CHAPTER II.

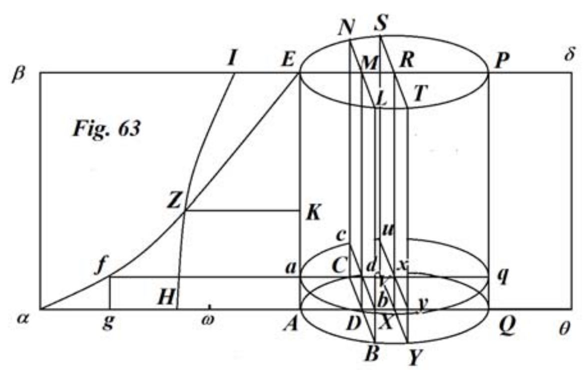
Concerning the pressures liquids exert on the sides of vessels and the strengths of pipes required for bearing these pressures.

In the above chapter we have considered only these weights of liquids, which are exerted on horizontal planes ; truly in this chapter the lateral pressures [*weights* in the original text] of liquids arising are to be examined in which the walls or sides of the vessels are affected. Indeed from such an investigation a rule will be provided for defining the strength of pipes for resisting the pressure of water, and in turn the forces by which liquids are trying to break through the walls of the vessels.

PROPOSITION VI. THEOREM.

263. *The external convex surface of the vessel or pipe* EAQP *will be pressed inwards by the same force from the surrounding liquid towards the axis from each direction of the openings* ETPS *and* AYQ , *and of the liquid* $\beta\alpha\theta\delta$ *sent in as high as* ETP, *as by which the internal cavity surface is acted on outwards by the liquid within the vessel.*

For if in place of the vessel EAQP a mass of liquid of the same constitution as the surrounding liquid is understood, and of the same shape and size and in the position of

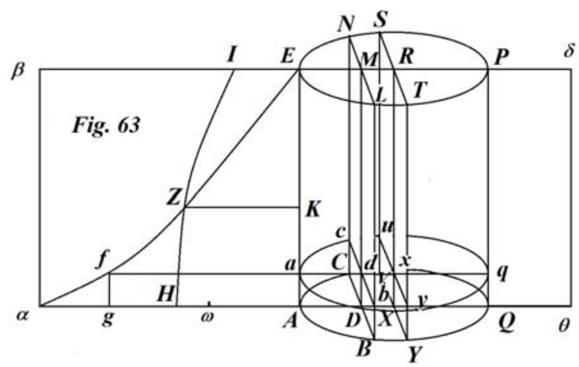


the vessel ; because (following the hypothesis) all remain at rest, the masses of the liquid EAQP outside the surface is acted on by the same force by the surrounding liquid $\beta\alpha\theta\delta$, as by which the surface of that within the cavity is acted on by the liquid EAYQP; otherwise, if either pressure should prevail on the other, the parts of the liquid may be stirred into motion, nor hence should the water or liquid remain at rest, contrary to the hypothesis. Now the surrounding liquid will exert the same impression on the surface of a rigid convex vessel, as a similar equal and similarly placed surface from the water EAYQP; as equally this liquid or water will exert on the surface in its cavity, as in a similar and equal cavity surface, but of a rigid vessel. Therefore the external and internal surfaces of the vessel are acted on by the surrounding and internal liquid, in the cavity of the vessel present, by equal forces in opposite directions, perpendicular along the directions of the surface or surfaces. Q.E.D.

PROPOSITION VII. THEOREM.

264. *If the right prism* NEABL *bounded by the curved surface* NCEBL, *by the rectangle* NLBC, *and by any two similar and equal figures* BAC, LEN, *shall be full of some heterogeneous liquid, and the plane cba shall be parallel to the base* CAB, *separated by the same interval Aa, immeasurable with respect to the whole height of the liquid* EA *or* MD [*i.e. Aa* is incremental]. *The inner cavity surface* C*ca*AB*b and the rectangle* BC*cb are pressed on by equal forces in opposite directions ; and the force itself, by which the surface and the rectangle are affected, will be equal to the weight of a prism of liquid, of which the base is the rectangle BbcC, and af the height ordinate of the graph of the weight EZα of the heterogeneous liquid, truly the same as* A*ω, the height of the liquid of uniform density equal to the mean density of the heterogeneous liqiud.* See Fig. 63.

The individual points of the curve *cab* sustain the same pressure, clearly equal to the weight of a filament of the homogeneous liquid (§.260), of which the height is the ordinate *af* of the graph of the weight, and the uniform density by the right line A*ω*, by which it represents the mean density of the heterogeneous density : and the individual points of the curve CAB undergo a pressure equal to the weight of the same filament (§.260.) as in the manner of a homogeneous liquid, of which the uniform density again shall be A*ω* , truly the height Aα the homologous ordinate of the graph of the pressures.



Indeed, because the difference *αg* of the ordinates *fa,* αA (following the hypothesis) is beyond measurement [*i.e.* incremental] and thus negligible in comparison with these ordinates, the individual points of the hollow surface C*a*B must be agreed to be affected by one and the same force, as the weight of a filament of liquid of height *af* or Aα and density Aω, and thus the force, which the said surfaces both curved and cylindrical experience from the fluid filled vessel (§. 63) is equivalent to the pressure, which the rectangle BC*cb* might undergo, if also the force *af* were to be applied to its individual points, and the weight on this rectangle is as *af.* BC.B*b,* or *af.*BC.A*a,* multiplied by Aω, since the weight of each filament *af* shall be as this filament, as a volume multiplied by the density or the specific gravity A*ω.* And the volume *af.*BC.B*b* multiplied by Aω denotes (§.33) the weight of the mass of fluid from this volume, as long as the volume is equal, of which the uniform density is equal to the mean density Aω of our heterogeneous liquid. Therefore the inner surface C*a*B and the rectangle C*b* are acted on by an equal force in opposite directions, and however great a size the weight of the prism of liquid shall be, the base of its prism shall be the infinitesimal rectangle B*c* subtending the curved surface C*a*B, truly the height *af* the ordinate of the weight EZα; and the homogeneous density Aω of the liquid shall be the mean density of the heterogeneous liquid. Q.E.D.

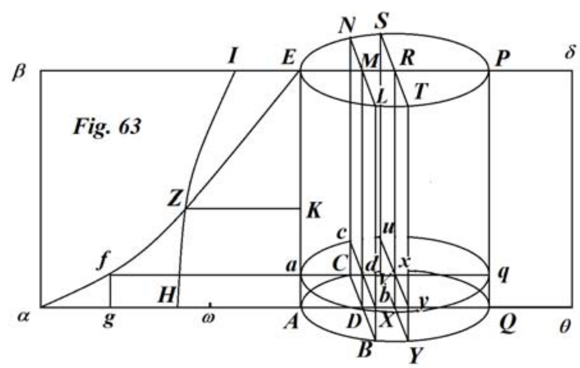
COROLLARY I.

265. The forces, by which the other inner surfaces C*cqyb*B and the subtending rectangle itself CB*b* are acted on in opposite directions, also are equal, if the same liquid were introduced into the whole of the prism EAQP, as by which the forces are set out by one and the same principle, evidently from the weight of the prism of liquid having the rectangular base CB*b* and the height *af*, of which the uniform density is the same as before, the mean density Aω of the heterogeneous fluid.

COROLLARY II.

266. Therefore the weight of the heterogeneous liquid acting on the whole inner surface CNEABL, or on the infinitesimal rectangle NCBL, likewise on the surface NCVQPTIB and again on the rectangle NCBL will be equivalent to the weight of a right prism of water, of which the base shall be the area of the three-sided figure EZαAE with the height CB, and the uniform density Aω.

For if the height AE of the vessel shall be divided into innumerable indefinitely small amounts A*a*, and if planes parallel to the horizontal are understood to be drawn through the individual points of the division *a*, such as *acqb*, by art. 264 any infinitesimal surface in the cavity *c*A*b* or *c*Q*b* and the elemental rectangle CB*b* undergo a pressure equal to the weight of the homogeneous liquid, of which the volume shall be a parallelepiped or right prism, of which the base shall be elemental rectangle CB*b* and the height *af*, evidently the homologous ordinate of the graph of the pressures EZα, and the density of which homogeneous liquid shall be Aω equal to the mean density of the homogeneous liquid of the vessel EQ or ECB, and the weight of the homogeneous liquid within the said volume may be expressed (§.33) by the product from the volume and the density of the liquid, which is Aω, or the mean density of the heterogeneous liquid, that is, by af.BC.Dd.Aω,= Aω.BC.incremental rect. *f*A [Note that *f*A is a vertical increment of the area, while BC is horizontal, so that their product is the volume of an elemental prism]; and the pressure of all the incremental surfaces C*a*B or of all C*q*B, which altogether are contained in CNELB, or in CNPLB, will be expressed by the product from Aω.BC by all the right lines *fA*, by which the areas EzαA can be inscribed, or because these increments of the inscribed area cease within that area itself, by the product from Aω.BC into the area EZαA. And this product (§.33) is equivalent to the weight which the homogeneous mass of liquid has, of which the density is Aω and the



volume is the prism from the base EZαA and the height BC. Therefore also the pressure of the heterogeneous liquid at the curved surface CNELB, or at CNPLBQ, is equivalent to the weight of the same homogeneous mass.

267. Truly the regions C*cab*B or C*cqb*B, the common height of which A*a* now may be put of finite magnitude, will undergo a pressure put in place from the weight of the liquid prism, of which the base shall be the trapezium or four lined figure *af*α*A*, with the height BC, and the uniform density *A*ω.

COROLLARY III.

268. From which if the plane EMDA crosses the rectangle NCBL at right angles at the middle position MD, at this plane the mean direction of the pressure will be found, which the individual points of the surfaces NAL and NQL undergo from the heterogeneous liquid, since in the said plane EMDA the middle shall be the direction of the pressure, which the individual points of the rectangle NCBL shall endure from the same heterogeneous liquid in the vessel, and the pressures of this liquid on the curved surfaces NAL and NQL shall be the same as the pressure, which the rectangle NCBL must endure. Therefore, if the centre of gravity of the area EZαA is considered to be drawn perpendicular to the rectangle NB, and the common section of which plane EMDA elongated shall be ZK, this KZ (§.54.) will be the mean direction of the weight of the liquid on the inner surface NAL. And now the point K can be called the c*entre of the pressure* or, *of the weight* of the heterogeneous liquid.

COROLLARY IV.

269. Hence the lateral pressure of the heterogenous liquid, which the inner surfaces NAL or NQL sustain, or also the equally high rectangle CBL, is to the absolute weight of the heterogeneous liquid NLBCE, as the product from the right prism, of which the base is the figure EZαA, and the height BC at Aω to the product from the prism NEABL by $A\omega$, or with this Aω omitted, just as that prism EZαA at BC to this prism NEABL. For (§.266.) the prism EZαA.BC multiplied into $A\omega$ indicates the pressure, which the curved surface CEB or CQPB experiences from the internal heterogenous liquid, and the prism NEABL taken by the same Aω (§§. 245 & 33) indicates the absolute weight of the heterogeneous liquid with the volume NEABL. Therefore, if the area EZαA were to the two lines BACB, the base of the prism or of the vessel NEAB, just as AE to BC, tge lateral pressure, which the inner surfaces NAL, NQL, or the rectangle NCB endure, will be equal to the weight of the absolute mass of the heterogeneous liquid NEAB; and that lateral pressure will be greater or less than this absolute weight, as the ratio of the three sided figure EZαA to the two sided figure BACB shall be in a greater or less ratio AE to BC.

COROLLARY V.

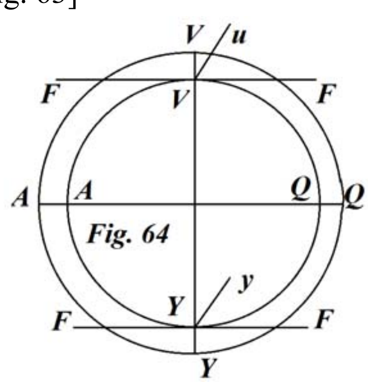
270. And thus, if our vessel may be placed to be the tube SAYPQ, in which the rectangle SVYT shall be the maximum right section of the base AYQV, the two parts of the inner surfaces SEAT and SPQT, with the adjacent rectangle SVYT, the *maximum* pressure will be experienced from the heterogeneous liquid, with the tube filled, and thus this liquid, with the parts of the tube considered trying in turn to separate themselves from each other, the *maximum* force will stretch out the lines SV and TY, at which the two parts SEAT and SPQT adhere to each other in turn ; thus so that, unless the tube [or pipe] shall be firm enough, whereby it may be able to resist the pressures of the liquid, to be broken at the lowest part of the lines SV or TY, or perhaps it may arise from cracks arising due to the pressure of the liquid. For since the product EZαA.VY.Aω expresses the lateral pressure, which the half-tube SAT or SQT endures, and on account of the maximum rectangle SVYT, of which the base VT is greater than the base CB of any other rectangle NCBL, the truth is apparent beyond doubt.

COROLLARY VI.

271. Therefore if once it may be established according to how the strengths of tubes proceed having various thicknesses, from the gathering together of these strengths with the weights of the liquids, the thickness of the tube or of the material will become known easily, from which the tube has been made, which it is required according to that, as a tube shall be able to bear the pressure from its contained liquid without the side rupturing. Truly the ratio of the strengths, exerted by two different tubes, will be shown in the following proposition.

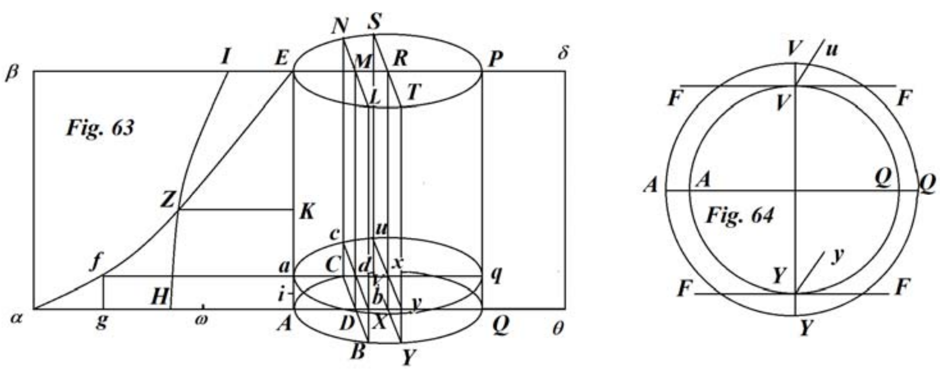
PROPOSITION VIII. THEOREM.

272. *The resistances or strengths of tubes are in a ratio composed from the ratio of the tenacity of the material to the tenacity of the material of the other, from the ratio of the thickness of the one to the thickness of other, and finally from the height of the first tube to the height of the other.* See Fig. 64. [*i.e.* a plan view of Fig. 63]



The tube, the opening of which shall be the interior figure AVQY, the thickness of the material, from which the tube has been made, VV, AA or QQ, and the height of the tube V*u* or Y*y*; the rectangles *u*VV & *y*YY will be the parts at which the half-tubes *u*A*y* and *u*Q*y* in turn are to be torn away from each other along the directions VF and YF by forces acting in the opposite directions, with themselves adhering to each other mutually. Now because they are joined together by so many chains, there are just as many physical points in these rectangles *u*VV and *y*YY, it is evident the resistance of the tube is equal to the force required for all the bonds or fibres to be torn apart; and the strength of this kind or the resistance of the tube is as the force, by which a single bond or fibre can be broken and the number of fibres joined together; truly the force, by which there is a need required to break a single fibre, is for us the tenacity of the material of the tube or at any rate proportional to this tenacity [This would now be considered in terms of the Young's modulus E.]; indeed the number of fibres is as the sum of the rectangles *u*VV and *y*YY, therefore by calling T the tenacity of the material of the tube [*i.e.* breaking stress or normal force per unit area E], the resistance of the same tube shall be as the product of T into twice the rectangle *u*VV, that is as 2.T.VV.V*u*, or also as 2.A.C.T, by calling A the height of the tube V*u*, and the either VV or YY the thickness C of the material. But thus in another tube, of which by calling the strength *f*, the thickness *c* of the material of the same tenacity *t*, and finally the height of the tube *a*, & F indicating equally the strength of the first tube, there will be *f* as 2*act*, and since a little before there was F as 2ACT, generally there will be $\mathrm{F} : f = \mathrm{TCA} : tca$. Q.E.D.

COROLLARY I.



273. Hence figures 63 and 64, are required to be examined jointly, because (§.267.) the pressure of the heterogeneous liquid, which is spread out into the parts *u*A*y* & *uQy*, is *afαA*.VY.*Aω*, this pressure also will be $= \mathrm{AMDS}$, by calling the height of the region *qua*ABQ, which is *Aa* above, as A, the four-lined figure $af\alpha\mathrm{A} = \mathrm{A.M}$, the diameter VY or $\mathrm{AQ} = \mathrm{D}$ and this $pr.\, u\mathrm{A}y + pr.\, u\mathrm{Q}y = 2.\mathrm{AMDS}$, on setting $\mathrm{A}\omega = \mathrm{S}$. Therefore by putting the pressure of the liquid in the region *a*AQ*q* to be exactly equal to its strength, or $\mathrm{F} = pr.\, u\mathrm{A}y + pr.\, u\mathrm{Q}y$, there will become (§.272.) $2.\mathrm{TCA} = 2.\mathrm{AMDS}$, or $\mathrm{CT} = \mathrm{MDS}$, and in another tube $ct = mds$,on indicating similar quantities by similar letters with capitals in the first and ordinary ones in the other. Therefore $\mathrm{CT} : ct = \mathrm{MDS} : mds$.

[Thus, $\mathrm{S} = \mathrm{M} \times \mathrm{A}\omega = pressure$ is proportional to $\frac{\mathrm{CT}}{\mathrm{D}} = \frac{thickness \times tenacity\ [i.e.\ breaking\ stress]}{diameter}$; the reader may wish to compare this result with Barlow's formula relating the internal

pressure P a pipe can withstand in terms of its dimensions and strength of material :
$P = \frac{2St}{D} = \frac{2 \times allowable\ stress \times thickness}{outer\ diameter}$.]

COROLLARY II.

274. Our pipes [or tubes] above may have circular openings, and contain homogeneous liquids, and in that case the line EZα makes a half right-angle with EA, thus so that there shall be $af = Ea$, truly $\mathrm{A}\omega$, by which S is called, designates the density or specific gravity of the liquid, and M will be E*i* evidently with *Aa* bisected at *i*; indeed since $\mathrm{M.A}a = af\alpha\mathrm{A}$, and in this case the four sided figure will become equal to the rectangular trapezium between E*i* and A*a*. With these in place the ratio of the preceding corollary will be changed into the following 1st $\mathrm{C} : c = (\mathrm{M.\ S.\ D} : \mathrm{T}) : (m.\ s.\ d : t.)$. That is, the thickness of the material in both pipes having the same tenacity with equally strong pressures of liquids, are as the product from the mean heights of the liquid and with the same specific gravities applied at the diameters of the openings to the tenacity of the material, from which the pipes have been made.

275. And thus if the liquids were of the same kind and the pipes were made of the same material, the thicknesses of equally strong pipes are in a ratio composed from the height of the liquids and the diameters of the tubes, that is in the ratio of the rectangles along the axes of the tubes.

276. IInd. $\mathrm{T} : t = (\mathrm{M.S.D} : \mathrm{C}) : (m.s.d : c)$. That is, the tenacities of the materials of the tubes of equal strength, are as the products from the mean heights of the liquids, the specific gravities of these, and the diameters of the pipes, to the thickness of the material, from which the pipes have been made.

SCHOLIUM.

277. So that the use of the preceding rule may become known, it pleases to bring forwards one or two examples. *Example* I. At the end of one of the works of the Royal Society of the Academy of Sciences Paris [1693], which is inscribed *Various mathematical and physical works of the members of the Royal Academy of Sciences*, there is on record a small dissertation by the Celebrated Dane Ole Romer [The Latin original can be downloaded from the *Gallica* website]: *Concerning the Thickness and Strengths of pipes in aquaducts, following the different heights of the sources, and the different diameters of the pipes*, in which the dissertation read from page 517 relates, a leaden pipe of diameter 16 inches, with a thickness of $6\frac{1}{3}$ lines (a line itself is the 12th part of an inch, or the 144th part of a Parisien foot) resisted sufficiently to bear the pressure from water 50 ft. in height assumed at one time from a certain experiment at Versailles, with which observation in place, the author was asked, of how great a thickness must another lead pipe be having an opening of 10 inches, so that it might bear a pressure of 40 ft. The rule

of the above article 275 will immediately satisfy that, since in this case the thicknesses shall be as the rectangles along the axis, for in the observed pipe by multiplying the height of 50 ft. of water by the diameter of the pipe 16 inches, the product 800 denotes the rectangle along the axis in the observed pipe, and by taking the height with the water 40 ft. by the diameter 10 inches of the other pipe, 400 arises for the rectangle along the axis for this pipe, which since it shall be only half of this 800, the thickness of this pipe according to the rule will be only half of the observed thickness in the pipe, which was $6\frac{1}{3}$ lines ; therefore the thickness sought of the other pipe will be only $3\frac{1}{4}$ lines, in place of the $4\frac{1}{2}$ lines approximately, which Mr. Romer had found, because he stated the resistance of the pipe to be in the square ratio of the thicknesses with everything else being the same, which in the above proposition appeared to be in the simple ratio and not in that square ratio.

278. *Example.* II. Mariott considers, as will be found in the same work which I have mentioned in the previous example, following page 513, a copper pipe six inches in diameter can support a water pressure of 30 ft. with a thickness of half a line. We may place these resistances of lead and copper pipes with the water pressure exactly equal, and it shall be required to find the proportion of the tenacity of the lead and copper. In the above formula (§.176.) with S and *s* omitted as it arises from the equality $\mathrm{T}:t=\left(\mathrm{M.D}:\mathrm{C}\right):\left(m.d:c\right)$ where the greater letters refer to the lead and the lesser ones to the copper, hence with the numbers nominated 50, 16, $6\frac{1}{3}$ arranged in place of M, D, C and with the numbers 30, 6 and $\frac{1}{2}$ in place of *m, d, c* ; there will be found $\mathrm{T}:t=\frac{2400}{19}:360 \;=240:684=20:57$, and thus the tenacity of lead becomes, according to these observations, a little greater than a third of the tenacity of copper [The Young's Modulus ratio lead : copper is approximately half of this amount].

PROPOSITION IX. THEOREM.

279. *If the parts* BAE, SAE *of the vessel* BAS, *full of some heterogeneous liquid, may be considered as an example of a vessel with expandable sides able to be moved away from the vertex* A, Fig.23 & 24 .
[Note :The upper vessels are considered to be filled with heterogeneous liquids, while the lower ones have the equivalent mean density. Figure 24 has an ambient fluid in the complementary chambers definded by the outer rectangles. These final theorems seem to have been inspired by an attempt at an perpetual motion machine based on a set of belows, an idea due originally to Denis Papus.]
The analogous volumes of the same parts OB*b*A, OS*s*A *may be indicated by the same signs* δ, χ, *or the analogous volumes of their complements,* 2B2A2E, 2S2A2E, *or* B2B2*b*2A, S2S2*s*2A, *as the centres of gravity of the analogous volumes of these also are designated.*
[Recall Hermann's convention, the name of a point and its value are the same, such as the position and value of the centre of gravity of a volume or of a mass.]

The area of the pseudo-wedge LPQ, *of the graph of the pressure LrQ of each side, may be designated by the symbol* ω *for each side of the common vessel, by which the centre of gravity of the same pseudo-wedge also may be designated.*
Certain forces, equal to the weights of the masses δ, χ *of the same homogeneous liquid, of which the density shall be the same as the mean density PV of the liquids in the heterogeneous vessel, act in their homologous directions* $\alpha\delta$, $\xi\chi$ *or* $\delta\chi$ *and* $\chi\xi$ *passing through the centres of gravity of the volumes* δ, χ *and applied normal to the plane BS of the vessel.*
Other forces γ and ω *equal to each other, as being equal to the single mean density of the heterogeneous fluid, are applied to the vessel in the directions* bu, su *, parallel to the plane BS, and passing through the centre of gravity* ω *of the graph.* [We are considering the hydrostatic case at present, for the lower homogeneous case] : *these four forces* δ, χ, γ and ω *are acting at the same time, evidently at any rate along their directions, exert the same force to be deduced for the sides of the vessel, as the same vessel had filled with heterogeneous liquid.*

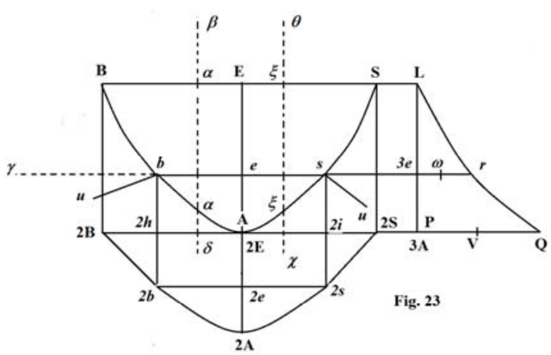


Fig. 23

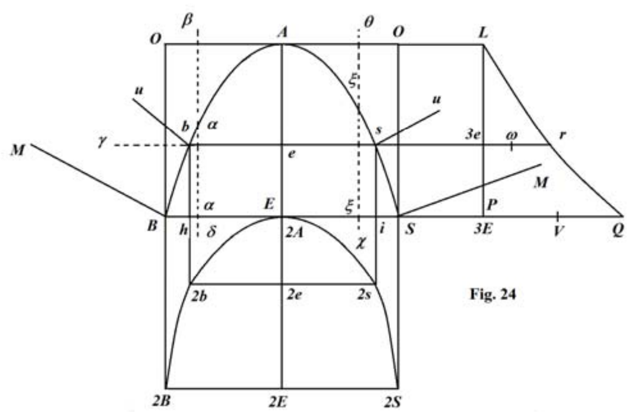


Fig. 24

This proposition is only a special case of the first corollary of Prop. X of the first book, which now generally in this proposition is called the graph of the applied forces allowed to be applied perpendicularly to the figure, now this graph [the Latin *scala* meaning *ladder* is used here, which we have called graph to be less pedantic] of the weights L*r*Q [*i.e.* pressures ; note that the graph essentially is one of weight of liquid, which can be converted into one of the pressure of the same shape, on multiplying the ordinate by the mean density of the fluid], since any point *b* on the surface A*b*B is forced along the direction *bu* perpendicular to the surface (§.260) by a force equal to the weight of a filament in turn equal to the volume of the liquid 3*er* by the mean density PV of the heterogeneous liquid, because the weight of the filament *r3e* is expressed by the factor *r3e*.PV (§.33) evidently from the volume by the density of the liquid, and thus the pressures of the liquid in the cavity of the vessel BAS maintain the same surface and the forces applied at the individual points of the surface, which the respective ordinates *r3e* of the pressures graph L*r*Q multiplied by the given PV designate. Therefore (§.81) the heterogeneous liquid of the vessel will provide the same effect, as the forces expressed by the volume $\delta, \chi, \&\ \omega$ multiplied by PV, and thus equal to the weight of the liquid of homogeneous density PV under these volumes $\delta,\ \chi,\ \omega$ applied to the volume BAS in the directions $\alpha\delta, \xi\chi$ or $\delta\alpha, \chi\xi$ and $b\gamma, s\omega$. Q.E.D.

COROLLARY I.

280. And thus the moments of the deformable sides of the vessel of the heterogeneous fluid exerted about the fulcrum A (§. 82.) by the side BAE will be $\mathrm{PV.}\ \delta.\alpha\mathrm{E}\ + PV.\gamma.\ \mathrm{A}e$, and for the side SAE they will be $\mathrm{PV.}\ \chi.\xi\mathrm{E} +\ \mathrm{PV.}\omega.\mathrm{A}e$. For these products $\mathrm{PV.}\delta, \mathrm{PV.}\gamma, \mathrm{PV.}\omega, \mathrm{PV.}\chi$ indicate the weights of the liquid having the density PV with the volumes $\delta, \gamma, \omega, \&\ \chi$.

COROLLARY II.

281. Just as in the preceding corollary §. 263 it prevails also, when the sides of the vessel are pressed out by some external ambient fluid, but with the directions of the pressures interchanged, that is , as if the pressures of the internal liquid of the sides of the vessel were trying to move away from each other; thus with the pressure of the ambient fluid interchanged it shall be the case also, that the same parts may be more strongly pressed together. From which the same formulas, by which the moments of the internal liquid designated in the preceding corollary were trying to spread out from the sides of the vessel, as equally the moments of the ambient liquid may be trying to compress the sides of the ambient liquid, but where the graphs of the exterior weights of the liquid apply.

SCHOLIUM.

And thus, towards estimating the pressures of liquids in deformable vessels, the whole difficulty may be reduced to this, that a rule may be found for finding the moments of the volumes δ and χ with respect of the right plane AE, and of the wedge-shape ω with respect to the plane PQ, or at once also with respect of the plane OAO of Fig. 24, evidently whenever the vertex A of the vessel may be considered upwards. Truly the calculation can be reduced easily to the quadratures of figures, as may be seen in the following two theorems:

PROPOSITION X. GEOMETRICAL THEOREM.

282. Fig.27. *A geometrical theorem of any volume* CBAD : *the sections of which cbd shall be parallel to the base* CBD*, and the volume composed of similar and similarly placed figures.*
[The volume is one of the wings or expandable sides of the deformable body above; it has the central vertical plane face C*c*A*d*D normal ot the line BEF from the vertex A, which is horizontal, while EA is in the vertical plane with the rectangle EMNA. CBD is the upper plane face, at right angles to C*c*A*d*D, while the other elemental sections *cdb* are parallel to this upper face. As situated, the solid is viewed in perspective, but the graph is drawn in the plane of the page, and its area may be considered as representing a form of the moment of inertia of the solid, and thus may be considered as a forerunner to Euler's work many years later in the different context of a body rotating about an axis. These diagrams are of interest also, as Hermann tried to adapt old fashioned geometric ideas to the then modern analytical ideas, one of which being how to represent Descartes' two dimensional graphs in three dimensions; as one can see, the outcomes were not entirely successful.]
The moment of the volume with respect to the plane CAD *of the right base* CBD*, will be equal to the product formed from a certain three-line form* AEF*f*A *by the rectangle under* EF, [*i.e.* the integral formed from the bilinear area elements by the distance of the centre of mass or gravity of each elemental plane from the plane CAD, in turn is equal to the distance of the centre of gravity of the whole body by the volume of the shape, taken as an integral;]
as we put EF *equal to* BE *itself, normal to* CD*, and with a given right line* D [, which has nothing to do with the point D, is not indicated on the original diagram, and which can be evaluated for special cases] *of that magnitude, so that the parallelepiped under the square of the right line* BE *or* EF *and this given* D*, will be made equal to the volume from the bilinear form* CBD *by* XE*, with this distance* XE *being the distance of the centre of gravity* X *of the bilinear form* CBD *from its base* CD*. Truly the trilinear form itself shall be described by this following rule: that the ordinates of that* EF*, ef shall be in the cubic proportion of the ordinates themselves in the direction of* BE*, be, put in place in the figure* A*b*BE*, by which the plane* CAD *is at right angles and crossing the axis* AE *of the figure* CAD*.*

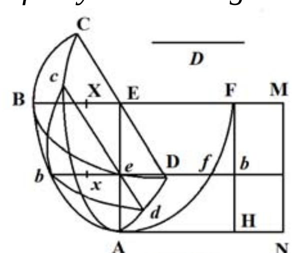


Fig. 27

[Thus, by forming a ratio for similar figures similarly placed, the introduction of common scaling factors such as fractions of π are avoided, as for semi-circular shapes.]

Let *x* be the centre of gravity of the figure *cbd,* and *xe* will be the distance of its centre from the base *cd* for the bilinear form *cbd* in the plane of the figure A*b*BE, since the figures CBD, *cbd* (following the hypothesis) are similar and similarly placed, and thus there will be $\text{CBDC.XE} : cbdc.xe = \text{BE}^3 : be^3$ that is by construction $= \text{EF} : ef = \text{D.EF}^2 : \text{D.EF}.ef$; but (from the construction) D.EF^2 or D.BE^2 is equal to the volume from CBDC by XE, therefore also D.EF.*ef* will be equal to the volume from *cbdc* by *xe,* or to the moment of the bilinear form *cbd* hung from the axis *cd*. Now, because the moment of the whole solid CBAD with respect to the plane CAD is equal to the moments of all of the bilinear formes *cbdc,* whch are contained in this volume, that is, with all the *cbdc* in *xe,* likewise the moment of the volume also will be equal to all the D.EF.*ef* ; of which the individual terms will be equal to the individual terms *cbdc.xe* ; but all the D.EF.*ef* = D.EF. all the *ef* = D.EF.area EA*f*F [*i.e.* the mean value of the sum or integral ; an example is provided below]. Therefore the moment of the volume CBAD is equal to the product from the area *EAfF* by the rectangle D.EF. Q.E.D.

COROLLARY I.

283. And thus the moment of the complement of the solid CBAD with respect to its plane CAD, or of the solid remaining CDBG, after the removal of CBAD itself from the right prism and with an equal height, will be equal to the three-sided figure AFH by the rectangle D.EF. For the moment of the prism CDBG is the product from the right line EH by the rectangle D.EF, and the moment of the volume CBAD is equal to the product AEF*f*A.D.EF, and thus the moment of the complement of this solid CBAD is equal to the entire product made from the right line EH, with the trilinear form EAF removed, by the rectangle D.EF, that is , from the product AFH by D.EF.

COROLLARY II.

284. From which, if our solid CBAD were the analogue to the solid BAE treated in the preceding proposition, and thus it shall have the same volume δ, in this case there will be $\delta.\alpha\mathrm{E}$ or the moment of the solid $\delta.\alpha\mathrm{E} = \mathrm{EA}f\mathrm{F.D.EF}$, in the case of the figure 23 where the vertex of the volume BAS is considered downwards; and $\delta.\alpha\mathrm{E} = \mathrm{HF}f\mathrm{A.D.EF}$ in the case of figure 24, where the vertex is considered upwards.

PROPOSITION XI. A GEOMETRICAL THEOREM.

285. Fig. 28. *If a certain curved prism on the base section* CAD *may be considered as the surface* Q*r*LCD, *thence a kind of wedge may be generated* QACD, *of which the edge will be on the line* CD, *and the shape of the curve adjacent to* CD *arises from a motion parallel to the line* CD, *thus so that the point* E *proves to be fixed always on the curve* L*r*Q *while it is moved* [Thus CD is the third axis of the surface coming out of the plane of the page, while E can be regarded as the origin.], *and the line* CD *shall be perpendicular to the plane LPQ, of which the moment of the wedge of all the other areas* CAD *with respect to the plane* PQ *at right angles will be equal to the product of the area, or of the bilinear shape,* EiA *by the rectangle* AE.CD; *where any bilinear ordinate ei will be to the homologous ordinate er of the figure* PLQ, *as the rectangle of the coordinates* A*e*, *cd in the figure* CAD, *shall be to the given rectangle* AE.CD.

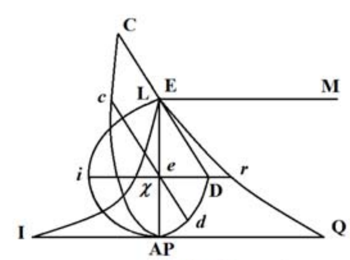


Fig. 28

For since (following the hypothesis) $ei : er = Ae.cd : AE.CD$, then $Ae.cd.er$, that is the moment of the rectangle *cd.er* by the distance A*e* appended to the plane PQ, $=$ to the volume $AE.CD.ei$. And the sum of the moments of all the rectangles *cd.er* contained in the pseudo-wedge are equivalent to the moment of this pseudo-wedge, therefore also to the sum of all the products EA.CD.*ei,* that is, the product from the rectangle AE.CD by the two-line form E*i*A*,* is equal to the moment of the pseudo-wedge QACD. Q.E.D.

[There does not appear to be any physical reason for this theorem, and it seems to be just a useful device in calculations. In any case, the assumption that the static pressure at some depth depends in some way on the width of the fluid in the vessel is quite fallacious; and of course, one cannot take moments in a liquid , as the author has done above, to equate pressures.]

COROLLARY I.

286. Whenever the vertex A of the figure CAD is pointed downwards, and thus it agrees with the point P, so also will the curve E*i*A pass through the points L and P, thus so that none of its ordinates shall be at these points. But if the A of the same figure CAD is seen to be upwards, thus so that the base CD may pass through the point P and the vertex may be joined with the point L, the curve L*q*I likewise will pass through the point L, truly not through the point P, since its ordinate at this point PI shall become equal to PQ itself or PL; since if CD is at P and A at L, the ratio $e\chi : er = Ae.cd : AE.CD$, becomes $PI : PQ = AE.CD : AE.CD$, and thus $PI = PQ = PL$, where clearly A*e* were made equal to AE. And in this case the moment of the volume with respect to the plane through L, by passing through the one parallel to PQ, being equally the product of the rectangle AE.CD by the area PLχI.

COROLLARY II.

287. Hence, if the figure CAD were a right section undergone by the volume BAS of the figures 23 & 24, and the curve L*r*Q is the graph of the pressure of the heterogeneous liquid, the volume QPCD will be the pseudo-wedge of the graph of the pressures, and its moment, or *w*.A*e,* in the case of figure 23, $= AE.CD.LiPL$, and ω.A*e* in the case of figure, $= AE.CD.L\chi IP$.

SCHOLIUM.

288. Therefore it may be apparent, in the computation of the moments of liquids in vessels with moveable walls, all of the pressures to be deduced according to an investigation of the areas AEF*f*A, L*i*P and LχIP in figures 27*,* 28 which always are able to be performed in particular examples, if not algebraically, then perhaps transcendentally. See Fig. 23, 24, 27. For if the vessel BAS shall be a cone filled by a homogeneous liquid, thus so that the curve L*r*Q will become a straight line, in which case

the volumes δ and χ will be similar and equal to the volumes BAE and SAE, and the volume CBAD from half a right cone, in which since there shall be

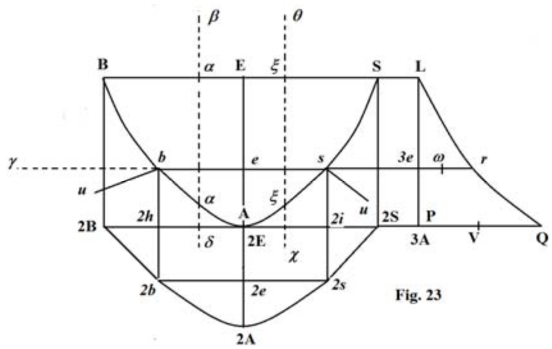


Fig. 23

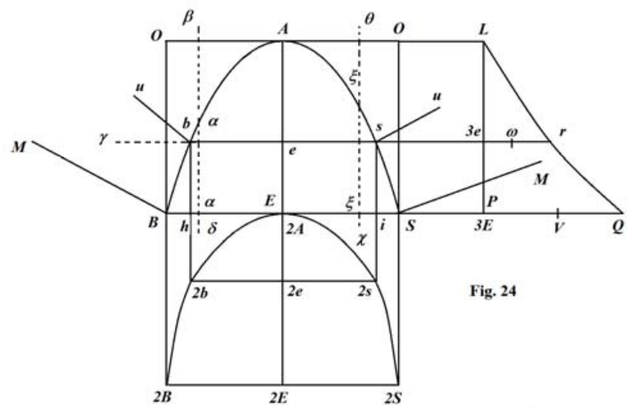


Fig. 24

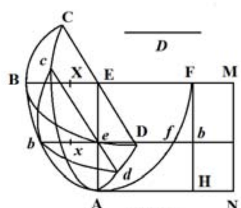


Fig. 27

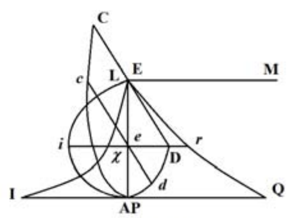


Fig. 28

$$\mathrm{EA}^3 : e\mathrm{A}^3 \left(= \mathrm{BE}^3 : be^3\right) = \mathrm{EF} : ef \ ,$$

it is apparent the curve A*f*F in this case to become a cubic parabola, and thus the [area of the] three-lined shape EAF will be $= \frac{1}{4}\mathrm{AE.EF}$ and $\mathrm{AFH} = \frac{1}{4}\mathrm{AE.EF}$. And because CBD is a semicircle, the centre of which is at E, there will be $\mathrm{CBDC.XE} = \frac{2}{3}\mathrm{BE}^3 = \mathrm{D.BE}^2$; and thus $\mathrm{D} = \frac{2}{3}\mathrm{BE}$. Hence (§.284)

$$\delta.\alpha\mathrm{E} = \chi.\xi\mathrm{E} = \mathrm{D.EF.AEF} = \tfrac{2}{3}\mathrm{BE.EF.AEFA} = \tfrac{2}{3}\mathrm{BE.EF}.\tfrac{1}{4}\mathrm{AE.EF} = \tfrac{1}{4}\mathrm{AE.BE}^3.$$

Fig. 28. Again with the right line L*r*Q present and $\mathrm{LP} = \mathrm{PQ}$, also there will be $\mathrm{L}e = er$, and thus everywhere : from which since (§.285) *ei* shall be to *er* as *Ae.cd* to AE.CD , everywhere the ordinate $ei = \mathrm{A}e.\ e\mathrm{E}.cd : \mathrm{AE.CD} = \mathrm{A}e^2.e\mathrm{E} : \mathrm{AE}^2$, by substituting the ratio *cd*:CD equal to the ratio A*e*:AE on account of the similar triangles A*cd* & ACD, since in the present case the figure ACD is a triangle. Hence the bilinear form may be found by the most noteworthy method of quadrature, $\mathrm{E}i\mathrm{A} = \frac{1}{12}\mathrm{AE}^2$: hence in the case of the figure 23, there will be found (§.287) $\omega.\mathrm{A}e (= \mathrm{AE.CD.L}i\mathrm{AL}) = \mathrm{AE.CD}.\ \frac{1}{12}\mathrm{AE}^2$, or, because CD is just as great as $2\mathrm{BE}, = \frac{1}{6}\mathrm{BE.AE}^3$.

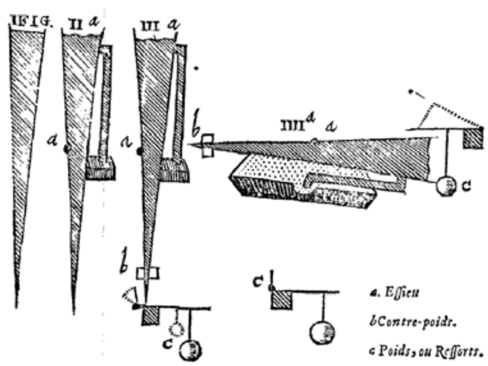
Therefore there will be :

$$\mathrm{PV}.\delta.\alpha\mathrm{E}+\mathrm{PV}.\omega.\mathrm{A}e=\tfrac{1}{6}.\mathrm{PV.AE.\ BE}^3+\tfrac{1}{6}.\mathrm{PV.\ BE.AE}^3$$
$$=\tfrac{1}{6}.\mathrm{PV.AE.\ BE.AB}^2=\mathrm{PV}.\chi.\xi\mathrm{E}+\mathrm{PV}.\omega.\mathrm{A}e$$

because in the present case the sides BAE and SAE, as the halves of the right cone BAS, are similar and equal.

289. If the cone BAS were placed to be pressed on by the air externally, and the height of the cone AE itself were sought, in assumption of the equilibrium of the pressure of the internal liquid in the cavity of the cone with the pressure of the external air on the convex surface, in this case the graph of the pressure L*r*Q will be a right line LP parallel to the axis or to AE, of which the distance from this line AE shall be PQ in the figures 23, 24, and 28, and EM or AN in figure 27. In this case the volume δ or χ will be a prism of the base CBDC and of the height EM or PQ, and the moment of this prism or $\delta.\alpha\mathrm{E}$ now becomes $=\mathrm{CBDC.XE.EM}$ (or as it has been found above $\mathrm{CBDC.XE}=\tfrac{2}{3}\mathrm{BE}^3)=\tfrac{2}{3}\mathrm{EM.\ BE}^3$. In place of the pseudo-wedge ω we will now have a prism with the base CAD and with the height PQ, from which, because generally there is $ei:er=\mathrm{A}e.\ cd:\mathrm{AE.CD}$ (or on account of the similar triangles CAD and *c*A*d*) $=\mathrm{A}e^2:\mathrm{AE}^2$, and therefore *er* equals PQ, since the line L*r*Q shall be supposed to be parallel to EA itself, and thus passing through Q, there will be $ei=\mathrm{A}e^2.\mathrm{PQ}:\mathrm{AE}^2$, and in this case the curve E*i*A shall be the parabola of the cone, of which the parameter is $\mathrm{AE}^2:\mathrm{PQ}$, thus so that the area E*i*AE in this case is going to become $=\tfrac{1}{3}\mathrm{AE.PQ}=\tfrac{1}{3}\mathrm{AE.EM}$; and thus
$\gamma.\mathrm{A}e=\omega.\mathrm{A}e(=\mathrm{AE.CD.E}i\mathrm{AE})=\tfrac{1}{3}\mathrm{AE}^2.\mathrm{CD.EM}=\tfrac{2}{3}\ \mathrm{BE.EM.AE}^2$. Therefore by calling the mean density of the atmosphere Z, the sum of the moments of the air pressure or $\mathrm{Z}.\delta.\alpha\mathrm{E}+\mathrm{Z}.\omega.\mathrm{A}e=\tfrac{2}{3}\mathrm{Z.\ BE.EM.BE}^2+\tfrac{2}{3}\mathrm{Z.BE.EM.AE}^2=\tfrac{2}{3}\mathrm{Z.BE.EM.AB}^2$.
Hence, because (following the hypothesis) in the case of equilibrium $\mathrm{PV}.\delta.\alpha\mathrm{E}\ +\ \mathrm{PV}.\omega.\mathrm{A}e=\mathrm{Z}.\delta.\alpha\mathrm{E}+Z.\omega.\mathrm{A}e$, also there will be $\tfrac{1}{6}\mathrm{PV.AE.BE.AB}^2=\tfrac{2}{3}\mathrm{Z.BE.BM.AB}^2$, and thus $\mathrm{PV.AE}=4.Z.\mathrm{BM}$. For we have found above, $\mathrm{PV}.\delta.\alpha\mathrm{E}+\mathrm{PV}.\omega.\mathrm{A}e=\tfrac{1}{6}.\mathrm{PV.AE.\ BE.AB}^2$, and a little before for the moments of the external air, or for $\mathrm{Z}.\delta.\alpha\mathrm{E}+\mathrm{Z}.\omega.\mathrm{A}e$ we have found $\tfrac{2}{3}\mathrm{Z.BE.BM.AB}^2$. But PV.AE denotes the weight of the column of liquid, of which the height is AE and the mean density PV, & Z.BM the pressure of the whole atmosphere, which we will indicate more simply be *p* ; therefore if the conical vessel BAS shall be full of mercury, it will require a height AE to be four times greater than the height of a column of mercury equivalent to the pressure of the atmosphere, and thus a minimum of 120 inches, since the weight of the atmosphere may be equivalent to 30 inches of mercury, towards obtaining an equilibrium between the external atmospheric pressure and the internal weight of the conical vessel of mercury from the parts in turn trying to move apart.

Truly if the vertex of the cone shall be pointed up, as in fig. 24, in that case the preceding theorems prevail $PV.AE = \frac{4}{3}Z.BM$ for the case of the equilibrium between the external pressure of the air and the internal pressure of the mercury, and thus for this equilibrium being obtained now it will suffice for a height AE of 6o inches. And these determinations are in agreement according to the precision with these, which the Celebrated James Bernoulli published on this matter in the *Actis Erudit. Lips.* 1686 and 1687, on the occasion of some proposition of Papin in the Nov.1685 issue about a perpetual motion machine in the *Nouvelles de la Republique des Lettres*, which consisted of a certain kind of triangular bellows, of both sides moveable about a horizontal axis fixed to the middle part, requiring to be filled up only with mercury, but only also to be moved in the right position in some manner as far as to be drained again. [See below.] The deficiency of this machine to be shown in a most elegant manner by the most praiseworthy James Bernoulli from the principles of hydrostatics and the law of the lever. Truly Papin, even if the approach may seem to be in doubt and the impossibility of its motion in some way may be shown from the imperfect judgment of the pressure, yet nevertheless the Bernoulli discussion of the machine judged the motion to be coming to an end ; by attributing a parallel motion to the sides, by which the account of the levers, by which Bernoulli had made his assault, to be going to cease, thus, Papin had considered matters wrongly. Truly the laws, by which the preceding machine above were moved about in each part, are these cited in the above *Actis Lips* article.



[Je suppose un soufflet de 40 pouces, qu'on l'ouvre seulement d'un tiers ou d'un quart; qu'on remplisse de Mercure cette capacité, sans y laisser d'air ni de communication avec l'air, qu'on tienne le soufflet tout droit la pointe en bas, la base en haut. Suivant l'experience de Torricelli, le Mercure doit quitter ces 40 pouces verticaux pour descendre à 27 pouces, ou environ, il ne le peut faire sans dilater davantage le soufflet, & laisser en haut un vuide de 10 ou 12 pouces. Vers le milieu du soufflet mettez un vase plein de Mercure. Il ne fera éloigné du haut de ce vuide que de 20 ou 22 pouces. Joignez donc ce vuide & ce vase par un tube de 20 ou 22 pouces plein de Mercure aussi. Suivant la même experience le Mercure du vase sera poussé par l'air dans ce vuide, & il le remplira entiérement, puisqu'il n'y a que 20 ou 22 pouces à monter. Le soufflet se dilatera & se remplira encore s'il le peut, parceque c'est toûjours la même raison.

Suspendez le soufflet par le milieu d'une de des ailes sur un essieu horizontal. La moitié d'en haut est beaucoup plus pesante que celle d'en bas, parcè que l'on donne au soufflet la figure de pyramide, dont la base est en haut & la pointe en bas. Ajoûtez donc à cette pointe un contrepoids. Là base ne descendra point qu'elle ne fait plus forte que la pointe & le contrepoids ensemble. Ajoûtez à cela un ressort, ou un poids attaché hors de la machine , lequel arrêtera la pointe, il faudra que la base du soufflet se remplisse suffisamment pour vaincre la pointe, le contrepoids & la détente du resort. Mais quand une fois la pointe sera dégagée de ce resort, ou du poids , la base en sera plus forte que la pointe & le contrepoid seuls , & par conséquent hors de danger de s'arrêter en son mouvement.

Que le rebord du vase ou quelque verge de fer arrête la base du soufflet, lorsqu'il fera devenu horizontal ; que le bout du tube fait tellement recourbé qu'il trempe encore dans le vase, & que ce bout & ce vase soient au dessous de tout le soufflet. Le Mercure sortira du soufflet par ce tube pour se décharger dans ce vase, parce qu'il ne fait ainsi que descendre, le soufflet se comprimera & se vuidera toujours tant qu'il sera horizontal. Que la base soi arrêtée par un ressort, ou un poids, le soufflet continuera à se vuider, jusqu'à ce que le contrepoids soit de vesu plus fort que la base & le ressort ou le poids. Alors le contrepoids l'emportera , dégagera la base du ressort ou du poids, deviendra plus fort que la base seule , remettra promptement le soufflet à là situation verticale, & engagera la pointe dans le ressort ou le poids d'en bas. Là le soufflet se remplira comme la premiére fois jusqu'à ce que la base soit plus forte que la pointe, le contrepoids,le ressort ou le poids d'en bas. Alors la base l'emportera, reviendra à la ligne horizontale, s'engagera dans le ressort ou le poids du milieu , le soufflet se vuidera & ainsi toujours.

On voit assez que la même chose se serait avec l'eau, en gardant la projection de l'eau au Mercure. Peut-être même que l'eau par le seul équilibre des liqueurs le feroit encore dans une màchine de quatre ou cinq pieds surtout si l'on aidoit son action par un contre poids.]

CAPUT II.

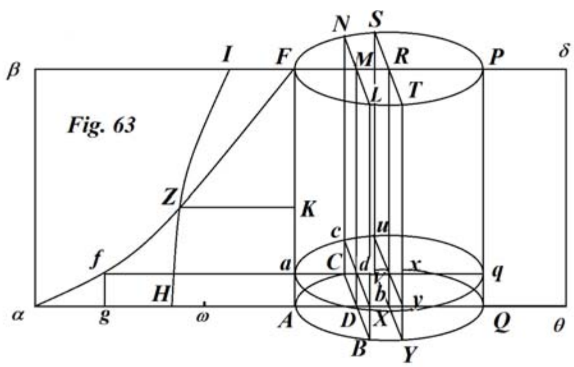
De gravitationibus Liquorum in Vasorum latera, & de Tuborum firmitatibus requisitis ad perferendas liquorum pressiones.

In Capite proxime antecedenti eas solum gravitationes liquorum contemplati sumus, quae in plana horizontalia exeruntur; in hoc vero examinandae veniunt pressiones liquorum laterales quibus parietes seu spondae vasorum afficiuntur. Talis enim indago praebebit regulam pro definiendis tuborum firmitatibus ad resistendum aqua pressionibus & viribus, quibus liquores vasorum latera ab invicem diducere conantur.

PROPOSITIO VI. THEOREMA.

263. *Vasis vel tubi* EAQP *ex utraque parte* ETPS & AYQ *aperti, & liquori* $\beta\alpha\theta\delta$ *immissi usque ad* ETP, *superficies externa & convexa ab ambiente liquore eadem vi introrsum versus axem premetur, quea cava vasis superficies extrorsum urgetur ab interno vasis liquore.*

Nam si loco vasis EAQP intelligatur moles liquoris ejusdem constitutionis cum liquore ambiente, ejusdemque figurae, magnitudinis & positioni cum vase ; quia (secundum



hypothesin) omnia sunt in statu manenti, massae liquidae EAQP superficies extrema eadem vi ab ambiente liquore $\beta\alpha\theta\delta$ urgebitur, qua superficies ejus cava a liquore EAYQP*;* alioqui, si alterutra pressio alteri praevaleret, partes liquoris ad motum concitarentur, nec proinde aqua vel liquor foret in statu manenti, contra hypothesin. Jam eandem impressionem exercebit liquor ambiens in superficiem vasis rigidam convexam, quam in similem aequalem & similiter positam superficiem aqua EAYQP; ac pariter hic liquor vel aquae in suam cavam superficiem eandem impressionem exseret, quam exsereret in similem & aequalem superficiem cavam, sed rigidam vasis. Ergo externa & interna vasis superficies a liquore ambiente & interno, in vasis cavitate existente, aequalibus viribus in oppositas partes, juxta directiones superficiei vel superficiebus perpendiculares, urgentur. Quid erat demonstrandum.

PROPOSITIO VII. THEOREMA.

264. *Si Prisma rectum* NEABL *terminatum superficie curva* NCEBL, *rectangulo* NLBC *& duabus figuris similibus ac aequalibus quibuscunque* BAC, LEN, *cujuslibet liquoris heterogenei plenum sit, & planum* cba *basi* CAB *parallelum, ab eadem distet intervallo Aa, insensibili respectu totius liquoris altitudinis* EA *vel* MD*. Superficies cava* CcaABb *& rectangulum* BCcb *viribus aequalibus in partes directe oppositas prementur; ipsaque potentia, qua superficies & rectangulum afficiuntur, aequabitur ponderi prismatis liquidi, cujus basis est rectangulum BbcC, & altitudo ordinata* af *scalae gravitationum EZ*α *liquoris heterogenei, densitas vero uniformis* $A\omega$ *eadem cum media densitate ejusdem liquoris heterogenei.* Fig. 63.

Singula curvae *cab* puncta eandem pressionem sustinent, aequalem (§.260.) scilicet ponderi filamenti liquoris homogenei, cujus altitudo est *af* ordinata scalae gravitationum, & densitas uniformis recta $A\omega$, qua liquoris heterogenei mediam densitatem repraesentat: & singula puncta curvae CAB pressionem subeunt aequalem ponderi (§.260.) filamenti ejusdem ac modo liquoris homogenei, cujus densitas uniformis sit iterum $A\omega$, altitudo vero Aα homologa ordinata scalae pressionum. Verum, quia differentia αg ordinatarum *fa,* αA (secundum hypothesin) insensibilis est atque adeo contemnenda prae his ordinatis, singula puncta superficiei cavae C*a*B una eademque potentia, quam exponit pondus filamenti liquidi altitudinis *af* vel Aα & densitatis $A\omega$ affici censeri debent, adeoque impressio, quam a fluido vasi indito patietur dicta superficies curva & cylindrica (§. 63.) aequivalet pressioni, quam subiret rectangulum BC*cb,* si etiam singulis ejus punctis potentia *af* applicata esset, atque gravitatio in hoc rectangulum est ut *af*. BC.B*b,* vel *af*. BC. A*a,* ductum in $A\omega$, cum pondus cujusque filamenti *af* sit ut hoc filamentum, tanquam volumen, ductum in densitatem seu specificam gravitatem $A\omega$. Atqui solidum *af*. BC. B*b* ductum in $A\omega$ denotat (§.33) pondus massae fluidae solido isti quoad volumen aequali, cujus uniformis densitas aequetur mediae densitati $A\omega$ liquoris nostri heterogenei. Ergo superficies cava C*a*B & rectangulum C*b* aequali vi in oppositas partes urgentur, & tanta quantum est pondus prismatis liquoris homogenei, cujus prismatis basis sit ipsum rec-lum B*c* superficiem curvam C*a*B subtendens, altitudo vero *af* ordinata scalae gravitationem EZα; & liquoris homogenei densitas sit media densitas $A\omega$ liquoris heterogenei. Quod erat demonstrandum.

COROLLARIUM I.

265. Vires, quibus altera superficies cava C*cqyb*B & rectangulum ipsam subtendens CB*b* in oppositas partes urgentur, etiam aequales erunt, si toti prismati EAQP idem liquor heterogeneus inditus fuerit, utpote qua vires exponuntur una eademque re, scilicet pondere prismatis liquidi basin rectangulam CB*b* & altitudinem *af* habentis, cujus densitas uniformis eadem est ac prius, media densitas $A\omega$ fluidi heterogenei.

COROLLARIUM II.

266. Propterea gravitatio liquoris heterogenei in totam superficiem cavam CNEABL vel in rec-lum NCBL, item in superficiem NCVQPTIB & idem rectangulum NCBL aequivalebit ponderi prismatis recti & liquidi, cujus basis sit area seu trilineum EZαAE altitudo CB, densitasque uniformis $A\omega$.

Nam si altitudo AE vasis in innumeras particulas indefinite parvas qualis A*a* divisa, & per singula divisionum puncta *a* plana horizonti parallela, quale *acqb,* ducta intelligantur, per art. 264 qualibet superficiecula cava *c*A*b* vel *c*Q*b* & rec-lum CB*b* pressionem subibunt aequalem ponderi liquoris homogenei, cujus volumen sit parallelepipedum seu prisma rectum, cujus basis rec-lum CB*b* ac altitudo *af,* scilicet homologa ordinata scalae pressionum EZα, & cujus liquoris homogenei densitas sit $A\omega$ par mediae densitati liquoris heterogenei vasis EQ vel ECB, atque pondus liquoris homogenei sub dicto volumine (§.33.) exponitur factum & volumine in densitatem liquoris, quae est $A\omega$, seu media densitas liquoris heterogenei, hoc est per af.BC.Dd.Aω, = Aω.BC.rec-lum fA ; & pressio omnium superficiecularum C*a*B vel omnium C*q*B, quae in tota CNELB, vel in tota CNPLB continentur, exponetur per factum ex $A\omega$.BC in omnia recta *fA,* qua areae EzαA inscribi possunt, vel quia haec recta areae inscripta in hanc ipsam aream desinunt, per factum ex $A\omega$.BC in aream EZαA. Atqui hoc factum (§.33) aequivalet ponderi quam habet massa liquoris homogenei, cujus densitas est $A\omega$ & volumen est prisma ex basi EZαA & altitudine BC. Ergo etiam pressio liquoris heterogenei in superficiem curvam CNELB, vel in CNPLBQ aequivalet ponderi ejusdem liquoris homogenei massae.

267. Zonae vera C*cab*B vel C*cqb*B*,* quorum communis altitudo A*a* nunc ponatur finitae magnitudinis, pressionem subibunt exponendam pondere prismatis liquidi, cujus basis sit trapezium vel quadrilineum *af*α*A,* altitudo BC, & densitas uniformis $A\omega$.

COROLLARIUM III.

268. Unde si planum EMDA rectangulo NCBL in media MD occurrat ad angulos rectos, in hoc plano reperietur media directio pressionum, quas singula puncta superficierum NAL & NQL subeunt a liquore heterogeneo, cum in dicto plano EMDA media fit directio pressionum, quas singula puncta rectanguli NCBL ab eadem in vase liquore heterogeneo sustinent, & pressiones hujus liquoris in superficies curvas NAL & NQL eaedem sint cum pressione, quam sustinere debet rectangulum NCBL. Idcirco, si per centrum gravitatis areae EZαA planum rec-lo NB perpendiculare ductum intelligatur, cujus & plani EMDA prolongati communis sectio sit ZK, haec KZ (§.54.) erit media directio gravitationum liquoris in superficiem cavam NAL. Et punctum K vocari nunc potest *Centrum pressionum* vel *gravitationum* liquoris heterogenei.

COROLLARIUM IV.

269. Hinc liquoris heterogenei pressio lateralis, quam sustinent superficies cavae NAL vel NQL aut etiam rec-lum aeque altum CBL , est ad pondus absolutum liquoris heterogenei NLBCE, sicut factum ex prismate recto, cujus basis est figura EZαA, & altitudo BC in $A\omega$ ad factum ex prismate NEABL in $A\omega$, vel omissa hac $A\omega$, velut prisma illud ex EZαA in BC ad hoc prisma NEABL. Nam (§.266.) prisma EZαA.BC ductum in $A\omega$ significat pressionem, quam superficies curva CEB vel CQPB ab interno liquore heterogeneo patitur, & prisma NEABL ductum in eandem $A\omega$ (§§. 245 & 33) significat pondus absolutum liquoris heterogenei sub volumine NEABL. Propterea, si area EZαA fuerit ad bilineum BACB, basin prismatis seu vasis NEAB, sicut AE ad BC, pressio lateralis, quam superficies cava NAL vel NQL sive rectangulum NCB subeunt, aequabitur ponderi absoluto massae liquoris heterogenei NEAB; & pressio illa lateralis hoc pondere absoluto major minorve existet, prout ratio trilinei EZαA ad bilineum BACB major aut minor fuerit ratione AE ad BC.

COROLLARIUM V.

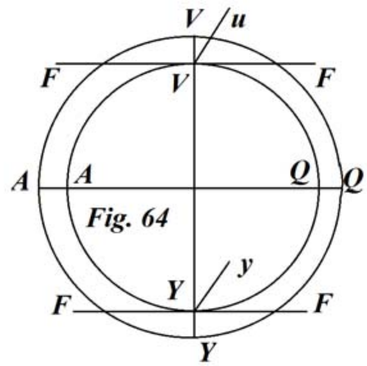
270. Adeoque, si vas nostrum ponatur esse tubus SAYPQ, in quo rectangulum SVYT sit maxima sectio basi AYQV recta, duae partes superficiei cavae SEAT & SPQT, rectangulo SVYT adjacentes, *maximam* pressionem a liquore heterogeneo, tubo indito, patientur atque adeo hic liquor, recensitas tubi partes a se invicem avellere conans, *maximam* impressionem in lineas SV & TY, in quibus duae partes SEAT & SPQT sibi invicem adhaerent, exseret; adeo ut, ni tubus satis firmus sit, qui liquoris pressionibus resistere valeat, in loco infimo linearum SV vel TY rumpi, vel saltem rimis pertundi debeat a liquoris pressura. Nam quia factum EAαA.VY exponit pressuram, quam semitubus SAT vel SQT a latere subit, & propter maximum rectangulum SVYT, basis ejus VT major est basi CB cujuslibet alius rectanguli NCBL, ultro liquet veritas asserti.

COROLLARIUM VI.

271. Si ergo semel constiterit juxta quam proportionem procedant firmitates tuborum varias crassities habentium, ex collatione harum firmitatum cum liquorum gravitationibus innotescent facili negotio crassities tubi seu spissitudo materiae, ex qua tubus paratus est, qua requiritur ad id, ut tubus citra sui rupturam liquoris ei inditi pressionem perferre possit. Firmitatum vero ratio, quibus duo diversi tubi pollent, demonstrabitur in sequenti propositione.

PROPOSITIO VIII. THEOREMA.

272. *Resistentiae seu firmitates tuborum funt in composita ratione ex ratione tenacitatis materiae unius ad tenacitatem materiae alterius tubi, ex ratione crassitiei ad crassitiem, & denique ex altitudinis tubi unius ratione ad altitudinem alterius.* Fig. 64



Sit tubus, cujus orificium figura interior AVQY, crassities materiae, ex qua tubus paratus est, VV vel AA aut QQ, & altitudo tubi V*u* seu Y*y;* erunt rectangula *u*VV & *y*YY partes, in quibus semitubi *u*A*y* & *u*Q*y* a se invicem avellendi juxta directiones VF & YF a potentiis in contrarias partes agentibus, sibi mutuo adhaerent. Jam quia tot vinculis semitubi conjunguntur, quot sunt puncta physica in rectangulis illis *u*VV & *y*YY, manifestum est resistentiam tubi aequalem esse vi requisitae ad omnia vincula seu fibras dilacerandas; atqui vis ejusmodi seu resistentia tubi est ut vis, qua unum vinculum seu una fibra rumpi potest & numerus fibrarum conjunctim; vis vero, qua opus est ad rumpendum unam fibram, est nobis tenacitas materiae tubi vel saltem huic tenacitati proportionalis; numerus vero fibrarum est ut summa rectangulorum *u*VV & *y*YY, ergo vocando tenacitatem materiae tubi T, erit ejusdem tubi resistentia sicut factum ex T in duplum rectangulum *u*VV, hoc est ut 2.T.VV.V*u*, vel etiam ut 2.A.C.T, vocando altitudinem tubi V*u,* A & crassitiem mataeriae seu VV vel YY, C. Sic etiam in alio tubo, cujus firmitas, crassities materiae ejusdemque tenacitas, ac denique tubi altitudo dicantur, *f, c, t* & *a,* & F indicet pariter firmitatem primi tubi, erit *f* ut 2*act,* & cum paulo ante fuent F ut 2ACT, erit omnino $\mathrm{F}: f = \mathrm{TCA}: tca$. Quod erat demonstrandum.

COROLLARIUM I.

273. Hinc figuras 63 & 64, junctim inspiciendo, quia (§.267.) pressio liquoris heterogenei, quae in partes *u*A*y* & *uQy* exseritur, est *af*α*A*.VY.*A*ω, haec pressio etiam erit $= \mathrm{AMDS}$, nominando altitudinem zonae *qua*ABQ, quae est *Aa* ut supra, A, quadrilineum $af\alpha\mathrm{A} = \mathrm{A.M}$, diametrum VY vel $\mathrm{AQ} = \mathrm{D}$ adeoque $pr.\,u\mathrm{A}y + pr.\,u\mathrm{Q}y = 2\mathrm{AMDS}$, existente $\mathrm{A}\omega = \mathrm{S}$. Ponendo igitur liquoris pressionem in zonam $a\mathrm{AQ}q$ praecise aequalem esse ejus firmitati, seu $\mathrm{F} = pr.\,u\mathrm{A}y + pr.\,u\mathrm{Q}y$, erit (§.272.) $2.\mathrm{TCA} = 2.\mathrm{AMDS}$, vel $\mathrm{CT} = \mathrm{MDS}$, & in alio tubo $ct = mds$, indicando similes res per similes litteras in utroque tubo majusculis in primo, & minusculis in altero. Ergo $\mathrm{CT}: ct = \mathrm{MDS}: mds$.

COROLLARIUM II.

274. Habeant insuper tubi nostri orificia circularia, atque contineant liquores homogeneos, eritque eo casu linea EZα recta angulum semirectum continens cum EA,

adeo ut sit $af = Ea$, $A\omega$ vero, qua dicta est S, designat densitatem seu gravitatem specificam liquoris , & M erit E*i* bisecta scilicet *Aa* in *i*; quandoquidem $\text{M.A}a = af\alpha\text{A}$, & hoc casu hoc quadrilineum abit in trapazium aequale rectangulo sub E*i* & A*a*. Hisce positis analogia corollarii praecedentiae transformabitur in sequentes
$1°.\ \text{C} : c = (\text{M. S. D} : \text{T}) : (m.\ s.\ d : t.)$. Hoc est, crassities materiae in ambobus tubis firmitates habentibus liquorum pressionibus aequipollentes, sunt ut facta ex altitudinibus liquorum mediis & gravitatibus eorumdem specificis in diametros orificiorum applicata ad tenacitatem materiae, ex qua tubi fabrefacti sunt.

275. Adeoque si liquores ejusdem speciei & tubi ex eadem materia facti fuerint, crassities tuborum aequaliter firmorum erunt in composita ratione altitudinum liquorum & diametrorum tuborum, hoc est in ratione rectangulorum per axem tuborum.

276. II^{o}. $\text{T} : t = (\text{M.S.D} : \text{C}) : (m.s.d : c)$. Hoc est tenacitates materiae tuborum aequalis roboris, sunt ut facta ex altitudinibus liquorum mediis & gravitatibus eorum specificis in diametros tuborum applicata ad crassities materiae, ex qua tubi parati sunt .

SCHOLION.

277. Ut usus praecedentium regularum innotescat, unum atque alterum exemplum afferre libet. *Exempl.* I. Ad Calcem Operis nonnullorum Academiae Reg. Parif. Scientiarum sociorum, quod inscribitur, *Divers Ouvrages de Mathematique & de Physique de Messierus de l'Academie Royale des scientes* , extat dissertatiuncula Celeb. Olai Romeri Dani De Crassitie & Viribus tuborum in aquaductibus, secundum diversas fontium altitudines, diversasque tuborum diametros, in qua dissertatione fol. 517 relatum legitur, tubum plumbeum diametri 16 pollicum, cum crassitie $6\frac{1}{3}$ linearum (linea ipsi est 12[a] pollicis vel 144[a] pedis Parisiensis pars) pressioni aqua 50 pedum in altitudine sufficienter restitisse in experimento quodam Versaliis olim sumpto, qua observatione posita, quaerit Autor qualis crassities alii tubo plumbeo tribui debeat orificium habenti 10 digitorum, ut aqua pressionem 40 pedum perferre queat. Canon articuli superioris 275 quaestioni illico satisfaciet, cum hoc casu crassities sint ut rectangula per axem, nam in tubo observationis multiplicando aqua altitudinem 50 pedum in diametrum tubi 16 pollicum, productum 800 denotat rectangulum per axem in tubo observationis, & ducendo altitudinem aqua 40 pedum in diametrum 10 pollicum alterius tubi proveniet 400 pro rectangulo per axem in hoc tubo, quod cum illius 800 tantum dimidium sit, hujus tubi crassities juxta canonem tantum dimidia erit crassitiei in tubo observationis, qua erat $6\frac{1}{3}$ linearum; ergo quasita alterius tubi crassities erit tantum $3\frac{1}{4}$ linearum, loco $4\frac{1}{2}$ lin. circiter, quam Dn. Romerus repererat, quia resistentae tuborum statuit esse in duplicata ratione crassitierum caeteris existentibus paribus, quas supra in propositione apparet esse in simplice non duplicata illa ratione.

278. *Exemp.* II. Mariottus censet, ut habetur in eodem Opere quod in antecedenti exemplo memoravi fol. 513, tubum cupreum sex pollicum in diametro aqua pressionem 30 pedum perferre posse cum crassitie dimidiae lineae. Ponamus has tuborum plumbei & cuprei resistentias aqua pressionibus praecise aequari, inveniendaque sit proporito tenacitatis plumbi & cupri. In formula superiore (§.176.) elisis S & *s* utpote aequalibus provenit $\mathrm{T}:t=(\mathrm{M.D}:\mathrm{C}):(m.d:c)$ ubi majusculae ad tubum plumbeum, minusculae vero ad cupreum pertinent, hinc subrogatis ordine loco M, D, C numeris 50, 16, $6\frac{1}{3}$ & loco *m, d, c*, numeris 30, 6 & $\frac{1}{2}$; invenietur $\mathrm{T}:t=\frac{2400}{19}:360 \ =240:684=20:57$, atque adeo tenacitas plumbi foret, juxta has observationes, paulo major quam subtripla tenacitatis cupri.

PROPOSITIO IX. THEOREMA.

279. *Si vasis* BAS, *liquoris cujuscunque heterogenes pleni, partes* BAE, SAE *instar alarum follis circa verticem* A *ab invicem diduci queant,* Fig.23,24, *earundemque partium vel complementorum* OB*b*A, OS*s*A *solida analoga* 2B2A2E, 2S2A2E, *vel* B2B2*b*2A, S2S2*s*2A *indicentur iisdem signis* $\delta,\chi,$ *quibus eorumdem solidorum analogorum centra gravitatis signantur, signoque pseudocuneus* LPQ *scalae gravitationum* L*r*Q *utrique vasis alae communis, quo ejusdem pseudocunei centrum gravitatis signatur, potentiae quadam aequales ponderi massarum* δ,γ *liquoris cujusdam homogenei, cujus densitas eadem sit cum PV media densitate liquoris in vase heterogenei, in directionibus suis homologis* $\alpha\delta$, $\xi\chi$ *vel* $\delta\chi$ & $\chi\xi$ *per centra gravitatis solidorum* δ, χ *inter se transeuntibus planisque BS normalibus vasi applicatae, & aliae potentiae* γ & ω *inter se aequales, utpote singulae aequalis mediae densitati liquioris heterogenei, vasi applicatae in directionibus* $b\gamma, su$ *, plano BS aequidistantibus, & per centrum gravitatis* ω *psuedocunei scalae gravitationum transeuntibus; hae quator potentiae* δ,χ,γ & ω *simul agentes, qualibet scilicet secundum suam directionem, eadem vi pollent ad diducendas vasis alas, quam habet eidem vasi inditus liquor heterogeneus.*

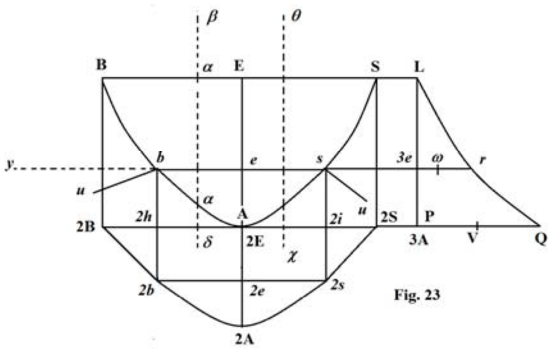


Fig. 23

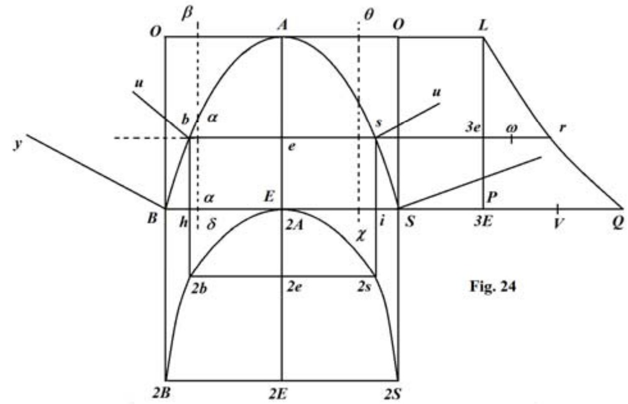


Fig. 24

Haec propositio tantum est casus particularis corollarii primi Propositionis X libri primi, nam quae in hac propositione generaliter dicitur scala potentiarum solido patienti perpendiculariter applicarum, nunc est scala gravitationum L*r*Q, quandoquidem quodvis punctum *b* in superficie A*b*B urgetur juxta directionem *bu* superficiei perpendicularem (§.260.) potentia aequali ponderi filamenti liquidi voluminis 3*er* & densitatis PV aequalis mediae densitati liquoris heterogenei, quod pondus filamenti *r3e* exponitur (§.33.) facto *r3e*. PV ex volumine scilicet in densitatem liquoris, atque adeo liquoris gravitationes in cavam vasis BAS superficiem idem praestant ac potentiae singulis punctis cavae isti superficiei applicatae, quas designant ordinatae respectivae *r3e* scalae gravitationum L*r*Q ductae in datam PV. Propterea (§.81) liquor heterogeneus vasis eundem effectum praestabit, quem potentiae expressae per solida $\delta, \chi, \& \ \omega$ ducta in PV, atque adeo aequales ponderi liquoris homogenei densitatis PV sub voluminibus illis $\delta, \ \chi, \ \omega$ applicatae solido BAS in directionibus $\alpha\delta, \xi\zeta$ vel $\delta\alpha, \chi\xi$ & $b\gamma, s\omega$. Quod erat demonstrandun.

COROLLARIUM I.

280. Adeoque momenta fluidi heterogenei vasis alas diducere conantis circa hypomochlion A (§. 82.) pro ala BAE erunt PV. $\delta.\alpha\mathrm{E} + PV.\gamma.$ A*e* , & pro ala SAE erunt PV. $\chi.\xi\mathrm{E} +$ PV.ω.A*e* . Haec enim facta PV.δ, PV.γ, PV.ω, PV.χ significant pondera liquoris densitatem PV habentis sub voluminibus $\delta, \gamma, \omega, \& \ \chi$.

COROLLARIUM II.

281. Antecedens corollarium juxta §. 263. etiam obtinet, cum vasis alae ab aliquo fluido ambiente extus premuntur, sed permutatis gravitationum directionibus, hoc est , sicut interni liquoris gravitationes vasis alas ab invicem diducere conantur, ita vice versa ambientis fluidi pressione fit, ut eadem alae ad se invicem constringantur fortiter. Unde eadem formulae, quae in corollario praecedenti momenta liquoris interni vasis alas dilatare conantis designabant, momenta pariter liquoris ambientis atasque constringere conantis expriment, sed scalae gravitationum liquoris exterioris applicatae.

SCHOLION.

Adeoque, ad aestimandas liquoris pressiones in vasis diducibilibus, tota difficultas eo reducitur, ut habeatur regula calculum subducendi pro inventione momentorum solidorum δ & χ respectu plani recti AE, & pseudocunei ω respectu plani PQ, vel subinde etiam respectu plani OAO figurae 24, quoties scilicet vasis vertex A sursum respicit. Calculus vero facile reduci potest ad quadraturas figurarum, ut ex sequentibus duobus theorematibus haud difficulter perspicietur.

PROPOSITIO X. THEOREMA GEOMETRICUM.

282. Fig.27. *Solidi cujusvis* CBAD, cujus *sectiones* cbd *basi* CBD *parallelae sint, figurae similes & similitur positae, momentum respectu plani* CAD *basi* CBD *recti, aequabitur facto ex trilineo quodam* AEFfA *in rectangulum sub* EF, *quam ipsi* BE *rectae* CD *normali aequalem ponimus, & data recta* D *ejus magnitudinis, ut parallelepipedium sub quadrato rectae* BE *vel* EF *& hac data* D, *aequale fiat solido ex bilineo* CBD *in* XE, *existente hac* XE *distantia centri gravitatis* X *bilinei* CBD *ab ejus basi* CD, *ipsum vero trilineum ea lege descriptum sit, ut ejus ordinatae* EF, ef *sint in triplicata proportione ordinatarum ipsis in directum positarum* BE, be, *in figura* AbBE, *qua plano* CAD *recta est, & per axem* AE *figurae* CAD *transit.*

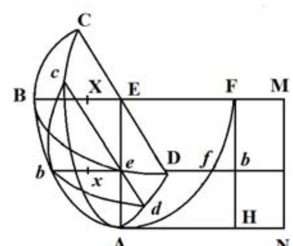


Fig. 27

Sit *x* centrum gravitatis figurae *cbd,* eritque *xe* distantia hujus centri a basi *cd* bilinei *cbd* in plano figurae AbBE, quandoquidem figurae CBD, *cbd* (secundum hypothesin) similes & similiter positae sunt, eritque adeo $\text{CBDC.XE} : cbdc.\chi e = \text{BE}^3 : be^3$ id est constr. $= \text{EF} : ef = \text{D.EF}^2 : \text{D.EF}.ef$; atqui (constr.) est D.EF^2 vel D.BE^2 aequale solido ex CBDC in XE, ergo etiam D.EF.*ef* aequabitur solido ex *cbdc* in χe, seu momento bilinei *cbd* axi *cd* appensi. Jam, quia momentum solidi totius CBAD respectu plani CAD aequatur momentis omnium bilineorum *cbdc,* quae in hoc solido continentur, id est, omnibus *cbdc* in *xe,* idem solidi momentum etiam aequale erit omnibus D.EF.*ef* ; quorum singula singulis *cbdc.xe* aequalia sunt; atqui omnia $\text{D.EF}.ef = \text{D.EF. omn}.ef = \text{D.EF.areamEA}f\text{F}$. Ergo momentum solidi CBAD aequatur facto ex area *EAfF* in rectangulum D.EF. Quod erat demonstrandum.

COROLLARIUM I.

283. Adeoque momentum complementi solidi CBAD respectu ejusdem plani CAD, seu solidi residui post detractionem ipsius CBAD a prismate recto & aequealto CDBG aequabitur ex trilineo AFH in rec-lum D.EF. Nam momentum prismatis CDBG est factum ex recto EH in rec-lum D.EF, & solidi CBAD momentum aequatur facto AEF*f*A.D.EF, adeoque momentum complementi hujus solidi CBAD aequatur omnino facto ex recto EH, demto trilineo EAF, in rectangulum D.EF, hoc est facto ex AFH in D.EF.

COROLLARIUM II.

284. Unde, si solidum nostrum CBAD fuerit analogum solido patienti BAE propositionis praecedentis, atque adeo idem sit cum solido δ, erit hoc casu $\delta.\alpha\mathrm{E}$ seu momentum solidi $\delta.\alpha\mathrm{E} = \mathrm{EA}f\mathrm{F.D.EF}$, in casu figurae 23 quo Vertex solidi BAS deorsum respicit, & $\delta.\alpha\mathrm{E} = \mathrm{EA}f\mathrm{F.D.EF}$ in casu figurae 24, quo vertex sursum respicit

PROPOSITIO XI. THEOREMA GEOMETRICUM.

285. *Si prisma rectum super basi* CAD *sectum intelligatur superficie quadam curva* Q*r*LCD, *nascetur inde cunei species* QACD, *cujus acies erit in linea* CD, *ejusque facies curva aciei* CD *adjacens oritur motu parallelo linea* CD, *ita ut fixum in ea punctum* E *semper in curva* L*r*Q *existat inter movendum, lineaque* CD *plano LPQ perpendicularis sit, hujus cunei momentum respectu plani* PQ *alteri* CAD *recti equabitur facto area seu bilinei* E*i*A *in rec-lum* AE.CD; *ubi bilinei ordinata quacunque ei fuerit ad ordinatam homologam er figurae* PLQ, *sicut rectangulum coordinatarum* A*e, cd in figura* CAD, *ad rectangulum gulum ex datis* AE, CD. Fig. 28.

Nam quia (secundum hypothesin) $ei : er = \mathrm{A}e.cd : \mathrm{AE.CD}$, erit *Ae.cd.er* hoc est momentum rec-li *cd.er* ad distantiam A*e* plano PQ appensi $=$ solido AE.CD.*ei*. Atqui momenta omnium rectangulorum *cd.er* contentorum in pseudocuneo aequivalent momento istius pseudocunei, ergo etiam omnia EA.CD.*ei,* hoc est, factum ex recto AE.CD in bilineum *EiA,* aequantur momento pseudocunei QACD. Quod erat demonstrandum.

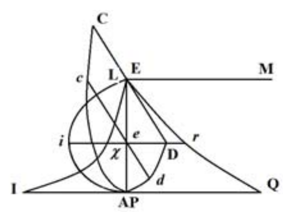


Fig. 28

COROLLARIUM I.

286. Quoties figurae CAD vertex A deorsum conversus est, atque adeo cum puncto P congruit, toties curva E*i*A per puncta L & P transibit, adeo ut ordinatae ejus in his punctis nullae sint. Sed si vertex *A* ejusdem figurae CAD sursum respicit, adeo ut basis CD per punctum P transeat & vertex cum puncto L confundatur, curva L*q*I transibit idem per punctum L, non vera per punctum P, cum ordinata ejus in hoc puncto PI ipsi PQ seu PL aequalis futura, sit, nam CD est in P & A in L, analogia $e\chi : er = \mathrm{A}e.cd : \mathrm{AE.CD}$, fiet $\mathrm{PI : PQ = AE.CD : AE.CD}$, atque adeo $\mathrm{PI = PQ = PL}$, ubi scilicet A*e* facta fuerit AE. Hocque casu erit momentum solidi respectu plani per L, transientis alterique PQ aequidistantis, aequale factoex rec-lo AE.CD in aream PL*q*I.

COROLLARIUM II.

287. Hinc, si figura CAD fuerit sectio recta in solido patiente BAS figurarum 23 & 24, & curva L*r*Q scala gravitationum liquoris heterogenei, solidum QPCD erit pseudocuneus scalae gravitationum, & momentum ejus, seu *w*.A*e*, in casu figurae 23 $= \mathrm{AE.CD.L}i\mathrm{PL}$, & *w*.A*e* in casu figurae 24, $= \mathrm{AE.CD.L}q\mathrm{IP}$.

SCHOLION.

288. Patet ergo computum momentorum liquorum in vasis diducibilibus gravitantium totum deduci ad investigationem arearum AE*sf*A, L*i*P & LχIP in figuris 27, 28 quae in particularibus exemplis semper haberi possunt, si non algebraice, saltem transcendenter. Fig. 23, 24, 27. Sit enim vas BAS conicum liquore homogeneo plenum, adeo ut curva L*r*Q abeat in lineam rectam, quo casu solida δ & χ erunt similia & aequalia BAE & SAE, & solidum CBAD erit semissis coni recti, in quo cum sit

$$\mathrm{EA}^3 : e\mathrm{A}^3 \left(= \mathrm{BE}^3 : be^3\right) = \mathrm{EF} : ef \; ,$$

liquet curvam AfF fore hoc casu parabolam cubicam, atque adeo trilineum EAF erit $= \frac{1}{4}\mathrm{AE.EF}$ & $\mathrm{AFH} = \frac{1}{4}\mathrm{AE.EF}$. Et quia CBD est semicirculis, cujus centrum in E, erit $\mathrm{CBDC.XE} = \frac{2}{3}\mathrm{BE}^3 = \mathrm{D.BE}^2$; atque adeo $\mathrm{D} = \frac{2}{3}\mathrm{BE}$. Hinc (§.284.)

$$\delta.\alpha\mathrm{E} = \chi.\xi\mathrm{E} = \mathrm{D.EF.AEF} = \tfrac{2}{3}\mathrm{BE.EF.AEFA} = \tfrac{2}{3}\mathrm{BE.EF}.\tfrac{1}{4}\mathrm{AE.EF} = \tfrac{1}{4}\mathrm{AE.BE}^3.$$

Fig. 28. Porro existente linea LrQ recta & $\mathrm{LP} = \mathrm{PQ}$, erit etiam $\mathrm{L}e = er$, & sic ubique: unde cum (§.285) *ei* sit ad *er* ut *Ae.cd* ad AE.CD , erit ubique ordinata $ei = \mathrm{A}e.\, e\mathrm{E}.cd : \mathrm{AE.CD} = \mathrm{A}e^2.e\mathrm{E} : \mathrm{AE}^2$, substituendo loco rationis *cd*:CD aequalem rationem A*e*:AE propter similitudinem triangulorum A*cd* & ACD, quandoquidem in praesenti casu figura ACD est triangulum. Hinc reperietur per notissimas quadraturarum methodos bilineum $\mathrm{E}i\mathrm{A} = \frac{1}{12}\mathrm{AE}^2$: hinc in casu figurae 23, habebitur (§.287) $\omega.\mathrm{A}e(= \mathrm{AE.CD.L}i\mathrm{AL}) = \mathrm{AE.CD.}\ \frac{1}{12}\mathrm{AE}^2$, seu, quia CD tantundem est ac $2\mathrm{BE}, = \frac{1}{6}\mathrm{BE.AE}^3$.

Erit ergo

$$\mathrm{PV}.\delta.\alpha\mathrm{E} + \mathrm{PV}.\omega.\mathrm{A}e = \tfrac{1}{6}.\mathrm{PV.AE.\ BE}^3 + \tfrac{1}{6}.\mathrm{PV.\ BE.AE}^3$$
$$= \tfrac{1}{6}.\mathrm{PV.AE.\ BE.AB}^2 = \mathrm{PV}.\chi.\xi\mathrm{E} + \mathrm{PV}.\omega.\mathrm{A}e$$

quia in praesenti casu alae BAE & SAE, utpote semisses coni recti BAS, similes &. aequates sunt.

289. Si conus BAS ab aere etiam extus premi ponatur, & quaratur altitudo AE ipsius coni, in suppositione aequilibrii internae liquoris pressionis in cavam coni superficiem cum externa aeris in convexam, erit hoc casu scala gravitationum L*r*Q linea recta axi LP

seu AE parallela, cujus ab hac AE distantia sit PQ in figuris 23, 24, & 28, & EM vel AN in figura 27. Hoc casu solidum δ vel χ erit prisma baseos CBDC & altitudinis EM seu PQ, hujusque prismatis momentum seu $\delta.\alpha\text{E}$ nunc fiet $=\text{CBDC.XE.EM}$ (vel quia supra inventum est $\text{CBDC.XE}=\frac{2}{3}\text{BE}^3$) $=\frac{2}{3}\text{EM. BE}^3$. Loco pseudocunei *w* nunc habebimus prisma ex basi CAD & altitudine PQ, unde, quia generaliter est $ei:er=\text{A}e.\,cd:\text{AE.CD}$ (vel propter triangula similia CAD & $c\text{A}d$) $=\text{A}e^2:\text{AE}^2$, & propter aequales *er* & PQ, cum linea L*r*Q ipsi EA parallela suppononda sit, atque per Q transiens, erit $ei=\text{A}e^2.\text{PQ}:\text{AE}^2$, hocque casu foret curva E*i*A parabola conica, cujus parameter est $\text{AE}^2:\text{PQ}$, adeo ut area E*i*AE hoc casu futura sit $=\frac{1}{3}\text{AE.PQ}=\frac{1}{3}\text{AE.EM}$; atque adeo

$$\gamma.\text{A}e=\omega.\text{A}e(=\text{AE.CD.E}i\text{AE})=\frac{1}{3}\text{AE}^2.\text{CD.EM}=\frac{2}{3}\text{ BE.EM.AE}^2.$$

Idcirco vocando athmosphaerae densitatem mediam Z, erit summa momentorum pressionis aeris, seu

$$\text{Z}.\delta.\alpha\text{E}+\text{Z}.\omega.\text{A}e=\frac{2}{3}\text{Z. BE.EM.BE}^2+\frac{2}{3}\text{Z.BE.EM.AE}^2=\frac{2}{3}\text{Z.BE.EM.AB}^2..$$

Hinc, quia (secundum hypothesin) in casu aequilibrii $\text{PV}.\delta.\alpha\text{E} + \text{PV}.\omega.\text{A}e=\text{Z}.\delta.\alpha\text{E}+Z.\omega.\text{A}e$, erit etiam $\frac{1}{6}\text{PV.AE.BE.AB}^2=\frac{2}{3}\text{Z.BE.BM.AB}^2$, adeoque $\text{PV.AE}=4.Z.\text{BM}$. Invenimus enim supra, $\text{PV}.\delta.\alpha\text{E}+\text{PV}.\omega.\text{A}e=\frac{1}{6}.\text{PV.AE. BE.AB}^2$, & paulo ante pro momentis aer extus prementis, seu pro $\text{Z}.\delta.\alpha\text{E}+\text{Z}.\omega.\text{A}e$ invenimus $\frac{2}{3}\text{Z.BE.BM.AB}^2$. Atqui PV.AE denotat pondus columnae liquidae, cujus altitudo AE & media densitas PV, & AE.BM gravitationem totius atmosphaerae, quam simplicius per *p* indicabimus; ergo si vas conicum BAS hydrargyri plenum sit , oportet altitudinem AE quadruplo majorem esse altitudine columnae mercurialis aequivalentis atmosphaerae pressioni, atque adeo minimum 120 digitorum, cum atmospaerae gravitatio aequivaleat 30 digitis Mercurii, ad obtinendum aequilibrium inter externam atmosphaerae pressionem & internam Mercurii gravitationem follis conici alas ab invicem diducere conantem.

Sin vero coni vertex sursum conversus esset, ut in fig. 24. eo casu praecedentia theoremata praeberent $\text{PV.AE}=\frac{4}{3}\text{Z.BM}$ pro casu aequilibrii inter pressuram externam aeris & internam Mercurii, atque adeo pro hoc aequilibrio obtinendo nunc sufficeret altitudo AE 6o digitorum. Atque hae determinationes ad amussim consentiunt cum iis, qua Celeb. Jac. Bernoullius super hac re prodidit in Actis Erudit. Lips. 1686 & 1687. occasione alicujus perpetui Mobilis in Novellis Reipublicae literatiae D.Baelii 1685 propositi, quod consistebat in quadam follis specie triangularis, circa axem horizontalem alterutrius alae medio affixum mobilis, Mercurio modo implendi, modoque etiam in situm rectum deducti iterum aliquo usque deplendi &c. Hujus machinae successum irritum fore ex principiis hydrostaticis & vectis natura ostendit eleganter modo laudatus Bernoullius. Papinus vero, etsi successum pro dubio habuisse videtur atque imperfecta pressionum aestimatione impossibilitatem motus ejus utcunque ostenderit, nihilominus tamen Bernoullianam machinae discussionem eludi posse existimavit; motum parallelum alis tribuendo, quo vectis rationem, in qua Bernoullii impugnatio sua data erat, cessaturam esse considebat perperam. Ea vero, qua super praedictam machinam in utramque partem agitata fuere, legi possunt in Actis Lips. supra citatis.