CHAPTER VI.

About the Elastic Force of the Air in general.

325. It is agreed from many experiments that the air itself has a particular condition, with no example in other fluids and liquids, by which not only does it try to expand itself continually, indeed also to a greater space or volume that it occupied before, and in truth it keeps on expanding as long as it is not impeded by a surrounding body. Attempts of this kind of expansion of the air is called its *Spring* or *Elastic force*. Experiments, which demonstrate conclusively this elastic property of the air [*i.e.* its pressure], several attempted by Boyle, Mariotte, James Bernoulli and others from careful observations and happily at the end they have deduced, which can be read in the works of the praiseworthy authors, or also in the *Aërometria* of the most distinguished Wolff, in which each of the most select experiments are described accurately. Among these experiments I will review here one or another, which I think will demonstrate the elastic force of the air to be observed.

I. A bull's or pig's bladder flaccid and folded up, but having the opening stopped up with the help of a string tied around, on receiving air drawn in at the same time from a pneumatic pump is observed to swell up slowly and slowly, and if the bladder shall be tender, it is observed to rupture at once by the spring of a little air inside the hidden wrinkles of the bladder.

II. From the same spring of the air, a glass vessel of the thinnest walls and properly blocked off, on receiving the evacuated air [from the pneumatic pump] suddenly bursts apart into minute fragments ; and similar to the phenomenon which frightens drinkers whenever it happens, since the drinker with good wine, wishing his wine to be drawn off from the flask, by moving his lips tightly around the opening, so that by being afraid of losing one single drop of its outpoured nectar, the communication of the external air with that which is present in the flask is removed, and by sucking strenuously the air in the flask becomes rarefied, so that it happens with the external air pressure prevailing, that the flask may be ground into small pieces, and the wine is lost.

III. The elastic force of the air is demonstrated also with an experiment with brass hemispheres, which with the air evacuated from their cavity, as they are verified to be adhering to each other, so that even with the greatest weight hung onto themselves, or by being pulled in opposite directions, they shall be scarcely able to be separated, which still full of air can be separated from each other with hardly any trouble.

326. Truly, without the experiments reviewed standing in the way, Mr. Parent, in *The Elasticity of the Air* in *Historia Acad. Reg. Paris. Scientiarum Academiae* 1708, may wish to call into doubt, if the article may be considered only in passing; but the text reveals on being examined more closely, that the spring of the air in the sense of the previous paragraph cannot be denied. [There were numerous theories at the time, most of which involved the aether of Descartes and pores, that attempted to explain the spring of the air : see, e.g. *Mixture and Chemical Combination. And related Essays*. Duhem] For

the author in the manner named, denies greatly that the particles of the air are to be considered in the image of folded plane shapes, or to have the form of tangled filaments in the shape of spirals, and themselves later requiring to be changed, or changed into some other form equivalent to these. But these rather he observes are to be considered as molecules of the material aether, beyond which it may be possible to understand, to be small and to be swimming in a most agile manner. Therefore according to the praiseworthy author, the molecules of the aether stand so much more apart in turn, and, because the kind of spring is said to be carried before themselves, by so much more are the molecules trying to recede from each other, by which the ethereal matter flowing through the movement of the air would be more abundant, and by which its motion would be more persistent. Indeed from this ethereal matter all the force is considered to be derived, by which the molecules of the air shall be able to act on other bodies. From all of which the Cel. Parent makes that tendency abundantly clear of the molecules of the air, by which they are trying to move away from each other, not to deny, nor hence to call into doubt that elastic force accepted by our perception. Or nevertheless this elasticity of the air, or the attempt of its particles of receding from each other, arising from the proper elasticity of the compressed particles, and then to restore themselves in the original state of struggling, or truly from the aether flowing between them, the questions are physical, which can be disputed in each part, and which all are surrounded by their difficulties.

327. The hypothesis explaining the elasticity of the air was set out by Parent several years before I too had thought out a similar hypothesis, but to whom I sent a message, because with the same in place, not only for the air, but in short all liquids must be endowed with an elastic force, I might consider to follow from that, but which liquids still are lacking. For, if thus the molecules of the air are trying to recede from each other, because with the most rapid motion of the incessantly circulating aether through the pores of the air, the molecules of the same air in turn may be repelled from each other and pushed away, why indeed I say, in the remaining liquids are the molecules not in the same manner repelled from each other by the aether flowing between, which no less must flow and be moved through the pores of the liquids, than the movements across the pores of the air? Perhaps the answer may be a disparity arising from the thickness of the molecules, from which the liquids are composed, and to be of a far greater density for liquids, than the density of the air shall be ; and on account of these two reasons, so that the molecules of liquids may not be successful, as the particles of air may be , in being driven away from each other by the aether flowing between, since the particles themselves are required to repel each other by a greater force and the abundance of the pores of the flow may be very small compared with that of the air. But honestly I doubt greatly, whether these philosophical reasons shall be sound ; for the narrowness of the pores in liquids, which is added for the restored account, why liquids are without the elastic force, may be considered to be judged rather to be the opposite. For indeed it is known, fluids are accustomed to be moving faster there where they flow through narrower locations, thus the velocities of flow of the river at different sections are inversely proportional to the sections ; and therefore this ratio, the velocity of the flow of the aether through the pore of some liquid L will be to the velocity of its same flow through a pore of air A, as the size of the pore of air to the pore of the liquid; and within different liquids the sizes of the pores put in place are similar, that is, the distances between the elements of neighbouring elements in

liquids are in the reciprocal cube root proportion of the densities ; for two similar masses and with equal weights of different liquids may have their volumes reciprocally proportional to their densities, and the volumes, which (following the hypothesis) are similar solids, are in the triplicate proportion of the homologous sides, and thus the densities are in the reciprocal triplicate proportion of the homologous sides, and thus the densities will be in the reciprocal triplicate ratio of the homologous sides in similar solids, and thus the sides of such homologies will be in the reciprocal sub triplicate ratio of the densities [*i.e.* as the cube roots]; truly the particles in both liquids, in account besides of being similarly placed, allow the gaps between the sides of the homologous volumes to be proportional, so that since the sides of the volumes shall be in the reciprocal cube root ratio of the densities, also the magnitudes of the pores, or the nearest distances of the particles of the fluids themselves will be in the sub triplicate ratio of the densities. And the velocity of the aether flowing across the pores of the liquid L is to the speed of the same flow through the pore of the air A, reciprocally, as the pore of the air or the space between two of its nearest molecules to the pore of the liquid, and thus directly as the sub triplicate ratio of the density of the liquid L to the density of the air A, that is, in the sub triplicate ratio of the specific gravity of the liquid to the specific gravity of the air. Now since, as said above (§.314.), the weight of water to the weight of air shall be as 800 to 1, and the cube root from 800 shall be as an approximation $\frac{92}{10}$; the velocity of the aether through water to the speed of the same through air will be approximately as 92 to 10; hence, because the pressures of the fluid on the bodies, on which they act, are in the square ratio of the velocities, as that will be shown in its place, in this case the force of the aether will be to the force of the fleeing molecules of water, for the same force to the fleeing force in turn of the air molecules, as the square of 92 to the square of 10 approximately, that is as 8464 to 100; and thus the water in Parent's hypothesis will be required to be more than eighty times a greater force than the force of the air, why therefore is there no such elasticity found in water ? in addition the elasticity of quicksilver and of other fluids of greater specific gravity must be greater.

In truth whatever shall be the physical cause of the spring of the air, for our understanding it suffices that an elastic force of the air is present, which besides the experiments related at the start of this chapter, also can be proven from the effects of the pumps of Otto von Guericke and by Robert Boyle after being perfected more, with the aid of which the air in some vessel, which are duly applied to the same, to be rarefied as much as you wish, and thus some great part of the air can be drawn off by repeated exhausts, which would succeed minimally if the air were without that expansive power.

[The description of von Guericke's pump appeared first as an appendix in a fascinating work by the Jesuit Gasper Schottus, entitled *Mechanica Hydraulico-Pneumatico,*~1660, and in which such rarities as an automated water-driven hymn-tune-playing pipe organ are described; Boyle's work appeared in the work : *New experiments physico-mechanical touching the air,* 1682. Von Guericke's pump had the disadvantage of requiring to be immersed in water, but it provided the inspiration for Boyle's pump, which it would appear was constructed by Robert Hook, who was Boyle's assistant at the time. An appendix has been added illustrating some of this at the end of the chapter.]

328. Indeed with *pneumatic pumps* of this kind the brass tube is perfectly smooth in its whole cavity, thus so that the piston, which is moving to and fro within the same cavity of the pump and everywhere must be accurately square, nor may it have any cracks left behind through which air may be able to be drawn in.

The *piston* is a pestle constructed from leather circles fitting perfectly with the opening of the pump, which with the help of a handle inserted at the centre, can be thrust into the bottom of the pump and afterwards withdrawn again to the head or opening of the same.

The *epistomium* of the pump is a small cylindrical close-fitting valve inserted into the curved cylindrical tube at right angles at the base of the pump, hence able to be turned freely, with a small round hole bored through about the same size as the bent tube, to this end, so that the same hole inside the curved tube communicates the air freely to the pump from the vessel, from which the air due to be expelled (which henceforth will be called, as briefly by others, the *receiving* vessel) through the curved tube into the pump may be permitted to pass, and conversely in the other direction it impedes and stops the action of the pump, since now time after time in this position, with the curved tube named not communicating the passage of the air from the receiving vessel in the manner mentioned. [Thus the *epistomium*, or literally *placed upon the opening* was a special kind of hand operated valve which allowed in turn either direct transmission of the air in the pump cavity into the receiver or into the outside air.]

The *spiraculum* [*i.e.* air hole or spiracle] is an opening from the top of the epistomium stopping up its transverse hole, and communicating at once with the inside of the curved tube ; being open requires the use of the small brass pin for stopping up the tube, so that the open air hole allows the cavity of the pump to communicate with the outside air.

The bent tube connected to the bottom of the pump generally ends in a male screw thread, to which a brass plate is connected at the centre to a female screw hollowed out around the hollow opening. The circular moistened leather is applied to the plate, receiving which so that most level space is imposed, and all around with the sides of the plate flooded with water, in order that its raised edges acts as an impediment to the ingression of the external air.

The movement of the piston consists of one stroke from the base of the pump as far as to the opening or its head, and with one return stroke from the opening as far as to the base.

329. In figure 79 AB is the body of the pump, EF the piston skillfully adapted to the cavity of the pump, so that nowhere shall air be able to flow across the piston ; BOH is the tube bent at O carefully joined to the base of the pump at B, and with both its cavity and the receiver M being joined through the opening H ;

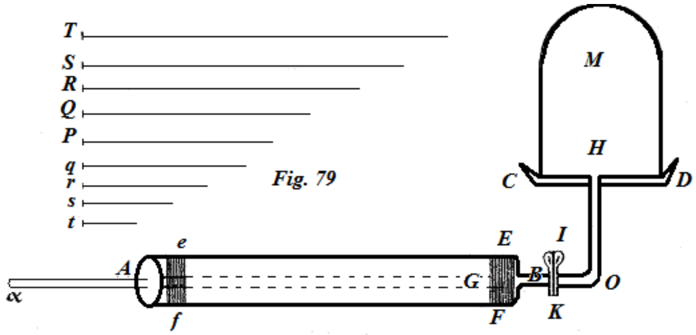


Fig. 79

the opening of this tube is inserted into the valve IK, so that as it is turned in this or that direction, turning off or on the communication of the pump with the recipient M. This valve IK has been bored all the way through to its transverse hole, and this opening is called the breathing hole, to which the small pin I is inserted, so that the communication of the pump with the external air is removed; indeed it can be removed and again the pump is drawing out the air, as often as there is a need for this communication, either for expelling the air from the cavity of the pump, or also for admitting external air, in that case, where the air of the receiver is required to be compressed. CD is the large flat dish, the edges of which raised a little at C and D are the counterpart of the edge of the bowl, containing water poured around the receiver ; a leather circle perforated in the middle is applied to the dish and made moist, to which henceforth the receiver M is imposed by hand and strongly bound, and, as in the manner said, flooded all around with water in the base, all according to that, least the external air by any crack in the receiver may be able to insinuate itself. And thus everything has been prepared for use. Not for me to hide anything otherwise about the artisans are accustomed to construct pumps, as long as certain circumstances, with regard to the inclination of the body may be, which yet is horizontal in our diagram ; but this is by accident, not made for its essential use. Thus also the spiracle described by us often is accustomed to be prepared otherwise, as well as the tube BOH; but in whatever way those may be constructed, always the same effect will have to produced, with that which is in the pumps of this description. Indeed in truth, if which may be prepared otherwise, that will be only with respect of the easier use of the pump.

330. With everything described which have been explained in the preceding section, the valve IK may be opened, so that the receiver M may be able to communicate with the pump AB through the tube HOB, clearly with the spiracle I stopped up and the piston EF drawn back as far as to the opening of the pump A at *ef*, from which it happens that the air which was filling the cavity of the receiver M and of the tube HOB, may spread itself out through the whole volume MHOB*ef*, composed from the cavity of the receiver M, of the tube HOB and of the valve B*ef*, and thus the remaining of the receiver air itself will be had to the air, which before it contained, as the cavities of the receiver M and of the tube HOB to the cavities M, BOH and B*ef* taken together, and therefore the rarity of the same residual air will be to the rarity of the whole air, which before belonged to the receiver, as MHOB*ef* to MHOI. Henceforth the valve IK is closed, and the air is expelled from the obstructing air-hole I, and with the intrusion of the piston *ef* towards the bottom

of the pump B the air from the cavity is expelled through the open air-hole, and with the air-hole I closed again the first movement of the piston will be carried out, the second stroke will be repeated with the same steady course ; the third, fourth, etc. strokes ; from which the following further flows

PROPOSITION XIX. THEOREM.

331. *If the volumes of the pump Bef, of the tube BH and of the receiver M taken at the same time, were to the volume M of the receiver alone as the right line Q to another P, rareness of the residual air after any movement of the piston will be as its rareness before this motion, as Q to P.* See Fig. 79.

For, because after the stroke of the piston going from B to *ef*, the air of the receiver M (§. 330) itself spreads out through the whole volume MHOB*ef*, the rarity of the air remaining in the volume M after the stroke of the piston, to its rareness before this stroke, will be as the volume MHB*ef* composed from the volume of the receiver M, of the tube HOB and of the pump B*ef* to the volume or the receiver alone M, that is (following the hypothesis) as Q to P. Q.E.D.

COROLLARY I.

332. If the ratio P to Q may be continued in some number of terms R, S, T, &c. and the first P establishes the rarity of the air in the receiver M present in its natural state, the following terms of our progression Q, R, S, T, &c. will indicate the rarity of the air remaining in the receiver, after the first, second, third, fourth, etc. stroke of the piston. Now, because the rarity of the air after the first stroke is to its rarity before this stroke as Q to P, and here the term P expresses the rarity of the air such as it was in its natural state before the first stroke, thus Q expresses the rarity of the air remaining in the receiver after the first stroke ; thus because the rarity of the air after the first to the rarity after the second stroke of the piston is as P to Q, or as Q to R, and Q expresses the rarity of the air after the second stroke, R expresses the rarity of the air after the second stroke of the piston ; and by the same argument S , T, &c. are put in place to express the rarities of the air remaining in the receive after the third, fourth, stroke etc.

COROLLARY II.

333. These also hold, with all changes required being made, in the compression of the air in the receiver. In which case in place of the stroke, which we have said to be the moving out of the piston from the base of the pump B as far as to *ef*, now by the stroke is understood the intrusion of the piston *ef* as far as to the base of the pump. Therefore, if the progression T, S, R, Q, P may be continued in just as many terms *q, r, s, t* ; &c. these terms will indicate the rarity of the air in the receiver M, after the first, second, third, fourth, etc. strokes of the piston or intrusion into the pump. For the air, which before the first intrusion was spread out in the volume MO*ef*, after the first intrusion will be driven back into the much narrower volume BOHM, or with the volume of the tube BOH ignored (which is negligible in comparison with the volume of the cavity M, and, because

the volume of the tube also can be taken away immediately, [found] by filling the volume EFOH with water) the rarity of the air before the first stroke of the piston, which is called P, to the rarity of the same air after the first stoke is as the volume MHO*ef* to the volume M, that is, as Q to P or as P to *q* ; therefore *q* represents the rarity of the air in the receiver M after the first stroke of the piston, and by a similar argument the following terms *r, s, t,* &c. will indicate the rarities after the second, third, fourth, &c. strokes of the piston.

COROLLARY III.

334. From which, since the rarities (§.20.) shall be in inverse proportions with the densities, the magnitudes Q, R, S, T, &c. which are present equally removed from the magnitude P ; *q, r, s, t,* &c. also inversely proportional, express the densities of the air in the receiver M, after the first, second, third, fourth, etc., strokes of the piston into the pump AB.

PROPOSITION XX PROBLEM.

335. *With the volumes of the pump and of the receiver, or just the ratio of the volumes of the pump and receiver, or perhaps in the ratio of one to the other given, to define, how many strokes of the piston shall be needed for the reduction of the air in the receiver to a given rarity or degree of density*. See Fig. 80.

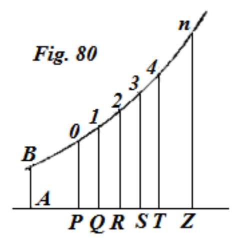


Fig. 80

To the extended axis AZ there shall be some logarithmic curve B1*n* , in which the magnitudes P, Q, R, S, &c. are attached as ordinates, as at P0, Q1, R2, S3, T4, &c. which since (following the hypothesis) they shall be in a geometric progression, the intervals PQ, QR, RS, ST, &c. will be equal. The numbers of the ordinates indicate how many strokes of the piston shall be needed for rendering the air in the receiver to that degree of rarity, which some ordinate expresses; thus P0 indicates no stroke of the piston required to reduce the air to the rarity P0 or P, truly S3 intimates 3 strokes of the piston to reduce the air to a rarity S3 or S, and thus with the remainder ; truly the letter *n* specifies the sought number of strokes of the piston, by which the air of the receiver is reduced to the given rarity Z which the ordinate Z*n* is showing. Here the rarity requiring to be represented shall be a ratio, so that some abscissa PS shall be to the abscissa PQ as the number 3 of the ordinate ascribed to the number 1 of the ordinate P1 ascribed, hence also there will be $\mathrm{PZ}:\mathrm{PQ}=n:1$, or simply $n=\mathrm{PZ}:\mathrm{PQ}$. And PZ is the logarithm of the ratio Z*n* to P0 or of Z to P, and PQ the logarithm of the ratio Q1 to P0 or Q to P ; therefore the number sought $n=\log.(\mathrm{Z}:\mathrm{P}):\log.(\mathrm{Q}:\mathrm{P})$. And hence the Bernoulli rule results.

The logarithm of the ratio (Z:P) which the rarity of the air desired has to the rarity of the natural air, may be divided by the logarithm of the ratio (Q:P) which the pump and the receiver together have, to the volume of the receiver alone, the quotient will indicate the number n of strokes sought. Which will indicate the quotient sought.

A publication was called for, the analysis of this rule developed by the Celeb. James Bernoulli at one time from the calculation of the exponentials without demonstration, now unless I am deceived published in the year 1693 by the most distinguished Varignon in the *Actis Acad. Reg. Scientiarum*, and that later with many additions in the *Actis* of the same Academy of the year 1705, again communicated to the public.

COROLLARY I.

336. Since the logarithm of any fraction shall be the difference of the logarithms of the numerator and the denominator, there will be

$$\text{logarithm } Z:P = \log.Z - \log.P, \ \ \& \ \log Q:P = \log.Q - \log.P,$$

$$\text{and thus } n(=\log.(Z:P):\log.Q:P) = (\log.\,Z - \log.P):(\log.Q - \log.P).$$

Hence if $Q:P = 2:1$, & $Z:P = 6:1$, there will be $\log.Z - \log.P = 0.7781512$, and $\log.Q - \log.P = 0.3010300$, and thus $n = 0.7781512:0.3010300 = 2\tfrac{2}{3}$ approximately. Again with the aid of the above rule from the three quantities Z:P ; Q:P and *n*, with any two given, the third always can be had by logs. But there is no time to tarry with these for long.

COROLLARY II.

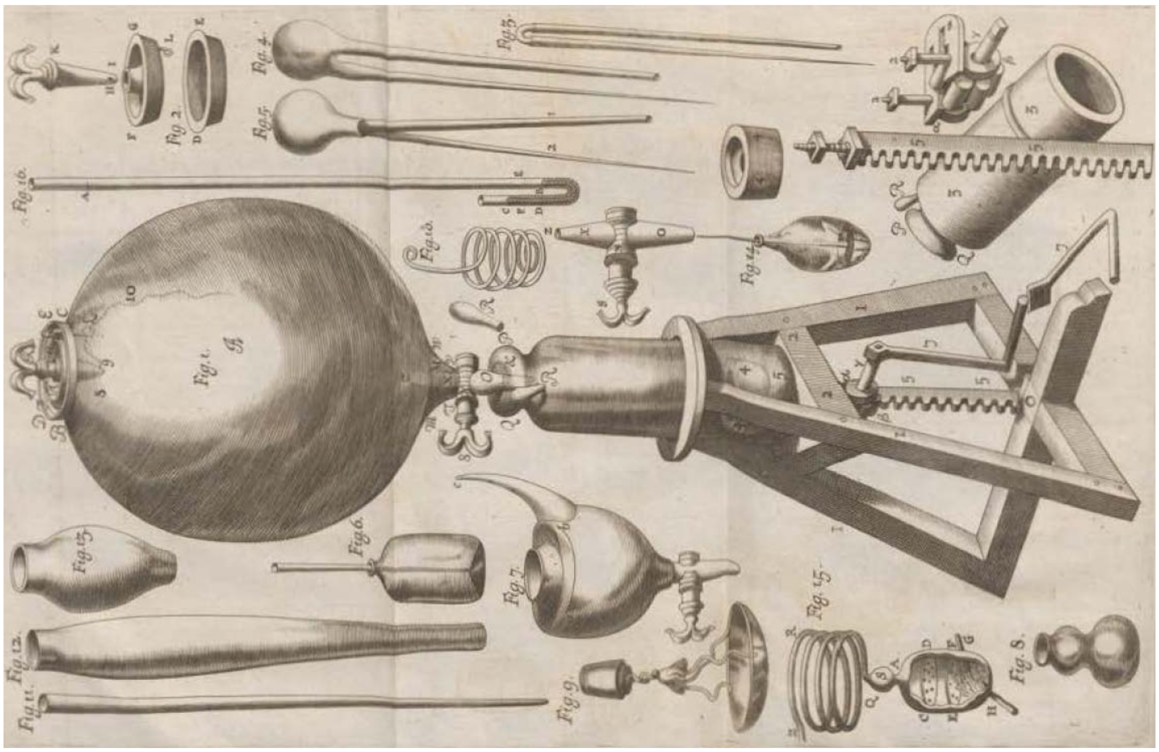
337. Although this problem only acts on the rarefaction of the air, yet the rule, which we gave come upon around the end of §.335, also can be applied in these cases, for which the air of the receiver indeed is not being rarefied, but compressed. So that if it may be asked, by how many strokes of the piston for air in the receiver in the natural state or constituted in some other state, of which the density and the density of the air in the natural state are given, shall be able to be reduced to the degree of density X. It arises that as X to P thus P to Z, and this Z expresses (§ 20) the rarity of the air of which the density is X, since P shall represent the density and rarity of the air in the natural state. For with Q: P, and Z: P given , the number of strokes *n* of the piston sought, from which the rarity of the air $Z = P^2 : X$ or the density X induced of the air. Truly in this case, where air is being forced in, the magnitude Z is less than P itself, and thus the log of the ratio Z: P will be negative, hence also the value of the number *n* present will be negative, from which yet another may not be indicated, unless the stroke of the piston in the case of compression itself has the contrary ratio, than in the case, in which the air of the receiver is being rarified, thus so that the sign may not be present.

SCHOLIUM.

338. The way has been set out of exploring the ratio of the volume of the pump to the volume of the receiver, if a certain cylindrical vessel upon the base or may be filled to some extent with water from it bottom, as much as the receiver and the pump at the same time can take and from that the height of the water will be noted by A ; then with the aid of a siphon so much of the water may be drawn off from the cylindrical vessel, as much water as the pump can take, and with the remainder in the cylindrical vessel the height

may be marked by B, I say that A shall be to B just as Q to P; for the cylinder from the height B is equal to the volume of the receiver alone, so that since the height A shall be to the height B , as the cylinder A to the cylinder B, that is, as the volume of the pump and the receiver in turn to the volume of the receiver alone, that is (following the hypothesis) as Q to P, the proposition is proven.

An Appendix added by the translator : Some pages taken : *New experiments physico-mechanical touching the air;* Robert Boyle ; London, 1682. The book is in the form of a letter to his nephew, Lord Dungarvan : The pump is shown with a number of extra



devices used in vacuum experiments, not discussed here. :

To give your Lordship then, in the first place, some account of the Engine itself: It consists of two principal parts; a glass Vessel, and a Pump to draw the Air out of it. The former of these (which we, with the Glass-men, shall often call a Receiver, for its affinity to the large Vessels of that name, used by Chymists) consists of a Glass with a wide hole at the top, of a cover to that hole, and of a Stop-cock fastned to the end of the Neck, at the bottom. The shape of the Glass, you will find express'd in the first Figure of the annexed Scheme. And for the size of it, it contain' d about 30 Wine carts, each of them containing near two Pound (of l6 Ounces to the Pound) of Water: We would have been better pleas'd with a more capacious Vessel, but the Glass-men professed themselves unable to blow a larger, of such a thickness and shape as was requisite to our purpose.

At the very top of the Vessel, (A) you may observe a round hole, whose Diametre (BC) is about four Inches; and whereof, the Orifice is incircled with a lip of Glass, almost an Inch high: For the making of which lip, it was requisite (to mention that upon the by, in case your Lordship should have such another Engine made for you) to have a hollow and tapering Pipe of Glass drawn out, whereof the Orifice above mentioned was the Basis, and then to have the Cone cut off with an hot Iron, within about an Inch of the Points (BC.)

The use of the lip, is to sustain the cover delineated in the second figure ; where (DF) points out a brass Ring, so cast, as that it doth cover the lip (BC) of the first Figure, and is cemented on, upon it, with a strong and close Cement. To the inward tapering Orifice of this Ring (which is about three Inches (p. 6) over) are exquisitely ground the sides of the Brass stopple (FG ;) so that the concave superficies of the one, and the convex of the other, may touch one another in so many places, as may leave as little access, as possible, to the external Air: And in the middle of this cover is left a hole (HI) of about half an Inch over, invironed also with a Ring or Socket of the same Metal, and fitted likewise with a Brass stopple (K) made in the form of the Key of a Stop cock, and exactly ground into the hole (HI) it is to fill ; so as that, though it be turn'd round in the cavity it possesses, it will not let in the air, and yet may be put in or taken out at pleasure, for uses to be hereafter mentioned. In order to some of which, it is perforated with a little hole, (8) traversing the whole thickness of it at the lower end; through which, and a little Brass Ring (L) fastened to one side (no matter which) of the bottom of the stopple (FG) a string (8, 9, 10) might pass, to be imploy'd to move some things in the capacity of the empty'd Vessel, without any where unstopping it.

The last thing belonging to our Receiver, is the Stop-cock, designed in the first Figure by (N,) for the better fastening of which to the neck, and exacter exclusion of the Air, there was soder' d on to the shank of the Cock (X) a Plate of Tin, (MTUW) long enough to cover the neck of the Receiver. But because the cementing of this was a matter of some difficulty, it will not be amiss to mention here the manner of it; which was, That the cavity of the Tin Plate was fill'd with a melted Cement, made of Pitch, Rosin, and Wood-ashes, well incorporated; and to hinder this liquid Mixture from getting into the Orifice (Z) of the shank (X,) that hole was stop'd with a Cock, to which was fastned a string, whereby it might be pull'd out at the upper Orifice of the Receiver; and then, the Glass neck of the Receiver being well warm'd, was thrust into this Cement, and over the shank, whereby it was effected, that all the space betwixt the Tin Plate and the Receiver, and betwixt the internal superficies of the Receiver, and the shank of the Cock, was fill'd with the Cement; and so we have dispatch'd the first upper part of the Engine. The (p.7) undermost remaining part consists of a Frame, and of a sucking Pump, or as we formerly call'd it, an Air Pump, supported by it: The Frame is of Wood, small, but very strong, consisting of three legs, (111) so plac'd, that one side of it may stand perpendicular, that the free motion of the hand may not be hindered. In the midst of which frame, is transversly nail'd a board, (222) which may not improperly be call'd a Midriff, upon which rests, and to which is strongly fastined, the main part of the Pump itself; which is the only thing remaining to be described.

The Pump consists of four parts, a hollow Cylinder, a Sucker, a handle to move that Sucker, and a Valve.

The Cylinder was (by a pattern) cast of Brass; it is in length about 14 Inches, thick enough to be very strong, notwithstanding the Cylindrical cavity left within it; this cavity is about three Inches Diametre, and makes as exact a Cylinder as the Artificer was able to bore. This hollow Cylinder is fitted with a Sucker, (4455) consisting of two parts; the one (44) somewhat less in Diametre than the Cavity of the Cylinder; upon which is nail'd a good thick piece of tann'd Shoe-leather, which will go so close to the Cylinder, that it will need to be very forcibly knock'd and ram'd in, if at any time it be taken out; which is therefore done, that it may the more exactly hinder the Air from insinuating itself betwixt it and the sides of the Cylinder whereon it is to move.

To the midst of this former part of the Sucker is strongly fastned the other; namely, a thick and narrow plate of Iron (55) somewhat longer than the Cylinder, one of whose edges is smooth, but at the other edge it is indented (as I may so speak) with a row of Teeth, delineated in the Scheme, into whose intervals are to be fitted, the Teeth of a small Iron Nut, $(\alpha\beta)$ (as Tradesmen call it) which is fastned by two staples (22) to the underside of the formerly mention'd transverse board (222) on which the Cylinder rests, and is turn'd to and fro by the third piece of this Pump, namely, the handle or *Manubrium*, (7) of which (p. 8) of which the figure gives a sufficient description.

The fourth and last part of this Cylinder, is the Valve, (R) consisting of a hole bored through at the top of the Cylinder, a little tapering towards the cavity; into which hole is ground a tapering Peg of Brass, to be thrust in, and taken out at pleasure.

The Engine being thus describ'd, it will be requisite to add that something is wont to be done before it be set on work, for the more easie moving of the Sucker, and for the better exclusion of the outward Air: which, when the Vessel begins to be exhausted, is much more difficult to be kept out, than one would easily imagine.

There must then be first pour' d in at the top of the Receiver, a little sallad Oyl, partly to fill up any small intervals that may happen to be betwixt the contiguous surfaces of the internal parts of the Stop-cock: And partly, that it may be the more easie to turn the Key (S) backwards and forwards. Pretty store of Oyl must also be pour'd into the Cylinder, both that the Sucker may slip up and down in it the more smoothly and freely, and that the Air might be the better hindred from getting in between them: And for the like reasons, a little Oyl is to be used also about the Valve. Upon which occasion, it would not be omitted (for it is strange) that oftentimes, when neither the pouring in of Water, nor even of Oyl it alone, prov'd capable to make the Sucker move easily enough in the Cylinder; a mixture of both those Liquors would readily (sometimes even to admiration) perform the desired effect. And lastly, the Brass cover of the Receiver, being put into the Brass ring formerly describ'd, that no Air may get between them, it will be very requisite to plaister *over* very carefully the upper edges of both, with the Plaister formerly mentioned, or some other as close, which is to be spread upon the edges with an hot Iron; that, being melted, it may run into and fill up all the cranies, or other little cavities, at which the Air might otherwise get entrance.

All things being thus fitted, and the lower shank (O) of the Stop-cock (9) being put into the upper Orifice of the Cylinder (and) into which it was exactly ground; the Experimenter is first, by turning the handle, to force the Sucker to the top of the Cylinder, that there may be no Air left in the upper part of it: Then shutting the Valve with the plug, and turning the other way, he is to draw down the Sucker to the bottom of the Cylinder; by which motion of the Sucker, the Air that was formerly in the Cylinder being thrust out, and none being permitted to succeed in its room, 'tis manifest that the cavity of the Cylinder must be empty in reference to the Air. So that if thereupon the Key of the Stop-cock be so turn'd, as that through the perforation of it, a free passage be opened betwixt the Cylinder and the Receiver, part of the Air formerly contain'd in the Receiver, will nimbly descend into the Cylinder. And this Air being, by the turning back of the Key, hinder'd from the returning into the Receiver, may, by the opening of the Valve, and forcing up of the Sucker to the top of the Cylinder again, be driven out into the open Air. And thus by the repetition of the motion of the Sucker upward and downward, and by opportunely turning the Key, and stopping the Valve, as occasion requires, more or less Air may be suck'd out of the Receiver, according to the exigency of the Experiment, and the intention of him that makes it.

Your Lordship will, perhaps, think that I have been unnecessarily prolix in this first part of my Discourse: But if you had seen how many unexpected difficulties we found to keep out the external Air, even for a little while, when some considerable part of the internal had been suck'd out; You would peradventure allow, that I might have set down more circumstances than I have, without setting down any, whose knowledge, he that shall try the Experiment, may not have need of. Which is so true, that, before we proceed any further, I cannot think it unseasonable to advertise Your Lordship, that there are two chief sorts of Experiments, which we design'd in our Engine to make tryal of: The one, such as may be quickly dispatch'd, (p. l0), and therefore may betry'd in our Engine, though it leak a little, because the Air may be faster drawn out, by nimbly plying the Pump, than it can get in at undecern'd leaks; I say at undiscern'd leaks, because such as are big enough to be discover'd, can scarce be uneasie to be stopt. The other sort of Experiments consist of those that require, not only that the internal Air be drawn out of the Receiver, but that it be likewise for a long time kept out of it. Such are the preservation of Animal and other Bodies therein, the germination and growth of Vegetables, and other trials of several sorts, which it is apparent cannot be well made, unless the external Air can, for a competent while, be excluded: Since, even at a very small leak, there may enough get in, to make the *Vacuum* soon lose that name; by which I here declare once for all, that I understand not a space wherein there is no Body at all, but such as is either altogether, or almost totally devoid of Air.

Now this distinction of Experiments I thought fit to premise to the ensuing Narratives, because, upon trial, we found it so exceeding (and scarce imaginably) difficult a matter, to keep out the Air from getting at all in at any imperceptible hole or flaw whatsoever, (in a Vessel immediately surrounded with the compressed Atmosphere,) that in spight of all our care and diligence, we never were able totally to exhaust the Receiver, or keep it when it was almost empty, any considerable time, from leaking more or less: although (as we have lately intimated) by unwearied quickness in plying the Pump, the internal Air can be much faster drawn out than the external can get in, till the Receiver come to be almost: quite empty. And that's enough to enable Men to discover hitherto unobserved *Phenomena* of Nature. The Experiments therefore of the first sort, will, I fear, prove the only ones wherewith my Avocations will allow me to entertain Your Lordship in this Letter. For till your further Commands shall engage me to undertake, by God's permission such an Employment, and more leasure shall better fit me (p.11) me for it, I know not whether I shall be in a condition to try what may be done, to enable me to give you some account of the other sort of Experiments also.

CAPUT VI.

De Vi Elastica Aëris in Genere.

325. Aër, quantum experimentis constat, nullae alio in caeteris fluidis & liquoribus exemplo, sibi peculiarem habet affectionem, qua non solum continuo se expandere conatur, sed etiam ad majus, quam antea occuparat, spatium seu volumen reapse se expandit quoties nullo corpore ambiente impeditur. Conatus ejusmodi expansivus aëris *Elater* seu *Vis elastica* ejus nuncupatur. Experimenta, quae hanc aëri elasticitatem inesse evincunt, plura a Boylio, Mariotto, Jac. Bernoullio aliisque cautis observatoribus tentata & feliciter ad exitum deducta sunt, quae apud laudatos Autores legi possunt, vel etiam in Aërometria Clariss. Wolfsi, in qua selectissima quaeque experimenta accurate describuntur. Inter haec experimenta unum alterumve hoc loco recensebo, quod aëris vim elasticam ad oculum demonstrare existimo.

I. Vesica bubula vel porcina flaccida & complicata, sed obstructum habens orificium ope circumligati fili, in recipiente sensim sensimque intumescere conspicitur, simul atque aër ex recipiente educitur ope antilae pneumaticae , & si vesica tenera sit, rumpi subinde observatur a solo elatere pauxilli aëris in vesicae rugis latentis.

II. Ab eodem aëris elatere phiala vitrea tenuiorum parietum & probe obstructa in recipiente ab aëre evacuato in minuta subinde fragmenta dissilit; & huic simile phaenomenum bibulos quandoque terret, cum generosum vini haustum facere volentes lagenam suam vitream hiscentibus labris tam arcte admovent, ut prae metu vel unius solum nectaris sui guttae effusionis communicationem aëris externi cum eo, qui lagenae inest, tollant, atque strenue sugendo lagenae aër rarefiat, quo fit, ut praevalente externi aëris pressione lagena in frustula conteratur, & merum pereat.

III. Vis elastica aëris probatur etiam experimento haemispheriorum aeneorum, quae educto ex eorum cavitate aëre, tam pertinaciter sibi invicem adhaërere comperiuntur, ut maximis etiam ponderibus ipsis appensis, vel in oppositas partes trahentibus, vix separari possint, quae tamen aëris plena nullo ferme negotio ab invicem diducuntur.

326. Verum, recensitis experimentis non obstantibus, D. Parent Elasticitatem aëris in Historia Acad. Reg. Paris. Scientiarum Academiae 1708. in dubium vocare velle videtur, si Historiae verba obiter tantum expendantur; ast textus diligentius inspectus postea manifestat, elaterem aëris in sensu paragraphi praecedentis non denegari. Nam Autor modo nominatus, tantum negat particulas aëris concipiendas esse instar lamellarum plicatilium, vel instar filamentorum in spiras contortarum, & sese postea evolventium, aut ullius rei instar hisce aequivalentis. Sed eas potius tanquam moleculas considerandas arbitratur, materiae aethereae, ultra quam cogitari possit, exili & agitatissimae innatantes. Idcirco juxta laudatum Autorem moleculae aëreae tanto magis ab invicem distant, & , quod elateris speciem prae se ferre dicit, tanto magis a se invicem recedere conantur, quo abundantior fuerit aetherea materia meatus aëris transfluens, & quo pernicior ejus motus. Ab hac enim materia aetherea vim omnem derivandam esse putat, qua aëris moleculae in

alia corpora agere possint. Ex quibis omnibus abunde liquet Cl. Parent tendentiam illam molecularum aërearum, qua a se invicem discedere conantur, non negare, nec proinde vim illam elasticam nostro sensu sumtam in dubium vocare. An autem haec aëris Elasticitas, seu conatus particularum ejus a se invicem recedendi, proveniat ab elasticitate propria particularum compressarum, & deinceps in pristinum statum se restituere molientium, an vero ab aethere interfluente, quaestiones sunt physicae, quae in utramque partem disputari possunt, quaeque omnes suis difficultatibus circundatae sunt.

327. Hypothesi Parentiantae explicandae elasticitatis aëris similem ante plures annos etiam excogitaram, sed cui postea nuncium misi, quod eadem posita, non aëra tantum sed omnes prorsus liquores vi elastica praeditos esse debere, qua tamen liquores carent, ex ea sequi viderim. Nam, si aëreae moleculae ideo a se invicem recedere conantur, quod rapidissimo aetheris motu per poros aëris indesinenter circulantis ejusdem aëris moleculae a se invicem abigantur & repellantur, cur quaeso in reliquis liquoribus moleculae non eadem modo ab invicem repelluntur ab interfluente aethere, qui non minus trans liquorum poros, quam trans aëris meatus fluere & moveri debet? Respondebitur forte disparitatem provenire a crassitie molecularum, quibus liquores componuntur, nec non a longe majori liquorum densitate, quam sit aëris densitas, & ob has duas rationes fieri, ut moleculae liquorum non eo successu, quo aëris particulae a se invicem ab interfluente aethere abigantur, cum ipsae particulae abigendae valde magnae & aetheris trans liquorum poros fluentis copia valde parva sint respectu aëris. Ast valde dubito, an hae rationes sincere philosophantibus satisfacturae sint ; nam angustia pororum in liquoribus, quae adducitur ad reddendam rationem, cur liquores vi elastica careant, potius contrarium probare videtur. Notum enim est, fluida eo velocius moveri solere quo angustiora sint loca, per quae fluant, sic fluminis in diversis sectionibus fluentis velocitaces sunt sectionibus reciproce proportionales; ac propter hanc rationeam, velocitas aetheris fluentis per porum alicujus liquoris. L erit ad velocitatem ejusdem fluentis per porum aëris A, ut amplitudo pori aëris ad porum liquoris; atqui in diversis liquoribus pororum similiter positorum amplitudines, hoc est, distantiae duorum vicinorum elementorum in liquoribus, sunt in reciproca subtriplicata proportione densitatum ; nam duae massae similes & aequalis ponderis diversorum liquorum volumina habeat densitatibus suis reciproce proportionalia, & volumina, quae (secundum hypothesin) sunt solida similia, sunt in triplicata proportione laterum homologorum, adeoque & densitates erunt in reciproca triplicata ratione laterum homologorum in solidis similibus, atque adeo latera ejusmodi homologa in reciproca subtriplicata ratione densitatum; verum particulae in ambobus liquoribus, praeter propter similiter positae, admittunt interstitia lateribus solidorum homologis proportionalia, unde cum latera solidorum sint in reciproca subtriplicata ratione densitatum, erunt etiam pororum amplitudines seu particularum fluidorum sibi invicem proximarum distantiae in reciproca subtriplicata ratione densitatum. Atqui velocitas aetheris fluentis trans porum liquoris L est ad celeritatem ejusdem fluentis trans porum aëris A, reciproce, ut porus aëris seu interstitium duarum ejus molecularum vicinarum ad porum liquoris, atque adeo directe in subtriplicata ratione densitatis liquoris L ad densitatem aëris A, id est, in subtriplicata ratione gravitatis specificae liquoris ad gravitatem specificam aëris. Jam cum, ut supra (§.314.) dictum, gravitas aquae ad gravitatem aëris sit ut 800 ad 1, & radix cubica ex 800

sit quam proxime $\frac{92}{10}$; erit velocitas aetheris trans aquam ad celeritatem ejusdem trans aërem proxime ut 92 ad 10; hinc, quia fluidorum impressiones in corporibus, in quae agunt, sunt in duplicata ratione velocitatum, ut id suo loco ostendetur, erit hoc casu vis aetheris ad abigendas moleculas aquae ad vim ejusdem ad abigendas a se invicem moleculas aëris ut quadratum ex 9l ad quadratum ex 10 proxime, id est ut 8464 ad 100; adeoque aquam oporteret habere in Parentii Hypothesi plus quam octoginta vicibus majorem vim elasticam quam aër, cur igitur nulla in aqua elasticitas deprehenditur? multo major adhuc deberet esse elasticitas hydrargyri & aliorum fluidorum aqua specifice graviorum.

Verum quicquid sit de causa physica elateris aëris, ad institutum nostrum sufficit aëri vim elasticam inesse, quod praeter experimenta ab initio hujus capitis relata, etiam probari potest effectibus antliae Guerikianae a Roberto Boylio postea magis perfectae, cujus opera aër in vasis quibuscunque, quae eidem rite applicari queunt, quantum velis rarefieri, atque adeo pars aëris quantacunque repetitis haustibus educi potest, quod minime succederet si aër virtute illa expansiva careret.

328. Est vero ejusmodi *Antlia Pneumatica* tubus aeneus in tota sua cavitate perfecte laevigatus, ita ut embolus, qui ultro citroque in eadem cavitate agitandus antliae cavitati ubique accuratissime quadret, nec ullas rimas relinquat per quas aër transpirare possit.

Embolus est pistillum ex circulis coriaceis orificio antliae perfecte quadrantibus constatum, quod antliae cavitati ope manubrii centro ejus inserti ad fundum antliae intrudi & dehinc iterum ad caput ejusdem seu orificium retrahi possit.

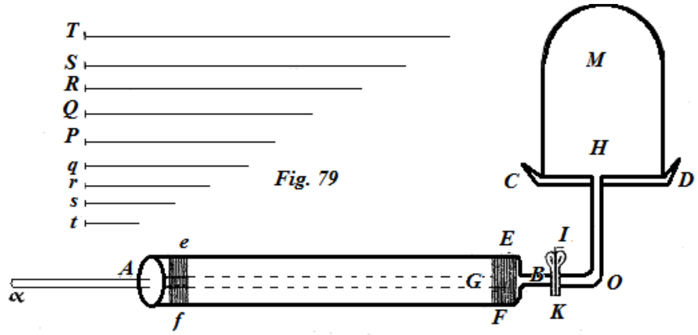
Epistomium Antliae est clavicula cylindrica tubo incurvato antliaeque fundo afferruminato ad angulos rectos inserta, libere hinc inde volvenda, foramine rotundo amplitudinis circiter tubi recurvi pertuso, eum in finem, ut foramen istud cum cavitate tubi recurvi antliae communicans liberum aëri ex vase, ex quo aër expelli debet (quod vas *Recipiens* cum aliis brevius deinceps vocabitur) per tubum recurvum in antliam transitum permittat, & in alium sensum conversa, & cum saepius jam nominato tubo recurvo non communicans modo commemoratum aëris transitum ex recipiente in antliam impediat atque tollat.

Spiraculum est cavitas ex summitate epistomii in foramen ejus transversum desinens, & cum tubi recurvi cavitate subinde communicans ; acicula aenea pro rei indigentia obturanda, ejus usus, ut spiraculum apertum cavitati antliae cum aëre externo liberam communicationem permittat.

Tubus inflexus fundo antliae afferruminatus plerumque desinit in cochleam marem, cui catinus aeneus in centro cochlea foemina excavatus circumvolvitur. Catino applicator circulus coriaceus madefactus, cui recipiens ut plurimum campaniforme imponitur, & circumcirca aqua affunditur marginibus catini, in circumferentia ejus elatis contenta ad impediendum externi aëris ingressum.

Agitatio emboli constat uno ejus *itu* ex fundo antliae usque ad orificium seu caput ejus, unoque *reditu* ab orificio usque ad fundum.

329. In figura 79 AB est corpus antliae, EF embolus cavitati antliae affabre adaptatus, ut antlia nusquam perfluere queat; BOH est tubus ad O inflexus antliae fundo in B afferruminatus, & cum ejus cavitate & cum recipiente M communicans per foramen H ;



hujus tubi foramini inseritur epistomium IK, quod, prout in hunc vel illum sensum convertitur, tollit vel aperit communicationem antliae cum recipiente M. Hoc epistomium IK perforatum est usque ad ejus foramen transversum, ac haec cavitas vocatur spiraculum cui acicula I inseritur, ut communicatio antliae cum externo aëre tollatur acicula vero removetur iterumque extrahitur, quoties hac communicatione opus est, vel ad expellendum aërem ex antliae cavitate, vel etiam ad admittendum externum aërem , eo casu, quo aër recipientis est condensandus. CD est catinus, cujus margines in C & D nonnihil elevati arginum instar sunt, aquam circa recipiens affusam continentes; catino circulus coriaceus in centro perforatus & madefactus applicatur, cui deinceps recipiens M imponitur manibusque fortiter astringitur, &, ut modo dictum , circumcirca aqua affunditur in catino, omnia ad id, ne aër exterior per ullam rimam in recipiens se insinuare queat. Et sic omnia ad usum erunt parata. Non me latet artifices nonnihil aliter antlias construere solere, quoad nonnullas circumstantias, cum ejus corpori situm inclinatum dent, qui tamen in schemate nostro est horizontalis ; sed hoc accidentale est, nec ad ejus essentiales usus facit. Sic etiam spiraculum subinde aliter quam a nobis descriptum est parare consueverunt & tubum BOH; sed quocunque modo id construant, idem semper effectus debebit praestari, cum eo qui est in antlia hujus descriptionis. Imo vero, si quae aliter parant, id duntaxat respicit faciliorem antliae usum.

330. Praeparatis omnibus quae §. praecaedenti descripta sunt, aperiatur epistomium IK, ut recipiens M cum antlia AB per tubum HOB communicare possit, obturato scilicet spiraculo I, & retrahatur embolus EF usque ad orificium antliae A in *ef*, quo fiet ut aër qui cavitatem recipientis M & tubi HOB impleverat, se diffundat per totum spatium MHOB*ef*, compositum ex cavitatibus recipientis M, tubi HOB & antliae B*ef*, atque adeo residuus recipientis aër se habebit ad aërem, quem antea continebat, ut recipientis M tubique HOB cavitates simul ad cavitates M, BOH & B*ef* simul sumtas, ac propterea raritas ejusdem aëris residui erit ad raritatem aëris totalis, qui antea recipienti inerat, ut MHOB*ef* ad MHOI. Dehinc clauditur epistomium IK, extrahiturque spiraculi I obturamentum, atque intrusione emboli *ef* versus fundum antliae B aër ex antliae cavitate per apertum spiraculum expellitur, ac clauso denuo spiraculo I prima emboli agitatio peracta erit, eodem tenore repetentur secunda; tertia, quarta, &c. agitationes ; ex quibus ultro fluit sequens

PROPOSITIO XIX. THEOREMA.

331. *Si cavitates antliae Bef, tubi BH & recipientis M simul, fuerint ad cavitatem solitus recipimtis M ut recta Q ad aliam P, raritas airis residui post quamlibet emboli agitationem erit ad raritatem ejus ante hanc agitationem, ut Q ad P.* Fig. 79.

Nam, quia post emboli itum ex B in *ef* aër recipientis M (§. 330.) se diffundit per totum spatium MHOB*ef*, raritas aëris residui in cavitate M post emboli itum, ad raritatem ejus ante hunc itum, erit ut spatium MHB*ef* compositum ex cavitatibus recipientis M tubi HOB & antliae B*ef* ad cavitatem recipientis solius M, id est (secundum hypothesin) ut Q ad P. Quod erat demonstrandum.

COROLLARIUM I.

332. Si ratio P ad Q continuetur in quotcunque terminis R, S, T, &c. & primus P exponat raritatem aëris in recipiente M in statu naturali existentis, termini sequentes progressionis nostrae Q, R, S, T, &c. indicabunt raritatem aëris in recipiente residui, post primam, secundam, tertiam, quartam, &c. emboli agitationem. Nam, quia raritas aëris post primam emboli agitationem est ad raritatem ejus ante hanc agitationem ut Q ad P, & hic terminus P exponit raritatem aëris in statu naturali qualis erat ante primam agitationem, ideo exponet Q raritatem aëris in recipiente residui post primam emboli agitationem; sic quia raritas aëris post primam ad raritatem post secundam emboli agitationem est ut P ad Q, vel ut Q. ad R, & Q.. exponit raritatem aëris post primam emboli agitationem, exponet R raritatem aëris residui post secundam emboli agitationem ; eodemque argumento conficitur exponere S , T, &c. raritates residui in recipiente aëris post tertiam, quartam, &c. agitationem.

COROLLARIUM II.

333. Haec etiam obtinent, mutatis mutandis, in condensationibus aëris recipientis. Quo casu loco itus, quem diximus esse eductionem emboli ex fundo antliae B usque in *ef*, nunc per itum intelligam intrusionem emboli *ef* usque ad antliae fundum. Propterea, si progressio T, S, R, Q, P continuetur in totidem terminis q, r, s, t; &c. hi termini significabunt raritatem aëris in recipiente M, post primum, secundum, tertium, quartum, &c. emboli itum seu in antliam intrusionem. Nam aër, qui ante primam intrusionem diffusus erat in spatio MO*ef*, post primam intrusionem redigetur in spatium multo angustius BOHM, vel neglecta cavitate tubi BOH (quae insensibilis est prae cavitate recipientis M, &, quae tubi cavitas etiam subinde tollitur, spatium EFOH aqua implendo) raritas aëris ante primam emboli intrusionem, quae dicitur P, ad raritatem ejusdem aëris post primum itum est ut spatium MHO*ef* ad spatium M, hoc est, ut Q ad P seu P ad *q* ; ergo *q* repraesentat raritatem aëris in recipiente M post primam emboli intrusionem, & pari argumento termini sequentes *r, s, t,* &c. indicabunt raritates post secundam, tertiam, quartam, &c. emboli intrustiones.

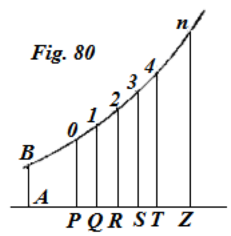
COROLLARIUM III.

334. Unde, cum (§.20.) densitatibus raritates reciproce sint proportionales, magnitudines Q, R, S, T, &c. quae a magnitudine P aeque remotis *q, r, s, t,* &c. etiam reciproce proportionales existunt, exponent densitates aëris in recipiente M, post primam, sccundam, tertiam, &c. emboli intrusionem in antlia AB.

PROPOSITIO XX PROBLEMA.

335. *Cavitatibus antliae & recipientis, vel saltem ratione unius ad Fig. 80. auteram datis, definire, quot emboli agitationibus opus sit ad reducendum recipientis aërem ad datum raritatis vel densitatis gradum.*

Ad axem AZ extructa sit quaecunque log-mica B1*n* , in qua magnitudines P, Q, R, S, &c. aptentur tanquam ordinatae, ut in P0, Q1, R2, S3, T4, &c. quae cum (secundum hypothesin) sint in progressione geometrica, intervalla axis PQ, QR, RS, ST, &c. erunt aequalia. Numeri ordinatis adscripti indicant, quot emboli agitationibus opus sit ad redigendum recipientis aërem ad eum raritatis gradum, quem ordinata quaeque exponit; sic Po significat nulla emboli agitatione aërem reduci ad rartitatem P0 seu P, S3 vera innuit 3 emboli agitationibus aërem reduci ad raritatem S3 vel S, & sic de reliquis; litera vero *n* denotat numerum quaesitum emboli agitationum, quibus aër Recipientis reducitur ad raritatem datam Z quae ordinata Z*n* repraesentatur. Hac raritates repraesentandi ratione sit, ut quaelibet abscissa PS sit ad abscissam PQ ut numerus 3 ordinatae adscriptus ad unitatem 1 ordinatae P1 adscriptam, hinc erit etiam $\mathrm{PZ}:\mathrm{PQ}=n:1$, vel simpliciter $n=\mathrm{PZ}:\mathrm{PQ}$.

Fig. 80


Atqui PZ est log-us rationis Z*n* ad P0 seu Z ad P, & PQ log-us rationis Q1 ad P0 seu Q ad P ; ergo numerus quaesitus
$n=\log.(\mathrm{Z}:\mathrm{P}):\log.(\mathrm{Q}:\mathrm{P})$. Atque hinc resultat regula Bernoulliana.

Log-us rationis (Z: P) quam habet raritas aëris desiderati at raritatem aëris naturalis, dividatur per log-um rationis (Q:P) quam habent antlia & recipiens simul, ad cavitatem solius recipiens, indicabit quotiens quaesitum agitationum numerum n. Qui erat inveniendus.

Hujus regulae a Celeb. Jac. Bernoullio olim sine demonstratione publicatae analysin ex calculo exponentialium petitam jam anno ni fallor 1693 promulgavit Cl. Varignon in Actis Acad. Reg. Scientiarum, eamque postea multis accessionibus auctam in Actis ejusdem Academiae anni 1705. denuo cum publico communicavit.

COROLLARIUM I.

336. Cum log-us cujusque fractionis sit differentia log-orum numeratoris & denominatoris erit $\log - \text{us}\ Z:P = \log.\ Z - \log.P$, & $\log : Q:P = \log.Q - \log.P$. adeoque $n(= \log.\ (Z:P) : \log.Q:P) = (\log.\ Z - \log.\ P) : (\log.Q - \log.\ P)$.

Hinc si $Q:P = 2:1$, & $Z:P = 6:1$, erit $\log.\ Z - \log.P = 0.7781512$, & $\log.Q - \log.P = 0.3010300$, adeoque $n = 0.7781512 : 0.3010300 = 2\frac{2}{3}$ circiter.
Porro ope canonis superioris ex tribus quantitatibus Z :P ; Q:P & *n*, datis quomodocunque duabus, tertia semper per logos haberi potest. Sed hisce non vacat diutius immorari.

COROLLARIUM II.

337. Quanquam hoc Problema tantum de rarefactione aëris agit, canon tamen, in quem circa finem §.335 incidimus, etiam casibus illis applicari potest, quibus aër recipientis non quidem rarefaciendus sed condensandus est. Ut si quaëratur, quot emboli pulsibus aër recipientis in naturali vel quocunque alio statu constitutus, cujus densitas ad densitatem aëris in naturali statu data sit, ad gradum densitatis X reduci possit. Fiat ut X ad P ita P ad Z, & haec Z exponet (§: 20.) raritatem aëris cujus densitas est X, quandoquidem P repraesentat densitatem & raritatem aëris in statu naturali. Jam datis Q: P, & Z: P invenietur per canonem citatum numerus quaesitus *n* pulsuum emboli, quibus aëri raritas $Z = P^2 : X$ seu densitas X inducitur. Verum hoc casu, quo aër recipienti est intrudendus, magnitudo Z est infra ipsam P, atque adeo logus rationis Z: P erit negativus, hinc etiam valor numeri *n* negativus existet, quo tamen aliud non indicatur, nisi emboli itus in casu condensationis contraria ratione se habere, quam in casu, quo recipientis aër est rarefaciendus , adeo ut ad signa non debeat attendi.

SCHOLION.

338. Modus est expeditus explorandi rationem cavitatis antliae ad cavitatem recipientis, si vas quoddam cylindriforme super basi seu fundo suo rectum aqua impleatur tanta, quantam recipiens & antlia simul capere possunt & dehinc aquae altitudo notetur per A; deinde ope siphonis tantum aquae exhauriatur ex vase cylindrico, quantum aquae capere potest antlia, residuaeque in vase cylindrico altitudo signetur B, dico A fore ad B sicut Q ad P; nam cylindrus ex altitudine B aequatur cavitati recipientis solius, unde cum sit altitudo A ad altitudinem B , ut cylindrus A ad cylindrum B, id est, ut cavitates antliae & recipientis simul ad cavitatem solius recipientis, hoc est (secundum hypothesin) ut Q ad P, liquet propositum.