CONCERNING THE MOTIONS AND FORCES OF BODIES

SECOND BOOK.

FLUID BODIES.

SECTION I.

The Forces on Fluids due to Gravity.

Bodies with hardness and consistency may be compared with a fluid in this regard, in that they are extended bodies and composed of hard particles or molecules; yet they differ from these same fluid bodies, because the molecules of fluids not only are so much smaller, so that they shall flee even from the sharpest eyes equipped with microscopes, but also they are separate and highly mobile, and which can follow easily the path of any solid body, and yet they are able to move about amongst themselves : thus they may be consistent with opposing hard bodies variously involved amongst themselves, and thus by being scarcely separated, so that none of the particles may appear to be moving according to the senses, because the whole sum of the parts of the body, that is the mass of the body, may share the same motion.

238. Now as it has been said above in §.25, a fluid body therefore is one*, the elements of which* (§.14*.*) *are able to be set in motion by some force acting, and just as so great a whole mass of the fluid may appear to be at rest, according to the eye.* Thus water, air, oil, quicksilver, etc are fluid bodies, likewise fused metals, as long as they are flowing.

And thus the distinct idea of fluid bodies involves the sum of the smallest of molecules, and such a configuration of these, so that thence equally they may be able to arise moving. And from these specified properties, fluids are distinguished from solids likewise in terms of the smallest particles dissolved away, yet not of so great a movement nor smallness, as with which the elements of the fluid are able to move about amongst themselves easily. Thus grains ground into the finest flour in the body of a fluid therefore do not dissolve away, because the particles of flour do not approach that fineness; nor hence do they reach that mobility that the nature of the fluid requires.

239. Since we undertake to deal with the forces of fluids, we do not have it in mind, so that the shapes of the particles or elements are to be defined and measured, thus so that I may say, neither do we believe to be able to show, nor hence will we look more closely into the shapes of the elements of the bodies, because these are accustomed to vary a great deal, as that shall be able to be addressed conveniently according to mathematical concepts ; for I consider nothing to stand in the way, why not of one and the same fluid particle both in the account of the magnitude, as well as in the account of the figure, that may be able to be changed in an infinite number of ways. Therefore the investigation of

the shapes, by which any fluid particles must be the described, I may leave to the physicists, and henceforth it will suffice for me to know, the shapes of the particles of each fluid, of whatever kinds these may have been, with no obstruction made to the mobility of these, since the particles may be of any fluid (following the hypothesis), that is most mobile bodies.

240. Nor also does it pertain, according to our principles, to investigate in a troubled manner, for truly the opinion of those shall be, who bestow a certain motion of any fluid, that they call *internal*, by which the particles of fluid are composed with various irregular motions to be put in motion either in one direction or the other, to the difference of the progressive motion of the fluid, by which the whole of its mass is transferred from one place to another. Thus when the river slides past, the motion is progressive by which the water is rolled down into its lower hollow ; truly the motion of hot water, that is the internal motion of its molecules, is called an internal motion: I present the example of hot water, because most certainly the particles are moved around by an internal motion of this kind, even if the motion itself cannot be seen by the eye; and thus the whole mass of water may be seen to be at rest. For truly all the fluid entirely will be carried around equally according to the philosophers, as I have said, I shall leave the rest requiring to be expended ; nor indeed do I have it in mind for me to become entangled with philosophical controversies in some way.

241. Fluid bodies composed of liquids differ as to genus and species, since indeed liquids shall be fluids, yet not necessarily vice versa. For they are liquids, which can flow when not excessively small in volume while the surface of these will have composed itself in a horizontal situation ; as neither flame, air, nor aether agrees with which property, these bodies therefore are called fluids only, but not truly liquids. In the following also we will attribute the name of fluid to liquids, because liquids are actually fluid bodies : therefore :

242. *Fluids are homogeneous,* or *uniformly heavy*, the density of which per unit mass [in the original Latin sense of particles being placed close together] is uniform, thus so that their weights shall be proportional to their masses. Such more or less are all the liquids known to us.

243. *Fluids are heterogeneous,* or *dissimilarly heavy*, the density of which is not the same for the whole mass of the fluid nor therefore are the weights of the masses of the fluid proportion. The atmosphere is such a fluid, of which the higher parts, that is the parts more distant from the surface of the earth are found to be rarer ; and indeed experiments performed in the ever flowing air of higher mountains produce rarer air than in the valleys. Truly the densities of the variation of the atmosphere can be expressed by various curves, and hence become known,

244. The *graph of the density* of each heterogeneous fluid is a figure with several curves, the ordinates of which set out the density in these planes of the fluid parallel to the horizontal, in which the ordinates are present. Therefore the axis of the scale, taking the ordinates at right angles, will be perpendicular to the horizontal.

245. The *mean density* of each heterogeneous fluid is the uniform density of some homogeneous fluid, which prevails to exert the same pressure as the heterogeneous fluid at the same altitude as the heterogeneous fluid, with itself lying in a horizontal plane. Such a mean density is known by applying the area of the scale of the densities for its axis, or the height of the fluid above the plane, which appears as the pressure.

246. The *graph of the pressure* or *of the weights* of heterogeneous liquids is a figure, of which the ordinates express the weights or pressures exercised in a horizontal plane, into which the ordinates are put in place, truly the abscissas corresponding to the ordinates designate the distances of the planes, which the pressures undertake from the upper surface of the liquid. The graph of the pressure also can be defined, because it shall be the quadrature of the graph of the densities; since afterwards (§. 258.) it will be shown the areas of the graph of the homologous densities prove to be proportional to the ordinates of the graph of the weights.

247. *Liquids are said to be in a permanent state, or standing in equilibrium,* when no part of the neighbouring liquid is expelled by its own action, but all the parts remain in perfect equilibrium amongst themselves.

248. The infinitesimals of each plane, also the weight of the particles of the surface, or the pressure of any liquid entering henceforth, will be called points as an abbreviation by us.

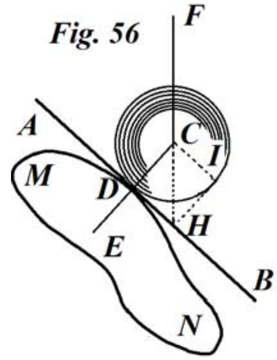
Truly we understand by the name *pressure* or weight the impression, which the fluid by its weight exerts on the underlying plane itself. Therefore, so that we are able to treat the matter generally, just as in the first book only we have done that everywhere, we consider any fluid to be heterogeneous, and for which the pressures and effects must come about from fluids of this kind, we will enfold with general theorems ; from which henceforth all will be able to be derived, which are observed for homogeneous liquids.

CHAPTER I.

Concerning the general laws of the weights of liquids subjected on a plane.

PROPOSITION I. LEMMA.

Fig. 56



249. *The pressures, which any solid or fluid body exercise on each other, are made along the directions perpendicular to the common planes touching the bodies, and pass through the common point of contact of the bodies.*
[We may presume the bodies to be at rest.]

MN shall be some body, Fig. 56, on which another body CD may be impressed along the direction FC ; the plane AB may pass through the common point of contact of each body D, both bodies touching at

the same point D, which thus may be called the common point of their contact. I say the driving force CD endured by MN to be exerted only along the direction CE perpendicular to the contingent plane AB, which may pass through the common point of contact of each body D.
Demonst. FC shall be produced as far as to the crossing H with the plane AB, and around the diameter CH the rectangular parallelogram CDHI may be understood to be described, or which the sides CD, IH are normal to the plane AB and CI being equidistant to the same. Now (§. 39.) the force or pressure along CH is equivalent to the forces along CD & CI. Truly, because CI is parallel to the plane AB, nothing can be exerted on that plane by the force along that direction, but only the force along CD perpendicular to the plane will act wholly on this plane. Therefore the force, which the pressing body C exerts on the body MN in the plane AB, is able to act only along the direction CD perpendicular to the plane AB, which passes through the point of contact D. Q.E.D.

PROPOSITION II. THEOREM.

250. *A liquid introduced into some vessel cannot remain standing in place, before its surface has acquired a horizontal resting position.*

BAC shall be the vessel, Fig. 57, of any shape and I say, however the liquid be poured into that, it cannot have a permanent position HGIA, unless its surface HGIF were horizontal. For it may have this surface EGCF remaining in the static position inclined to the horizontal : with which in place, because the mass of liquid in the plane GCIF is a weight overhanging the horizontal HGIF, and it can descent into the space GHEF, nor is there anything present which may be able to impede this flow, that actually will descend and that space GHEF is going to be filled ; therefore when the surface of the liquid GCE is inclined to the horizontal plane HGF, it is not in a permanent state, contrary to the hypothesis.

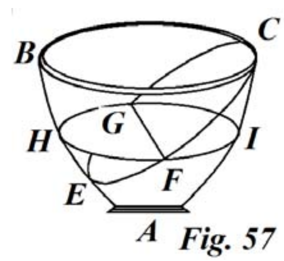


Fig. 57

COROLLARY.

251. Hence if the whole earth should be surrounded by ocean, nor would it be changed into a perfectly spherical surface by this itself. But because it is revolving in its daily motion around its centre, the surface of the water will not be spherical, but will have the surface of an oblate spheroid, because where the regions of water are closer to the equator, there they have a greater force of receding from the centre, and thus they have less weight, since the centrifugal forces, when they are greater, take away so much more from the weight of bodies. Truly on this account an inequality of the weights of the water arises, in order to obtain an equilibrium of the parts, the water cannot reach the same height everywhere, as otherwise it may come about, if the earth in its daily motion, from which the centrifugal forces of the water results, may be without [such a difference in height], but under the equator the greatest height and at the poles the maximum depression may emerge. Hence it is the case, as Newton, Huygens and others have

assigned the figure of the surface of the earth to be some kind of spheroid, arising from the revolution of some ellipse about its minor axis, with the major axis equal to the diameter of the equatorial earth. But with these in their place the discussion will continue.

PROPOSITION III. THEOREM.

252. Fig. 58. *Equal parts* (HI, IK) *of any horizontal plane* (FG) *within some liquid introduced* (DBCE) *into the vessel* (ABC) *as irregular as it pleases, remain standing in a permanent state, and equal pressures arise on account of the excess pressures from the liquid* (DFGE). *And the smallest parts of the sides of the vessel* (LF, MG*)end at the same horizontal plane* (FG) *experience the same equal pressures from the overlying liquid along the directions* (FP, GQ) *with the same perpendicular parts* (FL, GM), *which the equal small parts* (FR, GK) *experience to the horizontal plane along the normal directions themselves.*

I. Because (following the hypothesis) the liquid of the vessel is in a state of rest, its surface DE (§. 251.) will be parallel to the horizontal and hence to the plane FG, and on imagining the liquid DFG incumbent on the plane FG removed, for the remainder of the fluid in the vessel FBCG, the surface FG now also remains in a state of rest, because it is horizontal. And thus if, with the liquid DFGH restored that we imagined to be removed, the other part IK of the plane FG undergoes a smaller pressure than the neighbouring and contiguous equal small part HI, then the liquid in the location IK must be raised, or depressed, or moved to the right or left hand side or to be compressed somewhere else, since the force to move must always be greater or smaller, and the water or particles of the liquid are able to move away, and such without the motion of the parts, by which alone they are driven from their position to others, they are unable to perform, therefore the liquid cannot remain in a state of rest, contrary to the hypothesis. And therefore all the remaining equal parts HI, IK of the plane FG sustain equal pressures. Which is the first part.

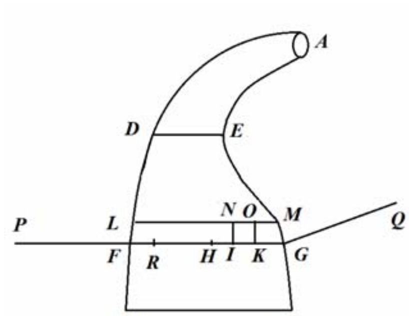


Fig. 58.

II. The plane LM shall be equidistant to the other FG, and all its equal small parts (§. 250.) NO, OM endure equally the same pressures from the above liquid DLME. Therefore the liquid between the two equidistant planes LM and FG, just as in the wine-press, are forced together strongly and indeed by equal forces in opposing directions, clearly by the incumbent liquid DLME perpendicularly downwards and from the reaction of the lower plane FG perpendicularly upwards ; for by how much that plane is pressed on by the force, by so great a force does the part pressed on react. From which since the planes NO and IK are being pressed on by forces in the opposite directions, so it is necessary that however great a pressure may act on any plane NI or OK, just as great a pressure must act on NO or IK, otherwise if it were allowed to be smaller, there the liquid ONIK might slip away through the plane NI contrary to the hypothesis. And on account of the contiguity of the water, the small lateral parts LF and MG of the vessel must undergo pressures equal to these, which the planes NI or OK endure, and these pressures are equal to these, which the planes NO, IK, &c. endure, and it is clear the pressures of the liquid DFGE on the small parts of the lateral sides of the vessel, FL and MG, are equal to the pressures, which the particles endure from the same liquid, equal to these HI, IK of the horizontal plane FG, (§.249.) along the directions FP, GQ perpendicular to these FL and GM, as therefore the small parts HI and IK. &c. are acted on likewise as along directions perpendicular to the plane FG. Q.E.D.

PROPOSITION IV. THEOREM.

253. *The pressures or weights of any homogeneous liquid on any horizontal planes are proportional to the heights of the liquid above these planes.*

DE shall be the surface of the liquid DBCE, poured into some vessel ABC, Fig. 59, and I extend the base of the vessel BC to the horizontal, as much as needed, by being stretched out together with the perpendicular ET dropped down, it is understood in addition some plane FG to be equidistant from the base BC and DE, which produced meets the right line ET at the point S. The pressures of the liquid exercised on the planes FG and BC may be designated by *pr*.FG and *pr*.BC, it must be proven that $pr.\mathrm{FG} : pr.\mathrm{BC} = \mathrm{ES} : \mathrm{ET}$. For ES is the height of the liquid DFGE above the plane FG, and ET the height of the liquid above BC.
I. ES shall be of a commensurable size to ET [*i.e.* the ratio of the heights can be expressed by a rational number], and E*p* of such a common measurable size to each, so that with drawn through the point *p* in the plane *dp*, within the space of the vessel D*de*E the small right line *eo* may be drawn parallel to E*p* or perpendicular to DE and *de* themselves, which always can happen however great the irregularity and greater narrowness the vessel ABC may itself be able to become. Again with the individual lengths *p*2*p*, 3*p*3*p*, &c. made equal to E*p* itself, and with the planes 2*d*2*p*, 3*d*3*p*, &c. put in place through the individual points 2*p*, 3*p*, ,&c., and because from the point *e* of the layer of the liquid *eo*, this point *e* will sustain as great a pressure, as the weight of the column *eo*, and (§. 252.) all the remaining points of the plane *de* will experience a pressure equal to the pressure of the point *e;* the point 2*f* on the base 2*d*2*e* of the second layer *e*2*d* will experience the pressure composed from the pressure, by which its highest point *d* is acted on, and from the weight of the small column *d*2*f* itself equal to *oe,* and thus the weight of the liquid at the point 2*f* and all the remainder of the plane 2*d*2*e* will be double of that on the plane *de*, and therefore it will be equivalent to the weight of the columns $oe + d2f$, that is E2*p* ; with a similar argument made the pressure of the liquid at some point in the plane 3*d*3*e* to be equivalent to the weight of the column itself equal to the height E3*p* ; hence the pressures

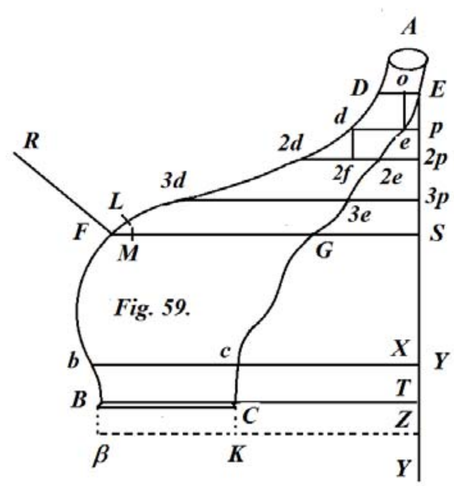


Fig. 59.

or weights of the liquids in the plane FG and BC will be equivalent to the columns, of which the heights are equal to the lines ES & EF, therefore generally there will be $pr.\mathrm{FG} : pr.\mathrm{BC} = \mathrm{ES} : \mathrm{ET}$.

II. ES shall be incommensurable to the other ET, and, if it were possible, the ratio *pr.*FG to *pr.*BC would be greater than ES to ET, for example put equal to the ratio ES: EX of which as a consequence EX shall be less than ET by the difference XT, from which deficiency a certain amount TV less than XT can be taken away, thus so that EV is made commensurable to ES. With which in place, it is understood the plane *b*V passes through the point V, and *pr.bc* will be less than *pr.*BC, since EV shall be less than ET, and $pr.\mathrm{FG} : pr.bc > pr.\mathrm{FG} : pr.\mathrm{BC}.$ Now because (from the construction) EV is commensurable to ES, by the first part of this section there will be , $pr.\mathrm{FG} : pr.\ bc = \mathrm{ES} : \mathrm{EV}$, and (following the hypothesis) there is $pr.\mathrm{FG} : pr.\mathrm{BC} = \mathrm{ES} : \mathrm{EX}$, therefore $\mathrm{ES} : \mathrm{EV} > \mathrm{ES} : \mathrm{EX}$, and thus EX is greater than EV, contrary to the hypothesis and thus *pr*.FG:*pr*.BC cannot be greater than ES: ET. Therefore if the ratio *pr*. FG: *pr*.BC shall be less than ES:ET, evidently equal to a certain ES:EY of which consequence EY shall be greater than ET, by the excess TY, again the portion YZ may be taken from this part TY of such a size that the remainder EZ may emerge commensurable to the other ES, a plane $Z\gamma\beta$ is imagined to pass through Z parallel to the base BC in which we will consider $\gamma\beta$ now to be the base. From which because EZ (by construction) is greater than AT, there will be $pr.\beta\gamma > pr.\mathrm{BC}$, and thus $pr.\mathrm{FG} : pr.\beta\gamma < pr.\mathrm{FG} : pr.\mathrm{BC}$; and because (by construction) EZ is commensurable to ES, there will be (from the first part of this) $pr.\mathrm{FG} : pr.\beta\gamma = \mathrm{ES} : \mathrm{EZ}$, truly from the hypothesis there is $pr.\mathrm{FG} : pr.\mathrm{BC} = \mathrm{ES} : \mathrm{EY}$, therefore $\mathrm{ES} : \mathrm{EZ} < \mathrm{ES} : \mathrm{EY}$, and thus EZ will be greater than EY, again contrary to the hypothesis, for EZ by necessity was less than EY, since from this EY with ZY taken away EZ was being left, therefore the ratio *pr*.FG: *pr*.BC cannot be less than the ES: ET and because neither can it be greater, it follows that *pr*.FG: *pr*: BC= ES: ET. Q.E.D.

COROLLARY I.

254. Hence besides it follow, because any part FM of any horizontal plane FG must endure just as much pressure from the incumbent liquid DFGE as the amount it would sustain from that column FM of the same liquid placed perpendicularly above for the height ES. For, because it has been shown in the proposition generally ; no account of the shape of the vessel has been given, the same prevails also with cylindrical vessels, in which it is apparent immediately the weights of a liquid to be in proportion to its height. From which, if the heights of the liquid shall be equal above some horizontal plane for a vessel with a regular [prismatic] shape and for a vessel with an irregular shape, equal parts of each plane will support equal pressures. But in the prismatic vessel any part of its horizontal plane will support the whole weight of the column rising above it perpendicularly, and thus some part FM of the plane FG will support only a part, as much as the weight of some prism, of which the base is FM and the height ES. And thus the

whole plane FG will support a load equal to the mass of the liquid in the shape of the prism, which shall have the plane FG for the base and ES for the height.

COROLLARY II.

255. And because it has been shown above (§.252.), equal minimal small parts FM and FL, of the plane FG and of the sides of the vessel, support equal pressures, each one along the direction to its normal, clearly FM vertically downwards, and FL perpendicular to the surface BFD at F, next to FR, hence it follows *any minimal part on the sides of a vessel must endure a pressure from the incumbent liquid equal to the weight of the column, of which the base is the minimal part, which the pressure can support, and the height is the distance of the particle from the surface of the liquid.*

And this is the general rule of hydrostatics elicited from its natural principles, which even if only in homogeneous liquids is it seen to prevail, yet for other liquids themselves, even heterogeneous, it will be proven to extend in the following.

SCHOLIUM.

256. Therefore from these it is clear, that homogeneous liquids weigh down on their subjective planes in the ratio of their heights above the planes upon which they press. From which the following unexpected property of fluids arises ; because evidently a small amount of each fluid may press down just as much on its subjective plane, as a mass of hundreds indeed thousands times as much at the same height. Fig. 60 : And indeed if for the pointed vessel BAC, the base of which BC shall be of a notable size, water may be poured in reaching as high as A, and by Corollary I of this section, the water will exert just as great a pressure on the base BC, as a cylinder of water IBCK would exert at the same height as the water in the vessel. Therefore the small amount of water in the sharp and slender tube ABC can have just as outstanding an effect, as a hundred times as great a mass of water, indeed many times greater at the same height, which without doubt will be seen to be a great paradox. Yet the truth of this has been shown by experiment, and thence it can be taken as proven ; for if the round stopper *a*BC*f* shall be placed in the base so accurately to fill the fourth part of the narrower vessel ABC, so that no cracks may remain, through which water may be able to pass through the small rounded end below [presumably the original pointed vessel is removed from the cylinder, or an equal cylindrical vessel is filled to the corresponding level, where the values of P are found to be the same]; having tied on a wire QM at the centre Q by which it may be raised, one end of which wire shall be connected at M to the head of the scales MDO; from the other side of the balance the weight P shall be attached as far as minimizing the increase while the bung B*f* begins to pull a little on the head of the scales MO turning in that direction, to which the weight has been attached; and it will be found this weight P to be able for a short time to be strong enough to raise the weight of water BIKC of the cylinder, with both arms of the scales DM and DO equal. From

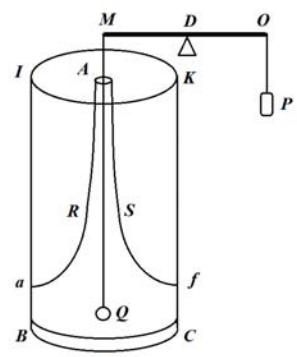


Fig. 60

which since the weight P must be just a little greater than the resistance, that it can overcome in the opposite direction, and this resistance shall be itself equal to just a little part of the weight of the water ABC at the bung *a*BC*f*, generally it must be concluded the weight of this water ABC to be equivalent to the weight of the cylinder of water I*af*K.

257. But you may argue strongly, if the water ABC exerts so great a weight on its base, it follows, that the vessel with its water influx must weigh so much more, as the amount by which the aqueous cylinder IC increases the weight of the vessel. For the vessel with the water introduced, and held in an upright position, must impose with so much weight on the plate, as much as the plate is pressed by the force ; but the plate will be pressed by that force, by which the base of the vessel is acted on by the increased weight of the vessel and of the cylinder KB at the same time : yet meanwhile, if the experiment may be seized on, it will be verified always the whole weight not to be greater than that of water influx and the weight of the vessel taken together, and thus it is seen to be against our rule of hydrostatic phenomena. But the objection rests on a false hypothesis, as if the weight of the liquid ABC at the base also were in excess over the whole [plate] and must be exerted on the plate of the scales ; which still is false, for if the bottom BC may be acted on by the load IBCK only from the liquid ABC, also the sides of the pointed tube BRA and CSA will be pressed at a height, for each of these sides is pressed by just as great a force (§.255.) as the weight of the filament of water equal to the distance of the point from the surface of the water, along a direction perpendicular to the surface of the vessel, and thus being applied to that, which generally above (§.81.) and which have been shown in the abstract, for the present case, may be verified for those before mentioned sides BRA and CSA, or rather the internal surface of the convex pointed tube, to be acted on by such a force from the liquid vertically at a height, as great as the weight of that water, by which the water cylinder IC is exceeded by the water in the tube ABC. From which, because the sides of the vessel and the bottom are joined together, only that force on the plate of the free scales MO can be in excess, from which the weight of water ABC on the base BC exceeds the force trying to raise the sides BRA and CSA of the vessel ABC arising from the pressures of the water, with the increased weight of the vessel; truly that force, or the excess of the cylinder IC above the mass of water, which in the manner named sets out the raising force, is the weight of the water only in the pointed tube ABC, therefore the force, by which the plate is depressed and must be acted on, is only the weight of the water ABC increased by the weight of the vessel, exactly as shown by experiment. Therefore as the objection lacks so much from the desired experiment to infringe any force of our proposition, as that is confirmed rather uncommonly well.

[Pascal had shown earlier that the pressure in a fluid is independent of the shape of the vessel, but proportional to the height of the fluid, see, e.g. his barrel experiment, uniform across any horizontal cross-section ; rather, the initial pressure in the narrow tube is thus far greater than it would be for a uniform cylindrical tube, and if the base were some sort of frictionless piston, it would measure this pressure; filling the rest of the cylindrical tube has no effect on this pressure, but increases the weight of the fluid within the cylinder. Thus, there is no paradox, as the total force on the base is the same in each case.]

PROPOSITION V. THEOREM.

258. *The weights of any liquids on the underlying horizontal plane, are proportional to the homologous areas in the graph of the densities, or also to the ordinates of the graphs of the pressure of the liquid.*

Fig. 61. ABC shall be some irregular filled with heterogeneous liquids as far as to DE,

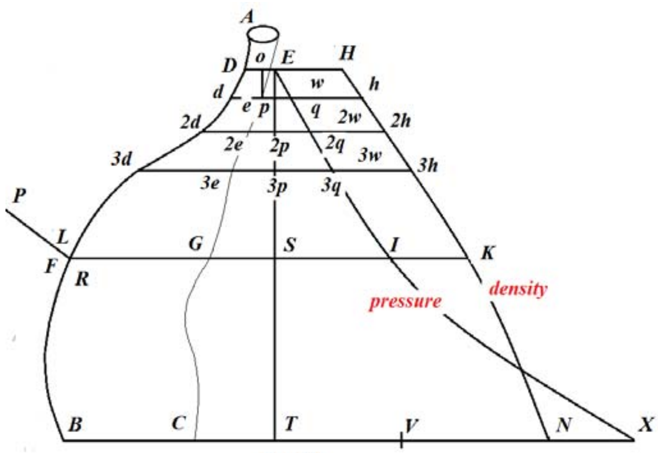


Fig. 61.

and around the axis ET of the surface of the liquid DE and perpendicular to the base BC, and thus designates the height of the fluid above the base, the curves HKN and EIX shall be placed together, of which the former HKN shall be the graph of the densities of the liquids, as the ordinates of which EH, 3*p*3*h,* SK, TN set out the densities of the liquids on these planes, on which the ordinates are found, truly the latter EIX is a graph of the pressure of the liquid, whose ordinates evidently 3*p*3*q*, SI, &c. denote the weights of the liquids in the planes 3*d*3*e*, FG, &c. And finally the area EHKNT corresponding to the height of this ET presents the mean density TV of the heterogeneous fluid. Truly the ordinates of the graph of the weights EIX (§.246) for the homologous graphs of the densities HKN are in proportion, and the final TX is put equal to the whole axis AT. With which understood it is required to prove, the weight of the liquid DFGE at the plane FG, to be to the weight of the liquid DBCE at the base BC, as the area ESKH to the area EMKH, or just as SI to TX, &c.

The liquid of the vessel may be understood to be divided, as in the preceding, indeed into all its equally high horizontal layers, but the infinitesimals of the heights, through the planes *de,* 2*d*2*e,* 3*d*3*e;* &c. are equidistant from the base BC. Therefore because the height of each layer shall be of an insensible amount, or it is indefinitely small, the density of the liquid through the whole layer can be regarded as uniform, clearly EH will signify the density of the first layer *de*ED*,* the ordinate *ph* the uniform density of the second 2*d*2*ed*, and thus again with the remainder. Now from the point *e* of the first layer a small column or filament *eo* may rise perpendicularly, thus it will support its weight : and the weight of *eo* itself (§.33) is as its volume by its density or jointly as the specific

gravity, that is it is equal to $eo.\mathrm{EH} = \mathrm{E}p.\mathrm{EH}$ or it is equal to the elemental rectangle of the inscribed area SHK adjacent to EH, and the pressure is the same, that enter all the remaining points of the plane *de;* truly the pressure that the base points 2*d*2*e* of the second layer will support 2*d*2*ed,* will be the sum of the weights of certain filaments of the second layer, and equal to the height of the filament *eo* in the first layer, of which this, as in the manner said, is expressed by the infinitesimal rectangle *w*, that truly by the rectangle 2*w* or *p*2*p.ph* : and the pressure in such will be $2d2e = w + 2w$
[Note : the numbers here are just labels for the layers; the quantities amount to $\Delta p = \rho\Delta h$ in conventional terms, where the density ρ is constant within a layer].
By the same argument the weight of the third layer in the third plane 3*d*3*e* to be expressed by the sum of the rectangles $w + 2w + 3w$; and thus the pressure, which the points of the plane FG will support, is expressed by all the rectangles $w, 2w, 3w, 4w$ which have inscribed the area ESKH, and on account of the infinitesimal height of each they vanish in this area, thus so that in place of the sum the area itself ESKH shall be required to be understood , and therefore the pressure, which the plane FG must endure from the incumbent liquid, is expressed by the homologous area ESKH, and the weight of the liquid at the base BC by the area EMKH; therefore generally there is
$pr.\mathrm{FG} : pr.\mathrm{BC} = \mathrm{ESKH} : \mathrm{ETNKH} = \mathrm{SI} : \mathrm{TX}$ by §. 246. Q.E.D.

COROLLARY I.

259. Hence the pressure, which any particles FR of the plane FG endure from the overhanging liquid, is equivalent to the absolute mass of the liquid of a certain homogeneity, of which the density shall be equal to the mean density TV of a heterogeneous liquid, truly the right volume from a prism above the base FR and with a height equal to SI of the homologous ordinate of the graph of the pressure SIX. For, following the present proposition, the pressure, which the particle FR endures, must be expressed by the product formed from the area SEHK by the particle FR of the plane FG; and (following the hypothesis) $\mathrm{SEHK} : \mathrm{TEHKN} = \mathrm{SI} : \mathrm{TX} = \mathrm{SI.TV} : \mathrm{TX.TV}$, and (§.245) the area $\mathrm{ETNKH} = \mathrm{ET.TV}$, since TV itself or the mean density of the liquid shall be that, which arises from the said area to the height ET, and thus made, we may suppose the factor to be used always thereafter, TX to be equal to ET, also the rectangle TX.TV may be equal to the area ETNKH ; the rectangle SI.TV will be found everywhere to be equal to the homologous area ESKH, and the product from this area into FR, expressing the weight or pressure on this particle, will be equal to the product SI.TV.FR, which designates a prism, of which the base FR is the plane supporting the pressure ; truly the height SI of the ordinate of the graph of the weight, multiplied into the mean density TV of the heterogeneous liquid and (§. 33.) with the prism SI.FR multiplied by TV expresses the weight of the homogeneous liquid, of which the density is TV and the volume itself SI.FR for the prism; therefore the particle FR of the plane FG endures a pressure from the incumbent heterogeneous liquid equal to the weight of a fluid prism, of which the base shall be sustaining the pressure from that particle itself, the height SI from the homologous ordinate of the graph of the pressure, and the mean uniform density TV of the heterogeneous liquid.

COROLLARY II.

260. From which, because particles of equal and minimal of sides FL and near FR in the horizontal plane FG (§.252.) sustain equal pressures from the heterogeneous pressure pressing down, whatever, along the direction to the perpendicular itself, also FL undergoes a pressure equal to the weight of a prism of fluid, of which the base shall be this particle itself FL taking the force of the fluid; truly the height SI taken from the homologous ordinate of the graph of the pressures, and finally TV the mean density of the heterogeneous liquid.

COROLLARY III.

161. If the liquid is homogeneous, the density of which shall be HE from the scale of densities with a right line drawn parallel to the axis ET through the point H, and thus in this case there will be $\mathrm{TV} = \mathrm{EH}$, and the curve EIX will be changed into a right line set at half a right angle to the axis ET; thus so that again hence it may be clear, the weights of each homogeneous liquid are in proportion to the heights of the liquid above the planes, which undergo the pressures. For in the right angled triangle ETX, the ordinates SI and TX, which indicate the weights of liquid in the planes FG and BC, with the abscissas ES and ET were proportional to the existing right line EIAE, such as we now suppose, nor in this particular case do we have the need to set out a new graph.

COROLLARY IV.

262. But if the homogeneous liquids of different densities or specific gravities are themselves brought together, the weights are in a composite ratio of the densities and the height ; and thus if in the leg ABC of the siphon AB*d* a liquid DC may be in there of which the uniform density shall be EH, and the height above the base above the base shall be equal to HN; truly for other leg *ab* inside which a liquid *dt* of density *eh* or *nc* rising to a height *ce* in the leg, the weight of this liquid will be expressed by the rectangle *he.et,* and the weight of the liquid DEC must be expressed by the rectangle EH.HN, and thus, if joined together by a horizontal tube *b*B, and the rectangles EH.HN and *he.et* were equal, the liquids remain in equilibrium; because thence it happens, where the heights of the liquid EC, *ec* were in inverse proportion with the densities or specific gravities (§.33.) *he,* HE.

DE VIRIBUS ET MOTIBUS CORPORUM

LIBERS SECUNDUS.

**DE
CORPORIBUS FLUIDIS.**

SECTIO I.

De Viribus Fluidorum a Gravitate.

Si corpora fluida cum duris & consistentibus in hoc conveniunt, quod extensa sunt & particulis seu moleculis duris componuntur; ab iisdem tamen differunt, quod moleculae fluidorum non solum exilissimae sunt eousque , ut omnem oculorum etiam microscopiis armatorum aciem fugiant, sed etiam disjunctae & vaIde mobiles, quae cuilibet corpori solido trajicienti facile cedere atque varie inter se agitari queunt: cum ex adverso dura corpora constent partibus sibi invicem varie implexis atque adeo aegre separandis ita ut nulla particularum sensibiliter moveri possit, quin totum partium aggregatum, hoc est corporis Massa eundem motum participet.

238. Est igitur *Corpus Fluidum, cujus elementa* (*§.14.*) *a vi quacunque in ea agente commoveri possunt quantumque tota fluidi Massa oculorum judico quescere videatur,* ut jam supra §.25. dictum. Sic Aqua, Aer, Olcum, Hydrargyrus &c. sunt corpora fluida, item metalla fusa, quandiu in fluore sunt.
Adeoque distinctus fluidorum corporum conceptus summam involvit molecularum exilitatem, talemque earum configurationem, ut par inde agilitas provenire queat. Et hisce specificis proprietatibus fluida distinguuntur a solidis in minutissimas itidem particulas discerptis, non tamen tantae agilitatis nec parvitatis, ut eadem qua fluidi elementa inter se agitari queant facilitate. Sic frumenta in subtilissimum pollinem contrita in corpus fluidum ideo non abeunt, quia hujusmodi pollinis particulae ad eam exilitatem non accedunt; nec proinde ad eam mobilitatem perveniunt quam fluidorum natura requirit.

239. Cum de fluidorum viribus agere suscipimus, non ea nobis est mens, ac si particularum seu elementorum figuras definiri atque digito, ut ita dicam, monstrari posse crederemus, nec proinde curiosius in has elementorum corporum figuras inquiram, quia hae nimis forte variare solent quam ut commode redigi possint sub mathematicos conceptus; nil enim impedire existimo, quominus unius ejusdemque fluidi particulae tam ratione magnitudinis, quam ratione figurae, infinitis modis variare possint. Idcirco figurarum indagationem, quibus cujuslibet fluidi particulae circumscriptae esse debent, Physicis relinquam, mihique deinceps sufficiet nosse, figuras particularum cujusque fluidi, qualescunque eae fuerint, mobilitati earum nihil officere ipso facto, cum particulae sint (secundum hypothesin) alicujus fluidi, id est summe mobiles.

240. Nec etiam ad institutum nostrum pertinet anxie disquirere, num vera sit illorum opinio, qui fluidis quibuslibet motum quendam, quem *intestinum* vocant, tribuunt, quo fluidi particulae variis irregularibus motionibus ultro citroque cieri finguntur; ad distinctionem motus progressivi fluidi, quo tota ejus massa de loco in locum transfertur. Sic cum labitur amnis, motus quo aqua in alveo suo ad inferiora devolvitur, est progressivus; motus vero aquae calidae, id est internus motus ejus molecularum, vocatur motus intestinus: exemplum adduco aquae calidae, quia certo certius ejusmodi motu intestino particulae ejus agitantur, etsi motus ipse in oculos non incurrit; atque adeo tota aquae massa quiescere videtur. Num vero omnia omnino fluida tali motu intestino agitentur philosophis pariter, ut dixi, expendendum relinquam; nec enim animus mihi est Philosophicis controversiis me quoquo modo irretire.

241. Corpora fluida a liquidis differunt ut genus a specie, cum liquida quidem fluida sint, non tamen vice versa. Liquida enim sunt, quae in volumine non nimium exiguo fluunt donec eorum superficies in situm horizontalem se composuerit ; unde, quia haec proprietas non flammae nec aeri aut aetheri competit; ideo haec corpora tantum fluida non vero liquida dicuntur. Nos in sequentibus etiam liquoribus. fluidi nomen tribuemus, quia revera liquores sunt corpora fluida : idcirco

242. *Fluida homogenea,* seu *uniformiter gravia* sunt, quorum densitas per universam massam uniformis est, adeo ut pondera ipsorum absoluta massis eorum proportionalia sint. Talia sunt praeterpropter omnes liquores nobis cogniti.

243. *Fluida heterogenea,* seu *difformiter gravia,* sunt, quorum densitas per universam fluidi massam non eadem est nec propterea pondera fluidi massis proportionantur. Tale fluidum est athmosphaera, cujus partes altiores , hoc est a terrae superficie remotiores rariores comperiuntur; etenim experimenta aera in montium editiorum jugis rariorem produnt quam in vallibus. Variae vero athmosphaerae densitates variis lineis exprimi possunt, atque hinc nascuntur,

244. *Scala densitatum* cujusque Fluidi heterogenei est figura mixtilinea, cujus ordinatae horizonti parallelae fluidi densitatem exponunt in planis illis, in quibus ordinatae existunt. Idcirco axis scalae, ordinatas ad angulos rectos excipiens, horizonti perpendicularis erit.
240. *Densitas media* cujusque fluidi heterogenei est densitas uniformis alicujus fluidi homogenei, quod in eadem altitudine cum fluido heterogeneo eandem pressuram exerere valet in subjectum sibi planum horizontale ac fluidum heterogencum. Talis media densitas innotescit applicando aream scalae densitatum ad ejus axem, seu fluidi altitudinem super plano, quod ejus pressuram subit.

246. *Scala pressionum* seu *gravitationum* liquoris heterogenei est figura, cujus ordinatae exponunt gravitationes seu pressuras fluidi in plana horizontalia exercitas, in quibus ordinatae positae sunt, abscissae vero ordinatis respondentes designant distantias planorum, quae pressiones subeunt a suprema liquoris superficie. Scala pressionum etiam definiri posset, quod fit quadratrix scalae densitatum; quandoquidem

postmodum (§. 258.) demonstrabitur areas scalae densitatum homologis ordinatis scalae gravitationum proportionales existere.

247. *Liquores in statu manente,* seu *in aequilibrio consistere* dicuntur, cum nulla liquoris pars vicinarum actione situ suo expellitur, sed omnes partes perfectum inter se aequilibrium servant.

248. Infinitesimae plani cujusque aut etiam superficiei particulae gravitationem vel pressuram cujusvis liquoris subeuntes *puncta* deinceps a nobis compendii gratia nominabuntur.

Pressionis vero vel *gravitationis* nomine intelligimus impressionem, quam fluidum gravitate sua in sibi subjectum planum exerit. Idcirco, ut rem generaliter possimus tradere, prout in primo libro tantum non ubique id fecimus, fluida utcunque heterogenea esse concipimus, atque quaenam pressiones atque effecta ab ejusmodi fluidis resultare debeant, generalibus theorematibus complectemur; ex quibus deinceps omnia derivari poterunt, quae ad liquores homogeneos spectant.

CAPUT I.

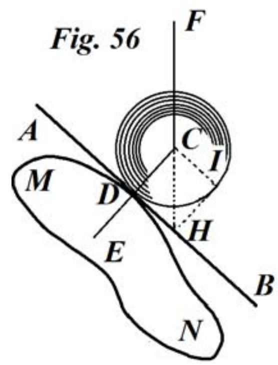
De generalibus legibus gravitationis Liquorum in subjecta plana.

PROPOSITIO I. LEMMA.

249. *Pressiones, quas corpora quaecunque solida vel fluida in se invicem exercent, fiunt juxta directiones communi plano contingenti corpora perpendiculares, atque transeunt per contingentiae punctum eorundem corporum.* Fig. 56.

Sit MN corpus quodcunque, in quod aliud corpus CD juxta directionem FC impingat; per commune contigentiae punctum D utriusque corporis transeat planum AB, ambo corpora in hoc eodem puncto D tangens, quod ideo commune eorum contingentiae punctum vocatur. Dico impressionem impellentis CD in patiens MN exeri tantum juxta directionem CE plano contingentiae AB perpendicularem, quae per commune utriusque corporis punctum contactus D transeat.

Fig. 56



Demonstr. Producatur FC usque ad occursum H cum plano AB, & circa diametrum CH descriptum intelligatur parallelogrammum rectangulum CDHI, cujus latera CD, IH plano AB normalia & CI eidem aequidistans sunt. Jam (§. 39.) conatus seu pressio juxta CH aequipollet impressionibus juxta CD & CI. Verum, quia CI plano AB parallela est, nulla in planum istud impressio juxta hanc directionem exeri potest, sed duntaxat impressio juxta CD plano perpendicularem tota in hoc planum redundabit. Ergo impressio, quam corpus impellens C, in planum AB seu in corpus MN exerit tantum fieri potest juxta directionem CD plano AB perpendicularem, quae per punctum contactus D transit. Quod erat demonstrandum.

PROPOSITIO II. THEOREMA.

250. Fig. 57. *Liquor vasi cuicunque inditus non potest in statu manenti consistere, priusquam ejus superficies situm horizontalem acquisiverit.*

Sit vas BAC figurae cujuscunque & quomodocunque positum dico liquorem ei infusum HGIA non posse situm manentem habere, nisi superficies ejus HGIF horizontalis fuerit. Habeat enim haec superficies in statu manenti positionem EGCF horizonti inclinatam: quo posito, quia liquidi moles GCIF plano horizontali HGIF imminens gravis est, atque in spatium GHEF defluere potest, nec quicquam adest quod hunc fluxum impedire queat, ea actu defluet spatium illud GHEF repletura; idcirco cum liquoris superficies GCE plano horizontali HGF inclinata est, non est in statu manenti, contra hypothesin.

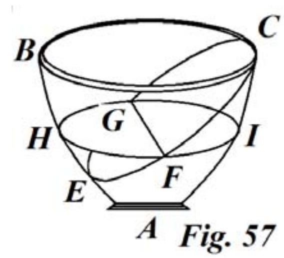


Fig. 57

COROLLARIUM.

251. Hinc si universa terra Oceano circundata esset, nec in se ipsam converteretur ejus superficie perfecte sphaerica foret. Sed quia motu diurno circa centrum suum revolvitur, aquae superficies sphaerica non erit, sed figuram habebit sphaeroidis latae, quia aquae partes quo propiores sunt aequatori, eo majorem habent conatum a centro recedendi, atque adeo minus graves sunt, quandoquidem conatus centrifugi tanto magis de corporum gravitate detrahunt quanto majores sunt. Ob hanc vero gravitatis inaequalitatem accidit, ut ad obtinendum partium aequilibrium, ad eandem ubique altitudinem aquae pervenire. nequeant, ut alias fieret, si terra motu illo diurno, ex quo conatus centrifugi partium aquae resultant, careret, sed sub aequatore altissima & in polis maxime depressa existat.
Hinc est, quod Newtonus, Hugenius aliique telluris superficiei figuram alicujus sphaeroidis assignarint, oriundae ex revolutione alicujus Ellipseos circa axem minorem, existente axe majore diametro aequatoris terrestris. Sed de hisce suo loco sermo recurret.

PROPOSITIO III. THEOREMA.

252. Fig. 58. *Partes aequales* (HI, IK) *cujuslibet plani horizontalis* (FG) *intra liquorem quemcunque* (DBCE) *in statu manenti consistentem, vasique,ut libet irregulari,* (ABC) *inditum, aequales pressiones subeunt ab imminente liquore* (DFGE). *Et partes minime laterum vasis* (LF, *MG) ad idem planum horizontale* (FG) *terminatae easdem pariter pressiones* ab *incumbente liquore patientur juxta directiones* (FP, GQ) *iisdem partibus* (FL, GM) perpendiculares, *quas patiuntur aequales particulae* (FR, GK) *plani horizontalis juxta directiones ipsis normales.*

I. Quoniam (secundum hypothesin) liquor vasis est in statu manenti, superficies ejus DE (§. 251.) horizontalis ac proinde plano FG parallela erit, & remoto cogitatione liquore

DFG plano FG incumbente, residui in vase FBCG superficies FG etiam nunc in statu manenti consistet, quia horizontalis est. Adeoque si, restituto quem cogitatione removimus liquore DFGH, pars aliqua IK plani FG minorem pressionem subiret quam contigua & aequalis particula HI, liquor in loco IK deberet attolli, aut deprimi, vel ad latera dextrorsum aut sinistrorsum deflecti sive alicubi condensari, quandoquidem vis minor majori semper cedere debet, & aquae vel liquoris particulae per naturam fluidi cedere possunt, atqui talia sine motu partium, quo unae ab aliis situ suo pelluntur, contingere nequeunt, ergo liquor non maneret in statu manenti, contra hypothesin. Partes igitur HI, IK omnesque reliquae aequales plani FG, aequales pressiones sustinent. Quod est primum.

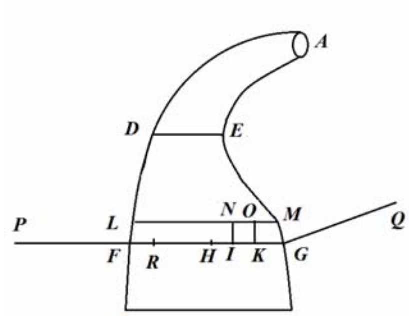


Fig. 58.

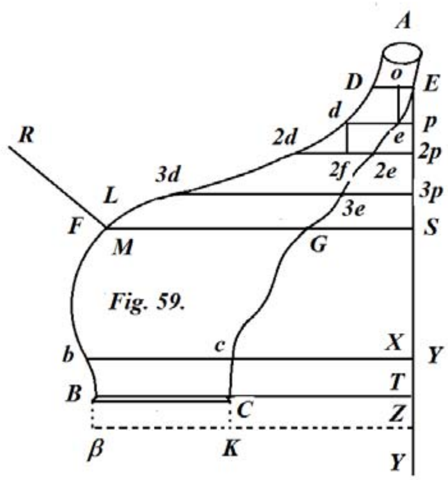
II. Sit planum LM alteri FG aequidistans , & (§. 250.) omnes ejus aequales particulae NO, OM aequales itidem pressuras ab imminente liquore DLME subibunt. Idcirco liquor inter duo plana aequidistantia LM & FG, velut in praelo, fortiter constringitur & quidem viribus aequalibus in partes oppositas, scilicet a liquore incumbente DLME perpendiculariter deorsum atque a reactione plani inferioris FG perpendiculariter in altum; nam quanta vi planum istud premitur, tanta vi in rem prementem reagit. Unde cum plana NO & IK aequalibus viribus in oppositas partes urgeantur, necesse est ut in quolibet plano NI vel OK tantam pressuram subeat quantam in NO vel IK, alioqui si minorem pateretur, ibi elaberetur per planum NI liquor ONIK contra hypothesin. Et quia ob contiguitatem partium aquae particulae laterales LF & MG vasis pressuras subire debent aequales illis, quas subeunt plana NI vel OK, & haec pressiones pares illis, quas plana NO, IK, &c. patiuntur, liquet pressuras liquoris DFGE in particulas laterales vasis FL & MG aequales esse pressionibus, quas ab eodem liquore patiuntur particulae illis aequales HI, IK plani horizontalis FG, (§.249.) juxta directiones FP, GQ particulis illis FL & GM perpendiculares, perinde ac particulae HI & IK. &c. itidem perpendicularibus plano FG directionibus urgentur. Quod erat demonstrandum.

PROPOSITIO IV. THEOREMA.

253. Fig. 59. *Pressiones seu Gravitationes cujuslibet liquoris homogenei in quaecunque plana horizontalia proportionales sunt altitudinibus liquoris super his planis.*

Sit DE superficies liquoris DBCE vasi cuilibet ABC infusi, fundoque vasis BC horizontali, quantum opus est, protenso demittatur ad cum perpendicularis ET, intelligatur insuper quodvis planum FG fundo BC & DE aequidistans, quod productum occurrat rectae ET in puncto S. Pressiones liquoris in plana FG & BC exercitae designentur per $pr.\mathrm{FG}$ & $pr.\mathrm{BC}$, probari debet fore $pr.\mathrm{FG} : pr.\mathrm{BC} = \mathrm{ES} : \mathrm{ET}$. Nam ES est altitudo liquoris DFGE super plano FG, & ET altitudo liquoris super BC.

I. Sit ES alteri ET commensurabilis, & E*p* communis utriusque mensura tantae magnitudinis, ut ducto per punctum *p* plano *dp*, intra spatium vasis D*de*E duci queat rectula *eo* parallela E*p* vel perpendicularis ipsis DE & *de,* quod semper fieri potest quantumvis irregulare & superius angustum esse queat ipsum vas ABC. Factis porro singulis *p*2*p,* 3*p*3*p,* &c. aequalibus ipsi E*p,* ductisque per singula divisionis puncta 2*p,* 3*p*, ,&c. planis 2*d*2*p,* 3*d*3*p*, &c. quibus positis, & quia puncto *e* primi strati liquoris imminet *eo,* hoc punctum *e* tantam pressuram sustinebit, quantum est pondus columnulae *eo,* & (§. 252.) omnia reliqua plani *de* puncta pressionem patientur aequalem pressioni puncti *e;* punctum 2*f* in basi 2d2e secundi strati *e*2*d* patietur pressionem compositam ex pressiem, qua punctum ejus supremum d urgetur, & ex pondere columnulae *d*2*f* aequalis ipsi *oe,* atque adeo gravitatio liquoris in punctum 2*f* & omnia reliqua, plani 2*d*2*e* erit dupla ipsius in plano *de*, & propterea aequivalet ponderi columnarum $oe + d2f$, hoc est E2*p* ; simili argumento conficietur pressionem liquoris in quodlibet punctum plani 3*d*3*e* aequivalere ponderi columnae ipsi E3*p* aequalis in altitudine; hinc pressiones seu gravitationes liquoris in plano FG & BC aequivalebunt columnis, quarum altitudines sunt aequales lineis ES & EF, ergo erit omnino $pr.\mathrm{FG} : pr.\mathrm{BC} = \mathrm{ES} : \mathrm{ET}$.



II. Sit ES alteri ET incommensurabilis , &, si fieri potest, ratio *pr.*FG ad *pr.*BC major quam ES ad ET, puta aequalis rationi ES: EX cujus consequens EX minor quam ET defectu XT, ex quo defectu auferri poterit quaedam TV minor quam XT, ita ut EV commensurabilis fiat ipsi ES. Quo posito, per punctum V planum *b*V transire intelligatur, eritque *pr.bc* minor quam *pr.*BC, cum EV minor sit quam ET, & $pr.\mathrm{FG} : pr.bc > pr.\mathrm{FG} : pr.\mathrm{BC}.$ Jam quia (constr.) EV commensurabilis est ipsi ES, erit per partem primam hujus, $pr.\mathrm{FG} : pr.\ bc = \mathrm{ES} : \mathrm{EV}$, & (secundum hypothesin) est $pr.\mathrm{FG} : pr.\mathrm{BC} = \mathrm{ES} : \mathrm{EX}$, ergo $\mathrm{ES} : \mathrm{EV} > \mathrm{ES} : \mathrm{EX}$, atque adeo EX major quam EV, contra hypothesin adeoque nequit *pr*.FG:*pr*.BC major esse quam ES: ET.

Sit igitur ratio *pr*. FG: *pr*.BC minor quam ES:ET, scilicet aequalis cuidam ES:EY cujus consequens EY major quam ET, excessu TY, auferatur iterum ex hac TY portio YZ talis ut residua EZ alteri ES commensurabilis evadat, per Z transire fingatur ·planum $Z\gamma\beta$ fundo BC parallelum in quo $\gamma\beta$ instar fundi nunc considerabimus. Unde quia EZ (constr.) major est quam AT, erit $pr.\beta\gamma > pr.\mathrm{BC}$, adeoque $pr.\mathrm{FG} : pr.\beta\gamma < pr.\mathrm{FG} : pr.\mathrm{BC}$; & quia (constr.) EZ commensurabilis est ipsi ES, erit (num. 1 hujus) $pr.\mathrm{FG} : pr.\beta\gamma = \mathrm{ES} : \mathrm{EZ}$, ex hypothesi vero est $pr.\mathrm{FG} : pr.\mathrm{BC} = \mathrm{ES} : \mathrm{EY}$, ergo $\mathrm{ES} : \mathrm{EZ} < \mathrm{ES} : \mathrm{EY}$, eritque adeo EZ major quam EY, iterum contra hypothesin, nam EZ necessario minor erat quam EY, quandoquidem ex hac EY ablata ZY relinquebatur EZ, nequit ergo ratio *pr*.FG: *pr*.BC minor esse quam ratio ES: ET & quia nec major, sequitur *pr*.FG: *pr*: BC= ES: ET. Quod erat demonstrandum.

COROLLARIUM I.

254. Hinc ultro sequitur, quod quaelibet portio FM plani cujuslibet horizontalis FG ab incumbente liquore DFGE tantam pressionem subire debeat quantam sustineret a columna ejusdem liquoris portioni illi FM ad altitudinem ES perpendiculariter imminentis. Nam, quod in propositione universaliter ostensum est; nulla habita ratione figurae vasis, idem valet etiam de vasis cylindricis , in quibus illico liquet gravitationes liquoris altitudinibus ejus proportionales esse. Unde, si altitudines liquoris super plano aliquo horizontali in vase prismatico & in vase irregulari utrinque aequales sint, aequales utriusque plani partes aequales pressiones sustinebunt. At in vase prismatico quaelibet cujusque horizontalis plani portio sustinet pondus totius columnae ipsi perpendiculariter imminentis, atque adeo portio quaecunque FM plani FG tantum onus sustinebit, quantum est pondus alicujus prismatis, cujus basis FM & altitudo ES. Adeoque totum planum FG sustinebit onus aequale massae liquoris in prisma conformatae, quae planum FG pro basi habeat & ES pro altitudine.

COROLLARIUM II.

255. Et quia supra (§.252.) demonstratum est, aequales particulas quam minimas FM & FL plani FG & lateris vasis, aequales pressiones sustinere, unamquamque juxta directonem sibi normalem, scilicet FM verticaliter deorsum, & FL juxta FR superficiei BFD in F perpendicularem, hinc sequitur *quamlibet particulam minimam in vasorum lateribus pressuram subire debere ab incumbente liquore aequalem ponderi columnae, cujus basis est particula, quae pressionem sustinet, & altitudo est distantia particulae a liquoris superficie.*

Atque haec est generalis hydrostaticae regula ex suis genuinis principiis eruta, quae etsi tantum in liquoribus homogeneis valere videtur, sese tamen ad quoscunque liquores, etiam heterogeneos, extendere in sequentibus probabitur.

SCHOLION .

256. Fig.60; Ex his ergo liquet, quod liquores homogenei gravitant in sua subjecta plana pro ratione altitudinis eorum super planis; in quae gravitant. Ex qua fluidorum proprietate sequens nascitur paradoxum ; quod scilicet parva liquoris cujusque copia tantundem in sibi subjectum planum gravitet, quantum ejusdem liquoris massa centies imo millies major iri eadem altitudine. Etenim si vasi acuminata BAC, cujus basis BC notabilis sit amplitudinis, aqua infundatur pertingens usque in A, & per Corollarium I. hujus, aqua tantam pressionem in fundum BC exeret, quantam exereret cylindrus aqueus IBCK in eadem altitudine cum vasis liquare. Idcirco pauxilla aqua in tubo acuminata gracilique ABC tantum effectum praestare potest, quantum aquae moles centuplo, imo pluries, major in eadem altitudine, quod haud dubie non paucis paradoxum videbitur. Ejus tamen veritas ipsa experientia comprobata est, atque deinceps probari potest ; nam si fundo vasis stricti ABC orbiculus *a*BC*f* inditus sit vasi tam

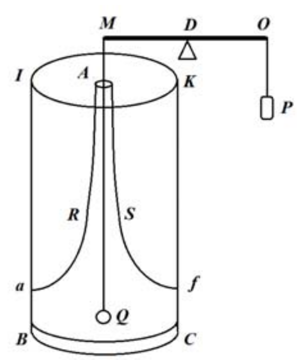


Fig. 60

accurate quadrans, ut nullas rimas relinquant, per quas infusa aqua infra orbiculum transire queat, in centro Q affixum habens filum QM quo attolli possit, cujus fili alteram caput staterae MDO annexum sit in M; ex altera bilancis parte applicetur pendus P eousque augendum minuendumve dum orbiculus B*f* non nihil attollere incipiat stateram MO eam in partem flectendo, cui pondus appensum est; reperieturque hoc pondus P orbiculum paulisper attollere valens satis accurate aequari ponderi cylindri aquei BIKC, existentibus jugi brachiis DM, DO aequalibus. Unde cum pondus P tantillo majus esse debeat resistentia, quam in parte opposita superare potest, & haec resistentia, & haec resistentia sit ipsa paucae aquae ABC gravitatio in orbiculum *a*BC*f*, omnino concludi debet hanc aquae ABC gravitationem aequivalere gravitationi cylindri aquei I*af*K.

257. Sed inquies forte, si aqua ABC tantam gravitationem in suam basim exerceret, sequeretur fore, ut vas cum infusa aqua tantum ponderare debeat, quantum cylindrus aqueus IC auctus pondere vasis. Nam vas cum infusa aqua lanci impositum atque erecto situ detentum tantum ponderare debet, quanta vi lancem premet, sed lancem urgebit ea vi, qua basis vasis ab interno liquore urgetur aucta pondere vasis, hoc est pondere cylindri KB & vasis simul: interim tamen, si experimentum capiatur, comperietur semper pondus totius non fore majus quam infusae aquae & vasis pondera simul sumta, atque adeo regula nostra hydrostatica phaenomenis adversari videtur. Sed objectio falsae innititur hypothesi, quasi liquoris ABC in basin gravitatio etiam tota redundare atque exeri debeat in librae lancem; quod tamen est falsum nam, si fundus BC urgetur onere IBCK a solo liquore ABC etiam latera tubi acuminati BRA & CSA in altum prementur, nam unumquodque horum laterum punctum tanta vi premitur (§.255.) quantum est pondus filamenti aquae aequalis distantiae puncti ab aquae superficie, juxta diretionem superficiei vasis perpendicularem, adeoque applicando ea, quae supra (§.81.) generaliter & in abstracto ostensa sunt, casui praesenti, comperietur praedicta illa latera BRA, CSA vel potius internam tubi acuminati superficiem convexam, tanta vi a liquore verticaliter in altum agi, quantum est pondus aquae illius, qua cylindrus IC aquam in tubo ABC excedit. Unde, quia latera vasis & fundus cohaerent, ea sola vis in lancem librae MO redundare potest, qua gravitatio aquae ABC in basin BC excedit vim attollere conantem latera BRA & CSA vasis ABC ab aquae pressionibus provenientem, aucta pondere vasis; vis vera illa, seu excessus cylindri IC supra massam aquae, quae modo nominatam vim attollentem exponit, est pondus solius aquae in tuba acuminata ABC , ergo vis, qua lanx deprimi atque urgeri debet, est solum pondus aquae ABC auctum pondere vasis, prorsus ut experientia id manifestat. Idcirco tantum abest, ut objectio ab experientia petita vim propositionis nostrae quicquam infringat ut eam potius egregie confirmer.

PROPOSITIO V. THEOREMA.

258. Fig. 61. *Gravitationes quorumcunque liquorum in subjecta plana horizontalia, sunt areis homologis in scala densitatum vel etiam ordinatis scalae pressionis liquorum proportionales.*

Sit vas quodcunque irregulare ABC liquoris heterogenei plenum usque ad DE, & circa

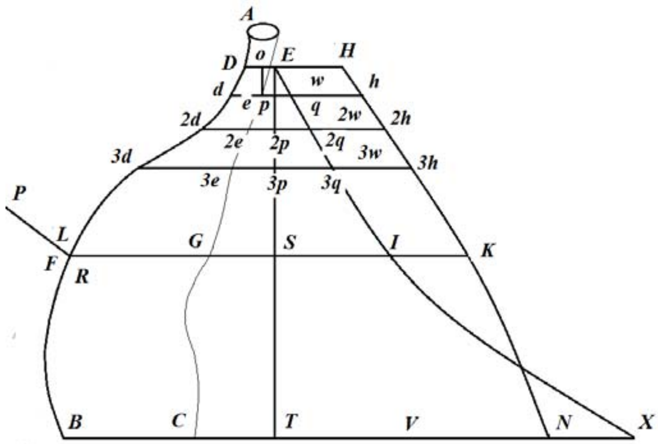


Fig. 61.

axem ET superficiei liquoris DE fundoque BC perpendicularem, atque adeo fluidi altitudinem super fundo designantem, extructae sint curvae HKN & EIX, quarum illa sit scala densitatum liquoris, utpote cujus ordinatae EH, 3*p*3*h,* SK, TN densitates liquoris exponunt in illis planis, in quibus ordinatae reperiuntur, haec vero EIX scala pressionis liquoris, cujus scilicet ordinatae 3*p*3*q*, SI, &c. gravitationes liquoris denotant in plana 3*d*3*e*, FG, &c. Ac denique area EHKNT applicata ad ejus altitudinem ET praebet mediam densitatem TV fluidi heterogenei. Ordinatae vero scalae gravitationum EIX (§. 246.) areis homologis scalae densitatum HKN proportionales sunt, ultimaque TX toti axi AT aequalis esse ponitur. Quibus intellectis probandum est, fore gravitationem liquoris DFGE in planum FG, ad gravitationem liquoris DBCE in fundum BC, ut area ESKH ad aream EMKH, vel sicut SI ad TX, &c.

Vasis liquor divisus intelligatur, ut in praecedenti, in sua strata horizentalia aeque alta quidem omnia, sed infinitesimae altitudinis, per plana *de,* 2*d*2*e,* 3*d*3*e;* &c. fundo BC aequidistantia. Quia igitur cujusque strati altitudo insensibilis seu indefinite parva est, densitas liquoris per totum stratum tanquam uniformis spectari potest, scilicet EH significabit densitatem primi strati *de*ED*,* ordinata *ph* densitatem uniformem secundi 2*d*2*ed*, atque ita porro de reliquis. Jam puncto e primi strati perpendiculariter imminet columnula seu filamentum eo, ejusque adeo pondus sustinebit: atqui pondus ipsius *eo* (§. 33) est ut ejus volumen & densitas seu gravitas specifica conjunctim, hoc est $= eo.\mathrm{EH} = \mathrm{E}p.\mathrm{EH}$ seu $=$ rec-lo areae SHK inscriptum ordinatae EH adjacens, eademque est pressio, quam subeunt omnia reliqua puncta plani *de;* pressio vero quam sustinebunt puncta baseos 2*d*2*e* secundi strati 2*d*2*ed,* erit aggregatum ponderum filamenti cujusdam secundi strati , & aeque alti filamenti *eo* in primo, quorum hoc, ut modo dictum, exponitur rec-lo *w*, illud vero rec-lo 2*w* seu *p*2*p.ph* : atque erit talis pressio in $2d2e = w + 2w$. Eodem argumento conficientur gravitationem liquoris in tertium planum 3*d*3*e* exponi aggregato rectangulorum $w + 2w + 3w$; atque adeo pressio, quam plani FG puncta sustinebunt, exponetur omnibus rectangulis $w, 2w, 3w, 4w$ quae areae ESKH inscripta sunt, atque ob infinitesimam cujusque altitudinem in hanc aream evanescunt, ita ut loco aggregati omnium area ipsa ESKH intelligenda sit , ac propterea pressio, quam planum FG a liquore incumbente subire debet, exponitur area homologa ESKH, & gravitatis liquoris in fundum BC per aream EMKH; est ergo omnino $pr.\mathrm{FG} : pr.\mathrm{BC} = \mathrm{ESKH} : \mathrm{ETNKH} = \mathrm{SI} : \mathrm{TX}$ per §. 246. Quae erant demonstranda.

COROLLARIUM I.

259. Hinc pressio, quam particula quaevis FR plani FG ab imminenti liquore patitur, aequivalet ponderi absoluto massae liquoris cujusdam homogenei, cujus densitas aequalis sit densitati mediae TV liquoris heterogenei, volumen vero prisma rectum super basi FR & altitudine aequali SI homologae ordinatae scalae pressionum SIX. Nam, juxta Propositionem praesentem, pressio, quam subit particula FR, exponi debet facto ex area SEHK in particulam FR plani FG; & (secundum hypothesin) $\mathrm{SEHK} : \mathrm{TEHKN} = \mathrm{SI} : \mathrm{TX} = \mathrm{SI.TV} : \mathrm{TX.TV}$, atque (§.245) area $\mathrm{ETNKH} = \mathrm{ET.TV}$, cum ipsa TV seu media liquoris densitas sit ea, quae oritur applicando dictam aream ad altitudinem ET, atque adeo facta, uti posthac semper factum supponemus, TX aequali ET, etiam rectangulum TX.TV area ETNKH aequetur; reperietur rectangulum SI.TV ubique aequale homolologae areae ESKH, & factum ex hac area in FR, exponens gravitationem seu pressionem in hanc particulam, erit aequale facto SI.TV.FR, quod designat prisma, cujus basis FR est planum pressionem sustinens; altitudo vero SI ordinata scalae gravitationum, ductum in mediam densitatem TV liquoris heterogenei: atqui (§. 33.) prisma SI.FR ductum in TV exponit pondus liquoris homogenei, cujus densitas est TV & volumen ipsum prisma SI.FR; ergo particula FR plani FG ab incumbente liquore heterogeneo pressionem subit aequalem ponderi prismatis fluidi, cujus basis sit ipsa particula pressionem sustinens, altitudo ordinata homologa SI scalae pressionum, & densitas uniformis TV media densitas liquoris heterogenei.

COROLLARIUM II.

260. Unde, quia aequales & minimae particulae lateris FL & contigua FR in plano horizentali FG (§.252.) aequales ab imminente liquore heterogeneo pressiones sustinent, quaelibet, juxta directionem ipsi perpendicularem, etiam FL pressionem subibit aequalem ponderi prismatis liquidi, cujus basis sit particula haec ipsa FL fluidi impressionem excipiens; altitudo vero SI ordinata homologa scalae pressionum & denique TV media densitas liquoris heterogenei.

COROLLARIUM III.

161. Si liquor est homogeneus, cujus densitas sit HE scala densitatum fiet recta axi ET parallela per punctum H ducta, eritque adeo hoc casu $\mathrm{TV} = \mathrm{EH}$, & curva EIX mutabitur in lineam rectam angulum semirectum cum axe ET continentem; adeo ut iterum hinc liqueat, cujusque liquoris homogenei gravitationes altitudinibus liquoris super planis, quae pressiones subeunt, proportionales esse. Nam in triangulo rectilineo ETX ordinatae SI, TX, quae gravitationes liquoris in plana FG, BC significant, abscissis ES, ET proportionales erunt existente linea EIAE recta, qualem nunc esse supponemus, ne pro hoc casu particulari novum schema delineare necessum habeamus.

COROLLARIUM IV.

262. Sed si liquores homogenei diversae densitatis vel gravitatis specificae inter se conferuntur, gravitationes sunt in composita ratione densitatum & altitudinem; adeoque si in syphonis AB*d* crure ABC insit liquor DC cujus densitas uniformis sit EH, & altitudo super fundum aequalis HN; alteri vero cruri *ab* insit liquor *dt* densitatis *eh* vel *ne* ad altitudinum *ce* in crure assurgens, erit hujus liquoris gravitatio exprimenda rectangulo *he.et,* & liquoris DEC gravitatio exponi debet rectangulo EH.HN, adeoque, si tubi per horizentalem *b*B communicantes, atque rectangula EH.HN, *he.et* aequalia fuerint, liquores in aequilibrio consistent; quod proinde continget, ubi altitudines liquoris EC, *ec* densitatibus seu (§.33.) gravitatibus specificis *he,* HE fuerint reciproce proportionales.