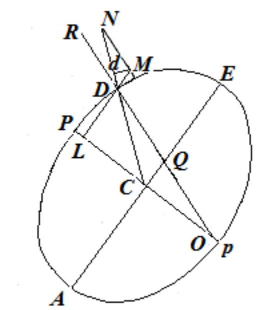


Fig. 145

COROLLARIUM II.

632. Propterea ND erit ad lineolam DM æquatoris diametro CE parallelam, sicut pondus absolutum corporis M ad ejus conatum centrifugum in D. Nam si ND exponit pondus corporis, lineola DM exponet vim, qua a directione ND retrahitur detineturque in directione NM. Vi enim ND æquipollent laterales NM & DM, & cessante motu conversionis circa axem P*p* pendulum NM se componet in situm ND, unde, ut detineatur in positione NM, ad id alia vi laterali juxta DM agente opus est; hæc vero vis lateralis est conatus centrifugus ex circulari motu sphæroidis oriundus, qui se in mobile exserit, secundum directionem LDM radii illius circuli, quem sphæroidis punctum D una conversione circa P*p* describit; ac, per consequens, gravitas corpusculi M se habet ad conatum ejus centrifugum sub æquatore in D, sicut ND ad DM.

COROLLARIUM III.

633. Si pondus corpusculi M sit ad conatum ejus centrifugum sub æquatore in E, ut P ad N, & recta DQO curvæ PDE perpendicularis in D, erit $LO:CO = P.EC:N.DC$. Nam gravitas est ad vim centrifugam in E, sicut P ad N, & conatus centrifugus in E, ad conatum centrifugum in D, ut EC ad DL, ergo ex æquo gravitas ad conatum centrifugum in D, id est (§. 632.) ND:DM, vel $DC:CQ = P.EC:N.DL$, adeoque etiam $N.DC:N.CQ = P.EC:N.DL$, vel permutando $N.DC:P.EC = N.CQ:N.DL = CQ:DL$ (vel propter parallelas EC, DL) $= CO:LO$, ergo invertendo sit $LO:CO = P.EC:N.DC$.

PROPOSITIO LXXX. PROBLEMA.

634. *Posita proprietate curvæ* PDE *corollario præcedente* (§.633.*) elicita, construere curvam, ejusque speciem definire.*

I. Sit PDE curva quæsita, & CB rectangulum ei circumscriptum, & hujus rectanguli latera producantur in A & M, ita ut $\mathrm{BA{=}BP = EC}$, & $\mathrm{PM = PC}$, & jungantur PA ac CM. Dein,

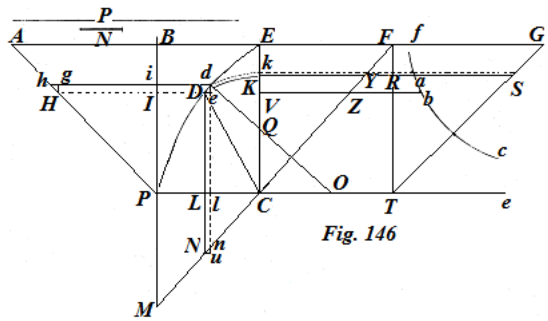


Fig. 146

in AE producta sumatur EF ad EC in data ratione P ad N, ductaque per F recta FT parallela EC protendatur EF in G, ita ut $\mathrm{FG = ST = EC}$. Sumto postea ubilibet curvæ elemento D*d*, ac centra C per elementi D*d* terminos descriptis arcubus circularibus DK, *dk*, agantur per K, *k* parallelæ KS , *ks* ipsi PT. Quibus præparatis & ducta TG, erunt.

II. Propter quadrata DL, LC æqualia quadrato DC vel TR, etiam triangula, quæ sunt quadratorum dimidia, scilicet PIH & CIN simul æqualia triangulo TRS, & triangula *Pih, CIn* simul æqualia triangulo T*rs*; idcirco, facta subductione minorum ex majoribus, erit $\mathrm{trapez.}h\mathrm{I} - \mathrm{trapez.\ N}l = \mathrm{trapez.R}s$, vel etiam $\text{rec-lum } g\mathrm{I} - \text{rec-lo N}l = \text{rec-lo S}r$, quia coeuntibus punctis D, *d* recensita trapezia & respondentia rectangula in rationes æqualitatis evanescunt.

III. Quia (secundum hypothesin) DO curvæ normanalis est, triangula elemetaria D*de* & DLO similia erunt, atque adeo $\mathrm{LO : LD}\left(= de : \mathrm{D}e\right) = \mathrm{I}i : \mathrm{L}l$, $\&\ \mathrm{LD : LC = HI : LN}$, ergo ex æquo, $\mathrm{LO : LC = HI.I}i : \mathrm{LN.L}l$, vel convertendo $\mathrm{LO : CO = HI.\ I}i : \mathrm{HI.\ I}i - \mathrm{LN.L}l$ (seu num II hujus) $= \mathrm{HI.I}i : \mathrm{RS.R}r$. Atqui(§.633.) $\mathrm{LO : CO = P.EC : N.DC}$, ergo $\mathrm{P.EC : N.DC} = \text{rec-lum H}i : \text{rec-lum R}t$ & , sed quia EF: FG (secundum hypothesin) $= \mathrm{P : N}$, atquo adeo $\mathrm{P.FG = P.EC = N.EF}$, erit $\mathrm{N.EF : N.DC = EF : DC}\left(\mathrm{RT}\right) = \text{rec-lum K}r : \text{rec-lo R}t :$ ac per consequens etiam $\mathrm{H}i : \mathrm{R}t = \mathrm{K}r : \mathrm{R}t,$ hinc rec-lum HJ. I*i*, seu H*i* æquatur ubique rec-lo KR.R*r* seu K*r*; ergo omnia H*i*, quæ in trapezia AHIB, continentur, æquantur omnibus *Kr,* quæ in rec-lo EKRF, id est, trapezium $\mathrm{AHIB} = \text{rec-lo EKRF.}$ Hinc assumto quolibet trapezia AHJB in triangulo APB, eique rec-lum EKRF æquale fiat, & arcus KD centro C intervalloque CK descriptus ordinatæ HI productæ ad partes I occurret in curvæ quæsitæ puncto D; ut adeo hinc constet, curvam PDE algebraicam esse, cum omnia ejus puncta geometrice inveniri queant. Quod erat inveniendum.

PROPOSlTIO LXXXI. PROBLEMA.

635. *Deducere constructionem curvæ* PDE *præcedente propositione exhibitam ex principio æquilibrii canalium, seu columnarum fluidi* DC, PC *secundum explicationem propositione* LXXIX *traditam.*

Posita constructione propositionis præcedentis, quia EF ad FG vel AB est ut P ad N, seu ut gravitas ad conatum centrifugum in E, conatus vero centrifugus in E ad conatum centrifugum in D, sicut EC ad DL, seu BP ad IP, id est, sicut AB ad HI; erit ex æquo gravitas, ad conatum centrifugum in D, sicut EF ad HI, adeo ut HI semper significet conatum centrifugum particulæ D in canali DL vel DC, adeoque omnes ordinatæ HI, quæ in triangulo HIP continentur, seu hoc ipsum triangulum, exponet conatum centrifugum totius canalis DL vel DC. Atqui excessus ponderis columinæ DC supra pondus columnæ PC æquivalet conatui centrifuge columnæ DC vel DL (harum enim duarum conatus centrifugi in singulis punctis ab axe PC æqualiter distantibus æquales, atque adeo ipsarum columnarum DC & DL conatus centrifugi æquabuntur), ergo rec-lum KVXR æquale est triangulo HIP, quandoquidem EF vel KR exponit gravitatis solicitationem, & facta $\mathrm{CV}=\mathrm{CP}$ recta KV, differentiam, qui columna DC excedit alteram PC & (§. 31.) volumen KV in gravitatem EF seu KR ductum, exponit ejus pondus, id est, excessum ponderum DC & PC. Propterea erit etiam totum triangulum PAB æquale rec-lo VF; ac per consequens ablatis ex hisce spatiis æqualibus PHI & VX, remanebit trapezium $\mathrm{AHIB}=\text{ rec}-\text{lo EKRF}$, ut in propositione antecedenti reperiebatur.

COROLLARIUM I.

636. Quoniam ergo triang. PBA = EKRF, erit
$\mathrm{EF}:\frac{1}{2}\mathrm{AB}\ (=\mathrm{P}:\frac{3}{2}\mathrm{N})=\mathrm{EC}:\mathrm{EV}=\mathrm{CE}:\mathrm{CE}-\mathrm{CP}$, erit convertendo $\mathrm{CE}:\mathrm{CP}=\mathrm{P}:\mathrm{P}-\frac{1}{2}\mathrm{N}$.

COROLLARIUM II.

637. Si dicantur CE, *a*; PC, *b*; CL, *x*; ID; *y*, & EF erit *ap*:*n*, quam vocabimus cum Hugenio *f*, & quia (§.§. 634, & 635.) $\frac{1}{2}\mathrm{AB}^2=\mathrm{EV.EF}$, fiet analytice
$af-f\sqrt{(xx+yy)}=\frac{1}{2}aa-\frac{1}{2}yy,$ quæ ab asymmetria liberata præbet
$y^4=4ffyy-4afyy+2aayy-4aaff+4a^3f-a^4+4ffxx$. Quæ ipsissima est æquatio, in quam lllustr. Hugenius incidit, calculum suum fundans in æquilibrio canalium DC & EC, non vero, ut nos fecimus in præsenti propositione, in æquilibrio columnarum PC & DC, quo calculus nonnihil simplicior emersisset. Vid. *Discours de la Cause de la Pesanteur par Mr.Huygens* pag.157.

SCHOLION.

638. Supposuimus vero in hisce cum laudato Hugenio gravitatem corporum uniformem seu ubique æqualem. Quod si vero gravitates ponantur in directa ratione distantiarum a centro C, curva PDE diversa erit ab ea, quam in duabus postremis propositionibus exhibuimus. Nam,

PROPOSITIO LXXXIL THEOREMA

639. *Si gravitas corporum distantiis locorum* D *a centro* C *proportionalis est, curva* PDE *erit ellipsis conica, cujus semiaxis* PC *erit ad radium æquaturis* EC *in subduplicata proportione* $\mathrm{P-N}$ *ad* P, *ubi* P *ad* N *est etiamnunc, ut gravitas sub æquatore in* E *ad vim centrifugam in eodem puncto.*

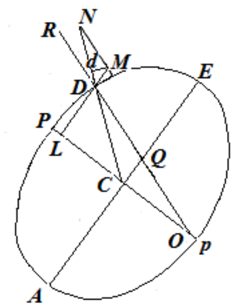


Fig. 145

Est (§.632.) ND ad DM, vel DC ad CQ, ut gravitas in D ad vim centr. in D, at (secundum hypothesin) grav.in $\mathrm{D : }$grav. in $\mathrm{E = DC : CE}$, & grav. in E ad conat. centr. in E (secundum hypothesin) $\mathrm{= P : N}$, ergo ex æquo grav. in D ad conat. centr. in $\mathrm{E = P.DC : N.CE}$, & conat. centr : in E ad conat. in $\mathrm{D = EC : DL = N.EC : N.DL}$, ergo denuo ex æquo gravit. in D ad conat. centrif. in D.DC:CQ vel P.DC ad $\mathrm{P.CQ = P.DC : N.DL}$, ergo $\mathrm{P.CQ = N.DL}$; atque adeo $\mathrm{DL : CQ = LO : CO = P : N}$, estque adeo ratio LO ad CO ubique eadem seu data.
Posita ergo constr. §.634 num. I. eritque (num. III. §. 634) $\mathrm{LO : CO = H}i : Rt$ (id est num. præc. hujus) $\mathrm{= P : N}$, ergo omn. H*i*, seu AHIB ad omnia R*t* seu $\mathrm{RFGS = P : N}$, vel. $\mathrm{2.AHlB : 2.RFGS = EC^2 - DL^2 : EC^2 - DC^2 = P : N}$, & convertendo

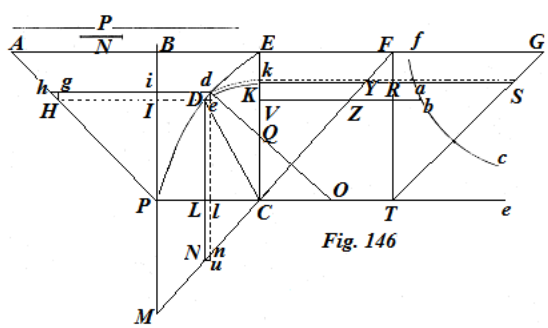


Fig. 146

$\mathrm{EC^2 - DL^2 : CL^2 = P : P - N}$. Ex quo constat, curvam PDE hoc esse ellipsin conicam, cujus semiaxis conjugati PC, CE sint ad se invicem in subduplicata proportione $\mathrm{P-N}$ ad P. Quod erat demonstrandum.

Aliter ex Principio Aequilibrii canalium DC, PC.

640. Ducatur CF, &, quia EF: FG (constr.)$\mathrm{= P : N}$; recta EF gravitatem in E exponere potest, & quoniam gravitas est distantiis a centro proportionalis, KY exponet gravitatem in D, & reliquæ ordinatæ in triangulo CKY exponent gravitatem in reliquis locis canalis DC, ergo gravitas totius canalis DC exponetur triangulo CKY. Pari argumento, si CV fuerit $\mathrm{= CP}$, exponet triangulum CVZ pondus seu gravitas canalis PC; differentia vera ponderum DC & PC, quæ exponitur per trapezium VKYZ , æquivalet conatui centrifugo

totius canalis DL vel DC. (§. 635.) exponendo per triangulum PHI, ergo $\mathrm{KYZV} =$ triang. PIH, adeoque $2.\mathrm{KYZV} = \mathrm{DL}^2$. Atqui $2.\mathrm{KYZ}V$ est ad $\mathrm{KC}^2 - \mathrm{VC}^2$ seu ad $\mathrm{DC}^2 - \mathrm{PC}^2$, vel quod idem est, ad $\mathrm{LC}^2 + \mathrm{DL}^2 - \mathrm{PC}^2$, ut EF ad FG, seu P ad N. Ergo $\mathrm{DL}^2 : \mathrm{DL}^2 + \mathrm{LC}^2 - \mathrm{PC}^2 = \mathrm{P} : \mathrm{N}$, & convertendo $\mathrm{DL}^2 : \mathrm{PC}^2 - \mathrm{LC}^2 = \mathrm{P} : \mathrm{P} - \mathrm{N}$. Ex quo nunc iterum liquet id, quod præcedenti paragrapho ostensum, scilicet curvam PDE ellipsin conicam esse, cujus semiaxis PC fit ad EC in subduplicata ratione $\mathrm{P} - \mathrm{N}$ ad P. Quod erat demonstrandum.

SCHOLION

641. Superest, ut rationem gravitatis absolutæ ad conatum centrifugum sub æquatore, id est, rationem P ad N numeris expressam exhabeamus, quod log-morum beneficia facile præstabitur. Sit A altitudo, quam grave a quiete casum incipiens motu naturaliter accelerato tempore unius minuti secundi in linea verticali & in vacuo perlabitur, quam Hugenius reperit esse 15 ped. 1 linæ; atque R significet radium æquatoris terrestris, retentisque P & N pro nominibus gravitatis & conatus centrifugi sed æquatore. Per (§.151.) $\sqrt{(2\mathrm{A} : \mathrm{P})}$ exponit tempus, quo altitudo A motu naturaliter accelerato conficitur, quod tempus, (secundum hypothesin) est unius minuti secundi; &, quia una revolutio diurna telluris, respectu fixarum, est 23. hor. 56. min. seu 86160 secundorum, nominetur hic numerus *n*, eritque $n\sqrt{(2\mathrm{A} : \mathrm{P})}$ expressio unius revolutionis diurnæ in minutis secundis, & quia hoc idem tempus (§.183.) etiam exponitur per $p\sqrt{(\mathrm{R} : \mathrm{N})}$, uti *p* est exponens rationis circumferenti ad radium, habebimus æquationem $n\sqrt{(2\mathrm{A}:\mathrm{P})} = p\sqrt{(R : \mathrm{N})}$ atque adeo, $\mathrm{P}:\mathrm{N} = 2nn\mathrm{A} : pp\mathrm{R}$. Jam, quia *n* significat 86160"; A, 15. ped. 1 lin. $p = 2.355 : 113 = 710 : 113$, & quia, secundum Piccarti dimensionem terræ unus gradus in Meridiano est 57060 hexapedarum, totius terræ ambitus log-us facile habebitur log-mo ex 57060 addendo log-um numeri 360; atque exinde etiam innotescet nullo ferme negotio log-us radii æquatoris R. Substitutis igitur numerorum logarithmis, erit log-us ex $2nnA = 10.5710475,$ & $\log. pp\mathrm{R} = 8.\,1108142$; ergo differentia horum log-morum, id est, log-us rationis $2nn\mathrm{A} : pp\mathrm{R} = 2.4612333$, cui log-mo in tabulis proxime convenit numerus 289. Propterea est $2nn\mathrm{A} : pp\mathrm{R},$ vel $\mathrm{P} : \mathrm{N} = 189 : 1.$ Quod erat inveniendum.

642. Hæc ratio $\mathrm{P} : \mathrm{N} = 289 : 1$, eadem est cum ea, quam primum Hugenius in tractatu De Caussa Gravitatis, pag. 146. & postea Illustris Newtonus in novissima editione Cantabrigiensi suorum Princ. Phil. Nat. Math. pag. 379, exhibuerunt. Quoniam igitur in casu propos. LXXX. & LXXXI, CE (§.636.) est ad PC, ut P ad $\mathrm{P} - \frac{1}{2}\mathrm{N}$, erit $\mathrm{CE} : \mathrm{CP} = 578 : 577$ etiam ut Hugenius reperit pag. 156 citati libri. In casu vero propositionis LXXXII. erat $\mathrm{PC} : \mathrm{EC} = \sqrt{\mathrm{P} - \mathrm{N}} : \sqrt{\mathrm{P}}$; seu $= \sqrt{288} : \sqrt{289}$, quæ etiam æquatur rationi $577 : 578$. Nam ratio $\sqrt{P - N} : \sqrt{\mathrm{P}} = \sqrt{(\mathrm{PP} - \mathrm{PN})} : \mathrm{P}$, atqui $\sqrt{(\mathrm{PP} - \mathrm{PN})} = \mathrm{P} - \frac{\mathrm{PN}}{2P} + \&\cdot \mathrm{c.} = \mathrm{P} - \frac{1}{2}\mathrm{N}$ proxime, ergo $\sqrt{(\mathrm{P} - \mathrm{N})} : \sqrt{\mathrm{P}} = \mathrm{P} - \frac{1}{2}\mathrm{N} : \mathrm{P} = 577 : 578.$ Verum Newtonus non determinata curvæ suæ

PAQB, seu sectionis telluris per axem PQ, specie, & multiplici usus calculo in prima editione Princ. Phil. Nat. pag. 424. invenit diametrum terræ secundum æquatorem ad diametrum per polos, ut 692 ad 689; in novissima vero invenit illam ad hanc ut 230 ad 229. Utraque multum abludit ab Hugeniana & nostra determinationibus ; nec mirum, cum calculum suum in principiis a nostris diversis fundarit, attractionibus illis usus, quarum leges, elegantibus theorematis in primo libro traditis, complexus est. In Propos. XX. Lib. III diserte ponit, pondera corporum æqualium in superficie telluris collocatorum, distantiis eorundem a centro reciproce proportionalia existere, idque ex eo deducit, quod canales EC & PC æquiponderantes sint, atque adeo earum particulæ quælibet similes & similiter positæ etiam ejusdem sint ponderis. Nam, quia pondera corporum sunt ut massæ & solicitationes gravitatis conjunctim, erunt solicitationes gravitatis omnino in reciproca ratione massarum; unde, cum massæ ipsis EC & PC proportionales sint, erit solicitatio acceleratrix in E ad solicitationem acceleratricem in P, ut PC ad EC, & sic de reliquis.

Huic eidem proprietati etiam institit CIar. David Gregorius in suis Elem. Astr. Phys. Propos. LII. fol. 269. sed nec ipse figuram sectionis terræ per axem definire curavit.

643. Hac vero proprietate posita, quod scilicet solicitationes gravitatis acceleratrices in E & D figuræ EP*p* circa P*p* revolventis, distantiis a centro EC ,DC reciproce proportionales sunt, quis crediderit gravitates absolutus corporum in iisdem punctis E & D eorum distantiis EC & DC directe proportionales esse? Id tamen ita est; nam solicitationes illæ acceleratrices distantiis locorum a centra reciproce proportionales, sunt gravitates corporum absolutæ in iis locis de quibus agitur, demtis conatibus centrifugis eorundem corporum, quatenus hi conatus exeruntur in corpora secundum illas directiones, secundus quas gravitas, sed contrario sensu, agit. Id est, si gravitas in E, demta vi centrifuga in eodem puncto, fuerit ad gravitatem in D, demta pariter vi centrifuga in hoc puncto D, sed ea, quæ in corpus exeritur secundum directionem CD eandem, secundum quam gravitas in corpus agit, ut DC ad EG, erit vis absoluta gravitatis in E ad vim absolutam in D, ut EC ad DC. Ad hoc demonstrandum oportet prius speciem curvæ PDE definite, Fig.145, 146, cuius analysis sic est ineunda. Gravitates absolutæ in E & D indicentur per *g*E, *g*D, & conatus centrifugi juxta directiones CE, CD in iisdem punctis per *c*E, *c*D. Eritque ex hypothesi $g\mathrm{E}-c\mathrm{E}:g\mathrm{D}-c\mathrm{D}=\mathrm{DC}:\mathrm{EC}$, atqui est $g\mathrm{E}:g\mathrm{E}-c\mathrm{E}=\mathrm{P}:\ \mathrm{P}-\mathrm{N}, \&\ c\mathrm{E}:g\mathrm{E}=\mathrm{N}:\mathrm{P}$, item conatus centrifugus in D secundum directionem OM est ad conatum centrifugum in E, ut DL ad EC, ac denique conatus centr. in D secundum DN est, ad conatum centr. secundum DM, ut DL ad DC*;* ergo ex æquo, id est, ductis omnibus antecedentibus in antecedentes ac consequentibus in consequentes, atque elisis elidendis, habebitur $c\mathrm{D}:g\mathrm{D}-c\mathrm{D}=\mathrm{N.DL}^2:(\mathrm{P}-\mathrm{N})\mathrm{EC}^2$, vel invertendo & componendo $g\mathrm{D}:c\mathrm{D}=(\mathrm{P}-\mathrm{N}).\mathrm{EC}^2+\mathrm{N.DL}^2:\mathrm{N.DL}^2$, sed $c\mathrm{D}:g\mathrm{E}=\mathrm{N.DL}^2:\mathrm{P.DC.EC}$, ergo denuo ex æquo fiet $g\mathrm{D}:\mathrm{gE}=(\mathrm{P}-\mathrm{N.})\mathrm{EC}^2+\mathrm{N.DL}^2:\mathrm{P.DC.EC}$. Est ergo gravitas in D ut hæc fractio $(\mathrm{P}-\mathrm{N}).\mathrm{EC}^2+\mathrm{N.DL}^2:\mathrm{P.DC.EC}$, adeoque & juxta methodum supra §.633. expositam, est gravitas in D ad conatum centrifugum in D juxta DM, id est,

$DC:CQ=(P-N).EC^2+N.DL^2:N.DC.DL=(EF-EC).EC+DL^2:DC.DL$, quia (constr.) in fig.146. $EF:EC=P:N$, seu in ratione gravitatis sub æquatore in E ad conatum centrifugum. Hinc $DL:CQ$ vel $LO:CO=(EF-EC).EC+DL^2:DC^2$ (§.634.num. III) $=$ rec-lum Hi : rec-lum Rt . et permutando ac invertendo $PI.Ii:(EF-EC).EC+DL^2=CK.Kk:CK^2\left(\text{vel } CD^2\right)$. Hinc ultro sequitur $EF.EC:(EF-EC).EC,+DL^2=EC^2: DC^2$, atque inde elicitur $\overline{EF-EC}$ in $\overline{EC^2-DL^2}=EF.\,CL^2$ (vel, quia $EF-EC:EF=P-N:P$) æquatio $\overline{P-N}.\overline{EC^2-DL^2}=P.CL^2$. Quæ eadem est cum ea, inquam supra (§.639.) incidimus; unde, quia in inquisitione curvæ PDE, quam citato loco ellipsin conicam esse vidimus, · supposuimus gravitates absolutas in E, D, &c. distantiis EC, DC proportionales esse, nunc illud ipsum, quod illic suppositio vel hypothesis erat; tanquam conclusio potuisset derivari ex principio, quod solicitationes acceleratrices in E & D distantiis EC , DC horum punctorum a centro C reciproce proportionales sint. Sed hac indirecta deductione non est opus, cum res directe probari possit: Nam, quia invenimus $(P-N).EC^2-DL^2.(P-N)=P.CL^2$, erit

$(P-N).\ EC^2,+N.DL^2=P.DL^2+P.CL^2=P.DC^2$; & , quia supra habuimus

$gD:gE=(P-N).EC^2,+N.DL^2=P.DC.EC$, erit omnino

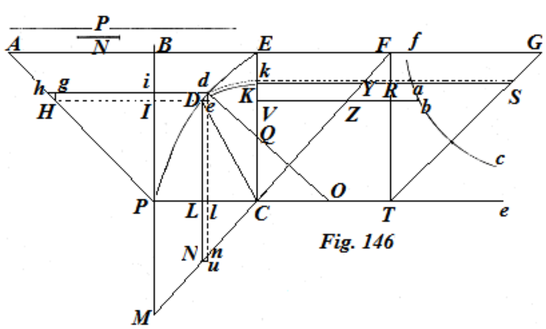


Fig. 146

$gD:gE=P.DC^2:P.DC.EC=DC:EC$. Quod erat demonstrandum.

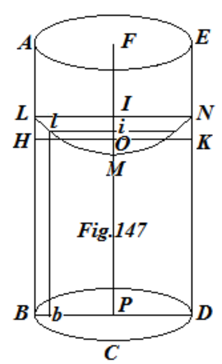
644. Idcirco insistendo principiis Illustris Newtoni , atque Celeb. Gregorii, sectio telluris per axem erit ellipsis conica, quam Propos. LXXXII. determinavimus, eritque adeo diameter terræ secundum; æquatorem ad diametrum per polos in subduplicata ratione P ad $P-N$, id est (§.642.) $=P:P-\frac{1}{2}N$, id est, ut $=578:577$. Quæ ellipsis perparum differt a curva Hugeniana, quam propositionibus LXXX. & LXXXI. duabus diversis viis demonstratam dedimus.

645. Facta iterum $CV = CP$, ductaque V*b* æquali longitudini penduli in polo P secunda minuta notantis, atque per punctum *b* descripta hyperbola *bc* inter asymptotas CE, CT, ejus ordinatæ K*a*, EF præbebunt longitudines pendulorum isochronorum, atque vibrationibus suis secunda minuta indicantium in locis D & E. Nam (§.178.) vires centrales, quibus pendula isochrona agitantur, sunt pendulorum longitudinibus directe proportionales, unde cum (secundum hypothesin) solicitationes acceleratrices in P, D & E distantiis horum punctorum a centro *c* reciproce proportionales sint vel ex natura hyperbolæ F*ab*, directe, ordinatis respectivis V*b*, K*a*, E; & cum ordinata V*b* jam repræsentet longitudinem pendulis in polo P secunda notantis, reliquæ K*a*, E*f*, &c .. omnino indicabunt longitudines pendulorum isochronorum in locis D, E, &c.
Hoc igitur principio haud difficulter tabulam construere liceret, qua penduli secunda minuta vibrationibus suis notantis longitudo ad singulos gradus latitudinis definiretur, si modo satis otii ad hunc ineundum calculum nobis suppeteret, & nisi summus Newtonius nos labore isto sublevasset, qui talem in propositione XX. Lib. III. Princ.Phil. Nat. novissimæ editionis exhibuit.

646. In hypothesi vero Hugeniana gravitatis uniformis, qualem in propositionibus LXXX. & LXXXI. sequuti sumus, longitudo penduli in polo P se habebit ad longitudinem penduli isochroni in quolibet loco D, ut $EF.PC + \frac{1}{2}DL^2$ ad $EF.PC - \frac{1}{2}DL^2$. Cujus demonstratio ex antecedentibus facilis est. Nam in quolibet loco longitudo penduli est proportionalis solicitationi acceleratrici penduli in eo loco, & hæc solicitatio acceleratriæ semper est excessus gravitatis absolutæ supra conatum centrifugum in eadem loco, sumtum in directione gravitatis.

PROPOSITIO LXXXIII. THEOREMA.

647. *Si tubus ABDE aqæ plenus usque ad H circa axem FP æquabili motu converti intelligatur, aqua hoc motu circulari ad latera tubi* BA, DE *attolletur, in medio vero deprimetur ad* M; *adeo ut superficies ejus, quæ in tubo quiescente sensibiliter plana erat, in superficiem cavam* LMN *abeat, quam retinebit quousque motus aqu*æ *eodem tenore perseveraverit.*



Nam,quia aquæ partes in gyrum actæ conatus habent recedendi a centra, & eo majores habet conatus a centro recedendi, quo majores fuerint circulationis celeritates, majores autem sunt prope latera vasis quam in media velocitates aquæ in orbem actæ, ergo & majores conatus centrifugi, qui, quoniam a lateribus istis impediuntur & aqua fluida est, exeruntur secundum directiones plana basis perpendiculares, ac per consequens, cum in lineis basi perpendicularibus prope latera conatus centrifugi maximi sint sequitur illic aquam altiorem esse debere quam in medio, ad id ut excessus pressionis a gravitate aquæ supra conatum centrifugum in ea linea æqualis manere queat pressioni aqua: in medio cylindri, alioqui aqua non posset in statu manenti consistere; propterea liquet, quod cum lineæ *bl*, lateribus vasis BL propiores, majores sint media MP, superficies aquæ LMN cava futura sit.

648. Ductis itaque ad axem PM ordinatis LI, *li*, abscissæ MI, M*i* exponent conatus centrifugos filamentorum aqueorum BL, *bl* circa axem PM revolventium, unde, si velocitates circulationis fuerint, ut dignitas quælibet *m* radiorum PB, P*b* &c. erit $\mathrm{MI:M}i=\mathrm{PB}^{2m-1}:\mathrm{P}b^{2m-1}$, atque adeo curva MIL una ex *infinitis* parabolis. Nam est generaliter quadratum circulationis in composita ratione conatus centrifugi & radii circulationis per formulam primam, §.183 adeoque conatus centrifugi MI erunt, ut quadrata celeritatum, seu (secundum hypothesin) ut PB^{2m} applicata ad radios PB circulitionis, atque adeo sunt directe, ut respectivæ quantitates PB^{2m-1}. Hinc, si $m=\frac{3}{2}$, id est, celeritates circulationum sint in sesquiplicata ratione radiorum, curva MIL erit parabola conica.

SCHOLION I.

649. Præsens propositio etiam experimento facili probari potest. Nam si situla ex fune prælongo pendens & aquæ semiplena, eo usque in orbem vertatur, dum funis valde rigescat; tum situla, sibi relicta vel derepente in contrarium sensum dextre impulsa, magna pernicitate motum circularem sequetur, talemque etiam aquæ imprimet; hoc proinde motu vorticoso aquæ in situla impresso, statim contingere observabitur, ut aqua ad parietes situlæ attollatur & in media subsidat, atque adeo ejus superficies figuram cava induat loco plana: superficiei, qualis prius erat.

SCHOLION II.

650. Quin imo ex propositione hac nostra facilis modus deduci potest explicandi, cur in quolibet vertice corpora solidiora quam vorticis partes ad vorticis centrum pellantur. Nam, quia in vase cylindrico AD aqua HBDK in gyrum acta ad parietes vasis nonnihil se attollit, adeo ut superficiem cavam LMN induat; ideo manifestum esse potest cuique, quod, si aqua opercula rigido KH operta esset, singula puncta operimenti rigidi excepto medio O ab aqua circulante ac propterea sese attollere conante pressiones diversas sint subitura, scilicet eo majores quo remotiora fuerint puncta a centro operimenti O. Atqui ea vi, qua puncta operimenti ab aqua sese attollere conante premuntur, eadem vi reactione sua in aquam aget, quod sane etiam de superficie cava cylindrica itidem est intelligentum; id est, qua vi hæc superficies cylindrica ab aqua HD in gyrum acta atque adeo axem MP fugiente, premitur, tanta etiam est ejus reactio, cujus vi versus axem MP aqua repellitur; adeo ut, si corpus aliquod in aqua sit, ut *b*, id pellendum fit versus P in directione BP superficiei cylindri perpendiculari.

Sic etiam, in vorticibus magna pernicitate in gyrum actis, fluidum elabendi conatum habens a vorticum superficiebus repellitur, & a repulso fluido corpora solida, quæ in vortice sunt, secundum directiones superficiei vorticum normales, atque hoc modo gravitas in hypothesi vorticum utcunque adumbratur. Utinam vero reliqua gravitatis phænomena eadem facilitate in hoc vorticum systemate explicare liceret.

CAPUT XXIII.

De Agitatione aëris in productione soni.

651. Ante Illustrem Newtonum nemo theoriam sonorum geometrice tractare ausus est, nec mirum, cum ejusmodi disquisitio iis difficultatibus circumsepta sit, quæ non nisi a sagacissimo Viro aliisve similibus geometris superari posse videbantur. Veruntamen elegantissimum nobis exhibuit theorema summus Geometra accelerationes pulsuum in aëre elastico concernens Propos. XLVIII. Lib. Sec. Princ. Phil. Nat. Math. in veleri & Propos. XLVII. in noviss. editione, posteaquam suam hypothesin de productione sonorum Propos. XLIII. exposuit. Ejus doctrina, ni fallor, huc redit.

652. Scilicet intelligit vibrationibus partium corporis sonori alternis circumjectum aërem elasticum propelli atque adeo densari nonnihil, dehinc relaxari & in partes contrarias regredi. Nam itu partium corporis tremuli contiguæ aëris partes propulsæ densabuntur, regressu vero, atque adeo amota vi comprimente, densatus aër iterum sese in omnes partes, quantum potest, elatere suo expandet, ita ut itu partium corporis tremuli aër condensetur & in reditu atque regressu iterum rarescat. Quod partes corporis tremuli in aëre ipsis contiguo efficiunt, idem etiam præstabunt aëris partes jam propulsæ in aëre ipsis contiguo, & hic in sibi proximo atque sic deinceps, sed non eadem ubique harmonia, ita ut, si quædam aëris partes propulsæ sint, propellantur & omnes reliquæ atque progrediantur, sed cum unæ propellantur atque densentur, aliæ redeant atque posteaquam densatæ erant, iterum rarescant & regrediantur, idque vicibus permutatis. Ejusmodi partium aëris propulsarum atque iterum, unde venerant, regrediendum, seu intervalla progressus atque regressus, vocentur pulsus aëris, quæ intervalla ob æqualia temporis intervalla præterpropter æqualia existent. Hisce positis sequens solvendum occurrit problema.

PROPOSlTIO LXXXIV. PROBLEMA.

653. *Si elasticitates aëris densitatibus respectivis proportionales fuerint, determinare celeritates aëris elastici in uno ejus pulso a vibratione corporis tremuli vel alia ratione excitato.*

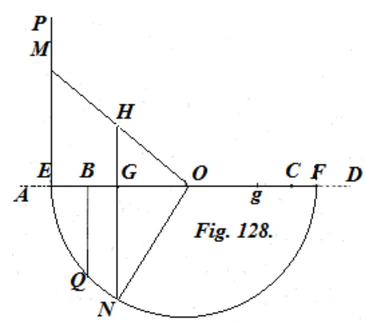


Fig. 128.

I. Sit EF latitudo unius pulsus, scilicet intelligatur uno itu vibrationis corporis tremuli contiguum aërem propulsum redactum esse in spatium AB, ex cujus puncto medio E excitata sit perpendicularis EP, atque in ea recta ME exponat elasticitatem aëris condensati AB. Regrediente parte corporis tremuli, vel potius remota vi compressione, quae aëri AB densitatem naturali majorem induxit, aër AB sui juris factus, in utramque partem (§.652.) æqualiter se diffundere conabitur ; sed considerando ejus dilatationem tantum quæ versus D sit, aërem BC ante se. propellet, atque eum condensando rediget in volumen CD ipsi AB circiter æquale.

Intervallum EF, inter puncta media portionum aëris densarum AB, CD, est intervallum unius pulsus, quoniam æquivalet itineri, quod eundo & redeundo conficitur. Ut innotescant celeritates aëris in singulis punctis intervalli EF, procedendum porro ut sequitur.

II. Dividatur EF bifariam in O, jungatur OM, ductaque per quodlibet punctum G intervalli EF recta GH parallela EP lineæ OM occurrente in H, fiat denique $\mathrm{F}g=\mathrm{EG}$. Ponatur aërem EB se extendesse in EG: quaëritur vis acceleratrix puncti progredientis G; hæc vero vis est virtus elastica aëris EG demta virtute elastica aëris FG, qui progredienti resistit. Jam aëris progredientis EG raritas est ut FG, & resistentis raritas ut FG, & quia densitates raritatibus sunt reciproce proportionales, & elasticitates directe densitatibus; elasticitates aëris raritatibus ejus erunt reciproce proportionales, unde, cum (secundum hypothesin) aëris AB elasticitas sit EM, aëris expansi EG elasticitas erit $\mathrm{EM.EB}:\mathrm{EG}$, aërisque expansi FG elasticitas $\mathrm{EM.EB}:\mathrm{FG}$. Ergo vis acceleratrix puncti G seu differentia elasticitatum $=\mathrm{EM.EB}:\mathrm{EG},-\mathrm{EM.EB}:\mathrm{FG}$ (vel quia gF& EG æquales) $=\mathrm{EM.EB.G}g:\mathrm{EG.FG}$. Jam, si ob angustiam pulsuum eorumque partium rec-lum EGF assumere licet æquale dato rec-lo BEF, erit vis acceleratrix puncti progredientis $\mathrm{G}=\mathrm{EM.EB.\,G}g:\mathrm{EB.\,EF}=\mathrm{EM.G}g:\mathrm{EF}=\mathrm{EM.GO}:\mathrm{EO}$, seu (propter triangula similia OEM & OGH) = GH. Est igitur vis acceleratrix puncti G circiter ut ordinata GH trianguli rectanguli OEM, hoc est, ut distantia ejus GO a medio vibrationis, ut Celeb. Newtonus, loco supra citato, primus invenit. Unde si centro O & radio OE descriptus sit semicirculus ENF, & EP media proportionalis inter EM & EO. Erit (§.148) celeritas puncti G, ut $\mathrm{GN.EP}:\mathrm{EO}$. Tempus vero, quo aër EB se extendit in spatium EG (§.149) exponetur arcu EN:EP. Quæ omnia erant invenienda.

COROLLARIUM I.

654. Hinc celeritates pulsuum erunt in composita ratione ex subduplicata intervallorum pulsuum & ex subduplicata densitatum. Vel brevius, celeritates sunt ut EP. Nam velocitates punctorum G in diversis EF similiter positorum, sunt ut homologæ EP, quia similitudo circulorum rationes omnes GN:NO vel EO æquales facit.

COROLLARIUM II.

655. Tempora vero pulsuum sunt in reciproca ratione celeritatum, seu ut EO:EP.

COROLLARIUM III.

656. Pendulum, cujus longitudo sit tertia proportionalis, ad EM & EO, unam oscillationem ex itu & reditu compositam eodem tempore conficiet, quo unus pulsus eundo & redeundo absolvitur.

SCHOLIUM.

657. Præcedentes determinationes circa vim acceleratricem, velocitatem, & tempus lationis puncti progredientis G, non nisi tanquam physice accuratæ haberi debent, non vera mathematice. Nam quia memorata vis acceleratriæ puncti G reperta est supra

proportionalis quantitati $EM.EB.Gg : EG.FG = 2EM.EB.GO : GN^2$, erit momentum solicitationis acceleratricis, ut

$$2EM.EB.GO - dGO : GN^2 = 2EM.BE.GN.dGN : GN^2 = 2EM.EB.dGN : GN$$

proportionale momento celeritatis acquisitæ, seu V*d*V, vocando hanc celeritatem V, & *d*V ejus elementum, & $-dGO$ significat elementum decrescens lineæ GO, alterum vero *d*GN elementum ordinatæ GN, hinc $EM.log.\left(GN^2 : BQ^2\right)$ est; ut quadratum celeritatis puncti progredientis G.

CAPUT XXIV.

De motu intestino fluidorum.

658. Hoc nomine non intelligitur hoc loco internus molecularum motus fluidi cujuscunque in suo statu naturali consistentis sed is particularum motus qui in fluidis a causis externis & accidentalibus excitari solet, quo calor præsertim est referendus, qui dubio procul ex concitatiore particularum motu in corpore calido a causis externis producitur. Utut vero ejusmodi motus intestinus admodum perturbatus fit, nihilo tamen minus regula physice satis accurata pro ejus mensura media tradi potest.

PROPOSITIO LXXXV. THEOREMA.

659. *Calor, cæteris paribus, est in composita ratione ex densitate corporis calidi, & duplicata rationi agitationis particularum ejusdem.*

Agitatio particularum est celeritas media inter celeritates particulares quibus calidi corporis particulæ agitantur. Vocetur hæc celeritas media V, & corporis densitas D. Jam, quia calor consistit in concitatiore particularum motu, calor erit, ut impressiones particularum corporis calidi in quopiam objecto corpore calorem excipiente, sed hæ impressiones sunt in composita ratione ex duplicata celeritatum & simpla densitatum, seu, ut $D.V^2$. Ergo etiam calor est ut $D.V^2$. Quod erat demonstrandum.

Dictum in propositione *cæteris paribus,* id est, in corporibus similis texturæ.

SCHOLIUM.

660. Ex hac propositione jam elicere licebit modum metiendi agitationem particularum aëris. Habeatur tubus AD M instar siphonis reflexus, cujus crus minus AD capsula vitrea ABC instructum sit, & majus DM hermetice sigillatum in M non nihil excedat longitudinem fistularum in ordinariis barometris adhiberi solitarum. Tempore hyberno tubus mercurio impleatur orificio aperto A deorsum spectante, atque ramo DM in verticali situ detento, parte D sursum respiciente, quod ope alius tubi inflexi atque aperto orificio A inserendi rite præstari potest. Totus tubus mercurio plenus inverti debet, ut denuo in situm, quem schema monstrat; reducatur, mercuriusque in ramo MD ad consuetam in ordinariis barometris altitudinem DH, 27 vel 28 digitorum se demittet, effluente per apertum orificium A superfluo hydrargyro. Sed, quia capsula ABC adhuc

mercurio plena erit, suctionis modo tanta ejus copia extrahenda, usque dum residui superficies FF sit circiter in medio capsulæ, vel paulo altior, ita ut BF aut CF sint $\frac{2}{3}$ totius altitudinis capsulæ BG. Clauso postea orificio A, atque adeo sublata communicatione aëris in ampulla cum exteriore, instrumentum usui aptum atque paratum erit. Ponamus enim aërem AFF calore extendi in spatium AEE, ita ut hydrargyrus in altero tubo ex H assurgat in I, columnæ IE pressio æquivalebit impressioni aëris AEE in sua basi EE, atqui hæc impressio (§. 659,) est, ut factum ex quadrato celeritatis mediæ particularum aëris in densitatem aëris. Vocetur, ut in propositionis expositione loco citato, velocitas media V, eritque aëris impressio in superficiem EE, ut $\mathrm{V}^2 : \mathrm{GE}$, nam densitas aëris AEE est ut I: AE vel I:GE; est ergo $\mathrm{IE} = \mathrm{V}^2 : \mathrm{GE}$, vel $\mathrm{IE.GE} = \mathrm{V}^2 = \mathrm{IG.GE} + \mathrm{GE}^2 = \mathrm{V}^2$. Hinc $\mathrm{V} = \sqrt{(\mathrm{IE.GE})}$ aut etiam $= \sqrt{(\mathrm{IG.GE} + \mathrm{GE}^2)}$. Propterea, si vocentur $\mathrm{FH} = a, \mathrm{EE} = b$. Diameter tubi $\mathrm{MD} = c, \&\ \mathrm{GF} = e$, ac denique variabilis $\mathrm{HI} = x$, invenietur V, seu velocitas media particularum aëris, ut $\sqrt{\left(ab^4 + b^4cx + bbccx + abbccx + bbccxx + c^4xx\right)} : bb.$ Hinc data x in aliqua observatione, innotescet ultro ipsa V.

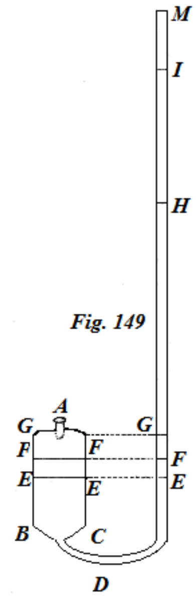


Fig. 149

Quod erat demonstrandum.

FINI .