CHAPTER XXI.

Concerning the motion of ships driven by the wind.

The motions of ships, that is their speeds, the distances traversed and the times of being rendered, can be deduced by the same method and from the same principles, as by which we have defined the motions of bodies in resisting mediums in the preceding chapters. Indeed the strength of the wind applied continually to the sail moving along is to be regarded as an instance of the accelerating force acting, and the water, in which the ship proceeds, an instance of the resisting medium. However the motions of ships are more difficult to be assigned than in the preceding motions of bodies in resisting mediums, because besides the absolute motions also the directions, both of the accelerations acting, or the curves along which the sails accept the forces of the wind, as well also as the paths traced out by the ship, which will depend on the shapes of the ships, are required to be determined initially. Yet all of which, as said, are able to be derived from the principles explained by us at this stage.

DEFINITIONS.

I. The path a ship BC leaves behind (French : *la Route du Vaisseau*), which a ship will describe permanently in the sea by the force of the wind, is called the line A*a*; without rotations about itself.

II. The axis of the ship shall be the spine of the keel BAC to which the foundation of the main mast is inserted. (French: *la Quille*).

III. The deviation of the ship (French: *la Derive)* is the right line *a*K, from some point *a* of the continued route sent perpendicular to the axis CK of the ship produced.

IV. The angle of deviation (*l'Angle de la Derive*) is the angle *a*AK; which the route of the ship maintains A*a* with the axis of the ship CB held towards K.

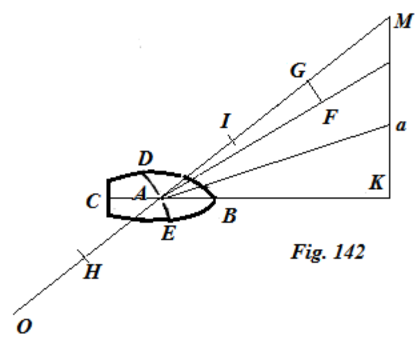

Fig. 142

PROPOSITION LXXVII. THEOREM.

624. *If the ship* ABC *may be carried along the permanent route* A*a*, *the mean direction of the force of the wind* AG *on the sail* DAE, *or on the sails, if several may be present, will be put in a straight line with the direction of the water resistance acting along* AO, *and with the ship BDC moving along the directions of lines parallel to* A*a*, *and with the rudder at C.*

If this were not the case, the mean direction of the force of the wind AF on the sail DAE shall be different from the line AM, which is drawn by extending the mean direction of the water AO, striking the ship BDC as it proceeds along A*a*. AF shows the force of the wind on this line, and AH the resistance of the water in the direction AO, and the perpendicular FG is dropped to OA, extended to M, and on AG there may be taken GI = AH. With which in place, because the sides AG and GF are equivalent to the force AF, and the force of the water resistance AH (constr.) is required to be taken away from the force AG; GI, the effectiveness of the wind towards moving the ship will be equivalent to the two remaining forces AI and FG; thence, because FG is not weakened by any contrary force, thus its effect will produced, on account of the force FG, along with the other AI, which accelerates the motion of the ship, the ship may be turning about itself according to the order of the letters K, F, G, while that is carried forwards in its motion along the line A*a;* therefore this line is not the path of the ship maintained, contrary to the hypothesis.

[It appears that the effect of the rudder has been neglected at this stage, which set at an angle, can cancel out any rotational torque.]

COROLLARY.

625. And thus from the mean direction of the impressed force of the wind on the sails, it is possible to know the mean direction of the water resistance to the ship, and vice versa. And from the mean direction of the water resistance with the aid of Prop. L. of this second Book, again the path of the ship may be found. Truly the mean direction of the wind inflating the sails can be found in the same way, as has been shown in §.474.

[Ignorance of what is now called the Bernoulli Principle, by which the sail becomes an aerofoil, renders Hermann's account incorrect. An account of Daniel Bernoulli's original investigation can be found in Section XII of his *Hydrodynamics*, and a translation of this work can be found on this website.]

PROPOSITION LXXVIII. PROBLEM.

626. *With the direction and speed of the wind given striking the sail* DAE, *and thus with the mean direction of the sail* AM, *to define the speed of the ship* BDC *along its permanent passage* A*a in still water or on the wide sea.*

I. Because (following the hypothesis) AM is the mean direction of the wind striking the sail DAE, the prolongation AO of this (§.624.) will give the mean direction of the water in the direction *a*A, or of the ship BDC moving along parallel to this, thus so that, with the motion of the ship endured, the angle MA*a* shall always remain the same.

Therefore the ratio A*a*:AM, or the sine of the complement of the angle MAK to the sine of the complement of the angle *a*AK, shall be as *b* to *a*; and the speed of the wind , along the initial direction AM at the start of the motion shall be *a*, truly the speed of the ship at the end of any distance measured out A*a*, *u;* thus *au*:*b* shall express the speed of the ship along AM,

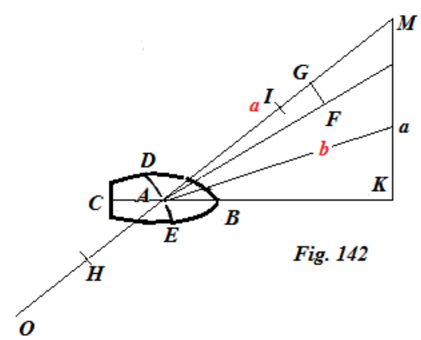

Fig. 142

[Initially, wind speed:boat speed $= a:b =$ rel. wind speed:u.] evidently which the ship acquires from the initial velocity of the following wind, thus therefore the speed, with which it acts on the sail, along AM in that instant, in which the ship has acquired the speed u in its journey, shall become

$a-,au:b$ [i.e. $a-\frac{au}{b}$, taken to be the speed of the ship relative to the wind.]; and thus

$(a-,au:b)^2 \cdot M$ expresses the force of the wind on the sail along AM, where M [a term for the mass of air] can be found with the aid of §. 474, Ch. 13. Truly the force of the water resistance acting, along the direction AO, is Nuu, where N can be found also by Prop. L, if not algebraically perhaps transcendentally and by approximations. Therefore $(a-,au:b)^2.\mathrm{M}-\mathrm{N}.uu$ is the acceleration acting on the ship.

[This treatment can be understood if we regard the air resistance of the wind as the square of the speed of the wind on the sail as the source of the ship's propulsion or acceleration, this force, with the opposing water resistance also taken in the ratio of the square of the speed, over an increment of distance, is put equal to the moment of the speed of the ship, or *mvdv*, or to the increase in the kinetic energy of the ship on integrating, as becomes apparent in the following proposition.]

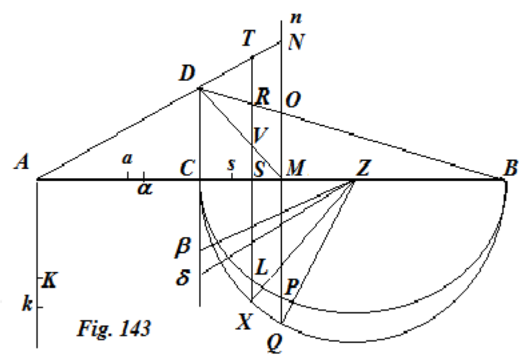

Fig. 143

II. Again let the equation be $(a,-au:b)^2.\mathrm{M}-\mathrm{N}.uu=\mathrm{N.SL}^2$, where SL is the ordinate of some curve CLP, to be found in the following manner, with AM being b; MS, u, and Mn, a, itself in place perpendicular to AM. [Note: a and b have now been interchanged from the above, which must be an error, and put at right angles to each other to ease the calculation.] With these in place, and because $a,-au:b=(b-u).a:b=\mathrm{AS.M}n:\mathrm{AM}$ there will be

$(a,-au:b)^2.\mathrm{M}=\mathrm{AS}^2.\mathrm{M}n^2.\mathrm{M}:\mathrm{AM}$, evidently if MN were to Mn in the square root

$=\mathrm{AS}^2.\mathrm{MN}^2.\mathrm{N}:\mathrm{AM}^2=\mathrm{N.ST}^2$

ratio M to N, and shall be multiplied by AN, thus so that the triangles AST and AMN shall be similar. Hence, $(a,-au:b)^2.\mathrm{M}-\mathrm{N}uu=\mathrm{N.ST}^2-\mathrm{N.SM}^2=$(following the hypothesis) $\mathrm{N.SL}^2$, and thus $\mathrm{ST}^2-\mathrm{SM}^2=\mathrm{SL}^2$. MD is drawn bisecting the right angle AMN, and from D DC may be sent normal to AM, and on account of the semi-right angle CMD, OC and CM shall be equal, and thus SM and SV ; therefore $\mathrm{SL}^2=\mathrm{ST}^2-\mathrm{SM}^2=\mathrm{ST}^2-\mathrm{SV}^2$ will be had. And with any intercept TV divided in two at R and with DR drawn, and with that produced as far as crossing the right line AM produced at B, there is a need for the same amount, and this DR also will pass through the centre O of MN itself; and $\mathrm{ST}^2-\mathrm{SV}^2$ will be 4SR.VR. Therefore also $\mathrm{SL}^2=4\mathrm{SR.RV}$. Truly because $\mathrm{SR}:\mathrm{MO}=\mathrm{SB}:\mathrm{MB}$, and $\mathrm{RV}:\mathrm{OM}=\mathrm{CS}:\mathrm{CM}$ by arranging the ratios there will be

$SR.RV : MO^2 = 4SR.RV$, that is, $SL^2 : NM^2 = SB.SC : MB.MC = SX^2 : MQ^2$, clearly with the semicircle CQB described on the diameter CB ; therefore we have $SL^2 : NM^2 = SX^2 : MQ^2$; or by permuting $SL^2 : SX^2 = NM^2 : MQ^2$ or $SL : SX = NM : MQ$; therefore all the homologous ordinates of the curve sought and of the semicircle are in a given ratio, and thus the curve CLP is an ellipse, of which the ordinate MP is equal to MN.

III Now A*a* shall be the distance traversed by the ship in a given time, $a\alpha$ the element of this distance and $MS = u$ the speed acquired by the ship, at the end of the distance A*a*, S*s* the element of the speed, and because (no.1. of this) we have found the acceleration [*c.f.* note above] of the ship to be, as $N.SL^2$, we may put that to be $= N.SL^2 : N.D$, or $= SL^2 : D$, where D, or with the magnitude given, taken according to the homogeneity required, and there will be (§. 484.) the moment of the action of the acceleration or its rectangle $SL^2.a\alpha : D$, from this action $SL^2 : D$ by the element of the distance $a\alpha$; equals the moment of the speed acquired MS.S*s* or to the rectangle from this speed MS by its element S*s*. Hence, since there shall be $SL^2.a\alpha : D = MS.Ss$ or $SL^2.a\alpha = D.MS.Ss$, there will be $MS.Ss : SL^2 = a\alpha : D$: There is indeed also $SL^2 : SX^2 = MP^2$ or $NM^2 : MQ^2$, therefore from the equation, $MS.Ss : SX^2 = NM^2.a\alpha : D : MQ^2$. And, if AK shall be made everywhere to A*a*, as MN^2 to D.MQ, there will be $Kk.a\alpha = NM^2 :$ D.MQ, & $NM^2 : a\alpha = D.MQ.Kk$, and $MS.Ss : SX^2 (= NM^2.a\alpha.D.MQ) = D.MQ.Kk : D.MQ^2 = Kk : MQ$. And, if finally SX may be put to be the mean proportional between MQ and a certain Z, thus so that Z may become $= SX^2 : MQ$, there becomes $MS.Ss : Z.MQ : \left(\text{or } SX^2\right) = Kk : MQ$; and thus $Z.Kk = MS.Ss$, and the problem has been reduced precisely to the case of proposition LXVII. of the second Book, since Z is, as the square of the ordinate SX in the circle, and now AK is required to be considers instead of the distance of the body being sent. Therefore, if in accordance with the precept of the proposition indicated, the right lines $C\beta$ and $C\delta$ shall be the tangents of the angles $CZ\beta$, $CZ\delta$ of the halves of these CZX and CZQ, there will be by (§.149)
$AK = \log.\left(MQ : SX\right) - MZ.\log.(C\delta : C\beta) : CZ$. And, because (by constr.) $AK : Aa = MN^2 : D.MQ$, there will be found
$Aa = D.MQ.\log.(MQ : SX) : MN^2 - D.MQ.MZ.\log\left(C\delta : C\beta\right) : CZ.MN^2$, on the logarithmic curve, of which the subtangent is MQ.

By the second part of the same proposition (§.549.) the time to travel through the distance $AK = \log.(C\delta : C\beta) : CZ$ *;* and $tAK : t.Aa = AK : Aa = MN^2 : D.MQ$, therefore the time for the ship to be sent through the distance A*a* will be
$D.MQ.\log\left(C\delta : C\beta\right) : CZ.\ MN^2$. Which were required to be found.

COROLLARY I.

627. Because the action of the acceleration of the ship everywhere is as SL^2, and because SL is zero at the point C, MC itself will designate the maximum velocity, which the ship may be able to acquire. It is truly on account of the similar triangles ACD and AMN and therefore $CD = CM;\ AC : CM\ (\text{or DC})$ (or, $= AM : MN = AM\sqrt{N} : MN\sqrt{N}$ because nM:NM (constr.) $= \sqrt{N} : \sqrt{M}$ and thus the rectangle $NM.\sqrt{N}$ equals $nM\sqrt{M}) = AM\sqrt{N} : nM\sqrt{M}$, and thus by adding $AM : CM = AM\sqrt{N} + nM\sqrt{M} : n\sqrt{M}\sqrt{M}$, that is, $b : CM = b\sqrt{N} + a\sqrt{M} : a\sqrt{M}$, and thus $MC = ab\sqrt{M} : a\sqrt{M} + b\sqrt{N}.$ For (constr.) there is $AM = a,$ and $nM = b.$

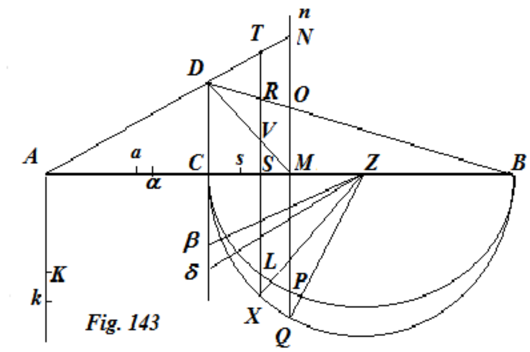

Fig. 143

COROLLARY II.

628. If D is of finite magnitude, the time, in which the maximum speed MC of the ship is acquired, and therefore the distance A*a* being transversed in this time, as that which has been shown until now by other means (§.550.).

But if D is indefinitely small, both the distance as well as the time, in which that moving body can acquire the maximum speed MC, will be of a finite magnitude.

SCHOLIUM

629. Because the maximum speed of the ship generally shall be acquired in a finite time, therefore in these cases the magnitude D, supplying the assumed homogeneity, always will be indefinitely ; but this does not prevent proportions between the distances traversed and between the times always able to be shown by the logarithms.

CAPUT XXI.

De Motu Navium vento impulsarum.

Motus Navium, hoc est eorum celeritates, spatia percursa & tempora lationis, eadem methodo & ex iisdem principiis deduci possunt, quibus in præcedentibus capitibus motus corporum in mediis resistentibus definivimus. Est enim venti velo allabentis virtus spectanda instar solicitationis acceleratricis navi continue applicatæ, & aquæ, in quibus navis incedit, instar medii resistentis. Verumtamen motus navium nonnihil difficilius assignantur quam in præcedentibus motus corporum in medio resistenti incedentium, quia præter motus absolutus etiam directiones, tum solicitationum acceleratricium, seu lineæ, secundum quas vela impressiones venti excipiunt, tum etiam itinera navis permanentia, quæ a figuris navium pendent, primum determinanda sunt. Quæ tamen omnia, ut dictum, ex principiis, haetenus a nobis explicatis derivari queunt.

DEFINITIONES.

I. Iter navis manens A*a* (Gallis *la Route du Vaisseau*) dicatur linea, quam navis BC vento impulsa in mari permanenter describit; absque rotationibus circa se ipsam.
II. Axis navis esto spina carinæ BAC cui inseritur stereobates mali principis. (Gallis *la Quille*).
III. Deviatio navis (Gallis *la Derive)* est recta *a*K,ex quolibet itineris manentis puncto *a* ad axem navis productum CK perpendiculariter demissa.
IV. Angulus deviationis *(l'Angle de la Derive)* est angulus *a*AK; quem iter navis permanens A*a* cum axe navis CB continet versus K.

PROPOSITIO LXXVII. THEOREMA.

624. *Si navis* ABC *tinere permanenti* Aa *feratur, media directio* AG *impressionum ventri in velo* DAE, *vel in velis, si plura adsint, in directum posita erit directione* AO *aquæ resistentis, & secundum directiones lineæ* A*a parallelas lateri navis BDC clavoque in C allabentis.*

Si negas, sit media directio impressionis venti secundum mediam directionem velariæ DAE recta AF diversa a linea AM, quæ est prolongatio mediæ directionis AO aquæ, navi BDC impingentis dum incedit itinere A*a*. Exponat AF impressionem venti in hac linea, & AH resistentiam aquæ in directione AO, & demittatur·ad OA, protractam in M; perpendicularis FG, &

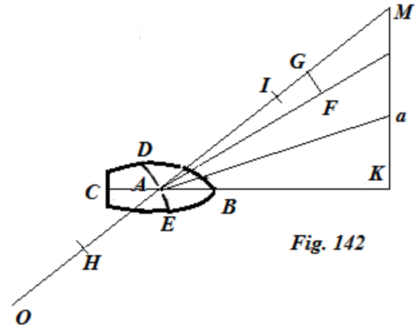

Fig. 142

in AG sumatur, GI = AH. Quibus positis, quia viribus AF æquipollent laterales AG & GF, & a vi AG auferenda est vis aquæ resistentis AH (constr.) GI, venti efficacia ad navim movendam æquipollebit duabus viribus residuis AI & FG; unde, quia FG nulla contraria vi retunditur, ideo suum effectum edet, propterea vi FG, juxta alteram AI, quæ motum navis accelerat, navis circa se ipsam convertetur secundum ordinem literarum K, F, G, dum ea progressivo motu fertur in recta A*a;* ergo hæc linea non est iter navis manens, contra hypothesin.

COROLLARIUM.

625. Adeoque ex media directione impressionum venti in velis, innotescere potest media directio aquæ navi resistentis, & vice versa. Et ex aquæ resistentis media directione ope Propos L. Lib. hujus secundi, invenietur iter navis permanens. Mediæ vero directiones venti vela inflantis obtineri possunt eo fere modo, quo §.474 ostensum est.

PROPOSITIO LXXVIII. PROBLEMA.

626. *Datis directione & celeritate venti velo* DAE *impingentis, atque adeo media directione veli* AM, *definire celeritates navis* BDC *itinere permanente* A*a in aqua stagnante seu mari latæ.*

I. Quia (secundum hypothesin) AM est media directio venti velo DAE impingentis, ejus prolongatio AO (§.624.) dabit mediam directionem aquæ in directione *a*A, vel huic parallela navi BDC allabentis, adeo ut, durante motu navis, angulus MA*a* constanter idem maneat.

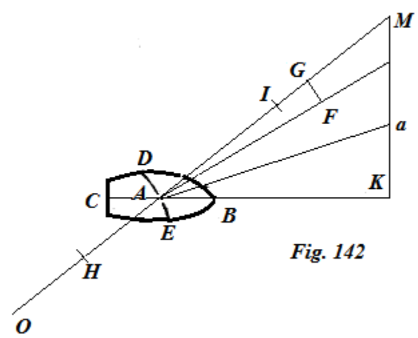

Fig. 142

Sint ergo ratio A*a*:AM, seu sinus comp. anguli MAK ad sin. compl. ang. *a*AK, ut *b* ad *a*; & celeritas venti, juxta mediam directionem AM initio motus, *u*, celeritas vero navis in termino *a* cujusvis spatii emensi A*a, u;* adeoque *au*:*b* exponet celeritatem juxta AM, qua scilicet navis insequentis venti velocitati initiali sese subducit, ut adeo celeritas, quacum in velum agit, juxta AM eo momento, quo navis in suo itinere velocitatem *u*, acquisivit, futura sit $a-, au:b$; adeoque $(a-, au:b)^2$; M exponet impressionem venti in velo secundum AM, ubi M inveniri potest ope §. 474.
Aquæ vero resistentis impressio, secundum directionem AO, est N*uu*, ubi N per Propos. L etiam inveniri potest,si non algebraice saltem transcendenter & per approximationes. Propterea est $(a-, au:b)^2.\mathrm{M}-\mathrm{N}.uu$ solicitatio acceleratrix navis.

II. Esto porro $(a,-au:b)^2.\mathrm{M}-\mathrm{N}.uu=\mathrm{N.SL}^2$, ubi SL est ordinata alicujus curvæ CLP sequenti modo inveniendæ, existentibus AM, *b;* MS, *u*, M*n* ipsi AM perpendiculariter insistens , *a*. His positis, & quia $a,-au:b=(b-u).a:b=\mathrm{AS.M}n:\mathrm{AM}$ erit $(a,-au:b)^2.\mathrm{M}=\mathrm{AS}^2.\mathrm{M}n^2.\mathrm{M}:\mathrm{AM}=\mathrm{AS}^2.\mathrm{MN}^2.\mathrm{N}:\mathrm{AM}^2=\mathrm{N.ST}^2$, scilicet si MN fuerit ad M*n* in subduplicata ratione M ad N, atque ducta sit AN, ita ut triangula AST & AMN similia existant. Hinc, $(a,-au:b)^2.\mathrm{M}-\mathrm{N}uu=\mathrm{N.ST}^2-\mathrm{N.SM}^2=$(secundum hypothesin) $\mathrm{N.SL}^2$, atque adeo $\mathrm{ST}^2-\mathrm{SM}^2=\mathrm{SL}^2$. Ducatur MD angulum rectum AMN bisecans, & ex D demittatur DC normalis ad AM, & ob angulum semirectum CMD erunt æquales OC & CM, perinde ac SM & SV ; propterea habetur $\mathrm{SL}^2=\mathrm{ST}^2-\mathrm{SM}^2=\mathrm{ST}^2-\mathrm{SV}^2$. Atqui divisa qualibet intercepta TV bifariam in R ductaque DR, eaque producta usque ad occursum B cum recta AM producta itidem quantum opus est, & hæc DR etiam per medium O ipsius MN transibit;

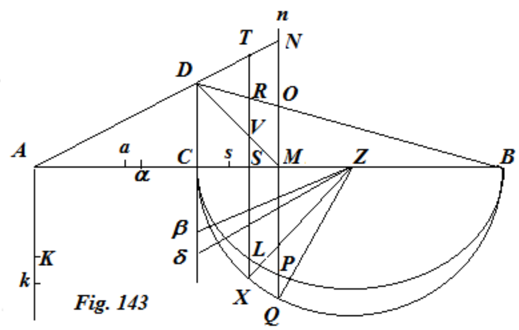

Fig. 143

ac $\mathrm{ST}^2-\mathrm{SV}^2$erit $=4\mathrm{SR.VR}.$ Ergo etiam $\mathrm{SL}^2=4\mathrm{SR.RV}.$ Verum propter $\mathrm{SR}:\mathrm{MO}=\mathrm{SB}:\mathrm{MB},\&\ \mathrm{RV}:\mathrm{OM}=\mathrm{CS}:\mathrm{CM}$ erit per compositionem rationum $\mathrm{SR.\,RV}:\mathrm{MO}^2=4\mathrm{SR.RV}$, id est, $\mathrm{SL}^2:\mathrm{NM}^2=\mathrm{SB.SC}:\mathrm{MB.MC}=\mathrm{SX}^2:\mathrm{MQ}^2$, descripto scilicet super diametro CB semicirculo CQB; habemus ergo $\mathrm{SL}^2:\mathrm{NM}^2=\mathrm{SX}^2:\mathrm{MQ}^2$; vel permutando $\mathrm{SL}^2:\mathrm{SX}^2=\mathrm{NM}^2:\mathrm{MQ}^2$seu $\mathrm{SL}:\mathrm{SX}=\mathrm{NM}:\mathrm{MQ}$; idcirco ordinatæ quæque homologæ curvæ quæsitæ & semicirculi sunt in data ratione, aque adeo curva CLP est ellipsis, cujus ordinata MP æquatur MN.

III Sint nunc A*a* spatium certo tempore navi percurrendum, $a\alpha$ elementum illius spatii & $\mathrm{MS}=u$ celeritas navi acquisita, in termino spatii A*a*, S*s* velocitatis elementum, & quia (num.1. hujus) invenimus solicitationem acceleratricem navis esse, ut $\mathrm{N.\,SL}^2$, ponamus eam $=\mathrm{N.SL}^2:\mathrm{N.D},$ vel $=\mathrm{SL}^2:\mathrm{D}$, ubi D, seu data magnitudo, assumitur ad supplenda homogenea, eritque (§. 484.) momentum solicitationis acceleratricis seu rec-lum $\mathrm{SL}^2.a\alpha:\mathrm{D}$, ex hac solicitatione $\mathrm{SL}^2:\mathrm{D}$ in elementum. spatii $a\alpha$; æquale momento celeritatis quisitæ MS.S*s* seu rec-lo ex hac celeritate MS in suum elementum S*s*. Hinc, cum sit $\mathrm{SL}^2.a\alpha:\mathrm{D}=\mathrm{MS.S}s$ vel $\mathrm{SL}^2.a\alpha=\mathrm{D.MS.S}s,$ erit $\mathrm{MS.S}s:\mathrm{SL}^2=a\alpha:\mathrm{D}:$ Est vero etiam $\mathrm{SL}^2:\mathrm{SX}^2=\mathrm{MP}^2$ vel $\mathrm{NM}^2:\mathrm{MQ}^2$, ergo ex æquo $\mathrm{MS.S}s:\mathrm{SX}^2=\mathrm{NM}^2.a\alpha:\mathrm{D}:\mathrm{MQ}^2.$ Et, si AK fiat ubique ad A*a*, ut MN^2 ad D.MQ, erit $\mathrm{K}k.a\alpha=\mathrm{NM}^2:\ \mathrm{D.MQ},\&\ \mathrm{NM}^2:a\alpha=\mathrm{D.MQ.K}k$, atque $\mathrm{MS.S}s:\mathrm{SX}^2(=\mathrm{NM}^2.a\alpha.\mathrm{D.MQ})=\mathrm{D.MQ.K}k:\mathrm{D.MQ}^2=\mathrm{K}k:\mathrm{MQ}.$ Et,si denique SX ponatur media proportionalis inter MQ & quandam Z, ita ut Z futura sit $=\mathrm{SX}^2:\mathrm{MQ}$,

fiet $MS.Ss : Z.MQ : \left(\text{seu } SX^2\right) = Kk : MQ$; atque adeo $Z.Kk = MS.Ss$, & problema præcise ad casum propositionis LXVII. hujus secundi Libri reductum est, quandoquidem Z est, ut quadratum ordinatæ SX in circulo, & jam AK consideranda instar spatii mobili transmittendi. Propterea, si juxta præceptum citatæ propositionis rectæ $C\beta$ & $C\delta$ sint tangentes angulorum $CZ\beta$, $CZ\delta$ dimidiorum ipsorum CZX & CZQ, erit (§.149) $AK = \log.\left(MQ : SX\right) - MZ.\log.(C\delta : C\beta) : CZ.$ Et, quia (constr.)
$AK : Aa = MN^2 : D.MQ$, invenietur
$Aa = D.MQ.\log.(MQ : SX) : MN^2 - D.MQ.MZ.\log\left(C\delta : C\beta\right) : CZ.MN^2$, in logarithmica, cujus subtangens est MQ.
Per secundam partem propositionis ejusdem (§.549.) erit tempus per spatium
$AK = \log.(C\delta : C\beta) : CZ$; atqui $tAK : t.Aa = AK : Aa = MN^2 : D.MQ$, ergo tempus per spatium A*a* navi transmissum erit $D.MQ.\log\left(C\delta : C\beta\right) : CZ.\ MN^2.$ Quæ erant invenienda.

COROLLARIUM I.

627. Quia solicitatio acceleratrix navis est ubique ut SL^2 ,& quia SL in puncto C nulla est, ipsa MC designabit maximam velocitatem, quam navis acquirere queat. Est vero propter triangula similia. ACD & AMN & propter
$CD = CM$; $AC : CM \left(\text{vel } DC\right) = AM : MN = AM\sqrt{N} : MN\sqrt{N}$ (aut,quia *n*M:NM (constr.) $= \sqrt{N} : \sqrt{M}$ atque adeo rec-lum $NM.\sqrt{N}$ æquale
$nM\sqrt{M}) = AM\sqrt{N} : nM\sqrt{M}$, adeoque componendo
$AM : CM = AM\sqrt{N} + nM\sqrt{M} : n\sqrt{M}\sqrt{M}$, id est, $b : CM = b\sqrt{N} + a\sqrt{M} : a\sqrt{M}$, atque adeo $MC = ab\sqrt{M} : a\sqrt{M} + b\sqrt{N}.$ Nam (constr.) est $AM = a,$ & $nM = b.$

COROLLARIUM II.

628. Si D est finitæ magnitudinis, tempus, quo navi maxima celeritas MC acquiritur, erit infinitum, perinde ac spatium A*a* hoc tempore percurrendum, ut id jam alibi (§.550.) quadantentus est ostensum.

Sed Si D est indefinite parva, cum spatium tum etiam tempus, quo id mobile transmittens celeritatem maximam MC acquirit, erunt finitæ magnitudinis.

SCHOLIUM

629. Quoniam navi plerunque tempore finito maxima celeritas acquiritur, ideo in hisce casibus magnitudo D, ad supplenda homogenea assumta, semper erit indefinite parva; sed hoc non obstante proportiones inter spatia transmissa & inter tempora per logarithmos semper exhibere possunt.