CHAPTER VIII.

Concerned with the densities of air in different places in the atmosphere under all possible hypotheses of the elasticities.

In this chapter the density is not to be disturbed by air of densities other than what may be induced by the weight of the incumbent air ; therefore in these the discussion will not be concerned with the rarefaction and condensation of the air arising from heat and cold. [*i.e.* an isothermal atmosphere is to be considered.]

DEFINITIONS.

I.

354. The curve C1C2C constructed near the axis OA2A, see Fig. 87 and 88, the ordinates of which AC, 2A2C, &c. express the variable weight of any body at the places A, 2A, &c. through which the ordinates pass, may be called the graph [scale] of the *variation of the weight* [with height].
[In retrospect, we may call this the acceleration of gravity at a height *h* above the surface of a spherical earth, which has the radius R, and where the acc. is g_0 : $g_0(h)/(1+h/R)^2$. As most of the atmosphere lies close to the surface of the earth, g can be taken as g_0, or the weight of a body is constant with height, as in Fig. 89 below.]

II.

355. Truly the curve B1B2B about the same axis A2A shall describe the graph of the densities of the atmosphere, because its ordinates AB, 2A2B, &c. express the densities of the atmosphere at the places A , 2A, &c. through which the ordinates have been drawn.
[At this stage, Hermann was unable to write down the differential equation expressing the variation of density or pressure with the height for an isothermal atmosphere, assuming a Boyle's law type equation of state, where the pressure is proportional to the density, and resorted instead to various possible relations between the pressure and density described below, although he conceded eventually that a logarithmic graph gave the best fit of pressure with altitude. Cassini and Mariotte independently had used a parabolic model for the variation, to measure the heights of mountains barometrically. See below.]

III.

356. And the curve A2DD below the horizontal line CB described about the same axis AO, shall be the graph of the elasticity of the air, evidently the abscissas of which AO, A2O, A1O represent the elasticities of the air, according to the densities OD, 2O2D, 1O1D.

IV.

357. With the ordinates of the graph of the elasticity of the air DO, 1D1O, 2D2O produced to E, 1E, 2E, &c., if the individual rectangles DOE, 1D1O1E, 2D2O2E were equal to each other in the given plane, the points E, 1E, 2E, &c. will be on the curve E1E2Ee, which hence we will call the *reciprocal of the graph of the elasticity*.

AXIOM.

358. The spring of the air is equal always to the force of compression, and thus at any place the elastic force of the atmosphere will be equivalent to the weight of the incumbent air. For the forces are directly opposed, as the force compressing the air and the spring of the air are equal, because everything is understood to be put in a state of rest.

[The interested reader can consult the translation of Newton's *Principia* on this website, Book II, Prop. XXII and corollaries, etc., which include extended notes by Leseur and Jacquier, etc., on the matters discussed below.]

PROPOSITION XXVI. THEOREM.

359. *Any equal ordinates* ab & od *in the graphs of the densities and elasticities of the air, produced beyond the common axes of the graphs* AT*, always cut off equal areas* ACca & OEeo *in the graphs of variable gravity and of the curve of the reciprocal elasticity of the air*. See Fig.87. [This prop.]

The ordinates $\alpha\beta$ & $\omega\delta$ may be moved indefinitely from their respective ordinates in the vicinity of *ab* & *od* at χ & f. Now the weight of the air [in the column] *a*T incumbent at the place *a*, and the density of the air induced at this place *ab* or *od* (§.358) is equal to the elasticity of the air set out by the abscissa A*o*, and the weight of the incumbent air at the place *a* is equal to the elasticity of the air at α, which the abscissa Aω is representing, and thus the difference of the weights at αT and *a*T, that is, the weight of the column of air αa will be equal to the difference of the elasticities Aω and A*o*, that is ωo. And (§. 32.) the weight of the column αa is expressed by the volume $ab.ac.\alpha a$; therefore this volume is equal to the other volume with ωo into the given plane EO.OD, or (§.357.) equal to the plane *eo.od* itself, that is, $ab \cdot ac \cdot a\alpha = eo \cdot od \cdot \omega o$; or with the equalities *ab* and *od* deleted (following the hypothesis), the rectangle $ac \cdot a\alpha =$ rectangle $oe \cdot \omega o$, and since by descending towards CB and ED it arises everywhere, that the individual elements of the areas AC*ca* and OE*eo* shall be equal, also they will be equal generally, that is every $ac \cdot a\alpha$ or the area AC*ca*, which they exhaust, will be equal to all the $oe \cdot \omega o$, or to the area OE*eo* [*i.e.* on integrating the small equal quantities treated as differentials]. Q.e.d.

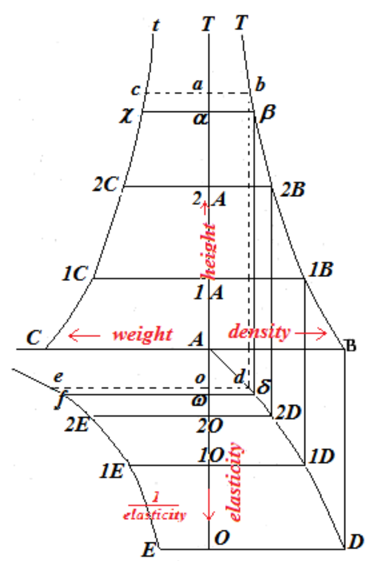


Fig. 87

[Thus, in terms of differentials, we can write $\rho kgdh = -\rho \frac{dp}{p}$; if g is constant, then we have $\log p = -kgh$ and $p = p_0 e^{-kgh}$ for some constant *k*, otherwise we have a numerical integration, as the author indicates. This integration could not be performed exactly at the time, as the exponential function lay in the future, and Hermann used his own scheme as indicated below.]

COROLLARY I.

360. If now the graph of the elasticities of the air A1D2D may be considered to be a parabola of indefinite degree, of which any ordinate 1O1D shall be as $\mathrm{A1O}^m$, or so that the power of the abscissa A1O denominated by the exponent *m*, will be on account of the rectangle 1D1O1E, equal to the given rectangle DOE, and thus on account of 1E1O being inversely proportional to the other 1D1O, 1E1O varies reciprocally as $\mathrm{A1O}^m$, and thus the curve E1E2E will be a hyperbola of indefinite degree *m*. In addition the curve C1C2C shall be another hyperbola of indefinite degree *n*, thus so that any ordinate of that 1C1A shall be inversely as the power of the abscissa *n*, that is, inversely as $\mathrm{O1A}^n$. Now if the equal ordinates 1A1B or 1O1D express the density of the air at 1A, (§. 359) the four-lined figures A1A1CC and O1O1EE will be equal, and following the method established above [See (§.92) Ch. 3, Section 1, Book 1 ; It is probably worthwhile to for the interested reader to re-examine this section.]
the four-lined figure $\mathrm{A1A1CC} = (\mathrm{OA.AC} - \mathrm{O1A.1A1C}) : n-1$, and the four-lined hyperbolic figure :
$\mathrm{OE1E1O} = (\mathrm{A1O.\ 1O1E} - \mathrm{AO.\ OE}) : m-1 = (\mathrm{AO.\ OE} - \mathrm{A1O.1O1E}) : 1-m.$ Therefore $(\mathrm{OA.AC} - \mathrm{O1A.1A1C}) : n-1 = (\mathrm{AO.OE} - \mathrm{A1O.1O1E}) : 1-m$. Now, because AC and OE themselves are of no determined magnitude, there may be put $\mathrm{AC} : \mathrm{EO} = n-1 : 1-m$, and the rectangle OA.AC to the rectangle AO.OE will be in the same ratio and consequently also the rectangle O1A.1A1C will be in the same ratio to the rectangle A1O.1O1E as $n-1$ to $1-m$ or AC to OE. But truly on the hyperbola C1C2C there is the ratio $\mathrm{OA.AC} : \mathrm{O1A.1A1C} = \mathrm{O1A}^{n-1} : \mathrm{OA}^{n-1}$, therefore from the equation and by the addition of the ratios there will be $\mathrm{OA.\ AC} : \mathrm{A1O.1O1E} = \mathrm{AC.O1A}^{n-1} : \mathrm{OE.OA}^{n-1}$, likewise in the hyperbola E1E2E, there is $\mathrm{A1O.1O1E} = \mathrm{AC.OE} = \mathrm{AO}^{m-1} : \mathrm{A1O}^{m-1}$ (that is on account of the parabola A2D1D) $= \mathrm{OD}^{m-1:m} : \mathrm{1O1D}^{m-1:m}$, therefore from the equation there is found anew $\mathrm{OA.AC} : \mathrm{AO.OE} = \mathrm{AC.O1A}^{n-1}.\ \mathrm{OD}^{m-1:m} : \mathrm{OE.OA}^{n-1}.\mathrm{1O1D}^{m-1:m}$. Truly from these it is deduced that $\mathrm{O1A}^{n-1}.\ \mathrm{OD}^{m-1:m} = \mathrm{OA}^{n-1}.\mathrm{1O1D}^{m-1:m}$, and with the quantities given removed, or replaced by constants which do not alter the proportions, evidently $\mathrm{OD}^{m-1:m}$ and OA^{n-1}, there will be $\mathrm{O1A}^{n-1}$ as $\mathrm{1O1D}^{m-1:m}$, or on dividing the exponents by $m-1:m$, the following will result 1O1D as $\mathrm{O1A}^{mn-m:m-1}$.
Hence :

1°. If the cube of the compressing force shall be proportional to the fourth power of the density and the weight shall be inversely as the square of the distance from the centre of gravity, there will be $m = \frac{3}{4}$ and $n = 2$, and thus $mn - m : m-1 = -3 : 1$, therefore 1O1D in this case will be as O_1A^{-3}, that is , the density will be inversely as the cube of the distance.

2°. If the cube of the compressing force shall be as the fifth power of the density, and the weight inversely as the square of the distance, there becomes $m = \frac{3}{5}$ and $n = 2$, therefore $mn - m : m - 1 = -\frac{3}{2}$, that is, the density $_1O_1D$ will be in the inverse three on two ratio of the distance O_1A.

3°. If the compressing force shall be as the square of the density, and the weight in the inverse square ratio of the distance, there arises $m = \frac{1}{2}$ *and* $n = 2$, therefore $mn - m : m - 1 = -1$, which indicates in this case, the density to be in the inverse ratio of the distance.

From these it is apparent the truth of some assertions are advanced without any proof by the illust. Newton in the Scholium after Prop.22, Lib.II. *Pr. Phil. Nat. Math.* And, though this corollary shall be only a particular case of our most general proposition, yet the same will be encountered in infinitely many diverse cases.

COROLLARY II.

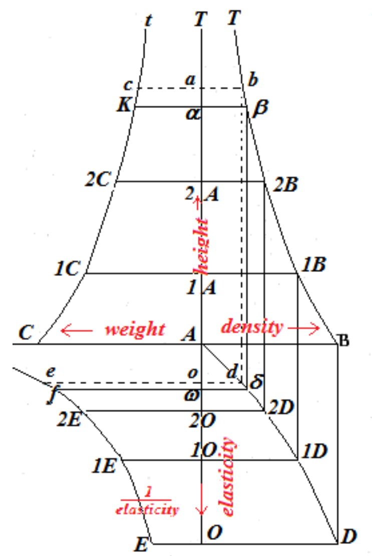


Fig. 87

361. Now if the graph of the elasticity of the air A_2D_1D were a straight line, that is, if the elasticity of the air, or the compressing force, shall be as the density, the curve E_1E_2E will be a conic [*i.e.* proper] hyperbola described within the asymptotes CA & AO ; for there is $_1E_1O : EO = OD : {_1O_1D}$ (on account of the right line A_2D_1D) $= AO : A_1O$. Hence, if the quadrilinear hyperbolic figures EO_1O_1E, $_1O_2O_2E_1E$, &c. shall be equal to each other, and from these just as many equal quadrilinear figures will be equal in the graph of the weight variable AC_1C_1A, $_1A_1C_2C_2A$, &c. of the homologous ordinates AB, $_1A_1B$, $_1A_2B$, &c. in the graph of the densities B_1B_2 will be continued proportionals. For on account of the equality of the hyberbolic trapaziums O_1E, $_1O_2E$, &c. the abscissas AO, A_1O, A_2O, &c. or from the abscissas and from the respective ordinates in the graph of the elasticity OD, $_1O_1D$, $_2O_2D$, &c. that is AB, $_1A_1B$, $_1A_1B$, &c., with the same arrangement assumed equal, they will be in continued proportion. Therefore, if the compressing forces, or the equivalent elastic forces of the air with its proportional densities, truly in the graph of the weights, the quadrilinear shapes AC_1C_1A, $_1A_1C_1C_1A$, &c. were equal, the densities of the air at the places A,$_1A$, $_2A$ &c. will be always in continued proportion.

COROLLARY III.

362. Therefore if the weight at any place 1A, or the ordinate 1A1C were inversely as $\mathrm{O1A}^{n}$, with a hyperbolic curve C1C2C present of the indefinite order *n*, and the magnitudes OA^{1-n}, $\mathrm{O1A}^{1-n}$, $\mathrm{O2A}^{1-n}$, &c. may be assumed in an arithmetic progression of ascending order, the ordinates of the graph of the densities AB, 1A1B, 2A2B, &c. will be continued proportionals. For since OA^{1-n}, $\mathrm{O1A}^{1-n}$, $\mathrm{O2A}^{1-n}$, &c are proportionals to the rectangles OA.AC, O1A.1A1C; O2A.2A2C, &c. thus these rectangles will be in an arithmetic progression, and thus that is $(\mathrm{O1A.1A1} - \mathrm{OA.AC}) : n-1$, following that which has been said in §.360 , the four-sided figure AC1C1A is equal to $(\mathrm{O2A.2A2C} - \mathrm{O1A.1A1C}) : n-1$, or to the four-sided figure 1A1C2C2A, and thus (§:361.) the ordinates AB, 1A1B, 2A2B, &c. will be continued proportionals.

COROLLARY IV.

363. If now *n* shall be 1, and thus the hyberbolic conic C1C2C having the centre at O, the equal four-sided figures A1C, 1A2C, &c. will have the abscissas OA, O1A, O2A, &c. in continued proportions, & vice versa, if these abscissas were in a geometric progression, the aforementioned four-sided figures will be equal, and consequently the densities AB, 1A1B, 2A2B, &c. will be continued proportionals, planely as Prop. 21, Lib. II *Princ. Ph. Nat. Math*. of the Cel. Newton indicates, as he has shown separately.

COROLLARY V.

364. If *n* shall be 2, and thus the curve C1C2C a square hyperbola ; that is, if the weights of bodies shall be in the inverse square ratio of the distances from the centre, and OA^{-1}, $\mathrm{O1A}^{-1}$, $\mathrm{O2A}^{-1}$, &c. shall be in an arithmetic progression, and thus the reciprocals of this series OA ,O1A, O2A shall be in a harmonic progression (§. 361, 362) the ordinates AB, 1A1B, 2A2B, &c. will be in a geometric progression. This demonstration the celebrated Newton himself gave by another method in Prop.22. Lib. II.

COROLLARY VI.

365. If $n = -1$, that is, if the weight is as the distance of the body from the centre O, the curve C1C2C will be changed into a right line passing through the centre O, and the series, which following §. 362., must be in the arithmetic progression OA^{1-n}, $\mathrm{O1A}^{1-n}$, $\mathrm{O2A}^{1-n}$ &c. now becomes OA^{2}, $\mathrm{O1A}^{2}$, $\mathrm{O2A}^{2}$, &c. and thus, if the squares of the distances OA, O1A, O2A, &c. were in an arithmetic progression, the rectilinear trapezia, A1C, 1A2C, &c. will be equal, and thus the ordinates (§.361) of the graph of the densities AB, 1A1B, 2A2B , will be in continued proportionals. As the praiseworthy Newton has asserted, but without any demonstration in the Scholium after Prop. XXII. Lib. II. *Princ. Phil. Nat. Math.*

COROLLARY VII.

366. Truly if *n* were 0, that is, if the weights of bodies were uniform or constant, such as is accustomed commonly to be considered, then the graph of the weight C1C2C will be a right line AT parallel to the axis, as in Fig. 88, and the series OA^{1-n}, $\mathrm{O1A}^{1-n}$, $\mathrm{O2A}^{1-n}$, &c. which must constitute an arithmetic progression to that, so that the ordinates AB, 1A1B, 2A2B, &c. shall be in continued proportion, now OA, O1A, O2A, &c. will be simply in an arithmetic progression, and thus the ordinates of the graph of these distances A1A, 1A2A of the densities of the atmosphere will be equal, and the graph itself B1B2B thence will be logarithmic. The wisest Edmund Halley first observed a corollary similar to this, which I know was now some time ago.

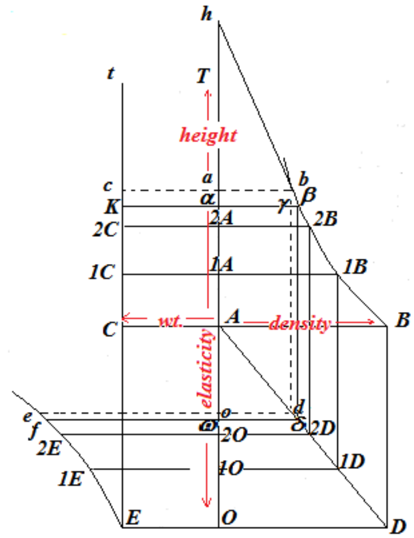


Fig. 88.

COROLLARY VIII.

367. With the same in place, as in the preceeding corollary, if the graph of the uniform gravity tC may be produces as far as to the crossing of the hyperbola E1E2E at E, and through this point E, EOD of the graph of the elasticity of the air is acting crossing A2D1D at D, the subtangent *ah* of the logarithmic B1B2B is equal to the abscissa AO of the graph of the elasticity, of the corresponding ordinate OD. Now (§.359) there is $ac \cdot a\alpha = o\omega \cdot eo$ and thus $o\omega : a\alpha = ac : eo = \mathrm{EO} : eo = od : \mathrm{OD} = \mathrm{A}o : \mathrm{AO}$ and $g\delta$ or $\gamma\beta$ is to *dg* or $o\omega = od$ or $ab : \mathrm{A}o$, therefore from the equation $\gamma\beta : a\alpha = ab : \mathrm{AO}$ and on account of the similar triangles $\beta\gamma b$ and bah, there is $\gamma\beta : a\alpha$ or $b\gamma = ab : ah$, therefore alos $ab : \mathrm{AO} = ab : ah$, and thus AO is equal to the subtangent *ah* of the logarithmic curve B1B2B.

COROLLARY IX.

368. Morever with the same in place, any four-sided hyperbolic figure OE1E1O will be to the given rectangle AOE, as A1A the distance between the ordinates AB and 1A1B of the equal ordinates OD, 1O1D of the graph of the applied hyperbola OE, 1O1E, in the put in place, to the subtangent *ah* of the logarithmic curve B1B2B. For, because (following the hypothesis) $\mathrm{OE1E1O} = \text{rectangle } \mathrm{AC1C1A}$,
the four-sided figure $\mathrm{OE1E1O} : \mathrm{AO.OE} = \mathrm{A1A.AC} : \mathrm{AO.AC} = \mathrm{A1A} : \mathrm{AO} = \mathrm{A1A} : ah.$

COROLLARY X.

369. Because the heights of the mercury in the barometer are, as the atmospheric pressures at the different distances from the horizontal, and these atmospheric pressures and elastic pressures of the supporting air, if the elasticities were proportional to the densities of the air, as indeed we have supposed in corollaries 2, 3, 4, 5, & 6 of this Proposition, then the heights of mercury in the barometer put at the places A, 1A, 2A, &c., will be as the ordinates AB, 1A1B, 2A2B, &c. in the graph of the densities B1B1B, if evidently the constant weights of bodies, or in general the distance from the centre of the earth O were the same, as that can be assumed without risk. Therefore with given amounts of mercury present in the barometer at the places A, 1A, and 2A, and for the height A1A , with the aid of logarithms the other distance or height A1A will be found. For, because A1A is to A2A as the log of the ratio AB to 1A1B, to the log of the ratio AB to 2A2B; if the log of the ratio AB:2A2B may be taken at the given height A1A, and the product is divided by the log of the ratio AB:1A1B; the quotient will show the height sought A1A. But if truly from the given heights A1A, A2A and the ratio AB:1A1B the ratio AB:2A2B is sought, or the ratio of the mercury located in the barometer at A to the amount of mercury present in the barometer at 2A; only the log of the ratio AB:1A1B is required to be multiplied by A2A and product being divided by A1A, and the quotient will show the logarithm of the ratio sought AB:2A2B, truly from the logarithm the ratio itself will become known at once from the usual table of logarithms.

SCHOLIUM.

370. So that the rule of corollary nine may be illustrated by another example, the height of the mercury at A or AB to be 28 inches, and at the height of the place 1A above A, that is A1A, 63 feet to be one line of mercury less, thus so that 1A1B shall be 335 lines only, compared with AB, as 28 inches contains 336 lines. Truly at the height A2A or at the place 1A the amount of quicksilver shall be deficient by the amount $16\frac{1}{3}$ from the amount at A or AB, thus so that 2A2B shall be as great as $319\frac{2}{3}$ lines. The height A2A is sought from these given. Following the rule, (§. 369.) the log of the ratio AB: 2A2B or now the log of $336:\ 319\frac{2}{3}$ must be multiplied by A1A or 63 and the product divided by the log of AB:1A1B or the log. 336:335, and the quotient will indicate the quantity sought ; and the product from $\log.336 - \log.319\frac{2}{3}$ into 63 divided by log.336−log. 335, produces $1053\frac{1}{4}$; therefore this number expresses how many feet the height sought shall be. We have obtained this example from Mariotte's *Tentamine De Natura aëris*, [*The Nature of the air from Experiment*] who refers on pages 194 and 195, the most celebrated Cassini to have observed at one time on the summit of some French mountain in Provence ; the height of which he found with care with a measure of 1070 feet, the mercury of the barometer was lowered by $16\frac{1}{3}$ lines, as at the base of the mountain where it was supporting 28 inches, therefore the height of the mountain elicited from our calculation to be around 17 feet deficient from the observed 1070 feet, but this difference thence arises by chance, because with Mariotte we have put the quick silver to be deficient by one line at a height

of 63 feet with a mercury barometer on the horizontal raised to 28 inches. Truly if the height of the mountain at which the mercury in the barometer decreases by one line, or becomes greater by one foot, evidently becomes 64 feet, then the calculation according to the above rule at once will provide a height of the mountain of $1069\frac{44}{95}$ feet, thus so that it may be deficient by one 95th part of a foot from the observed height.

371. Even if the most enlightened Mariotte makes his hypothesis so that the density of the air shall be proportional to the compressive forces of the air, and it is agreed the heights of places being elicited from the differences of heights of quicksilver and with the calculation by logarithms ; nevertheless in his calculation adapted from the same example of the preceding paragraph, it is done without the use of logarithms, but indeed easier, and thus truly by a less exact calculation, the height of the mountain may be elicited to be 1080 feet, and that by the following method. Since the atmospheric pressure at the lowest place is equivalent to 28 inches of mercury or 336 lines, the whole atmosphere is to be divided into 336 parts of equal weight, of which each one is equivalent to the weight of a single line of mercury ; but the heights of these parts of the air will be unequal, thus indeed, so that height which corresponds to the half division 168 of 336, shall be twice as high or greater than the lowest 63 ft. of the first mercury line agreeing towards the horizontal, and thus it shall be 126 feet ; the height or the differences of the parts of the atmosphere, homologous to the individual lines of mercury, is imagined to increase upwards in an arithmetic progression, rather than by a geometric one, as it can be judged correctly, the increments of the parts can be considered to hardly change ; therefore on dividing 63 by 168, the increment of the second division of the atmosphere above the first to the horizontal will be found to be approximately $\frac{63}{168}$, and on taking 63 by $16\frac{1}{3}$, 1029 feet is found for the total height, or for the height of the mountain, but only if the corrresponding individual divisions of the atmosphere of $16\frac{1}{3}$ lines of mercury may appear to be the same, indeed because they increase in arithmetical proportion, thus all the increments found add up to a height of 1029 ; hence in the end the number 136, which is the sum of all the numbers of the natural progression l, 2, 3 as far as to 16 inclusive, multiplied by $\frac{63}{168}$, and the product 51 itself shows the sum of all the increments, and thus this number 51 added to the other 1029 gives the sum 1080 expressing the height of the mountain sought.

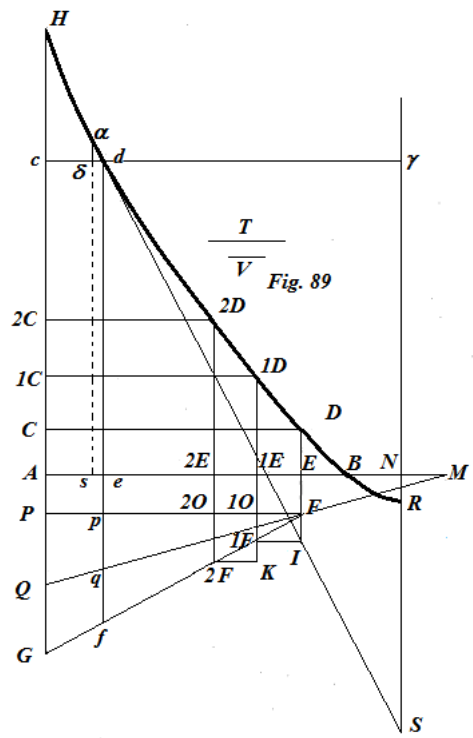
372. The most distinguished Maraldi, [Giacomo Filippo or Jacques Phillipe Maraldi 'Maraldi I' was the nephew of the original Giovanni Domenico Cassini, then employed by the French king as an astronomer; in 1700 a project was initiated to measure out the Meridian through the Royal Observatory from Dunkirk to the south of France ; Maraldi so aiding Cassini in his cartographic activities by his measurements of the heights of French mountains from his barometric calculations ; these activities were carried on by several generations of the two families to give an ordinance survy map of France. See Cassini de Thury's *Description geometrique de la France* (1783)], likewise separates the atmosphere into 336 parts of equal weight as well as Mariotte, and he imagines the

heights of these parts to increase in an arithmetic progression, yet recognising the spans of the air not to be precisely in the inverse ratio of the incumbent weight, as Mariotte's hypothesis required, then also he has departed from the number of Mariotte, setting up the first part of the atmosphere of the weight of one line of mercury contiguous to the horizontal agreeing to be 61 feet, and after this the numbers following to increase almost as 1, 2, 3, 4, &c. thus so that the second, third, fourth, etc. parts shall become 62, 63, 64, &c. feet. This progression to be made from all the above barometric observations, for the different French mountains, the praiseworthy man indicates to be calculated in a proper enough manner, with the advancement of the Meridian thence from the Royal Observatory through the south of France district, the work being directed by the celebrated Cassini, along the way he might put his aid at the disposal of others [This sentence is obscure] ; and thus within the limits of half a French mile, within which he said his observations fell, the benefit of this progression of the positions of the heights to be able to be had accurately scarcely with an error exceeding one or two hexapods [a hexapod being 6 feet or two English yards]; and according to the same progression, the height of the atmosphere appeared to be 12796 hexapods, or, what amounts to almost the same, 6 and a half French miles. This progression was calculated aptly by the celebrated Maraldi, but with the observation of Cassini, with regard to Mariotte related above §.370, it may be observed that it did not agree well enough, for the height from all the $16\frac{1}{3}$ divisions or from the sum of all the parts will be found to be 1122 feet ; and yet the height of the mountain was 1070 feet, the excess is 52 feet, or more than 8 hexapods.

Moreover so that it may be agreed with certainty, the hypothesis of how great a part of the atmosphere shall be of equal weight to a single line of mercury, and along the arithmetic progression increasing upwards, it will agree or disagree from the hypothesis of Mariotte & Boyle, by which the densities of the atmosphere are in different places, as the compressing forces or the elasticities of the air ; first the curve is to be found, the axis of which shall be divided into parts constituting an arithmetic progression, truly the ordinates drawn through the individual divisions of the axis likewise may effect an arithmetic progression, but descending ; hence from that this cudrve is found, requiring to define the graph of the elasticity of the air, and from this the graph of the density of the atmosphere is to be elicited. Here the following problems are solved.

PROPOSITION XXVII. PROBLEM.

373. *If the axis, or the linea* AH, *shall be divided into the parts* AC, C1C, 1C2C, *&c. making an increasing arithmetic progression, and the ordinates* AB, CD, 1C1D, 2C2D, *&c. following a decreasing arithmetic progression, define the curve* BD1D2D *passing through the ends* B, D, 1D, 2D, *of the individual terms.* Fig.89.



Lines parallel to AH are acting through the points D, and on the first line DE producted there is taken $\mathrm{EF = DE}$, and through F with FP drawn parallel to AB, on the line 1E1O from the line FP there is put downwards 1O1F equal to the excess, by which the second division C1C exceeds the first AC, or the third exceeds the second, or the fourth the third, and thus henceforth, and the right line FG is drawn through the points F and 1F; and there will be $\mathrm{AC = DE = EF}$, likewise $\mathrm{C1C = 1E1F}$, & $\mathrm{1C2C = 2E2F}$, and thus so forth. And thus $\mathrm{A2C = 2E2F + 1E1F + EF}$. Truly

$$\mathrm{2E2F = rect.\ 2E2F.2E1E : 2E1E},$$
$$\text{\&}\ \mathrm{1E1F = rect.\ 1E1F.1EE : 1EE};$$

therefore from $\mathrm{2E1E = 1EE = EB}$ from the hypothesis since AB, CD, 1E1D, 2C2D (according to the hypothesis) are in an arithmetical progression, there will be $\mathrm{2E2F + 1E1F = (\ rect.\ K2E + rect.\ I1E) : BE}$; truly however many of these rectangle shall have been taken at the same time, are equal to the trapezium EF2F2E together with the triangles IF1F, K1F2F, which are outside the trapazium EF2F2E, and the said triangle likewise are equal to the rectangle within half 2O2F and 2E1E or BE: therefore $\mathrm{2E2F + 1E1F = \left(EF2F2E + \tfrac{1}{2}\cdot 2O2F.BE\right) : BE = EF2.F2E : BE, + \tfrac{1}{2}\cdot 2O2F}$; and hence $\mathrm{A2C = EF2F2E : BE, + \tfrac{1}{2}\cdot 2O2F + EF = 2E2F + 1E1F + EF}$; by the same argument A*c* is shown to be indefinite or $ed = \mathrm{EF}fe : \mathrm{BE}, + \tfrac{1}{2}\,pf + \mathrm{EF}$ either *ep*, or bisects *pf* at *q* and with the right line F*q*Q drawn through *q*, there will be $ed = \mathrm{EF}fe : \mathrm{BE}, + pq + ep = \mathrm{EF}fe : \mathrm{BE}, + eq$; and thus $ed.\mathrm{BE} = \mathrm{EF}fe + eq.\mathrm{BE}$; and the trapezium $\mathrm{EF}fe = \text{rect. } eq.e\mathrm{E}$; therefore $ed.\mathrm{BE} = eq.\mathrm{E}e + eq.\mathrm{BE} = \mathrm{B}e.eq$; hence this is a property of the curve southt, that *de* or *c*A everywhere shall be to the homologous *eq* as B*e* to BE, which agrees with a common property of the parabola.

II. For if indeed T may become to EB, just as EM (which results from the meeting of the line QF produced at M with the line AB likewise extended) to EF; hence there will be also $eq : e\mathrm{M} \ (= \mathrm{EF} : \mathrm{EM}) = \mathrm{BE} : \mathrm{T}$, truly in the manner of the curve reviewed shows the property $de : eq = \mathrm{B}e : \mathrm{BE}$, therefore from the equation $de : e\mathrm{M} = \mathrm{B}e : \mathrm{T}$ & $de.\mathrm{T} = \mathrm{M}e.\mathrm{B}e = \mathrm{N}e^2 - \mathrm{NB}^2$ clearly with BM bisected at N; hence $de.\mathrm{T} + \mathrm{NB}^2 = \mathrm{N}e^2 = rd^2$; and if V shall be the third proportional to T and BN it will become $\mathrm{T.V} = \mathrm{BN}^2$, and thus $de.\mathrm{T} + \mathrm{T.V} \left(= de.\mathrm{T} + \mathrm{NB}^2\right) = rd^2$, therefore, if at rN for the perpendicular produced to AB producted towards S, taking $\mathrm{NR} = \mathrm{V}$, there will be $\mathrm{T.R}r = rd^2$, and thus the curve sought RBD1D is a parabola, of which the parameter is T, and the vertex at R; in which T is the fourth proportional for the given EF, EM and EB, and V the third proportional to T and the given BN. Therefore the parabola is of the kind and with the magnitude given, and its part sought BD2DH is satisfied. Q.E.D.

COROLLARY I.

374. Because the parts AC, C1C, 1C2C, &c. are arithmetical proportionals, thus so that the differences of these shall be equal, and the ordinates AB, CD, 1C1D, 2C2D, &c. also follow in arithmetical proportion, that is, they are diminished by equal decrements, these parts AC, C1C, 1C2C, &c. will represent the divisions of the atmosphere, of which each one is equivalent to the weight of a column of homogeneous liquid of height BE, or E1E, &c. in the method of the most celebrated Mariotte and Maraldi, and these BE, E1E, &c. will denote the equal decrements of the liquid in the barometer used, and finally the ordinates AB, CD, 1C1D, 2C2D, &c. express the amounts of mercury or liquid in the barometer situated at the places A, C, 1C, 2C, &c.

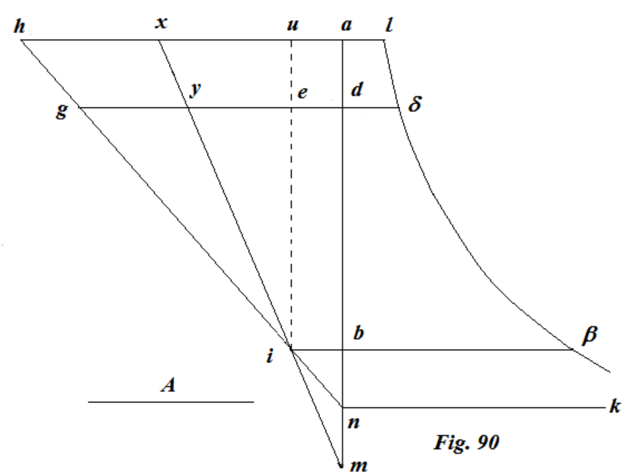
COROLLARY II.

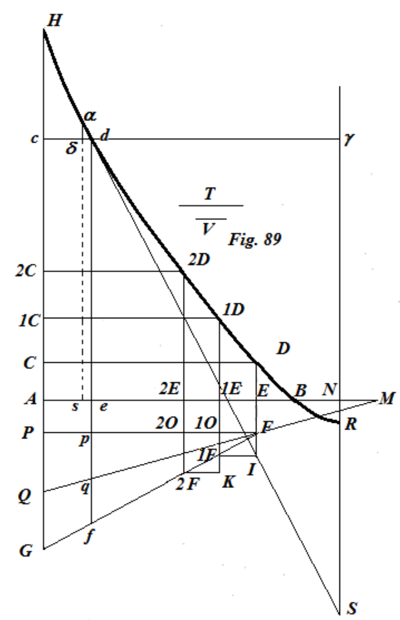
375. Because in Mariotte's hypothesis the densities are proportional to the incumbent weights of the air, and the amounts of quicksliver in the barometer are equivalent to the incumbent weights of the compressed air in different places, and because (§. 374) the ordinates *cd* express the amounts of quicksilver at the places *c*, it follows (§.355) the part of the parabola B2DH to be the graph of the densities of the atmosphere in Mariotte's suppositionss ; and yet it has been shown above (§.366); this graph of the densities in the Mariottian hypothesis to be logarithmic ; from which it appears the progression to be arithmetical, as otherwise the most perspicacious man therefore assumed easier to calculate in place of the geometric progression, and to be present differing little from the geometrical progression, the curve required for the whole atmosphere being different from the logarithmic, as the geometric hypothesis of its hypothesis, also by agreeing with that, that may usually be brought forwards. Therefore because, with the progressions used by the most celebrated of men in place, Mariotte and Maraldi, the densities of the air with the compressions or its elasticities are unable to be in proportion, remaining to be investigated, just as how the progression of the densities may increase the elasticities, or,

what is the same, the graph of the elasticities of the atmosphere which were to be given everywhere, will be determined from the graph of the increasing densities.

PROPOSITION XXVIII. PROBLEM.

376. *If in the parabola* BDH *found in the preceding proposition, Fig. 89,* AC, A1C, A2C, &c. *may designate the heights of the places, and the ordinates* CD,1C1D, 2C2D *may represent the amount of mercury present in the barometer at the places* C, 1C, 2C, &c. ; *to find the graphs of the elasticity of the air and the density*.





In figure 90. the right lines [corresponding to those in Fig. 89] shall be :

$$am = \text{AM},\ an = \text{AN},\ ab = \text{AB},\ \text{and } ad = \text{A}e,$$

and there will be $db = e\text{B}$, and these *bn, mn* will be equal with respect to BN & MN. Through the point *b* of the right line *ab* the normal *ib* acts, which shall be to *mb* as the given right line A, which expresses the constant weight T to the parameter of the parabola B2DH, the right line *nih* joining the points *n* and *i* will be inversely proportional to the graph of the elasticity, therefore by producing *ib* at β and by drawing through this point the hyperbola $l\delta\beta$ within the asymptotes *an* & *nk*,

this hyperbola will be the graph of the elasticity of the air according to the suppositions of Mariotte and Maraldi, and by applying the trapezium *bigd* to the given right line A, the height *ed* will result, at the end of which the density *d* of the air of the atmosphere is as $\alpha\delta$.

Demonstr. I. The line *mix* is drawn through the points *m* and *i* , and these individual lines *hu, gz*, &c. parallel to *ib* divided into two parts at *x, y*, &c. clearly if the right line *iu* were drawn equidistant to the other *am* ; hence any *yd* will bge the arithmetic mean between *gd* and *zd*, or between *gd* and *ib*, and thus the rectangle *bdy* will be equal everywhere to the homologous *bdgi* : with which in place the similar triangles *mdy* and *mbi* provide the analogous $md : dy$ $(= mb : bi,$ from the construction$) = \mathrm{T} : \mathrm{A}$, and thus $md.bd : dy.bd = de.\,\mathrm{T} : de.\,\mathrm{A}.$ and since *md* and *bd* in fig. 90. shall be equal to the homologous M*e* and B*e* in Fig. 89, and (§.373, n.II) the rectangle $\mathrm{M}e.\,\mathrm{B}e = \mathrm{T}.ed$, there will be also $md.bd = de.T$; therefore also the rectangle *dy.bd*, that is, as said a little before, the trapazium *bdgi*, to which the rectangle *bdy* is equal, to the rectangle A.*de* from the uniform weight A at the height of any place *de*, and since this thus shall be the same with the remainder, (§.319) the line *nih*, or the part of it *ih*, will be the reciprocal of the graph of the elasticity of the air.

II. On account of the hyperbola $l\alpha\beta$ there will be found $b\beta : d\delta = nd : nb = gd : ib$, and thus the individual rectangles $gd\delta,\ ib\beta,\ hal$, &c. are equal, and hence (§.357) the curve $l\alpha\beta$ is the graph of the elasticity, since the curve *ih* of its reciprocal shall be shown.

IlI. Because (no.I of this) $ibdg = \mathrm{A}.de,$ there will be $de = ibdg : \mathrm{A} = bd.yd : \mathrm{A}$, and thus there will be the height of the place *d*, at which the density is $d\delta$, the fourth proportonal to A, *bd* or B*e* and *yd* , and with that always able to hve the same, and hence the graph of the densitiy of the atmosphere in this hypothesis always can be described geometrically through the same point. Which all were required to be found.

COROLLARY I.

377. Again much appears to be missing, as the forces of the air compressing or its elasticity shall be proportional tp the densities, with an arithmetical progression of the heights standing AC, C1C, 1C2C equivalent to the heights of cylinders of mercury or of other homogeneous liquids, the heights of which are expressed by BE, E1E, 1E2E, &c.

COROLLARY II.

378. The density of the air for the horizontal will be to its density at the extreme of the atmosphere thus as $b\beta$ to *al*, or as *an* ad *bn*, that is in the other figure (89.) as AN to BN or MN.

COROLLARY III.

379. Hence if AC, AP or 1E1O, may be called *a* ; the excess of 1E1F more than EF, that is 1O1F, *e*; while $\mathrm{AB} = b,$ and $\mathrm{BE} = \mathrm{E1E} = l$, there will be found $\mathrm{BN} = \mathrm{NM} = 2al - el : 2e,$ and $\mathrm{AN} = b, +\left(2al - el\right) : 2e, = \left(2be + 2al - el\right) : 2e$; and hence there will be AN to BN, that is, the density of the atmosphere at the horizontal to the density of the same of the same in the continuation of the whole atmosphere or to its end, will be as $2be + 2al - el$ to $2al - el$. Therefore if in this general ratio with the celebrated Maraldi in place of *a, e , b* and *l* putting 61, 1, 336 & 1, the ratio becomes AN to BN as 793 to 121. But if indeed with Mariotte they shall become $a = 63$ feet, $e = \frac{61}{268},$ $b = 2.8$ inches $= 336$ lines and $l = 1$ line, there will be $2be + 2al - el$ to $2al - el$, as 1007 to 335 or approximately in the triple ratio.

SCHOLIUM.

380. Therefore it is clear, in Maraldi's suppositions, the air near the horizon to be less than seven times denser than the air at the end of the atmosphere, nor from Mariotte's suppositions the same to be made four times denser at the horizon, than it shall be at the top of the atmosphere, and yet at that place by virtue of losing all that elasticity ; since nothing will succumb to it, because of that expansion, if it may have that, it may prevail to hinder ; which beyond all doubt is a paradox. For if from the experiments of the Academy of Florence *Del Cimento*, of Robert Boyle, and of others it may be agreed, the air by the force of lifting is able to be made indeed to 60 or indeed to 152 miles ; why not as in the suppositions of Maraldi of seven times nor in the four times less dense of Mariotte make it a thousand times less dense, than it shall be in the natural state, destitute of all elastic force ? It is indeed true, that the distinguished Maraldi had been expressly advised of the danger of not extending his progression beyond half a French mile.

PROPOSITION XXIX. PROBLEM.

381. *If the graph of the elasticity of the air* A1DD *were a common hyperbola between the asymptotes containing the right angle* FPQ, *and the individual rectangles* EOD, 1E1O1D *were equal to the given rectangle* PFA, *the reciprocal line* E1E *of the graph of the elasticity is sought, and the graph of the densities of the atmosphere* B1B *under the supposition of uniform or constant gravity, the graph of which shall be* C1C *parallel to the axis* A1A. See Fig. 91.

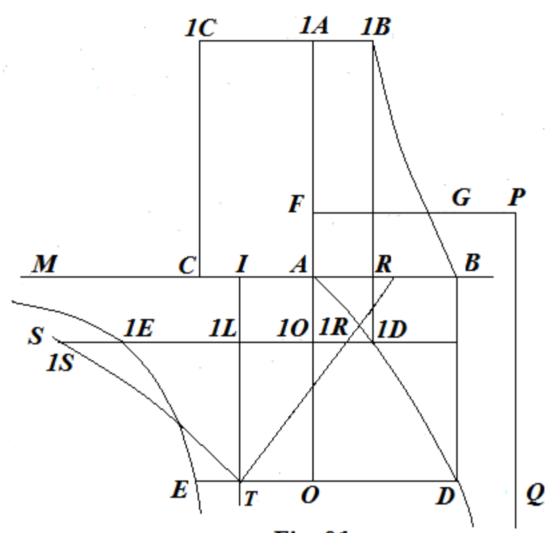


Fig. 91

On the indefinite right line AM, AI is taken equal to the given AF, and with the right line IT drawn through the point I parallel to AO, it is understood the hyperbola E1E described between the lines MI and IT as asymptotes, thus so that any rectangle in that ILE will be equal to the given square AI; and this hyperbola E1E will be the reciprocal graph of the elasticity A1DD; thus so that with the rectangle AC1C made equal to the four-lined figure OE1E1O; the density of the air at the place 1A shall become by §.359, 1O1D or 1A1B.

Demonstr. Because (following the hypothesis) $EO.OD = FP.FA$, there will be $EO : FA$ or $LO = FP : OD$ or FG, therefore by re-arranging, $EO : EL = FP : GP$ (that is on account of the hyperbola A1DD) $= GD$ or $FO : FA$, and by dividing $LO : EL = AO$ or $IL : AF$ or AI or LO: therefore the rectangle of the extremes and means will be equal, that is $LO^2 = AI^2 = IL.LE$, and thus the point E is on the hyperbola E1E between the asymptotes MI and IT ; and therefore the reciprocal of the graph of the elasticity in the present hypothesis is a common hyperbola. Hence, if a certain rectangle A1C shall be made equal to any four sided figure O1O1EE on this hyperbola, the line 1C1A extended to the other part of the axis 1A1B will have been made equal to the ordinate 1O1D, the point 1B (§. 359) will be on the graph of the density of the atmosphere B1B. Q.E.D.

SCHOLIUM.

382. This problem is only an application of our general theory, shown above (§.359), for a particular hypothesis, as we have set out in Proposition 23 of this chapter ; since we have now seen in the above, the densities do not always exist in proportion with the forces compressing the air; but the present hypothesis agrees more closely with the observations. Otherwise also it is apparent, that, since the reciprocal E1E of the graph of the densities is a hyperbola also, and therefore in the hypothesis of the proportionalities of the densities with the forces of compression, with this distinction only, which in this hypothesis one of its asymptotes shall be AO itself, indeed in the present case the right line IT, in the manner named parallel to AO, the curve or the graph of the densities B1B shall be able to be describped with the aid of logarithms. For the logarithmic curve L1SS drawn through the point L to the asymptote AM, of which the subtangent shall be equal to AI, and thence AC, which expresses uniform weight, the line LR makes a semi-right angle lLR contained with IL, and cutting 1L1D itself at 1R; if C1C may become equal to the intercept 1R1S always between the logarithmic curve L1SS and the right line L1RR, and 1C1B drawn indefinitely through the point 1C may cut the other axis, so that in the first place 1A1B equals the ordinate 1O1D in the graph of the density of the atmosphere B1B. And with these we have pursued something for a long time, because a knowledge of the density of the atmosphere is not without its use ; for besides because a most accurate way will be had of measuring the heights of mountains and of other objects on the surface of the earth with the aid of barometers, recognised by a law, following how the densities of atmospheres decrease upwards, the curve will be known more accurately how the rays of the sun or of stars may be in curved trajectories in the atmosphere, because certainly an excellent use will be had in astronomy. The other difficulty, if not an impossibility, is to know from the beginning what densities of the atmosphere are to be collated with the incumbent weights, the law to follow must account for a thousand anomalies, which happen in the atmosphere, therefore the matter must be derived from observations well and carefully put in place in the following manner.

383. On a certain right line indefinite lines are cut off having the same proportion between each other, as the heights of the places to be observed most diligently, and through the individual points of the division perpendiculars are acting having the same proportion between each other, as the quantities of mercury in a barometer situated at the aforesaid places, and thus just as many points will be had, as the number of places are of the observed heights, through which points hence a certain regulated line can be drawn there in a straightforwards manner, which the Celebrated Newton set out in Lemm. V. Lib. III. *Pr. Ph. Math.* without any demonstration; and such a suitable curve may arise from the experimental relation, which is between the heights of the places and of the

quantity of quicksilver in the barometer placed successively at these places. From this curve the reciprocal curve of the graph of the elasticity can be elicited, and from the shape of this graph the reciprocal can be deduced itself of the elastic property in the air ; from this, with the aid of our theorem §. 359 itself also the graph of the density of the atmosphere through the point can be described. Truly there is a need for many observations, because where many points are in given positions, from that a more accurate curve will itself be defined, about which I have spoken before ; how indeed the calculation following from this curve may be related in some other more convenent manner, when we will give a demonstration of Newton's afore-mentioned Lemma V with some other relateed matters.

CAPUT VIII.

De Densitatibus aëris in diversis Atmosphaerae locis in omni possibili elasticitatum hypothesi.

In hoc capite non agetur de aliis aëris densitatibus quam quae aëri a pondere incumbente inducuntur ; propterea in hisce sermo non erit de rarefactione & condensatione aëris; quae a calore & frigore proveniunt.

DEFINITIONES.

354. Curva C1C2C circa axem OA2A extructa, cujus ordinatae AC, 2A2C, &c. exponunt gravitatem variabilem cujusque corporis in locis A, 2A, &c. per quae ordinatae transeunt, dicatur *scala gravitatis variabilis*.

II.

355. Curva vera B1B2B circa eundem axem A2A descriptae esto scala densitatum atmosphaerae, quia ordinatae ejus AB, 2A2B, &c. exponunt densitates atmosphaerae in locis A , 2A, &c. per quae ordinatae ductae sunt.

III.

356. Et curva A2DD subter CB horizontem circa eundem axem AO descripta, sit *scala elasticitatis* aëris, cujus scilicet abscissae AO, A2O, A1O repraesentent elasticitates aëris sub densitatibus OD, 2O2D, 1O1D.

IV.

357. Productis ordinatis scalae elasticitatis aëris DO, 1D1O, 2D2O in E, 1E, 2E, &c. si singula rectangula DOE, 1D1O1E, 2D2O2E inter se & dato plano aequalia fuerint, puncta E, 1E, 2E, &c. erunt in curva E1E2Ee, quam *reciprocam scalae elasticitatis* deinceps vocabimus.

AXIOMA.

358. Elater aëris aequatis semper est vi comprimenti, atque adeo in quolibet atmosphaerae loco aëris vis elastica aequivalet ponderi aëris incumbentis. Nam vires directe contrariae, ut vis aërem comprimens & elater aëris, aequales sunt, quia omnia in statu manenti posita esse intelligentur.

PROPOSITIO XXVI. THEOREMA.

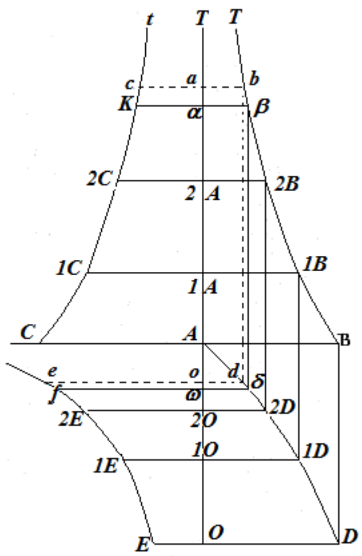


Fig. 87

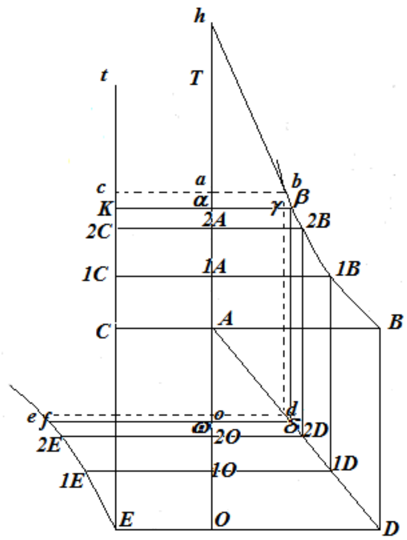


Fig. 88.

359. *Ordinatae quaecunque aequales* ab & od *in scalis densitatum & elasticitatum aëris ultra communem scalarum axem* AT *productae, in scala gravitatis variabilis & curva scalae elasticitatis aëris reciproca perpetuo areas aequales* ACca & OEeo *abscindent.* Fig.87

Ordinatae $\alpha\beta$ & $\omega\delta$ suis respectivis *ab* & *od* indefinite vicinae protrahantur in χ & f. Jam gravitas aëris *a*T loco *a* incumbentis, aërique in hoc loco densitatem *ab* vel *od* inducens (§.358.) aequatur elasticitati aëris expositae per abscissam A*o*, & pondus aëris loco incumbentis aequatur elasticitati aëris in α, quae repraesentatur abscissam Aω, adeoque differentia ponderum αT & *a*T, id est, pondus columnulae aëreae αa aequabitur differentiae elasticitatum Aω & A*o* hoc est ωo. Atqui (§. 32.) pondus columnulae αa exponitur solido $ab.ac.\alpha a$; ergo hoc solidum aequatur alii solido ex ωo in datum planum EO.OD, vel (§.357.) ipsi aequale planum *eo.od* , id est, $ab\cdot a\alpha = eo\cdot od\cdot \omega o$; seu deletis (secundum hypothesin) aequalibus *ab* & *od*, rec – lum $ac\cdot a\alpha$ = reclo $oe\cdot \omega o$, & cum descendendo versus CB & ED hoc ubique eveniat, ut singula elementa arearum ACca & OE*eo* aequalia sint, aequabuntur etiam universa, id est omnia $ac\cdot a\alpha$ seu area ACca, quam illa exhauriunt, erunt aequalia omnibus $oe\cdot \omega o$, seu areae OE*eo*. Quod erat demonstrandum.

COROLLARIUM I.

360. Si nunc scala elasticitatum aëris A1D2D ponatur esse parabola indefiniti gradus, cujus quaelibet ordinata 1O1D sit ut $\mathrm{A1O}^m$, seu ut dignitas abscissae A1O ab exponente *m* denominata, erit propter rec-lum D1O1E, aequale dato rectangulo DOE, atque adeo

propter 1E1O alteri 1D1O reciproce proportionalem, 1E1O reciproce ut $A1O^m$, atque adeo curva E1E2E hyperbola indefiniti gradus *m*. Sit in super curva C1C2C alia hyperbola indefiniti gradus *n*, adeo ut quaelibet ejus ordinata 1C1A sit reciproce ut abscissae dignitas *n*, id est, reciproce ut $O1A^n$. Jam si aequales ordinatae 1A1B vel 1O1D exponunt densitatem aëris in 1A, (§. 359) quadrilinea A1A1CC & O1O1EE aequalia erunt, atqui juxta methodum supra (§. 92.) expositam invenitur Quadrilineum
$A1A1CE = (OA.AC - O1A.1A1C) : n-1$, & quadrilineum hyperbolicum
$OE1E1O = \left(A1O.\ 1O1E - AO.\ OE\right) : m-1 = \left(AO.\ OE - A1O.1O1E\right) : 1-m.$
Ergo $\left(OA.AC - O1A.1A1C\right) : n-1 = (AO.OE - A1O.1O1E) : 1-m$. Jam, quia ipsae AC & OE nullius sunt determinatae magnitudinis, fiat $AC : EO = n-1 : 1-m$, eritque rec-lum OA.AC ad rec-lum AO.OE in hac eadem ratione ac consequenter etiam rec-lum O1A.1A1C ad rec-lum A1O.1O1E in eadem erit ratione $n-1$ ad $1-m$ seu AC ad OE. At vero in hyperbola C1C2C est rec-lum $OA.AC : O1A.1A1C = O1A^{n-1} : OA^{n-1}$, ergo ex aequo & per compositionem rationum erit $OA.\ AC : A1O.1O1E = AC.O1A^{n-1} : OE.OA^{n-1}$, item in hyperbola E1E2E, est $A1O.1O1E = AC.OE = AO^{m-1} : A1O^{m-1}$ (hoc est propter parabolam A2D1D)
$= OD^{m-1:m} : 1O1D^{m-1:m}$, ergo denuo ex aequo habetur
$OA.AC : AO.OE = AC.O1A^{n-1}.\ OD^{m-1:m} : OE.OA^{n-1}.1O1D^{m-1:m}$. Ex hac vero deducitur $O1A^{n-1}.\ OD^{m-1:m} = OA^{n-1}.1O1D^{m-1:m}$, & rejectis quantitatibus datis, seu constantibus, quae proportiones non alterant, scilicet $OD^{m-1:m}$ & OA^{n-1}, erit $O1A^{n-1}$ ut $1O1D^{m-1:m}$, vel dividendo exponentes per $m-1 : m$, resultabit sequens 1O1D ut $O1A^{mn-m:m-1}$. Hinc
1°. Si cubus vis comprimentis proportionetur quadrato-quadrato densitatis & gravitas sit reciproce ut quadratum distantiae a centro gravium, erunt $m = \frac{3}{4}$ & $n = 2$, adeoque $mn - m : m - 1 = -3,$ ergo 1O1D erit hoc casu, ut $O1A^{-3}$, hoc est, densitas erit reciproce ut cubus distantiae.
2°. Si cubus vis comprimentis sit ut quadratocubus densitatis, & gravitas reciproce ut quadratum distantiae, fient $m = \frac{3}{5}$ & $n = 2$, ergo $mn - m : m - 1 = -\frac{3}{2},$ id est, densitas 1O1D erit in reciproca sesquiplicata ratione distantiae O1A.
3°. Si vis comprimens sit ut quadratum densitatis, & gravitas in reciproca duplicata ratione distantiae, fient $m = \frac{1}{2}$ & $n = 2$, ergo $mn - m : m - 1 = -1,$ quod indicat densitatem hoc casu esse in reciproca ratione distantiae.

Ex hisce ergo liquet veritas assertionum nonnullarum sine ulla demonstratione prolatarum ab Illustr. Newtono in Scholio post Prop.22, Lib.II. *Pr. Phil. Nat. Math*. Et, quanquam hoc corollarium tantum casus sit particularis propositionis nostrae generalissimae, idem tamen infinities infinitos casus diversos in se continet.

COROLLARIUM II.

361. Nam si scala elasticitatum aëris A_2D_1D fuerit linea recta, hoc est, si elasticitas aëris, aut vis comprimens, est ut densitas, curva E_1E_2E erit hyperbola conica intra asymptotas CA & AO descripta; est enim $_1E_1O : EO = OD : _1O_1D$ (propter rectam A_2D_1D) $= AO : A_1O$. Hinc, si hyperbolae quadrilinea EO_1O_1E, $_1O_2O_2E_1E$, &c. sint aequalia inter se, & hisce aequalibus quadrilineis aequentur totidem in scala gravitatis variabilis AC_1C_1A, $_1A_1C_2C_2A$, &c. homolgae ordinatae AB, $_1A_1B$, $_1A_2B$, &c. in scala densitatum B_1B_2 erunt continue proportionales. Nam propter aequalitatem trapaziorum hyperbolicorum O_1E, $_1O_2E$, &c. abscissae AO, A_1O, A_2O, &c. vel abscissis hisce respectivae ordinatae in scala elasticitatis OD, $_1O_1D$, $_2O_2D$, &c. hoc est AB, $_1A_1B$, $_1A_1B$,&c. ipsis eodem ordine sumtis aequales erunt in continua ratione. Idcirco, si vires comprimentes, vel aequivalentes vires elasticae aëris densitatibus ejus proportionales, in scala vero gravitatis quadrilinea AC_1C_1A, $_1A_1C_1C_1A$, &c. aequalia fuerint, densitates aëris in locis A,$_1$A, $_2$A &c. erunt semper in continua ratione.

COROLLARIUM III.

362. Propterea si gravitas in loco quolibet $_1$A, seu ordinata $_1A_1C$ fuerit reciproce ut O_1A^n, existente curva C_1C_2C hyperbola indefiniti gradus exponentis *n*, & magnitudines OA^{1-n}, O_1A^{1-n}, O_2A^{1-n}, &c. sumantur in progressione arithmetica ascendentae ordinatae scalae densitatum AB, $_1A_1B$, $_2A_2B$, &c. erunt continue proportionales. Nam quia OA^{1-n}, O_1A^{1-n}, O_2A^{1-n}, &c proportionales sunt hisce sequentibus eodem ordine sumtis O_2A^{n-1}, O_1A^{n-1}, OA^{n-1}, &c. hisce jam proportionantur rectangula OA.AC, $O_1A._1A_1C$; $O_2A._2A_2C$, &c. ergo haec rectangula sunt etiam in progressione arithmetica, atque adeo $(O_1A._1A_1 - OA.AC) : n-1$ id est, juxta ea quae §.360 dicta sunt, quadrilineum AC_1C_1A aequatur $\left(O_2A._2A_2C - O_1A._1A_1C\right) : n-1$, seu quadrilineo $_1A_1C_2C_2A$, adeoque (§:361.) ordinatae AB, $_1A_1B$, $_2A_2B$, &c. erunt continue proportionales.

COROLLARIUM IV.

363. Si nunc *n* sit 1, adeoque curva C_1C_2C hyperbola conica centrum habens in O, quadrilinea aequalia A_1C, $_1A_2C$, &c. habebunt abscissas OA, O_1A, O_2A, &c. continue proportionales, & vice versa, si hae abscissae fuerint in progressione geometrica, quadrilinea praedicta erunt aequalia, ac consequenter densitates AB, $_1A_1B$, $_2A_2B$, &c. erunt continue proportionales, plane ut habet Prop. 21, Lib. II *Princ. Ph. Nat. Math*. Cel. Newtoni, quam seorsim demonstravit.

COROLLARIUM V.

364. Si *n* sit 2, atque adeo curva C1C2C hyperbola quadratica ; id est, si gravitates corporum sint in reciproca duplicata ratione distantiarum a centro, & OA^{-1}, $\mathrm{O1A}^{-1}$, $\mathrm{O2A}^{-1}$, &c. in progressione arithmetica, atque adeo seriei hujus reciproca OA ,O1A, O2A, in progressione harmonica, (§. 361, 362) ordinatae AB, 1A1B, 2A2B, &c. erunt in progressione Geometrica. Hoc ipsum demonstratum dedit alia ratione Cel. Newtonus Prop.22. Lib. II.

COROLLARIUM VI.

365. Si $n = -1$, hoc est, si gravitas est ut distantia corporis a centro O, curva C1C2C mutabitur in lineam rectam transeuntem per centrum O, seriesque, quae juxta §. 362. debet esse in arithmetica progressione OA^{1-n}, $\mathrm{O1A}^{1-n}$, $\mathrm{O2A}^{1-n}$ &c. nunc fiet OA^{2}, $\mathrm{O1A}^{2}$, $\mathrm{O2A}^{2}$, &c. adeoque, si quadrata distantiarum OA, O1A, O2A, &c. fuerint in progressione arithemetica, trapezia rectlinea, A1C, 1A2C, &c. erunt aequalia, atque adeo (§. 361.) ordinatae scalae densitatum AB, 1A1B, 2A2B , erunt continue proportionales. Ut asseruit, sed absque ulla demonstratione, laudatissimus Newtonus in Scholio post Prop. XXII. Lib. II. *Princ. Phil. Nat. Math*.

COROLLARIUM VII.

366. Sin vero *n* fuerit *o*, id est, si gravitas corporum uniformis seu constans fuerit, qualis communiter considerari solet, tunc scala gravitatis C1C2C erit linea recta axi AT parallela, ut in fig. 88. & series OA^{1-n}, $\mathrm{O1A}^{1-n}$, $\mathrm{O2A}^{1-n}$, &c. quae constitituere debet progressionem arithmeticam ad id, ut ordinatae AB, 1A1B, 2A2B, &c. sint in continua proportione, nunc erit simpliciter OA, O1A, O2A, &c. in progressione arithmetica, atque adeo ipsae distantiae A1A, 1A2A ordinatatum scalae densitatum atmosphaerae erunt aequales, scalaque ipsa B1B2B proinde erit logarithmica. Huic corollario simile quid jam olim primus, quod sciam, observavit acutissimus Edmundus Hallejus.

COROLLARIUM VIII.

367. Iisdem, quae in corollario praecedenti, positis, si scala gravitatis uniformis tC producatur usque ad occursum E hyperbolae E1E2E, & per hoc punctum E agatur EOD scalae elasticitatis aëris A2D1D occurrens in D, erit subtangens *ah* log-micae B1B2B aequalis abscissae AO scalae elasticitatis, ordinatae OD correspondenti. Nam (§.359) est $ac \cdot a\alpha = o\omega \cdot eo$ atque adeo $o\omega : a\alpha = ac : eo = \mathrm{EO} : eo = od : \mathrm{OD} = \mathrm{A}o : \mathrm{AO}$ atqui $g\delta$ vel $\gamma\beta$ est ad dg vel $o\omega = od$ vel $ab : \mathrm{A}o$, ergo ex aequo $\gamma\beta : a\alpha = ab : \mathrm{AO}$ & propter triangula similia $\beta\gamma b$ & bah, est $\gamma\beta : a\alpha \text{ vel } b\gamma = ab : ah$, ergo etiam $ab : \mathrm{AO} = ab : ah$, atque adeo AO aequatur subtangenti ab logarithmicae B1B2B.

COROLLARIUM IX.

368. Praeterea erit, iisdem positis, quodlibet quadrilineum hyperbolicum OE_1E_1O ad datum rec-lum AOE, ut A_1A distantia ordinatarum AB & $_1A_1B$ aequalium ordinatis OD, $_1O_1D$ scalae elaesticitatis applicatis hyperbolae OE, $_1O_1E$, in directum positis, ad subtangentem
ah log-micae B_1B_2B. Nam, quia (secundum hypothesin) $OE_1E_1O = \text{rec} - \text{lo } AC_1C_1A$, erit quadrilineum $OE_1E_1O : AO.OE = A_1A.AC : AO.AC = A_1A : AO = A_1A : ah.$

COROLLARIUM X.

369. Quia altitudines Mercurii in barometro sunt, ut pressiones atmosphaerae in diversis ab horizonte distantiis, & pressiones atmosphaerae & elasticitates aëris pressiones illas sustinentis si elasticitates densitatibus aëris proportionales fuerint, ut quidem in corollariis 2, 3, 4, 5, & 6 hujus Propositionis supposuimus, altitudines Mercurii in barometro in locis A, $_1A$, $_2A$, &c. positi, erunt ut ordinatae AB, $_1A_1B$, $_2A_2B$, &c. in scala densitatum B_1B_1B, si scilicet gravitas corporum uniformis, seu in omnia centro terrae O distantia eadem fuerit, ut tuto id assumere licet. Propterea datis quantitatibus Mercurii in barometro in locis A, $_1A$, & $_2A$ existente, & altitudine A_1A; invenietur logaërithmorum ope altera distantia seu altitudo A_1A. Nam, quia A_1A est ad A_2A ut log-us rationis AB ad $_1A_1B$, ad log-us rationis AB ad $_2A_2B$; si log-us rationis $AB{:}_2A_2B$ ducatur in datam altitudinem A_1A, & productum dividatur per log-um rationis $AB{:}_1A_1B$; quotiens manifestabit altituditem quaesitam A_1A. Sin vero ex datis altitudinibus A_1A, A_2A & ratione $AB{:}_1A_1B$ quaeratur ratio $AB{:}_2A_2B$, seu ratio Mercurii in barometro collocato in A ad quantitatem Mercurii barometro existente in $_2A$; log-us rationis $AB{:}_1A_1B$ tantum multiplicandus est per A_2A & productum dividendum per A_1A, & manifestabit quotiens log-um rationis quaesitae $AB{:}_2A_2B$, invento vero log-mo ipsa ratio ex tabulis usualibus logarithmorum illico innotescet.

SCHOLION.

370. Ut regula noni corollarii exemplo aliquo illustretur, ponatur altitudinem Mercurii in A seu AB esse 28 digitorum, & in altitudine loci $_1A$ supra A,id est A_1A, 63 pedum Mercurium una linea deficere, ita ut $_1A_1B$ tantum sit 335 linearum, cum AB, seu 18 pollices contineant 336 lineas. In altitudine vero A_2A seu in loco $_1A$ quantitas argenti vivi $16\frac{1}{3}$ lineis deficiat a quantitate in A seu AB, adeo ut $_2A_2B$ tantum sit $319\frac{2}{3}$ lin. Quaëritur in hisce datis altitudo A_2A. Juxta canonem (§. 369.) log-us rationis AB: $_2A_2B$ seu nunc log-us 336: $319\frac{2}{3}$ multiplicare debet per A_1A seu 63 & productum dividi per log-um $AB{:}_1A_1B$ seu log. 336:335, & quotiens indicabit quaesitum; atqui productum ex $\log.336 - \log.319\frac{2}{3}$ in 63 divisum per log.336−log. 335, praebet $1053\frac{1}{4}$ ergo hic numerus exprimit, quot pedum sit altitudo quaesita. Hoc exemplum sumsimus ex Mariotti *Tentamine De Natura aëris*, qui pag. 194 & 195 refert, Celeberrimum Cassinum

observasse olim in summitate alicujus Montis in Provincia Galliae; cujus altitudinem diligenti dimensione 1070 pedum invenerat, Mercurium barometri $16\frac{1}{3}$ lineis depressiorem fuisse, quam ad radicem montis ubi ad 28 digit subsistebat, idcirco altitudo montis calculo nostro elicita 17 circiter pedibus deficit ab observata 1070 pedibus, sed haec differentia forte inde venit, quod cum Mariotto posuimus in altitudine 63 pedum argentum vivum una linea deficere a Mercurio barometri in horizonte ad 28 digitos sublato. Verum si altitudinem in qua Mercurius una linea in barometro decrescit, vel uno tantum pede major fiat, scilicet 64 pedum, calculus, juxta Canonem superiorem subductus, praebebit altitudinem Montis $1069\frac{44}{95}$, adeo ut unica $95^{\text{ma.}}$ pedis ab observata altitudine deficiat.

371. Cl. Mariottus, etsi hypothesin quod densitates aëris viribus comprimentibus proportionales sint, suam fecit, atque logarithmico calculo altitudines locorum ex argenti vivi differentiis eliciendas censuit, in calculo suo, eidem exemplo paragraphi praecedentis aptato, logarithmis tamen non est usus, sed faciliori quidem, minus exacto vero computo altitudinem montis elicuit 1080 pedum idque sequenti ratione. Quoniam atmosphaerae pressio in loco infimo aequivalet 28 digitis Mercurii seu 336 lineis, totam atmosphaeram in 336 partes aequiponderantes dirimit, quarum unaquaeque unius lineae Mercurii gravitationi aequipollet ; sed harum aëris partium altitudines inaequales erunt, adeo quidem, ut ea, quae divisioni 168 semissi ipsius 336 respondet, duplo altior seu major futura sit quam infima altitudo 63 ped. primae Mercurii lineae versus horizontem conveniens, atque adeo sit 126 pedum; altitudinum seu partium atmosphaerae differentias, singulis Mercurii lineis homologas, sursum crescere fingit in arithmetica progressione, quam a geometrica, juxta quam incrementa partium fieri recte judicat, perparum abludere arbitratur; idcirco dividendo 63. per 168, secundae atmosphaerae divisionis incrementum supra primam horizonti proximam reperit $\frac{63}{168}$, & ducendo 63 in $16\frac{1}{3}$, invenit 1029 ped. pro altitudine totali, seu montis altitudine, si modo singulae atmosphaerae divisiones $16\frac{1}{3}$ lineis Mercurii respondentes aequales extitissent, verum quia crescunt in proportione arithmetica, ideo omnia incrementa repertae altitudini 1029 pedum adjicit ; hunc in finem numerum 136 qui est aggregatum omnium numerorum naturalis progressionis l, 2, 3 usque ad 16 inclusive, ducit in $\frac{63}{168}$, & productum 51 ipsi exhibet summam omnium incrementurum, atque adeo hic numerus 51. alteri 1029 additus dedit summam 1080 exprimentem altitudinem montis quaesitam.

372. Cl. Maraldus atmosphaeram itidem in 336 partes aequiponderantes distinguit perinde ac Mariottus, & harum partium altitudines in progressione arithmeca crescere fingit, agnoscens tamen extensiones aëris non esse praecise in reciproca ratione ponderum incumbentium, ut Mariotti hypothesis requirit, deinde etiam a Mariotti numeris discessit, statuens primam eamque horizonti contiguam atmosphaerae partem gravitati unius lineae Mercurii convenientem esse 61 pedum, & post hanc sequentes crescere Juxta numeros 1, 2, 3, 4, &c. ita ut partes secunda, tertia, quarta, &c. futurae sint 61, 63, 64, &c. pedum. Hanc progressionem omnibus observationibus barometro factis super,

diveriis Galliae montibus, satis prope quadrare se deprehendisse indicat laudatus vir, cum protrahendae inde ab Observatorio Regia per Meridionalem Galliae partem Meridianae Cel. Cassino opus dirigenti operam suam juxta alios commodaret; atque adeo intra limites semissis Milliaris Gallici, quibus observationes suas terminui dicit, hujus progressionis beneficio locorum altitudines satis accurate haberi posse errore vix unam alteramque hexapodum excedente; ac juxta eandem progressionem altitudo atmosphaerae foret 12796 hexapodatum, seu, quod idem fere est, 6 Milliarium Gallicorum cum semisse. Haec Cel. Maraldi progressio calculo aptissima est, sed cum observatione Cassiniana, ex Mariotto supra §.370 relata, non satis convenire videtur, nam altitudo ex omnibus $16\frac{1}{3}$ divisionibus seu partibus composita invenietur esse 1122 pedum ; & tamen altitudo montis inventa erat 1070, excessus est 52 pedum, seu plus quam 8 hexapodarum.

Ut autem certo constet, quantum hypothesis partium atmosphaerae uni Mercurii lineae aequiponderantium, & juxta arithmeticam progressionem sursum crescentium, conspiret aut discrepet a Mariotti & Boylii Hypothesi, qua densitates atmosphaerae in diversis locis sunt, ut vires comprimentes seu elasticitates aëris; quaërenda prius est curva, cujus axis in partes arithmeticam progressionem constituentes divisus sit, ordinatae vero per singulas axis divisiones ductae efficiant progressionem itidem arithmeticam, sed descendentem ; dehinc inventa hac curva, definienda est scala elasticitatis aëris, & ex hac elicienda scala densitatum atmosphaerae. Huc faciunt sequentia Problemata.

PROPOSITIO XXVII. PROBLEMA.

373. *Si axis, seu linea* AH, *divisus sit in partes* AC, C1C, 1C2C, *&c. arithmeticam progressionem ascendentem constituentes , & per singularum terminos transeant ordinatae* AB, CD, 1C1D, 2C2D, *&c. juxta progressionem arithmeticam decrescentes, definire curvam* BD1D2D *per singularum ordinatarum terminos* B, D, 1D, 2D, *transeuntem.* Fig.89.

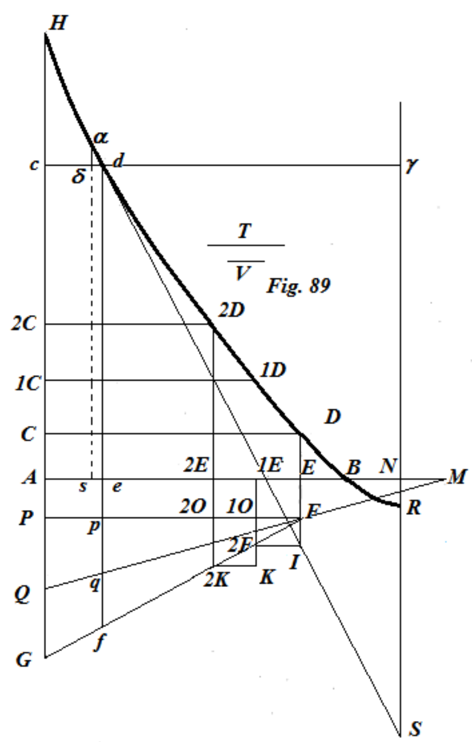


Fig. 89

Per puncta D agantur parallelae ipsi AH, & in prima DE producta sumatur $EF = DE$, ac per F ducta FP parallela AB, in linea 1E1O a linea FP deorsum ponatur 1O1F aequalis excessui, quo secunda divisio C1C primam AC, vel tertia secundam, seu quarta tertiam, atque ita deinceps, excedunt, ducaturque per puncta F & 1F, recta FG; eruntque $AC = DE = EF$, item $C1C = 1E1F$, & $1C2C = 2E2F$, atque sic porro. Adeoque $A2C = 2E2F + 1E1F + EF$. Verum $2E2F = \text{rec-lo } 2E2F.2E1E : 2E1E$, & $1E1F = \text{rec-lo } 1$;
ergo quia $2E1E = 1EE = EB$ ex hypothesi quandoquidem AB, CD, 1E1D, 2C2D (secundum

hypothesin) sunt in progressione arithmetica,
erit $_2E_2F + {_1E_1F} = (\text{ rec} - \text{lum } K_2E + \text{rec} - \text{lo } I_1E) : BE$; haec vero rectangula simul sumpta quotcunque eorum fuerint, aequantur trapezia EF_2F_2E simul cum triangulis IF_1F, K_1F_2F, quae sunt extra trapazium EF_2F_2E, & dicta triangula simul aequantur rec-lo sub dimidia $_2O_2F$ & $_2E_1E$ vel BE:
ergo $_2E_2F + {_1E_1F} = \left(EF_2F_2E + \frac{1}{2} \cdot {_2O_2F}.BE\right) : BE = EF_2.F_2E : BE, + \frac{1}{2} \cdot {_2O_2F}$; ac proinde $A_2C = EF_2F_2E : BE, + \frac{1}{2} \cdot {_2O_2F} + EF = {_2E_2F} + {_1E_1F} + EF$; eadem argumento probatur esse indefinite Ac vel $ed = EFfe : BE, + \frac{1}{2}\, pf\; + EF$ vel ep, vel bisecta pf in q ductaque per q recta FqQ, erit $ed = EFfe : BE, + pq + ep = EFfe : BE, + eq$;
adeoque $ed.BE = EFfe + eq.BE$; atqui trapezium $EFfe = \text{rec} - \text{lo } eq.eE$; ergo $ed.BE = eq.Ee + eq.BE = Be.eq$; hinc curvae quaesitae haec est proprietas, ut de vel cA ubique sit ad homologam eq ut Be ad BE, quae proprietas parabolae communi competit.

II. Nam si quaedam T fiat ad EB, sicut EM (quae resultat a concursu M rectae QF productae cum recta AB itidem protensa) ad EF; hinc erit etiam
$eq : eM \;(= EF : EM) = BE : T$, curvae vera modo recensitae proprietas exhibet
$de : eq = Be : BE$, ergo ex aequo $de : eM = Be : T$ & $de.T = Me.Be = Ne^2 - NB^2$ bisecta scilicet BM in N; hinc $de.T + NB^2 = Ne^2 = rd^2$; & si V sit tertia proportionalis ad T & BN erit $T.V = BN^2$, atque adeo $de.T\; + T.V \;\left(= de.T\; + NB^2\right) = rd^2$, idcirco, si in rN perpendiculari ad AB producta versus S, sumatur $NR = V$, erit $T.Rr = rd^2$, atque adeo curva quaesita RBD_1D est parabola, cujus parameter est T, & vertex in R; in qui T est quarta proportionalis ad datas EF, EM & EB, & V tertia proportionalis ad T & datam BN. Propterea parabola est specie & magnitudine data, ejusque portio BD_2DH quaesito satisfacit. Quod erat inveniendum.

COROLLARIUM I.

374. Quoniam partes AC, C_1C, $_1C_2C$, &c. sunt arithmetice proportionales, adeo ut earum differentiae aequales sint, ordinataeque AB, CD, $_1C_1D$, $_2C_2D$, &c. etiam juxta proportionem arithmeticam, id est, per aequalia decrementa imminuuntur, partes illae AC, C_1C, $_1C_2C$, &c. repraesentabunt atmosphaerae divisiones, quarum unaquaeque aequivalet ponderi columnulae liquoris homogenei altitudinis BE, vel E_1E, &c. in methodo Mariotti & Cl. Maraldi, ipsaeque BE, E_1E, &c. denotabunt aequalia decrementa liquoris in barometro adhibiti, ac denique ordinatae AB, CD, $_1C_1D$, $_2C_2D$, &c. exponent quantitates Mercurii seu liquoris in barometro in locis, A, C, $_1C$, $_2C$, &c. versante.

COROLLARIUM II.

375. Quoniam in hypothesi Mariotti densitataes aëris ponderibus incumbentibus proportionales sunt, & quantitates argenti vivi in barometro aequivalent ponderibus aëri compresso in diversis locis incumbentibus, & quoniam (§. 374) ordinatae cd exponunt

quantitates argenti vivi in locis *c*, sequitur (§.355) portionem parabolae B2DH esse scalam densitatum atmosphaerae in suppositionibus Mariotti; & tamen supra (§.366) est ostensum; hanc scalam densitatum in hypothesi Mariottiana logarithmicam esse; ex quo liquet progressionem arithmeticam, quam Vir alioqui perspicacissimus loco progressionis geometricae facilioris calculi ergo assumsit , atque a geometrica parum abludere existimavit, curvam suppeditare toto coelo diversam a log-mica, quam progressio geometricain hypothesi ejus, ipso etiam consentiente, adhibenda produxisset. Quoniam igitur, stantibus progressionibus a Celeb. viris Mariotto & Maraldo adhibitis, densitates aëris viribus comprimentibus aut elasticitatibus ejus nequeunt proportionales esse, disquirendum superest, juxta quam densitatum progressionem elasticitates crescant, vel, quod idem est, quaëri debet scala elasticitatum atmosphaerae quae ubi data fuerit, scala densitatum ultra determinabitur.

PROPOSITION XXVIII. PROBLEMA.

376. *Si in parabola* BDH *propositione praecedenti reperta* AC, A1C, A2C, &c. *designent altitudines locorum, & ordinatae* CD,1C1D, 2C2D *repraesentent quantitatem Mercurii barometro versante in locis* C, 1C, 2C, &c. *invenire scalas elasticitatis aëris & densitatum*.

In figura 90. sint linea recta $am = \mathrm{AM},\ an = \mathrm{AN},$ $ab = \mathrm{AB}, \&\ ad = \mathrm{A}e,$ eritque $db = e\mathrm{B}$, & ipsae *bn, mn* respectivis BN & MN aequabuntur. Per punctum *b* agatur rectae ab normalis *ib*, quae sit ad *mb* ut data recta A, quae gravitatem uniformem exponit ad T parametrum parabolae B2DH, linea recta *nih* jungens puncta *n* & *i* erit reciproca scalae elasticitatis, idcirco producendo *ib* in β & per hoc punctum ducendo hyperbolam $l\delta\beta$ intra asymptotas *an* & *nk*, hyperbola ista erit scala elasticitatis aëris in suppositionibus Mariotti atque Maraldi, & applicando trapezium *bigd* ad datam rectam A, resultabit altitudo *ed*, in cujus termino *d* densitas aëris atmosphaerae est sicut $\alpha\delta$.

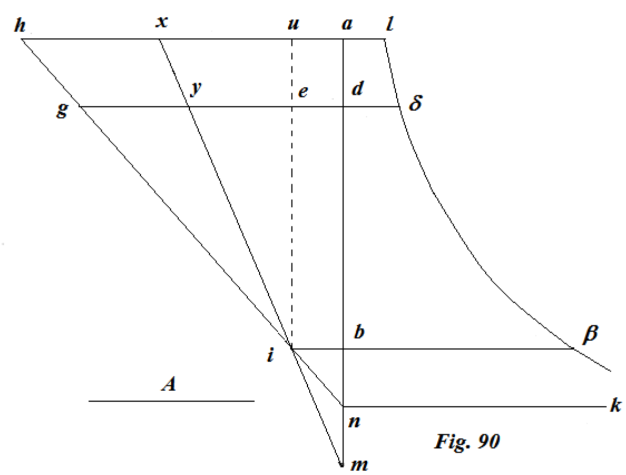


Fig. 90

Demonstr. I. Ducatur linea *mix* per puncta *m* & *i* , & haec singulas *hu, gz*, &c. ipsi *ib* parallelas bifariam dividet in *x, y*, &c. si scilicet recta *iu* alteri *am* aequidistans ducta fuerit; hinc quaelibet *yd* erit media arithmetica inter *gd* & *zd*, vel inter *gd* & *ib*, atque adeo re-lum *bdy* aequabitur ubique trapezio homologo *bdgi* : quibus positus triangula similia *mdy* & *mbi* suppeditant analogiam $md : dy\ (= mb : bi, \text{constr.}) = \mathrm{T} : \mathrm{A}$, atque adeo $md.bd : dy.bd = de.\,\mathrm{T} : de.\,\mathrm{A}.$ atqui cum *md* & *bd* in fig. 90. aequales sint homologis M*e*

& B*e* in Fig. 89 & (§.373, n.11) rec-lum M*e*. B*e* = T.*ed* , erit etiam $md.bd = de.T$; ergo aequabitur quoque rec-lum *dy.bd*, id est, ut paulo ante, dictum trapazium *bdgi*, cui rec-lum *bdy* aequale est, rec-lo A.*de* ex gravitate uniformi A in altitudinem alicujus loci *de*, & cum hoc ita sit de reliquis, (§. 319) linea *nih*, vel ejus portio *ih*, erit reciproca scalae elasticitatis aëris.

II. Propter hyperbolam $l\alpha\beta$ habetur $b\beta : d\delta = nd : nb = gd : ib$, atque adeo singula rec-la, $gd\delta$, $ib\beta$, hal, &c. sunt aequalia, ac proinde (§. 357) curva $l\alpha\beta$ est scala elasticitatis, cum lineam *ih* ejus reciprocam esse ostensum sit.

IlI. Quia (num.l. hujus) $ibdg = \mathrm{A}.de$, erit $de = ibdg : \mathrm{A} = bd.yd : \mathrm{A}$, eritque adeo altitudo loci *d*, in quo densitas est $d\delta$, quarta proportonalis ad A, *bd* seu B*e* & *yd*. eaque semper haberi potest, ac proinde scala densitatis atmosphaerae in hac hypothesi semper per puncta describi geometrice poterit. Quae omnia erant invenienda.

COROLLARIUM I.

377. Patet iterum multum abesse, ut vires comprimentes aëris ejusve elasticitates densitatibus proportionales sint, stante progressione arithmetica altitudinum AC, C1C, 1C2C aequivalentium aequalibus cylindris Mercurii aliusve liquoris homogenei, quorum altitudines exponuntur per BE, E1E, 1E2E, &c.

COROLLARIUM II.

378. Densitas aëris ad horizontem erit ad densitatem ejus in extremitate atmosphaerae sicut $b\beta$ ad *al*, vel sicut *an* ad *bn*, hoc est in altera figura (89.) sicut AN ad BN vel MN.

COROLLARIUM III.

379. Hinc si dicantur AC vel AP aut 1E1O, a ; excessus ipsius 1E1F supra EF, hoc est 1O1F, *e*; tum $\mathrm{AB} = b$, & $\mathrm{BE} = \mathrm{E1E} = l$, invenietur $\mathrm{BN} = \mathrm{NM} = 2al - el : 2e$, & $\mathrm{AN} = b, +(2al - el) : 2e, = (2be + 2al - el) : 2e$; eritque proinde AN ad BN, hoc est, densitas atmosphaerae in horizonte ad densitatem ejusdem in continio atmosphaerae totius ejusve termino, ut $2be + 2al - el$ ad $2al - el$. Igitur si in hac ratione generali cum Cel. Maraldo loco *a, e , b* & *l* ponatur 61, 1, 336 & 1, fiet AN ad BN ut 793 ad 121. Sin vero cum Mariotto sint $a = 63$ ped., $e = \frac{61}{268}$, $b = 2.8$ poll. $= 336$ lineis & $l = 1$ lin., erit $2be + 2al - el$ ad $2al - el$, ut 1007 ad 335 seu quam proxime in ratione tripla.

SCHOLION.

380. Liquet igitur, in suppositionibus D. Maraldi, aërem prope horizontem non septuplo densiorem esse aëre in termino atmosphaerae, nec Mariotti suppositiones eundem quadruplo densorem facere ad horizontem, quam sit in summitate atmosphaerae, & tamen omni illic elastica virtute destitui oportere; quandoquidem nihil ipsi sucumbit, quod ejus expansionem, si quam haberet, impedire valeat; quod procul omni dubio paradoxum est. Nam si experimentis Academiae Florentinae *Del Cimento*, Roberti Boylii, aliorumque

constat, aërem vi elaterii 60 es, 152es imo & millies rariorem fieri posse, quam in statu naturali, qui fit, ut in suppositionibus Maraldi non septies, in Mariotti vero non quidem, quater rarior in summitate atmosphaerae quam in statu naturali , omni vi elastica destituatur? Verum quidem est, Clariss. Maraldum non ultra extensionem semissis Milliaris Gallici progressionis suae periculum fecisse, nec adeo ulterius eam extendi debere, expresse monuisse.

PROPOSITION XXIX. PROBLEMA.

381. *Si scala elasticitatis aëris* A1DD *fuerit hyperbola communis inter asymptotas* FPQ *angulum rec-lum continentes, atque singula rectangula* EOD, 1E1O1D *dato rectangulo* PFA *aequalia fuerint, quaeritur reciproca linae* E1E *scalae elasticitatum, & scala densitatum atmosphaerae* B1B *in suppositione gravitatis uniformis seu constantis, cujus scala sit* C1C *axi* A1A *parallela.* Fig. 91.

In recta indefinita AM sumatur AI aequalis datae AF, ductaque per punctum I recta IT parallela AO, intra lineas MI & IT tanquam asymptotas descripta intelligatur hyperbola E1E, ita ut quodlibet in ea rec-lum ILE aequetur quadrato datae AI; eritquc haec hyperbola E1E reciproca scala elasticitatis A1DD; adeo ut facto rec-lo AC1C aequali quadrilineo OE1E1O; densitas aëris in loco 1A futura sit per §.359, 1O1D vel 1A1B.

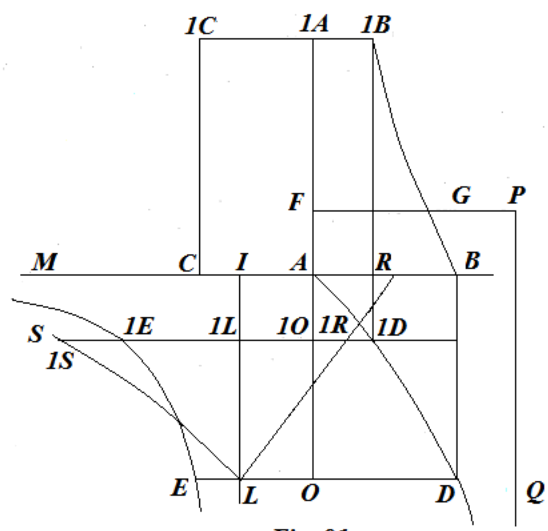


Fig. 91

Demonstr. Quia (secundum hypothesin) $EO.OD = FP.FA$, erit $EO : FA$ seu $LO = FP : OD$ seu FG, ergo convertendo $EO : EL = FP : GP$ (id est propter hyperbolam A1DD) $= GD$ seu $FO : FA$, ac dividendo $LO : EL = AO$ vel $IL : AF$ seu AI vel LO: ergo rec-la extremorum & mediorum aequabuntur, id est $LO^2 = AI = IL.LE$,
atque adeo punctum E est in hyperbola E1E inter asymptotas MI & IT ; ac propterea reciproca scalae elasticitatis in praesentae hypothesi est hyperbola communis. Hinc, si rectangulum quoddam A1C cuilibet in hac hyperbola quadrilineo O1O1EE aequale factum, atque linea 1C1A protensa ad alteram axis partem 1A1B aequalis facta fuerit ordinatae 1O1D, punctum 1B (§. 359) erit in scala densitatis atmosphaerae B1B. Quae erant demonstranda.

SCHOLION.

382. Hoc problema est tantum applicatio theorematis nostri generalis, supra (§.359) exhibiti, hypothesi particulari, quam Propositione 23 hujus excussimus; quoniam in superioribus jam vidimus, densitates aëris viribus comprimentibus non semper proportionales existere; sed praesentem hypothesin cum observationibus propius conspirare. Caeterum liquet etiam, quod, quia scalae densitatum reciproca E1E etiam est hyperbola, perinde ac in hypothesi densitatum viribus comprimentibus proportionalium , hoc solo cum discrimine, quod in hac hypothesi ejus asymptota una sit ipsa AO, in praesenti vero recta IT, modo nominatae AO aequidistans, linea seu scala densitatum B1B ope log-micae cujusdam per puncta describi possit. Nam ad asymptotam AM ducta per punctum L logarithmica L1SS, cujus subtangens sit aequalis AI, perinde ac AC, quae gravitatem uniformem exponit, fiat recta LR angulum semirec-lum lLR continens cum IL, atque ipsam 1L1D secans in 1R; si C1C constanter aequalis fiat ipsi 1R1S interceptae inter log-micam L1SS & rectam L1RR, atque per punctum 1C ducta indefinita 1C1B in ea ultra axem abscindatur, ut prius 1A1B aequalis ordinatae 1O1D in scala densitatum atmosphaerae B1B. Hisce diu nonnihil institimus, quia cognitio densitatum atmosphaerae. utilitate sua non caret ; nam praeterquam quod accuratissimus haberetur modus mensurandarum altitudinum montium aliorumque in superficie terrae altiorum objectorum ope barometrorum, cognita lege, juxta quam densitates atmosphaerae sursum decrescunt , accuratius innotesceret quam radii solares siderumve atmosphaeram trajicientes incurvantur, quod utique utilitatem haberet eximiam in astronomicis. Caeterum difficillimum; si non impossibile, est a priori noscere quamnam densitates atmosphaerae cum ponderibus incumbentibus collatae, legem sequi debeant propter mille anomolias, quae in atmosphaera contingunt, idcirco res ex observationibus bene multis & diligenter institutis derivari debet modo sequenti.

383. In recta quadam indefinita abscindantur lineae eandem inter se proportionem habentes, quam altitudines locorum diligentissime observatae, & per singula divisionum puncta agantur perpendiculares eandem inter se proportionem habentes, quam quantitates Mercurii in barometro in praedictis locis versante, & sic tot habebuntur puncta, quot sunt observatae locorum altitudines, per quae puncta proinde duci potest linea quaedam regularis eo prorsus modo, quem Celeb. Newtonus Lemm. V. Lib. III. *Pr. Ph. Math*. absque ulla demonstratione exponit; & talis curva apta nata esset ad exprimendam relationem, quae est inter altitudines locorum & argenti vivi quantitatem barometro successive in his locis versante. Ex hac curva elici potest linea reciproca scalae elasticitatum, & ex figura huius scalae reciproca deduct potest ipsae scala elasticae virtutis in aëre; dehinc, ope theorematis nostri §. 359 ipsa etiam scala densitatum atmosphaerae per puncta describi potest. Verum multis observationibus opus est, quia, quo plura sunt puncta positione data, eo accuratius definietur ipsa curva, de qua ante dixi; quomodo vero calculus subducendus sit pro hac curva alias commodius aliquando dicetur, quando demonstrationem praefati V. Lemmatis Newtoniani demonstratum dabimus cum aliis nonnullis affinibus materiis.