CHAPTER IV.

Concerning the shapes which fluids at rest in flexible bodies must induce on these flexible bodies.

Up to the present we have considered vessels containing liquids as rigid and inflexible, the figures of which are unable to be changed by the pressures of the liquids. But if the material of the vessels were considered to be soft and flexible, it does happen, that the liquid within the cavity of those may not be able to come to rest immediately, whatever the shape attributed to them, but only after the sides of these are bent in various ways that the pressure of the liquid induces on the shape of the vessel, as the complete equilibrium of all the pressure related forces requires.
Vessels of this kind occur to a great extent in the structure of animals, since generally the vessels are soft and pliable, as in the manner of fleshy fibers, in which vessels the various humors and fluids circulate around periodically. Therefore the investigation is about the kind of organs [*i.e.* vessels] animals must have in place that they may be both comfortable and useful. Indeed in fact such an investigation about the forces and movements of muscles by the Cel. Joh. Bernoulli is not out of place here, in his most elegant dissertation upon this matter, the particular contents of which may be found in the *Actis Lips.* 1694, following p. 200. Thus also speculations of this kind by the distinguished Scottish doctor and mathematician Archibald Pitcairn, in place of the multitude of these fermentations, by which the matters were considered before he was believed about secretions in animal bodies, differing from the teaching of the distant Arabs [Guamanians in the text] and Indians ; attempt to bring everything into that form, as he tried to prove in his learned dissertation on the *Circulation of the Blood* [*Circulatione Sanguinis*], the openings of vessels and channels of the glands, and the parts of our bodies having other than round shapes, nor yet hence from the shapes, but also by the size of shapes differing from each other ; from which he concludes then, *nothing peculiar to fermentation to remain stored, and no fermentation to be present in animals.*
The circular shape of vessels follows from a general principle of mechanics : Because any fluids communicate their pressure along a line through the surface of the body. and at some point the force of the fluid to be sustained by perpendicular impulses ; above also (§. 249.) it is contended in a demonstration provided by us, regarding the cross-section of a channel at right angles to the axis as if it were a polygon of an infinite number of sides, the small sides of which polygon may support the pressure of the fluid along directions normal to themselves, and because these pressures on the individual equal sides are equal to the circumstances in the work of the author, he represents these by small right lines perpendicular to the sides of the polygon. To this extent all is well, but while from that, because these perpendiculars of the sides of the polygon in turn provide an angle between each other, it is wished to conclude, or perhaps it may seem to infer, all the said perpendiculars must meet in one and the same point, that is mistaken, even if most certainly these perpendiculars shall in truth be converging at one and the same point, for from that, because any two perpendiculars concur, it does not follow that all are going to join together at the one point. In all curves the right lines, which are perpendicular to neighbouring elements, will contain an angle, that is, they will be concurrent, but from

thence can it be inferred that all perpendiculars to the curve meet at one and the same point ? Therefore from these principles, which the most cel. man has put in place, and which I think on the other hand on granting as being true, he has not yet shown that matter to be deduced well enough, so that a curve may be found whose subtangents all concur at a point, as he writes on page 28 of the Dissert. Medic. Really, he wished to say the matter to be deduced according to the discovery or the investigation of a curve, of which all the subnormals of the curve end at one and the same point ; indeed no curve is given, of which the subtangents concur, since all the subtangents are taken on the same line, that is, on the axis of the given curve, or on another line, which shall be present on the axis, which shall be in place of the axis. Indeed I admit, because, if all the perpendiculars to the curve meet at a point, all the subnormals are going to ends at a certain one and the same point, but also it must be admitted, that there is no need for that reduction of the problem to a consideration of subnormals ; for if all the perpendiculars may converge at a certain point, immediately this curve is required to be concluded to be a circle ; thus so that now the solution of the problem needs no further reduction, either according to the named property of the subnormals, or according to the method, that Pitcairn called that of inverse fluxions. Therefore we will try in our following problem to show clearly to be that in truth, all the perpendiculars of which may be inclined towards the same point or centre, not withstanding, what the solution to this same problem above shall be now (§.102.), but by a different account towards an illustration of the general theorem, which has solved infinitely many other problems of this kind.

[See e.g. *Apollo Mathematicus: A Story of Resistance to Quantification in the Seventeenth Century* by Stephen M. Stigler: *Proceedings of the American Philosophical Society*, Vol. 136, No. 1 (Mar., 1992), pp. 93- 126.]

PROPOSITION XVI. PROBLEM.

303. *With the tube AB made from soft or flexible material closed at B, and with the end A open, erected vertically to the horizontal, and full of water or some other liquid, to determine the shape, that some horizontal section in the tube will acquire from the liquid pressure.* Fig. 69 & 70.

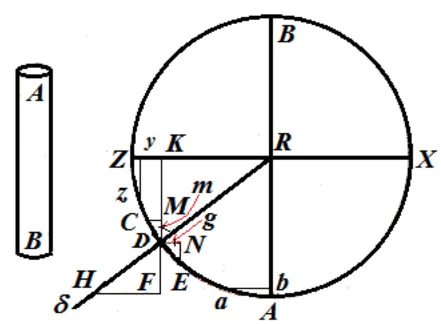

Fig. 69 & 70

AZBX shall be the figure sought, for which the right lines ZX, AB crossing each other in turn at right angles at R meet at the points A, X, B, Z. The contiguous adjacent elements of the curve DC, DE & Z*z*, A*a*, shall be equal to each other at the individual points Z, A, and the right lines *zγ*, DK may be drawn normal to ZX, and EN is parallel to these, again the right lines CM & DN are drawn through the points C & D parallel to the right line ZX, & through *a* the small line *ab* parallel to the same ZX ; and finally there becomes $\mathrm{D}m = \mathrm{EN}$ & $\mathrm{D}g = \mathrm{CM}$. With which in place, because the individual points of the section of the tube ZAXB are equally distant from the surface of the liquid, they endure an equal pressure, and because of its force the point D (§. 255.) will sustain a pressure equal to the weight of a filament of liquid, whose length, as we have indicated by the line DH perpendicular to the curve at the point D, is equal to the height of the fluid above the section ZAXB, along the direction Dδ normal everywhere to the curve; therefore the individual DH, which are taken on the lines perpendicular to the line of the curve, will represent the forces applied to the same curve, and thus the rectangle DH.DC refers to the force acting on the whole element DC. Truly the force DH (§.39.) is equivalent to the sides DF & FH, of which the one is parallel to ZX and indeed the other is perpendicular to the same ; therefore also the force acting on the whole element of the curve CD, which is set out by the rectangle DH.DC, is equivalent to the forces from the sides CD.DF & CD.FH, or because the triangles CDM & DHF are similar, they are provided by the lateral forces $\mathrm{CD.DF = CM.DH}$ & $\mathrm{CD.\ FH = DM.DH}$ with CM.DH and DM.DH set out along the directions DF & FH, or acting parallel to these.

[In modern terms we expect pressure to be the force per unit area, so here we may assume another unit length perpendicular to the plane of the page, to give the correct units.]

Now (§. 98.) generally there is $\int \mathrm{FH}$, as the sum of the forces along the direction FH, or acting parallel along the directions of the line ZX on all the elements of the curve CD or DE contained in the arc of the curve ZDE or ZD, that is by the method shown, all the rectangles DM.DH of the respective arc ZD, which compose the rectangle DK.DH, to $\mathrm{DN - Zy}$, or $\mathrm{CM - Zy}$, just as the strength of the curve at the point A, as we designate the strength by this letter A, as a constant element of the curve CD : that is $\mathrm{DK.DH : CM - Zy = A : CD}$, or because (following the hypothesis) ZX meets the curve at Z at right angles, and thus Zy vanishes with respect of $\mathrm{Z}z$ & CM, making $\mathrm{DK.DH : CM - Zy = A : CD}$ and on interchanging, $\mathrm{DK.DH : A = CM : CD.}$

Likewise all the DF or $\int \mathrm{DF}$, that is, in the present case all the relevant rectangles CM.DH for the arc of the curve DA, that is, the rectangle KR.DH, since all the CM compose KR, or the sine of the arc DA, are to $\mathrm{dM - A}b$ just as A to CD, or also because RA (following the hypothesis) is normal to the curve at A, and thus Ab vanishes before Aa , CD or MD, also there will be $\mathrm{KR.DH : DM = A : DC}$, and on interchanging $\mathrm{KR.\ DH : A = DM : DC}$; but a little before we had $\mathrm{DK : DH : A = CM : CD}$ or by inverting $\mathrm{A : DK.DH = DC : CM}$; therefore from the equation $\mathrm{KR.DH : DK.DH = KR : DK = DM : CM = FH : DF}$, and thus with DR joined, the triangles DKR & DFH are similar, and thus the right lines DR & DH point in every direction, and hence all the DH of the curve ZDA produced normal concur at the same point R, hence the curve sought ZDA is a circle. What was required to be found and demonstrated.

COROLLARY I .

304. The ratios of the preceding paragraph $\mathrm{DK.DH : A = CM : CD}$, applied to the element of the curve Aa adjacent to the vertex A, will present $b\mathrm{R.\ DH : A} = ab : a\mathrm{A}$, in truth because RA is perpendicular to the curve, making $ab = a\mathrm{A}$ therefore also

$A = b$R.DH or AR.DH. Therefore the strength of the tube at some point of the section, is as the rectangle from the radius of the section AR or ZR by the force applied DH at the point of the section, that is by the height of the liquid above this point. Therefore 1°. in tubes of equal orifices the strengths or degrees of firmness required for bearing the pressures of the fluid will be as the heights of the homogeneous fluid multiplied by their densities or specific gravities. 2°. If the product from the heights of the liquids by the densities were equal ; the strengths of the tubes will be as the radii of the openings. 3°. If the products of this kind were reciprocally proportional to the radii of the sections of the tubes normal to the axes, the strengths will be equal.

COROLLARY II.

305. Because equal forces applied to the same or similar subjects only are able to produce the same effects, thus there is no concern, whether the tube or vessel were extended by the weight of the heavy liquid in its cavity, or truly may be pressed along directions by some exterior elastic fluid, and likewise it is produced by a fluid perpendicular to the curve. For the pressures of an elastic fluid can always be recalled according to the equivalent pressures of some liquid, the heights of which were being represented by DH. Hence indeed there is the case, with a boy playing at blowing soapy water through straw tubes, that the water bubbles which form suddenly from their protuberances shall become rounded into hollow bubbles. Therefore the viscosity of the water [really the surface tension *T*] in bubbles of this kind requires a force for halting the flow of the included air, for this the inclusive elastic force [i.e. the excess air pressure *p* in the bubble] is as this elastic force [*T*] multiplied by the radius of the bubble or rather by the radius of the same cavity.

[From elementary considerations we have $p.\pi r^2 = 2\pi r.2.T$; hence $pr = 4T$.]

SCHOLIUM.

306. The strengths of flexible tubes will not be had in the same straight forwards manner in the matter being observed here, as for the strength of rigid tubes (§. 275.) that indeed we have shown to be as the rectangles of the tube along the axis, or as the product from the heights of the liquids among themselves and the absolute diameters of the homogeneous tubes, because we have shown the thicknesses of the tubes required to be made in this ratio, and, with all else being equal, the resistances are as the thicknesses ; whereby in flexible tubes the true resistance or strength at any point is as the force in place, or as the height of the liquid multiplied by the radius of the tube, and indeed the firmness of the tube at two diametrically opposite points and likewise in rigidity is the product from the height of the liquid by the whole diameter. Hence I do not see, how the account of the most ingenious Mr. Parent estimated the ratio of the periphery to the diameter in his principles for the strength of tubes introduced in the *Commentariis Academiae Reg. Scientiarum* 1707.

PROPOSITION XVII. PROBLEM.

307. *If some heterogeneous liquid may remain at rest within the sail-cloth ZDAX with its ends Z and X fixed, of which the graph of the pressure shall be the curve ROS; to find the shape maintained by the sail.* Fig. 71.

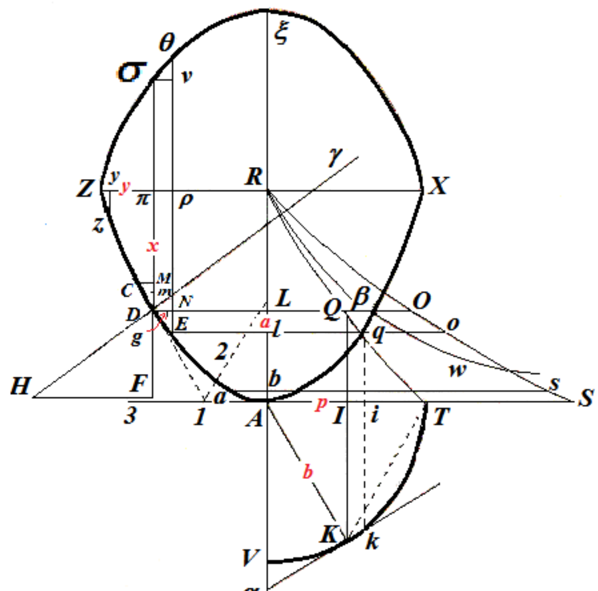

Fig. 71.

Again, as in the preceding problem, Z*z*, CD, DE, A*a* shall be equal elements of the curve ZDA, & DL, E*l* the right ordinates to the axis AR, D_π and E_β indeed parallel to the same axis ; and they are prepared with the rest drawn equal. Now, because by hypothesis all are in equilibrium and by (§.260), the element of the curve CD sustains a pressure equal to the weight of the prism of liquid CD.LO, since LO shall be the ordinate of the graph of the pressure, the force by which the element CD may be acted on along a direction perpendicular will be DC.DH with DH made equal everywhere to LO, and because above on account of all the perpendicular forces on the curve (§. 96.) the strength of the sail is the same at all points, and therefore it can be designated by the given amount A, there will be (§.98.) $\int FH$ or all the $\mathrm{FH} : \mathrm{DN} - \mathrm{Z}y = \mathrm{A} : \mathrm{DC}$; or because Z*y* vanishes before DN itself, the sum becomes $\int FH : \mathrm{DN} = \mathrm{A} : \mathrm{DC}$, and by interchanging $\int FH : \mathrm{A} = \mathrm{DN} : \mathrm{DC}$. Now, because the force FH, which has been derived from DH, not only at the point D, but also is considered in the whole element DC, FH.DC must be put in place of FH, and in this case DH.DC must be understood to be in place of DH, and because moreover FH.DC (on account of the similar triangles DHF and CDM) is equal to the rectangle DH.MD, or DH.NE, that is LO.L*l*, and the sum of all the LO.L*l* will be equal to this inscribed area RLO ; therefore $\int FH$, [we can regard this integral as $\int P(x)\,dx$, where *P* is the pressure at the depth *x*, to which the 'bending' $\mathrm{DN} : \mathrm{DE}$ is taken to be proportional at the depth *x* ; The area under such a curve has the dimensions of work done, and hence the assumption has been made that the bending as measured by the sine or cosine of an angle, is proportional to the work done, rather than involving the square of the extension, as would be the case if we consider a spring or a beam bending;] that is, the area $\mathrm{RLO} : \mathrm{A} = \mathrm{DN} : \mathrm{DE}$ or DC, and this ratio in respect of an element of the curve A*a* becomes the area RO*sb* or

$\mathrm{ROSA} : \mathrm{A} = ab : a\mathrm{A}$, thence, because at the point A, the element $a\mathrm{A} = ab$, there will be also $\mathrm{A} = \text{area ROSA}$, and thus the preceding ratio $\mathrm{RLO} : \mathrm{A} = \mathrm{DN} : \mathrm{DE}$ will be changed into $\mathrm{RLO} : \mathrm{RAS} = \mathrm{DN} : \mathrm{DE}$ [$\mathrm{RLO} : \mathrm{RAS} = \mathrm{DN} : \mathrm{NE}$ in the text; how this change happens is not explained; however, the rest of the proof depends on the tangent of the angle; perhaps the initial assumption should have used this gradient rather than the sine…..].

Finally the curve RQT shall be of this kind, so that the rectangle SA.AT shall = the three-sided figure ROSA, and SA.LQ = the three-sided figure ROL, and there will be $\mathrm{ROL} : \mathrm{RAS} = \mathrm{AS.\,LQ} : \mathrm{AS.\,AT} = \mathrm{LQ.AT}$: from which since we have found $\mathrm{ROLR} : \mathrm{RASR} = \mathrm{DN} : \mathrm{NE}$, also there will be $\mathrm{LQ} : \mathrm{AT} = \mathrm{DN} : \mathrm{NE}$. Hence with the centre A and with the radius AT with the quadrant of the circle TDV described, and with the right line QK drawn through Q parallel to the quadrant crossing at K, and finally with AK acted on, there will be also $\mathrm{AI} : \mathrm{AK} = \mathrm{DN} : \mathrm{DE}$, and thus the triangles DNE and AIK are similar and similarly placed ; and hence the radius AK, will be parallel to the tangent of the curve sought at the point D. And equally there will be $\mathrm{DN} : \mathrm{NE} = \mathrm{AI} : \mathrm{IK}$. Which were required to be found.

COROLLARY I.

308. If on the right line LO there may be taken $L\beta = \mathrm{K}\alpha$, and thus with respect of any other point of the curve, drawn through the point of the quadrant K with the tangent Kα, all the β will be on a certain curve $R\beta\omega$ having the asymptote AS, of which the area $\mathrm{SAL}\beta\omega$ for the asymptotic parts will be equal to the rectangle under the radius AT and with the ordinate DL of the curve ZDA . For the similar triangles AIK and AKα give rise to $\mathrm{AI} : \mathrm{IK} = \mathrm{K}\alpha : \mathrm{AK}$, but $\mathrm{AI} : \mathrm{IK} = \mathrm{DN} : \mathrm{NE}$, therefore $\mathrm{K}\alpha : \mathrm{AK}$ or $\mathrm{AT} = \mathrm{DN} : \mathrm{NE}$, therefore $\mathrm{AT.DN} = \mathrm{K}\alpha.\mathrm{NE} = \mathrm{L}\beta.\,\mathrm{L}l$, therefore all the AT.DN, that is the rectangle AT.DL = to all the right lines, $\mathrm{L}\beta.\,\mathrm{LI}$, which have inscribed the area $\mathrm{AL}\beta\omega\mathrm{S}$, that is, for this area $\mathrm{AL}\beta\omega\mathrm{S}$; and thus $\mathrm{DL} = \mathrm{AL}\beta\omega\mathrm{S} : \mathrm{AT}$.

COROLLARY II.

309. The ratio, that we found near the end of paragraph 307, $\mathrm{DN} : \mathrm{NE} = \mathrm{AI} : \mathrm{IK}$ immediately gives rise to the differential equation of the curve sought $\mathrm{DN} = \mathrm{NE.AI} : \mathrm{IK}$. For if on calling $\mathrm{Z}\pi = \mathrm{y},\ \pi\mathrm{D} = \mathrm{RL} = \mathrm{x},\ \mathrm{AI} = \mathrm{LQ} = p$, and thus $\mathrm{IK} = \sqrt{(bb - pp)}$ with $\mathrm{AT} = \mathrm{AK} = b,\ \mathrm{RA} = \mathrm{AS} = a$ present, and finally $\pi\rho = \mathrm{DN} = dy,\ \mathrm{NE} = \mathrm{L}l = dx$, and from the above equation $\mathrm{DN} = \mathrm{NE.AI} : \mathrm{IK}$ with the substitutions due made along the denominations of the lines put in place in this manner, the ratio will be changed into $dy = pdx : \sqrt{(bb - pp)}$, which is the differential equation of the curve sought agreeing with the equations according to the precision, which the most celebrated Bernoulli brothers have found, each by their own method. If the line ROS were right, the curve RQT will be a conical parabola having the equation $app : b = xx$, and with the value *bxx*: *aa* substituted in place of *p* in the differential equation found before

$dy = pdx : \sqrt{(bb - pp)}$, it will become $dy = pdx : \sqrt{(a^4 - x^4)}$ which again agrees with that which we have deduced above (§. 104).

COROLLARY III.

310. Because (§.96.) the strength of the sail is the same at all points, the right line 12 bisecting the angle D1A formed from the tangents D1 and AI of the curve (§.110.) will be the mean direction of the pressure of all the fluid on the curve DA, and the line KT subtended by the angle TAK equal to the angle LDI (§.112.) is present parallel to the mean direction 12 ; and thus (§.110.) the force along the mean direction 21 will be to the strength of the sail at D as the sine of the angle D1A to the sine A12, that is, as the sine of KAT to the sine of KTA, that is, just as KT to AK. From which since the strength of the sail (§.307) shall be the three-sided figure $\mathrm{ROSA}(\text{by constr.}) = \mathrm{AS.AT.}$ or AK , the force along the direction $21 = \mathrm{AS.AK.KT} : \mathrm{AK} = \mathrm{AS.KT} = \mathrm{AR.KT}$. And because in the symbols of the preceding corollary $\mathrm{KT} = \sqrt{(2bb - 2bp)}$ & $\mathrm{AR} = a$, the force along the mean direction $21 = a\sqrt{(2bb - 2bp)}$.

SCHOLIUM.

311. The curves of this proposition ZAX provide a solution to one part of the famous problem about isoperimetric curves solved by the most ingenious Bernoulli's. For if from the individual points of this D, E, &c. the indefinite lines $\mathrm{D}\sigma, \mathrm{E}\theta$, &c. are acting parallel to the axis AR, and on these there is made $\pi\sigma = \mathrm{LQ}$; $p\theta = lq$, &c. the curve $\mathrm{Z}\xi\mathrm{X}$ arises, which since from the base ZX a greater distance will be contained than by any other curve from other isoperimetric curves ZDAX described by a similar law, as in the *Actis Lips*. 1701. page 213. & *Comm. Acad. Reg. Scient. Paris*. 1706. d. 17. Apr. has been shown by the most select geometers.

Besides the equation $dy = pdx : \sqrt{(bb - pp)}$, James Bernoulli found others of this form $dy = (p - b)dx : \sqrt{(2pb - pp)}$ which contain a *minimum*, and others a *maximum*. But indeed one may be elicited from the other without any difficulty, if in the first $dy = pdx : \sqrt{(bb - pp)}$ in place of p there may be put $p - b$, or its complement for the maximum *b*, and the other equation arises, and if in the other for $p - b$ there may be substituted *p*, the first will be returned.

312. The radius of the curve of the osculating circle at any point on it D, or the radius of the evolute $\mathrm{D}\gamma$, is the fourth proportional to LO the homologous ordinate of the graph of

the pressures ROS, and RA & AT given. The demonstration of which matter is easy from our construction.

CAPUT IV.

De Figuris , quas fluida in corporibus flexibilibus stagnantia hisce corporibus flexibilibus inducere debent.

Consideravimus hactenus vasa liquores continentia tanquam rigida & inflexibilia, quorum figurae a liquorum gravitationibus mutari nequeant. Sed si vasa materia molli & flexili constent, non continget, ut liquor intra eorum cavitatem stagnans in statu manenti statim consistere queat, quaecunque figura ipsis tributa fuerit, sed posteaquam ipsorum latera variis modis inflexa fuerint, eam liquoris pressurae figuram vasis inducent, quam perfectum inter omnes potentias gravitantes aequilibrium deposcet.

Ejusmodi vasa in oeconomia animali magno numero occurrunt, cum pleraque flexilia & mollia , utpote ex fibris carnosis contexta sint, in quibus vasis varii humores & fluida periodicis motibus circumeunt. Quales igitur figuras ejusmodi vasa induere debeant non injucunda nec inutilis est disquisitio. Revera enim talis indagatio non contemnenda Celeb. Joh. Bernoullio produxit circa Vires & Motus musculorum in Dissertatione ejus super hac materia oppido eleganti, cujus praecipua contenta in Actis Lips. 1694 pag. 200 seq. continentur. Sic etiam insignis Medicus & Geometra Scottus Archibaldus Pitcarnius illius generis speculationes in Physiologiam loco multiplicium illorum fermentorum, quibus secretionum in oeconomia animali negotium antehac transigi credebatur, ad Garamantas procul & Indos ablegandarum, invehere conatus totus in eo est, ut probet in sua docta Dissertatione de *Circulatione Sanguinis*, orificia vasorum & poros glandularum partiumque corporis nostri alias quam circulares figuras non habere, nec proinde figuris, sed figurarum amplitudine inter se differre; ex quo deinceps concludit, *nulla fermentis peculiaria superesse promptuaria, fermentaque ipsa in Animali nulla.*

Figuram vasorum circularem ex generali Mechanicae principio, *Quod fluida quaecunque pressionem suam communicent per lineam corpori continenti & fluidi actionem sustinenti in quovis puncto impulsus per perpendicularem*, supra etiam (§. 249.) a nobis demonstrato derivare contendit, sectionem canalis axi rectam tanquam polygonum infinitilaterum respiciens, cujus polygoni latuscula indefinite parva impressiones fluidi juxta directiones ipsis normales sustineant, & quia hae impressiones in singula latera aequalia poligoni in circumstantiis apud Autorem aequales sunt, eas per lineolas rectas & poligoni lateribus perpendiculares repraesentat. Hucusque omnia bene, sed dum ex eo, quod hae perpendiculares laterum poligoni cum se invicem angulos efficiunt, concludere vult, aut saltem inferre videtur, omnes dictas perpendiculares in unoeodemque puncto concurrere debere, fallitur, etsi caeterum certissimum sit perpendiculares illas reapse in uno eodemque puncto convergere, nam ex eo, quod duae quaeque perpendiculares concurrunt, non sequitur omnes in uno puncto coire. In omni curva rectae, quae duobus curvae contiguis elementis perpendiculares sunt, angulos continebunt, hoc est, concurrent, sed quis inde inferret omnes perpendiculares curvae in uno eodemque puncto convenire? Ex principiis ergo illis, quae Cl.Vir posuit, & quae tanquam vera ultro admittenda puto , nondum satis ostendit rem eo deduci, *ut inveniatur curva cujus omnes subtangentes in puncto concurrant*, prout scribit pag. 28. *Dissert. Medic*. Voluit dicere rem deduci ad inventionem seu investigationem curvae, cujus omnes *subperpendiculares* curvae ad unum idemque punctum terminentur; nulla enim datur curva, cujus subtangentes concurrant, quandoquidem omnes subtangentes sumuntur in eadem linea, id est, in axe datae curvae, seu in alia linea, quae instar axis sit. Fateor quidem, quod, si omnes curvae perpendiculares in puncto concurrant, omnes subnormales ad unum idemque punctum quoddam terminatum iri, ast etiam est fatendum, tunc ea reductione problematis ad considerationem subnormalium non opus esse; nam si omnes curvae perpendiculares in quodam puncto convergant, ilico concludendum hanc curvam esse circulum; adeo ut problema jam solutum sit absque ulteriore reductione, sive ad proprietatem nominatam subperpendicularium, sive ad methodum, quam Pitcarnius dicit inversam fluxionem. Idcirco nos sequenti Problemate perspicue ostendere conabimur curvam quaesitam reapse eam esse, cujus cunctae perpendiculares in idem punctum seu centrum vergant, non obstante, quod hoc idem problema jam supra (§.102.) solutum sit, sed diversa ratione ad illustrationem generalis theorematis, quod infinita ejusmodi alia problemata solvit.

PROPOSITIO XVI. PROBLEMA.

303. *Dato tubo AB ex materia molli seu flexili clauso in B, apertoque A ad horizontem verticaliter erecto, & aquae aliusve liquoris pleno, determinare figuram, quam sectio quaelibet horizontalis in tubo a liquoris pressuris acquiret.* Fig.69.

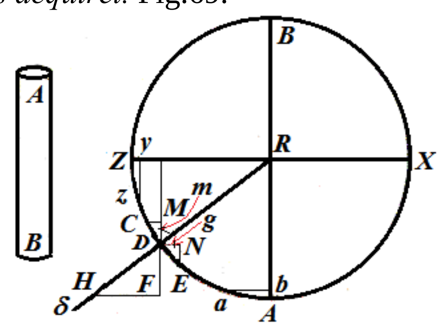

Fig. 69 & 70

Fig.70. Sit AZBX figura quaesita, cui rectae ZX, AB ad angulos rectos in R se invicem decussantes perpendiculariter occurrant in punctis A, X, B, X. Sint contigua curvae elementa DC, DE & Z*z*, A*a*, punctis Z, A adjacentia singula inter se aequalia, ducanturque *zγ*, DK rectae ZX normales, ipsisque aequidistans EN, nec non per puncta C & D rectae CM & DN ipsi ZX parallelae, & per *a* lineola *ab* eidem ZX aequidistans; ac denique fiant $\mathrm{D}m = \mathrm{EN}$ & $\mathrm{D}g = \mathrm{CM}$. Quibus positis, quia sectionis tubi ZAXB singula puncta a liquoris superficie aequaliter distant singula puncta aequalem pressionem subibunt, & quod vis ejus punctum D (§. 255.) pressionem sustinebit aequalem ponderi filamenti liquoris, cujus longitudo, quam per lineam DH curvae perpendicularem in puncto D indicabimus, aequetur fluidi altitudini super sectione ZAXB, juxta directionem curvae ubique normalem Dδ; ergo singulae DH, quae sumuntur

in lineis curvae perpendicularibus, potentias eidem curvae applicatas repraesentabunt, atque adeo rec-lum DH.DC refert potentiam in totum elementum DC agentem. Verum potentia DH (§.39.) aequipollet lateralibus DF & FH, quarum haec ipsi ZX parallela illa vero eidem perpendicularis est; ergo etiam potentia agens in totum curvae elementum CD, quae exponitur rec-lo DH.DC, aequipollet potentiis lateralibus CD.DF & CD.FH, vel quia triangula similia CDM & DHF praebent potentiis lateralibus per $CD.DF = CM.DH$ & $CD.FH = DM.DH$ rectangula CM.DH ac DM.DH exponendis juxta directiones DF & FM vel huic parallelam, agentibus.

Jam (§. 98.) est generaliter $\int FH$, hoc est aggregatum potentiarum juxta directiones FH, seu juxta directiones rectae ZX parallelas agentes in omnia elementa curvae CD vel DE in curvae arcu ZDE vel ZD contenta, id est per modo ostensa, omnia rectangula DM.DH respectae arcus ZD, quae componunt rectangulum DK.DH, ad $DN - Zy$, seu $CM - Zy$, sicut firmitas curvae in puncto A, quam firmitatem hac litera A designamus, ad constans curvae elementum CD; hoc est $DK.DH : CM - Zy = A : CD$, vel quia (secundum hypothesin) ZX curvae ad angulos rectos occurrit in Z, atque adeo Zy respectu Z*z* & CM evanescit , fiet $DK.DH : CM - Zy = A : CD$ & permutando $DK.DH : A = CM : CD$.

Item omnes DF seu $\int DF$, hoc est, in praesenti casu omnia rectangula CM.DH pertinentia ad arcum curvae DA, hoc est, rectangulum KR.DH, quandoquidem omnes CM componunt KR, seu sinum arcus DA, sunt ad $dM - Ab$ sicut A ad CD, vel quia etiam RA (secundum hypothesin) curvae normalis est in A, atque adeo A*b* prae A*a* vel CD aut MD evanescit, erit etiam $KR.DH : DM = A : DC$, & permutando $KR.DH : A = DM : DC$; atqui paulo ante etiam habuimus $DK : DH : A = CM : CD$ seu invertendo $A : DK.DH = DC : CM$ ergo ex aequo $KR.DH : DK.DH = KR : DK = DM : CM = FH : DF$, adeoque juncta DR triangula DKR & DFH similia sunt, atque adeo rectae DR & DH ubique in directum jacent, ac proinde omnes DH curvae ZDA normales productae in eodem puncto R concurrunt, hinc curva quaesita ZDA est circulus. Quod erat inveniendum & demonstrandum.

COROLLARIUM I .

304. Analogia praecedentis paragraphi $DK.DH : A = CM : CD$, elemento curvae A*a* vertici A adjacenti applicata, praebebit $bR.DH : A = ab : aA$, verum quia RA curvae, perpendicularis est, fiet $ab = aA$ ergo etiam $A = bR.DH$ seu AR.DH. Est igitur firmitas tubi in quolibet sectionis puncto, ut rectangulum ex radio sectionis AR vel ZR in potentiam sectionis puncto applicatam DH, hoc est in liquoris altitudinem super hoc punctum. Propterea 1°. in tubis aequalium orificiorum tenacitates seu firmitates requisitae ad perferendas fluidi pressuras erunt ut altitudines fluidi homogenei ductae in suas densitates seu gravitates specificas. 2°. Si facta ex altitudinibus liquorum in densitares aequalia fuerint; firmitates tuborum erunt ut radii orificiorum. 3°. Si ejusmodi facta radiis sectionum tuborum axibus normalium reciproce proportionalia fuerint, firmitates erunt aequales.

COROLLARIUM II.

305. Quia vires aequales eidem vel similibus subjectis similiter applicatae non nisi similes effectus producere possunt, ideo nihil refert, an tubus seu vas flexile a pondere liquoris in ejus cavitate gravitantis, an vera a fluido quodam elastico extrorsum prematur juxta
directiones , perinde ac fluidum facit, curvae perpendiculares. Nam fluidi elastici pressiones semper revocari possunt ad aequivalentes pressiones alicujus liquoris, cujus altitudines per DH repraesentabantur. Hinc enim est, quod bullae aqueae, quas pueri loturam saponis per fistulas stramineas flatu suo protrudentes subinde ludendo formant, in globulos intus cavos rotundentur. Idcirco viscositatis aequae in ejusmodi bullis vis requisita ad resistendum aeris inclusi elasticitati est ut haec vis elastica, ducta in radium bullae vel potius in radium cavitatis ejusdem.

SCHOLION.

306. Non abs re fuerit hoc loco observasse firmitates tuborum flexibilium eodem prorsus modo se habere, quo firmitates tuborum rigidorum, has enim ostendimus esse (§. 275.) ut rectangula tuborum per axes, seu ut facta ex altitudinibus liquorum inter se & absolute homogeneorum in diametros tuborum, quoniam crassities tuborum in hac factorum ratione esse oportere ostendimus, &, caeteris paribus, resistentiae sunt ut crassities, in flexibilibus vera resistentia seu firmitas in quolibet puncto est ut vis instans, seu ut liquoris altitudo ducta in tubi semidiametrum, adeoque firmitas tubi in duobus punctis diametraliter oppositis perinde ac in rigidis est factum ex liquoris altitudine in totam diametrum. Hinc non video, quibus rationibus Vir ingeniosus D. Parent rationem peripheriae ad diametrum formulis suis pro firmitatibus tuborum rigidorum aestimandis introduxerit in Commentariis Academiae Reg. Scientiarum 1707.

PROPOSITIO XVII. PROBLEMA.

307. *Si in linteo ZDAX in terminis suis Z & X fixo stagnet quilibet liquor heterogeneus, cujus scala gravitationis sit curva ROS; invenire figuram lintei manentem.* Fig. 71.

Sint iterum, ut in praecedenti problemate, Z*z*, CD, DE, A*a* elementa curvae ZDA aequalia, & DL, El ordinatae axi AR rectae, D*g* vero & Eβ eidem axi parallela; & reliquis ductis aequales in figura apparent. Jam, quia omnia in aequilibrio sunt per hypothesen & (§.260.) elementum curvae CD pressuram sustinet aequalem ponderi prismatis liquidi CD.LO, cum LO sit ordinata scalae gravitationum,

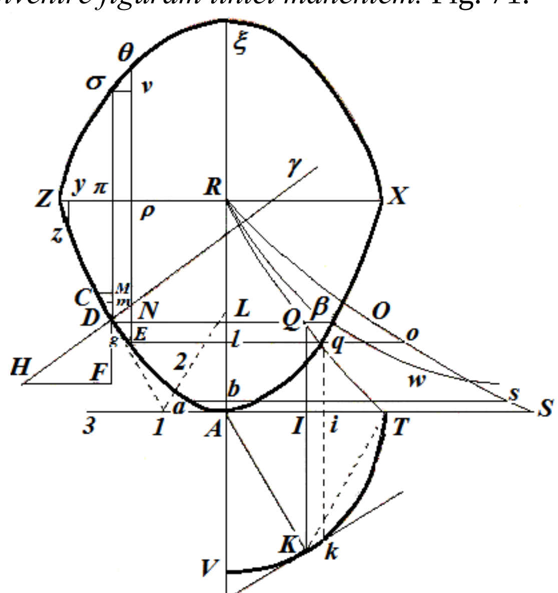

Fig. 71.

potentia qua elementum CD urgetur juxta directionem perpendicularem erit DC.DH facta DH ubique aequali homologae LO, & quia insuper ob omnes potentias curvae perpendiculares
(§. 96.) firmitas lintei in omnibus punctis eadem est, atque adeo per datam quantitatem A designati potest, erit (§. 98.) $\int FH$ seu omnes

$\mathrm{FH}:\mathrm{DN}-\mathrm{Z}y=\mathrm{A}:\mathrm{DC}$; vel quia Z$y$ evanescit prae ipsae dN, fiet $\int FH:\mathrm{DN}=\mathrm{A}:\mathrm{DC}$, &

permutando $\int FH:\mathrm{A}=\mathrm{DN}:\mathrm{DC}$. Jam, quia potentia FH, quae ex DH derivata est, non solum punctum D, sed integrum elementum DC respicit, loco FH poni debet FH.DC perinde ac loco DH in hoc casu intelligi debet DH.DC, & quia praeterea FH.DC (propter triangulorum DHF & CDM similitudinem) aequale est rec-lo DH.MD, vel DH.NE, id est LO.LI, erunt omnia LO.LI areae RLO inscripta aequalia huic areae; ergo $\int FH$, id est, area $\mathrm{RLO}:\mathrm{A}=\mathrm{DN}:\mathrm{DE}$ vel DC, & haec analogia respectu elementi curvae Aa fiet area ROsb vel $\mathrm{ROSA}:\mathrm{A}=ab:a\mathrm{A}$, unde, quia in A, elementum $a\mathrm{A}=ab$, erit etiam $\mathrm{A}=$ areae ROSA, adeoque praecedens analogia $\mathrm{RLO}:\mathrm{A}=\mathrm{DN}:\mathrm{NE}$ mutabitur in $\mathrm{RLO}:\mathrm{RAS}=\mathrm{DN}:\mathrm{NE}$.

Sit denique curva RQT ejus naturae; ut rec-lum $\mathrm{SA.AT}$ sit $=$ trilineo ROSA, & $\mathrm{SA.LQF}=$ trilineo ROL, eritque $\mathrm{ROL}:\ \mathrm{RAS}=\mathrm{AS.\,LQ}:\mathrm{AS.\,AT}=\mathrm{LQ.AT}$: unde cum invenerimus $\mathrm{ROLR}:\mathrm{BASR}=\mathrm{DN}:\mathrm{NE}$, erit etiam $\mathrm{LQ}:\mathrm{AT}=\mathrm{DN}:\mathrm{NE}$. Hinc centro A intervalloque AT descripto quadrante circuli TDV, ductaque per Q recta parallela QK quadranti occurrente in K, ac denique acta AK, erit etiam $\mathrm{AI}:\mathrm{AK}=\mathrm{DN}:\mathrm{DE}$, atque adeo triangula DNE & AIK sunt similia & similiter posita ; ac proinde radius AK, parallelus erit tangenti curvae quaesitae in puncto D. Eritque pariter $\mathrm{DN}:\mathrm{NE}=\mathrm{AI}:\mathrm{IK}$. Quae erant invenienda.

COROLLARIUM I.

308. Ducta per quadrantis punctum K tangente Kα, si in recta LO sumatur $L\beta=\mathrm{K}\alpha$ & sic respectu cujusvis alius curvae puncti, omnia puncta β erunt in curva quadam, $R\beta\omega$ asymtotam AS habente, cujus area $\mathrm{SAL}\beta\omega$ ad partes asymptotae aequabitur rectangulo sub radio AT & ordinata DL curvae ZDA . Nam triangula similia AIK & AKα praebent $\mathrm{AI}:\mathrm{IK}=\mathrm{K}\alpha:\mathrm{AK}$, sed $\mathrm{AI}:\mathrm{IK}=\mathrm{DN}:\mathrm{NE}$, ergo $\mathrm{K}\alpha:\mathrm{AK}$ vel $\mathrm{AT}=\mathrm{DN}:\mathrm{NE}$, ergo $\mathrm{AT.DN}=\mathrm{K}\alpha.\mathrm{NE}=\mathrm{L}\beta.\,\mathrm{LI}$, ergo omnia AT.DN hoc est rec-lum $\mathrm{AT.DL}=$ omnibus rectis, $\mathrm{L}\beta.\,\mathrm{LI}$, quae areae $\mathrm{AL}\beta\omega\mathrm{S}$ inscripta sunt, id est, huic areae $\mathrm{AL}\beta\omega\mathrm{S}$; atque adeo DI!=ALd3cwS: AT. $\mathrm{DL}=\mathrm{AL}\beta\omega\mathrm{S}:\mathrm{AT}$.

COROLLARIUM II.

309. Analogia, in quam paragrapho 307 circa finem incidimus, $\mathrm{DN}:\mathrm{NE}=\mathrm{AI}:\mathrm{IK}$ immediate praebet aequationem differentialem curvae quaesitae $\mathrm{DN}=\mathrm{NE.AI}:\mathrm{IK}$. Nam si dicantur $\mathrm{Z}\pi=\mathrm{y}$, $\pi\mathrm{D}=\mathrm{RL}=\mathrm{x}$, $\mathrm{AI}=\mathrm{LQ}=p$, adeoque

$\mathrm{IK} = \sqrt{(bb - pp)}$ existente $\mathrm{AT} = \mathrm{AK} = b,\ \mathrm{RA} = \mathrm{AS} = a$, & denique $\pi\rho = \mathrm{DN} = dy,\ \mathrm{NE} = \mathrm{LI} = dx$, superiorque aequatio $\mathrm{DN} = \mathrm{NE.AI} : \mathrm{IK}$ factis debitis substitutionibus juxta denominationes linearum modo institutas, mutabitur in $dy = pdx : \sqrt{(bb - pp)}$, quae est aequatio differentialis curvae quaesitae ad amussim conveniens cum aequationibus, quas Celeberrimi Bernoullii Fratres, quisque sua methodo, invenerunt. Si linea ROS fuerit recta, curva RQT erit parabola conica aequationem habens $app : b = xx$, & substituto valore bxx: aa, loco p in aequatione differentiali ante inventa $dy = pdx : \sqrt{(bb - pp)}$, habebitur $dy = pdx : \sqrt{(a^4 - x^4)}$ quae iterum coincidit cum ea quam supra (§. 104.) dedimus.

COROLLARIUM III.

310. Quonian (§.96.) tenacitas lintei in omnibus punctis eadem est, recta12 angulum D1A a tangentibus curvae D1 & AI formatum bisecans (§.110.) erit media directio omnium fluidi gravitationum in curvam DA, & subtensa anguli TAK aequalis angulo LDI (§.112.) parallela existet mediae directioni 12 ; adeoque (§.110.) erit potentia juxta mediam directionem 21 ad tenacitatem lintei in D ut sinus anguli D1A ad sinum A12, hoc est, ut sinus KAT ad sinum KTA, id est, sicut KT ad AK. Unde cum tenacitas lintei (§.307.) sit trilineum $\mathrm{ROSA}(\mathrm{constr.}) = \mathrm{AS.AT.}$ vel AK, erit potentia juxta $21 = \mathrm{AS.AK.KT} : \mathrm{AK} = \mathrm{AS.KT} = \mathrm{AR.KT}$. Et quia in symbolis corollarii antecedentis $\mathrm{KT} = \sqrt{(2bb - 2bp)}$ & $\mathrm{AR} = a$, erit potentia juxtae mediam directionem $21 = a\sqrt{(2bb - 2bp)}$.

SCHOLION.

311. Curva hujus propositionis ZAX uni parti famosi problematis circa isoperimetras ab ingeniosissimis Bernoulliis soluti, satisfacit. Nam ii ex singulis ejus punctis D, E, &c. indefinitae $\mathrm{D}\sigma, \mathrm{E}\theta,$ &c. axi AR parallelae agantur, & in iis fiat $\pi\sigma = \mathrm{LQ};\ p\theta = lq$, &c. nascetur curva $\mathrm{Z}\xi\mathrm{X}$, quae cum basi ZX majus spatium continebit quam quaelibet alia curva ex alia isoperimetra ZDAX simili lege descripta, ut in Actis Lips. 1701. pag.213. & Comm. Acad. Reg. Scient. Paris. 1706. d. 17. Apr. ab eximiis Geometris est ostensum.

Praeter aequationem $dy = pdx : \sqrt{(bb - pp)}$ Dn. Jac. Bernoullius invenit aliam hujus formae $dy = (p - b)dx : \sqrt{(2pb - pp)}$ quae *minimum* continet, & altera maximum. Sed una ex altera nullo negotio elicitur etenim, si in priore $dy = pdx : \sqrt{(bb - pp)}$ loco p ponatur $p - b$, seu complementum ejus ad maximam b, orietur altera aequatio, & si in altero pro $p - b$ substituatur p, redibit prior.

312. Radius circuli curvam osculantis in ejus puncto quolibet D, seu radius evolutae $\mathrm{D}\gamma$, est quarta proportionalis ad LO homologam ordinatam scalae gravitationum ROS, & datas RA & AT. Cujus rei demonstratio ex nostra construction facilis est.