SECTION IV.

On the motions of bodies in resisting media.

Whenever Galileo, Torricelli and the other celebrated geometers of that time dealt with motion, the bodies were supposed to be carried in a vacuum, *i.e.* in a medium which was unable either to accelerate or retard the motion, not because these geometers were unaware of a resisting medium to be given on our earth of this kind with such an ability, but because they were unable to see clearly, how the resistances could be adapted to the geometrical laws of motion. Therefore with such a willing mind, these resistances were abstracted from the media by the supposition of a vacuum, since these might be estimated as small errors only from the supposed vacuum, able to be introduced later and to be perceptible to the senses for their determination. Truly among the more recent of the best geometers, who have come upon the most perfect theory of motion from the outstanding Galileo, yet who were not content and wondered how much this theory might be developed by the inclusion of the difficult matter for the geometer of motions in resisting media, but being of more abstruse nature, the precept was to be established by a succession of strokes of good luck, and the geometers have been conveyed into regions not known before. Indeed the thoughts of most outstanding of men in this regard, *Newton, Leibniz, Huygens* & *Wallis,* were published before many years partially without proofs and only indicated briefly, and partially also strengthened with demonstrations, although very short, thus not accommodating enough for the understanding of beginners, with the exception of Wallis, who in his dissertation on this matter set out all the minute details, but only under the particular hypothesis of the resistance put in place to be in the ratio of the speed of the bodies bringing about the impediment. After these praiseworthy men the distinguished *Varignon* disseminated the same reasoning in a learned manner, and treated in a clear manner in the *Actis Academ. Reg. Paris. Scient*. of the years 1707, 1708, 1709 & 1710 thus so that also this business would be made clear to the following mathematicians; truly, because this matter relates so directly to our established methods, so that if I might leave out something to be acted on concerning that, I might think to omit some precepts from my little work, thus I will not be omitting anything from this treatment after all the outstanding authors, but as far as I will be able to set it out more briefly and clearly. I will add some of my own findings, and I shall insist on the simple principle, to which nearly everything I have put in place above, which pertains to the actual motion of bodies, evidently the *moment of any force acting to be equivalent to the moment of the speed.* [What Hermann has in mind is the work done over an infinitesimal distance to equal the change in kinetic energy of the immersed body.] None of the most praiseworthy men has used this principle in the teaching of moving bodies considered in resisting media, as far as can be judged from the publication of their papers, even if they may have obtained this principle much more briefly and in a natural manner, than from other fundamentals.

CHAPTER XIV.

Containing the general principles which pertain to the theory of the motion of bodies in resisting media, and several geometrical lemmas necessary in this theory.

DEFINITIONS.

There are two kinds of resistance, the one *absolute* and the other *with respect to or depending on the speed.*

I.

477. An *absolute resistance* is one which removes just as much of the strength of a moving body, whether the same body may be carried by a small or large velocity. Glutinous media and rough surfaces are examples of resistances of this kind: for in a glutinous fluid there is a need for the same force to be separating its parts, with whatever speed at last the moving body may be trying to separate these parts. Thus also with the frictions of a body advancing on a rough plane the same obstructions are to be overcome, whether the mobile body may be moving with a small or a large velocity, and in whatever manner the impediments to the motion may be had, either in the case of elastic hairs being depressed, and afterwards righting themselves again at once ; or after the manner of filaments requiring to be broken off, or by some other manner equivalent to these, for all these there is a need for a certain definite and determined force, which does not depend on the velocity acting.

II.

478. Truly a *resistance* is *with respect of the speed*, which arises from the striking of the particles from the fluid medium on the anterior parts of the moving body. This resistance depends on the density of the medium and on the speed of the mobile body ; for the parts of the fluid (§.422.) are agreed to strike that body itself with the speed of the body moving in that fluid, with which the body may impinge on these particles themselves, and the amount of fluid bodies striking (§. 423) are in a ratio composed from the density and the speed.

III.

479. The fluid medium and the resistance of the air may be simply signified by name, since the air may be understood to be of such a subtle nature, that it may produce the same outstanding effect as a resisting medium.

IV.

480. *The fundamental motions* are those which are made in a vacuum. Thus the fundamental uniform motion is a motion of equality, by which a body proceeds in a vacuum by some impulsive force. And the fundamental motion of acceleration or retardation from uniform gravity are those motions, which Galileo showed for a weight

to be falling or rising in a vacuum, the kind of motion that we have considered in Book I, Sect. II. Cap. I., §.150, 151.

V.

481. The action of the acceleration of gravity applied to a weight ascending in air, is the excess, by which the weight exceeds the resistance of the medium, & and the whole or absolute resistance opposing the ascent of a body in air, is the sum of the actions of the gravity and of the resistance of the air. For these two are struggling against each other in the ascent of the body in air. Indeed with a falling weight it falls downwards by no other force, other than that, by which it overcomes the resistance of the air in which the body is moving : therefore here the excess of the acceleration of gravity over the air resistance of the body falling in air is to be examined.

VI.

482. And, because these accelerative actions in air decrease continually on account of the increased resistance with the speed of the moving air, and still without reaching that, so that each may be completely cancelled out, as will be shown further in the following, thus the speed indeed also will increase, as it has not yet reached a certain level of speed, beyond which it cannot pass. Such a velocity that weights at no time can attain by falling, and if they approach more and more towards that, Huygens considered the *terminal velocity* , and indeed Leibniz called the same exclusively the *maximum speed*.

The graphs of the accelerating forces, of the total resistances, and of the speed acquired or of the remaining speeds, are assumed in the same manner as in Section II of Book I ; Thus so that no ambiguity of the names may be able to slip into this account. Likewise equally it is required to be understood about the moments of these forces or of the whole resistances, and the speed of increasing or decreasing.

VII.

483. If right lines of indefinite length may be said to be carried along thereupon with an equal motion, and on these lines bodies may be placed to proceed along these and sideways as well, these moving lines are called the *carrying lines*, and the motion of these lines is called the *common motion*; clearly the motion of the moving line and of the body being carried along on that line, and thus the motion of the body on the carrying line hence may be called its own motion, and the *absolute motion* of the same body is that, which arises from the common and individual motions, and is the sum of these, if the carrying line and on that the body by its proper motion are carried in the same direction, the difference truly if carried in opposite directions. Therefore, from the individual and common motion of the body the absolute motion always will be known. [Thus, a body may be at rest on a line and may be carried with a uniform speed on the line ; the line itself is the rest frame for the body at some instant, and this frame can be moving with a uniform speed w.r.t. the observer, here called the common motion. On the other hand, the body may be accelerated along its line if initially placed at rest in the moving fluid, and eventually its speed tends towards the speed of the fluid, depending on the law of friction between the two; Hermann makes use of this idea to produce two motions which he considers identical at any instant.]

PROPOSITION LIV. LEMMA.

484. *The moment of any accelerating force of a body falling in air, or of the whole resistance of the same body ascending in air in some manner, is equal to the moment of the speed increase acquired, or to the moment of the speed decrease remaining.*

This proposition is the same as Prop. XVII. Book I. §.131. so that there may be no need for a new demonstration, for some actions called dead forces, are able to be applied continuously [This was the name of workless forces, such as action reaction pairs, so designated by Leibniz]. Therefore, if for the sake of convenience, the body may be considered to be moving on a vertical line, and the distances traversed may be called x, gravity g, the resistance of the air r, and the velocity acquired by falling or remaining from ascending u ; the action of the acceleration of gravity falling (§.481.) will be $g-r$, and the moment of its action $gdx-rdx$ is equal to the moment of the increase of the velocity udu. Hence $gdx-rdx=udu$.

The whole resistance of the body ascending (§. 481.) is $g+r$, and the moment of its resistance $gdx+rdx$ is equal to the moment of the decrease of the speed $-udu$, thus so that there may be had $gdx+rdx=-udu$. Or also by assuming the distance not actually traversed, but subsequently to be traversed, as far as the whole motion of the ascent becomes zero, there will be $-gdx-rdx=-udu$, or $gdx+rdx=udu$, indeed with this agreed upon the distances to be resolved, after being now traversed, decrease with the speeds.

Therefore by including each case of the vertical descent and ascent of a body in air by one general formula, there will be found $gdx\mp rdx=udu$. Where the upper sign − of the descents, truly the other + of the ascents of the body with respect to the air resistance ; where it is necessary to remember the distance x not to be the actual distance traversed by a body ascending, but its complement to the total or maximum height, that an ascending body can traverse.

[Hermann had stumbled across a form of the conservation of energy principle from his calculus, relating work done against gravity and frictional forces to the change in kinetic energy; but of course he could not recognise it as such at the; subsequently his work was largely forgotten, otherwise he would certainly have been advocated as a co-founder of this principle.]

COROLLARY.

485. If now the element of time, in which the incremental distance dx is moved through, may be called dt, there will be $dt=du:g\mp r$. For, because (§.128.) $dt=dx:u$, and (§.484.) $gdx\mp rdx=udu$, there will be $dt\left(=dx:du\right)=du:g\mp r$. And thus $t=\int du:g\mp r$, where t indicates the time, in which a weight by moving initially from rest slips through the distance x, or the time in which with the weight by ascending will be able to complete the complement of the distance now actually ; thus, if the time, in which the maximum height A, as the weight with a given speed can be projected to a height to

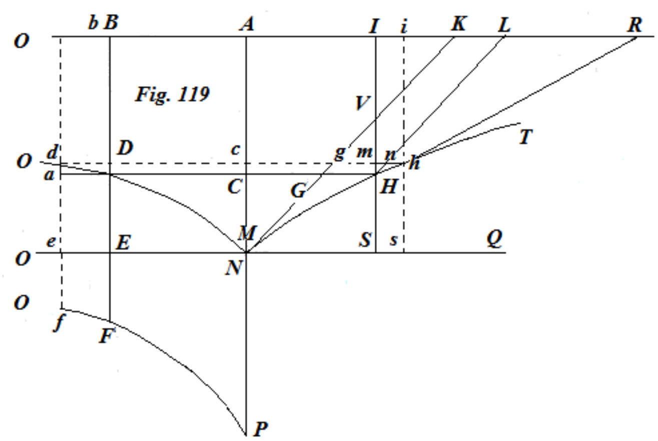
be measured, may be called T, the amount $T-t$ expresses the time, in which a moving body has past through the actually distance by ascending.

SCHOLIUM.

486. Since at this time no attention shall be paid to the origin of the resistance of the medium, or how it may depend on the actual speeds of the moving body or truly on anything else, it is clear enough that the variations of the laws of motion are contained in the preceding general principles, for any absolute or relative hypothesis for the resistance of the medium.

PROPOSITION LV. LEMMA.

487. *The motions of a body in air, to be varied from the fundamental uniform motion by some law of resistance, are the same as those which may result either from the common motion of the body [through the air, with the line or air at rest], or from the uniform motion of the line itself, with the body being carried along on the line by its own motion, arising from the interaction of the air on the body along this line .* [Essentially the motion can be considered in two different physical situations : the one in which the air is at rest, and the body M is introduced suddenly moving to the right along the line NQ, initially with the speed V, subsequently being slowed down by the air resistance to its terminal velocity ; and a complementary situation in which M is at rest initially and the air is moving past M along the line NO, with the constant speed V, which gradually accelerates M to this terminal velocity. In the first case, the air acts as a resistance to slow down the initial motion of the body; while in the second case, the air provides an accelerating force to be diminished as the body reaches the velocity of the air, or zero in the frame of the moving air.]



The moveable body [we will henceforth simply call this the body] M shall begin to move from N towards O with the given velocity AN, indeed thus, so that if it were to proceed in a vacuum, the motion would be perfectly uniform ; but, because it is carried in a medium with resistance, the motion of our body appears to vary continuously on the right line NO, thus so that the velocity may always become smaller and smaller. It is required to show that this varied motion to be the same as that, which arises from the motion of the common line NO travelling with the constant speed AN from N towards O, and with the mobile M dislodged from rest on this line, and the motion of the body M itself to carried along on the line NO, but with the motion of this carrying line clearly equal and opposite to that from N towards O [so that M is initially at rest], such as arises from the continual flying past of the air on the mobile M, and which (following the hypothesis) is assumed to be moving freely beyond the end of the carrying line, without any impediment taken into account of the common motion or the motion of the carrying line.

Demonstration. Because the carrying line NO is moved by a motion equal to that [of the body initially] towards Q, see Fig. 119, in as far as the moving body M is carried along with it in this context, but because the air is striking the mobile continually, and the body itself (following the hypothesis) shall itself begin to move freely on the carrying line NO , and because the air exerts the same force on the mobile body, (§422.) as if it were striking the body itself at rest with that same speed, by which the mobile body is carried in the air with an absolute motion ; therefore it is clear the body M itself is acted on by continuous impulses from the air towards O, the motion going entirely from N towards O, according to the motion of the carrying line itself, but by a motion contrary to its carrying line [in the sense of being less than] ; and it is agreed above that the motion of the moving body itself arises from the impulses of the air, to be decreasing the absolute motion of the body in the [moving] air, and thus decreasing the effect of the air resistance itself, as thus it shall be evident that the absolute motion of the body M shall be had in the air [*i.e.* at rest], if from the motion of the carrying line NQ the motion of the body M itself may be taken away, thus indeed, as if at a some certain exact time and with its own motion, the mobile body will be sent across the distance NE on the carrying line, and at its end E a change of the velocity DE will be acquired, and the absolute velocity of the body moving in the air (§.483.) shall become BD, clearly the excess of the speed BE, or AN, by which the carrying line will be moving, beyond the speed DE on the carrying line of the moving body acquired from the forces of the air resistance. [Thus, the speed lost in the left hand case with still air is the same as the speed gained in the right hand case with moving air.] Therefore the absolute motion and the variation of the speed of the body M in the air [at rest] will be the same as that, which arises from the motion of the common line NO, and from the motion of the mobile body itself on this carrying line. Q.E.D.

COROLLARY I.

488. Now the curve NDO shall be a graph of the speed acquired by the mobile M on the carrying line NO, and the curve PFO the graph of the force of the air exerted on the mobile M ; and the initial velocity of the mobile M , which was AN, will be reduced to the speed BD in that time, in which the mobile on the carrying line has moved through a distance NE. And thus generally by the preceding there will be had $\mathrm{EF.E}e = \mathrm{DE.}ad$. That is, *the moment of the action of the deceleration* EF *on the carrying line is equivalent to the moment of the* [decrease in the] *speed* ED *acquired on the same line.*

Therefore the time to pass along NE is equal to either $\int \mathrm{E}e : \mathrm{DE}$ or $\int ad : \mathrm{EF}$, (§.128.).

COROLLARY II.

489. Hence the absolute distance, which the body M resolves in this time $\int ad : \mathrm{EF}$, will be found generally to be : $\mathrm{AN}.\int da : \mathrm{EF} - \mathrm{NE}$. For the carrying line NO completes the distance $\mathrm{AN}.\int ad : \mathrm{EF}$, by its uniform motion and with the velocity AN, and the distance $\mathrm{AN}.\int ad : \mathrm{EF}$ is completed in the time $\int ad : \mathrm{EF}$, and the mobile M is moved along the distance NE on the carrying line in this time by the accelerating force. [Note that $\int ad : \mathrm{E} = \int dv : a = \int dt$ in modern terms.]

SCHOLIUM.

490. If the absolute resistance of the medium in all the points of the medium traversed may be assumed to be the same, the line PFO will be a right line parallel to NO or AO ; and the graph of the speed NDO will be a parabola. For the moment of the accelerating force EF in this case shall be $\mathrm{NP.E}e = \mathrm{DE}.ad$, and the sum of all the NP.E*e* that is NP.NE is equal the sum of all the DE.*da* or $\frac{1}{2}\mathrm{DE}^2$, hence $2\mathrm{NP.NE} = \mathrm{DE}^2$, hence NDO is a parabola, the parameter of which is 2NP.

Hence, because $t\mathrm{NE} = \int ad : \mathrm{EF} = \int ad : \mathrm{NP}$, there will be
$t\mathrm{NE} = \mathrm{DE} : \mathrm{NP} = \mathrm{DE}^2 : \mathrm{DE.NP} = 2\mathrm{NP.NE} : \mathrm{DE.NP} = 2\mathrm{NE} : \mathrm{DE}$. Hence $\mathrm{NA}.\int ad : \mathrm{EF}$ or $\mathrm{NA}.t\mathrm{NE} = 2\mathrm{AN.NE:DE}$, and the absolute distance, which the mobile traverses in the medium with this absolute resistance in the time 2NE:DE, will be

$$\mathrm{NA}.t\mathrm{NE} - \mathrm{NE} = 2\mathrm{AN.NE} : \mathrm{DE} - \mathrm{NE} = (2\mathrm{AN} - \mathrm{DE}).\mathrm{NE} : \mathrm{DE} = \left(2\mathrm{NA.DE} - \mathrm{DE}^2\right) : 2\mathrm{NP}.$$

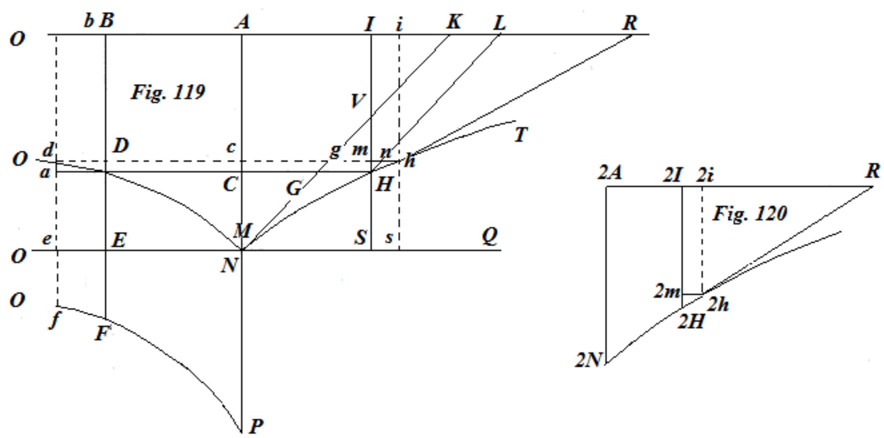
Therefore the speed remaining of the mobile, in the time elapsed DE:NP or 2.NE:DE, will be BD, as that now has been said above (§.488.). Therefore, in this hypothesis, the mobile M with the speed AN will be forced to return to rest in the time AN:NP. This account agrees with those, which the celebrated Varignon published in Probl. III. Coroll. I. *Act. Acad. Reg. Science*. 13th. Aug., 1707. by a different method.

Truly where the resistance of the medium is obtained with respect to the speed, and thus the resistance of the air depends on its density and on the velocities of the mobile, EF themselves always will have some relation with the line BD of the velocities of the mobile being represented in air, as will be made clear in the following.

PROPOSITION LVI. LEMMA.

491. *Showing some properties of the logarithmic curve to be used henceforth.*

I. The element of any ordinate of the logarithmic curve is to the ordinate itself as the element of the axis to the subtangent of the logarithmic.
For HI shall be some ordinate of the logarithmic curve NHT, of which the subtangent



shall be IR, and on putting H*m* to be an element of the ordinate HI, *hi* is acting parallel to HI, and *mh* parallel to the axis AR, with which in place, the similar triangles H*mh* and HIR produces $\mathrm{H}m : \mathrm{HI} = \mathrm{I}i : \mathrm{IR}$. Q.E.D.
[Now, $\mathrm{H}m : \mathrm{HI} = \mathrm{I}i : \mathrm{IR}$ is the differential equation defining the logarithmic curve, where $V = HI$ is the ordinate; $dV = Hm$; $dx = Ii$; and V/subtan.$=(dV / dx)$ gives $V/IR=(dV / dx)$; the subtangent is found from $V/(dV / dx)$; and applying these results , we have $\mathrm{H}m : \mathrm{HI} = \mathrm{I}i : \mathrm{IR} = dx : V / (dV / dx) = dV / V$.]

II. The rectangle HI*i*, under the ordinate HI and of the adjacent element I*i* of the axis of the ordinate HI, is equal everywhere to the rectangle under the subtangent IR and to the element H*m* of the ordinate HI. For the ratio of the preceding numbers $\mathrm{H}m : \mathrm{HI} = \mathrm{I}i : \mathrm{IR}$, gives rise to $\mathrm{HI.I}i = \mathrm{IR.H}m$ [or $Vdx = V / (dV / dx) \cdot dV$].

III. The third proportional to the subtangent and some ordinate of the logarithmic with the element of the adjacent axis makes a rectangle equal to the rectangle under its ordinate and its element. Let VI be the third proportional to IR and IH, and reconsidering the ratio of the first number $\mathrm{H}m : \mathrm{HI} = \mathrm{I}i : \mathrm{IR}$, or by transposing (following the hypothesis)$= \mathrm{IV} : \mathrm{HI}$. Therefore $\mathrm{VI.I}i = \mathrm{HI.H}m = VdV = \mathrm{VI.}dx$

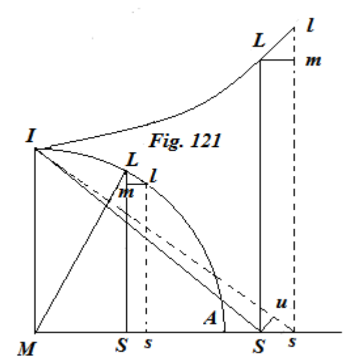
IV. If, with the two logarithmic curves NHT & 2N2H2T, the ordinate NA [Fig. 119 & 120] were to HI, just as 2N2A to 2H2I, AI will be to 2A2I as the subtangent IR to the subtangent 2I2R. For I*i* & 2I2*i* shall be the similar elements of the wholes AI & 2A2I, and there will be $\mathrm{HI} : hi = 2\mathrm{H}2i : 2h2i$; because these ratios are submultiples of the equal ratios following the same number of multiples ; therefore there will be $\mathrm{H}m:\mathrm{HI} = 2\mathrm{H}2m:2\mathrm{H}2\mathrm{I}$, and (no. I. of this) H*m*: HI=I*i*: IR , & $2\mathrm{H}2m:2\mathrm{H}2\mathrm{I} = 2\mathrm{I}2i:2\mathrm{I}2\mathrm{R}$, therefore $\mathrm{I}i:\mathrm{IR} = 2\mathrm{I}2i:2\mathrm{I}2\mathrm{R}$, and on interchanging $\mathrm{I}i:2\mathrm{I}2i = \mathrm{IR}:2\mathrm{I}2\mathrm{R}$; truly , because I*i* & 2I2*i* are similar small parts of the wholes AI, 2A2I, there will be $\mathrm{I}i:2\mathrm{I}2i = \mathrm{AI}:2\mathrm{A}2\mathrm{I}$, therefore also $\mathrm{AI}:2\mathrm{A}2\mathrm{I} = \mathrm{IR}:2\mathrm{I}2\mathrm{R}$. Q.E.D.

COROLLARY.

492. Hence the logarithm of the ratio of two magnitudes divided by the subtangent of the logarithm, from which the logarithm of the said ratio has been taken, is the same as for the logarithm of the same ratio taken from any other logarithms, but divided by the subtangent of this other logarithm. And hence it is apparent by what ratio the logarithms taken from some given logarithms can be transferred into the logarithms which are found in common tables of logarithms, and vice versa.
[It is appearent that Hermann considers velocity versus time curves for certain kinds of motion can be derived from the inverse of the graph of common logarithms; the exponential function or logarithmic curve on which such curves depend as indicated above, and indeed the concept of function itself had been indicated already by luminaries such as Huygens, Leibniz, and Varignon.]

PROPOSITION LVII. LEMMA.

493. *If , from the same centre* M, *and with the radius* MI *the quadrant of a circle* ILA *and the equilateral hyperbola* IL *shall be described*, *the ordinates of which* LS *refer to the axis* MS, *the rectangle* MS.S*s* *will be on the circle and on the hyperbola, under the abscissa and with the element of that, equal to the homologous rectangle* LS.L*m* or *lm* *under the ordinate of that element.* See Fig. 121.



On the circle. Join ML, the similar triangles MLS and L*lm* present $\mathrm{MS}:\mathrm{LS} = \mathrm{L}m:lm$ or S*s* ; and thus $\mathrm{MS}.\mathrm{S}s = \mathrm{LS}.\mathrm{L}m$. [The circle shall be $x^2 + y^2 = 1$ giving $xdx + ydy = 0$ and the hyperbola $y^2 - x^2 = 1$ giving $ydy - xdx = 0$, with origin at *M*.]

On the hyperbola. IS & I*s* are joined, and with the centre I and with the interval IS the arclet S*u* is understood to be drawn. Now from the nature of the hyperbola and $\mathrm{LS} = \mathrm{IS}$, & $ls = \mathrm{I}s$; therefore $us = lm$. And the similar triangles ISM & S*su* supply the ratio $\mathrm{S}s:su$ or $lm = \mathrm{IS}$ or $\mathrm{LS}:\mathrm{MS}$. Therefore $\mathrm{MS}.\,\mathrm{S}s = \mathrm{LS}.\,lm$ Q.E.D.

GENERAL SCHOLIUM.

494. Now with the general principles examined, from which whatever pertains to the motion of bodies in resisting media, can be deduced, and with the application of geometrical lemmas relating to small areas necessary ; it remains now to begin this application. And indeed in the first place, what motion arises concerned with the absolute

resistance ; with that in the above paragraph 490, as well also in the second section of Book I as far as has been useful, and indeed in the paragraph mentioned the various resulting motion is defined from the original uniform motion under the hypothesis of absolute resistance ; truly in the first chapter in the first sections of Book I §;§. 150,151 the motions are determined in the hypothesis of uniform gravity, to which case also absolute resistance can be placed, where there is the same amount of resistance with bodies passing through the individual points of the distance ; from which if the resistance shall be constant everywhere it will be removed from the constant gravity, the remaining action of the acceleration of gravity also will be constant, with which discussed in place shown for uniform gravity, these accelerations of this kind are to be applied. Therefore we will not tarry long over absolute resistance or from the motions thence arising. Respective resistances depend on the speed, with which the moving body advances in a medium of this kind. There are three particular hypothesis of this resistance, which we will run through one by one in the following chapters, then we will treat the analysis, by which motions will be determined generally in a variety of mediums of this kind.

SECTIO IV.

De Motibus corporum in mediis resistentibus.

Quoties Galilæus, Torricellius aliique illius ævi Celeberrimi geometræ de motu egerunt, corpora in vacuo, id est, in medio, quod nec motum accelerare nec retardare queat, ferri supposuerunt, non quod nescirent ejusmodi medium resistendi facultate carens apud tellurem nostram non dari, sed quod non satis viderent, quo pacto resistentiæ geometriæ legibus possent subjici. Ab his ergo resistentiis medii tanto libentius animum abstraxerunt, quod existimarent non nisi per-exiguos eas errores vixque sensibus perceptibiles determinationibus suis, ex suppositione vacui, inducere posse. Inter recentiores vero summi quique geometræ præclara Galilæi reperta circa motus doctrinam perfectisse & mirum quantum amplificasse non contenti, difficilem de motibus in mediis resistentibus materiam ad geometria, sed reconditioris, normam exigere fortunato successu aggressi & in regiones non antea cognitas delati sunt. Extant enim eximia hac de re Virorum Celeberrimorum *Newtoni, Leibnitii, Hugenii* & *Wallisii* meditata ante complures annos partim sine demonstrationibus edita, & breviter tantum indicata, partim etiam demonstrationibus munita, sed perbrevibus, nec ideo tyronum captui satis accommodatis, nisi Wallisium excipias, qui in suo schediasmate super hac re omnia minutim exponit, sed tantum in hypothesi particulari substitit resistentiarum in ratione celeritatum corporibus remoram afferentium. Post laudatos viros Cl. *Varignon* idem argumentum fuse, docte, atque perspicue pertractat in Actis Academ.Reg. Paris. Scient. annorum 1707, 1708, 1709 & 1710 adeo ut secuturis Mathematicis etiam hac in re fecisse videatur; verum, quia hæc materia tam directe ad institutum nostrum pertinet, ut si de ea agere omitterem, ex præcipuis aliquid opusculo meo deesse putarem, ideo non detrectabo post tot præclaros Autores etiam hanc doctrinam, quantum potero breviter simul & perspicue, proponere. Illorum inventis nonnulla propria addam, atque simplici principio insistam, cui ferme omnia, quæ ad motum actualem corporum attinent, superstruxi, scilicet *momentum cujuslibet solicitationis agentis æquivalere momento celeritatis.* Quo principio nemo ex laudatissimis viris usus est in doctrina motuum corporum in mediis resistentibus latorum, saltem quatenus ex publicis eorum scriptis judicare licet, etsi hoc principio multa brevius & naturalius, quam ab aliis fundamentis, obtineantur.

CAPUT XIV.

Complectens generalia, quæ ad theoriam motus corporum in mediis resistentibus pertinent, & nonnulla Lemmata Geometrica in hac theoria necessaria.

DEFINITIONES.

Duæ sunt resistentiæ species, una *absoluta* altera *respectiva.*

I.

477. *Resistantia absoluta* est, quæ tantundem, virium mobili detrahit, sive magna sive parva velocitate idem mobile feratur. Hujusmodi resistentiæ absolutæ exempla præbent media glutinosa & superficies asperæ : nam in fluido glutinosa eadem vi opus est ad separandas ejus partes, quacumque demum celentate mobile partes eas separare conetur. Sic etiam in frictionibus corporis in plano aspero incedentis eadem obstacula sunt superanda, sive magna sive parva velocitate mobile in plano aspero moveatur, & quocunque modo ipsa motus impedimenta se habeant, sive instar pilorum elasticorum deprimendorum, & postea sponte sua sese iterum erigentium ; sive filorum perrumpendorum ad instar, vel quoquo alio modo hisce æquivalente, ad hæc omnia definita quadam & determinata vi opus est, nec refert quæ sit agentis velocitas.

II.

478. *Resistantia* vero *respectiva* est, quæ provenit ab allisione medii fluidi partibus anterioribus mobilis. Hæc resisitentia pendet a densitate medii & celeritate mobilis; partes enim fluidi (§.422.) ea ipsa celeritate corpori in eo fluido delato allabi censendæ sunt, qua corpus hisce particulis impingit, & quantitates fluidi corporibus allabentis (§. 423) sunt in composita ratione densitatum & celeritatum.

III.

479. Medium fluidum atque resistens *aëris* nomine simpliciter insignietur, cum aër tantæ subtilitatis intelligi possit; ut eundem effectum præstare valeat, quem medium resistens.

IV.

480. *Motus primitivi* sunt illi, qui in vacuo fierent. Sic motus primitive uniformis est motus æquabilis, quo corpus vi quacunque impulsum in vacuo incederet. Et motus primitive accelerati vel retardati a gravitate uniformi, sunt motus illi, quos gravi in vacuo cadenti vel ascendenti convenire Galilæus demonstravit, cujusmodi motus Lib. I.Sect. II. Cap. I.~§.150, 151 consideravimus.

V.

481. *Solicitatio acceleratricis* gravi in aëre aescendenti applicata, est excessus, quo gravitas medii resistentiam superat, & *resistentia totalis* seu *absoluta* corpori in *aër*e ascendenti opposita, est aggregatum solicitationis gravitatis & resistentiæ aëris. Nam hæc duo corpori in aëre ascendenti conjunctim renituntur. Grave vero cadens non alia vi deorsum ruit, quam ea, qua gravitas resistentiam aëris, in quo corpus movetur, superat: propterea hic *excessus* solicitatio acceleratricis gravis in aëre decidentis audit.

VI.

482. Et, quia hæ solicitationes acceleratrices in aëre continue decrescunt ob crescentem cum velocitate mobilis resistentiam aëris, absque tamen eo, ut unquam, penitus extinguatur, ut in sequentibus fusius demonstrabitur, ideo etiam celeritas continue quidem crescet, non tamen certum quendam velocitatis gradum unquam attinget, nedum prætergredietur. Talis velocitas quam gravia nunquam cadendo assequi possunt, et si ei semper magis magisque accedunt, Hugenio *velocitas terminalis* audit, Leibnitius vero *celeritatem maximam* exclusive eandem appellat.

Scalae solicitationum acceleratricium, resistentiarum totalium,& celeritatum acquisitarum vel residuarum, eodem sensu sumuntur, quo in Sectione secunda Libri primi; adeo ut nulla ambiguitas vocabulorum in hac materia irrepere queat. Idem pariter intelligendum de momentis harum solicitationum aut resistentiarum totalium, & celeritatum crescentium aut decrescentium.

VII.

483. Si lineæ rectæ indefinitæ longitudinis, æquabili motu ferri subinde dicentur, atque in hisce lineis corpora libere etiam ultro citroque incedere ponentur, ipsæ lineæ mobiles lineæ *deferentes*, earum motus, *motus communis*, scilicet lineæ mobilis & corporis secum abrepti, & denique corporis in deferenti linea motus, ejus *motus proprius* deinceps dicentur, corporisque ejusdem *motus absolutus* est is, qui resultat ex motu communi & proprio, estque horum aggregatum, si linea deferens & in ea corpus proprio motu ad easdem partes feruntur differentia vero si in partes oppositas. Idcirco, ex motu proprio & communi corporis ejus motus absolutus semper innotescet.

PROPOSITIO LIV. LEMMA.

484. *Momentum cujuslibet folicitationis acceleratris corporis in aëre cadentis, aut resistentiæ totalis ejusdem corporis in aëre quomodocunque ascendentis, æquatur momento celeritatis acquisitæ crescentis, vel momento celeritatis residuæ & decrescentis.*

Haec propositio eadem est cum prop. XVII. Lib. I. §. 131. ut adeo nova demonstratione non indigeat, nam solicitationum nomine quæcunque vires mortuæ, sed continue applicatæ, intelligi queunt. Propterea, si facilitatis gratia corpus in linea verticali moveri ponatur, spatiaque transmissa dicantur x, gravitas g, resistentia aëris r, velocitas acquisita corpori decidenti vel residua ascendentis u ; solicitatio acceleratricis gravis cadentis (§.481.) erit $g-r$, & momentum hujus solicitationis $gdx-rdx$ æquatur momento velocitatis crescentis udu. Hinc $gdx-rdx=udu$.

Resistentia vero totalis corporis ascendentis (§. 481.) est $g+r$, momentumque hujus resistentiæ $gdx+rdx$ æquale est momento celeritatis decrescentis $-udu$, adeo ut habeatur $gdx+rdx=-udu$. Vel etiam sumendo spatia non actu percursa, sed deinceps percurrenda, usque ad totalem motus ascensionalis extinctionem, erit $-gdx - rdx = -udu$, seu $gdx + rdx = udu$, hoc enim pacto spatia absolvenda, post jam percursa, decrescent cum celeritatibus.

Idcirco utrumque casum descensus ascensusque verticalis corporum in aëre uni generali formulæ includendo, invenietur $gdx \mp rdx = udu$. Ubi signum superius − descensum, alterum vero + ascensum corporum in aëre resistente respicit; ubi meminisse oportet spatia *x* non esse spatia acu transmissa a corpore ascendente, sed eorum complementa ad totam seu maximam altitudinem, quam ascendens mobile percurrere potest.

COROLLARIUM.

485. Si jam elementum temporis, quo spatiolum *dx* transmittitur, dicatur *dt,* erit $dt = du : \mathrm{g} \mp r$. Nam, quia (§.128.) $dt = dx : u$, & (§.484.) $gdx \mp rdx = udu$, erit $dt\left(= dx : du\right) = du : g \mp r$. Adeoque $t = \int du : g \mp r$, ubi *t* significat tempus, quo grave motu a quietem incepto spatium *x* perlabitur, vel tempus quo grave ascendens complementum spatii actu jam transmissi ascendendo conficere posset; unde, si tempus, quo maxima altitudo A, quam grave data celeritate in altum projecti emetiri potest, dicatur T, quantitas $\mathrm{T} - t$ exponet tempus, quo mobile spatium ascendendo actu absolvit.

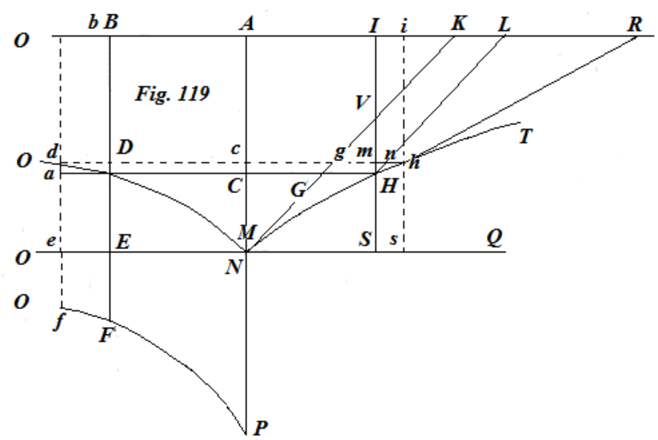
SCHOLION.

486. Cum nulla adhuc attentio habita sit ad originem resistentiæ medii, an pendeat a celeritatibus mobilis actualibus an vera aliunde, satis manifestum est, in præcedentibus canonibus generalibus contineri leges motuum variatorum pro quacunque resistentiæ medii absolutæ vel respectivæ hypothesi.

PROPOSITIO LV. LEMMA.

487. *Motus variati corporis in aëre quacunque lege resistente ex motu primitive uniformi sunt iidem, qui resultarent ex motu communi ac uniformi lineæ mobile secum deferentis, & ex motu proprio mobilis in linea deferente, orto ab allisione aëris corpori in linea deferente delato.*

Mobile M incipiat moveri ex N versus O velocitate data AN, ita quidem, ut si in vacuo incederet, motus perfecte æquabilis esset; sed, quia in medio resistente fertur, mobilis nostri motus in recta NO continue variatus existet, ita ut semper magis magisque



languescat. Probari debet hunc motum variatum eundem fore cum eo, qui resultaret ex motu communi lineæ NO celeritate AN ex N versus Q æquabili motu latæ, atque mobile M secum abripientis, & ex motu proprio mobilis M in deferenti linea NO, sed contrario motui æquabili lineæa deferentis, tendente scilicet ex N versus O, utpote orto ex continua allapsu aëris mobili M, quod (secundum hypothesin) ultra citroque in linea deferente liberrime moveri posse supponitur, absque ullo impedimento ratione motus communis seu motus lineæ deferentis.

Demonstratio. Quatenus linea deferens NO æquabili motu versus, Fig. 119. Q fertur,eatenus ea mobile M ipsi inhærens secum abripit, sed quia aër mobili isti continue alliditur, & corpus ipsum (secundum hypothesin) libere in deferenti linea NO moveri potest, aërque eandem in mobile impressionem exserit, (§422.) ac si in ipsum quiescens ea ipsa celeritate impingeret, qua mobile in *aër*e motu absoluto fertur ; ideo per se liquet corpus M ab aëre continue versus O impulsum omnino motum iri ex N versus O, motu proprio in linea deferente, sed contrario motui hujus lineæ deferentis; ac constat insuper motus proprios mobilis M ab impulsu aëris ortos, esse decrementa motus absoluti corporis in aëre, atque adeo effecta resistentiæ aëris, ut adeo hinc manifestum sit motum absolutum corporis M in aëre haberi, si a motu lineæ deferentis NQ subducatur motus proprius corporis M in hac deferente, adeo quidem, ut si, exacto quodam tempore atque motu proprio, mobile spatium NE in linea deferente transmiserit, & in ejus termino E gradum velocitatis DE acquisiverit, velocitas absoluta mobilis in aëre (§.483.) futura sit BD, excessus scilicet celeritatis BE vel AN, qua linea deferens movetur, supra celeritatem DE in linea deferente mobili acquisita ab aëris resistentis impressionibus. Propterea motus absolutus & variatus mobilis M in aëre idem erit cum eo, qui resultaret ex motu communi lineæ deferentis NO, & ex proprio mobilis in hae linea deferente. Quod erat demonstrandum.

COROLLARIUM I.

488. Sit jam curva NDO scala celeritatum mobilis M in linea deferente NO acquisitarum, & curva PFO scala impressionum aëris in mobile M exertarum ; & velocitas initialis mobilis M , quæ erat AN, reducta erit ad celeritatem BD eo tempore, quo mobile in linea deferente spatium NE transmittit. Adeoque per præcedenten habetur generaliter $\mathrm{EF.E}e = \mathrm{DE}.ad$. Id est *momentum solicitationis acceleratricis* EF *in linea defermte æquivalet momento ceteritatis* ED *in eadem linea deferente acquisitæ.*

Tempus vero per NE seu (§.128.) $\int \mathrm{E}e : \mathrm{DE} = \int ad : \mathrm{EF}$.

COROLLARIUM II.

489. Hinc spatium absolutum, quod mobile M hoc tempore $\int ad : \mathrm{EF}$ in aëre absolvit, reperietur generaliter $\mathrm{AN}.\int da : \mathrm{EF} - \mathrm{NE}$. Nam linea deferens NO absolvit motu suo æquabili & velocitate AN spatium $\mathrm{AN}.\int da : \mathrm{EF}$ tempore $\int da : \mathrm{EF}$, & mobile M in linea deference spatium NE.

SCHOLION.

490. Si resistentia medii absoluta in omnibus spatii percurrendi punctis eadem assumatur, linea PFO erit recta ipsi NO vel AO parallela; & scala celeritatum NDO parabola erit. Nam momentum vis acceleratricis EF fit hoc casu $\mathrm{NP}.\mathrm{E}e = \mathrm{DE}.ad$, & omnia NP.E*e* hoc est NP.NE æqualia omnibus DE.*da* seu $\frac{1}{2}\mathrm{DE}^2$, hinc $2\mathrm{NP}.\mathrm{NE} = \mathrm{DE}^2$, ergo NDO est parabola, cujus parameter est 2NP.
Hinc, quia $t\mathrm{NE} = \int ad : \mathrm{EF} = \int ad : \mathrm{NP}$, erit
$t\mathrm{NE} = \mathrm{DE} : \mathrm{NP} : \mathrm{DE}^2 : \mathrm{DE}.\mathrm{NP} = 2\mathrm{NP}.\mathrm{NE} : \mathrm{DE}.\mathrm{NP} = 2\mathrm{NE} : \mathrm{DE}$. Hinc
$\mathrm{NA}.\int ad : \mathrm{EF}$ seu $\mathrm{NA}.t\mathrm{NE} = 2\mathrm{AN}.\mathrm{NE}:\mathrm{DE}$, & spatium absolutum, quod mobile in medio hoc absolute resistente tempore 2NE:DE transmittit, erit
$\mathrm{NA}.t\mathrm{NE} - \mathrm{NE} = 2\mathrm{AN}.\mathrm{NE} : \mathrm{DE} - \mathrm{NE} = \left(2\mathrm{AN} - \mathrm{DE}\right).\mathrm{NE} : \mathrm{DE} = \left(2\mathrm{NA}.\mathrm{DE} - \mathrm{DE}^2\right) : 2\mathrm{NP}.$

Celeritas vero mobilis, elapso tempore DE:NP vel 2.NE:DE, residua erit BD, ut jam supra (§. 488.) dictum. Propterea in ista hypothesi mobile M celeritate AN impulsum tempore AN:NP ad quietem redigetur. Hæc probe consentiunt cum iis, quæ Cl. Varignon Probl.III. Coroll. I. Act. Acad. Reg. Scient. 1707. d. 13. Aug. diversa tamen methodo, tradidit.

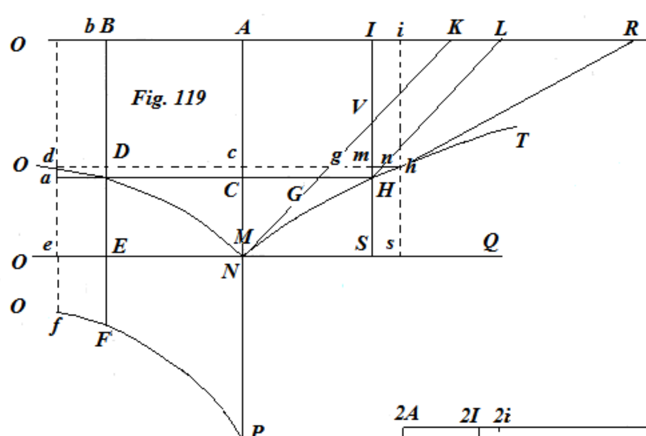
Ubi vero resistentia medii respectiva obtinet, atque adeo resistentiæ aëris ab ejus densitate & velocitatibus mobilis pendent, ipsæ EF semper aliquam relationem habebunt cum lineis BD velocitatum mobilis in aëre repræsentatricibus, ut ex sequentibus dilucide patebit.

PROPOSITIO LVI. LEMMA.

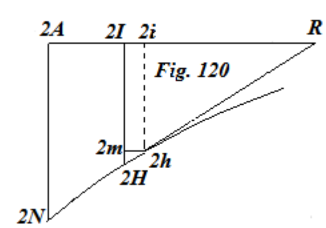
491. *Exhibens nonnnullas logarithmicæ proprietates deinceps adhibendas.*

I. Elementum cujusque ordinatæ logarithmicæ est ad ipsam ordinatam ut elementum axis ad subtangentem log-micæ.

Sit enim HI ordinata quæcunque log-micæ NHT,cujus subtangens sit IR, positoque H*m* elemento ordinatæ HI, agatur *hi* parallela HI, & *mh* parallela axi AR , quibus positis, triangula similia H*mh* & HIR præbent $\mathrm{H}m : \mathrm{HI} = \mathrm{I}i : \mathrm{IR}$. Quod erat demonstrandum.



II. Rectangulum HI*i*, sub ordinata HI & elemento axis I*i* ordinatæ HI adjacenti, æquatur ubique rectangulo sub subtangente IR & elemento H*m* ordinatæ HI. Nam analogia præcedentis numeri $\mathrm{H}m : \mathrm{HI} = \mathrm{I}i : \mathrm{IR}$, præbet $\mathrm{HI}.\mathrm{I}i = \mathrm{IR}.\mathrm{H}m.$



III. Tertia proportionalis ad subtangentem & quamlibet ordinatam log-micæ cum adjacente axis elemento efficit rectangulum æquale rectangulo sub ordinata ejusque elemento. Esto VI Tertia proportionalis ad IR & IH, ac resumatur analogia numeri primi $\mathrm{H}m:\mathrm{HI}=\mathrm{I}i:\mathrm{IR}$, vel permutando $\mathrm{H}m:\mathrm{I}i=\mathrm{HI}:\mathrm{IR}$ (secundum hypothesin) $=\mathrm{IV}:\mathrm{HI}$. Ergo $\mathrm{VI.I}i=\mathrm{HI.H}m$.

IV. Si in duabus logarithmicis NHT & 2N2H2T ordinata NA Fig. 119 & 120; fuerit ad HI, sicut 2N2A ad 2H2I, erit AI ad 2A2I sicut subtangens IR ad subtangentem 2I2R. Sint enim I*i* & 2I2*i* elementa similia totarum AI & 2A2I, eritque $\mathrm{HI}:hi=2\mathrm{H}2i:2h2i$; quoniam hæ rationes sunt submultiplices æqualium rationum secundum eundem multiplicitatis numerum; idcirco erit $\mathrm{H}m:\mathrm{HI}=2\mathrm{H}2m:2\mathrm{H}2\mathrm{I}$, atqui (num. I. huius) $\mathrm{H}m:\mathrm{HI}=\mathrm{I}i:\mathrm{IR}$, & $2\mathrm{H}2m:2\mathrm{H}2\mathrm{I}=2\mathrm{I}2i:2\mathrm{I}2\mathrm{R}$, ergo $\mathrm{I}i:\mathrm{IR}=2\mathrm{I}2i:2\mathrm{I}2\mathrm{R}$, & permutando $\mathrm{I}i:2\mathrm{I}2i=\mathrm{IR}:2\mathrm{I}2\mathrm{R}$; verum , quia I*i* & 2I2*i* sunt particulæ similes totarum AI, 2A2I, erit $\mathrm{I}i:2\mathrm{I}2i=\mathrm{AI}:2\mathrm{A}2\mathrm{I}$, ergo etiam $\mathrm{AI}:2\mathrm{A}2\mathrm{I}=\mathrm{IR}:2\mathrm{I}2\mathrm{R}$. Quod erat demonstrandum.

COROLLARIUM.

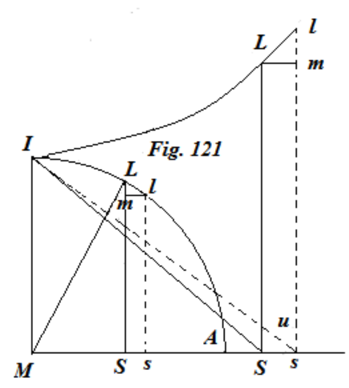
492. Hinc log-us rationis duarum magnitudinum divisus per subtangentem logarithmicæ, ex qua log-us dictæ rationis desumtus est, est idem cum log-mo ejusdem rationis desumto ex qualibet alia logarithmica, sed diviso per subtangentem hujus alterius logarithmicæ. Atque hinc liquet, qua ratione logarithmi desumti ex data quadam log-mica transferri queant in log-micam, quæ vulgares tabularum logarithmos continet, & vicissim.

PROPOSITIO LVII. LEMMA.

493. *Si ex eodem centro* M, *& semilatere transverso* MI *descriptus fit* Fig. 122. *sit quadrans circulu* ILA *& hyperbola æquilatera* IL, *quorum ordinatæ* LS *referantur ad axem* MS, *erit m* circulo & in *hyperbola rec-lum* MS. S*s, sub abscissa ejusque elemento, æquale homolo rec-lo* LS.L*m* vel l*m sub.ordinata ejusque elemento.*

In circulo. Juncta ML, triangula similia MLS & L*lm* præbent $\mathrm{MS}:\mathrm{LS}=\mathrm{L}m:lm$ vel S*s*; adeoque $\mathrm{MS}.Ss=\mathrm{LS.L}m$.

In hyperbola. Jungantur IS & I*s*, & centro I intervalloque IS descriptus intelligatur arculus S*u*. Jam ex natura hyperbolæ et $\mathrm{LS}=\mathrm{IS}$, & $ls=\mathrm{I}s$; ergo $us=lm$. Atqui triangula similia ISM & S*su* suppeditant analogiam $Ss:su$ vel $lm=\mathrm{IS}$ vel $\mathrm{LS}:\mathrm{MS}$. Ergo $\mathrm{MS}.\ \mathrm{S}s=\mathrm{LS}.lm$ Quod erat demonstrandum.



SCHOLION GENERALE.

494. Traditis jam generalibus principiis, ex quibus quicquid ad motus corporum in mediis resistentibus pertinet, deduci potest, & normullis lemmatis geometricis in applicatione principiorum necessariis; ad hanc applicationem jam veniendum restat. Et primum

quidem, quod motus ex resistentia absoluta concernit; id cum in paragrapho superiore 490, tum etiam in secunda sectione Libri primi quadantenus jam præstitum est, etenim in citato paragrapho definitus est motus variatus resultans ex primitive uniformi in hypothesi resistentiæ absolutæ; in Capite vero primo Sect. Sec. Libr. I. §;§. 150,151. determinati sunt motus gravium in hypothesi gravitatis uniformis, ad quem casum etiam revocari potest resistentia absoluta, qua in singulis percurrendi spatii punctis eadem quantitate corporibus resistitur; unde si constans ubique resistentia a constanti gravitate subducetur, residua solicitatio acceleratricis etiam constans erit, unde quæ citato loco respectu gravitatis uniformis demonstrata sunt, ea solicitationi ejusmodi acceleratrici etiam applicari possunt & debent. Idcirco non est quod resistentiæ absolutæ seu motibus inde orituris diutius immoremur. Resistentia medii respectiva pendet a velocitate, qua mobile in ejusmodi medio incedit. Tres sunt præcipuæ hujus resistentiæ hypotheses, quas in sequentibus capitibus primum sigillatim percurremus, deinde analysin trademus, qua motus in ejusmodi mediis variati generaliter determinabuntur.