SECTION III.

Concerned with the effects of striking fluids.

Until now we have attended to the motions of the fluid, which result from the pressure of its weight, but also requiring to be scrutinized are the effects on the same fluid on being struck, as often by other fluids, as well as by colliding with solid bodies at other times, and also indeed the effects on solids on being struck by fluids. Effects of this kind must refer to the diminutions of the motions, which solid bodies undergo both on taking into account the shapes being thrust into fluids, as well also as with account of the motions themselves, that is, the resistances of bodies endured in fluids. Which all generally and some especially are to be reviewed here in this third section.

DEFINITION.

42l. When a fluid is impinging on another fluid or solid body, the action of the fluid on the other body is called *Striking Force,* and the effect, which is endured by the body receiving the percussion is called the *Impressed Force.*

In the above repetition of the impressed forces we have supposed to discuss matters in a slightly wider sense, also by designating the effect, which arises from the pressure of the fluid.

AXIOM.

422. Any solid body will receive the same impressed force from a fluid, that either may be carried in the fluid at rest in some given direction with a given speed, or the fluid travelling with the same speed in a given direction may strike the same body at rest in the fluid.

CHAPTER XI.

Concerning the general relations of striking fluids.

Proposition XLII. THEOREM.

423. *Of the fluids* ABb, CDd *meeting the plane* MN *with a uniform motion at right angles, thus yet, so that the parts of the same may be able by rebounding to recede freely from the plane, and since there because the arriving quantities may be impeded, in equal times at the plane* MN, *they will be in a ratio composed from the ratio of the speeds* AB , CD, *of the lengths* Bb, Dd , *and of the densities* M, N. See Fig. 102.

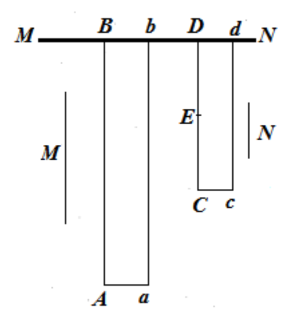


Fig. 102

ABb and *CDd* shall be the quantities of fluid arriving in equal times at the plane MN with an equal motion, and the approach speeds shall be as AB ad CD. Truly the quantities of fluid at the plane MN likewise shall be accustomed to be designated in a ratio composed of the volumes and densities, and the volumes composed from the ratio of the speeds AB, CD and of the lengths, therefore the mass ABb is to the mass COd, just as AB.B*b*.M to CD.D*d*.N or in the composite ratio from the ratio of the speed AB to the speed CD, of the width Bb to the width D*d*, and finally of the density M to the density N. Q.E.D.

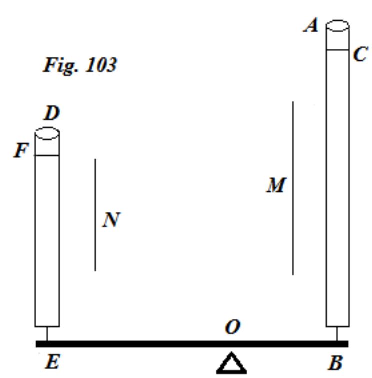
COROLLARY I.

424. Hence the forces, which the masses of fluids AB*b*, *CDd* may exert on the plane MN, will be in the ratio composed from the square of the velocities, AB, CD, and from both the simple ratio of the widths B*b*, D*d*, as well as of the densities M, N, that is, the force of the mass AB*b* will be to the force of the other CD*d*, just as AB^2.B*b*.M to CD^2.D*d*.N, or with ED taken as the third proportional to AB and CD, just as AB.B*b*.M to ED.D*d*.N. For the forces are in the motions of the fluid and the motion in the composite ratio of the masses and of the velocities.

COROLLARY II.

425. Hence also, if the tubes AB, DE were filled with liquids of some kind, of which the densities of the liquids shall be M and N, and the liquids may flow out through the equal openings B, E, and may strike at the ends of the lever EB turning about O, the momentum of the liquid CB to the momentum of the other FE, shall be as M.CB.BO to N.FE.EO.

For the forces at B and E are as the motions of the fluid, and thus (§.424.) in the ratio composed from the square of the velocities and from the simple densities, with the presence of tubes with equal openings ; truly the velocities of the fluids escaping from the tubes (§.386) will be in the square root ratio of the heights CB and FE, or in the square of the same velocities, which are the heights CB and FE; and therefore the force at B will be to the force at E just as CB.M to SE at N. Truly these forces, in as much as they act on the lever BE turning about O, have the ratio of the applied motive forces to the lever, therefore since they may have proportional moments of the lever made from these forces themselves at the distances of these from the fulcrum O, the moment of the force at will be had at B to the moment of the force at E, just as M.CB.BO to N.FE.EO.

Fig. 103


COROLLARY III.
See Fig. 104

426. Therefore the torrents DG, HL, approaching the plane EL with the speeds *a*A, *b*B, will remain in equilibrium with the third ON striking the same plane EL with the speed *Cc*, if with the densities of the torrents put R, S , T, and with the widths EG, KL, MN , if there were $R.Aa^2.EG.AC = S.Bb^2.KL.BC$ & $R.Aa^2.EG + S.Bb^2.KL = T.Cc^2.MN$.
For if the plane EL may be considered in place of the lever, the equal striking moments will become (§.425.) $R.Aa^2.EG.AC$ & $S.Bb^2.KL.BC$, and thus they will remain in equilibrium with the third OMN since (following the hypothesis) its mean direction *c*C passes through *C*, and its force or $T.Cc^2.MN$, is equal to the two opposite forces taken together, evidently $R.Aa^2.EG + S.Bb^2.KL$.

In this corollary are related the particular noteworthy collisions and striking forces of fluids amongst themselves which can happen.

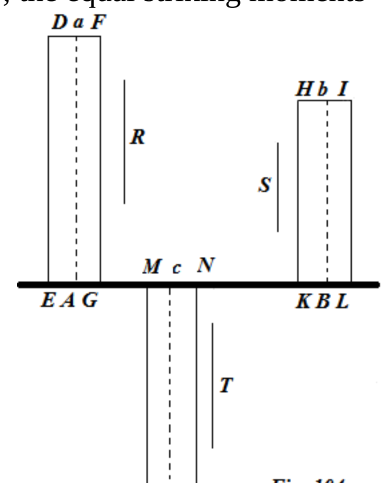


Fig. 104

COROLLARY IV.

427. The resistances, which solid bodies carried in fluids will experience, will be in the ratio composed of the densities and of the square of the velocities: and thus these resistances in one and the same fluid will be in the square ratio of the speeds. For, because (§.422.) bodies advancing in fluids endure the same forces from the fluid, which they undergo, if the fluid impinged with that speed, by which the bodies are carried in that fluid, and in which the bodies are at rest, and because these forces are as the striking forces of the fluid, that is, in the composite ratio from the square of the speed and of the simple density, truly the resistances which bodies endure are as the forces, by which they are resisted, undergo in the fluid ; generally it is clear the resistances, which bodies will experience in the mean fluid flow, to be in the composite ratio of the densities and of the square of the velocities, or if the bodies may be carried in the same medium, in the square ratio of the speed.

PROPOSITION XLIII. THEOREM.

428. *If a fluid, or stream CABD, may impinge perpendicularly on the plane* AB *with a certain given speed, then also the impressed force will be with the same speed, but at the oblique angles* KOF, KOE *for another plane* EF *equal to the first* AB; *for the impressed force the first plane* AB *will endure, will be to the impressed force of the other plane* EF *along the normal directions OP*, *OQ of the plane, in the square proportion of the*

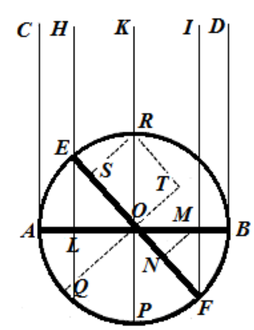


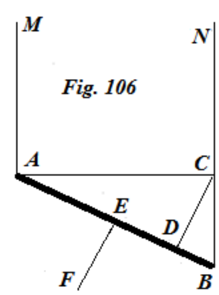
Fig. 105

whole sine to the angle of incidence KOE ; *or, with the perpendiculars* HL, KOP, IMF *drawn through* E, O *&* F *to the plane* AB, *&* MN *to the normal plane* EF*, just as* OF *to* ON. See Fig. 150.

Because it may be agreed from §.249, the fluid forces CABD, HEFI exerted on the planes AB and EF are along the directions OP, OQ normal to the planes. From the point R , RS and RT are acting perpendicular and parallel to the plane EF. Now, because the filament of the fluid KO falls upon the plane LM at right angles, truly at the oblique angles KOE, KOF in the plane EF; and if RO may express the speed of the filament, it will be itself evident, that the filament will be unable to exert its whole force on the oblique plane EF, because the direction RO is not directly opposite to this plane, but with the speed RO resolved along the equivalent sides RS and RT, merely will act on the plane EF with the speed RS directly opposite and contrary to this direction, with the other RT as it is with its direction parallel to the plane EF, therefore may exert no force on the same plane. Therefore the force of the filament KO on the plane LM to the force of the same on the plane EF shall be as RO to RS, or as OF to OM*;* from which because the same number of filaments flows in with the two planes LM and EF, the force of the stream HLMI on the plane LM will be to the force of the same on the plane EF, as OF to OM; indeed the force of the stream CABD on the plane AB is to the force of the stream HLMI on the plane LM, just as AB to LM, that is, as OF to OM, or OM to ON; therefore from the equation the force of the stream CABD on the plane AB is to the force of the stream HEFI on the plane EF, as OF to ON, that is, in the square ratio of the whole sine OF to the sine of the angle of incidence IFO or KOE. Q.e.d.

COROLLARY.

429. See Fig. 106. Hence if the fluid MABN may be moving towards the plane AB with the given speed V, the force will be, that the plane encounters from the fluid striking that, the product from the square of the velocity by the right line AD ; evidently with AC the perpendicular to AM, BN themselves, with the perpendicular CD dropped from C above AB. For the force (§. 424.), which the plane AB may endure from the fluid striking perpendicularly, is as $V^2.AB$, truly by the present proposition the force of the perpendicular fluid to the plane AB to the force of the same, but by striking the plane AB obliquely, shall be as AB to AD, or $V^2.AB$ to $V^2.AD$; therefore $V^2.AD$ expresses the force of fluid MABN incident on the plane AB with the speed V, and this force is exerted along the direction EF perpendicular to the plane and passing through its centre of gravity, by §.249 & §.54.



SCHOLIUM I.

430. Fig. 107. The force of the tiller or rudder is deduced readily from the present proposition, with the aid of which ships, in which you may wish to be present, can change direction ; about which a few things is required to be indicated. Thus B shall be

the stern of the ship AB, the rudder of which BH is attached to a hinge moveable about B, to which XY shall be drawn normal through the centre of gravity X. Now we may consider the ship to be progressing along BA from B towards *A* with some given velocity, and the rudder BH to be fixed, so that with AB produced to T, it may constitute some acute angle HBT : Therefore with the ship progressing towards E, the water ABHG, along directions parallel to the right line AB, will itself impinge on the rudder with that speed on the rudder BH, by which the ship is carried off in the opposite sense towards E ; and because the same effect may result (§.422.), if the water ABHG were to impinge with the given speed on the rudder of the ship at rest, as if with the ship progressing the rudder would be dashed against by water with that speed, and the force of the water striking on the rudder acts on BS, with unity put for the velocity, and with BR drawn normal to GH itself, and RS normal to BH, which force being set out through BS is exerted on the rudder BH along the direction XY drawn through the centre of gravity. Therefore with the motion of the ship along the direction BA, the resisting water will perform the same effect, as if the rudder were being impelled by a certain force acting on BS along the direction XY; and thus hence it is apparent, this force exerted on BS is going to turn the ship around itself. But because the direction XY for another HG, as along which the ship is moving, not agreeing nor directly opposite, nor also is perpendicular, the force BS in as much as it is applied along the direction XY, is not the absolute force, which may be the cause of the turning of the ship; but with BN drawn perpendicular to BH, and made equal to BS itself, and with NO parallel to BV, this force NO expresses the full force of the ship turning; since ON is normal to the direction of the ship OA, and the force $\mathrm{BN} = \mathrm{BS}$ is resolved into the equivalent forces BO and ON, of which BO, placed along the direction of OA, confers nothing to the rotation of the ship, but only delay the progression of the ship a little, thus so that only the force ON may remain, which performs the rotation of the ship. Similarly if the rudder may be placed at BF, KI will denote the force of rotation of the ship, evidently with QL and BI drawn normally to the rudder BF, and by making $\mathrm{BI} = \mathrm{BL}$, also IK will be drawn perpendicular to the right line AT.

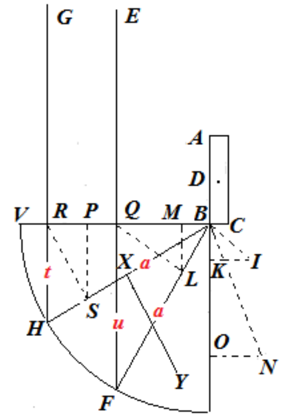


Fig. 107

431. From a consideration of the figure it is apparent at once, KI and ON themselves can be equal, even if the angles IBK and NBO shall be different [Fig. 107 does not show this particular case]. From which it follows, two different positions of the rudder shall be equally effective in turning of the ship about itself. Indeed SP and LM are acting parallel to TB, which themselves respectively are equal to ON and KI, since the triangles PSB and ONB, and also MLB and KIB emerge similar and equal. Now
$\mathrm{HR} : \mathrm{PS}(=\mathrm{HB} : \mathrm{SB}) = \mathrm{HB}^2 : \mathrm{RB}^2$, [for $\mathrm{RB:SB} = \mathrm{HB} : \mathrm{RB}$] & $\mathrm{ML} : \mathrm{QF} = \mathrm{QB}^2 : \mathrm{FB}^2$;
therefore if PS and ML or KI and ON shall be equal, there will be, [since HB=FB ,]
$\mathrm{HR} : \mathrm{FQ} = \mathrm{QB}^2 : \mathrm{RB}^2$, from the equation, and thus $\mathrm{HR.RB}^2 = \mathrm{FQ.QB}^2$. Hence, if BH or BF were called a ; RH, t; QF, u , there will be $\mathrm{RB}^2 = aa - tt$, & $\mathrm{QB}^2 = aa - uu$, and thus

$\mathrm{HR.RB}^2 = \mathrm{FQ.QB}^2$ becomes $aat - t^3 = aau - u^3$; or $u^3 - t^3 = aau - aat$, and thus on dividing by $u - t$, there will be $uu + ut + tt = aa$, which is an equation making available infinitely many different positions of the rudder of which there are two, and two equal suitable ones for turning the ship.

432. Therefore, if $u = t$, there will be had $3tt = aa$, and thus $t = \sqrt{\frac{1}{3}aa}$; and this value of the sine of the angle VBH will give the position of the rudder BH, in which the force of rotation of the ship will be the maximum present. Thence, if the angle VBH were around 35°, 16' , or the angle HBT 54°, 44', the position of the rudder BH will be at its most useful for turning the ship. [Mention should have been made here of some limiting process.]

SCHOLIUM II.

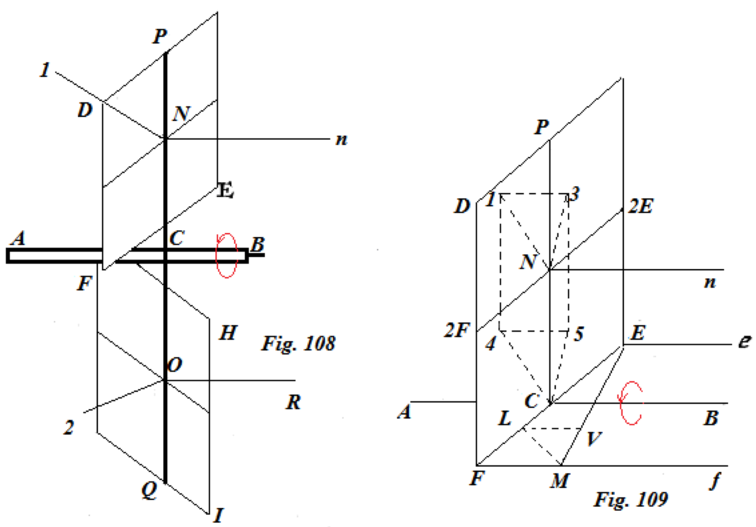


Fig. 108
Fig. 109

433. The matters considered in the preceding scholium, can also be applied readily to the working of windmill sails, see Fig.'s 108 & 109. For DE shall be a vertical sail being turned around by the wind, set at some acute angle ECB to the small axle AB of the mill ; with the speed of the wind taken as 1, and to be moving past the sail DE in the direction *n*N parallel to CB, and E*e*, F*f* shall be parallel to AB, and FC = CE , and with the normals EM, ML drawn to *Ff* and FE, and LV drawn parallel to the right line F*f*. Through the centre N of the sail, N1 is understood to be drawn equal to the perpendicular LE in the plane of the sail DE, and this N1 expresses the force of the wind and its direction, as any line 2F2E will approach parallel to the other FE, thus so that the total force, which may be received from the wind, along the direction normal to the sail, shall become PC.N1. Again, if through the point 1 the right line 13 may pass parallel to N*n* or AB, and through the point N the right line N3 shall be normal to the same N*n*, the sides 31 and N3 [of triangle 13N] will be equivalent to the force N1, of which the first 31, as it is parallel to AB or to the axle, contributes nothing to the rotation of the sail; but as N3 shall be perpendicular to the axle of the mill AB: and thus the force of rotation of the sail from the

force derived along N1 will be PC.N3. Truly, because the right lines NI , 13 and N3 with just as many others LM, LV and MV parallel in the plane EFM, and the triangles LEV and MLV are similar, also MLV and 1N3 are equiangular and equal, because above there is $\mathrm{LE} = \mathrm{N1}$. Hence also $\mathrm{LV} = \mathrm{N3} = \text{force}$ of rotation of the sail DE. [Recall that LE is the projection of the maximum force, when sail is normal to the wind, on to the surface of the sail.] Hence likewise $\mathrm{PC.N3} = \mathrm{PC.LV}.$ Therefore if there may be called FE, *a* ; PC, *b* ; FM, *t* ; there will be found $\mathrm{LV}=\left(aat - t^3\right) : aa$ and $\mathrm{PC.LV} = \left(aabt - bt^3\right) : aa$. In addition it is understood the sail DE has another position and in this manner it will have, with respect to which FM shall be *u*, with the others remaining the same as before, in this case there will be the homologous : $\mathrm{PC.LV} = \left(aabu - bu^3\right) : aa$

Thence, in both these two different positions the sail may be turned by the same force, by putting $\left(aabt - bt^3\right) : aa = \left(aabu - bu^3\right) : aa$. Hence $u^3 - t^3 = aau - aat$, which divided by $u - t$, produces $uu + tu + tt = aa$, which is the same equation as before (§. 431). Thus by setting $t = u$, again there will be found $t = \sqrt{\tfrac{1}{3}aa}$, and thus the angle FEM of 35°, 16' ; and the sail DE in this position will be turned by the wind in the most efficient manner.

Thence, if the vertical sails DE and GI under the opposite situation with the same angle or equal to ECB and HCB evidently may be inclined at 54°, 44' to AB, likewise to the horizontal, is understood not to be expressed in the diagram ; the windmill will be turned in the most efficient manner and led around by the wind by striking the sails along directions parallel to AB. [Clearly something has gone amiss in the lower diagram labeling, but it can be seen what is meant.] But sails placed in opposing positions are required to be in the contrary positions, otherwise, if they may be present in the same plane, evidently the mill will not be able to be carried around, since the force of the wind on one sail will be destroyed by an equal force on the opposite sail.

SECTION III.

De Effectis Fluidorum ex percussione.

Expendimus hactenus motus fluidorum, qui resultant ex pressione gravitatis, modo etiam excutiendi sunt effectus eorundem ex percussione, quoties ad alia fluida, aut ad dura corpora , alliduntur vel etiam, quoties solida fluidis impinguntur. Ad hujus generis effectus referri debent motus imminutiones, quas corpora solida in fluidis protrusa tum ratione figurae , tum etiam ratione motus ipsius subeunt, id est, resistentiae corporum in fluidis delatorum. Quae omnia, & nonnulla alia huc spectantia, sigillatim in hac tertia Sectione ad examen revocabuntur.

DEFINITIO.

42l. Cum fluidum in aliud fluidum aut solidum corpus impingitur, fluidi actio in alterum corpus dicitur *Percussio,* & effectus, qui in corpore excipiente percussionem editur, dicitur *Impressio.*

In superioribus subinde impressionis vocabulum latiori paulo sensu sumsimus, etiam pro designando effectu, qui à *pressione* fluidorum provenit.

AXIOMA.

422. Eandem à fluido impressionem excipiet corpus quodcunque solidum, sive id data cum celeritate dataque in directione in fluido feratur, sive fluidum eadem celeritate & directione in idem corpus solidum, sed quiescens, impingat.

CAPUT XI.

De generalibus Affectionibus percussionis fluidorum.

PROPOSlTIO XLII. THEOREMA.

423.*Fluidorum* ABb, CDd *plano* MN *aequabili motu ad angulos rectos occurrentium, ita tamen, ut partes eorum à plano repercussae libere recedere queant, absque eo quod advenientes impediant, quantitates, aequalibus temporibus ad planum* MN *accedentes, erunt in composita ratione celeritatum* AB , CD*, latitudinem* B*b*, D*d & densitatum* M, N. Fig. 102.

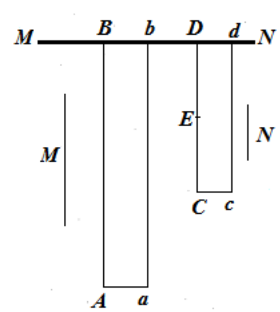
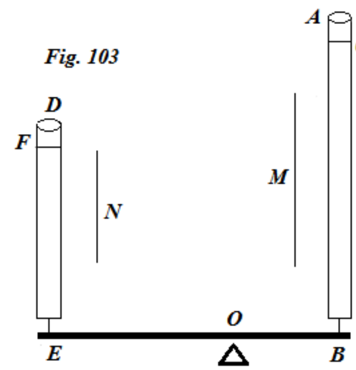


Fig. 102

Sint *ABb*, & *CDd* quantitates fluidorum aequalibus temporibus ad planum MN aequabili motu accedentes, eruntque celeritates accessus ut AB ad CD. Quantitates vero fluidorum ad planum MN simul appellentium sint in composita ratione voluminum & denisatum, & volumina in composita ratione celeritatum AB, CD & latitudinum, ergo massa AB*b* est ad massam CO*d*, sicut AB.B*b*.M ad CD.D*d*.N seu in composita ratione ex rationibus celeritatis AB ad celeritatem CD, latitudinis B*b* ad latitudinem D*d*, & denique densitatis M ad densitatem N. Quod erat demonstrandum.

COROLLARIUM I.

424. Hinc impressiones, quas massae fluidae AB*b*, *CDd* in planum, MN exerent, erunt in composita ratione ex duplicata velocitatum, AB, CD, & ex simplici tum latitudinum B*b*, D*d*, tum densitatum M, N, id est, impressio massae AB*b* erit ad impressionem alterius CD*d*, sicut AB^2.B*b*.M ad CD^2.D*d*.N, vel sumta ED tertia proportionali ad AB & CD, sicut AB.B*b*.M ad ED.D*d*.N. Nam impressiones sunt ut motus fluidorum & motus in composita ratione massarum & velocitatum.

Fig. 103


COROLLARIUM II.

425. Hinc etiam, si fuerint tubi AB, DE liquoribus quibuslibet pleni, quorum liquorum densitates sint M & N , effluantque liquores per foramina aequalia, E, impingantque in extremitates vectis EB circa O convertibilis, erit momentum liquoris CB ad momentum alterius FE, ut M.CB.BO ad N.FE.EO.

Nam percussiones in B & E sunt ut motus fluidorum, adeoque (§.424.) in composita ratione ex duplicata velocitatum & simpla densitatum,. existentibus tuborum foraminibus aequalibus; velocitates vero fluidorum ex tubis erumpentium (§.386) sunt in subduplicata ratione altitudinum CB & FE, seu duplicata velocitatum eadem, quae altitudînum CB & FE; ac propterea percussio in B erit ad percussionem in E sicut CB.M ad SE in N. Verum hae percussiones, quatenus in vectem BE circa O convertibilem agunt, habent rationem potentiarum motricium vecti applicatarum, ergo cum potentiae vectis habeant momenta proportionalia factis ex potentiis ipsis ia distantias earum ab hypomochlio O, habetur momentum percussionis in B ad momentum percussionis in E, sicut M.CB.BO ad N.FE.EO.

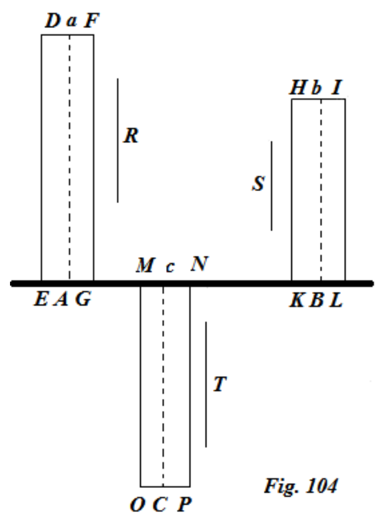


Fig. 104

COROLLARIUM III.

Fig. 104

426. Propterea torrentes DG, HL, celeritatibus *a*A, *b*B ad planum EL accedentes, in aequilibrio consistent cum tertio ON celeritate *Cc* in idem planum EL impingente, si positis torrentum densitatibus R, S , T, & latitudinibus EG, KL, MN , fuerint

$\text{R.A}a^2.\text{EG.AC} = \text{S.B}b^2.\text{KL.BC}$ & $\text{R.A}a^2.\text{EG} + \text{S.B}b^2.\text{KL} = \text{T.}Cc^2.\text{MN}.$

Nam si planum EL consideretur instar vectis, erunt (§.425.)

$\text{R.A}a^2.\text{EG.AC}$ & $\text{S.B}b^2.\text{KL.BC}$ momenta percussionis aequalia, atque adeo in aequilibrio consistent cum tertio OMN quandoquidem (secundùm hypothesin) ejus media directio *c*C transit per *C*, & ejus percussio, seu $\text{T.C}c^2.\text{MN}$, aequatur duabus oppositis potentiis simul sumtis, scilicet $\text{R.A}a^2.\text{EG} + \text{S.B}b^2.\text{KL}$.

In hoc corollario praecipua continentur quae circa collisiones & percussiones fluidorum inter se notatu digna occurrere possunt.

COROLLARIUM IV.

427. Resistentiae, quas corpora solida in fluidis delata patientur, erunt in composita ratione densitatum & duplicatae velocitatum: adeoque hae resistentiae in uno eodemque fluido erunt in duplicata ratione celeritatum. Nam, quia (§.422.) corpora in fluidis incedentia easdem a fluido impressiones subeunt, quas subirent, si fluidum ea celeritate, qua corpora in eo feruntur, in hae corpora quiescentia impingeret, & quia hae impressiones sunt ut percussiones fluidi, id est in composita ratione ex duplicata celeritatum & simplici densitatum; resistentiae vero ut impressiones, quas corpora, quibus resistitur, in fluido subeunt; liquet omnino resistentias, quas corpora in medio fluido mota patientur, esse in composita ratione densitatum & duplicata velocitatum, aut si corpora in eodem medio ferantur, in duplicata ratione celeritatum.

PROPOSITIO XLIII. THEOREMA.

428. *Si fluidum, seu torrens CABD, data quadam celeritate impingat perpendiculariter in planum* AB, *deinde etiam eadem celeritate, sed sub angulis obliquis* KOF, KOE *alteri plano* EF *primo* AB *aequali, erit impressio, quam subibit planum* AB , *ad impressionem alterius plani* EF *juxta directiones OP, OQ planis normales, in duplicata proportione sinus totius ad sinum anguli incidentiae* KOE ; *vel, ductis per* E, O *&* F *perpendicularibus* HL, KOP, IMF *ad planum* AB, *&* MN *normali plano* EF, *sicut* OF *ad* ON. Fig.150.

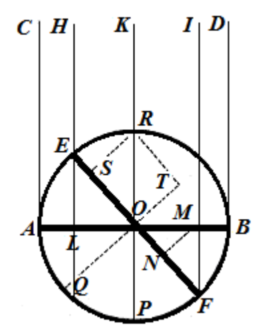


Fig. 105

Quod fluida CABD, HEFI impressiones in plana AB & EF exerant juxta directiones OP, OQ planis normales, constat ex §.249. Ex puncto R agantur RS, RT perpendiculares & parallelae plano EF. Jam, quia filamentum fluidi KO ad angulos rectos in planum LM incidit, sub obliquis vero KOE, KOF in planum EF; & si RO celeritatem filamenti exponat, per se claret fore, ut filamentum totam suam vim in planum obliquum EF exerere nequeat, quia directio RO plano isti directe opposita non est, sed resoluta celeritate

RO in laterales aequipollentes RS & R T, duntaxat aget in planum EF celeritate RS plano isti directe opposita atque contraria, cum altera RT utpote cujus directio plano EF . parallela est, nullam prorsus impressionem in idem planum exerere possit. Est igitur impressio filamenti KO in plano LM ad impressionem ejusdem in plano EF ut RO ad RS, seu ut OF ad OM*;* unde quia idem est filamentorum numerus binis planis LM & EF allabentium, erit impressio torrentis HLMI in plano LM ad impressionem ejusdem in plano EF, sicut OF ad OM; impressio vero torrentis CABD in plano AB est ad impressionem torrentis HLMI in plano LM, sicut AB ad LM, id est, sicut OF ad OM, aut OM ad ON; ergo ex aequo impressio torrentis CABD in plano AB est ad impressionem torrentis HEFI in plano EF, sicut OF ad ON, id est, in duplicata ratione sinus totius OF ad sinum, anguli incidentiae IFO vel KOE. Quod erat demonstrandum.

COROLLARIUM

429. Fig.106. Hinc si fluidum MABN plano AB allabatur celeritate data V, erit impressio, quam planum istud à fluido in id impingente excipiet, factum ex quadrato velocitatis in rectam AD ; ducta scilicet AC perpendiculari ipsis AM, BN & demissa ex C super AB perpendiculari CD. Nam (§. 424.) impressio, quam subiret planum AB à fluido perpendiculariter impingente, est ut $V^2.AB$, per propositionem vero praesentem est impressio perpendicularis fluidi in plano AB ad impressionem ejusdem, sed oblique impingentis plano AB, ut AB ad AD, seu $V^2.AB$ ad $V^2.AD$; ergo $V^2.AD$ exponit impressionem fluidi oblique incidentis MABN in planum AB celeritate V, haecque impressio exseritur juxta directionem EF plano perpendicularem & per ejus centrum gravitatis transeuntem, per §. 249 & §. 54.

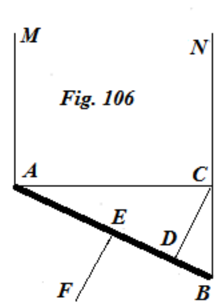


Fig. 106

SCHOLION I.

430. Fig. 107. Ex praesenti propositione facile deducitur vis clavi seu gubernaculi, cujus ope naves, in quem volueris situm, converti possunt; quod paucis est indicandum. Sint itaque AB navis, B puppis cujus cardini inseritur clavus BH convertibilis circa B, cui per centrum gravitatis X normalis, ducta sit Xy. Ponamus jam navem progredi juxta BA ex B versus *A* quacunque data velocitate, clavumque BH firmatum esse, ut cum AB producta in T quemcunque angulum acutum HBT constituat: Progrediente igitur navi versus E, aqua ABHG, juxta directiones rectae AB parallelas, ea ipsa celeritate in clavum BH impinget, qua navis contrario sensu versus E fertur; & quia (§.422.) idem effectus resultat, si aqua ABHG data celeritate in clavum navis quiescentis impingat, quae si navi progrediente clavus eadem illa celeritate aquae allidatur, & aquae clavo impingentis impressio est BS, posita unitate pro velocitate, ductisque BR

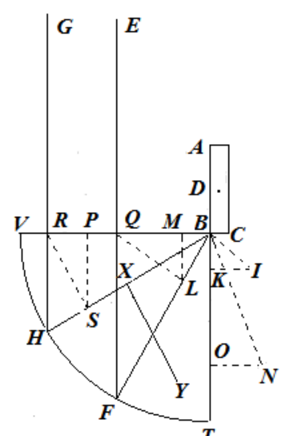


Fig. 107

normali ipsi GH, & RS normali BH, quae impressio exponenda per BS exeritur in clavum juxta directionem XY per centrum gravitatis clavi BH ductam. Idcirco progrediente navi juxta BA, aqua resistens eundem effectum praestabit, ac si clavus impelleretur juxta XY potentia quadam BS; atque adeo hinc liquet, hac potentia BS navim in seipsam conversum iri. Sed quia directio XY alteri HG, juxta quam navis progreditur, non conspirans nec directe contraria, nec etiam perpendicularis est, potentia BS quatenus directioni XY applicatur, non est absoluta potentia, quae navis conversionem causetur; sed ductis BN perpendiculari BH, aequalique facta ipsi BS, & NO parallela BV , haec NO exponet vim plenam conversionis navis; quoniam ON normalis est directioni navis OA, & potentia $BN = BS$ resolvitur in aequipollentes BO & ON, quarum BO, in directum posita ipsi OA, nihil ad gyrationem navis conferre, sed tantum modo ejus motum progressivum aliquantum morari potest, adeo ut sola potentia ON restet, quae conversionis effectum praestet. Similiter si clavus habeat situm BF, denotabit KI vim rotationis navis, si scilicet ductis QL & BI normalibus clavo BF, factaque $BI = BL$, etiam IK rectae AT perpendicularis ducta fuerit.

431. Ex contemplatione figurae statim apparet, ipsas KI & ON aequales esse posse, etsi anguli IBK & NBO diversi sint. Ex quo sequitur, duas diversas clavi positiones aequaliter aptas esse posse ad navis conversionem circa se ipsam. Agantur enim SP & LM parallelae TB, quae ipsis ON & KI respective aequales erunt, quandoquidem triangula PSB & ONB, nec non MLB & KIB similia & aequalia existunt. Jam $HR : PS (= HB : SB) = HB^2 : RB^2$, & $ML : QF = QB^2 : FB^2$; ergo si PS & ML vel KI & ON aequales sint, erit ex aequo $HR : FQ = QB^2 : RB^2$, atque adeo $HR.RB^2 = FQ.QB^2$. Hinc, si BH vel BF dicatur a; RH, t; QF ,u, erunt $RB^2 = aa - tt$, & $QB^2 = aa - uu$, adeoque $HR.RB^2 = FQ.QB^2$ fiet $aat - t^3 = aau - u^3$; seu $u^3 - t^3 = aau - aat$, adeoque dividendo per $u - t$, erit $uu + ut + tt = aa$, quae est aequatio indeterminata infinitas suppeditans diversas clavi positiones, quarum binae & binae aequaliter aptae sunt ad navis conversionem.

432. Idcirco, si $u = t$, habebitur $3tt = aa$, atque adeo $t = \sqrt{\frac{1}{3}aa}$; & hic valor sinus anguli VBH dabit positionem gubernaculi BH, in qua vis rotationis navis *maxima* existet. Unde si angulus VBH fuerit graduum 35, 16' circiter , vel angulus HBT graduum 54, min. 44, positio clavi BH ad conversionem navis aptissima erit.

SCHOLION II.

433. Quae in praecedenti scholio ostensa sunt, etiam molis alatis Fig. 108, 109, facili negotio possunt applicari. Sit enim DE ala verticalis vento circumagenda, cum axicula molae AB quemlibet angulum acutum ECB continens; ventusque celeritate ut 1, & in directione nN parallela CB, alae DE allabatur,

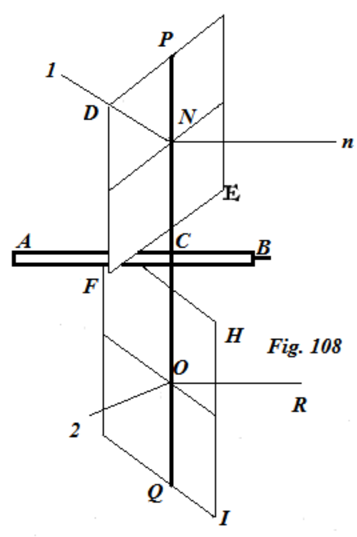


Fig. 108

sintque E*e*, F *f* parallelae AB, & $FC = CE$, ductisque EM, ML normalibus ad *Ff* & FE, & LV parallela rectae Ff. Per alae centrum N ducta intelligatur N1 aequalis LE plano alae DE perpendicularis, & exponet haec N1 venti impressionem ejusque directionem, quam quaelibet linea 2F2E alteri FE parallela subibit, adeo ut totalis impressio, quam ala à vento excipiet, juxta directionem alae normalem, futura sit PC.N1. Porro si per punctum 1 transeat recta 13 parallela N*n* vel AB, & per punctum N recta N3 eidem N*n* normalis, impressioni N1 aequipollebunt laterales 31 & N3, quarum prima 31, utpote AB seu axiculo parallela, ad alae conversionem nihil confert; sed tanquam N3 eidem axiculo molae AB perpendicularis: adeoque vis rotationis alae ex impressione juxta N1 derivata erit PC. N3. Verum, quia rectae NI , 13 & N3 totidem aliis LM , LV & MV parallelae sunt in plano EFM, & triangula LEV &. MLV similia, erunt etiam MLV & 1N3 aequiangula & aequalia, quoniam insuper est $LE = N1$. Hinc etiam $LV = N3 =$ vi rotationis alae DE. Hinc itidem $PC.N3 = PC.LV$.

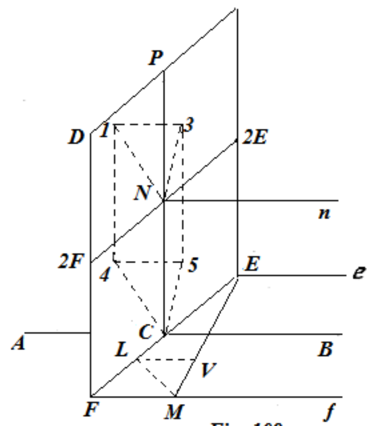


Fig. 109

Igitur si dicantur FE, *a* ; PC, *b* ; FM, *t* ; reperietur

$LV = \left(aat - t^3\right) : aa$ & $PC.LV = \left(aabt - bt^3\right) : aa.$

Intelligatur insuper alam DE aliam habere positionem ac modo habuit, cui respectiva FM sit *u*, reliquis iisdem manentibus, quae antea, erit hoc casu homologa $PC.LV = \left(aabu - bu^3\right) : aa$

Unde, si in binis hisce diversis positionibus ala debeat eadem vi circumagi, ponendum $\left(aabt - bt^3\right) : aa = \left(aabu - bu^3\right) : aa$. Hinc $u^3 - t^3 = aau - aat$, quae divisa per $u - t$, praebet $uu + tu + tt = aa$, quae est eadem aequatio ac antea (§. 431). Adeoque facta $t = u$, reperitur iterum $t = \sqrt{\tfrac{1}{3}aa}$, atque adeo angulus FEM graduum 35 min: 16; alaque DE in hac positione a vento efficacissime circumagetur. Unde, si alae DE & GI verticales sub contrario situ eodem angulo seu aequali ECB & HCB scilicet 54 gr. 44' ad AB inclinentur,idemque de horizontalibus, in schemate non expressis, intelligatur; mola alata efficacissime versabitur & circumducetur à vento juxta directiones AB parallelas in alas impingente. Alas autem oppositas situm subcontrarium habere oportet, alioqui, si in eodem plana existerent, prorsus circumagi non possent, quandoquidem venti impressio in una ala destrueretur aequali impressione in ala opposita.