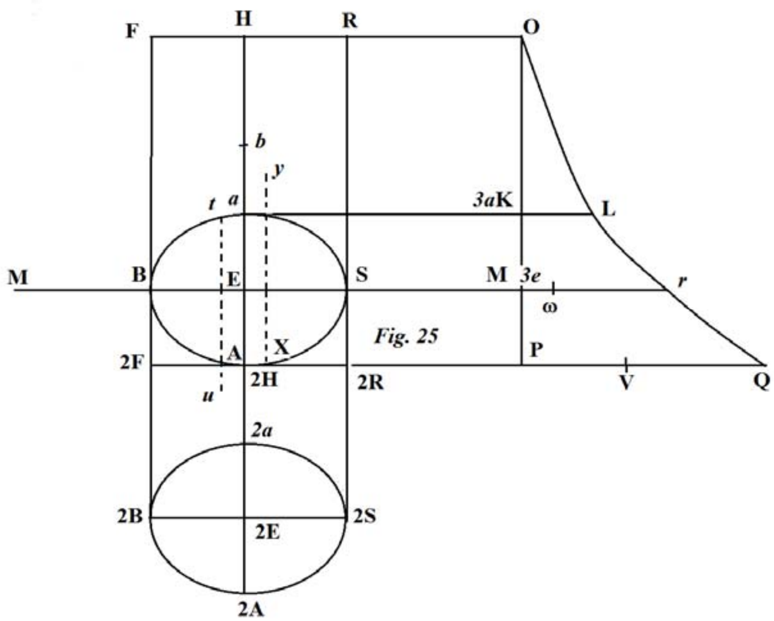
CHAPTER III.

Concerning the Equilibrium of Solid Bodies immersed in some Fluids, or floating in the same Fluids.

PROPOSITION XII. THEOREMA.

290. *Any solid body immersed completely in a heterogeneous fluid, or floating on the same, is trying to be raised upwards by just as great a force as the weight of the liquid of the same homogeneity corresponding to the volume of the solid or of its part immersed, of which the density is equal to the mean density of the heterogeneous fluid, along the direction normal to the surface of the fluid passing through the centre of gravity of the analogous solid.*

Fig. 25. Generally this theorem is none other than that composed in §.83 ; therefore with



the graph of the said paragraph being resumed and by putting the line OLQ in place, which there may be called the graph of the forces applied to the solid now to be the graph of the pressure or of the weight of the heterogeneous fluid, and the plane, which is called FO in place of the upper plane cited, now will be the surface of the fluid in the discussion, in which the body *a*BAS has been immersed, and from the said paragraph 83 the solid body *a*BAS will be pressed upwards at once by that force, which is equal to the weight of the mass of the liquid 2*a*2B2A2S enclosed by an analogous part of the volume submerged, of which the equal uniform density shall be PV, or the mean density of the heterogeneous fluid. For (§.260) the individual points of the surface of the body *a*BAS undergo a pressure equal to the weight of the same homogeneous fluid or of a filament, of which the length is the same as the homologous ordinate of the graph of the gravity OLQ, and the uniform density of the homogeneous fluid is equal to the mean density PV of the heterogeneous fluid in which the body *a*BAS has been immersed, because the weight of a filament (§. 33.) is equal to the product made always from its volume proposed by the ordinate of the graph of the gravity by its density PV, from which, because in the case of the present force with the individual points of the surface *a*BAS are applied perpendicularly to the respective ordinates of the graph OLQ multiplied by the given PV, it is apparent (§. 83) the force trying to raise the body *a*BAS in the heterogeneous fluid is by the product from the volume 2*a*2B2A2S into PV requiring to be set out, that is (§.33) from the weight of the mass of the homogeneous liquid, of which the density PV and the volume shall be in the manner of the solid reviewed, and the direction *xy* of the said forces in the plane FO, that is normal to the surface of the fluid, passing through the centre of gravity of the solid 2*a*2B2A2S. Which was required to be shown first concerning the immersion of the body *a*BAS.

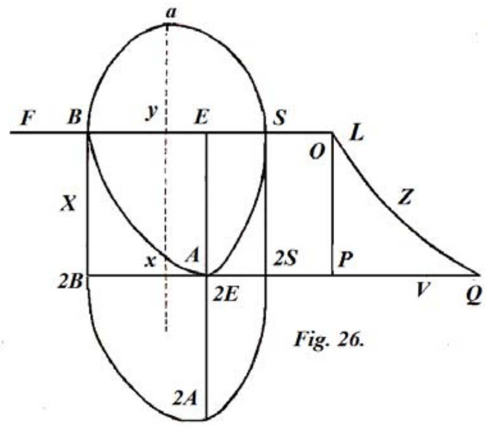


Fig. 26.

Fig. 26. The demonstration concerning a body *a*BAS floating on a heterogeneous fluid XZ will not be different. For the graph of the gravitational [*i.e.* pressure] curve of this fluid shall be LQ, 2*a*2B2A2S the proportional part of the immersed solid BAS, and even now the force striving to lift the solid *a*BAS will be the weight of the mass of the fluid 2*a*2B2A2S, of which the uniform density PV is equal to the mean density of the heterogeneous liquid, and the direction FL of that perpendicular to the surface shall pass through the centre of gravity of the same solid 2B2A2S, by § 83. Q.E.D.
[Thus the upthrust of the liquid is presented as the weight of an analogous or proportional volume of the liquid equal to that of the object, acting upwards on the body, whether partially or completely submerged.]

COROLLARY I.

291. Hence bodies present in fluids do not weigh by their whole amount, or by their absolute weight, but only by the amount, by which the absolute weight of these exceeds the weight of the heterogeneous fluid, of which the volume shall be in a given proportion, and the density equal to the mean density of the fluid, in which the body has beam submerged. And thus, if the weight of the mass of fluid may be equal in the said manner to the absolute weight of the body, this body will have no weight in the fluid, that is, it will neither descent nor ascend, and will weigh so much negatively, that is, it will rise in the fluid, if the nominated weight of the fluid within the volume of the solid, shall exceed the weight of this in a given ratio to the body.

COROLLARY II.

292. Now in the preceding corollary it is supposed the line joining the centre of gravity of the given body and of a solid of the same ratio alongside the graph of the pressure, to be normal to the surface of the fluid, otherwise the body immersed in the fluid in any case itself will be turned around, as far as until the said line joining the joining the centres of gravity were made perpendicular to the surface Fig. 25. For if the line *xy* produced to the

surface of the fluid FO cross at right angles, and may pass through the centre of gravity 2*a*2B2A2S, hence the solid *a*SAS will be acted on upwards by the force of the present proposition along the direction *xy*, itself truly by its weight is struggling to fall along the direction *tu* parallel to *xy* passing through the centre of gravity of the body *a*BAS. Now, if the line connecting the centre of gravity of the bodies *a*BAS and 2*a*2B2A2S shall be oblique to the plane FO, it is necessary that that the direction *tu* of the body *a*BAS trying to descend and *xy* the direction of the force of elevation shall be different, or not the same, from which, because the same body *a*BAS may be pressed on by two forces acting in opposite directions *tu* and *xy*, it is apparent to be going to rotating about itself in the direction of the order of the letters *a*BAS, nor can a motion of this first kind of rotation be able to stop, even when that body shall be found placed within the fluid, in which the directions *tu* and *xy* agree with each other in turn, and thus the line joining the centres of gravity of the solids *a*BAS and 2*a*2B2A2S shall be made perpendicular to the line FO.

Up to the present we have generally considered the equilibria and motion of bodies of all kinds of fluids put in place, there follows some corollaries about homogeneous fluids, and some easier problems.

COROLLARY III.

293. If the body *a*BAS were immersed in a homogeneous fluid, the graph of the pressure of the fluid will be the right angled isosceles triangle OPQ, and the volume 2*a*2B2A2S analogous to the body *a*BAS, will be similar and equal to the same ; and thus the body immersed or floating on the same fluid is trying to rise with just as great a force or strength, as the weight of the similar and equal mass of liquid contained within the volume of an immersed or of a floating body. Therefore according to which a body is pulled upwards only by its weight within a fluid of this kind, as much as the mass of the fluid weighed in the manner reckoned. And in this corollary nearly all the rules are established, which authors relate one after the other concerning the equilibria of homogeneous solids.

COROLLARY IV.

294. Hence also (§. 292.) the line joining the centres of gravity of some body floating and its homogeneous fluid and there in a state of equilibrium, will emerge normal to the surface of its submerged fluid part. Otherwise the floating body will oscillate hither and thither until it will have composed itself into this situation.

COROLLARY V.

295. Different parts of one and the same body immersed in different homogeneous liquids in the case of equilibrium are in the reciprocal ratio of the densities or of the specific gravities of the liquids, in which the same body is put to be immersed in successively.

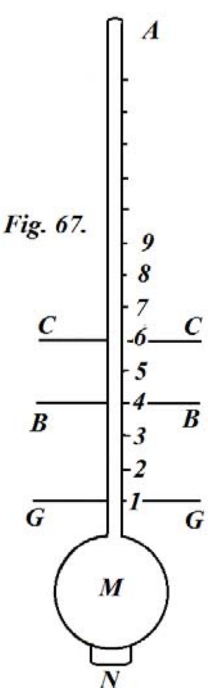
Liquids L, *l*, of specific gravities S, *s*, are said to sent in to immerse successively the volumes P, *p* of a body ; and I say there shall be $\mathrm{P} : p = s : \mathrm{S}$. For because the part of the body immersed is P of the liquid L and S the specific gravity of the liquid, the product from P and S (§.33) expresses the absolute weight of the mass of liquid within the volume P, and the same on account of the ratio *p.s* will denote the weight of the mass of liquid *l* within the volume *p*; but the weights P.S and *p.s*, which in the case of the equilibrium of one and the same body immersed in the liquids L, *l* are equal, also will be equal between themselves ; thus $\text{P.S} = p.s$, and as a consequence $\text{P} : p = s : \text{S}$.

SCHOLIUM.

296. The preceding corollary contains the basis for various little hydrostatic machines by which specific gravities of various liquids are usually investigated. The most useful device consists of a glass bubble M with a smooth neck and equipped with a cavity MA, into which at times a little mercury is accustomed to be poured, so that the small device always may stand upright in liquids ; truly the bubble M as it usually ends in a small bag in N, in which the mercury poured in may be able to collect. The neck of the device MA is usually divided into equal parts by artisans, but wrongly, for indicating equal weights of different liquids. In truth any account of the division of the collar shall be returned suitably with the specific gravities of all liquids being indicated by the help of several observations. Indeed these observations must be carried out according to the following account. Fig. 67. First the weight of the whole device with the mercury in place may be observed, which we will call P, and we may suppose that expressed in grains. Then the device is put into a certain liquid L to be submerged as far as the first division thence, or as far as to GG; which can happen by adding or subtracting some of the mercury. Again so many whole grains may be weighed out to the device in successive turns, to the extent that it may be immersed to all the following divisions 2, 3, 4, 5, 6, for the same liquid and the whole weight, which the device shall be able to made to descent in the same liquid as far as to the said divisions $\text{P, P}+\text{a, P}+\text{b, P}+\text{c, etc.}$, such as are seen in the adjoining list ; and which weights in this list also are denoted by P, 2P, 3P, 4P respectively; in which expressions the number prefixed to the letter P do not denote multiples of P, but are only general characteristic notes indicating to which division of the scale the number of grains shall be referred to by the letter P indicated by its prefixed number. And thus P, 2P, 3P, 4P &c. indicate certain different numbers of grains $\text{P, P}+\text{a, P}+\text{b, P}+\text{c, etc.}$ which become known through observation, and thus in a certain table, as in the above list, are required to be noted carefully, but not at all in an arithmetic progression, such as the series P, 2P, 3P, &c. seems to show ; from all the observations of this kind prepared in the table they assign the a useful way of providing all the remaining weights of liquids being investigated ; for with all the grains *a*,*b*, *c*,*d*, &c. removed so that only the device with the mercury placed within [initially] may remain, that may be immersed in a certain liquid A, as far as to BB, that is, as far as to the

Divisions of the neck.	Weight of the instrument immersed by the liquid.
1	$\text{P} = \text{P}$
2	$2\text{P} = \text{P} + \text{a}$
3	$3\text{P} = \text{P} + \text{b}$
4	$4\text{P} = \text{P} + \text{c}$
5	$5\text{P} = \text{P} + \text{d}$
6	$6\text{P} = \text{P} + \text{e}$
etc.	etc.

Fig. 67.



fourth division of the scale, and in another liquid O it may immersed as far as to 6P or to CC, and the specific gravity of the liquid A to the specific gravity of the other O, will be as the number of grains 6P agreeing with the sixth division of the scale to the number of grains 4P, which agrees with the fourth division ; that is the specific gravity of the liquids are in the inverse ratio of the numbers taken from the table constructed, which agree with the divisions of the scale, to which the device descents in the liquids. The demonstration is easy ; for (§.295) the specific gravity of the liquid A is to the specific gravity of another liquid O as N6 to N4 that is, in the reciprocal ratio of the parts of the device immersed in the liquids, and N6 is to N4, as the mass of the liquid L of which the volume is N6 to the mass of the same liquid of which the volume is N4; truly these masses of liquid L, N6 and N4 , will be weighing 6P and 4P grains respectively, since as many grains were in equilibrium with the masses of the fluid in the manner indicated by the force of the observation described a little before; therefore the specific gravity of the liquid A is to the specific gravity of the liquid O, as 6P to 4P, as was being said: Other instruments of this kind can be constructed from the same principles.

SCHOLIUM.

297. Even if the equilibria of fluids both amongst themselves may not be hidden from me, as also the equilibria of solid bodies in homogeneous fluids may be deduced more briefly from other principles, it is evident that all bodies brought together act on the principle of the maximum descent of the centre of gravity ; or, what amounts almost to the same, it follows from the equality of the moments of the bodies moving around among themselves to be used, according to the principles of Pascal and others. Truly, except that such principles are indirect, whether troublesome or not, by which we have deduced the preceding propositions from these approximate principles, and indeed which can be seen to be applied universally to heterogeneous fluids without long involved discussions ; I have preferred to pursue the method which we have explained in the first book, with the fundamentals concerning the forces applied to the individual points of any body, as in which elegant manner the pressures of heterogeneous fluids supplied are reduced to the equivalent pressures of homogeneous fluids.

PROPOSITION XIII. PROBLEM.

298. *With the diameter of some metal ball given, and with the ratio of the specific gravity of the metal, from which the ball has been made, to some homogeneous liquid, to find the diameter of the cavity of the ball for that required, so that the ball may be immersed in that homogeneous liquid to a given depth.*

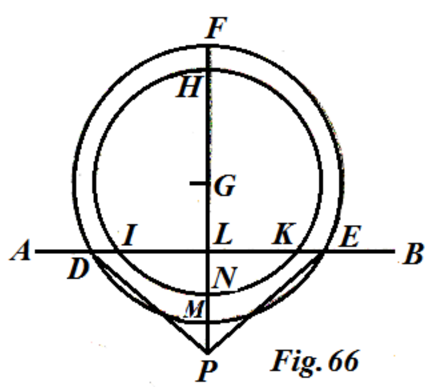


Fig. 66

FDE shall be the ball, and from its diameter FM, with the depth LM, to which the ball must be immersed in the liquid APB, and clearly with the ratio 1 to n of the specific gravity of the metal to the weight of that of the liquid, it is required to find the diameter HN of the hollow ball HIK of the concentric surface FDE.

Because in the case of the equilibrium of the ball with the liquid, the weight of the ball (§.293) must be equal to the weight, or of the force of gravity of the mass of fluid DME, and the weight of the ball is the product from the $\text{sphere FDE} - \text{sphere HIK}$ by the specific gravity of the metal, which is as 1, and the weight of the mass of liquid, is as the product from its volume DME by its specific gravity n :
therefore $\text{sph. FDE} - \text{sph. HIK} = n.\text{segment. DME.}$ Calling FM, a; the circumference FDE, b; LM, c and finally HN, x; there will be
$\text{sph. FDE} - \text{sph. HIK} = \left(a^3b - bx^3\right) : 6a$; and the spherical segment
$\text{DME} = (3abcc - 2bc^3) : 6a$, which segment multiplied by n, produces
$\left(3abccn - 2bcn^3\right) : 6a = \left(a^3b - bx^3\right) : 6a$, from which there is elicited
$x^3 = a^3 - 3accn + 2c^3n,$ or $\sqrt[3]{(a^3 - 3accn + 2c^3n)} = x,$ or putting $c = a : m,$ for some number m arising, also there will be $x = a\sqrt[3]{(m^3 - 3mn + 2n : m^3)}.$ Q.E.D.

COROLLARY.

299. Hence, if the whole ball must be immersed in the fluid; there becomes $c = a,$ or $m = 1$, and the final equation will be changed into $x = a.\sqrt[3]{(1-n)}$. Therefore, if a copper ball may be placed to float in air eight hundred times lighter than water, it will be, with the specific gravity of copper nine times that of water, $n = 1 : 7200$, and thus the formula $x = a.\sqrt[3]{(1-n)}$ will become $x = a\sqrt[3]{(7199 : 7200)}$, and by the per short-cut of logarithms the value of the ratio itself $a\sqrt[3]{(7199 : 7200)}$ may be found between the decimal fractions $0.999995a$ and $0.99996a$, and thus $a - x$ or twice the thickness of the metal is less than five hundred thousandths of the diameter FM, and thus the thickness itself of FH or MN to be less than one forty thousandth part of the same diameter FM. From which, if a hollow copper globe must be prepared, its thickness MN shall be only of one scruple or of 144$^{\text{th}}$ part of a foot, the diameter of the globe shall be required to be greater than 277 ft., so that it may stand freely in air. Truly if with Father Francisco de Lanis we may assume a diameter of a ball of 8 ft., the thickness of which will be required to be less than one five thousandths part of a foot ; and finally if the diameter may be put in place as 25 ft. by the same author in Book II. p. 291. of *Magisterii Naturae & Artis*, the thickness of the metal to become less than three four hundredth parts of an inch. And from which also it is considered worthy to note from the Illus. Leibniz, Book I of the *miscellaneous berolinensis* pag. 127, from which with everything made abundantly clear, all hope is to be abandoned that aerial navigation by some fortunate circumstance could be undertaken, on which account the distinguished de Lanis seemed to have wished to raise our hopes.

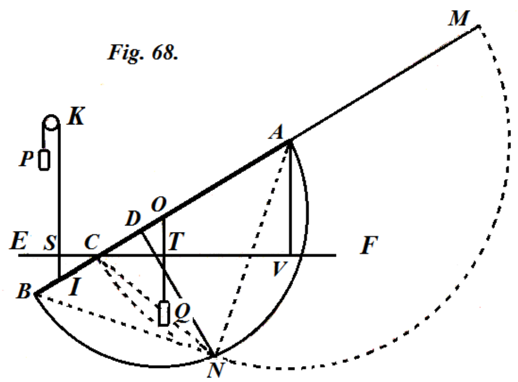
PROPOSITION XIV. PROBLEM.

300. *With the specific gravity of a smooth and slender rod* AB *given, and that of some homogeneous liquid, to determine how far from its end A the rod may be suspended by a string, thus immersed in the liquid so that the other end B may hang freely.* Fig. 68. [Thus, the general equilibrium of a supported rod is changed on immersing one end in a liquid, due to the upthrust of the liquid, while the remainder of the rod is unaffected.]

The specific gravity of the rod AB is itself considered to be to the specific gravity of the liquid ENF as BD is to the length of the rod AB, and the part BC of the rod to be immersed in the liquid, the magnitude is sought of AC itself or BC from the given AB and DB. [Thus, the densities or specific gravities are related by : $\frac{d_{rod}}{d_{liq}} = \frac{BD}{AB}$]

Geometrical Analysis. By §. 293 the rod immersed as far as C in the liquid is acted on to rise again, by a force equal to the weight of the liquid contained in the volume BC of the part of the rod immersed, along the direction IK through the centre of gravity, or passing through the centre I of the same BC, and normal to the surface of the liquid EF; and thus if a weight P were hung from a string KP surrounding the pulley K, equal to the weight of the mass of liquid BC, this weight P strives to fall in the direction KP and exerts the same force on the rod along the direction IK as the fluid ENF straining to raise the stick ; truly the rod by its own weight is trying to descent along the direction OQ likewise perpendicular to the surface of the liquid EF and through the centre of gravity of the rod, which is passing through its midpoint O ; therefore the liquid trying to raise the rod, and the weight perform the same effect, as the weights Q and P put in place, of which the one designates the absolute weight of the rod, and the other truly, as said now, the weight of liquid contained in the volume BC ; flexible and weightless strings are applied to AB in the directions IK and OQ parallel to each other and normal to the surface of the liquid EF; from which, with the rod immersed in some liquid as far as possible (according to the hypothesis) in a state of rest, or in equilibrium, it may be agreed, from the principle of levers that there will be (§. 55) $P.SV = Q.TV$, evidently with the normal AV dropped from A above EF, or

Fig. 68.



$P:Q = TV:SV = 2OA:2IA$ (or with BA produced to M, so that AM shall be equal to IA) $= BA:BM$. For, as IA is the arithmetic mean between BA and CA or AM, its double or 2IA will be equal to the sum of the extremes BA and CA or AM, that is the right line BM, and the double of OA is BA itself. Truly, the weight P is to the weight Q (§.33) as the product from the first with the volume BC by its specific gravity BD to the product from the volume of the other Q or AB by the density or the specific gravity of the same AB [as defined initially], that is $P:Q = BC.BA:AB.BD = BC:BD.$ But a little before we had $P:Q = BA:BM$, therefore it arises thence,

$BA:BM = BC:BD$; & $BM.BC = BA.BD$. Hence, if with centre O, and with the radius

OB or OA, the semicircle BNA is described, and through the point D given on its diameter DN shall be drawn perpendicular to the diameter cutting the semicircle at N, there will be $\mathrm{BN}^2\left(=\mathrm{BA.BD}\right)=\mathrm{BM.BC}$, and thus the right line BN touches the semicircle CNM with centre A and with the radius AC or AM described at the point N, which hence will be the common intersection of the semicircles BND and CNM, and thus the equal lines AC and AN are present ; but this AN is the mean proportional between the given BA and DA, therefore also AC, and thus this AC is given. Which was required to be found.

[We have to admit that Hermann has not got this proposition correct, though the comments about the vertical forces are beyond reproach; however, the taking of moments is not a correct procedure, as at equilibrium, the sum of the moments about any point is zero for the rod ; Hermann seems to have chosen a special point for equating moments, which could only correspond to a constant rate of rotation of the rod about this point; clearly the rod has an unbalanced moment about its centre of mass, and will rotate and oscillate about its equilibrium position, until at last, due to viscous forces, it lies horizontally at rest on the surface of the fluid. Besides, there is no reason why the rod should adopt the unusual position shown, being uniform, and no experiment could be performed to produce this outcome. Generally in this proposition, and in the following one, the idea of the metacentric height, or of the centre of gravity of the displaced fluid, being vertically above the centre of gravity of the body for stability, is completely lacking, and hence these propositions are incorrect; of course they may be of interest to historians of science for this reason.]

COROLLARY.

301. And indeed also from the part of the rod BC immersed in some liquid and from the length of the rod AB itself, the ratio of the specific gravity of the liquid to the specific gravity of the rod will become known ; and indeed as far as taking DA as the third proportion for the length of the rod BA and its part CA extending from the liquid, and always BA to BD will be as the specific gravity of the liquid to the specific gravity of the rod for some extent of the immersion. And thus the different lengths BD, which result from different liquids, by which the rod can be successively immersed, will be in the inverse ratio of the specific gravity of some liquids. Therefore it is clear such a rod demonstrates conveniently the hydrostatic instrument (Accustomed to be called the *Pese-liqueur* by the French) by which the specific gravities are able to be examined. On which account the rod must be divided, so that from its immersed parts the specific gravities of liquids are able to be discerned, from this corollary it is easy to deduce, and thus I refrain from further explanation for the same.

Thus so far we have shown some examples of the hydrostatic rule indicating how far solid bodies must be immersed in liquids, so that they shall become in equilibrium with these liquids; there still remains a single example to be included, from which other hydrostatic rules, respecting the setting up of solid bodies at rest in fluids will be shown, hence in the end we will select the easiest of all propositions to be attended to.

PROPOSITION XV. PROBLEM.

301. Fig. 65. *With the ratio of the specific gravity of the triangular prism ABG to the specific gravity of some liquid XBZ, to determine the position of the prism, in which with the preceding vertex B of the triangle ABG, that may remain in equilibrium with that sent into the liquid.*

MBN shall be the part immersed in the liquid and the point D its centre of gravity, truly C the centre of gravity of the whole triangle ABG, and thus (§. 294.) the line DC, joining the centres of gravity of the whole and if the part of the triangle immersed, is required to be perpendicular to the surface XZ of the liquid in the case of equilibrium. Now from the centres of gravity it is agreed, the centres of gravity C, D of the triangles ABG & MBN are to be found on the lines BP and BQ, the bases AG and MN of the triangles being bisected at the points P and Q, and the lines or sections PC, QD being a third of the whole PB and QB ; hence with PQ, PM and PN drawn, PQ and CD themselves will be parallel, since PC and QD shall be similar parts of PB and QB themselves. Now since it shall be required that CD be perpendicular to the right line XAE, it is necessary that also PQ likewise shall be normal to XZ or MN, from which, because now QM and QN have been shown to be equal, it is required PM and QN likewise are equal.

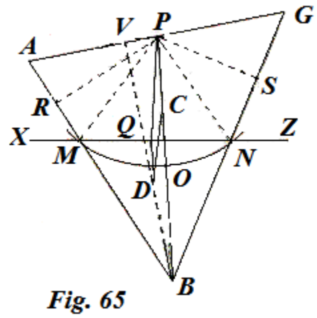


Fig. 65

Again, because in the case of the equilibrium of the mass of liquid MBN the weight (§.293) is equal to the absolute weight of the prism ABG, by §.33 the specific gravity of the liquid to the weight of the prism will be as the triangle ABG to the triangle MBN; and thus this given ratio of the triangles, since the ratio of the specific gravity of the liquid to the prism ABG (according to the hypothesis) has been given. Therefore from that the problem is reduced, so that it is described by the circle MON with the centre O and with a certain radius PM or PN, which may cut off such segments MB and NB from the sides AB and GB, so that with the line MN drawn, the triangle ABG shall be to the triangle MBN, or the rectangle ABG to the rectangle MBN in the given ratio of the specific gravity of the liquid to the weight of the prism ABG, that we will call $a:f$. Therefore with the perpendiculars sent from P , PR to BA, PS to BG and finally BV to AG, if they may be called AB, a; BG, b; BR, l and BS, m; truly the unknown BM, x; from the comparison of the right angled triangles PRM and PSN, in which just above the hypotenuses PM and PN are shown to be equal, the equation may be elicited, which will be shown to be reduced to that same biquadratic $x^4 - 2lx^3 + 2bfmx - bbff = 0$, the roots of which will determine BM or BN, and thus the point M or N and the radius PM of the circle being described MON, of which the intersections M and N with the right lines BA and BG determine the position of the right line MN. Q.E.I.

Similarly we would have arrived at a biquadratic equation, if we had assumed a scalene cone in place of the prism ABG.

CAPUT III.

De Equilibrio Corporum solidorum in Fluidis quibuscunque demersorum, vel iisdem Fluidis innatantium.

PROPOSITION XII. THEOREMA.

290. *Omne fluidum heterogeneum corpus quodcunque solidum in ipso demersum, vel eidem innotans, tanta vi in altum pellere conatur, quantum est pondus masse liquoris cujusdam homogenei sub volumine solidi analogi corpori demerso ejusve parti immersae, cujus densitas aequet mediam densitatem fluidi heterogenei, secundum directionem fluide superficiei normalem per centrum gravitatis solidi analogi transeuntem.*

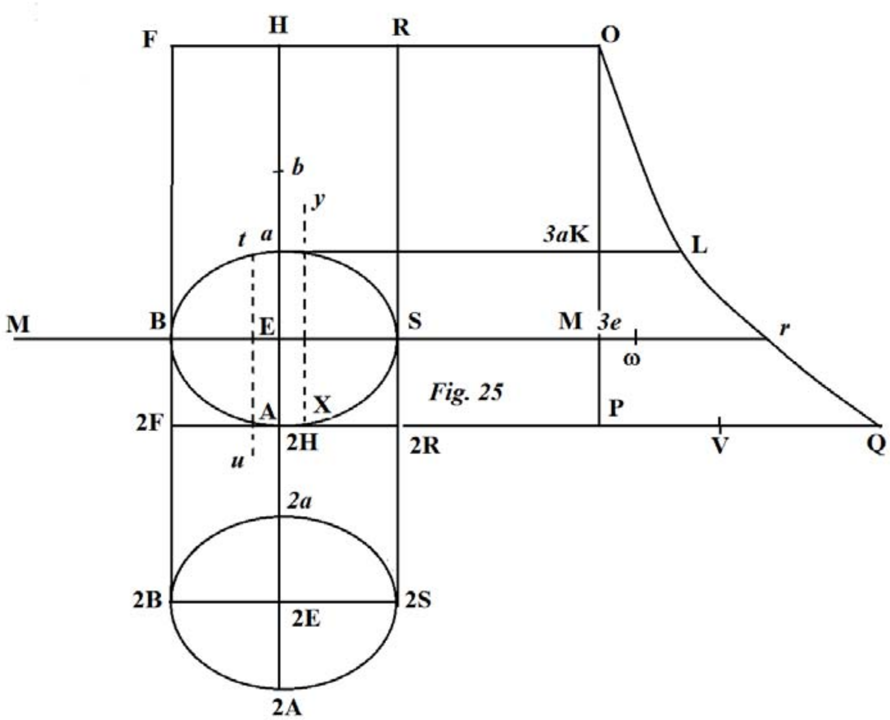


Fig. 25. Generale istud theorema aliud non est quam §. 83. in concreto sumtus ; idcirco resumendo schema dicti paragraphi ponendoque lineam OLQ, quae illic dicebatur scala potentiarum solido parienti applicatarum nunc esse scalam pressionum seu gravitationum fluidi heterogenei, planumque FO, quod supra in citato loco planum sublime vocabatur, nunc in concreto erit superficies fluidi, in quo corpus *a*BAS demersum est, & ex dicto paragrapho 83. illico liquebit solidum corpus *a*BAS ea vi in altum urgeri, quae aequetur ponderi massae liquidae 2*a*2B2A2S sub volumine solidi demenso corpori analogi, cujus densitas uniformis aequalis sit PV seu mediae densitati fluidi heterogenei. Nam (§.260.) singula puncta superficiei corporis *a*BAS pressionem subeunt aequalem, gravitati seu ponderi filamenti cujusdam fluidi homogenei, cujus filamenti longitudo eadem

est cum homologa ordinata scalae gravitationum OLQ, & fluidi homogenei densitas uniformis aequalis mediae densitati PV fluidi heterogenei in quo corpus *a*BAS demersum est, quod pondus filamenti (§. 33.) aequatur perpetuo facto ex volumine ejus exposito per ordinatam scalae gravitationum in densitatem ejus PV, unde, quia in casu praesenti potentiae singulis punctis superficiei *a*BAS perpendiculariter applicatae sunt ordinatae respectivae scalae OLQ in datam PV ductae, liquet (§. 83) potentiam corpus *a*BAS in fluido heterogeneo attollere conantem facto ex solido 2*a*2B2A2S in PV exponendam esse, hoc est (§.33) pondere massae liquoris homogenei, cujus densitas PV & volumen sit modo recensitum solidum, dictaeque potentiae directionem *xy* plano FO, hoc est fluidi superficiei normalem, transire per centrum gravitatis solidi 2*a*2B2A2S. Quod primum de corpore demerso *a*BAS erit demonstrandum.

Fig. 26. Non diversa erit demonstratio circa corpus *a*BAS fluido heterogeneo XZ innatans. Sit enim hujus fluidi scala gravitationum curva LQ, & 2*a*2B2A2S solidum analogum parti immersae BAS, potentia solidum *a*BAS attolere connitens etiamnunc erit pondus molis fluidae 2*a*2B2A2S cujus densitas uniformis PV aequatur mediae densitati liquoris heterogenei, ejusque directio superificiei liquoris heterogenei FL perpendicularis, transibit per centrum gravitatis ejusdem solidi 2B2A2S per §. 83. Quod erit demonstrandum.

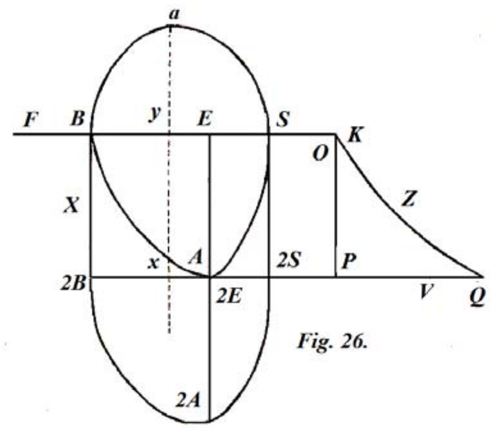


Fig. 26.

CORollARIUM I.

291. Hinc corpora in fluidis existentia non gravitant toti sua absoluta gravitate, seu pondere absoluto, sed duntaxat pro quantitate, qua pondus illorum absolutum superit pondus massae fluidi heterogenei, cujus volumen sit solidum corpori dato analogum, densitasque par mediae densitati fluidi, in quo corpus demersum est. Adeoque, si pondus massae fluidae modo dictae aequetur ponderi absolutio corporis, hoc corpus in fluido non gravitabit, hoc est, non descendet nec ascendet, & tantum negative gravitabit, hoc est, in fluido ascendet, si nominata fluidi pondus sub volumine solidi, dato corpori analogi, hujus pondus excedit.

COROLLARIUM II.

292. In corollario praecedenti jam supponitur lineam jungentem centra gravitatis corporis dati & solidi eidem analogi, juxta scalam gravitationum, fluidi superficiei normalem esse, alioqui corpus in fluido demersum in omni casu circa seipsum convertetur, usque dum dicta linea centra gravitatis conjungens fluidi superficiei perpendicularis Fig.25 facta fuerit. Nam si linea *xy* producta superficiei fluidi FO ad angulos rectos occurrat, atque per centrum gravitatis 2*a*2B2A2S transeat, vi praesentis propositionis solidum *a*SAS juxta directionem : hanc *xy* sursum agetur, ipsam vero proprio pondere descendere nititur juxta directionem *tu* parallelam xy per centrum gravitatis corporis *a*BAS transeuntem. Jam, si linea connectens centra gravitatis corporum *a*BAS & 2*a*2B2A2S plano FO obliqua fit,

necesse est ut directio *tu* corporis *a*BAS descendere conantis & *xy* directio potentiae attollentis sint diversae, seu non congruentes, unde, quia idem corpus *a*BAS duabus potentiis oppositas in partes agentibus juxta *tu* & *xy* urgetur, liquet id in se ipsum conversum iri juxta ordinem literarum *a*BAS, nec motum ejusmodi conversionis prius posse cessare, quam corpus cum intra fluidum situm nactum sit, quo directiones *tu* & *xy* sibi invicem congruant, atque adeo linea jungens centra gravitatis solidorum *a*BAS & 2*a*2B2A2S plano FO perpendiculars facta fit.

Hactenus generalia circa aequilibria & motus corporum in omnis generis fluid s positorum, sequuntur unum alterumve corollarium circa fluida homogenea, & faciliora aliquot problemata.

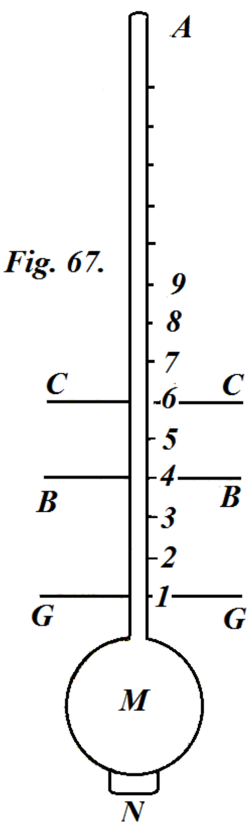
COROLLARIUM III.

293. Si corpus *a*BAS fluido homogeneo immersum est, fluidi scala gravitationum erit triangulum rectangulum isosceles OPQ & solidum 2*a*2B2A2S, corpori *a*BAS analogum, idem simile & aequale erit ; atque adeo fluidum homogeneum tanta vi seu potentia corpus in ipso demersum vel innatans attollere conatur, quantum est pondus massae liquoris sub, volumine corporis demersi simili & aequali vel partis fluido immersae. Idcirco cuilibet corpori tantum de suo pondere intra ejusmodi fluidum decedit, quantum ponderat massa fluidi modo recensita. Atque in hoc corollariolo fundantur ferme omnes regulae, quas Autores circa aequilibria solidorum cum fluidis homogeneis subinde tradunt.

COROLLARIUM IV.

294. Hinc etiam. (§. 292.) linea jugens centra gravitatis corporis cujusque fluido homogeneo innatantis & in eo in aequilibrio consistentis, ejusque partis fluido immersae superficiei fluidi normalis existet. Alioqui corpus fluido innatans hinc & inde vacillaret, donec in hunc situm se composuerit.

Fig. 67.



COROLLARIUM V.

295. Diversae partes unius ejusdemque corporis diversis liquoribus homogeneis immersae in casu aequilibrii sunt in reciproca ratione densitatum seu gravitatum specificarum liquorum, quibus idem corpus successive immersum esse ponitur.

Dicantur liquores L, *l*, eorum gravitates specificae S, *s*, corporis liquoribus successive immissi partes immersae P, *p* ; & dico fore $\mathrm{P} : p = s : \mathrm{S}$. Nam quia corporis liquori L immersa pars est P & S specifica gravitas liquoris, factum ex P in S (§.33.) exprimet pondus absolutum massae liquidae sub volumine P, & eandem ob rationem.
p.s denotabit pondus massae liquoris *l* sub volumine *p*; sed pondera P.s & *p.s*, quae in casu aequilibrii uni eidemque corpori liquoribus L, *l* immerso aequalia sunt, etiam inter se aequabuntur; adeo $\mathrm{P.S} = p.s$., & per consequens $\mathrm{P} : p = s : \mathrm{S}$.

SCHOLION.

296. Corollarium praecedens fundamentum continet diversarum machinularum hygrostathmicarum, quibus diversorum liquorum specificae gravitates explorari solent.

Divisiones colli.	Pondera machinulam liquori immergentia.
1	$P = P$
2	$2P = P + a$
3	$3P = P + b$
4	$4P = P + c$
5	$5P = P + d$
6	$6P = P + e$
etc.	etc.

Usitatissima constat bulla vitrea M collotereti & cavo MA instructa, cui subinde pauxillum Mercurii infundi solet, ut machinula in liquoribus situ semper erecto consistat; bulla vero M ut plurimum desinit in sacculum N, in quo infusus Mercurius se colligere possit. Machinulae collum MA ab artificibus in partes aequales dividi solet, sed perperam, ad indicandas aequales gravitatis liquorum differentias. Verum quacunque ratione collum divisum fit, ope nonnullarum observationum machinula apta redditur indicandis omnium liquorum specificis gravitatibus. Ipsae vero observationes sequenti ratione peragi debent. Fig.67. Notetur primum totius machinulae cum indito Mercurio pondus, quod nominabimus P, idque granis expressum supponemus. Machinula deinde liquori cuidam L immissa mergatur usque a primam divisionem, seu usque ad GG; quod semper fieri potest adedendo vel demendo nonnihil Mercurii. Appendantur porro machinulae successivis vicibus tot grana, usque dum eadem liquori L immergatur ad divisiones omnes sequentes 2, 3, 4, 5, 6, ponderaque totalia, quae machinulam in eodem liquore usque ad dictas divisiones descendere faciunt sint $P, P+a, P+b, P+c,$ etc. qualia in adjecto laterculo conspiciuntur; & quae pondera in hoc laterculo etiam signantur per P, 2P, 3P, 4P respective; in quibus expressionibus numeri literae P praefixi non denotant multipla ponderis P, sed sunt duntaxat notae generales characteristicae significantes ad quamnam colli divisionem referendus sit granorum numerus per literam P cum suo praefixo numero significatus. Adeoque P, 2P, 3P, 4P &c. significant quidem diversos numeros granorum $P, P+a, P+b, P+c,$ etc. qui ex observatione innotescunt, atque adeo in tabella quadam, ut in superiori laterculo, diligenter notandi sunt, sed minime progressionem arithmeticam, qualem series P, 2P, 3P, &c. exhibere videtur; ex ejusmodi observationibus parata tabella machinulam omnium reliquorum liquorum gravitati investigandae aptam reddunt; nam remotis omnibus granis *a,b, c,d,* &c. ut sola machinula cum indito Mercurio remaneat, immergatur ea in liquore quodam A, usque in BB, id est ad quartam colli divisionem, & in alio liquore O immergatur usque ad 6P seu in CC, eritque gravitas specifica liquoris A ad gravitatem specificam alterius O, ut numerus granorum 6P conveniens sextae colli divisioni ad numerum granorum 4P, qui convenit quartae divisioni; id est gravitates specificae liquorum sunt in reciproca ratione numerorum ex constructa tabula excerptorum, qui colli divisionibus, ad quas in liquoribus machinula descendit, conveniunt. Demonstratio facilis est; nam (§.295.) gravitas specifica liquoris A est ad gravitatem specificam alterius O sicut N6 ad N4 hoc est, in reciproca ratione partium machinulae liquoribus immersarum, atqui N6 est ad N4, ut massa liquoris L cujus volumen est N6 ad massam ejusdem liquoris cujus volumen est N4; hae vero massae N6 & N4 liquoris L ponderabunt 6P & 4P grana respective, cum tot grana cum fluidi L massis modo indicatis in aequilibrio fuerint vi observationis paulo ante descriptae; ergo gravitas specifica liquoris A est ad gravitatem specificam liquoris O, ut 6P ad 4P, prout dicebatur: Hujus generis aliae machinaeex iisdem principiis construi possunt.

SCHOLIUM.

297. Etsi me non lateat aequilibria fluidorum cum inter se, tum etiam solidorum corporum cum fluidis homogeneis ex aliis principiis nonnihil brevius posse deduci, scilicet ex fundamento maximi descensus centri gravitatis, quem omnia corpora inter se commissa affectant ; seu, quod ferme eodem redit, ab aequalitate momentorum corporum inter se agitandorum, cujusmodi principiis Pascalius aliique usi sunt. Verum, praeterquam quod talia principia indirecta sunt, ea vix ac ne vix quidem absque longis ambagibus fluidis heterogeneis applicari posse videntur in ea universalitate, in qua praecedentes propositiones ex principiis suis proximis directe deduximus ; malui methodo & ffundamentis circa potentias singulis punctis cujusque corporis applicatis, quae in primo libro exposuimus, insistere, utpote quae modum non inelegantem subministrarunt pressiones fluidorum heterogeneorum ad aequivalentes pressiones fluidorum homogeneorum reducendi.

PROPOSITIO XIII. PROBLEMA.

298. *Datis diametro alicujus pilae metallicae cavae*, & *ratione specificae gravitatis metalli, ex quo pila parata est, ad aliquem liquorem homogeneum, invenire diametrum cavitatis pilae ad id requisitae, ut pila in liquore illo homogeneo ad datam profunditatem immergatur.*

Sit pila FDE, & ex diametro ejus FM, profunditate LM, ad quam pila liquori APB immergi debet, & ex ratione 1 ad n scilicet gravitatis specificae metalli ad liquoris gravitatem, invenire oportet diametrum HN cavitatis pilae HIK superficiei FDE concentricae.

Quia in casu aequalibrii pilae cum liquore pondus pilae (§.293) aequari debet ponderi, seu gravitati massae fluidae DME, atqui pilae gravitas est factum ex sphaera FDE – sphaer. HIK in specificam metalli gravitatem, quae est ut 1, & gravitas massae liquoris, est ut factum ex ejus volumine DME in gravitatem ejus specificam n ergo sphaer. FDE – sph. HIK = n.segm. DME..

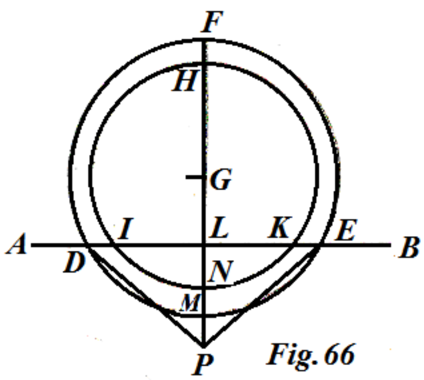


Fig. 66

Dicantur FM, a;
circumfer. FDE, b; LM, c & denique HN, x;
eritque sphaera FDE – sph. HIK = $\left(a^3b - bx^3\right):6a$;
& segmentum sphaericum
DME = $(3abcc - 2bc^3):6a$, quod segmentum ductum in n, praebet

$\left(3abccn-2bcn^3\right):6a=\left(a^3b-bx^3\right):6a$, ex qua elicitur
$x^3=a^3-3accn+2c^3n,$ vel $\sqrt[3]{(a^3-3accn+2c^3n)}=x,$ seu posita $c=a:m,$ existente m numero quocunque, erit etiam $x=a\sqrt[3]{(m^3-3mn+2n:m^3)}.$ Quod erit demonstrandum.

COROLLARIUM.

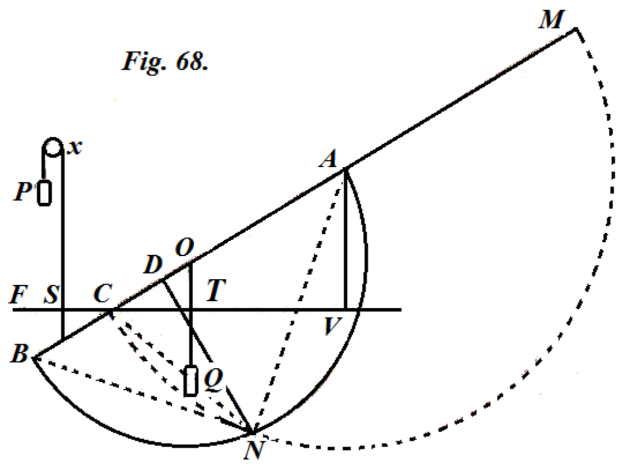
299. Hinc, si tota pila fluido debet immergi; fiet $c=a,$ seu $m=1$, & postrema aequatio mutabitur in $x=a.\sqrt[3]{(1-n)}$. Idcirco, si pila aenea in aere octingenties aqua leviore natare ponatur, erit, posita gravitate specifica aeris noncupla gravitatis aquae, $n=1:7200$, atque adeo formula $x=a.\sqrt[3]{(1-n)}$ abit in $x=a\sqrt[3]{(7199:7200)}$, atqui per compendium logarithmorum invenietur valor rationalis ipsius $a\sqrt[3]{(7199:7200)}$ inter fractiones decimales 0.999995*a* & 0.99996*a*, adeoque $a-x$ seu dupla metalli crassitities est minor quinque centies millesimis diametri FM, atque adeo crassities ipsa FH vel MN minor una quadragesies millesima ejusdem diametri FM particula. Unde, si globus aeneus cavus parari deberet, cujus crassities MN tantum sit unius scrupuli seu 144[mae] pedis partis, pilae diametrum majorem 277 pedibus esse oporteret, ut in aere librata consistere possit. Sin vero cum P. Francisco de Lanis Pilae diametrum 8 pedum assumere velimus, ejus crassities minor requiretur, quam una quinquies millesima pedis pars ; ac denique si diameter statuatur cum eodem Autore Tom. II. fol 291. *Magisterii Naturae & Artis*, 25 pedum, crassities metalli foret minor quam tres quadringentesimae pollicis partes. Hisce similia etiam exhibere dignatus est Illustris Leibnitius Tom.I. *Miscellaneorum Berolinensium* pag. 127. ex quibus omnibus abunde liquet, spem abjiciendam esse omnem, fore ut navigatio aerea fortunato aliquando eventu suscipi queat, in quam egregius ille de Lanis nos erigere voluisse videtur.

PROPOSITIO XIV. PROBLEMA.

300. *Data specifica gravitate bacilli teretis & gracilis* AB *& liquoris cujusque homogenei, determinare quousque bacillus extremitate sua A e filo suspensus, ita ut altera extremitate B liber pendeat, liquori debeat immergi.*

Fig. 68. Ponatur gravitatem specificam bacilli AB se habere ad gravitatem specificam liquoris ENF ut BD ad bacilli longitudinem AB, atque bacilli partem BC liquori immergi, quaeritur magnitudo ipsius AC vel BC ex datis AR & DB.

Analysis Geometrica. Per §. 293. bacillus liquori immersus usque ad C perpendiculariter sursum agitur vi aequali ponderi liquoris sub volumine BC partis bacilli liquori immersae duxta directionem IK per centrum gravitatis, seu per medium I ejusdem BC transeuntem liquorisque superficiei EF normalem; adeoque si filo IKP trochleam K amplectendi appensum fuerit pondus P, aequale gravitati massae liquidae BC, hoc pondus P in directione KP descendere nitens eandem vim in bacillum juxta directionem IK exeret ac fluidum ENF bacillum attollere connitens ; ipse vero bacillus proprio suo pondere descendere conator juxta directonem OQ superficiei liquoris EF itidem perpendicularem & per centrum gravitatis bacilli, quod est in ejus medio O transeuntem; idcirco liquor baculum attollere conans & gravitas eundem effectum praestabunt, quem praestarent pondera P & Q, quorum hoc bacilli gravitatem absolutam, illud vero, ut jam dictum, pondus liquoris sub volumine BC designant; lineae inflexili & gravitatis experti AB in directionibus IK & OQ inter se parallelis ac superficiei liquoris EF normalibus, applicata; unde, cum bacillus liquori aliquousque immersus (secundum hypothesin) in statu manenti, seu in aequilibrio, consistat, ex principio vectis erit (§. 55) $P.SU = Q.TV$, demissa scilicet ex A normali AU super EF, vel



$P : Q = TV : SV = 2OA : 2JA$ (seu producta BA in M, ut AM aequalis fit CA) $= BA : BM$. Nam, quia IA est media arithmetica inter BA & CA vel AM, ejus dupla seu 2IA aequabitur aggregato extremarum BA & CA vel AM, hoc est rectae BM, & dupla ipsius OA est BA. Verum, pondus P est ad pondus Q (§.33) ut factum ex illius volumine BC in gravitatem ejus specificam AB ad factum ex volumine alterius Q seu AB in densitatem seu gravitatem specificam ejusdem BD, hoc est $P : Q = BC.BA : AB.BD = BC : BD$. Atqui paulo ante habuimus $P : Q = BA : BM$, ergo inde nascitur $BA : BM = BC : BD$; & $BM.BC = BA.BD$. Hinc si centro O & intervallo OB vel OA descriptus semicirculus BNA, atque per punctum in ejus diametro datum D perpendicularis diametro DN ducta sit semicirculum DNA secans in N, erit $BN^2 (= BA.BD) = BM.BC$, atque adeo recta BN tanget semicirculum CNM centro A & intervallo AC vel AM descriptum in puncto N, quod proinde communis erit intersectio semicirculorum BND & CNM, atque adeo aequales existent AC & AN; atqui haec AN est media proportonalis inter datas BA & DA, ergo etiam AC, atque adeo haec AC data est. Quod erit inveniendum.

COROLLARIUM.

301. Adeoque etiam ex parte bacilli BC cuilibet liquori immersa & bacilli ipsius longitudine AB, semper innotescet ratio gravitatis specificae liquoris ad gravitatem specificam bacilli; etenim ad longitudinem bacilli BA ejusque partem CA extra liquorem extantem duntaxat sumenenda est tertia proportionalis DA, eritque BA ad BD semper ut gravitas specifica liquoris ad gravitatem specificam bacilli liquori aliquousque immersi. Atque adeo portiones diversae BD, quae resultant a diversitate liquorum, quibus bacillus successive immergi potest, erunt in reciproca ratione gravitatum specificarum ipsorum liquorum. Liquet ergo talem bacillum aptum exhibere instrumentum hygrostathmicum (Gallis *Pese-liqueur* dici solitum) quo liquorum specificae gravitates examinari queant. Qua ratione bacillus dividi debeat, ut ex partibus ejus immersis gravitates liquorum dignosci queant, facile colligitur ex hoc corollario, adeoque eidem ulterius explicandae supersedeo.

Hactenus aliquot exemplis illustravimus hydrostaticae regulam indicantem quousque corpora solida liquidis immergi debeant, ut cum hisce liquidis aequilibrium faciant; restat adhuc unicum exemplum adducendum, quo aliae hydrostaticae regula, situm corporum solidorum in fluidis consistentum respiciens, illustretur, hunc in finem facillimum omnium eligemus propositione sequenti excutiendum.

PROPOSITIO XV. PROBLEMA.

301. Fig.65. *Data ratione specificae gravitatis prismatis triangularis ABG ad gravitatem alicujus liquoris XBZ, determinare situm prismatis, in quo praecedente vertice B trianguli ABG, id liquori immissum cum eo in equilibrio maneat.*

Sit MBN pars liquori immersa & punctum D ejus centrum gravitatis, C vero centrum gravitatis trianguli totius ABG, adeoque (§. 294.) lineam DC, jungentem centra gravitatis totius & partis immersae triangulorum, oportet esse liquoris superficiei XZ perpendicularem in casu aequilibrii. Jam ex centrobaricis constat, centra gravitatis. C, D triangulorum ABG & MBN reperiri in lineis BP & BQ, bases AG & MN triangulorum bifariam dividentibus in punctis P & Q, lineasque vel intervalla PC, QD trientes esse totarum PB & QB ; hinc ductis PQ, PM & PN, ipsae PQ & CD aequidistantes erunt, cum PC & QD sint partes similes ipsarum PB & QB. Jam cum oporteat ipsam CD perpendicularem esse rectae XAE, necesse est ut etiam PQ eidem XZ vel MN normalis sit, unde,quia jam QM & QN aequales ostensae sunt, ipsas PM & QN pariter aequari oportet.

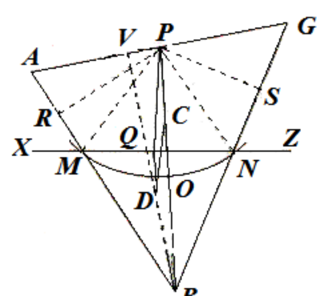


Fig. 65

Porro, quia in casu aequilibrii massae liquoris MBN pondus (§.293) aequale est ponderi absoluto prismatis ABG, per §. 33 erit gravitas specifica liquoris ad gravitatem prismatis ut triangulum ABG ad triangulum MBN; atque adeo haec triangulorum ratio data, quandoquidem ratio gravitatis specifica liquoris ad prisma ABG (secundum hypothesin) data est. Propterea problema eo reducitur, ut describatur centro O & intervallo quodam PM vel PN circulus MON, qui ex lateribus AB & GB abscindat segmenta MB & NB talia, ut ducta linea MN, triangulum ABG sit ad triangulum MBN, vel rectangulum ABG ad rec-lum MBN in ratione data specificae gravitatis liquoris ad gravitatem prismatis ABG, quam dicemus $a:f$. Idcirco demissis ex P perpendicularibus PR ad BA, PS ad BG ac denique BV ad AG, si dicantur AB, a; BG, b; BR, l & BS, m; incognita vero BM, x; ex comparatione triangulorum rectangulorum PRM & PSN, in quaebus juxta superius ostensa hypothenusae PM & PN aequantur, elicietur aequatio, quae reducta exhibebit istam biquadraticam $x^4 - 2lx^3 + 2bfmx - bbff = 0$, cujus radices determinabunt BM vel BN, atque adeo punctum M vel N & radium PM circuli describendi MON, cujus intersectiones M & N cum rectis BA & BG determinant positionem rectae MN. Quod erit inveniendum.

Similiter incidissemus in aequationem biquadratam, si loco prismatii ABG sumsissemus conum scalenum.