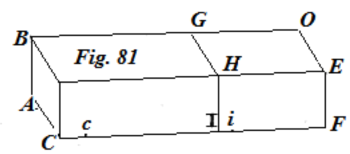
CHAPTER VII.

Concerning the elastic forces of the air compared with its densities.

Not only by experiments, of which I have reviewed several at the beginning of the previous chapter, it has been agreed the air to be endowed with an elastic force ; but also it has become known by the philosophers this expansive force of air to be greater there, where the density of the air shall be greater, and vice versa; therefore at this point it remains to be examined whether the elasticities increase precisely along proportional with the densities, or whether indeed according to some other ratio. In this chapter we will present some theorems towards resolving this matter.

339. Since the elasticity of the air shall agree with these pressures, by which the molecules of the air are trying to recede mutually from each other, it is evident that the pressure is equal to the whole force of the individual particles pressing, which any plane endures, by which the expansion of the air is impeded

PROPOSITION XXI. THEOREM.



340. *If the air within the volume of any prism* BOF *is driven into smaller and smaller volumes* GOF*, in which the proximity of any molecules is* C, c *and* I, i *trying in turn to recede from each other, they shall be in the reciprocal ratio of the density, of which the exponent of the intervals* Ce *and* Ii *of the same molecules is* n*, that is, in the ratio* Cc^{-n} *to* Ii^{-n}*, or* Ii^{n} *to* Cc^{n}*. The forces by which the planes* BC *and* GI *are pressed by the elastic force of the air* BF *and* GF*, will be as the power* n *of the density of the air* BF*, to a similar power of the air* GF*. That is, as* d^{n} *to* D^{n} *by calling the density of the air in the volume* BF*, d; and the density in the volume* GF*,* D*.*

For the force, by which the plane BC is acted on by the particles of the air in contact with the plane itself, is to the force, by which GHI is acted on by the air particles near to itself, is as all the particles in the plane BC to all, which are in the plane GHI, or rather, as the force resulting of all, by which the particles of the plane exert a force adhering to that, to the force resulting from the forces, by which all the molecules of this plane GHI are endowed, and the force of all C, which are in the plane BC, is to the force of all I, which are in the plane GHI, as the force of a single C to the force of a single I, that is (following the hypothesis) as Ii^{n} to Cc^{n}; therefore the force, by which the plane BC is acted on, is to the force, by which the plane GHI is acted on, as Ii^{n} to Cc^{n}. Truly Ii is to Ce as IF to CF, or as the prism BF to the prism GF, that is, as the density of the air BF to the density of the air GF or, just as *d* to D, therefore Ii^{n} ad Cc^{n}, that is, the force, by which the plane BC is acted on, to the force, by which the plane GHI is acted on by the included air

GF, is as d^n to D^n, or directly, as the power of the density of which the index is *n*. Q.E.D.

COROLLARY.

341. And thus, if the forces, by which the particles C, c, and I, i in turn are trying to flee from each other, will be inversely proportional to the distances between them Ce, Ii , the elastic forces of the air will be proportional to the densities.

PROPOSITION XXII. THEOREM.

342. *If the air, which is within the volume* CFE, *is forced back into a smaller volume* cfe *similar to the earlier, and the forces, by which the molecules of the air in each volume are trying to recede from each other in turn, shall be as in the preceding proposition, in the reciprocal ratio of the powers with the index* n *of the intervals* GH, gh *by which the molecules* G, g *from the adjacent planes* BAE *and* bae *a from the nearby* H *&* h *stand apart, the force will be, by which the whole plane* BAE *is acted on by the elasticity from the air* FCE, *to the force, by which the plane* bae *may be pressed by the air* fce, *as the side of the cube with the power of the density d of the air* FCE *denominated by the index n, to the side of the cube with the density D of the air* D *of the air* cfbe*; that is, as the cube root of* d^n *to the cubic root of* D^n. Fig.82.

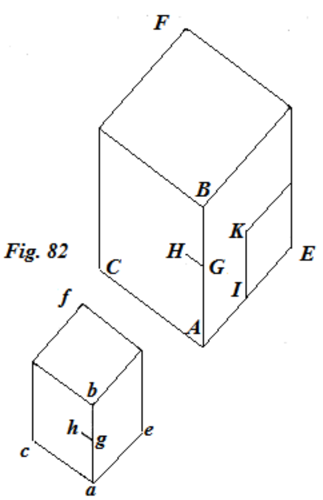


Fig. 82

Because the volumes CFE, *cfe* are similar solids, also the planes BAE, *bae* will be similar figures and particular areas on these will be considered similar. Therefore the elastic force of the air CFE, which it exerts on the plane BZ, will be to the elastic force of the air *cfe* superfluous on the plane *bae*, as the force, by which a single particle G presses on the plane BAE to the force of a single particle *g* acting on the plane *bae*, that is (following the hypothesis) as gh^n to GH^n, or because GH and *gh* in similar solids CSE & *eft* are similarly short lines put in place, so that ab^n to AB^n. Now because the volume *cfe* is to the volume CFE as the density of the air in FA to the density of the air in the volume *fa*, that is, as *d* to D, then also as the volume *fa* to the volume FA, or because these volumes are similar (following the hypothesis) as ab^2 to AB^2, there will be $ab : \mathrm{AB} = \sqrt{\mathrm{C}}.d : \sqrt{\mathrm{C}}.\mathrm{D}$ and $ab^n : \mathrm{AB}^n = \sqrt{\mathrm{C}}.d^n$, ad $\sqrt{\mathrm{C}}.\mathrm{D}^n$. Therefore the force of the air, which it exerts on the plane BAE, is to the force, which it exerts on the plane *bae*, as the side of the cubes from d^n to the side of the cube D^n. Q.e.d.

COROLLARY I.

343. Truly the figure KIE in the plane BAE similar and equal to the figure *bae* submits to the pressure from the air with the elasticity FA, itself having to the pressure, which the elastic air *fa* exerts on the plane *bae*, as the side of the cubes from d^{n+2} to D^{n+2}. For the pressure of the figure KIE is to the pressure of the figure BAE, as the figure KIE, or if the equivalent *bae* to the figure BAE, that is, on account of the similitude of the figures, as ab^2 to AB^2, and (§. 342.) the pressure of the figure BAE to the pressure of the figure *bae*, as $\sqrt{\mathrm{C}}.\, d^n$ to $\sqrt{\mathrm{C}}.\mathrm{D}^n$, therefore from the equation and through the composition of the ratio of the pressure in figure KIE to the pressure had in figure *bae* itself, as the side of the cube from d^{n+2} to D^{n+2}. As the celebrated Newton has in Sch. Prop. 23. Lib. II. *Pr. Ph. Nat. Math.*

COROLLARY II.

344. Hence, if again there were $\mathrm{n} = 1$, or the centrifugal forces of the air molecules were in turn inversely proportional to their distances apart, the elastic forces or pressures, which the equal figures KIE and *bae* in turn each will be impressed on by its air, in the ratio of the densities, and in this circumstance the matter agrees with the corollary of the previous proposition. An conversely, if the elastic forces were proportional to the densities, the forces of the molecules, or the centrifugal trials, will be inversely proportional to their distances from nearby molecules.

PROPOSITION XXIII. THEOREM.

345. *If the elastic forces of the air were as the ratios, which the quantities of air have to the volumes remaining, within which the air may be contained [i.e. the densities], the same forces [i.e. pressures] will be as the ordinates to the other line parallel to the asymptote of a rectangular hyperbola, truly the densities, as the abscissas of homologous ordinates, of which the origin shall be on that hyperbolic curve itself.* See Fig. 83.

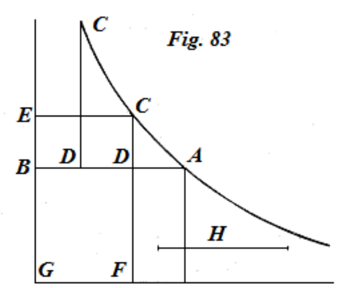


Fig. 83

If the segments AD of the given line AB express the amounts of air contained in the same volume, with the whole contained expressed by AB, likewise AD will indicate the densities of the same air; for following §16, the densities are as the ratios, which the quantities of matter have to the volumes, in which they are contained, and in this situation different quantities of air are considered to be enclosed in the same volume. Truly the segments DB of AB itself are the volumes of air remaining from the volume AB, clearly with the volumes of air AD, AD compressed from this volume of air AB &c. At the points D on AB, the perpendiculars DC expressing the elastic forces of the air to be set up, having the homologous densities AD ; with which in place, and because (following the hypothesis) the elasticity of the air with a given density AD is, as the ratio AD to DB; the ratio arises, with a certain density H given, $DC:H = AD:DB$, [for any values of D as shown] and by adding there will be $DC + H:H = AB:DB$, and thus the right line EB produced normal to the given AB at G, as GB may become equal to the assumed given line H, and with another line GF drawn through the point G parallel to the other line AB, there will be $CF:DF = AB:DB$ [here the l. h. ratio is one of densities rather than volumes]; or with CE drawn through C parallel to AB itself, there $EG:BG = AB:CE$, from which the curve AC is a hyperbola between the asymptotes GF and GE, of which the ordinates DC express the elastic forces of the air, and indeed the abscissas AD the densities respectively. Q.E.D.

COROLLARY.

346. If the AD themselves were of very small magnitudes, with respect to the whole of AB, a trilinear shape or shapes ADC are able to be viewed just as straight lines without perceptible error, and in this case the elasticities DC of this kind will emerge as approximately proportional to the densities AD ; but if indeed the densities shall be greater, which in order that AD may vanish before the whole AB ; the elasticities of the air will increase for a long time by a greater proportion than the densities. The celebrated James Bernoulli thought out the present hypothesis and has tried to give a physical reason in his tract *De Gravitate Aetheris*,[Concerning the Weight of the Aether, 1683], page 97 onwards, which the reader will be able to examine and read there. Also from this hypothesis his celebrated brother Johann proposed this Proposition XIII of ours in his elegant dissertation *De Motu Musculorum* [Concerning the Motion of the Muscles] now praised above.

SCHOLIUM.

347. The Leibnizian hypothesis, where the illustrious man decided air to be considered as a mixture of compressible and incompressible matter can perhaps be examined not incongruously according to the present proposition. In that hypothesis of pure air or matter agreeing only to be compressible with elasticities proportional to the densities, truly in a mixture of air from compressible and incompressible matter the forces of elasticity do not follow the ratio of the densities, but in the majority of these they are present in a ratio between themselves, thus so that for the variation in proportion of the mixture from the compressible and incompressible materials, the law must be varied for the densities of the combined elasticities, and thus an agreement for the ratio shall be reached, the elastic forces to be in the proportion of the ratios amongst themselves, which the quantities of the material contained compressed within some volume have to its remaining volume of material fully compressed, according to the hypothesis had of this Proposition XXIII. Otherwise from the accurate experiments of Boyle, Mariotte, Bernoulli, Amontonius, Joh. Polenus and others the densities of the air were found from the forces of compression, or from the elasticities of that which emerge approximately proportional. One way amongst others in which the experiment may be performed, is what follows.

[Thus Leibniz tried to accommodate deviations from Boyle's Law, as Van der Waals did much later for the general gas equation of state; the volume occupied by the actual molecules is of course incompressible; the residual attraction between the particles accounts for the extra pressure, as gases can be liquefied ; we note the missing component of the gas laws, namely the absolute temperature, in the present description; there being no mention of the measurements being made at the same temperature.]

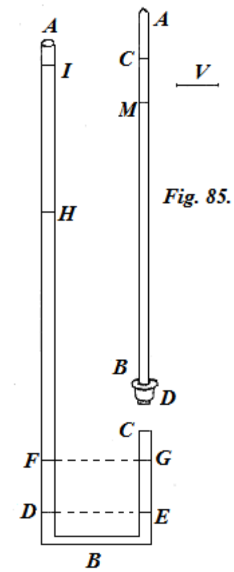


Fig. 85.
Fig. 84

348. Fig. 84. The tube ABC shall be an example of a reflex siphon open at A and hermetically sealed at C, quicksilver is poured through the opening A into the shorter leg reaching as far as to E, and in the longer AB as far as to H, and where all will remain standing, with the base D from the incumbent mercury HD, by the increased pressure of the whole atmosphere, or of 28 inches, will undergo the same pressure, as the base E from the elasticity of the air enclosed in the volume CE ; if again mercury may be poured through the opening A so that at last it remains at I, and in the other branch CB it may reach as far as to G, thus so that the air CE shall be forced back to the smaller volume CG, the base F will sustain the pressure of the column IF increased by the weight of the atmosphere of 28 inches, and the other G will endure the equal pressure CG from the elasticity of the air. Therefore if $\mathrm{IF}+28$ inches of mercury to $\mathrm{HD}+28$ inches were as CE to CG, the compressing forces or elasticities of the same air will be directly proportional to the densities, as has been proven by several experiments by Mariotte, Amontonio & Poleno. See Mariotte *Tentamen de Natura Aëris*, as well as *Tractatus de Motu Aquarum*, in which he has set out carefully the experiments performed, and likewise Amontonio published later in the *Commentariis Acad. Scient. Paris*. But because commonly the volumes CE, CG, &c. and the measures of these are extremely small and with the proportions requiring to be investigated, it is easy to make an error ; therefore Mariotte now being required to investigate these same proportions between the elastic or compressing forces, and the density of the air, with the merit added of another desired from the observations of the barometer, which James Bernoulli then improved more in his tract *De Gravitate Aetheris*. Here the way is indirect, because in this that is assumed, which is required to be proven in the form of the principle, and the conclusion is brought together with the phenomena observed, with which if it will agree in retrospect with the order adopted it is concluded the principle is assumed to be true. For mercury is poured into the tube of the barometer in this other way of doing the experiments far as a certain indicator, with the rest of the tube left consisting of natural air, not indeed, as is accustomed to happen in the usual construction of barometers, where the whole tube is filled with mercury ; hence with the tube inverted, with its opening obstructed, and by being sent below the surface of the mercury at rest in some vessel, the obstruction of the opening is removed, so that the mercury in the tube may be able to descent perpendicularly into the vessel, which indeed will not flow altogether from the tube, but yet will itself drop lower in the tube, by means of which air is introduced into the

ordinary barometer ; from which, as much as can happen, all the air is accustomed to be carefully excluded. Now if the mercury put in place by this operation, may fall to another mark, and to remain at that, which the calculation has indicated it must halt, which is established according to the hypothesis, because the densities are proportional to the compressing forces, is anyone going to call into question the goodness of the hypotheses? Because truly the mercury may reach that precise mark, which the calculation based on this memorable hypothesis has determined, that agreed upon from accurate measurements by Mariotte and James Bernoulli. Truly just how far the mercury must fall in the aforementioned circumstances, the following problem will show.

PROPOSITION XXIV. PROBLEM.

349. *With the pressure of the atmosphere given, and with the length of some tube AB open at B, truly closed at A, to assign at what height the mercury within the tube must remain suspended, if, after filling the tube there only as far as a part of the length, V the part empty of mercury, but full of the natural air it may retain, and after the obstruction of the opening B by the fleshy part of a finger or by some other means, and with the tube inverted the same obstruction shall be immersed in the vessel D of quicksilver at rest, with the tube present in the perpendicular position, thus so that the air enclosed by the tube at the head of the tube AB shall be able to gather together to equal V at first, then with the finger removed obstructing the opening B, the mercury shall be able to descend freely.* Fig. 85/86.

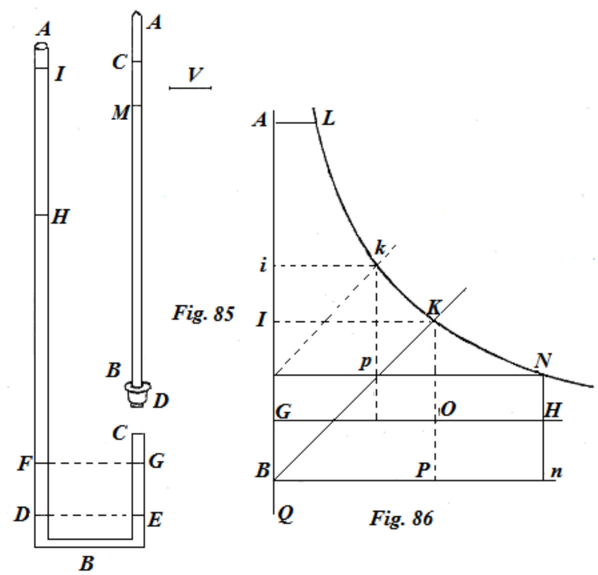


Fig. 84 Fig. 85 Fig. 86

AG shall be equal to the column of mercury equivalent to the atmosphere, or approximately 28 inches, and to that at G the line GH of indefinite length may be prepared for the right angle AGO, and on the right line AL passing through A, parallel to GH there is taken AL equal to V, or to the small column of air AC introduced into the tube AB full of mercury as far as C, and with the hyperbola LKN described through the point L within the asymptotes AGH ; and the line AB taken on the asymptote AG equal to the given length of the tube AB, the line BK is drawn through the point B constituting a semi right angle; with the asymptote AG, and the line IK is acting parallel to the asymptote GH from the point K of the hyperbola, on which the right line BK crosses with that, AI will be equal to the height of the column of quicksilver BM, at which it exists in equilibrium with the external atmospheric pressure, and BI or IK expresses the volume AM, into which the air of a natural consistency will stretch itself out in the tube AC.

Demonstr. Because the density of the air AM is to the density of the air AC as AC to AM, that is, as AL to IK, or by means of the hyperbola LKN, as GI to GA, and the densities of that air with the elastic forces within these levels of density are (following the hypothesis) as these densities, the elastic force of the air AM will be to the elastic force of the air AC, as GI to AG; and the air of a natural consistency AC has the elastic force expressed equal to the pressure of the atmosphere AG, therefore the elastic force of the air AM will be equivalent to the column of mercury GI, and thus the pressure composed from the elasticity of the air AM and from the weight of the column $\mathrm{BM} = \mathrm{AI}$, AG is composed from AI and IG; therefore the internal pressure is equal, resulting from the elastic force of the air AM and from the weight of the mercury MB , to the external weight of the atmosphere AG, and therefore the column of quicksilver hangs freely at the height MB within the tube. Q.E.D.

COROLLARY I.

350. Now from these the value of the right line BI may be elicited easily or equally of IK or AM. But in this determination the cases are to be distinguished. For the length of the tube AB can be greater or less than AG. If the former, and because the semi right angle IBK makes the line BI equal to the other line IK, thus so that the figure IKPB shall be a square from the two right rectangles IO and GP taken together, that is $\mathrm{IK}^2 = \mathrm{IK.BG} + \mathrm{GH.HN} = \mathrm{IK.BG} + \mathrm{AG.AL}$, therefore $\mathrm{IK} = \frac{1}{2}\mathrm{BG} + \sqrt{\left(\frac{1}{4}\mathrm{BG}^2 + \mathrm{AG.AL}\right)}$.

Hence with BG being nothing from the coincidence of the points B and G, the equality becomes $\mathrm{IK} = \sqrt{\mathrm{AG.AL}}$

But indeed with the same in place if the length of the tube AB were less than AG, Aβ becomes equal to the length of the tube, and with the construction of the problem treated by the previous paragraph, by a similar argument it will be found :
$$ik = -\tfrac{1}{2}\beta\mathrm{G} + \sqrt{\left(\tfrac{1}{4}\beta\mathrm{G}^2 + \mathrm{AG.AL}\right)}.$$

COROLLARY II.

351. If now with the celebrated James Bernoulli, who also treated this problem in his treatise T*ractatu De Gravitate Aetheris* pag. 117. *seqq*., we may put for AC or AL, a; BC, b; and thus $\mathrm{AB} = a + b$, $\mathrm{AG} = b + c$, from which $\mathrm{AG} - \mathrm{AB} = \beta\mathrm{G} = c - a$, and finally $\mathrm{CM} = y$; and with these values substituted in the final equation of the paragraph above, it will be found :
$$ik = a + y = \tfrac{1}{2}a - \tfrac{1}{2}c + \sqrt{\left(\tfrac{1}{4}aa + \tfrac{1}{2}ac + \tfrac{1}{4}cc + ab\right)}$$ and thus
$$y = -\tfrac{1}{2}a - \tfrac{1}{2}c + \sqrt{\left(\tfrac{1}{4}aa + \tfrac{1}{2}ac + \tfrac{1}{4}cc + ab\right)}$$, which is the very equation itself, which the distinguished man himself found also. We would have fallen on the same equation, if in the other formula of the preceding corollary there were substituted $a - c$ in place of BG.

With numbers the tube AB shall be 21 inches, AG now 27 inches, AC, 1 inch, and *ik* or AM will be found to be 3 inches and thus BM, 18 inches.

PROPOSITION XXV. PROBLEM.

352. *With the column of quicksilver AG given, by being equivalent to the atmospheric pressure, and with AB the given length of the tube, to define how great a volume AC must be full with the consistency of natural air must be introduced to the top of the tube, so that after the air may extend itself from AC into the volume AM, the mercury may remain suspended at the given height BM within the tube.* Figs.85, 86.

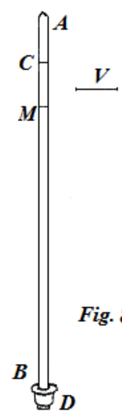


Fig. 85

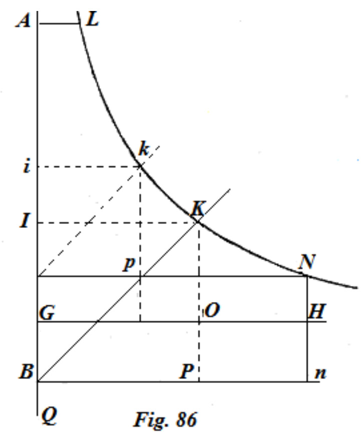


Fig. 86

In this problem AL, AC or V is sought from the given AG, AB, and either AI or BM. Therefore with the semi right angle ABK made, and with AI = to the given , in the tube with the height of the mercury remaining BM, IK acts parallel to the asymptote GH crossing the line BK at K, through which point now the hyperbola NKL being drawn, for the right line AL itself parallel to GH crossing at L, AL will be the volume sought. The demonstration is completely the same with the previous demonstration, and thus here it is not required to be repeated. AL will be had in numbers by applying the given rectangle GIK to the line AG, which expresses the pressure of the atmosphere : for GI is the excess, by which AG exceeds the given AI or BM, and either IK, BI or AM is also (following the hypothesis) given, and thus the rectangle GI.IK will be equal to the given rectangle AG.AL; thus so that by dividing the rectangle GI. IK by AG, the quotient, or by the geometrical phrase the fourth proportional for AG, IG & IK shall become AL, denoting the amount of air sought to be introduced into the tube AB, so that the mercury may remain at the given height BM. Q.E.D.

SCHOLIUM.

313. Now if the determinations of this and of the preceding propositions shall agree with the phenomena, to which they are taken to agree well enough with these ; generally it has to be inferred, the assumed hypothesis, that the density of the air shall be the proportionals of that with its compressions or elasticities, to be in enough harmony with nature, provided it is not taken at a greater latitude, as the experiments, with which it may be seen to agree. Indeed experiments of this kind with air of medium density have been taken; therefore, if anyone may wish to extend the aforementioned hypothesis to the densest air, he would be very much mistaken, since along with what has been said above (§. 346.) in denser air the elasticities have a greater proportion between themselves than the densities agreeing with these.

CAPUT VII.

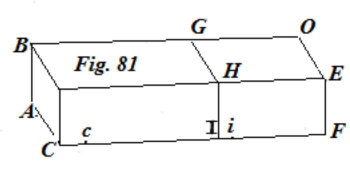
De Viribus elasticis aëris cum densitatibus ejus comparatis.

Non solum per experimenta, quorum nonnulla initio capitis proxime antecedentis recensui, constitit aëra vi elastica praeditum esse; sed etiam Philosophis innotuit hanc vim expansivam aëris eo majorem esse, quo major sit aëris densitas , & vice versa; idcirco
examinandum adhuc restat an elasticitates crescant praecise juxta proportionem densitatum, an vero secundum aliam quamcunque rationem. In hoc capite nonnulla exhibebimus theoremata ad hanc rem facientia.

339. Cum aëris elasticitas consistat in nisibus illis, quibus aëris moleculae a se mutuo recedere conantur, manifestum est, quod pressio, quam quodlibet planum, quo aëris expansio impeditur ae particulis aëris plano contiguis subit, aequalis est universae vi singularum particularum urgentium.

PROPOSITIO XXI. THEOREMA.

340. *Si aër sub volumine cuiusvis prismatis* BOF *redigatur in minus volumen* GOF, *minusque, quibus proximae quaeque moleculae C,* c *&* I,i *a se invicem recedere conantur , sint in reciproca ratione densitatis, cujus exponens est* n, *intervallorum* Ce *&* Ii *earundem molecularum, hoc est, in ratione* Cc^{-n} *ad* Ii^{-n}, vel Ii^{n} *ad* Cc^{n}. *Erunt vires, quibus plana* BC *&* GI *ab aëre elastico* BF *&* GF *premuntur, ut potestas* n *densitatis aëris* BF, *ad similem potestatem aëris* GF. *Hoc est, ut* d^{n} *ad* D^{n} *vocando densitatem aëris in volumine* BF, *d & densitatem in volumine* GF, D.



Nam vis, *qua p*lanum BC a particulis aëris ipsum tangentibus urgetur, est ad vim, qua *planum* GHI a particulis aëris ipsi contiguis urgetur, est ut omnes particulae in plano BC ad omnes, quae sunt in plano GHI, vel potius, ut vis ex omnibus, quibus particulae plano illi adhaërentes pollent, resultans, ad vim resultantem ex viribus, quibus omnes moleculae plani hujus GHI praeditae sunt, atqui vis omnium C, quae sunt in plano BC, est ad vim omnium I, quae sont in plano GHI, ut vis unius C ad vim unius I, id est (secundum hypothesin) ut Ii^{n}ad Cc^{n}; ergo vis, qua planum BC est ad vim, qua planum GHI urgetur, ut Ii^{n}ad Cc^{n}. Verum est Ii ad Ce ut IF ad CF, seu ut prisma BF ad prisma GF, id est, ut densitas aëris BF ad densitatem aëris GF vel, sicut *d* ad D, ergo Ii^{n}*ad* Cc^{n}, id est, vis, qua urgetur planum BC, ad vim, qua ab incluso aëre GF planum GHI urgetur, est ut d^n ad D^n, seu directe, ut densitatum dignitas cujus index est *n*. Quod erat demonstrandum.

COROLLARIUM.

341. Adeoque,si vires,quibus particulae C, c, & I, i a se invicem aufugere conantur, fuerint interstitiis Ce, Ii reciproce proportionales vires elasticae aëris erunt densitatibus proportionales.

PROPOSITIO XXII. THEOREMA.

342. Fig.82. *Si aër, qui est sub volumine* CFE, *redigatur in volumen minus* cfe *priori simile, ac vires, quibus moleculae aërae in ultroque volumine a se invicem recedere conantur, sint, ut in propositione praecedenti, in reciproca ratione potestatis ex i*ndice n *intervallorum* GH, gh *quibus moleculae,* G, g *planis* BAE *&* bae *adjaccntes a proximis* H *&* h *distant, erit vis, qua totum planum* BAE *ab aëre elastico* FCE *urgetur, ad vim, qua planum* bae *ab aëre* fce *prematur, ut latus cubicum ex potestate densitatis d aëris* FCE *ab indice n denominata, ad latus* cubicum*um ex pari dignitate densitatis* D *aëris* cfbe*; hoc est, ut radix cubica ex* d^n *ad radicem cubicam ex* D^n.

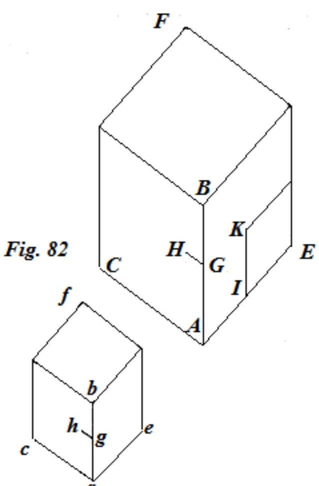


Fig. 82

Quia volumina CFE, *cfe* sunt solida similia, etiam plana BAE, *bae* erunt figurae similes & particulae aëreae in iis similiter positae erunt. Propterea vis elastica aëris CFE, quae exeritur in planum BZ, erit ad vim elasticam aëris *cfe* redundantem in planum *bae*, ut vis, qua unica particula G premit planum BAE ad vim unius particulae *g* urgentis planum *bae*, id est (secundum hypothesin) ut gh^n ad GH^n, seu quia GH & *gh* in solidis similibus CSE & est sunt lineolae similiter positae, ut ab^n ad AB^n. Jam quia solidum *cfe* est ad solidum CFE ut densitas aëris in FA ad densitatem aëris in volumine *fa*, id est, ut *d* ad D, tum etiam ut solidum *fa* ad solidum FA, vel quia haec solida (secundum hypothesin) similia sunt ut ab^2 ad AB^2, erit $ab : \mathrm{AB} = \sqrt{\mathrm{C}}.d : \sqrt{\mathrm{C}}.\mathrm{D}$ & $ab^n : \mathrm{AB}^n = \sqrt{\mathrm{C}}.d^n$, ad $\sqrt{\mathrm{C}}.\ \mathrm{D}^n$.
Ergo vis aëris,quae exeritur in planum BAE, est ad vim, quae exeritur in planum *bae*, ut latus cubicum ex d^n ad latus cubicum ex D^n. Quod erat demonstrandum.

COROLLARIUM I.

343. Figura vera KIE in plano BAE similis & aequalis figurae *bae* pressionem subibit ab aëre elastico FA, se habentem ad pressionem, quam aër elasticus *fa* exerit in planum *bae*, ut latus cubicum ex d^{n+2} ad D^{n+2}. Nam pressio figurae KIE est ad pressionem figurae BAE, ut figura KIE, vel aequalis *bae* ad figuram BAE, id est, propter figurarum similitudinem ut ab^2 ad AB^2, & (§. 342.) pressio figurae BAE ad pressionem figurae

bae, ut $\sqrt{C}.\,d^{n}$ ad $\sqrt{C}.D^{n}$, ergo ex aequo & per compositionem rationum pressio in figura KIE ad pressionem in figura bae se habet, ut latus cubicum ex d^{n+2} ad D^{n+2}. Ut habet Celeb. Newtonus Sch. Prop.23. Lib. II. Pr. Ph. Nat. Math.

COROLLARIUM II.

344. Hinc, si iterum fuerit $n = 1$, aut vires centrifugae molecularum aëris suis distantiis ab invicem reciproce proportionales, erunt vires elasticae, seu pressiones, quas aequales figurae KIE & *bae* a suo quaeque aëre subibunt, in ratione densitatum, atque in hac circumstantia res convenit cum Corollario Propositionis praecedentis. Et conversim, si vires elasticae densitatibus proportionales fuerint, molecularum vires, seu conatus centrifugi, distantiis suis a proximis moleculis reciproce proportionales erunt.

PROPOSITIO XXIll. THEOREMA.

345. *Si vires elasticae aëris fuerint, ut rationes, quas habent quantitates aëris ad residua voluminum, sub quibus aër continetur, eadem vires erunt ut ordinatae ad lineam alterutri hyperbolae aequilaterae asymoptotam aequidistantem, densitates vero, ut abscissae ordinatis homologae, quarum abscissarum origo sit in ipsa hyperbolae curva.* Fig.83

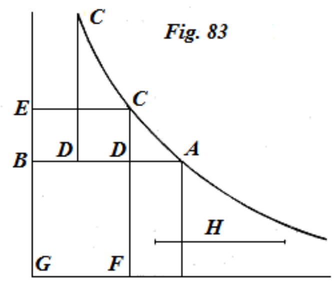


Fig. 83

Si segmenta AD lineae datae AB exponant quantitates aëris in eodem volumine, per totam AB exposito, contentas, eaedem AD densitates aëris simul indicabunt; nam juxta §.16, densitates sunt ut rationes, quas materiae quantitates habent ad volumina, in quibus continentur, & hoc loco diversae aëris quantitates eidem volumini inclusae spectantur. Segmenta vero DB ipsius AB sunt residua voluminis AB, detractis scilicet ex hoc volumine aëris quantitatibus AD, AD, &c. In punctis D ad AB excitatae sint perpendiculares DC exponentes vires elasticas aëris, densitates AD homologas habentis; quibus positis, & quia (secundum hypothesin) elasticitas aëris sub densitate AD est, ut ratio AD ad DB; fiat, assumta quadam data H, ratio $DC : H = AD : DB$, eritque componondo $DC + H : H = AB : DB$, atque adeo producta recta EB datae AB normali in G, ut GB fiat aequalis assumtae datae lineae H, ductaque per punctum G linea GF alteri AB aequedistanti, erit $CF : DF = AB : DB$; vel ducta per C parallela CE ipsi AB, fiet $EG : BG = AB : CE$, unde curva AC cujus ordinatae DC exponunt vires elasticas aëris, abscissae vero AD densitates respectivas, est hyperbola inter asymptotas GF & GE. Quod erat demonstrandum.

COROLLARIUM.

346. Si ipsae AD fuerint exiguae magnitudinis, respectu totius AB, trilineum, vel trilinea, ADC spectari possunt tanquam rectilinea absque errore sensibili, ac hoc casu elasticitates DC ejusmodi densitatibus AD quam proxime proportionales existerent; sin vero densitaties majores sint, quam ut AD prae tota AB evanescat; elasticitates aëris in longe

majori crescent proportione quam densitates. Hypothesin praesentem excogitavit Celeb. Jac. Bernoullius ejusque rationem physicam dare conatus est in tractatu suo de *Gravitate Aetheris* pag.97. seqq. quas Lector ibi legere atque examinare poterit. Ex hac hypothesi etiam Celeb. ejus Frater Johannes hanc nostram Propositionem XIII. proposuit in eleganti sua Dissertatione *De Motu Musculorum* jam supra laudata.

SCHOLION.

347. Leibnitiana hypothesis, qua Illustris Vir statuit aërem ut plurimum mixtum esse ex materia *comprimibili & incomprimibili* fortasse non incongrue ad praesentem propositionem exigi potest. In ista hypothesi aëris puri seu materia tantum comprimibili constantis elasticitates essent densitatibus proportionales, in aëre vero mixto ex comprimibili & incomprimibili vires elasticae non sequu tur rationem densitatum, sed in majori harum ratione existunt inter se, ita ut pro varia proportione mixturae ex comprimibili materia & incomprimibili elasticitatum cum densitatibus collatarum lex variare debeat, atque adeo rationi consentaneum sit, vires elasticas inter se esse in proportione rationum, quas quantitates materiae comprimibilis sub aliquo volumine contentae habent ad residua hujus voluminis materiae incomprimibilis plena, ut habet hypothesis hujus Propositionis XXIII. Caeterum Boylius, Mariottus, Bernoullii, Amontonius, Joh. Polenus & alii accuratis experimentis comperierunt densitates aëris viribus comprimentibus, seu elasticitatibus ejus quam proxime proportionales existere. Modus unus inter alios quo experimentum sumserunt, est qui sequitur.

348. Fig.84. Sit tubus ABC instar siphonis reflexus apertus in A & hermetice sigillatus in C , per orificium A argentum vivum infunditur in breviore crure pertingens usque in E, & in longiore AB usque in H, & ubi omnia in statu manente fuerint, basis D ab incumbente Mercurio HD, aucto pressione totius atmosphaërae, seu 28 digitorum, eandem pressuram subibit, quam basis E ab elasticitate aëris spatio CE inclusi ; si porro Mercurius infundatur per orificium A ut tandem subsistat in I, & in altero ramo CB pertingat usque ad G, ita ut aër CE in spatium angustius CG redactus sit, basis F pressuram sustinebit columnae IF auctae atmosphaërae gravitatione 28 digitorum, alteraque G ab aëre elastico CG parem pressionem subibit. Idcirco si $IF+28$ digit. Mercurii ad $HD+28$ digit. fuerit, ut CE ad CG, erunt vires comprimentes seu elasticitates aëris ejusdem densitatibus directe proportionales, ut a Mariotto, Amontonio & Poleno pluribus experimentis comprobatum est. Vid. Mariotti *Tentamen de Natura Aëris*, tum etiam Tractatus de Motu Aquarum, in quibus ejusmodi experimenta a se capta distincte exponit, perinde ac post ipsum fecit Amontonius in Commentariis Acad. Scient. Paris. Sed quia communiter spatia CE, CG, &c. perexigua sunt in eorumdem mensuris & proportionibus explorandis, facile est errorem committere; idcirco Mariottus modo isti proportiones investigandi inter vires elasticas, seu vires comprimentes, & densitates aëris, merito

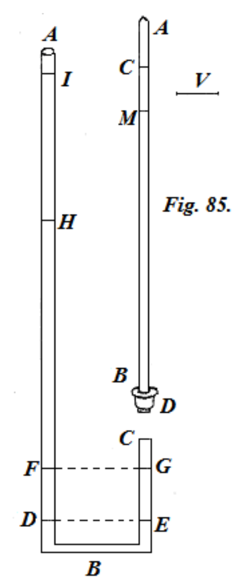


Fig. 85.
Fig. 84

alium adjecit ex observationibus barometri petitum, quem Jac. Bernoullius deinceps magis excoluit in suo Tractatu de Gravitate Aetheris. Hic modus est indirectus, quia in eo assumitur id, quod est probandum instar principii, & conclusio confertur cum phaenomenis, cum quibus si conspiraverit retrogrado ordine concluditur principium assumtum esse verum. Nam fistulae barometri in hoc altero experimentali modo infunditur Mercurius ad certum signum usque, reliquo tubi aëri naturalis consistentiae relicto, non vero, ut in communi barometrorum constructione fieri solet, totus tubus hydrargyro impletur; dehinc inverso tubo, obstructo ejus orificio, & demisso infra superficiem Mercurii in vasculo quodam stagnantis, aperitur obstructum orificium, ut Mercurius in fistula vasculo perpendiculariter insistenti descendere queat, qui non totus quidem ex tubo effluet, sed tamen humilius se in fistula demittet, propter aërem fistulae introductum quam in barometro ordinario; ex quo, quantum fieri potest, omnis aër diligenter excludi solet. Jam si mercurius hac operatione posita, ad illud ipsum signum delabitur, & in eo subsistit, in quo ipsum subsistere debere calculus indicavit, qui in hypothesi fundatur, quod densitates viribus comprimentibus proportionales, ecquis de bonitate hypotheseos ambiget? Quod vero Mercurius ad ea praecise signa perveniat, quae ipse calculus in hac memorata hypothesi fundatus determinavit, id constitit Mariotto & Jac. Bernoullio ex observationibus accuratis. Quousque vero Mercurius in praememoraris circumstantiis se demittere debeat, manifestabit sequens Problema.

PROPOSITIO XXIV. PROBLEMA.

349. *Datis atmosphaërae pressione, & longitudine alicujus fistule AB apertae in B, clausae vero in A, definire ad quam altitudinem intra fistulam suspensus manere debeat Mercurius, si, post impletionem fistulae eo usque ut sola pars longitudinis datae, V aequales, Mercurii vacua, sed aëris naturalis plena retineatur, & postquam apertum orificium B digiti pulpa vel alia ratione obstructum, inversoque tubo idem obstructum orificium argento vivo in vasculo D stagnanti immersum fuerit, tubo existente in situ perpendiculari, ita ut aër tubo inclusus in summitatae fistulae AB aequali V primam se colligere possit, dein remoto digito orificium B obstruente, Mercurius libere descendere queat.* Fig. 84/85.

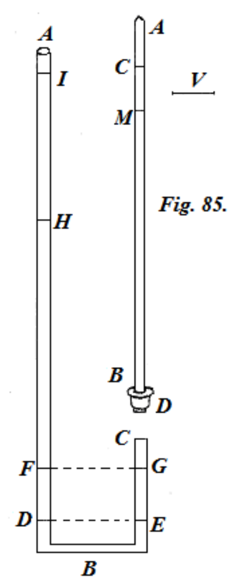


Fig. 85.
Fig. 84

Fiat AG aequalis columnae argenti vivi aequivalentis atmosphae, seu 28. digitorum circiter, eique in G ad angulum rectum AGO aptetur linea indefinite longa GH, & in recta AL per A transeunte, ipsi GH parallela sumatur AL aequalis V, seu columnulae aëreae AC tubo AB Mercurii pleno usque in C intromissae, descriptaque intra asymtotas AGH hyperbola LKN per punctum L; sumtaque in asymtota AG linea AB aequalis fistula AB longitudini datae, ducatur per punctum B linea BK angulum semirectum constitituens Fig. 85; cum asymtota AG, agaturque ex puncto K hyperbolae, in quo recta BK ei occurrit, linea IK parallela asymtotae GH, eritque AI aequalis altitudini columnae argenti vivi BM, in qua in aequilibrio existet cum externa atmosphaërae pressione, & BI vel IK exponet spatium AM, in quod aër naturalis consistentiae in fistula AC sese extendet.

Demonstr. Quia densitas aëris AM est ad densitatem aëris AC ut AC ad AM, id est, ut AL ad IK, seu propter hyperbolam LKN, ut GI ad GA, & densitates aëris viribus elasticis ejus sub illis densitatis gradibus sunt (secundum hypothesis) ut hae densitates, erit vis elastica aëris AM ad vim elasticam aëris AC, ut GI ad AG; atqui aër naturalis consistentiae AC vim elasticam habet atmosphaërae pressioni per AG expositae aequalem, ergo vis elastica aëris AM aequipollet columnae Mercuriali GI, atque adeo pressio compositae ex elasticitate aëris AM & gravitatione columnae $\mathrm{BM} = \mathrm{AI}$, est AG composita ex AI & IG; aequaliae ergo est interna pressio, resultans ex vi elastica aëris AM & gravitatione Mercurii MB , externae atmosphaërae AG, ac propterea columna argenti vivi in altitudine MB intra fistulam librata haeret. Quod erat demonstrandum.

COROLLARIUM I.

350. Ex hisce nunc facile elicietur valor rectae BI aut aequalis IK vel AM. Sed in hac determinatione casus distingui debent. Nam fistulae longitudo AB major esse potest quam AG, vel minor. Si illud, & quia angulus semirectus IBK lineam BI alteri IK aequalem efficit, adeo ut figura IKPB sit quadratum aequale duobus rectangulis IO & GP simul, hoc est $\mathrm{IK}^2 = \mathrm{IK.BG} + \mathrm{GH.HN} = \mathrm{IK.BG} + \mathrm{AG.AL}$, ergo $\mathrm{IK} = \frac{1}{2}\mathrm{BG} + \sqrt{\left(\frac{1}{4}\mathrm{BG}^2 + \mathrm{AG.AL}\right)}$. Hinc existente BG nulla coincidentibus punctis B & G, fiet $\mathrm{IK} = \sqrt{\mathrm{AG.AL}}$

Sin vero iisdem positis fistulae AB longitudo minor fuerit quam AG, fiat Aβ eidem fistulae longitudini aequalis, positaque constructione problematis antecedente paragrapho tradita, simili fere argumento invenietur $ik = -\frac{1}{2}\beta\mathrm{G} + \sqrt{\left(\frac{1}{4}\beta\mathrm{G}^2 + \mathrm{AG.AL}\right)}$.

COROLLARIUM II.

351. Si jam cum Cl. Jac. Bernoulli, qui etiam in suo Tractatu De Gravitate Aetheris pag. 117. seqq. hoc problema tractavit, ponamus AC vel AL, *a*; BC, *b*; adeoque $\mathrm{AB} = a + b$, $\mathrm{AG} = b + c$, unde $\mathrm{AG} - \mathrm{AB} = \beta\mathrm{G} = c - a$, & denique $\mathrm{CM} = y$; substitutis hisce valoribus in ultima aequalitate paragraphi proxime antecedentis, invenietur $ik = a + y = \frac{1}{2}a - \frac{1}{2}c + \sqrt{\left(\frac{1}{4}aa + \frac{1}{2}ac + \frac{1}{4}cc + ab\right)}$ adeoque

$y = -\frac{1}{2}a - \frac{1}{2}c + \sqrt{\left(\frac{1}{4}aa + \frac{1}{2}ac + \frac{1}{4}cc + ab\right)}$, quae est ipsissima aequatio, quam insignis vir etiam reperit. In eandem incidissemus aequationem, si in altera formula corollarii praecedentis loco BG substitutum suisset $a - c$.

In numeris sit fistula AB, 21 digit. AG nunc 27. digit. AC, 1. dig. invenieturque *ik* vel AM, 3 digitorum adeoque BM, 18. digitorum.

PROPOSITIO XXV. PROBLEMA.

352. *Datis columna AG argenti vivi, aequivalente pressioni atmosphaërae, & longitudine fistulae AB , definire quantum spatium AC aëris naturalis consistentiae plenum in summitatem fistulae debeat introduci ut, postquam aër ex AC se extendit in spatium AM, Mercurius in data altitudine BM intra fistulam suspensus maneat.* Figs.85,86.

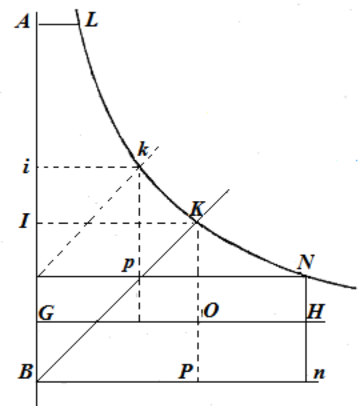


Fig. 86

In hoc problemate ex datis AG, AB, AI vel BM quaëritur AL vel AC aut V. Idcirco facto iterum angulo semirecto ABK, & $AI = \text{datae}$, in fistula altitudine Mercurii manentis BM, agatur IK parallela asymtotae GH rectae BK occurrens in K, per quod punctum nunc ducenda est hyperbola N KL, rectae AL ipsi GH parallelae occurrens in L, erit AL spatium quaesitum. Demonstratio eadem penitus est cum demonstratione propositionis antecedentis, atque adeo hoc loco non repetenda. In numeris habebitur AL applicando rectangulum datum GIK ad lineam AG, quae atmosphaërae pressionem exponit: nam GI est excessus, quo AG excedit datam AI vel BM, & IK vel BI aut AM est etiam (secundum hypothesin) data, adeoque & rec-lum GI.IK eique aequale rec-lum AG.AL datum erit; adeo ut diviso recto GI.IK per AG, quotiens, vel in geometrica phrasi quarta proportionalis ad AG, IG & IK futura sit AL denotans quantitatem quaesitam aëris in fistulam AB intromissi, ut Mercurius in data altitudine BM consistat. Quod erat inveniendum.

SCHOLION.

313. Si jam determinationes hujus & antecedentis propositionis cum phaenomenis convenerint, ut satis egregie cum iis conspirare deprehensae sunt; omnino inferri debet, assumptam hypothesin, quod densitates aëris viribus comprimentibus seu elasticitatibus ejus proportionales sint, naturae satis consentaneam esse, modo ea non in majore latitudine sumatur, quam experimenta, cum quibus convenire videtur. Ejusmodi vero experimenta. cum aëre mediocris densitatis capta sunt; idcirco, si quis praememoratam hypothesin ad aërem densissimum vellet extendere, insigniter hallucinaretur, quandoquidem juxta superius dicta (§. 346.) in aëre densiore elasticitates majorem habent inter se proportionem quam densitates ipsis convenientes.