CHAPTER XII*a*.

The resistance of figures moving in fluids.

DEFINITION.

434. Fig. 110. If CBABC were some figure, with the line CBABC composed from curves and straight lines, and terminated by the right line CC, and this figure may be carried in the fluid in the direction of the axis MA from A towards S; indeed from the posterior part on the adjacent base CC there shall be another figure CYC, of which the area taken by the curve CYC and the right line CC, shall be to the rectangle from the base for the same CC at CL or MO : which expresses the square of the speed, with which the figure falls into the fluid - shall be as the resistance which the figure CAC undergoes in the fluid, with its vertex A going along the right line MA, to the resistance, which the same figure endures with its base going first : or, which amounts to the same, as the resistance of the figure or rather of the curved line CBABC is to the resistance of the right line FF parallel to the base CC, and equal to the forces of the fluid received at right angles. The curved line CBABC is called the curve enduring the resistance [henceforth to be called the *resisting curve*], and the line CIYIC *the graph of the resistance of the resisting figure.*

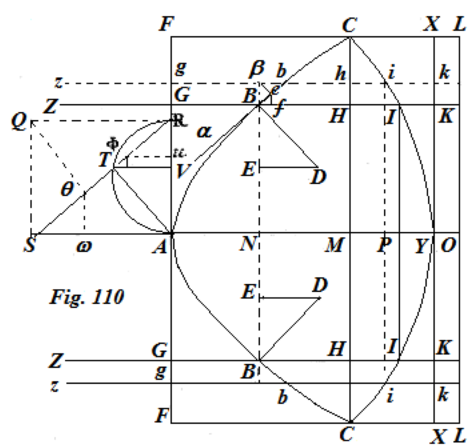

Fig. 110

PROPOSITION XLIV. THEOREM.

435. Fig. 110. *If, with the right line* AR *drawn through the end* R*, perpendicular to the axis* AM *of the resisting curve* CBABC, *and* RS *is parallel to the tangent* Bα *of the resisting curve at* B, *then from* A, *with* AT *drawn normal to* RS, *and through* T *the right line* TV *is drawn parallel to the axis* AM, *and* BH *is parallel to the same axis,* HI *is taken everywhere equal to the respective* RV, *and the points* I *will be on the graph* GIYIC *of the resistance endured by the figure* CBABC; *the resistance of the curve* CBABC *moving in the direction* MA *towards* S, *will be to the resistance of the right line* FF *normal to the axis* [*in turn moving towards S*], *as the two-fold curve* CIYIC *is to the rectangle* CCLL. *And the resistance of the surface of rotation from the rotation of the curve* CBA *about* AM *shall be to the resistance of the circle from the rotation of the right line* AR *about the same* AM, *as the volume from the rotation of the figure* CIYM *about* MY, *to the cylinder from the rotation of the rectangle* CMOL *about the same* MY *or* MO.
[Fig. 110 is multi-functional, and it is not obvious at first what the purpose of the lines and curves shall be.]
I. The curve ABC may be considered to be divided into its [differential] elements, through the ends of one of which B*b* the parallel lines ZK, *zk*, shall be acting parallel to

the axis AM, normal to the right line AF at the points *G, g; zb* itself crosses the ordinate BN at the point β, from which the perpendicular βe shall be sent to the element of the curve B*b*. In addition putting in place BD normal to the curve, ED parallel to the axis AM, with the lines QR, QS drawn through the points R and S parallel to the lines AS and AR, the perpendicular Qθ falling from Q onto RS, and $\theta\omega$ perpendicular to AS. Thus so that, if in addition, the line increment *ef* were drawn parallel to the ordinate BN, the figures $b\beta Bfe$ and RQS$\omega\theta$ will become similar, since the individual sides of the one have been put similar and parallel to the other.
[For ΔQRθ is similar to the incremental $\Delta\beta be$; ΔQSθ to $\Delta\beta$B*e* ; and ΔRSA to ΔeB*f*.]

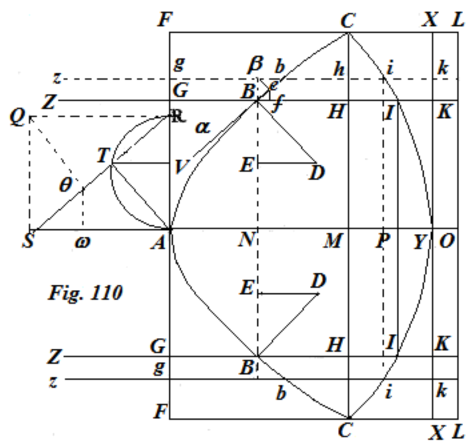
Fig. 110

II. And thus in the similar figures $b\beta Bfe$ and RQS$\omega\theta$ there will be $\mathrm{RA} : \theta\omega = \mathrm{B}\beta : fe$, and $\mathrm{RA}.fe = \theta\omega.\mathrm{B}\beta$. Truly, because the triangles QSθ and RAT are similar, and QS and RA are equal, there will be also $\mathrm{S}\theta = \mathrm{RT}$, as a consequence $\theta\omega = \mathrm{RV}$, also therefore $\theta\omega.\mathrm{B}\beta = \mathrm{RV}.\mathrm{B}\beta$ that is, by construction $= \mathrm{HI}.\mathrm{H}h$; and therefore $\mathrm{RA}.fe =$ the inscribed elemental rectangle IH*h* [*i.e.* the differential] for the area CIYIC.

III. Therefore, because the force of the impinging filament of fluid *ZBbz* on this element of the curve *Bb* along the [normal] direction BD is (§.429.) AR.B*e*, since AR expresses (following the hypothesis) the square of the speed of the moving fluid for the curve. Therefore the force, along BD, virtually contains the equivalent lateral forces themselves, along the directions BE and ED perpendicular and parallel to the axis AM, of which the forces perpendicular to the axis BE, as which are contrary and directly opposite, in as much as they may be derived from two elements placed similarly but on opposite parts B*b*, B*b* about the axis AM, mutually destroy each other, thus so that no account of these shall be had, but only the forces along the directions ED parallel to the axis may come to be examined. Truly the force along BD (§. 39) to the force along ED is as BD to ED, or, on account of the similar triangles EBD & *f*B*e*, just as *Be* to *ef*, that is, as RA.B*e* to RA. *fe* or (no. II of this) to the elemental rectangle IH*h*. And, as said at the start of this paragraph, the force of the fluid, along BD is RA.B*e;* therefore the force of the fluid on the element B*b*, along the direction parallel to the axis AM, is the elemental rectangle IH*h*. And thus the force, which all B*b*, or the whole curve CBABC endures, that is, the resistance of this curve is expressed by all the rectangles IH*h*, which are contained in the bilinear curve CIYIC ; and they are evanescent in that bilinear form; that is, the resistance of the curve CBABC, along the direction MA, for the motion in the fluid being expressed is the bilinear form CIYIC. Truly the force, which the line FF receives from the fluid at right angles, is expressed by the rectangle from FF by RA, or (from constr. $\mathrm{CL} = \mathrm{RA}$) by the rectangle CLLC. Therefore the resistance, which the curve CBABC will experience in the fluid, will be to the resistance of the line FF, as the area CIYIC to the rectangle CLLC. Which was the first part of the Proposition to be shown.

Concerning the second part of the proposition, since the force, which the element of the curve *Bb* endures from the fluid along the direction ED parallel to the axis, (no. III of this paragraph) may be shown by the elemental area IH*h*, and by the rotation of the plane CACY about the axis AY, the element of the curve B*b* shall describe an incremental conical zone, which element of the surface will appear equally to arise from the rotation of the curve CBA about the axis AM, indeed the elemental rectangle IH*h* by the same rotation will describe a cylindrical tube by the volume inscribed by the figure CIYM rotated about MY ; it is apparent generally the force of the fluid, which the conical regions will endure from B*b* around AM, in the direction parallel to the axis AM, must be expressed by the tube from the elemental rectangle IH*h* about MY. Therefore the resistance, which the whole surface will receive from the curve CBA about AM, from the striking fluid along AM, is expressed by all the cylindrical tubes from the elemental IH*h* rectangles around MY, that is, by the volume from the figure of revolution CIYM about MY, in which all quantities are evanescent. Indeed the force which the circle endures arising from the rotation of the line AF about AM, will be expressed by a cylinder, of which the base shall itself be a circle, receiving the force of the fluid at right angles, truly the height AR, CL, or MO, which express the square of the speed of the moving fluid; or also the cylinder from the rotation of the rectangle CMO about MO. And thus the resistance of the solid of rotation in the fluid CBABC is to the resistance of the circle FF, as the spherical volume CIYIC to the cylinder CLLC. Which was the second part of the proposition to be shown.

COROLLARY.

436. Since (from the constr.) there shall be
$\mathrm{RA}=\mathrm{HK}\ \ \&\ \ \mathrm{RV}=\mathrm{HI},\ \ \&\ \ \mathrm{AR}:\mathrm{RV}=\mathrm{RS}:\mathrm{RT}=\mathrm{RS}^2:\mathrm{RA}^2$, [Thus, the triangle BED is similar in turn to the triangles STA, ATR, and TVR; and :
$\mathrm{RA}:\mathrm{RT}=\mathrm{RS}:\mathrm{RA}$ gives $\mathrm{RS}:\mathrm{RT}=\mathrm{RS}^2:\mathrm{RA}^2$, and thus $\mathrm{RA}:\mathrm{RV}=\mathrm{RS}^2:\mathrm{RA}^2$] also there will be $\mathrm{HK}:\mathrm{HI}=\mathrm{RS}^2:\mathrm{RA}^2$, and on dividing [*i.e.* taking 1 from each ratio,]
$\mathrm{IK}:\mathrm{HI}=\mathrm{AS}^2:\mathrm{AR}^2$, therefore there will be [*i.e.* the gradient] $\mathrm{RA}:\mathrm{AS}=\sqrt{\mathrm{HI}}:\sqrt{\mathrm{KI}}$. Thus, from the two curves either from the given resisting curve, or from the graph of its resistance, the other will be known always, if not algebraic then perhaps transcendental; and certainly the graph of the resistance will be algebraic always, if the resisting curve were algebraic.

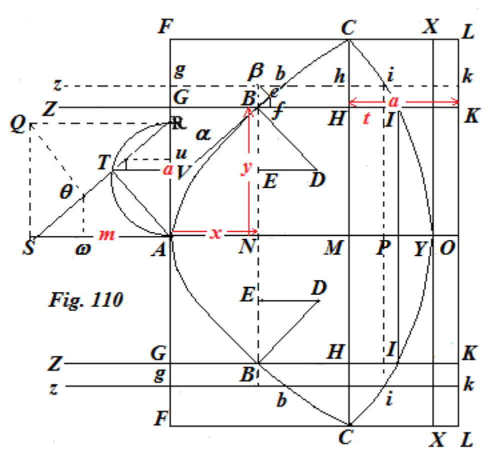

Fig. 110

SCHOLIUM.

437. Therefore, if on calling AN, *x*; BN or HM, *y;* HI, *t;* HK or RM, *a*; AS, *m* ; the analogous

terms of the preceding corollary $\mathrm{RA} : \mathrm{AS} = \sqrt{\mathrm{HI}} : \sqrt{\mathrm{KI}}$ will become

$$a : m = \sqrt{t} : \sqrt{a-t} \ \ \& \ \ t = a^3 : aa + mm.$$

I^{o}. Then also $m = a\sqrt{(a-t:t)}$.

II^{o}. And because the triangles $b\beta$B & ASR are similar, there will be also

$$dx = dy\sqrt{(a-t:t)} .$$

III^{o}. And finally also $dy = dx\sqrt{(t:a-t)}$.

IV^{o}. These latter can be elucidated from the above without any difficulty, merely by substituting the proportional elements *dy & dx* in place of *a & m.* Now with the aid of these four equations a variety of more general problems can be solved, several of which it pleases to propose. [The simplest of which two dimensional shapes is an isosceles triangle.]

PROPOSITION XLV. PROBLEM.

438. *To assign the proportion between the resistances, which the isosceles triangle* BER *will endure in the fluid, if as in the preceding, on the one hand the base* BR, *and on the other, the vertex* E *may be carried forward in the fluid in the direction* AE. Fig. 111. [Not all the points and line in the diagram are needed immediately.]

From the construction of the square AM on half the base AB, the perpendicular AF may be sent from A to the side of the triangle EB, and FG is acting parallel to the axis EA through the point F, and with AI or BH made equal to BG, the right line HI will be equal to the resistance of the line EB, and thus the resistance of the line EB to the resistance of the line BA will be as the rectangle BI to the rectangle BP, or as AI to AP; and thus in the square ratio of the line FB to BA, or to the sum of the sides BE and ER to the base BR of the isosceles triangle BER; and, as the resistance of the line FB to the resistance of the line BA, thus the resistance of the lines BE, RE likewise, or of the whole triangle, with its preceding vertex E, to the resistance of the base BR, therefore the ratio of these resistances will be equivalent to the square of the ratio of the sides BE, RE to the base BR. For, if BA shall represent the [resistance proportional to the] square of the speed of the triangle, BG or AI (§.435) will express the constant ordinate of the graph of the resistances of the triangle HI, or rather of the resisting right line BE. Which was to be shown and found. [Since the same force is exerted by the fluid on BE as on BA, but only the normal component FA contributes, etc, and the ratio BA : BE corresponds to the square of the speed on each side; from similar triangles, $\mathrm{BA} : \mathrm{BE} = \mathrm{BF}:\mathrm{BA}$. Note : the original text has EB rather than FB.]

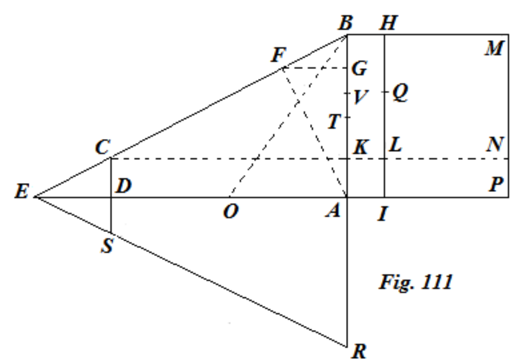

Fig. 111

COROLLARY I.

439. And thus the resistance of a square, carried along in the direction of a side in the fluid, will be to the resistance of the same, but in the direction of the diagonal advancing with an equal speed, as the diameter is to the side of the same square. For if the isosceles triangle BER may be considered to be right-angled at E, the resistance of the sides (§.438.) BE, ER along AE to the resistance of the base BR, which diameter is half of the square of this triangle BER, is as AI to AP, or as BF to BE; truly the resistance of the square of the diameter BR of advancing perpendicularly above AE, is to the resistance of the side of the square BE likewise carried forwards above AE, just as BR to BE or 2BA to BE, or 2BE to BR, that is BE to BA; therefore from the equation the resistance of the sides BER, or of the square, along the direction of the diagonal AE moving forwards in the fluid, to the square preceding with the vertex E, to the resistance of the side BE of the fluid, with the force received at right angles, is as BF to BA, or as the side BE to the diagonal BR, and thus by inverting, the resistance of the side BE above AE moved perpendicularly in the fluid, to the resistance of the square preceding along the diagonal, is as the diameter to the side.

COROLLARY II.

440. The resistance of the right cone BER along the direction of the axis AE carried into the fluid, yet with its vertex E preceding, will be to the same resistance, with its base BR preceding, again as AI to AP, or as BF to BE. For (§. 435.) the resistance in the first case is to the other, as the cylinder from the rectangle AH about AP to the cylinder from the square AM about the same AP, but the first cylinder is to the other as AI to AP, since the base of each cylinder shall be the same. Therefore, &c.

PROPOSITION XLVI. PROBLEM.

441. *To assign the resistances of the sections of a cone moving along the direction of the axis in a fluid.* Fig. 110.

If ABC shall be an *ellipse* or a *hyperbola,* the centre of which shall be at M ; and the transverse half-line AM, may be called *b;* the parameter *c;* the abscissas AN, *x* ; the semi-ordinate NB ,*y;* the equation of the ellipse and hyperbola will be

$$byy = 2bcx \mp cxx$$

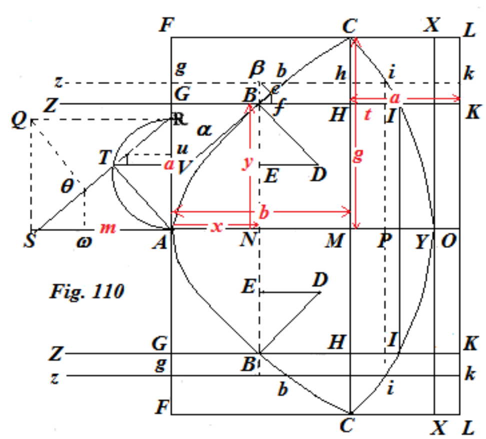

Fig. 110

[*i.e.* $byy=\mp c(x\mp b)^2\pm cb^2$; for the ellipse we have
$byy+c(x-b)^2=cb^2$ or $\frac{(x-b)^2}{b^2}+\frac{y^2}{bc}=1,$ and $\frac{(x-b)^2}{b^2}-\frac{y^2}{bc}=1$
for the hyperbola, in place of the modern standard forms. At
this time the focus: directrix property for conics was unknown.]

in the differential equation $bydy=bcdx\mp cxdx$, in place of the differentials *dy*, *dx* the proportionals of these *a* and *m* may be substituted, which are the magnitudes of the lines RA and AS , and $aby=bcm\mp cmx$, and thus
$m=aby:bc\mp cx,\&\ mm=aabbyy:bbcc\mp 2bccx+ccxx$ (or by substituting from the equation of the curve its value of $\mp 2bccx+ccxx,$ at $y)=aabbyy:bbcc\mp bcyy$ [from above: $bcyy=2bccx\mp ccxx$]; hence
$aa+mm=[aa(1+bbyy:bbcc\mp bcyy)]=(aabyy+aabcc\mp aacyy):bcc\mp cyy$; hence, because (§.437) $t=a^3:aa+mm,$ there will be $t=abcc\mp acyy:bcc+byy\mp cyy$ (or by putting *e* for $b\mp c$) $=abcc\mp acyy:bcc+eyy$, and this becomes the equation of the graph of the resistance CIY; in addition there may be put
$bcc=eff$, [to produce a known integrand], and the equation of the curve CIY will be
$t=aeff\mp acyy:eff+eyy=[\frac{aeff}{eff+eyy}\mp\frac{acyy}{eff+eyy}]=\mp ac:e;\pm acff+aeff:eff+eyy$ (and by substituting its value *b* in place of $\pm c+e$ into the numerator of the final fraction,)
$t=\mp ac:e;+abff:eff+eyy=[\frac{\mp\frac{ac}{e}+abff}{eff+eyy}]$.

Therefore the element of the arc MHIY, or the infinitesemal rectangle
IH$h=\mp acdy:e;+abffdy:eff+eyy=[\frac{\mp acdy}{e}+\frac{abffdy}{eff+eyy}=\frac{\mp acdy}{e}+\frac{abd\alpha}{e}]$; by calling the circular arc length α, of which the tangent is *y* and the radius *f*; since (§.166.) the element of this arc, or $d\alpha$, is $=ffdy:ff+yy;$[*i.e.* $d\alpha=\frac{ffdy}{ff+yy}$; see *below]. Therefore the area itself MHIY $=\left(ab\alpha\mp acy\right):e$. And thus, if the base CM is called *g*, and the arc described by the radius *f*, of which the tangent is *g* , will be called A, the whole area will be
MCIY $=(abA\mp acg):e.$ Truly the rectangle CO $=ag$.
Therefore the resistance, which the curve CBABC will undergo in the fluid, will be to the resistance of the line FF, just as the magnitude $abA:e$; $\mp acg:e$ to ag; [*i.e.* $\frac{abA}{e}\mp\frac{acg}{e}:ag$.] that is, just as $bA\mp cg$ to eg.

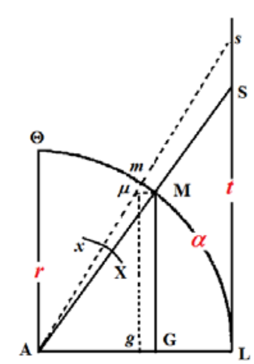

Fig. 38.

[At this time the function notation was still unknown, so the arc A is written down rather than making use of the inverse tangent function, and the integration has been performed without the use of an integral sign with limits.]
Q.E.I.

[*COROLLARY §166. from Book I, CH. 2.
Therefore, if the following may be designated : the radius AL, r ; the tangent LS , t; the arc LM, α, the secant will be $\mathrm{AS}=\sqrt{(rr+tt)}$, and thus $d\alpha = rrdt : rr+tt$.
And conversely from this equation converted into a series, the individual members of the series will be summable, thus so that from the sum of these an expression can actually be had of the arc of the circle α, for an infinite series expressed in terms of the tangent and radius.
[For $\tan\alpha=\frac{t}{r};\ \sec^2\alpha d\alpha=\frac{dt}{r}; d\alpha=\frac{dt}{r}\times\frac{rr}{tt+rr}=\frac{rdt}{tt+rr}$, where here α is the angle in radians; thus the associated arc length is $\frac{rrdt}{tt+rr}$.]

This indeed is the ratio of the resistances, which the Celebrated James Bernoulli stated finally in so many words without demonstration in the Act. Erudit. Lips. 1693. pag. 253, art. 5. He obtained this proportion of such a magnitude, with the conic section to be moving along the direction of the major axis, and with regard to the upper sign for the ellipse, and the lower for the hyperbola.

COROLLARY I.

442. If b shall be infinite in comparison with the parameter c, there becomes $e=b\mp c=b$, and thus the ratio $b\mathrm{A}\mp cg$ to eg , will be the same, as bA to bg or A to g. From which since the transverse width of the ellipse is infinite, the curve will be changed into a *parabola*, it is evident of a parabola moving in the fluid along the direction of the axis, by proceeding in the manner with the vertex followed by the base, to be resisted in the ratio of the arc of the circle to the semi-parameter of the parabola described, of which the tangent of the arc is equal to half the base of the parabola, to half this base.

COROLLARY II.

443. But if indeed ABC were *a circle*, the ratio $bA\mp cg$ to eg will be changed into a ratio with its terms have been given algebraically, which generally is worthy of mention, as may be shown here. Because the circle is an ellipse, of which the latus rectum is equal to the parameter, in this case there shall be $b=c$, and thus $b-c=e=0$; and in this case the ratio $b\mathrm{A}-cg$ to eg is required to be used, or $\frac{b\mathrm{A}}{e}-\frac{cg}{e}$ to g , truly $-\frac{cg}{e}=-cg:b-c=g-\frac{bg}{b-c}=g-\frac{bg}{c}$, the ratio $\frac{b\mathrm{A}}{e}-\frac{cg}{e}$ to g will be equal to the ratio $\frac{b\mathrm{A}}{e}+g-\frac{bg}{e}$ to g . And, following paragraph 166, there is

$$\mathrm{A}=g-\frac{g^3}{3ff}+\frac{g^5}{5f^4}-\&\mathrm{c.};$$

[recall that $\tan^{-1}z=z-\frac{1}{3}z^3+\frac{1}{5}z^5-\frac{1}{7}z^7+....\mathrm{etc.}$ for the angle z in radians; here Hermann considers α to be the arc length, hence $d\alpha/r=rdt:rr+tt$; this can be converted into the series

$d\alpha = \frac{rrdt}{rr+tt} = \frac{dt}{\left(1+\frac{t^2}{r^2}\right)} = dt\left(1 - \frac{t^2}{r^2} + \frac{t^4}{r^4} - \frac{t^6}{r^6} + ...\right)$; on integrating, this becomes:

$\alpha = t - \frac{t^3}{3r^2} + \frac{t^5}{5r^4} - \frac{t^7}{7r^6} + ...]$

therefore $\frac{b\mathrm{A}}{e} + g - \frac{bg}{e} = g - \frac{bg^3}{3eff} + \frac{bg^5}{5f^4e}$ or (because $bcc = eff$)

it becomes also $= g - \frac{bg^3}{3bcc} + \frac{bg^5}{5bccff} - \text{etc.} = g - \frac{g^3}{3cc} + \frac{g^5}{5ccff} - \text{etc.}$;
or, (because ff shall be infinite with respect to *cc* or *gg*, and thus the fraction $g^5 : 5ccff$ is indefinitely small or evanescent) $= g - \frac{g^3}{3cc}$. Therefore the resistance of the segment of the circle CBABC to the resistance of its base, or equal to its right line FF, as $g; -g^3 : 3cc$, to g , that is, as $cc - \frac{1}{3}gg$ to *cc*, or $bb - \frac{1}{3}gg$ to *bb*, or, what returns to the same, as the square of the diameter, with one third of the square of the base segment removed, to the square of the diameter ; in short, as the Cel. Bernoulli found in the place mentioned above (§.141.)

SCHOLIUM.

444. The solid of rotation CIYIC is to the cylinder CLLC just as all the MH.HIH*h* to $\frac{1}{2}\mathrm{MC}^2.\mathrm{CL}$, or as $\int tydy$ to $\frac{1}{2}ayy$. And since *t* shall be (§.441.) $= \mp ac : e; +abff : eff + eyy = a - \frac{ab}{e} + abff : eff + eyy$, there will be $tydy = aydy; -abydy : e\ ; +abffydy : eff + eyy;$ therefore

$$\int tydy = \frac{1}{2}ay^2; -abyy : 2e\ ; + \frac{abff}{e}.\log\sqrt{(ff + yy : ff)}.$$

For the integral of $ydy : ff + yy$ is $\log.\sqrt{(ff + yy : ff)}$. Therefore the solid of rotation from MHIY about MY, to the cylinder HKKH, or the ratio resistance of the solid BAB to the resistance of the base BB, by proceeding first with the vertex A at one time, at another time of the base BB in the direction MA, is as
$\frac{1}{2}ay^2; -abyy : e\ ; + \frac{abff}{e}.\log\sqrt{(ff + yy : ff)}$ to $\frac{1}{2}ayy$, or as
$yy; -byy : e\ ; + \frac{2bff}{e}.\log\sqrt{(ff + yy : ff)}$ to *yy ;* or also finally, as
$\frac{2bff}{e}.\log.\sqrt{(ff + yy : ff)} \mp \frac{cyy}{e}$ to yy. Where again the upper sign considers the ellipse, truly the lower the hyperbola.

Therefore the conic paraboloid, carried in the direction of its axis is resisted, by proceeding with the vertex in one way and with the base in another, in the ratio $2ff.\log.\sqrt{(ff + yy : ff)}$ to *yy*, or $2cc.\log.\sqrt{(cc + yy : cc)}$ to *yy*.

445. For the segment of a sphere is resisted the same in these directions as in the preceding circumstances; in the ratio $cc-\frac{1}{2}yy$ to *cc;* and thus the resistance of a hemisphere, carried along the direction of the axis MA, will be only half the resistance of the line FF.

For in the sphere there shall be $b=c\,\&\,e=0$, but $\log.\sqrt{(ff+yy:ff)}=\frac{yy}{2ff}-\frac{y^4}{4f^4}+\&c.$ hence the ratio $yy;-byy:e;+\frac{2bff}{e}.\log.\sqrt{(ff+yy:ff)}$ to yy will be changed into the ratio $yy-\frac{by^4}{2eff}$ to *yy ;* or $2eff-byy$ to $2eff$ (or because $eff=bcc$), into

$$2bcc-byy \text{ to } 2bcc=2cc-yy \text{ to } 2cc=cc-\tfrac{1}{2}yy \text{ to } cc.$$

PROPOSITION XLVII. PROBLEM.

446. *To find the resisting curve* ABC, *which shall be its own graph of the resistance.*

Therefore it is necessary for the curves CAC and CYC to be similar and equal, thus so that $\mathrm{BH}=\mathrm{HI}$, and $\mathrm{AM}=\mathrm{MY}$, and so that $\mathrm{AN}=\mathrm{PY}$. And by putting, as in the preceding, RS parallel to the element of the curve sought *Bb,* there will be $\mathrm{RV}=\mathrm{HI}=\mathrm{BH}$, & $\mathrm{R}u=hi=bh$, and thus $\beta b=\mathrm{V}u$, and with the right line $u\sigma$ drawn through *u*, crossing RS at the point σ, and through this the line element $\sigma\pi$ is drawn parallel to RA; and upon the diameter RA the described circle will be pass through the point T, since the angle RTA (constr.) shall be right; with which in place, and since now it shall be said $\mathrm{V}u=\sigma\pi$ to equal βb, there will be $\mathrm{T}\pi=\mathrm{B}\beta=\mathrm{H}h$; and with $u\sigma$ produced as far as to the crossing with the semicircle ϕ, from this point $\varphi\rho$ is acting parallel RA ; $\mathrm{T}\pi=T\rho+\rho\pi=T\rho+\varphi\sigma$, or, as it will be shown elsewhere (§. 463, no. III.) $=T\rho+T\varphi$ from which all $\mathrm{T}\pi$ or $\mathrm{B}\beta$ that is the ordinate $\mathrm{N}=$ all $\mathrm{T}\rho$, or to the ordinate $\mathrm{TV}+$ all the arclets $T\varphi$, or to the arc AT, therefore the ordinate of the curve sought BN is the sum of the arc *AT* and of its sine, and thus the curve ABC is the ordinary cycloid.

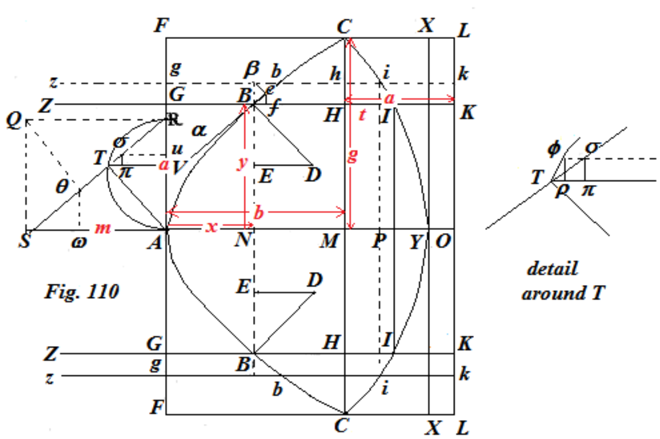

Fig. 110

Q.E.I.

Hence the resistance of the cycloid CBABC will be to the resistance of its base, as the area of the cycloid to the circumscribed rectangle, and thus as 3 to 4.

PROPOSITION XLVIII: PROBLEM.

447. *From all the conic frustums* BCSR *on the same given base* BR *and of the same height* AD, *to find the vertex* E *of these, for which it may be resisted minimally, if it may advance in the direction of the axis* AD *in a fluid with a certain given speed.*

With the square AM made upon AB, from A above EB the perpendicular AF may be dropped, and FG may be drawn parallel to the axis of the cone EA, and making $BH = BG$, HI shall be parallel to BA, and (§.435) HI will be the graph of the resistance of the triangle BER, and the cylinder from the rotation of the rectangle AH about AI expresses the resistance of the cone BER in the fluid, and the cylinder from AL the resistance of the cone CES , and thus the annulus from the rotation of the rectangle KH about AP expresses the resistance of the truncated cone BCSR, only with the resistance which the circle CS endures in the fluid, of which it takes the forces at right angles, because the resistance of the circle CS is expressed by the cylinder from AN around AP. Therefore the resistance of the truncated cone BCSR must be expressed from the two solids, evidently from the annulus KH about AP, and from the cylinder from AN about the same, and (following the hypothesis) these two solids must effect a certain *minimum*. V shall be the mid-point of BK, and VA will be the distance of the centre of gravity of the rectangle BL from the axis AP, and thus *p*.AV will express the circumference, which the centre of gravity describes by one rectangle rotated about *p*A, where *p* indicates the constant ratio established of the circumference of the circle to the radius. From which with the rotated solid (§. 47) there may be found from the drawing of the figure of rotation about some axis in the path of its centre of gravity, made from the rectangle BL by *p*.AY, that is *p*.AV.CK.BH will be equal to the annulus from the rectangle BL about AP, truly the circumference, which the centre of gravity of the rectangle AN will describe, will be $\frac{1}{2}p.\text{AK}$, and thus $\frac{1}{2}p.\text{AK}^2.\text{NK}$ will be the value of the cylinder from AN about AP. And thus $p.\text{AK.BK.BH} + \frac{1}{2}p.\text{AK}^2.\text{NK}$ expresses the resistance which the truncated cone BCSR will undergo in the fluid, and therefore, it must effect some *minimum.* Or also the annulus from the rectangle LM must be by being rotated about AP a *maximum* in its kind, since it is the complement of the aforementioned volume from KH and AN to the given cylinder from the square AM rotated about AP. And thus because $p.\text{QI.HL.HM}$ expresses that solid annulus from LM, it must be a *maximum.* Now, if there may be called AB, *a;* AD, 2*b*, or AO and OD, and each one separately *b*; AE, *x;* by an easy calculation the solid will be found $p.\text{QI. HL.HM} = (x-b).2a^3bp : aa + xx$, from which with the constant quantity omitted in the numerator, it is multiplied by the indeterminate $x-b$, $(x-b) : aa + xx = \text{maximum}$. Hence $\log.(x-b) - \log.(aa+xx) = \text{constructed curve}$, and thus $dx : x-b = 2xdx : aa + xx$, hence

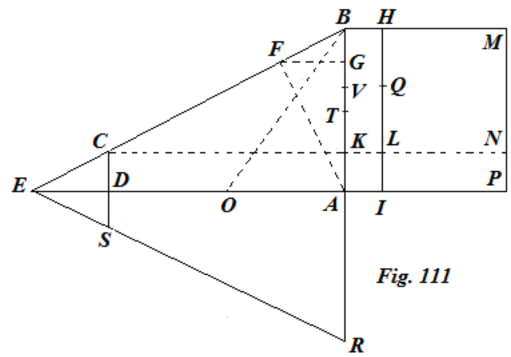

Fig. 111

$aa + xx = 2xx - 2bx$, that is, $xx - 2bx = aa$, or $xx - 2bx + bb = aa + bb$, or by extracting the root $x - b = \sqrt{(aa + bb)}$, or $x = b + \sqrt{(aa + bb)}$. Which supplies the construction of that very equation, which the Cel. Newton introduced without analysis and demonstration in a Scholium after Prop. 35. Bk. II. *Princ. Phil. Nat. Math.* which is given thus: from the mid-point O of the height AD of a truncated cone OB may be considered, to which OE shall be equal, and E will be the vertex of the cone sought BER.

PROPOSITION XLIX. PROBLEM.

448. *With the position given for a right line* CI, *and with two points A and B given, to find on this right line with the given position a third point* C, *so that the lines drawn from* AC, CB *to the given point with the right lines* AD, BF *normal to* CI, *and with* DF *normal to the parallel lines, form the rectilinear figure* DACBF, *which in a rotated motion around the axis* DF *produces a solid, whose surface described on the lines* AD, BF, *may suffer a* minimum *in the fluid resistance, with circles excluded, with the solid preceding along the direction of the axis* FD. Fig. 112.

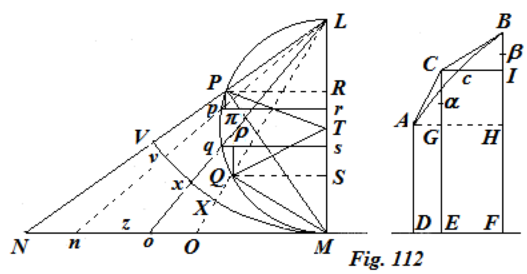

Fig. 112

Analysis. The given points A, B may be put to be equidistant from the right line CI, so that CG and BI may be equal. From the midpoint T of the given line LM (setting out the square of the speed, with which the fluid is accustomed to be forced against the solid), so that from the centre the semicircle LPM may be described, which the line MO adjoined to the other line DF along the same direction at M, and thus LM will be parallel to AD and BF themselves. Through L, LO and LN are acting parallel to the lines AC and BC, cutting the semicircle at P and Q, through which points PR and QS will pass, parallel to MN; the right lines MP and MQ will be perpendicular to PL and QL themselves. To the point C, another *c* indefinitely near may be understood, then also the right lines *c*A, *c*B shall be drawn, and from these the parallel lines L*o* and L*n*, cutting the semicircle at the points *q* and *p*, through which equally the ordinates *qs* and *pr* may pass, and with Pπ and Qρ drawn parallel diameter LM, $p\pi$ & $q\rho$ will be [differential] elements of the ordinates PR and QS. Again *p* sets out the ratio of the circumference of the circle to the radius, and with CG bisected at α, and BI at β, p.Eα will denote the circumference of the radius Eα, and p.Fβ the circumference of the radius Fβ, and thus the annuli generated by the lines CG, BI about DF by rotation, will be p.Eα.CG and p.Fβ.BI .

II. Now by means of those, which have been shown in the preceding (§. 435.), the resistance, which the conic annulus undergoes in the fluid from the right line AC described in a circle about DF, is put in place by $p.\mathrm{E}\alpha.\mathrm{CG.LS}$, and thus the resistance of the annulus from BC about DF $p.\mathrm{F}\beta.\mathrm{BI.LR}$, and thus the resistance of the annulus AC + the resistance of the annulus BC $= p.\mathrm{E}\alpha.\mathrm{CG.LS} + p.\mathrm{F}\beta.\mathrm{BI.LR}$. Similarly the resistance of the annulus from A*c* + the resistance of the annulus B*c* $= p.\mathrm{E}\alpha.\mathrm{CG.LS} + p.\mathrm{F}\beta.\mathrm{BI.L}r$. Now, as from the nature of the *minimum* resistance of the annulus AC + the resistance of the annulus BC = resistance of the annulus A*c* + resistance of the annulus B*c*, also there will be $p.\mathrm{E}\alpha.\mathrm{CG.LS} + p.\mathrm{F}\beta.\mathrm{BI.LR} = p.\mathrm{E}\alpha.\mathrm{CG.LS} + p.\mathrm{F}\beta.\mathrm{BI.L}s$, or since (following the hypothesis) CG and BI are equal, on dividing by *p*.CG or *p*.BI; there will be $\mathrm{E}\alpha.\mathrm{LS} + \mathrm{F}\beta.\mathrm{LR} = \mathrm{E}\alpha.\mathrm{L}s + \mathrm{F}\beta.\mathrm{L}r$, and thus $\mathrm{E}\alpha.\mathrm{S}s = \mathrm{F}\beta.\mathrm{R}r$, or $\mathrm{S}s : \mathrm{R}r = \mathrm{F}\beta : \mathrm{E}\alpha$.

III. On account of the parallel lines LO, AC, and L*o*, A*c*; there will be

$$\mathrm{O}o : \mathrm{LM} = c\mathrm{C} : \mathrm{C}G, \text{ or } \mathrm{O}o : \mathrm{C}c = LM : CG \text{ and } \mathrm{N}n : \mathrm{C}c = \mathrm{LM} : \mathrm{BI} \text{ or CG},$$

therefore, $\mathrm{O}o = \mathrm{N}n$. And $\mathrm{N}n : \mathrm{P}p$ shall be as triangle NL*n* to 2.triangle PT*p*, clearly of which the base is P*p*, truly with the altitude *p*T*;* for it is the double of this triangle PT*p* ; of which the base also is P*p*, but with the height twice that of *p*T, or equal to the diameter LM ; and thus of the same altitude as triangle LN*n*: and the angle $\mathrm{NL}n =$ angle (in the manner of the named triangle) to the vertex on the circumference of the other semicircle not expressed in the figure ; therefore $\mathrm{N}n : \mathrm{P}p = \mathrm{NL}^2 : \mathrm{LM}^2 = \mathrm{NL} : \mathrm{PL} = \mathrm{LM} : \mathrm{LR}.$ By the same argument there is $\mathrm{O}o : \mathrm{Q}q = \mathrm{LM} : \mathrm{LS}$, or on inverting $\mathrm{Q}q : \mathrm{O}o$ or $\mathrm{N}n = \mathrm{LS} : \mathrm{LM}$; therefore from the equation, $\mathrm{Q}q : \mathrm{P}p = \mathrm{LS} : \mathrm{LR}$. But on account of the similar triangles Q*qρ* & TQS , there is *Ss* or $\mathrm{Q}\rho : \mathrm{Q}q = \mathrm{QS} : \mathrm{QT}$, therefore from the equation, $Ss : \mathrm{P}p = \mathrm{LS.\,QS} : \mathrm{LR.QT}$. Likewise on account of the similar triangles $\mathrm{P}p\pi$, TPR there is $\mathrm{P}p : \mathrm{P}\pi$ or $\mathrm{R}r = \mathrm{PT}$, or $\mathrm{QT} : \mathrm{PR} = \mathrm{LR.QT} : \mathrm{LR.PR}$, therefore again from the equation, there shall be $\mathrm{S}s : \mathrm{R}r = \mathrm{LS.QS} : \mathrm{LR.PR}$; and above (no. II at the end) we find $\mathrm{S}s.\mathrm{R}r = \mathrm{F}\beta : \mathrm{E}\alpha$ therefore $\mathrm{F}\beta : \mathrm{E}\alpha = \mathrm{LS.QS} : \mathrm{LR.PR},$ and $\mathrm{E}\alpha.\mathrm{LS.QS} = \mathrm{F}\beta.\mathrm{LR.PR}.$

IV. Therefore, on calling LM, a; $\mathrm{E}\alpha, e$; $\mathrm{F}\beta, f$ and MO, m*;* the fourth proportional MZ to the given BH, HA & LM may be called *g*, and this *g* or MZ will be taken to be the arithmetic mean between the unknowns MO and MN, from which, since MO shall be *m*, the other MN will be found to be $= 2\mathrm{g} - m.$

And with these values substituted into the proceedings a little before finding $\mathrm{E}\alpha.\mathrm{LS.QS} = \mathrm{F}\beta.\mathrm{LR.PR}$, an equation of the fifth dimension will be found, which, with the calculation set out, will be as follows :

$$\begin{array}{llllll}
+em^5 & -8eg.\,m^4 & +24egg.m^3 & -8.aage.mm & +a^4e.m & -2a^4 fg \\
+f & -2fg & +2aaf & -4aafg & +a^4 f & \\
 & & +2aae & -32g^3e & +16g^4e & \\
 & & & & -8aaegg = 0. &
\end{array}$$

The roots of which determine the values sought MO ; with which found and with OL drawn, if AC may be drawn through the given point A ; parallel to OL itself, which AC meets the line CI; for that occurs at the optimum point C. Q.E.I.

An equation of the fifth dimension may also emerge, even if we may have assumed especially the lines CG and BI to be unequal.

COROLLARY I.

449. Now the solution of the problem for the solid with the minimum resistance is contained in the workings found above $\mathrm{L}\alpha.\mathrm{LS.QS} = \mathrm{F}\beta.\mathrm{LR.PR}$. For in this case CG and BI and likewise AG and CI are supposed to be required to be indefinitely small, from which it arises, that αE & AD, likewise βF & CE must be treated as equal and with all the solids $\mathrm{L}\alpha.\mathrm{LS.QS};\ \mathrm{F}\beta.\mathrm{LR.PR}$ &c. clearly considering the individual elements of the curve AC, CB, &c., shall be equal, each one of these shall be equal to the given cube LM^3 , and thus there will be $\mathrm{L}\alpha.\mathrm{LS.QS} = \mathrm{LM}^3$, or by calling Eα or DA, y ; CG, dy ; AG, dx; and as before LM, a and finally MO, m; $a^5 my : (aa+mm)^2 = a^3$, hence $y = (aa+mm)^2 : aam = aa : m\ ;+2m\ ;+\ m^3 : aa, \&\ dy = -\ aadm : mm\ ;+2dm\ ;+3mmdm : aa,$ and due to the similarity of the triangles CGA and LMO, there is $mdy = adx$, thus also there will be found $dx = -adm : m\ ;+2mdm : a\ ;+3m^2dm : a^3$; therefore $x = mm : a\ ;+\ 3m^4 : 4a^3 - lm$, where lm signifies the logarithm of the indeterminate m in the logarithms, of which the subtangent is a. And this solution agrees with that according to the precision, which the Cel. Joh. Bernoulli established in the Act. Erud. Lips. 1699. p.515, & which he explained more fully in the Actis of the following year.

COROLLARY II.

450. In the equation $y = (aa+mm)^2 : aam,$ or $y : a = (aa+mm)^2 : a^3 m$, if in place of $aa+mm,\ a,$ and m the differentials $ds^2,\ dy,$ and dx , may be substituted, which are proportional to these, there will come about $y : a = ds^4 : dxdy^3;$ and thus $ads^4 = ydxdy^3,$ which is the differential equation of the curve sought in which the most praiseworthy Bernoulli happened to mention in the place mentioned, and from that he elicited the later expressions of the coordinates found in the previous corollary. And with all these the illustrious Marquis de L' Hôpital had a solution also, agreeing with that, which he had seen fit to publish in the *Actis* Lips. and in the Commentaries of the Royal Academy of Science for the year 1699 .

Until now we have considered only these resistances which bodies carried in fluids undergo, when they are moved along the direction of the axis. But if they may advance in other directions, the matter is resolved with a little more investigation, as will be abundantly clear from the following elegant problem .

CAPUT XII.

De Resistentiis figurarum in fluidis motarum.

DEFINITIO.

434. Fig. 110. Si fuerit figura quæcunque CBABC, linea CBABC ex curvis vel rectis composita, & recta CC terminata, & hæc figura in directione axis MA in fluido feratur ex A versus S; à postica parte vero super basi CC apposita sit alia figura CYC, cujus area curva CYC & recta CC comprehensa, sit ad rec-lum ex basi eadem CC in CL vel MO, quæ quadratum celeritatis exponit, qua figura in fluida incedit, sicut resistentia quam patitur figura CAC in fluido, præeunte ejus vertice A in recta MA, ad resistentiam, quam eadem figura subiret præeunte ejus basi : vel, quod eodem recidit, sicut resistentia figuræ aut potius lineæ curvæ CBABC ad resistentiam rectæ FF basi CC parallelæ & æqualis fluidi impressiones ad angulos rectos excipientis. Linea CBABC dicatur patiens, lineaque CIYIC *scala resistentiæ figuræ patientis.*

PROPOSITIO XLIV. THEOREMA.

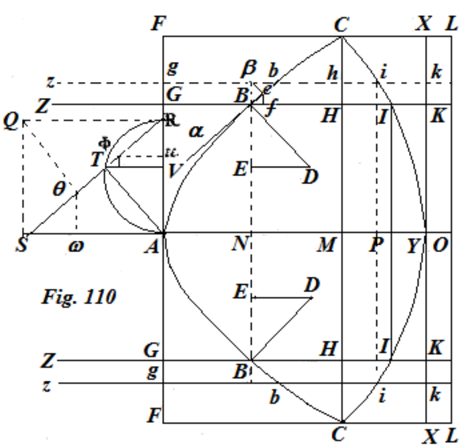

Fig. 110

435. Fig. 110. *Si. ducta per terminum* R *rectæ,* AR *axi* AM *curvæ patientis* CBABC *perpendicularis linea* RS, *parallela tangenti* Bα *curvæ patientis in* B, *deinde ex* A, *normali* AT *super* RS, *& per* T *recta* TV *equidistanti axi* AM, *in* BH *eidem axi paralla, sumatur ubique* HI *æqualis respectivæ* RV, *puncta* I *erunt in scala* GIYIC *resistentiarum figuræ patientis* CBABC, *erit resistentia curvæ* CBABC *in directione* MA *versus* S *promotæ, ad resistentiam lineæ rectæ* FF *axi normalis, sicut bilineum* CIYIC *ad rectangulum* CCLL. *Et resistentia superficiei rotundæ ex conversione curvæ* CBA *circa* AM *ad resistentiam circuli ex conversione rectæ* AT *circa eandem* AM, *ut solidum ex revolutione figuræ* CIYM *circa MY, ad cylindrum ex conversione rec-li* CMOL *circa eandem* MY *seu* MO.

1. Curva ABC in sua elementa divisa intelligatur, quorum B*b* unum, per cujus terminos agantur ZK, *zk* axi AM æquidistantes, rectæ AF in punctis *G, g;* ipsi *zb* ordinata BN occurrat in puncta β, ex quo ad elementum curvæ B*b* perpendicularis β*e* demissa sit. Ponantur insuper BD curvæ normalis, & ED axi AM parallela, ac ductis per puncta R & S lineis QR, QS parallelis AS & AR, & ex Q cadat Qθ perpendicularis super RS, & θω perpendicularis super AS. Adeo ut, si insuper lineola *ef* ordinatæ BN parallela ducta fuerit, figuræ $b\beta Bfe$ & RQS$\omega\theta$ similes futuræ sint, quandoquidem singula latera unius singulis alterius parallela sunt & similiter posita.

II. Adeoque in figuris similibus $b\beta Bfe$ & $\mathrm{RQS}\omega\theta$ erit $\mathrm{RA}:\theta\omega=\mathrm{B}\beta:fe$, atque $\mathrm{RA}.fe=\theta\omega.\mathrm{B}\beta$. Verum, quia triangula $\mathrm{Q}S\theta$ & RAT similia & QS ac RA æquales sunt, erit etiam $\mathrm{S}\theta=\mathrm{RT}$, & per consequens $\theta\omega=\mathrm{RV}$, ergo etiam $\theta\omega.\mathrm{B}\beta=\mathrm{RV}.\mathrm{B}\beta$ id est, per constructionem $=\mathrm{HI}.\mathrm{H}h$; ac proinde $\mathrm{RA}.fe=$ rec-lo IH*b* areæ CIYIC inscripto.
III. Igitur, quia filamentum fluidi *ZBbz* in curvæ elementum *Bb* impingentis impressio in hoc elementum juxta directionem BD est (§.429.) AR.B*e*, quia AR exponit (secundùm hypothesin) quadratum celeritatis fluidi curvæ allabentis. Impressio vero, juxta BD, virtualiter continet impressiones ipsi æquipollentes laterales, juxta BE & ED directiones axi AM perpendicularem & parallelam, quarum perpendiculares axi BE, utpote quæ contrariæ & directe oppositæ sunt, quatenus ex duobus elementis circa axem AM similiter, sed ad oppositas partes sitis B*b*, B*b* derivantur, se mutuo elidunt, ut adeo nulla earum ratio habenda sit, sed solæ impressiones, juxta directiones ED axi parallelas, considerandæ veniant. Est vero (§. 39) impressio juxta BD ad impressionem juxta ED ut BD ad ED, seu, propter triangla similia EBD & *f*B*e*, sicut *Be* ad *ef*, id est, ut RA.B*e* ad RA.B*e* seu (num. 11 hujus) rec-lum IHh. Atqui, ut initio hujus numeri dictum, impressio fluidi, juxta BD est RA.B*e;* ergo impressio fluidi in elementum B*b*, juxta directionem axi AM parallelam, est rec-lum IH*h*. Atque adeo impressio, quam omnia B*b*, seu tota curva CBABC à fluido subibit, id est, resistentia hujus curvæ exponetur omnibus rectangulis IH*h*, quæ in bilineo CIYIC continentur; & in bilineum istud evanescunt; hoc est, resistentia curvæ CBABC, juxta directionem MA, motæ in fluido exponenda est bilineo CIYIC. Impressio vero, quam lineà FF a fluido ad angulos rectos excipit, exponitur rectangulo ex FF in RA, seu (quia constr. $\mathrm{CL}=\mathrm{RA}$) rec-lo CLLC. Ergo resistentia, quam curva CBABC in fluido patietur, erit ad resistentiam lineæ FF*;* ut area CIYIC ad rectangulum CLLC. Quod erat primum.

Quoad secundam partem Propositionis, cum impressio, quam elementum curvæ *Bb* à fluido subit juxta directionem ED axi parallelam, (num. 111 hujus) exponatur rec-lo IH*h*, atque revolutione plani CACY circa axem AY, elementum curvæ Bb describat zonulam conicam, quæ pariter elementum existet superficiei ex conversione curvæ CBA circa AM ortæ, rec-lum vero IH*h* eadem conversione describat tubum cylindricum solido ex figura CIYM circa MY conversa inscriptum; liquet omnino impressionem fluidi, quam zona conica ex B*b* circa AM subibit, in directione axi AM parallela, exponi debere tubo ex rec-lo IH*h* circa MY. Ergo resistentia, quam universa superficies ex curva CBA circa AM excipiet, juxta AM à fluido impingente, exponitur omnibus tubis cylindricis ex rec-lis IH*h* circa MY, id est, solido ex figura CIYM circa MY revolvente, in quod omnes evanescunt. Impressio vero quam circulus ex conversione lineæ AF circa AM ortus subibit, exponetur cylindro, cujus basis sit ipse circulus, impressionem fluidi ad angulos rectos excipiens, altitudo vero AR vel CL, seu MO, quæ quadratum celeritatis fluidi allabentis exponunt; vel etiam cylindra ex conversione rectanguli CMO circa MO. Adeoque est resistentia solidi rotundi in fluido CBABC ad resistentiam circuli FF, ut solidum rotundum CIYIC ad cylindrum CLLC. Quod erat secundum.

COROLLARIUM.

436. Cum (constr.) sint $\mathrm{RA} = \mathrm{HK}$ & $\mathrm{RV} = \mathrm{HI}$, & $\mathrm{AR} : \mathrm{RV} = \mathrm{RS} : \mathrm{RT} = \mathrm{RS}^2 : \mathrm{RA}^2$, erit etiam $\mathrm{HK} : \mathrm{HI} = \mathrm{RS}^2 : \mathrm{RA}^2$, & dividendo $\mathrm{IK} : \mathrm{HI} = \mathrm{AS}^2 : \mathrm{AR}^2$, propterea erit $\mathrm{RA} : \mathrm{AS} = \sqrt{\mathrm{HI}} : \sqrt{\mathrm{KI}}$. Adeoque, data alterutra ex duabus curvis patiente, vel scala resistentiarum ejus, innotescet semper altera, si non algebraice saltem transcensdenter; & quidem scala resistentiarum semper algebraica erit, si patiens curva fuent.

SCHOLION.

437. Ergo, si vocentur AN, *x*; BN seu HM, *y;* HI, *t;* HK vel RA, *a*; AS, *m* analogia præcedentis corollarii $\mathrm{RA} : \mathrm{AS} = \sqrt{\mathrm{HI}} : \sqrt{\mathrm{KI}}$, præbebit
$a : m = \sqrt{t} : \sqrt{a-t}$ & $t = a^3 : aa + mm.$
I^{o}. Tum etiam $m = a\sqrt{(a-t:t)}.$
II^{o}. Et quia triangula $b\beta$B & ASR similia sunt, erit etiam $dx = dy\sqrt{(a-t:t)}$.
III^{o}. Ac denique etiam $dy = dx\sqrt{(t:a-t)}$.
IV^{o}. Hæ postremæ nullo negotio ex secunda eliciuntur, substituendo duntaxat loco *a & m* elementa proportionalia *dy & dx.* Jam ope harum quatuor æquatio num generalium varia problemata solvi possunt, quorum nonnulla proponere libet.

PROPOSITIO XLV. PROBLEMA.

438. *Assignare proportionem inter resistentias, quas triangulum isosceles* BER *in fluido patietur, si præcedente modo basi BR, modo vertice* E *in directione* AE *id feratur.* Fig. 111.

Exstructo super dimidia basi AB quadrato, AM demittatur ex A perpendicularis AF ad latus trianguli EB, & per per punctum F agatur FG parallela axi EA, & facta AI vel BH æquali BG, erit recta HI scala resistentiarum lineæ EB, eritque adeo resistentia lineæ EB ad resistentiam lineæ BA ut rec-lum BI ad rec-lum BP, seu ut AI ad AP; atque adeo in duplicata ratione lineæ EB ad BA,vel aggregati laterum BE & ER ad basin BR trianguli isoscelis BER; atqui, ut resistentia lineæ EB ad resistentiam lineæ BA, ita resistentia linearum BE, RE simul, seu totius trianguli, præcedente ejus vertice E, ad resistentiam basis BR, ergo harum resistentiarum ratio æquivalet duplicatæ rationi laterum BE, RE ad basin BR.

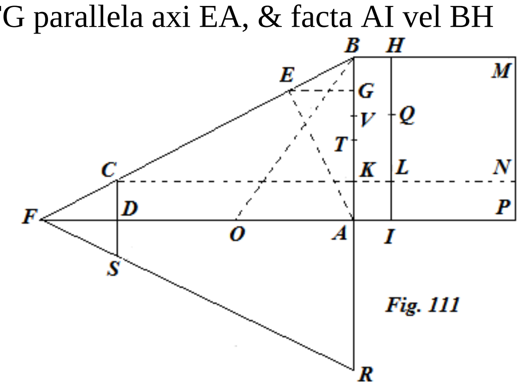

Fig. 111

Nam; si BA repræsentet quadratum celeritatis trianguli, BG vel AI (§.435) exponet ordinatam constantem scalæ resistentiarum HI trianguli, vel potius lineæ rectæ patientis BE. Quod erat inveniendum ac demonstrandum.

COROLLARIUM I.

439. Adeoque resistentia quadrati, juxta directionem lateris in fluido delati, erit ad resistentiam ejusdem, sed in directione diagonalis pari celeritate incedentis, ut diameter quadrati est ad latus ejusdem. Nam si triangulum isosceles BER cogitetur rectangulum esse in E, erit (§.438.) resistentia laterum BE, ER juxta AE ad resistentiam basis BR, quæ diameter est quadrati cujus triangulum BER est semissis, ut AI ad AP ,seu BF ad BE; resistentia vero diametri quadrati BR perpendiculariter incedentis super AE, est ad resistentiam lateris quadrati BE itidem normaliter super AE delati, sicut BR ad BE seu 2BA ad BE, seu 2BE ad BR, id est BE ad BA; ergo ex æquo resistentia laterum BSR, seu quadrati, juxta directionem diagonalis AE promoti in fluido, præcedente quadrati vertice E , ad resistentiam lateris BE fluidi, impressionem ad angulos rectos excipientis, est ut BF ad BA, seu ut latus BE ad diagonalem BR, adeoque invertendo, resistentia lateris BE super AE perpendiculariter in fluido delati, ad resistentiam quadrati juxta diagonalem incedentis, est ut diameter ad latus.

COROLLARIUM II.

440. Resistentia coni recti BER juxta directionem axis AE in fluido delati, præcedente tamen ejus vertice E, erit ad resistentiam ejusdem, præeunte ejus basi BR, iterum ut AI ad AP, seu ut BF ad BE. Nam (§. 435.) resistentia primi casus est ad alteram, ut cylindrus ex rectangulo AH circa AP ad cylindrum ex quadrato AM circa eandem AP, sed prior cylindrus est ad alterum ut AI ad AP, cum utriusque cylindri basis sit eadem. Ergo &c.

PROPOSITIO XLVI. PROBLEMA.

441. *Assignare resistentias Sectionum Conicarum juxta directionem axis in fluido motarum.* Fig. 110.

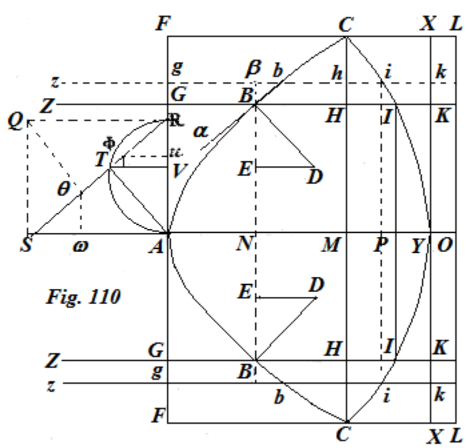

Fig. 110

Si ABC sit *ellipsis* vel *hyperbola,* cujus centrum sit in M ; dicanturque semilatus transversum AM, *b;* parameter *c;* abscissæ AN, *x* ; semiordinatæ NB ,*y;* æquatio ellipseos & hyperbolæ erit $byy = 2bcx \mp cxx$, in differentiata æquatione $bydy = bcdx \mp cxdx$, loco differentialium *dy, dx* earum proportionales *a* & *m* substituantur,quæ sunt nomina linearum RA & AS , fietque $aby = bcm \mp cmx$, atque adeo $m = aby : bc \mp cx,$ & $mm = aabbyy : bbcc \mp 2bccx + ccxx$ (vel substituto ex æquatione curvæ valore ipsius $\mp 2bccx + ccxx,$ in $y) = aabbyy : bbcc \mp bcyy$; hinc $aa + mm = (aabyy + aabcc \mp aacyy) : bcc \pm cyy$; hinc, quia (§.437) $t = a^3 : aa + mm,$ erit $t = abcc \mp acyy : bcc + byy \mp cyy$ (seu ponendo *e* pro $b \mp c$) $= abcc \mp acyy : bcc + eyy$, & hæc foret æquatio scalæ resistentiarum CIY; ponatur insuper

$bcc = eff$, & æquatio curvæ CIY erit
$t = aeff \mp acyy : eff + eyy = \mp ac : e; \pm acff + aeff : eff + eyy$ (& substituendo in numeratore postremæ fractionis, loco $: \pm c + e$ suum valorem b) $= \mp ac : e; + abff : eff + eyy$.
Igitur elementum arcæ MHIY, seu rec-lum
$\text{IHh} = \mp acdy : e; + abffdy : eff = \mp acdy : e; + abdu : e$; nominando arcum circularem *a*. cujus tangens est *y* & radius *f*; quandoquidem (§.166.) hujus arcus elementum, seu $d\alpha$, est $= ffdy : ff + yy$. Ergo area ipsa erit $\text{MHIY} = (ab\alpha \mp acy) : e$. Adeoque, si basis CM dicatur *g*, & arcus radio *f* descriptus, cujus tangens est *g* , nominetur A, erit universa area $\text{MCIY} = (abA \mp acg) : e.$ Rectangulum vero $\text{CO} = ag$. Ergo resistentia, quam curva CBABC in fluido patietur, erit ad resisentiam lineæ FF, sicut quantitas
$ab\text{A} : e$; $\mp acg : e$ ad ag; idest, sicut $bA \mp cg$ ad eg. Quod erat inveniendum.

Hæc ipsissima est resistentiarum ratio, quam Celeb. Jac. Bernoullius pluribus verbis, ast gine demonstratione & analysi, declarat Act. Erudit. Lips. 1693. pag. 253, art.5. Hæc proportio tantum obtinet, cum sectio conica juxta directionem axis majoris movetur, signumque superius ellipsin, hyperbolam vero inferius respicit.

COROLLARIUM I.

442. Si *b* sit infinita præ parametro *c*, fiet $e = b \mp c = b$, atque adeo ratio $b\text{A} \mp cg$ ad eg , erit eadem , quæ *b*A ad *bg* vel A ad *g*. Unde cum ellipsis cujus latus transversum est infinitum; abeat in *parabolam*, manifestum est, parabolæ in fluido latæ juxta directionem axis, præeunte modo vertice mox basi, resisti in ratione arcus circuli radio semiparametro parabolæ descripti, cujus arcus tangens æquetur dimidiæ basi parabolæ, ad dimidiam hanc basin.

COROLLARIUM II.

443. Sin vero ABC fuerit *circulus*, ratio $bA \mp cg$ ad *eg* abit in rationem cujus termini algebraice dati sunt, quod omnino meretur, ut hoc loco ostendatur. Quia circulus est ellipsis , cujus latus rectum æquatur parametro, sit hoc casu $b = c$, atque adeo
$b - c = e = 0$; & hoc casu adhibenda est ratio $b\text{A} - cg$ ad *eg*, seu $\frac{b\text{A}}{e} - \frac{cg}{e}$ ad g , verum
$-\frac{cg}{e} = -cg : b - t = g - \frac{bg}{b-c} = g - \frac{bg}{c}$, ratio $\frac{b\text{A}}{e} - \frac{cg}{e}$ ad *g* æquabitur rationi
$\frac{b\text{A}}{e} + g - \frac{bg}{e}$ ad g . Atqui, juxta paragraphum 166, est
$\text{A} = g - \frac{g^3}{3ff} + \frac{g^5}{5f^4} - \&\text{c.}$; ergo $\frac{b\text{A}}{e} + g - \frac{bg}{e} = g - \frac{bg^3}{3eff} + \frac{bg^5}{5f^4e}$ vel (quia $bcc = eff$)
fiet etiam : $= g - \frac{bg^3}{3bcc} + \frac{bg^5}{5bccff} - \text{etc.} = g - \frac{g^3}{3cc} + \frac{g^5}{5ccff} - \text{etc.}$;
seu, (quia *ff* respectu *cc* aut *gg* infinita, atque adeo fractio $g^5 : 5ccff$ indefinite parva seu evanescens est) $= g - \frac{g^3}{3cc}$. Erit ergo resistentia segmenti circularis CBABC ad resisistentiam ejus basis, seu ei æqualis rectæ FF, sicut $g; -g^3 : 3cc,$ ad g , id est,

ut $cc-\frac{1}{3}gg$ ad cc, vel $bb-\frac{1}{3}gg$ ad bb, aut, quod eodem recidit, velut quadratum diametri, demta una triente quadrati baseos segmenti, ad quadratum diametri; prorsus ut invenit Cl. Bernoullius in loco supra (§.141.) citato.

SCHOLION.

444. Solidum rotundum CIYIC est ad cylindrum CLLC sicut omnia MH.HIHh ad $\frac{1}{2}\mathrm{MC}^2.\mathrm{CL}$, vel sicut $\int tydy$ ad $\frac{1}{2}ayy$. Atqui cum t sit (§.441.)
$=\mp ac:e;+abff+abff:eff+eyy=a-\frac{ab}{e}+abff:eff+eyy$, erit
$tydy=aydy;-abydy:e\ ;+abffydy:eff+eyy;$ ergo

$$\int tydy=\tfrac{1}{2}ay^2;-abyy:2e\ ;+\frac{abff}{e}.\log\sqrt{(ff+yy:ff)}.$$

Nam integrale ipsius $ydy:ff+yy$ est $\log.\sqrt{(ff+yy:ff)}$. Ergo solidum rotundum, ex MHIY circa MY, ad cylindrum HKKH, vel ratio resistentiae solidi BAB ad resistentiam baseos BB, præeunte modo vertice A, modo basi BB in directione MA, est sicut
$\frac{1}{2}ay^2;-abyy:e\ ;+\frac{abff}{e}.\log\sqrt{(ff+yy:ff)}$ ad $\frac{1}{2}ayy$, vel sicut
$yy;-byy:e\ ;+\frac{2bff}{e}.\log\sqrt{(ff+yy:ff)}$ ad yy; aut denique etiam, ut
$\frac{2bff}{e}.\log.\sqrt{\left(ff+yy:ff\right)}\mp\frac{cyy}{e}$ ad yy. Ubi iterum signum superius respicit ellipsin, hyperbolam vero inferius.

Idcirco conoidi parabolico in directione axis suæ lato resistitur, præeunte modo vertice modo basi, in ratione $2ff.\log.\sqrt{(ff+yy:ff)}$ ad yy, vel $2cc.\log.\sqrt{(cc+yy:cc)}$ ad yy. ·

445. Sphæræ segmento resistitur, in iisdem ac in præcedentibus circumstantiis; in ratione $cc-\frac{1}{2}yy$ ad cc; atque adeo resistentia hemisphærii, juxta directionem axis MA lati, erit tantum semissis resistentiæ lineæ FF.

Nam in sphaera sit $b=c$ & $e=0$, at $\log.\sqrt{(ff+yy:ff)}=\frac{yy}{2ff}-\frac{y^4}{4f^4}+$&c. hinc ratio
$yy;-byy:e;+\frac{2bff}{e}.\log.\sqrt{(ff+yy:ff)}$ ad yy abibit in rationem $yy-\frac{by^4}{2eff}-$ ad yy; vel
$2eff-byy$ ad $2eff$ (aut quia $eff=bcc$), in
$2bcc-byy$ ad $2bcc=2cc-yy$ ad $2cc=cc-\frac{1}{2}yy$ ad cc.

PROPOSITIO XLVII. PROBLEMA.

446. *Invenire curvam patientem* ABC, *quæ sui ipsius sit scala resistentiarum.*

Oportet ergo curvas CAC & CYC similes & æquales esse, adeo ut $\mathrm{BH}=\mathrm{HI}$, & $\mathrm{AM}=\mathrm{MY}$, ut & $\mathrm{AN}=\mathrm{PY}$. Positaque, ut in præcedentibus, RS parallela elemento curvæ quæsito Bb, erit $\mathrm{RV}=\mathrm{HI}=\mathrm{BH}$, & $\mathrm{R}u=hi=bh$, adeoque $\beta b=\mathrm{V}u$, ductisque per u rectula $u\sigma$, ipsi RS occurrente in puncto σ, & per hoc recta $\sigma\pi$ parallela RA; & super diametro RA descriptus circulus transibit per punctum T, cum angulus RTA

(constr.) rectus sit; quibus positis, & cum jam dictum sit $Vu = \sigma\pi$ aequari βb, erit $T\pi = B\beta = Hh$; atqui producta $u\sigma$ usque ad occursum cum semicirculo Φ ex hoc puncto agatur $\varphi\rho$ parallela RA; $T\pi = T\rho + \rho\pi = T\rho + \varphi\sigma$, seu, ut alibi (§. 463, num. 111.) ostendetur $= T\rho + T\varphi$ unde omnes $T\pi$ seu $B\beta$ id est ordinata $N = \text{omn. } T\rho$, seu ordinatæ $TV + \text{Omnibus arculis} T\varphi$, seu arcui AT, est ergo ordinata curvæ quæsitæ BN aggregatum arcus *AT* ejusque sinus, atque adeo curva ABC est cyclois ordinaria.
Quod erat inveniendum.

Hinc resistentia cycloidis CBABC erit ad resistentiam basis ejusdem, ut area cycloidis ad rec-lum circumscriptum, atque adeo ut 3. ad 4.

PROPOSITIO XLVIII: PROBLEMA.

447. *Ex omnibus frustis conicis* BCSR *super eadem data basi* BR *ejusdemque altitudinis* AD *invenire verticem* E *illius, cui minime resistatur*, *si in directione axis* AD *in fluido incedat data quadam celeritate.*

Super B , facto quadrato AM ex A super EB, demittatur perpendicularis AF, & ducatur FG parallela axi coni EA, factaque $BH = BG$ agatur HI parallela BA, & (§.435) HI erit scala resistentiarum trianguli BER, cylindrusque ex conversione rec-li AH circa AI exponit resistentiam coni BER in fluido, & cylindrus ex AL resistentiam coni CES , atque adeo annulus ex conversione rectanguli KH circa AP exponet resistentiam curticoni BCSR, excepta resistentia, quam circulus CS subibit in fluido, cujus impressiones ad angulos rectos excipit, quæ circuli CS resistentia exponitur cylindra ex AN circa AP. Ergo resistentia universi frusti conici BCSR exponi debet duobus solidis, scilicet annulo ex KH circa AP, & cylindra ex AN circa eandem, &*:* (secundum hypothesin) hæc duo solida *minimum* quoddam efficere debent. Sit V punctum medium ipsius BK, eritque VA distantia centri gravitatis rectanguli BL ab axe AP, adeoque *p*.AV exprimet circumferentiam, quam centrum gravitatis una rec-li convesione describet circa *p*A, ubi *p* significat exponentem rationis peripheriæ circuli ad radium. Unde cum solida rotunda (§. 47 .) inveniantur ex ductu figurae rotantis circa aliquem axem in viam centri ejus gravitatis, factum ex rec-lo BL in *p*.AY, id est *p*.AV.CK.BH æquabitur annulo ex rec-lo BL circa AP, peripheria vero; quam centrum gravitatis rectanguli AN describit, erit $\frac{1}{2}p.AK$, adeoque $\frac{1}{2}p.AK^2.NK$ erit valor cylindri ex AN circa AP. Atque adeo $p.AK.BK.BH + \frac{1}{2}p.AK^2.NK$ exponit resistentiam quam curticonus BCSR in fluido subibit, ac propterea, efficere debet aliquod, *minimum.* Aut etiam annulus ex rectangulo LM circa AP rotato debet esse in suo genere *maximus* , quandoquidem prædictorum solidorum ex KH & AN complementum est ad datum cylindrum ex quadrato AM circa AP rotato. Adeoque $p.QI.HL.HM$ quod solidum annulum illum ex LM exprimit, debet esse *maximum.* Jam., si dicantur AB, *a;* AD, 2*b*, seu AO & OD ;. unaquæque seorsim *b;* AE, *x;* facili calculo reperietur solidum

p.QI. HL.HM $= (x-b).2a^3bp : aa+xx$, unde omissa quantitate constante in numeratore, quacum indeterminata $x-b$ multiplicata est, $(x-b) : aa+xx = \text{maximo}$. Hinc $\log.(x-b) - \log.(aa+xx) = \text{constructio}$, atque adeo $dx : x-b = 2xdx : aa+xx$, hinc $aa+xx = 2xx-2bx$, id est, $xx-2bx = aa$, vel $xx-2bx+bb = aa+bb$, vel extrahendo radicem $x-b = \sqrt{(aa+bb)}$, seu $x = b+\sqrt{(aa+bb)}$. Quæ æquatio ipsissimam constructionem suppeditat, quam Celeb. Newtonus sine analysi & demonstratione tradidit *in* Schol. post Prop.35. Lib. Sec. *Princ. Phil. Nat. Math.* quæ ita habet: ex puncto medio O altitudinis AD frusti conici ducatur OB, cui æqualis fiat OE, eritque E vertex coni quæsiti BER.

PROPOSITIO XLIX. PROBLEMA.

448. *Datis positioni recta* CI *duobusque punctis* A & B, *invenire in hac recta positione data tertium punctum* C, *ut lineæ ex eo ad data puncta ducta* AC, CB *cum rectis* AD, BF *rectæ* CI *ejusque parallelæ* DF *normalibus, figuram rectilineam* DACBF *forment, quæ circa axem* DF *in gyrum acta producat solidum, cujus superficies exclusis circulis a lineis* AD, BF *descriptis,* minimam *in fluido resistentiam patiatur, incedente solido juxta directionem axis* FD.

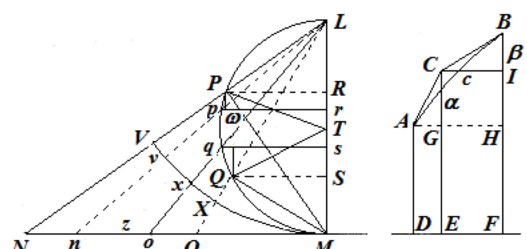

Fig. 112

Analys. Puncta data A, B à recta CI æqualiter distare ponantur, ut sint æquales CG & BI. Ex puncta T medio datæ linea LM (quadratum celeritatis exponentis, qua fluidum solido patienti alliditur) tanquam centro descriptus sit semicirculus LPM, quem linea MO alteri DF in directum posita contingat in M, eritque adeo LM parallela ipsis AD & BF. Per L agantur LO, LN parallelæ lineis AC, BC, secantes semicirculum in P & Q, per quæ punctæ transeant PR; QS parallelæ MN; rectæ MP & MQ ipsis PL & QL perpendiculares erunt. Puncto C, aliud *c* indefinite vicinum intelligatur, tum etiam rectæ *c*A, *c*B ductæ sint, & hisce parallelæ L*o*, L*n* semicirculum secantes in punctis *q* & *p*, per quæ pariter transeant ordinatæ *qa* & *pr*, ductisque $P\pi$ & $Q\rho$ parallelis diameter LM, $p\pi$ & $q\rho$ erunt elementa ordinatarum PR & QS. Sit iterum *p* exponens rationis circumferentiæ circuli ad radium, bisectisque CG: in α, & BI in β, denotabunt p.Eα circumferentiam radii Eα, & p.Fβ circumferentiam radii Fβ, adeoque annuli à lineis CG, Bi circa DF revolventibus geniti, erunt p.Eα.CG & p.Fβ.BI

II. Jam per ea, quæ in præcedentibus (§. 435.) sunt ostensa, resistentia, quam in fluido subibit annulus conicus à recta AC circa DF in orbem acta descriptus, ex ponitur per $p.\mathrm{E}\alpha.\mathrm{CG.LS}$, adeoque resistent annuli ex BC circa DF $p.\mathrm{F}\beta.\mathrm{BI.LR}$, adeoque resistentia annuli AC + resistent. annuli BC $= p.\mathrm{E}\alpha.\mathrm{CG.LS} + p.\mathrm{F}\beta.\mathrm{BI.LR}$. Similiter resistent. annuli ex A*c* + resistent. annuli B*c* $= p.\mathrm{E}\alpha.\mathrm{CG.LS} + p.\mathrm{F}\beta.\mathrm{BI.L}r$. Jam, quia ex natura *minimi* resistentia annuli AC + resistent. annuli BC = resistent. annuli A*c*.+ resistent. annuli B*c*, erit etiam $p.\mathrm{E}\alpha.\mathrm{CG.LS} + p.\mathrm{F}\beta.\mathrm{BI.LR} = p.\mathrm{E}\alpha.\mathrm{CG.LS} + p.\mathrm{F}\beta.\mathrm{BI.L}s$, vel quia (secundum hypothesin) CG & BI æquales sunt, dividendo per $p.\mathrm{CG}$ aut $p.\mathrm{BI}$; erit $\mathrm{E}\alpha.\mathrm{LS} + \mathrm{F}\beta.\mathrm{LR} = \mathrm{E}\alpha.\mathrm{L}s + \mathrm{F}\beta.\mathrm{L}r$, atque adeo $\mathrm{E}\alpha.\mathrm{S}s = \mathrm{F}\beta.\mathrm{R}r$, vel $\mathrm{S}s : \mathrm{R}r = \mathrm{F}\beta : \mathrm{E}\alpha$.

III. Ob parallelas LO, AC, & L*o*, A*c*;
erit $\mathrm{O}o : \mathrm{LM} = c\mathrm{C} : \mathrm{C}G$, vel $\mathrm{O}o : \mathrm{C}c = LM : CG$ & $\mathrm{N}n : \mathrm{C}c = \mathrm{LM} : \mathrm{BI}$ vel CG,
ergo, $\mathrm{O}o = \mathrm{N}n$. Atqui $\mathrm{N}n : \mathrm{P}p$ ut triangulum NL*n* ad 2. triangula PT*p*, quorum scilicet basis P*p*, altitudo vero *p*T*;* nam trianguli hujus PT*p* duplum est triangulum ; cujus basis etiam P*p*, sed altitudo dupla ipsius *p*T, seu diametra LM æqualis ; atque adeo ejusdem altitudinis cum triangula LN*n*: & angulus $\mathrm{NL}n =$ angula (modo nominati trianguli) ad verticem in circumferentia alterius semicirculi in figura non expressi;
ergo $\mathrm{N}n : \mathrm{P}p = \mathrm{NL}^2 : \mathrm{LM}^2 = \mathrm{NL} : \mathrm{PL} = \mathrm{LM} : \mathrm{LR}$. Eodem argumenta est $\mathrm{O}o : \mathrm{Q}q = \mathrm{LM} : \mathrm{LS}$, seu invertenda $\mathrm{Q}q : \mathrm{O}o$ vel $\mathrm{N}n = \mathrm{LS} : \mathrm{LM}$; ergo ex æquo $\mathrm{Q}q : \mathrm{P}p = \mathrm{LS} : \mathrm{LR}$. Sed propter triangula similia Q*qρ* & TQS , est *Ss* vel $\mathrm{Q}\rho : Qq = \mathrm{QS} : \mathrm{QT}$, ergo ex æqua $Ss : \mathrm{P}p = \mathrm{LS.\ QS} : \mathrm{LR.QT}$. item propter triangula similia
$\mathrm{P}p\pi$, TPR est $\mathrm{P}p : \mathrm{P}\pi$ vel $\mathrm{R}r = \mathrm{PT}$ vel $\mathrm{QT} : \mathrm{PR} = \mathrm{LR.QT} : \mathrm{LR.PR}$, ergo denuo ex æquo fit, $\mathrm{S}s : \mathrm{R}r = \mathrm{LS.QS} : \mathrm{LR.PR}$; atqui supra (num.11. in fine) invenimus
$\mathrm{S}s.\mathrm{R}r = \mathrm{F}\beta : \mathrm{E}\alpha$ ergo $\mathrm{F}\beta : \mathrm{E}\alpha = \mathrm{LS.QS} : \mathrm{LR.PR}$, & $\mathrm{E}\alpha.\mathrm{LS.QS} = \mathrm{F}\beta.\mathrm{LR}.PR$.

IV. Igitur, si dicantur LM, *a*; Eα, *e*; Fβ, *f* & MO, *m ;* quarta proportionalis MZ ad datas BH, HA & LM dicatur *g*, & hæc *g* vel MZ deprehendetur esse media arithmetica inter incognitas MO & MN , unde, cum MO sit *m*, altera MN invenietur $= 2\mathrm{g} - m$.
Et substitutis his valoribus in canone paulo ante reperto $\mathrm{E}\alpha.\mathrm{LS.QS} = \mathrm{F}\beta.\mathrm{LR.PR}$, pervenietur ad æquationem quinque dimensionem, quæ, salvo calculo, erit ut sequitur:

$$\begin{array}{llllll} +em^5 & -8eg.\,m^4 & +24egg.m^3 & -8.aage.mm & +a^4e.m & -2a^4fg \\ +f & -2fg & +2aaf & -4aafg & +a^4f & \\ & & +2aae & -32g^3e & +16g^4e & \\ & & & & -8aaegg = 0. & \end{array}$$

Cujus radices determinant valores quæsitæ MO ; qua inventa ductaque OL, si per datum punctum A ducetur AC; parallela ipsi OL, quæ AC occurrat rectæ CI; ei occurret in optato puncto C. Quod erat inveniendum.

Exiisset pariter æquatio 5.dimensionum, si vel maxime lineas CG & BI inæquales assumpsissemus.

COROLLARIUM I.

449. In canone supra invento $\mathrm{L}\alpha.\mathrm{LS.QS} = \mathrm{F}\beta.\mathrm{LR.PR}$ jam continetur solutio problematis de *solido minimæ resistentiæ*. Hoc enim casu CG & BI perinde ac AG & CI supponendæ sunt indefinite parvæ, quo fiet, ut αE & AD, item βF & CE instar æqualium tractare debeant & cum omnia solida $\mathrm{L}\alpha.\mathrm{LS.QS}$; $\mathrm{F}\beta.\mathrm{LR.PR}$ &c: singula scilicet curvæ elementa AC, CB, &c. respicientia, æqualia sint, ponatur unumquodque eorum dato cubo LM^3 æquale, eritque adeo $\mathrm{L}\alpha.\mathrm{LS.QS} = \mathrm{LM}^3$, aut vocando E$\alpha$ vel DA, y ; CG, dy ; AG ,dx; & ut prius LM, a & denique MO, m; $a^5my:(aa+mm)^2 = a^3$, hinc
$y = (aa+mm)^2 : aam = aa:m \ ;+2m \ ;+ m^3:aa,$ & $dy = -\ aadm:mm \ ;+2dm \ ;+3mmdm:aa,$
& quia propter similitudinem triangulorum CGA & LMO, est $mdy = adx$, ideo invenietur quoque $dx = -adm:m \ ;+2mdm:a \ ;+3m^2dm:a^3$; ergo
$x = mm:a \ ;+ \ 3m^4:4a^3 - lm$, ubi lm significat log-um indeterminatæ m in log-mica, cujus subtangens est a. Et hæc solutio ad amussim convenit cum ea, quam Celeb. Joh. Bernoullius in Act. Erud. Lips. 1699. p.515. exhibuit, & in Actis anni sequentis uberius explicuit.

COROLLARIUM II.

450. In *æquatione* $y = (aa+mm)^2 : aam,$ vel $y:a = (aa+mm)^2 : a^3m$, si loco $aa+mm,\ a,$ & m substituantur differentialia $ds^2,\ dy,$ & dx, quæ illis proportionalia sunt, proveniet $y:a = ds^4 : dxdy^3$; adeoque $ads^4 = ydxdy^3$, quæ est æquatio differentialis curvæ quæsitæ in quam laudatissimus Bernoullius citato loco incidit, & ex ea postea expressiones coordinatarum in præcedenti corollario repertas elicuit. Hisce omnibus Illustris Marchionis Hospitalii solutio etiam, consona ei, quam in *Actis* Lips. & in Commentariis. Acad. Reg. Scient. anni 1699 publicari voluit.

Solas hactenus resistentias illas contemplati sumus, quas corpora in fluidis delata subeunt, cum juxta directionem axis moventur. Sed si in aliis directionibus incedant, res evadet paullo altioris indaginis, ut ex sequenti problemate eleganti abunde patebit.