CHAPTER VI.

Concerning the rules of motion in the collision of bodies.

DEFINITIONS.

I.

Two bodies are said to be running to meet each other when they are moving along the right line joining the centres of gravity, and passing through the point of contact of the same bodies.

II.

The *proper velocities* of the bodies are those with which the bodies are moved. Indeed the *relative velocity* of the same is that, with which they approach each other. Therefore the relative speed of the bodies meeting from opposite directions mutually opposed to each other is the *sum* of the proper speeds of the bodies. But truly if a slower body may follow a swifter body in the same direction along a line, the relative velocity of these will be the *excess* of the greater proper velocity over the lesser proper velocity.

III.

212. The bodies A, B are understood to be moving in a horizontal plane and to collide with each other according to this, so that the motions of these before and after the meeting may emerge equal, nor with the same motion acted on by gravity : as it is hindered by the plane, in which the moving bodies approach, in whatever way they may be altered. The proper speeds of the bodies before and after the meeting will be designated by the right lines AD, BD in the directions of these assumed, thus so that in whatever manner the moving bodies A and B approach, provided that they are not carried along on lines at an angle, the relative speed of these shall be AB. Truly the directions of the particular bodies, or rather the directions of the impact, may be designated henceforth by the order in which the letters A, D and B, D are written. Hence AD indicates the moving body A to be carried with a uniform speed AD from A towards D, and B to be carried with a uniform speed expressed by this line BD from B also towards D. And thus whenever the letter D falls between A and B, both these bodies likewise are moving in the contrary sense, that is, they come together from opposite directions both with the relative speed AB, which in this case has been composed from the particular speeds AD, BD; but truly if the letter D falls outside AB, both the bodies will be moving in the same direction, in which the letter D has been put equal to the relative speed AB in such a case with the proper velocity of the one body more than the proper speed of the other.

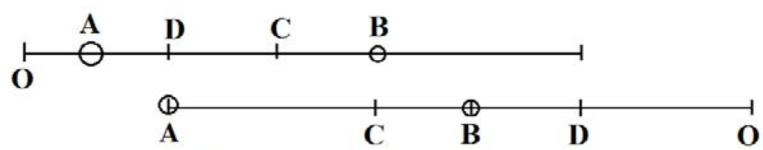


Fig. 48.

213. And because the directions of the motions also are accustomed to be designated by the following signs + & −, it is to be understood properly +AD to be the same as − DA, and + DA the same as − AD, and thus etc. For the motion +AD is contrary, or is directed in the opposite direction, as − AD, as well as DA also, and thus these two expressions are equivalent *;* equally because both + DA and −AD are opposite to the motion + AD, the notations + DA & −AD signify one and the same thing, thus so that one will suffice always for the other, which finally understood well, will be verified to be useful.

IV.

214. We will consider the speeds of bodies whenever before colliding with each other as acquired by falling vertically beginning from rest with the natural acceleration of gravity ; and the speeds of the same after meeting each other as the initial speeds of the bodies, which carried vertically at a height with the motion slowed by the natural retardation prevail to pass through certain heights before the motion of their ascent is destroyed completely.

V.

215. The absolute *power* of the bodies before the impact is the height, [*i.e.* originally *vis* in Latin , usually in the sense of military strength or power; the concept of energy was known only in a intuitive way at this time, and the actual word was introduced by Thomas Young a century later from the Greek *ενεργον* , meaning 'active or busy' : see *A Greek Lexicon*, p. 478, Liddel & Scott; of interest perhaps also, this word in Greek is the opposite of our word, 'inertia' also derived from the Greek ; Leibniz made use of the term *vis viva*, or living force, to describe the quantity we now call kinetic energy, but without the factor of ½ present] which the common centre of gravity of the moving bodies is able to pass through, when each body by falling through its height acquires its velocity. And the absolute power of the moving bodies is the height, to which the same common centre of gravity of the bodies can rise, when each of these bodies with that initial speed as it has acquired in the collision, can spend in ascending to its height.

216. Thus with the moveable body A to acquire its particular speed AD by falling with an acceleration from the height EA, and B its particular speed by falling with an acceleration from the height FB, the centre of gravity of the mobile bodies A and B at the points E , F arising at the point G, falls from the height GC. And if the speed of the mobile body A after the collision shall be of such a uniform amount D*a* , so that as it were, the mobile A would be able to cause it rise to the height *ae* with that initial speed, and the other B rises to the height *bf* with the initial speed *Db,* which is the uniform speed, which that mobile body B itself, has acquired in its collision ; the common centre of gravity *c* of the bodies A and B is

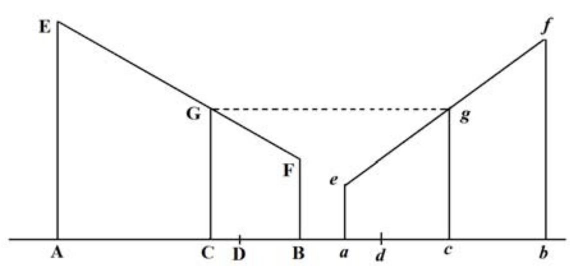


Fig. 50.

considered to be able to ascend by the absolute power of the bodies after the collision along the right line *cg* as far as to *g*.

VI.

217. Bodies, which are endowed with the force of inertia, thereafter may be called inertial bodies. Truly bodies, which have the active power of repelling bodies by which force they themselves are impelled, may be said to be active bodies. Inertial bodies otherwise are accustomed to be called *inelastic*, and active bodies *elastic*.

HYPOTHESIS.

218. *When elastic bodies are collided together the same absolute power remains between themselves after the collision, which there was before the same collision.*

We disregard inelastic bodies, the absolute power of which is never the same after the collision, as it was before the collision, since this power is used up completely in the collision itself, clearly since the moving bodies meet against each other with speeds in turn inversely proportional to their masses. But perhaps there are no bodies given of this kind with complete inelasticity, for generally even the most elastic bodies may not be endowed with perfect elasticity, because the compression of these may not prevail to be restored to its former state, perhaps the elements of these are able to be perfectly elastic, to which being considered none of the more outstanding geometers of this age appear to abhor. Truly whatever shall be the cause of this, it is restored for elastic bodies, whose absolute power remains the same according to the powers of this hypothesis, if the common centre of gravity of these rises to the same height *cg* after the collision, from which the fall was before the running together of these, that is, with which position above (§. 216.), if $cg = \mathrm{GC}$.

219. We estimate the absolute powers of bodies against the heights, to which on being slowed by their weight they are able to attain by their motion, or more accurately by the product from their heights by the masses of the bodies; in this estimation of the powers we have the preceding judgment of the illus. Leibniz, who indeed has indicated the same approach in more than one place in the *Actis Eruditorum Lips*., which yet he did not prove, even if it an explanation could be provided, as perhaps we will show that on another occasion. I do not know anyone to be more significant, for which the account as such forces requiring to be estimated may not raise a smile, judging that from the quantities of motion being desired. Certainly the Celeb. Papin disputed with Leibniz elegantly for a long time, who denied the powers of bodies to be in proportion to the heights which bodies are able to attain by rising [This was an argument about the *vis viva* or kinetic energy as we know it, taking a different form, which Denis Papin, an early inventor of the steam engine, denied in its Liebnizian form] ; he thought why not indeed should such powers not be proportional to the times in which the said heights were reached, or even equal to the initial speeds of the moving bodies. But plainly he assumes a false principle by making an incautious slip himself in trying to prove his assertion ; and this is the false principle, because he considers that bodies with different initial speeds to be rising in equal times to equal heights from gravity itself, with the air resistance removed. [This does not correspond with what Papin himself said in his

correspondence with Leibniz at one stage : *if we are dealing with bodies which are raised to a certain height by a motion acquired through a previous descent from the same height, we cannot assume that these heights are proportional to the vires motrices, since the forces are diminished by the resistance they must overcome, and not by the distance they traverse.*] But if several learned geometers, with the common prejudiced view of the account that the powers are to be estimated by the amount of motion retained, think differently from us, yet there are other of the most acute geometers present, who admit besides the Leibnizians, that now indeed several years ago Huygens used the same principle in a certain way in his demonstration of the laws of motion for bodies colliding, assuming these principles in place, not to be able for the common centre of gravity of bodies amongst themselves to rise higher after the collision, than it fell before the collision ; yet he did not suppose the ascent of the centre of gravity to be equal to the descent of the same; truly at other times he had assumed the hypothesis, which are equivalent to ours of the equality of the ascent and descent of the centres of gravity after and before the collision of bodies. Therefore from our established rule we will thence derive the laws of motion easily; but before we may approach this treatment, it will please us to establish a small proposition about the motion of inelastic bodies after the collision.

PROPOSITION XXXVII. THEOREM.

220. *The speed and direction of the centre of gravity of these inelastic bodies, moving directly towards each other, is the same in turn after the meeting as it was before.*
AD shall be the uniform speed of the body A, and BD the velocity of the body B before the collision, it is required to be shown the motion of each body A and B after the meeting is going to be with the speed CD, which is the speed, with which the centre of gravity C of the moving bodies A and B were being carried along before the collision, with AC being to BC, as the mass B is to the mass A, or as the weight B to the weight A of the other.

Demonst. Because the motion of bodies in a collision does not change except on account of the inelasticity of the material, and in the collision itself the bodies depart with the same, and if the bodies are joined together, and just as in turn they shall be required to be moved stuck together with the magnitude of the motion equal to the excess, by which the motion of the greater one from the bodies shall exceed the other smaller one; and if the bodies approach against each other from the opposite directions in the way to themselves in turn, or also to be equal to the sum of the motions of each body, if both tend towards the same direction. From which with the opposite motion and in the case, in which the centre of gravity C falls between the points D and B (§. 44) [recall the mass of the body and its label are taken to be the same] there shall be $B.BD - A.AD = (A + B).CD = A.CD + B.CD$, whenever $B.BD - A.AD$ shall be the excess, by which the motion B.BD of the stronger B exceeds the motion of the other A, it is apparent both bodies A and B after the contact are going to be carries from C towards

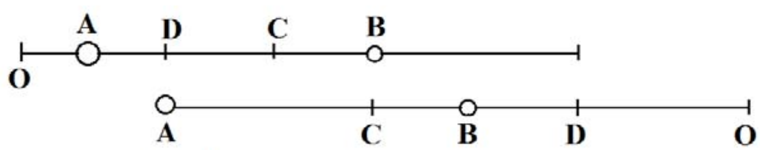


Fig. 48.

O with a speed equal to CD, that is with the speed and direction, by which the centre of gravity of these was being carried before the collision.

Truly if both bodies A, B may be moving in the same direction towards O with their particular speeds AD and BD, there will be $\mathrm{A.AD} + \mathrm{B.BD} = (\mathrm{A}+\mathrm{B}).\mathrm{CD} = \mathrm{A.CD} + \mathrm{B.CD}$, that is the sum of the motions of the bodies A and B before the collision is equal to the quantity of motion of the same, with the speed CD of the centre of gravity before the impulse requiring both to move and therefore each is carried forwards with this speed CD towards O. Q.E.D.

PROPOSITION XXXVIII. THEOREM.

221. *If the bodies* A *and* B, *moving in the oblique directions* AD *and* BD *containing some angle* ADB, *may meet each other mutually, and with the speeds for these expressed by the lines* AD, BD *themselves, the common centre of gravity of these* C *may be carried forwards after the collision with the same velocity as* CD *produced to d.* Fig. 49.
[We are assuming inelastic collisions at this stage.]

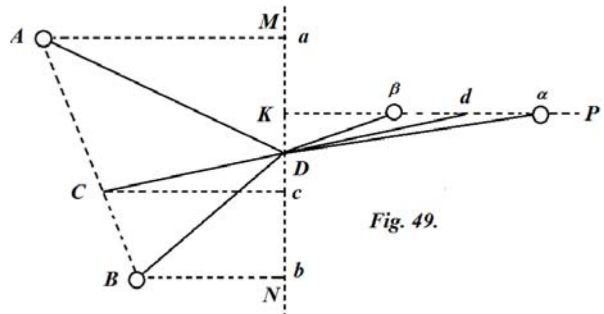


Fig. 49.

MN shall be a line joining the centres of the moving bodies A and B in the case of contact, and to that shall be dropped the normals A*a*, B*b*, & C*c* from the centres of the bodies A and B and from the common centre of gravity C of these ; the motion along AD may be resolved along the sides A*a* & *a*D, and the motion BD along the sides B*b* & *b*D; but truly, since the motions along A*a* and B*b* shall be parallel, the bodies are unable to react between themselves from these motions, therefore there remain only the motions *a*D and *b*D, by which they can act mutually between themselves. Now if the bodies A & B were present at *a* & *b*, and mutually came together with the speeds *a*D, *b*D, after the contact each will be moving with the speed (§. 220.) $\mathrm{DK} = c\mathrm{D}$, with *c* being the centre of gravity of these, since there shall be $\mathrm{AC} : \mathrm{BC} = ac : bc = \mathrm{B} : \mathrm{A}$, from which because the motions *a*D and *b*D are maintained virtually in a slanting manner [*i.e.*, this occurs in the reference frame of the centre of mass], thus this modification of the motion also is considered to have happened virtually : therefore with the indefinite line KP drawn through the point K, on that there may be taken $\mathrm{K}\alpha = \mathrm{A}a$ & $\mathrm{K}\beta = \mathrm{B}b$, and because the motions A*a*, B*b* undergo no change themselves in the collision, the body A will be moving after the collision, or will be trying to move, with the lateral motions DK, $\mathrm{K}\alpha$, from which the motion $\mathrm{D}\alpha$ emerges; truly the other body B with the lateral movements DK & $\mathrm{K}\beta$, from which the motion $\mathrm{D}\beta$ arises; *d* shall be the centre of gravity of the bodies A, B arising at α & β, and there will be (§.44.)

$\mathrm{A.K}\alpha + \mathrm{B.K}\beta = (\mathrm{A}+\mathrm{B}).\mathrm{K}d$. likewise $\mathrm{A.A}a + \mathrm{B.B}b = (\mathrm{A}+\mathrm{B}).\mathrm{C}c$, and (from the constr.) $\mathrm{A.K}\alpha + \mathrm{B.K}\beta = \mathrm{A.A}a + \mathrm{B.B}b$, therefore also $(\mathrm{A}+\mathrm{B}).\mathrm{K}d = (\mathrm{A}+\mathrm{B}).\mathrm{C}c$,or $\mathrm{K}d = \mathrm{C}c$: and by the preceding also there is $c\mathrm{D} = \mathrm{DK}$ [$\mathrm{C}c = \mathrm{DK}$ in the text], therefore $\mathrm{D}d = \mathrm{CD}$, and thus after the collision the centre of gravity of the bodies A, B will be moving with the speed *Dd* equal to that C*c,* which was incident before the meeting. Q.E.D.

Only from inelastic bodies, or with the elastic forces ignored, do the laws of motion of elastic bodies follow.
[There are a number of difficulties with this demonstration, although the principle that the centre of mass continues to move in a straight line is of course correct; the most obvious perhaps being that the bodies are assumed to lie on a horizontal line with the centre of mass after the collision: this peculiarity has arisen because a completely inelastic collision has occurred in the vertical direction, yet the horizontal motion is assumed to continue unchanged, so that the bodies drift apart after the collision horizontally, while moving upwards with the same constant vertical speed according to the laws of inelastic collisions. Normally, totally inelastic collisions are considered in one dimension; here such a collision is considered in two dimensions, in the absence of gravity. Whether such collisions as mentioned here are physically possible it is hard to say, but probably not, as the bodies have collided inelastically vertically, and as of not at all horizontally.]

PROPOSITION XXXIX. THEOREM.

222. *With the same magnitude of the absolute power* [*i.e. vis viva*] *of the colliding bodies put in place between themselves before and after the encounter, the products from the masses of the bodies by the squares of the velocities taken together before and after the collision are equal.*

That is $\text{A.AD}^2 + \text{B.BD}^2 = \text{A.}da^2 + \text{B.}db^2$, with the speeds *da, db* of the bodies A, B after the collision , and with the speeds AD and BD before the collision.

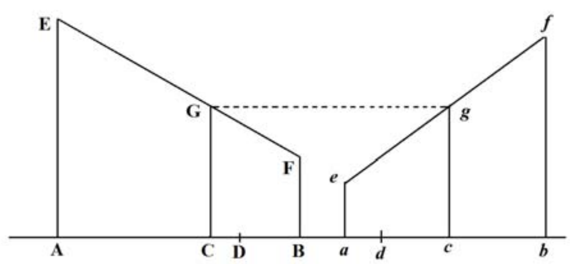


Fig. 50.

Demonstr. With A put to have acquired its speed AD in the case from the height EA, and the other B from the height FB, and with the right line EF joining the points E and F in the reciprocal ratio of the weights A and B ; GC will denote the descent of the line of the centre of gravity of the mobile bodies A and B ; and *ea, fb* will denote the heights, with which accelerated motion transmitted to the bodies they acquire the speeds *da* and *db,* with which henceforth they are able to begin to be carried high and to rise again to the same heights *ae* and *bf* ; from which motion of their ascending the centre of gravity *c* ascends through the height *cg.* Now because (§.218.) the same absolute *vis viva* of the bodies A, B remains before and after their collision, there will be $\text{GC} = cg$, or on putting C to indicate $\text{A} + \text{B}$, there will be $C.GC = C.cg$; and (§.44.) :
$\text{C.GC} = \text{A.EA} + \text{B.FB}, \ \& \, \text{C.}cg = \text{A.}ae + \text{B.}bg$, therefore also $\text{A.EA} + \text{B.FB} = \text{A.}ae + \text{B.}bf$;
truly $\text{AD}^2, \text{BD}^2, \ da^2, \& \ db^2$ are proportional to the right lines (§.150.) EA, FB*, ae, bf* ,

with which being substituted in place of these others in the nearby preceding ratio, there will be found $\mathrm{A.AD}^2 + \mathrm{B.BD}^2 = \mathrm{A.}da^2 + \mathrm{B.}db^2$. Q.E.D.

[The original text uses D*a* and D*b* for the speeds after the collision, which has been taken to mean that D and *d* are the same collision point, which of course is the case.]

COROLLARY.

223. Therefore also there will be $\mathrm{A.AC.AB} + \mathrm{C.CD}^2 = \mathrm{A.}ac.ab + \mathrm{C.}dc^2$.

For $\mathrm{A.AD}^2 = \mathrm{A.AC}^2 + 2\mathrm{.A.AC.CD} + \mathrm{A.CD}^2$, & $\mathrm{B.BD}^2 = \mathrm{B.BC}^2 - 2\mathrm{.B.BC.CD} + \mathrm{B.CD}^2$,

therefore $\mathrm{A.AD}^2 + \mathrm{B.BD}^2 = \mathrm{A.AC}^2 + \mathrm{B.BC}^2 + (\mathrm{A}+\mathrm{B}).\mathrm{CD}^2 + 2\mathrm{.A.\ AC.CD} - 2\mathrm{.B.\ BC.CD}$

(or because the centre or gravity C gives rise to :

$\mathrm{A.AC} = \mathrm{B.BC}$ or $2\mathrm{A.AC} - 2\mathrm{B.BC} = 0$, & $\mathrm{C} = \mathrm{A} + \mathrm{B}$, it will become),

$= \mathrm{A.AC}^2 + \mathrm{B.BC}^2 + \mathrm{C.CD}^2$

(or because $\mathrm{B.BC} = \mathrm{A.AC}$ or $\mathrm{B.BC}^2 = \mathrm{A.AC.BC}$, also) $= \mathrm{A.AC.BC} + \mathrm{C.CD}^2$.

By a similar argument it may be inferred: $\mathrm{A.}da^2 + \mathrm{B.}db^2 = \mathrm{A.}ac.ab + \mathrm{C.}dc^2$. And thus since we have $\mathrm{A.AD}^2 + \mathrm{B.BD}^2 = \mathrm{A.}da^2 + \mathrm{B.}db$; there will be also

$\mathrm{A.AC.AB} + \mathrm{C.CD}^2 = \mathrm{A.}ac.ab + \mathrm{C.}dc^2$.

PROPOSITION XL. THEOREM.

224. *The centre of gravity of colliding elastic bodies is carried forwards between themselves with the same speed and direction after the collision as it advanced before the collision, and the mutual relative speed between the bodies will be the same after the collision, as it was before in approaching.*

Fig. 50 : With which put in place as in the preceding proposition, it is required to prove $dc = \mathrm{CD}$ and $ab = \mathrm{AB}$, for AB is the relative speed before, and *ab* the relative speeds of the same bodies A, B, after the collision and the points C, *c* indicate the centres of gravity in each case.

[While D and *d* represent the same collision point.]

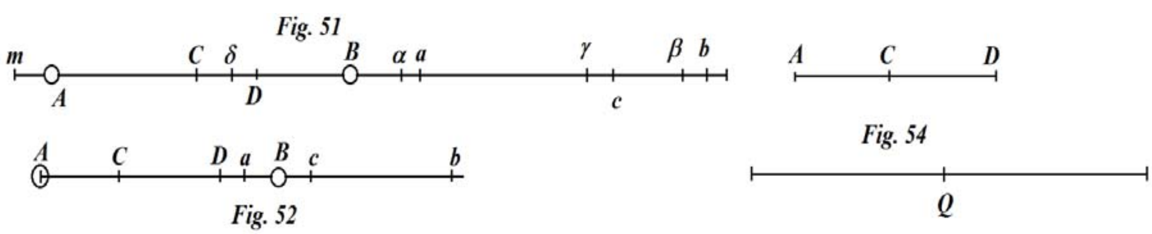


Fig. 51: *Demonstr.* The right line may be considered to be infinitely long, on which the bodies are moving, and to be moved towards *m* by an infinitely small velocity A*m*, and

also the bodies A, B share in this common motion advancing on the infinite line ; from which with the individual $\mathrm{D}\delta, a\alpha, c\gamma, b\beta$ made equal to *Am,* the ball A will be moving only with the absolute velocity $\mathrm{A}\delta$ in place of AD encountered before ; since from its own proper motion AD, as the opposite part $\mathrm{D}\delta$ is required to be removed : indeed B approaches with the velocity $\mathrm{B}\delta$ before colliding, with the proper approach of its motion BD agreeing to be changed to the other by the increment $\mathrm{D}\delta$. Moreover after the collision the speeds *Da, Db,* will be changed into $\mathrm{D}\alpha$ & $\mathrm{D}\beta$; indeed because all the increments $b\beta, c\gamma, a\alpha, \mathrm{D}\delta$ (following the hypothesis) are equal, there will be $\alpha\gamma = ac$, & $\alpha\beta = ab$ *;* and thus the velocities AC, *ac,* and also AB & *ab* remain the same in the above equation (§.223.) with only the changes CD & D*c* entering. Therefore in this equation $\mathrm{A.AC.AB} + \mathrm{C.CD} = \mathrm{A}.ac.ab + \mathrm{C.D}c^2$ in place of CD & D*c* there may be substituted $\mathrm{C}\delta$ & $\mathrm{D}\gamma$, becoming $\mathrm{A.AC.AB} + \mathrm{C.CD}^2 = \mathrm{A}.ac.ab + \mathrm{C.D}\gamma^2$, which taken from the other equation leaves $\mathrm{C.CD}^2 - \mathrm{C.C}\delta^2 = \mathrm{C.\ D}c^2 - \mathrm{C.D}\gamma^2$, or $\mathrm{CD}^2 - \mathrm{C}\delta^2 = \mathrm{D}c^2 - \mathrm{D}\gamma^2$, but also $2\mathrm{CD.D}\delta - \mathrm{D}\delta^2 = 2\mathrm{D}c.c\gamma - c\gamma^2$, that is because $\mathrm{D}\delta = c\gamma$, there will be $2\mathrm{CD.D}\delta = 2\mathrm{D}c.c\gamma$, and thus $\mathrm{CD} = \mathrm{D}c$.

Again from the equation $\mathrm{A.AC.AB} + \mathrm{C.CD}^2 = \mathrm{A}.ac.\ ab + \mathrm{C.D}c^2$ with the equal terms $\mathrm{C.CD}^2$ & $\mathrm{C.D}c^2$ removed, there will remain $\mathrm{A.AC.AB} = \mathrm{A}.ac.ab$, or $\mathrm{AC.AB} = ac.ab$. Truly there is $\mathrm{AB} : \mathrm{AC} = ab : ac$, or $\mathrm{AB}^2 : \mathrm{AC.AB} = ab^2 : ac.ab$, therefore because $\mathrm{AC.AB} = ac.ab$, there will be $\mathrm{AB}^2 = ab^2$ and thus $\mathrm{AB} = ab$. Q.E.D.

PROPOSITION XLI. PROBLEM.

225. *With the given* [*elastic*] *bodies* A, B *unequal in some manner, meeting each other with given speeds, to find the speeds of the same bodies after the collision.*

The solution of this problem is most easily found from the preceding proposition. For by making $\mathrm{D}c = \mathrm{CD}$, likewise $ca = \mathrm{CA}$, & $cb = \mathrm{CB}$, where the point D is required to be observed, where the right lines representative of the velocities are ended before the collision, must always fall between the points C, *c* by which the centre of the moving bodies is indicated before and after the collision, and the points *a* & *b* with respect to the point *c,* to be put in the same order, where the other C has been placed with respect to the bodies A, B. With which observed there will be also D*a*, the velocity of the body A, and D*b* the velocity of B after the collision.

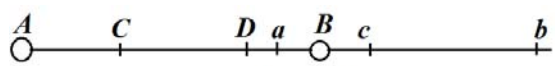


Fig. 52

Demonst. Because (from the construction) $ac = \mathrm{AC}$ & $bc = \mathrm{BC}$, there will be $ab = \mathrm{AB}$, and because again (construction) $\mathrm{D}c = \mathrm{CD}$, that is, because the relative speed of the bodies after the collision equals the relative speed before the collision, and equally the speed and the direction of the centre of gravity is the same in each case, during the preceding the bodies A and B are to be moving with no other velocities than those after the collision D*a*, D*b*. Q.E.D.

COROLLARY I.

226. Hence 1: if the bodies A and B are carried in the same direction, thus so that the point D falls beyond AB, but according to the direction of the body B ; there will be $\mathrm{D}a = \mathrm{CD} - \mathrm{AC}$ & $\mathrm{D}b = \mathrm{CD} + \mathrm{CB}$.
Indeed if the point D for the direction of the body A falls beyond AB there becomes $\mathrm{D}a = \mathrm{CD} + \mathrm{AC}$, & $\mathrm{D}b = \mathrm{CD} - \mathrm{CB}$.

2. If the bodies A, B coming from opposite directions are colliding with each other, thus so that as D may fall between A and B, and the point C shall be between D and A, there will be $\mathrm{D}a = \mathrm{CD} - \mathrm{AC}$, & $\mathrm{D}b = \mathrm{CD} + \mathrm{CB}$. And on the other hand there will be $\mathrm{D}a = \mathrm{CD} + \mathrm{AC}$, & $\mathrm{D}b = \mathrm{CD} - \mathrm{CB}$, whenever the centre of gravity has been placed between D and B.

COROLLARY II.

227. Thus if the body A with the speed AD may strike the resting body B, thus so that the points D and B are placed together, and to this resting body B the striking body A is required to give a speed twice that of CB. And thus if both A and B were equal, in the collision itself the motion of the striking body A stops, and the whole motion AB henceforth will be carried over to B, with which A had struck on that itself.

COROLLARY III.

228. Truly if the bodies may approach each other from opposite directions with their speeds AD, BD inversely proportional to the masses of the bodies B, A thus so that D may be incident at C, each body stops with which velocity it arrived. And indeed in this case CD vanishes.

COROLLARY IV.

229. Generally, if the [masses of the] bodies A, B may be called *m, n,* the velocities of these before the collision *u* & *r*, the speed of the body A may be found D*a* after the collision $= (mu - nu + 2nr) : m + n, = u, + (u - r).2n : m + n$ [This last term should be $(mu - nu + 2nr) : m + n, = u, + (r - u).2n : m + n$, subsequent terms have been changed ; see below;] and D*b,* or the speed of the body B after the collision $(2mu + mr - nr) : m + n, = r, + (u - r).2m : m + n$. These expressions have been derived from the values of the lines CD, AC & BC due to be substituted into the expressions of corollary I: $Da = \mathrm{CD} - \mathrm{AC}$, & $\mathrm{D}b = \mathrm{CD} + \mathrm{CB}$, for the case, in which the bodies may be moving in the same direction before the collision. The formulas for the other case of the bodies rushing towards each other from opposite directions may be elicited from these now shown to be effected by changing the sign of the letter *r* only, and there will be

$$\mathrm{D}a = u, -(u + r).2n : m + n, \& \ \mathrm{D}b = -r, +(u + r).2m : m + n.$$

[With the absolute velocities given for bodies travelling in the same direction :

$v_A; v_B; v_C; v_a; v_b; v_c$. The velocity of the centre of mass rel. to D will be, from the conservation of linear momentum :

$$v_C = \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{mu+nr}{m+n} = CD.$$

The velocity of A rel. to the C.of M. before the collision will be:

$$v_A - v_C = v_{AC} = v_A - \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{m_B(v_A - v_B)}{m_A + m_B} = \frac{m_B}{m_A + m_B}(v_A - v_B) = \frac{n}{m+n}(u-r) = AC;$$

The velocity of B rel. to the C.of M. before the collision will be:

$$v_B - v_C = v_{BC} = v_B - \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{m_A(v_B - v_A)}{m_A + m_B} = \frac{m_A}{m_A + m_B}(v_B - v_A) = \frac{m}{m+n}(r-u) = BC;$$

A's final ab. vel. will be :

$$CD - AC = \frac{m_A v_A + m_B v_B}{m_A + m_B} - \frac{m_B(v_A - v_B)}{m_A + m_B} = \frac{m_A v_A + 2m_B v_B - m_B v_A}{m_A + m_B} = \frac{m_A v_A + m_B v_A + 2m_B v_B - 2m_B v_A}{m_A + m_B}$$

$$= v_A + \frac{2m_B}{m_A + m_B}(v_B - v_A) = u + \frac{2n}{m+n}(r-u) = Da.$$

B's final ab. vel. will be :

$$CD - BC = \frac{m_A v_A + m_B v_B}{m_A + m_B} - \frac{m_A(v_B - v_A)}{m_A + m_B} = \frac{m_B v_B + 2m_A v_A - m_A v_B}{m_A + m_B} = \frac{m_B v_B + m_A v_B + 2m_A v_A - 2m_A v_B}{m_A + m_B}$$

$$= v_B + \frac{2m_A}{m_A + m_B}(v_A - v_B) = r + \frac{2m}{m+n}(u-r) = Db.$$

Similarly, the momenta of A before and after the collision relative to the C. of M. ar:

$$p_A^i = \frac{m_A m_B}{m_A + m_B}(v_A - v_B) = \frac{mn}{m+n}(u-r);\ p_A^f = m_A v_A + \frac{2m_A m_B}{m_A + m_B}(v_B - v_A) = mu + \frac{2mn}{m+n}(r-u);$$

and, the momenta of B before and after the collision relative to the C. of M. are:

$$p_B^i = \frac{m_A m_B}{m_A + m_B}(v_B - v_A) = \frac{mn}{m+n}(r-u); p_B^f = m_B v_B + \frac{2m_A}{m_A + m_B} m_B (v_A - v_B) = nr + \frac{2mn}{m+n}(u-r).]$$

PROPOSITION XLII. THEOREM.

230. *The body* A *carried with the speed* AD *to the intervention of the middle body* X, *will give the same velocity to the resting body* B, *as the intervention of a certain other middle body* Y, *if the striking body* A *were to* X *as the other* Y *were to the resting body* B.

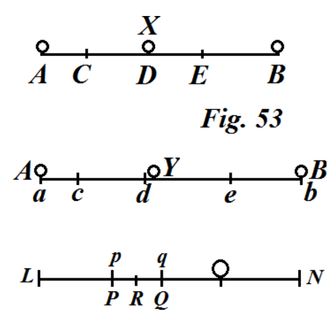


Fig. 53

Fig. 53. That is, if the body A first rushes towards the body X with the speed AD, and this body with that speed taken from the impact then may run towards the body B at rest; I say this body B is then going to take the same speed, as if from the other body Y the impulse were from that speed, as equally Y at rest may take from the interaction with the body A with the speed AD, clearly being in the ratio A to X as Y to B. In order that the proposition may be shown, the construction of Proposition XL is required to be put in place twice, and a figure is required to be considered for each intermediate body X

and Y separately, by which the lines of capital letters are marked, regarding the intermediate body X, and which by the same, but in lower case letters are notated, pertaining to the figure of the other intermediate body Y.

For from §. 227 it is agreed the body X is going take a speed DB, twice that of CD [$\frac{mu+nr}{m+n}=\frac{mu}{m+n}$] itself from the striking body A, [*i.e.* the speed DB is given by: $r+\frac{2m}{m+n}(u-r)=\frac{2mu}{m+n}$, as r is zero initially; thus, whatever the masses, in an elastic collision between a moving body and one at rest in the lab. ref. frame, the latter moves off with twice the speed of the C. of M., which remains constant,] with C the centre of gravity of the bodies A and X present, and thus AC is to CD in the ratio of the [mass of the] body X to that of the body A; and with DB divided at the point E, thus so that DE shall be to EB just as the mass of the body B to that of the middle body X, with the moveable body X going to give to the other resting body B, on which it strikes with the speed DB, twice the speed of EB itself; [*i.e.* in analogy with the first motion.]

By a similar construction, but with respect to the bodies *a,* Y and *b,* between which *a* and *b* are equal to the bodies A and B, it is understood to be repeated, and the body *b* will accept the speed 2*eb* from the body Y. It is required to show that $\mathrm{EB}=eb$.

Demonst. Because (following the hypothesis) there is $\mathrm{A}:\mathrm{X}=\mathrm{Y}:\mathrm{B}$, and on account of the centre of gravity C, $\mathrm{A}:\mathrm{X}=\mathrm{CD}:\mathrm{AC}$; also $\mathrm{Y}:b(\mathrm{B})=be:de$, there will be $\mathrm{CD}:\mathrm{AC}=be:de$, or by inverting and adding $\mathrm{AD}:\mathrm{CD}=2.cd\,(db):be$;

[$\mathrm{CD}:\mathrm{AC}=be:de;\therefore\frac{\mathrm{AC}}{\mathrm{CD}}+1=\frac{de}{be}+1,\text{or }\frac{\mathrm{AD}}{\mathrm{CD}}=\frac{db}{be}=\frac{2cd}{be}$;] and thus also, from the consequence of the duplications, $\mathrm{AD}:\mathrm{DB}(2\mathrm{CD})=2cd:2be=cd:be$. Truly also there is $\mathrm{B}:\mathrm{X}=\mathrm{Y}:\mathrm{A}$, that is $\mathrm{DE}:\mathrm{BE}=ac:cd$, and therefore by adding $\mathrm{DB}:\mathrm{BE}=ad:dc$, hence also $ad\,(\mathrm{AD}):\mathrm{DB}=cd:\mathrm{BE}$, but a little before there was $\mathrm{AD}:\mathrm{DB}=cd:be$; therefore also $cd:be=cd:\mathrm{BE}$, and thus $be=\mathrm{BE}$. Q.E.D.

231: *Otherwise.* Let $\mathrm{LN}=\mathrm{AD}=ad$, $\mathrm{PN}=\mathrm{CD}$; and $p\mathrm{N}=cd$, and finally LN may be divided at Q and *q*, so that LN shall be to QN thus as $\mathrm{B}+\mathrm{X}$ to X, & LN to qN as $\mathrm{B}+\mathrm{Y}$ to Y, and it may be shown that the points P, *q*, and likewise *p*, Q coincide. For because (following the hypothesis) $\mathrm{A}:\mathrm{X}=\mathrm{Y}:\mathrm{B}$, there will be $\mathrm{A}+\mathrm{X}:\mathrm{A}=\mathrm{B}+\mathrm{Y}:\mathrm{Y}$, that is $\mathrm{AD}:\mathrm{CD}=\mathrm{LN}:\mathrm{PN}=\mathrm{LN}:q\mathrm{N}$, therefore $\mathrm{PN}=q\mathrm{N}$ and thus the points P and *q* coincide. Likewise because $\mathrm{B}:\mathrm{X}=\mathrm{Y}:\mathrm{A}$, & $\mathrm{B}+\mathrm{X}:\mathrm{X}=\mathrm{A}+\mathrm{Y}:\mathrm{A}$, there will be (from the construction) $\mathrm{LN}:\mathrm{QN}(=ad:cd)=\mathrm{LN}:p\mathrm{N}$, and hence also $\mathrm{QN}=p\mathrm{N}$; therefore the points *p* and Q coincide. With these in place, and because

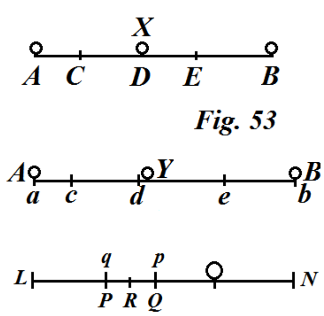


Fig. 53

$\mathrm{AD}:\mathrm{DB}(2\mathrm{CD})=\mathrm{LN}:2\mathrm{PN}$; & $\mathrm{DB}:2.\mathrm{EB}=\mathrm{LN}:2\mathrm{QN}$; there will be from these equations $\mathrm{AD}:2.\mathrm{EB}=\mathrm{LN}^2:4.\mathrm{PN}.\mathrm{QN}$. From the same argument there will be found

$ad:2eb=\mathrm{LN}^2:4\mathrm{PN}.q\mathrm{N}$, therefore there will be from the equation by inverting again $2.eb:2.\mathrm{EB}=4.p\mathrm{N}.q\mathrm{N}:4.\mathrm{PN}:\mathrm{QN}$, or $eb:\mathrm{EB}=p\mathrm{N}.q\mathrm{N}:\mathrm{PN.QN}$: from which since there shall be shown to be $p\mathrm{N}=\mathrm{QN}$ and $q\mathrm{N}=\mathrm{PN}$, generally there will be $eb=EB$. Q.E.D.

COROLLARY I.

232. If in place of each X and Y from the middle, a body M may be taken to be the mean proportional between A and B, and thus between X and Y, since A shall be to X as $\mathrm{Y}:\mathrm{B}$; and LN may be divided at R, so that the whole LN shall be to RN just as $\mathrm{A}+\mathrm{M}$ to A, [as $\mathrm{LR:RN}=\mathrm{M:A}$; hence $\mathrm{LR}+\mathrm{RN:RN}=\mathrm{M}+\mathrm{A:A}=\mathrm{LN:RN}$,] and the initial speed of the moveable body A will be to the speed of the body B with the intervention of the acquired mean M, as LN^2 to $4.\mathrm{RN}^2$. And thus the speed, that the body B acquired from either of the intervening bodies X or Y will be to the speed, as acquired by the intervention of the mean M, as 4PN.QN to $4.\mathrm{RN}^2$, or as PN.QN to RN^2. For since $\mathrm{A}+\mathrm{M}:\mathrm{M}=\mathrm{LN}:\mathrm{RN}$, & $\mathrm{A}+\mathrm{M}:\mathrm{A}=\mathrm{B}+\mathrm{M}:\mathrm{M}$, in this case there will be both PN, QN since between themselves, as both equal RN, from which generally there shall be $\mathrm{AD}:2\mathrm{EB}=\mathrm{LN}^2:4.\mathrm{PN.QN}$, in this case there shall be the particular speed of the body A, to the speed of B taken from M $=\mathrm{LN}^2:4.\mathrm{RN}^2$, and because (§.231.) the velocity of B itself taken from either X or Y, is to the initial speed of the body A, as 4.PN.QN to LN^2; from the equation there will be the speed of the body B taken from either X or Y, to the speed of the same acquired from the body M as 4.PN.QN to $4.\mathrm{RN}^2$ that is, as PN.QN to RN^2.

COROLLARY II.

233. Because (from the construction) $\mathrm{PN}:\mathrm{LN}=\mathrm{A}:\mathrm{A}+\mathrm{X}$, & $\mathrm{LN}:\mathrm{RN}=\mathrm{A}+\mathrm{M}:\mathrm{A}$, from the equation there will be $\mathrm{PN}:\mathrm{RN}=\mathrm{A}+\mathrm{M}:\mathrm{A}+\mathrm{X}$. Similarly there will be found $\mathrm{QN}:\mathrm{RN}=\mathrm{A}+\mathrm{M}:\mathrm{A}+\mathrm{Y}$, therefore through the composition of the ratios there will be

$$\mathrm{PN.QN}:\mathrm{RN}^2=(\mathrm{A}+\mathrm{M})^2:(\mathrm{A}+\mathrm{X}.\mathrm{A}+\mathrm{Y})=\mathrm{AA}+2.\mathrm{AM}+\mathrm{MM}:\mathrm{AA}+(\mathrm{X}+\mathrm{Y}).\mathrm{A}+\mathrm{XY}$$
$$=\mathrm{AA}+2.\mathrm{A.M}+\mathrm{AB}:\mathrm{AA}+(\mathrm{X}+\mathrm{Y}).\mathrm{A}+\mathrm{AB}=\mathrm{A}+2\mathrm{M}+\mathrm{B}:\mathrm{A}+\mathrm{X}+\mathrm{Y}+\mathrm{B},$$

for $\mathrm{A.B}=\mathrm{MM}=\mathrm{XY}$, and thus it can be substituted into the other place, as has been done. Now because X, M and Y are in continued proportion, $\mathrm{X}+\mathrm{Y}$ will be greater than 2.M and thus $\mathrm{A}+\mathrm{X}+\mathrm{Y}+\mathrm{B}>\mathrm{A}+2\mathrm{M}+\mathrm{B}$; hence there will be $\mathrm{RN}^2>\mathrm{PN.QN}$; from which since (§.132) the speed of the body B taken from either X or Y, shall be to its same speed expressed by the body M itself, to be as PN.QN to RN^2, it is clear this speed is always be greater than that. Indeed this speed of the body B will be deduced to be greater than that for the body M, as likewise the body B was being impelled by the speed AD of the body A itself, for in place of the body X it can be understood as the vis [viva] of the body; and thus the one equal to A, because from the impulse A will take the total speed (§217.) CD, thus so that the same shall be, if the body A with its speed AD immediately impinges on the resting B, or impels that with the aid of an intermediate itself equal to A, to which its whole speed may be brought together; from which since it may be shown generally, by the intervention of some body X, different from M, a smaller velocity is going to be gathered by the body B than from the mean proportional M between the end values A and B ; by necessity it follows also the body A to be giving a smaller speed to the other body B than M, even if it may impinge immediately on that. And this corollary contains an easy demonstration of the penultimate proposition of the tract of Huygens, *de Motu Corporum ex Percussione*, Concerning the Motion of Bodies from Percussion.

COROLLARY III.

234. Hence in the first place from bodies in continual proportion the last will be given the greatest speed by some number of bodies interposed continually proportional, as if the same may be encountered at rest.

SCHOLIUM.

Fig. 54. 235. Now from the preceding the rules determining the velocity may be elicited readily, which the final body resting will take from however many bodies placed continually in proportion in series with the motion transmitted from the first to the second, the third, the fourth, etc. The first body may be called A, of which the speed from the start shall be AD, and the final speed V : the initial speed AD may be divided at the point C, in the inverse ratio of the mass of the second body to the first, or, which is the same, in the ratio of any body in the series of proportionals to the body, which precedes that nearest in the same series, thus so that AC shall be to CD in that same ratio, thence the ratio of the whole AD to twice CD may be continued in just as many terms less by one as the series of proportional bodies, of which the final shall be Q, I say this Q to express the speed V for the final body from all the preceding ones brought together. And thus if the number of terms were $n+1$, the speed AD of the first body will be to Q the speed acquired by the final body U, as AD^n to $(2\mathrm{CD})^n$, hence there will be had $\mathrm{Q}=(2\mathrm{CD})^n:\mathrm{AD}^{n-1}=(2\mathrm{CD})^n$, clearly with unity taken in place of AD, which is allowed to occur in this situation.

236. *Example.* One hundred bodies shall be doubled in a continuous ratio, and the first motion begins from the maximum [mass], Q is sought, or the speed for the smallest mass V by being brought together. In this case there will be $\mathrm{AC}=\frac{1}{3}$, and $\mathrm{CD}=\frac{2}{3}$, and thus $(2.\mathrm{CD})^n=\left(\frac{4}{3}\right)^{99}$; is indeed $n=99$. Therefore we have $\mathrm{Q}=\left(\frac{4}{3}\right)^{99}$, and $\mathrm{Log\,Q}=99.\log\frac{4}{3}$; and $\log(4:3)=0.1249388$ which log. multiplied by 99 gives $99\cdot\log(4:3)=12.3689412=\log\mathrm{Q}$, therefore $\mathrm{Q}=2338520732310$ as an approximation [indeed this number also should be rounded off to 6 or 7 places], for this number corresponds to that, whose characteristic is agreed to be nearest 12; and thus the speed, which the hundredth body must accept, is to the first speed A as 2338520732310 to 1 approximately.

Indeed if the motion may begin from the smallest value, its speed from the beginning to the speed which from the hundredth and maximum will give from the 98 interposing bodies in continued proportion, will be just as the number 27103713483146067 to one approximately. Huygens in the final proposition treated above (§.133.) extolled a number shown for these cases less than ours, perhaps from transcription or from a slip in the calculation.

PROPOSITION XLIII. THEOREM.

237. *The centre of gravity of two spherical elastic bodies meeting each other in turn obliquely, will be carried after the collision in the same direction with the same speed with which it was being carried before the collision.* Fig. 55.

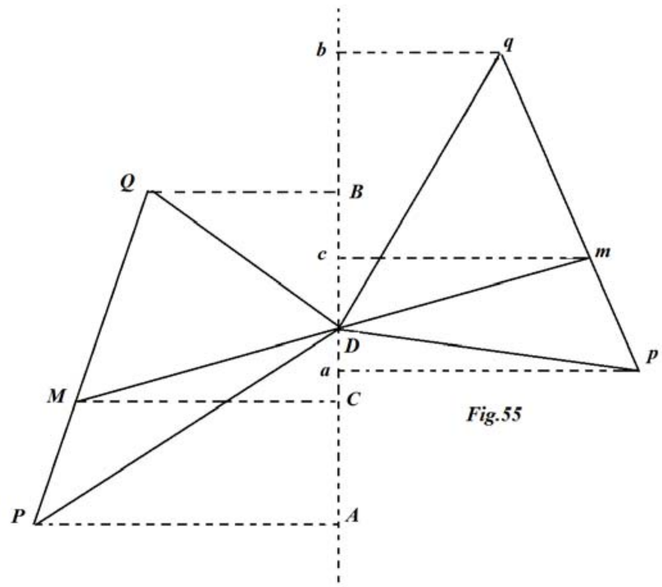


Fig.55

The elastic spheres P and Q meet each other mutually at D with their individual speeds PD, QD, and after the collision they rebound with the individual speeds D*p* and D*q*, thus so that these lines are resolved in the same time by uniform motions, as the right lines PD and QD before the conjunction ; and the points M and *m* shall be the centres of gravity of the spheres at P and Q, likewise arising at *p* and *q* ; it is required to show that both the right lines MD and D*m* are equal, while also placed in the same direction ; since MD sets out the velocity and direction of the centre of gravity M before the collision, D*m* truly displays the velocity and direction of the common centre of gravity after the collision. D shall be the point of contact of the spheres, and BA the right line joining the centres of the same, which, where the spheres mutually touch each other, pass through the point of contact ; from the centres of the spheres P and Q the perpendiculars PA and QB shall be sent to AB, and from M the right line MC parallel to the remaining QB and AP, with which in place, it is evident from art. 29, the sides PA and AD to be equivalent to the motion along PD, and the sides QB and BD to be equivalent to the motion QD. Now because the motions along PA and QB are parallel, as far as the spheres are disturbed by these motions, are not able mutually to disturb each other, but only the motions along AD and BD, come as directly opposite, in the case of the collision requiring to be considered with the remaining parts to be set aside for some time and later to be put together again. Again because M is the centre of gravity of the spheres present at P and Q, also C will be the centre of gravity of the spheres placed at A and B; and thus by making $\mathrm{D}c = \mathrm{CD}$, also $ca = \mathrm{CA}$ and $cb = \mathrm{CB}$, (§. 225) D*a* will be the appropriate speed of the sphere A, and D*b* the speed of the other B, after the collision on the indefinite line *ab*. Again the right lines $ap = \mathrm{AP}$ and $bq = \mathrm{BQ}$ themselves normal to AB will be multiplied by *a* and *b*, with which in place and because in the collision itself the motions along PA and QB, as to be parallel, are unable to undergo any change, these unimpaired motions after the impulse are to be conserved ; and thus after coming together the body P recedes from the point D with the resultant motion D*p* from the sides D*a* and *ap,* and the other Q with the motion D*q* arising from the sides D*b* and *bq.* Indeed Q because (by the construction) $ca = \text{CA};\ cb = \text{CB};\ ap = \text{AP};\&\ bq = \text{BQ}$, the figure *abqp* will be similar and equal to the other figure ABQP, and thus $cm = \text{CM}$: from which since (by the construction) also there shall be $\text{D}c = \text{CD}$, and the triangle *Dcm* will be similar and equal to the triangle DEM, and thus the lines MD and *Dm,* are put in the direction, hence the centre of gravity of the spheres *P* and Q after the collision will progress with the same speed and direction with which it progressed before. Q.E.D.

CAPUT VI.

De Regulis motus in collisione Corporum.

DEFINITIONES.

I.

Corpora duo sibi directe ocurrere dicuntur cum moventur in linea recta centra gravitatis jungente, atque per contactum eorumdem corporum transeunte.

II.

Velocitates corporum *propriae* sunt illae, quibus corpora moventur. *Velocitas* vero eorundem *relative* est ea, qua cum ad se mutuo accedunt. Idcirco celeritas relativa corporum ex oppositis plagis sibi mutuo obviam venientium est *aggregatum* celeritatum eorundem propriarum. Sin vero velocius corpus alterum tardius ad eandem partem latum insequatur, velocitas eorum relativa erit *excessus* velocitatis majoris propriae supra minorem.

III.

212. Corpora A, B in planis horizontalibus moveri atque inter se collidi intelliguntur ad id, ut motus eorum ante & post occursum aequabiles evadant, nec iidem motus a gravitatis actione: a plano utpote sufflaminanda, in quo mobilia incedunt, quicquam alterentur.

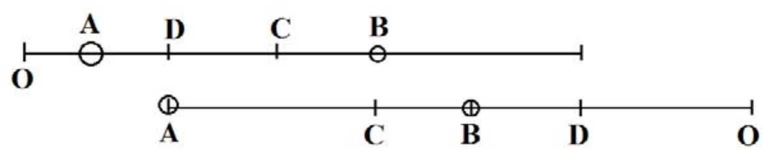


Fig. 48.

Celeritates mobilium propriae ante occursum designabuntur posthac per lineas rectas AD, BD in ipsorum directionibus sumtas, adeo ut quocunque modo mobilia A, B, incedant, dummodo non in rectis angulum continentibus ferantur, ipsorum celeritas relativa futura sit AB. Directiones vero particulares corporum, aut potius directionum plagae , designentur deinceps per ordinem, quo literae A, D & B , D scribuntur. Proinde AD significat mobile A ferri celeritate uniformi AD ex A versus D, & B ferri velocitate hac lineA & BD expressa ex B etiam versus D. Adeoque quoties signum D inter A & B cadit, toties ambo haec corpora contrario sensu moventur, id est, ex oppositis partibus sibi invicem obviam veniunt celeritate relativa AB, quae hoc casu composita est ex particularibus AD, BD; sin vero signum D cadit extra AB, ambo corpora movebuntur ad eandem partem, in qua signum D positum est celeritate relativa AB aequali in tali casu excessui velocitatis propriae alterutrius mobilis supra celeritatem propriam alterius.

213. Et quia motuum plagae etiam per signa + & − subinde designari solent, probe sciendum +AD idem esse ac − DA, & + DA idem ac − AD, & sic de ceteris. Nam motus +AD contrarius est, seu in plagam contrariam dirigitur, tum − AD, tum etiam DA, atque

adeo hae duae expressiones sunt aequivalentes *;* pariter quia & + DA & −AD eidem motui + AD contrarii sunt, notationes + DA & −AD unum idemque significant, adeo ut una alteri semper sufficere liceat, quod aliquando bene notatum, utile esse comperietur.

IV.

214. Celeritates corporum inter se collisorum ante conflictum considerabimus quandoque tanquam acquisitas lapsu verticali a quiete incepto motu gravium naturaliter accelerato; & celeritates eorundem post occursum tanquam velocitates initiates corporum, quae verticaliter in altum lata motu naturaliter retardato certas altitudines emeteri valent priusquam motus eorum ascensionalis penitus extinguatur.

V.

215. Vis corporum absoluta ante conflictum est altitudo, quam centrum gravitatis commune mobilium perlabi potest, quando unumquodvis corpus per suam altitudinem cadendo velocitatem suam propriam acquirit. Visque mobilium absoluta post collisionem, est altitudo, ad quam assurgere potest idem commune centrum gravitatis corporum, quando unumquodque horum corporum celeritate initiali ea, quam in conflictu acquisivit, suam altitudinem ascendendo absolvit.

216. Sic posito mobile A acquirere celeritatem suam propriam AD lapsu accelerato ex altitudine EA, & B suam celeritatem propriam descensu accelerato ex altitudine FB, centrum gravitatis G mobilium A, B in punctis E , F existentium cadet ex altitudine GC. Et si mobilis *A* post conflictum celeritas uniformis sit D*a* talis, ut ea tanquam velocitate initiali rectam *ae* ascendendo conficere possit mobile A, alterumque B altitudinem *bf* celeritate initiali *Db,* quae est velocitas uniformis, quam mobile istud B, in ipso conflictu acquisivit; commune corporum A, B gravitatis centrum *c* censetur ascendere posse vi absoluta mobilium post conflictum in recta verticali *cg* usque ad *g*.

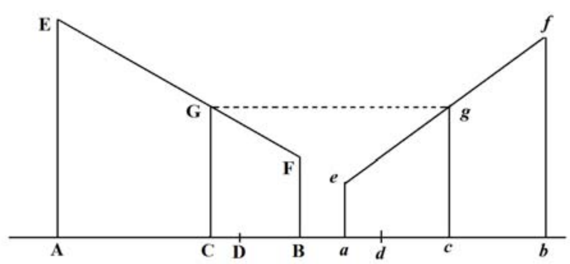


Fig. 50.

VI.

217. Corpora, quae nonnisi vi inertiae materiae praedita sunt, dicantur deinceps corpora inertia. Corpora vero, quae vim actuosam habent repellendi corpora ea ipsa vi qua impulsa fuerunt, dicantur corpora actuosa. Corpora inertia alias etiam nominari solent *non elastica,* ac corpora actuosa *elastica.*

HYPOTHESIS.

218. *Cum corpora actuoso inter se colliduntur eadem manet vis eorum absoluta post conflictum, quae erat ante eundem conflictum.*

Excipimus corpora inertia, quorum vis absoluta post collisionem nunquam eadem est, quae erat ante concursum, quandoquidem haec vis in ipso conflictu quandoque penitus absorbetur, scilicet cum mobilia celeritatibus massis suis reciproce proportionalibus sibi invicem obviam veniunt. Sed ejusmodi corpora penitus inertia forte nulla dantur, nam etiamsi pleraque corpora perfecta actuositate vel elasticitate praedita non sunt, quod particulae eorum compressae sese in pristinum statum restituere non valeant, eorum saltem elementa perfecte elastica esse queunt, a quo statuendo nullus ex praestantioribus Geometris hujus aevi abhorrere videtur. Verum quicquid rei sit, revertor ad corpora actuosa, quorum vis absoluta eadem manet vi hujus hypotheseos, si commune eorum centrum gravitatis ad eandem altitudinem *cg* ascendit post conflictum , ex qua delapsum erat ante occursum eorum, hoc est, positis quae supra (§. 216.) si $cg = \mathrm{GC}$.

219. Vires absolutas corporum aestimamus juxta altitudines, ad quas retardato suo a gravitate motu pertingere possunt, vel accuratius per facta ex his altitudinibus in massas corporum; in hac virium aestimatione praeeuntem habemus Illust. Leibnitium, qui eandem non uno loco in Actis Eruditorum Lips. indicavit quidem, non tamen demonstravit, etsi apodictice demonstrari potest, ut forte alia id occasione ostendemus. Non ignoro perplures esse insignissimos viros, quibus talis vires aestimandi ratio non arrideat, existimantes eam a quantitatibus motus petendam esse. Celeb. Papinus sane diu multumque cum Leibnitio disputans rotunde negavit corporum vires altitudinibus, quas corpora ascendendo conficere possunt, proportionales esse ; quin imo vires eas temporibus proportionari putat, quibus altitudines modo nominatae absolvutur vel etiam celeritatibus initialibus mobilium aequalium. Sed in probando hoc suo asserto incautus labitur ipse, principium assumens aperte falsum; & hoc principium falsum est, quod existimet corpora cum diversis velocitatibus initialibus ascendentia aequalem a gravitate ipsis resistente ictuum numerum exceptura esse temporibus aequalibus. Sed si plures docti Geometrae, praejudicara opinione communis vires aestimandi rationis per quantitatem motus detenti, a nobis diversa sentiunt, adsunt tamen alii acutissimi Geometrae, qui Leibnitianam ultro admittunt, quin imo jam ante complures annos Nob. Hugenius eadem quodammodo usus est in demonstrationibus suarum regularum motus ex percussione, principii loco in iis assumens, non posse commune centrum gravitatis corporum inter se collisorum altius ascendere post occursum, quam descenderat ante conflictum; non tamen supponit ascensum centri gravitatis descensui ejusdem aequalens esse; alias vero assumsit hypotheses, quae aequivalentes sunt nostrae de aequalitate ascensus atque descensus centri gravitatis post & ante conflictum corporum. Idcirco hac nostra hypothesi posita regulas motus corporum actuosorum facili ratione inde derivabimus; sed priusquam hanc tractationem aggrediamur, unicam propositiunculam praemittere libet circa motum corporum inertium post conflictum.

PROPOSITIO XXXVII. THEOREMA.

220. *Corporum inertium, atque sibi invicem directe occurrentium, celeritas & directio post conflictum est eadem, quae erat anti communis ipsorum centri gravitatis occursum.*
Fig.48 . Sint AD celeritas uniformis corporis A, & BD velocitas corporis B ante conflictum, probandum est utrumque corpus A & B motum iri post occursum celeritate

CD, quae est celeritas, qua centrun gravitatis C mobilium A, B ferebatur ante collisionem, existente AC ad BC, sicut massA & B ad massam A, seu ut pondus B ad alterius A pondus.

Demonstr. Quia motus corporum in collisione aliter non variantur quam pro ratione inertiae materiae, in ipso conflictu res eodem recidit, ac si corpora conjuncta, ac velut sibi invicem agglutinata movenda essent quantitate motus aequali excessui, quo motus major alterutrius ex mobilibus minorem alterius excedit, si corpora ex oppositis partibus sibi invicem obviam veniant, vel etiam aequali aggregato quantitatum motus utriusque corporis, si ambo ad easdem partes tendant. Unde cum in motus oppositione & casu, quo corporum centrum gravitatis C cadit inter puncta D & B (§. 44) sit $\mathrm{B.BD}-\mathrm{A.AD}=(\mathrm{A}+\mathrm{B}).\mathrm{CD}=\mathrm{A.CD}+\mathrm{B.CD}$, cumque $\mathrm{B.BD}-\mathrm{A.AD}$ sit excessus, quo motus B. BD fortioris B excedit motum alterius A, patet ambo corpora A & B post contactum celeritate aequali CD latum iri ex C versus O, id est celeritate & directione , qua centrum eorum gravitatis ante impactum ferebatur.

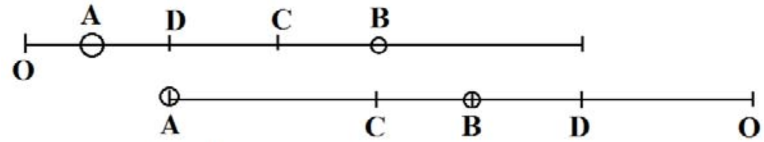


Fig. 48.

Sin vero ambo corpora A, B ad eandem partem versus O suis celeritatibus particularibus AD, BD moveantur, erit $\mathrm{A.AD}+\mathrm{B.BD}=(\mathrm{A}+\mathrm{B}).\mathrm{CD}=\mathrm{A.CD}+\mathrm{B.CD}$, hoc est aggregatum motuum corporum A, B ante conflictum aequatur quantitati motus eorundem, sed celeritate CD centri gravitatis ante ictum movendorum feretur ergo utrumque hac celeritate CD versus O. Quod erat demonstrandum.

PROPOSITIO XXXVIII. THEOREMA.

211. Fig. 49. *Si mobilia* A, B *in directionibus obliquis* AD, BD *quemlibet angulum* ADB *continentibus, atque celeritatibus hisce ipsis lineis* AD, BD *expressis sibi mutuo occurrant, commune illorum gravitatis centrum* C *eadem velocitate cum* CD *atque in directione hujus* CD *productae in d post conflictum feretur.*

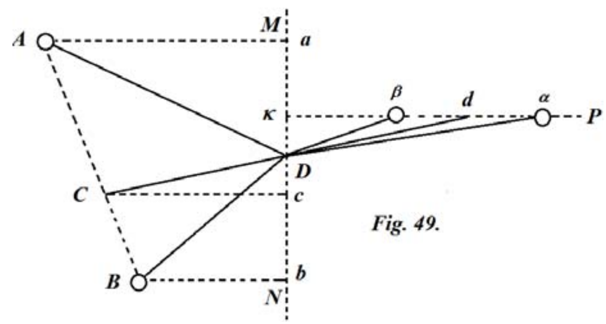


Fig. 49.

Sit MN linea jungens centra mobilium A, B in casu contactus, atque ad eam demissae sint ex centris corporum *A,* B eorumque communi centro gravitatis C normales A*a,* B*b*, & C*c,* motus juxta AD resolvetur in laterales *Aa,* & *aD,* motusque BD in laterales *Bb, bD*; at vero, cum motus juxta A*a* & Bb paralleli sint, corpora hisce motibus in se invicem agere nequeunt, restant ergo soli motus *a*D & *b*D, quibus in se mutuo agunt. Jam si corpora A & B existerent in *a* & *b,* atque celeritatibus *a*D, *b*D sibi mutuo obviam venirent, post contactum utrumque moveretur (§. 220.) celeritate $\mathrm{DK}=c\mathrm{D}$, existente *c* eorum centro gravitatis, cum sit $\mathrm{AC}:\mathrm{BC}=ac:bc=\mathrm{B}:\mathrm{A}$, unde quia motus *a*D & *b*D in obliquis virtualiter continentur, ideo haec motuum modificatio etiam in conflictu virtualiter accidisse censenda est : propterea ducta per punctum K indefinita KP, in ea sumantur $\mathrm{K}\alpha=\mathrm{A}a$ & $\mathrm{K}\beta=\mathrm{B}b$, ac quia motus A*a*, B*b* in ipso conflictu nullam mutationem subierunt, mobile A post conflictum movebitur, aut moveri nitetur, motibus lateralibus $D\mathrm{K}, \mathrm{K}\alpha$, ex quibus resultat motus $\mathrm{D}\alpha$; alterum vero B motibus lateralibus $D\mathrm{K}\ \&\ \mathrm{K}\beta$, ex quibus nascitur motus $\mathrm{D}\beta$. Sit *d* centrum gravitatis corporum A, B in $\alpha\ \&\ \beta$ existentium, eritque (§.44.) $\mathrm{A.K}\alpha + \mathrm{B.K}\beta = (\mathrm{A}+\mathrm{B}).\mathrm{K}d$. item $\mathrm{A.A}a + \mathrm{B.B}b = (\mathrm{A}+\mathrm{B}).\mathrm{C}c$, & (constr.) $\mathrm{A.K}a + \mathrm{B.K}\beta = \mathrm{A.A}a + \mathrm{B.B}b$, ergo etiam $(\mathrm{A}+\mathrm{B}).\kappa d = (\mathrm{A}+\mathrm{B}).\mathrm{C}c$, vel $\kappa d = \mathrm{C}c$: atqui per praecedentem est etiam $\mathrm{C}c = \mathrm{D}\kappa$, ergo $\mathrm{D}d = \mathrm{CD}$, atque adeo post conflictum centrum gravitatis *mobilium* A, B movetur celeriate *Dd* aequali illi C*c,* qui ante occursum incedebat. Quod erat demonstrandum.

Tantum de corporibus inertibus seu vi elastica carentibus sequuntur leges motus corporum elasticorum.

PROPOSITIO XXXIX. THEOREMA.

222. *Posita eadem quantitate vis absolutae corporum inter se collisorum ante & post occursum eorum directum, facta ex massis corporum in quadrata velocitatum collective sumta ante & post conflictum aequantur.*

Hoc est $\mathrm{A.AD}^2 + \mathrm{B.BD}^2 = \mathrm{A.D}a^2 + \mathrm{B.D}b^2$, existentibus D*a,* D*b* celeritatibus corporum A, B post collisionem, & AD ac BD velocitatibus ante conflictum.

Demonstr. Posito A acquisivisse celeritatem suam AD casu ex altitudine EA, alterumque B ex altitudine FB, divisaque recta EF jungente puncta E, F in ratione reciproca ponderum A, B, lineae GC denotabit descensum centri gravitatis mobilium A, B ; & *ea, fb* denotent altitudines, quibus accelerato motu transmissis mobilibus acquiruntur celeritates D*a,* D*b,* cum quibus deinceps in altum ferri incipientia iterum ad easdem altitudines *ae, bf* assurgere possunt; quo motu ascensionali eorum centrum gravitatis *c* ascendet per altitudinem *cg.* Jam quia (§.218.) eadem manet vis absoluta corporum A, B ante & post conflictum eorum, erit $\mathrm{GC} = cg$, vel posito C significare $\mathrm{A}+\mathrm{B}$, erit $C.GC = C.cg$; atque (§.44.) $\mathrm{C.GC} = \mathrm{A.EA} + \mathrm{B.FB}$, $\&\ \mathrm{C.}cg = \mathrm{A.}ae + \mathrm{B.}bg$, ergo etiam $\mathrm{A.EA} + \mathrm{B.FB} = \mathrm{A.}ae + \mathrm{B.}bf$; verum (§.150.) rectis EA, FB,*ae, bf* proportionalia sunt $\mathrm{AD}^2, \mathrm{BD}^2, \mathrm{D}a^2, \&\ \mathrm{D}b^2$, unde sufficiendo haec illarum loco in proxime antecedenti analogia , reperietur $\mathrm{A.AD}^2 + \mathrm{B.BD}^2 = \mathrm{A.D}a^2 + \mathrm{B.D}b^2$. Quod erat demonstrandum.

COROLLARIUM.

223. Idcirco erit etiam $\mathrm{A.AC.AB} + \mathrm{C.CD}^2 = \mathrm{A.}ac.ab + \mathrm{C.D}c^2$.

Nam $\mathrm{A.AD}^2 = \mathrm{A.AC}^2 + 2.\mathrm{A.AC.CD} + \mathrm{A.CD}^2$, & $\mathrm{B.BD}^2 = \mathrm{B.BC}^2 - 2.\mathrm{B.BC.CD} + \mathrm{B.CD}^2$, ergo $\mathrm{A.AD}^2 + \mathrm{B.BD}^2 = \mathrm{A.AC}^2 + \mathrm{B.BC}^2 + (\mathrm{A}+\mathrm{B}).\mathrm{CD}^2, +2.\mathrm{A.\ AC.CD} - 2.\mathrm{B.\ BC.CD}$ (vel quia centrum gravitatis C efficit

$\mathrm{A.AC} = \mathrm{B.BC}$ vel $2\mathrm{A.AC} - 2\mathrm{B.BC} = 0, \&\ \mathrm{C} = \mathrm{A}+\mathrm{B}$, erit),

$= \text{A.AC}^2 + \text{B.BC}^2 + \text{C.CD}^2$

(seu quia $\text{B.BC} = \text{A.AC}$ vel $\text{B.BC}^2 = \text{A.AC.BC}$, etiam) $= \text{A.AC. AB} + \text{C.CD}^2$.

Pari argumento inferetur $\text{A.D}a^2 + \text{B.D}b^2 = \text{A.}ac.\ ab + \text{C.D}c^2$. Adeoque cum habeamus $\text{A.AD}^2 + \text{B.BD}^2 = \text{A.D}a^2 + \text{B.D}b$; erit etiam

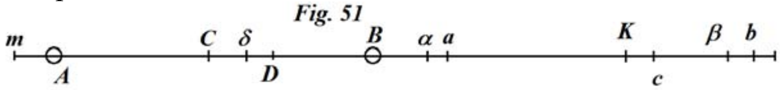
$\text{A.AC.AB} + \text{C.CD}^2 = \text{A.}ac.ab + \text{C.D}c^2$.

PROPOSITIO XL. THEOREMA.

224. *Centrum gravitatis corporum actuosorum inter se collisorum eadem post occursum celeritate ac directione feretur, quae ante incedebat, eademque erit post impulsum a se mutuo recedentium corporum celeritas relativa, quae ante conflictum erat accedentibus.*

Fig. 50 :Positis quae in praecedenti propositione, probandum est fore $\text{D}c = \text{CD}$ & $ab = \text{AB}$, nam AB est celeritas relativa ante, & *ab* celeritas relativa eorundem corporum A, B, post occursum, punctaque C, *c* centra gravitatis indicant in utroque casu.

Fig. 51



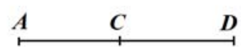


Fig. 54

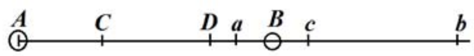


Fig. 52

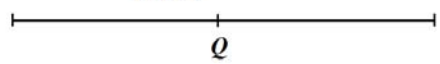


Fig. 51: *Demonstr.* Fingatur lineam rectam, in qua corpora moventur, infinite longam esse, atque moveri versus *m* velocitate infinite parva *Am,* huncque motum communem etiam corpora A, B in linea infinita incedentia participabunt; unde factis singulis $\text{D}\delta, a\alpha, c\text{K}, b\beta$ aequalibus *Am,* globus A movebitur tantum celeritate absoluta $\text{A}\delta$ loco ipsius AD ante occursum; quandoquidem ex motu ejus proprio AD communis $\text{D}\delta$ utpote contrarius est auferendus : B vero incedet velocitate $\text{B}\delta$ ante conflictum, accedente ejus motui proprio BD communi $\text{D}\delta$ alteri BD conspirante. Post conflictum autem celeritates *Da, Db,* mutabuntur in $\text{D}\alpha$ & $\text{D}\beta$; verum quia omnes $b\beta, c\text{K}, a\alpha, D\delta$ (secundum hypothesin) aequales , erunt $\alpha\text{K} = ac$, & $\alpha\beta = ab$ *;* atque adeo velocitates AC, *ac,* nec non AB & *ab* inalteratae manent in aequalitate superiore (§.223.) solis CD & D*e* variationem subeuntibus. Idcirco si in ea aequalitate

$\text{A.AC.AB} + \text{C.CD} = \text{A.}ac.ab + \text{C.D}c^2$ loco CD & D*c* substituantur $\text{C}\delta$ & DK , fiet

$\text{A.AC.AB} + \text{C.CD}^2 = \text{A.}ac.ab + \text{C.DK}^2$, quae ex altera subducta

relinquet $\text{C.CD}^2 - \text{C.C}\delta^2 = \text{C. D}c^2 - \text{C.DK}^2$, vel $\text{CD}^2 - \text{C}\delta^2 = \text{D}c^2 - \text{DK}^2$, aut

etiam $2\text{CD.D}\delta - \text{D}\delta^2 = 2\text{D}c.c\text{K} - c\text{K}^2$, hoc est quia $\text{D}\delta = c\text{K}$, erit $2\text{CD.D}\delta = 2\text{D}c.c\text{K}$, atque adeo $\text{CD} = \text{D}c$.

Porro ex aequalitate $\text{A.AC.AB} + \text{C.CD}^2 = \text{A.}ac.\ ab + \text{C.D}c^2$ abjectis aequalibus C.CD^2 & $\text{C.D}c^2$, restabit $\text{A.AC.AB} = \text{A.}ac.ab$, vel $\text{AC.AB} = ac.ab$. Verum est $\mathrm{AB}:\mathrm{AC}=ab:ac$, vel $\mathrm{AB}^2:\mathrm{AC.AB}=ab^2:ac.ab$, igitur propter $\mathrm{AC.AB}=ac.ab$, erit $\mathrm{AB}^2=ab^2$ atque adeo $\mathrm{AB}=ab$. Quae erant demonstranda.

PROPOSITIO XLI. PROBLEMA.

225. *Datis corporibus* A, B *utcunque inaequalibus sibi directe occurrentibus cum datis celeritatibus, invenire velocitates eorumdem corporum post conflictum.*
Ex praecedenti propositione hujus Problematis solutio facillima est. Nam factis $\mathrm{D}c=\mathrm{CD}$, item $ca=\mathrm{CA}$, & $cb=\mathrm{CB}$, ubi observandum punctum D, quo rectae velocitatum ante occursum repraesentatrices terminantur, semper cadere debere inter puncta C, *c* quibus centrum mobilium anti & post occursum signatur, punctaque *a* & *b* respectu puncti *c*, eodem ordine posita esse, quo alteram C situm est respectu corporum A, B. Quibus observatis erit semper D*a*, velocitas corporis A, & D*b* corporis B post conflictum.
Demonst. Quia (constr.) $ac=AC$ & $bc=\mathrm{BC}$, erit $ab=\mathrm{AB}$, & quia porro (constr.) $\mathrm{D}c=\mathrm{CD}$, hoc est, quia celeritas relativa corporum post collisionem aequalis relativae ante eandem , & pariter celeritas atque directio centri gravitatis utroque casu eadem est, praecedentem corpora A, B non aliis quam velocitatibus D*a*, D*b* moveri possunt post collisionem. Quod erat demonstrandum.

COROLLARIUM I.

226. Hinc 1°. si mobilia A, B ad eandem partem ferantur, adeo ut punctum D cadat extra AB, sed ad partes corporis B ; erit $\mathrm{D}a=\mathrm{CD}-\mathrm{AC}$ & $\mathrm{D}b=\mathrm{CD}+\mathrm{CB}$. Sin vero punctum D ad partem corporis *A* extra AB cadat fient $\mathrm{D}a=\mathrm{CD}+\mathrm{AC}$, & $\mathrm{D}b=\mathrm{CD}-\mathrm{CB}$.
2°. Si corpora A,B ex adversis partibus venientia inter se collidantur, ita ut D cadat inter A & B, & punctum C sit inter D & A, erit $\mathrm{D}a=\mathrm{CD}-\mathrm{AC}$, & $\mathrm{D}b=\mathrm{CD}+\mathrm{CB}$. Ac vice versa erit $\mathrm{D}a=\mathrm{CD}+\mathrm{AC}$, & $\mathrm{D}b=\mathrm{CD}-\mathrm{CB}$, quoties centrum gravitatis corporum situm est inter D & B.

COROLLARIUM II.

227. Adeoque si corpus A celeritate AD impingatur inquiescens B, ita ut puncta D & B confundantur, huic quiescenti B dabit impellens A celeritatem duplam ipsius CB. Atque adeo si ambo A & B fuerint aequalia, in ipso conflictu sistetur motus corporis impellentis A, alterumque B tota velocitate AB deinceps feretur, qua agens A in ipsum impegerat.

COROLLARIUM III.

228. Sin vero corpora ex oppositis partibus obviam veniant celeritatibus propriis AD, BD corporibus B, A reciproce proportionalibus, ita ut D incidat in C, utrumque corpus, qua advenerat velocitate, resiliet. Etenim hoc casu evanescrit CD.

COROLLARIUM II.

229. Generaliter, si corpora A, B dicantur *m,n,* eorum velocitates propriae ante occursum *u* & r, invenietur D*a* celeritas corporis A post conflictum $=(mu-nu+2ur):m+n,=u,+(u-r).2n:m+n$ & D*b* vel celeritas corporis B post conflictum $(2mu+mr-nr):m+n,=r,+(u-r).2m:m+n$. Hae expressiones elicitae sunt ex debitis substitutionibus valorum linearum CD, AC & BC in expressionibus Corollarii I: $Da=\mathrm{CD}-\mathrm{AC}$, & $\mathrm{D}b=\mathrm{CD}+\mathrm{CB}$, pro casu, quo mobilia ante concursum ad easdem partes moventur. Formulae pro altero casu corporum ex oppositis partibus in se mutuo irruentium ex hisce jam exhibitis elicientur mutando duntaxat signa quibus litera *r* affecta est, eritque
$\mathrm{D}a=u,+(u+r).2n:m+n,$ & $\mathrm{D}b=-r,+(u+r).m+n.$

PROPOSITIO XLII. THEOREMA.

230. *Corpus* A *celeritate* AD *latum interventu corporis medii* X *eandem corpori quiescenti* B *velocitate dabit*, *quam interventu alterius cujusdam corporis medii* Y, *si ipsam impellens A fuerit ad X sicut alterum* Y *ad quiesens corpus* B.

Hoc est, si corpus A celeritate AD primuim in corpus X irruat, & hac celeritate ab impellente accepta deinceps incurrat in corpus quiescens B ; dico hoc corpus B eandem celeritatem accepturum, quam si ab alio corpore Y impulsum fuisset celeritate ea, quam Y pariter quiescens a corpore A itidem celeritate AD ipsi occurrente accepisset, existente scilicet A ad X ut Y ad B. Ut propositum demonstretur, constructio Propositionis XL. est geminanda, & pro utroque corpore intermedio X & Y seorsim ponendae propterea figura, in qua lineae majusculis litteris signantur, respicit corpus intermedium X, & quae iisdem, sed minuculis literis notantur, pertinent ad figuram corporis alterius intermedii Y.

Jam ex §.227. constat corpus X accepturum ab impellente A celeritatem DB duplam ipsius CD, existente C centro gravitatis corporum A, X, adque adeo AC ad CD in ratione corporis X ad corpus A; ac divisa DB in puncto E, ita ut DE sit ad EB sicut corpus B ad mediam X, mobile X daturum alteri quiescenti B, in quod impingat velocitae DB, celeritatem duplun ipsius EB. Similis constructio, sed respectu corporum *a,* Y & *b,* inter quae ipsa *a* & *b* corporibus A & B aequantur, repetita intelligatur, accipietque corpus *b* a medio Y celeritatem 2*eb.* Probandum fore $\mathrm{EB}=eb$.

Demonstr. Quia (secundum hypothesin) est $\mathrm{A}:\mathrm{X}=\mathrm{Y}:\mathrm{B}$, & propter centrum gravitatis C, $\mathrm{A}:\mathrm{X}=\mathrm{CD}:\mathrm{AC}$; nec non $\mathrm{Y}:b(\mathrm{B})=be:de$, erit $\mathrm{CD}:\mathrm{AC}=be:de$, vel invertendo & componendo $\mathrm{AD}:\mathrm{CD}=2.cd\,(db):be$; adeoque etiam duplicatis consequentibus $\mathrm{AD}:\mathrm{DB}(2\mathrm{CD})=2cd:2be=cd:be$. Verum est etiam $\mathrm{B}:\mathrm{X}=\mathrm{Y}:\mathrm{A}$, hoc est $\mathrm{DE}:\mathrm{BE}=ac:cd$, ergo & componendo $\mathrm{DB}:\mathrm{BE}=ad:dc$, hinc etiam $ad\,(\mathrm{AD}):\mathrm{DB}=cd:\mathrm{BE}$, sed paulo ante erat $\mathrm{AD}:\mathrm{DB}=cd:be$; ergo etiam $cd:be=cd:\mathrm{BE}$, atque adeo $be=\mathrm{BE}$. Quod erat demonstrandum.

231: *Aliter.* Sint $\mathrm{LN = AD} = ad, \mathrm{PN = CD};$ ac $pN = cd$,& denique dividatur LN in Q & *q*, ut LN sit ad QN sic ut $\mathrm{B + X}$ ad X, & LN ad *q*N ut $\mathrm{B + Y}$ ad Y , & ostendetur puncta P, *q*, item & *p*, Q coincidere. Nam quia (secundum hypothesin) $\mathrm{A : X = Y : B}$, erit $\mathrm{A + X : A = B + Y : Y}$, id est $\mathrm{AD : CD = LN : PN = LN :} q\mathrm{N}$, ergo $\mathrm{PN} = q\mathrm{N}$ atque adeo coincidunt puncta P & *q*. Item quia $\mathrm{B : X = Y : A}$, & $\mathrm{B + X : X = A + Y : A}$, erit (constr.) $\mathrm{LN : QN}\left(= ad : cd\right) = \mathrm{LN} : p\mathrm{N}$, ac proinde etiam $\mathrm{QN} = p\mathrm{N}$; coincidunt ergo puncta *p*, Q. Hisce positis, & quia
$\mathrm{AD : DB}\left(\mathrm{2CD}\right) = \mathrm{LN : 2PN};$ & $\mathrm{DB : 2.EB = LN : 2QN}$; erit ex aequo
$\mathrm{AD : 2.EB = LN^2 : 4.PN.QN}$. Eodem argumento invenietur $ad : 2eb = \mathrm{LN}^2 : 4\mathrm{PN}.q\mathrm{N}$, ergo invertendo erit ex aequo $2.eb : 2.\mathrm{EB} = 4.p\mathrm{N}.q\mathrm{N} : 4.\mathrm{PN : QN}$,
vel $eb : \mathrm{EB} = p\mathrm{N}.q\mathrm{N} : \mathrm{PN.QN}$: unde cum ostensum sit esse $p\mathrm{N = QN}$ & $q\mathrm{N = PN}$, erit omnino $eb = EB$. Quod erat demonstrandum.

COROLLARIUM I.

232. Si loco alterutrius ex mediis X, Y sumatur corpus M medium proportionale inter A & B, atque adeo inter X, & Y, cum A sit ad $\mathrm{X = Y : B}$; atque LN dividatur in R, ut tota LN sit ad RN sicut $\mathrm{A + M}$ ad A , celeritas initialis mobilis A erit ad celeritatem corpori B interventu medii M acquisitam, sicut $\mathrm{LN^2}$ ad $\mathrm{4.RN^2}$. Atque adeo celeritas, quam acquiret corpus B interventu alterutrius X vel Y erit ad celeritatem, quam acquiret interventu medii M; sicut 4PN.QN ad $\mathrm{4.RN^2}$,vel sicut PN.QN ad $\mathrm{RN^2}$. Nam quia
$\mathrm{A + M : M = LN : RN},$ & $\mathrm{A + M : A = B + M : M}$, erunt hoc casu ambae PN, QN cum inter se, tum etiam RN aequales, unde cum generaliter sit $\mathrm{AD : 2EB = LN^2 : 4.PN.QN}$, erit in hoc casu particulari celeritas corporis A, ad celeritatem ipsius B abs M acceptam $= \mathrm{LN^2 : 4.RN^2}$, & quia (§.231.) velocitas ipsius B ab alterutro X vel Y accepta, est ad celeritatem initialem corporis A,sicut 4.PN.QN ad $\mathrm{LN^2}$; erit ex aequo celeritas corporis B ab alterutro X vel Y accepta, ad celeritatem ejusdem a corpore M acquisitam sicut 4.PN.QN ad $\mathrm{4.RN^2}$ hoc est, sicut PN.QN ad $\mathrm{RN^2}$.

COROLLARIUM II.

233. Quia (constr.) $\mathrm{PN : LN = A : A + X},$ & $\mathrm{LN : RN = A + M : A}$, erit ex aequo $\mathrm{PN : RN = A + M : A + X}$. Similiter invenietur $\mathrm{QN : RN = A + M : A + Y}$, ergo per compositionem rationum erit

$$\mathrm{PN.QN : RN^2} = \left(\mathrm{A + M}\right)^2 : \left(\mathrm{A + X.A + Y}\right) = \mathrm{AA + 2.AM + MM : AA + (X + Y).A + XY}$$

$$= \mathrm{AA + 2.A.M + AB : AA} + \left(\mathrm{X + Y}\right)\mathrm{.A + AB = A + 2M + B : A + X + B},$$

nam $\mathrm{A.B = MM = XY}$, adeoque unum alterius loco, ut factum est, substituere licet. Jam quia X, M & Y sunt in ratione continua, erit $\mathrm{X + Y}$ major quam 2.M atque adeo $A+X+Y+B>A+2M+B$; hinc erit $RN^2>PN.QN$; unde cum (§.132.) celeritas corporis B ab alterutro X vel Y accepta, sit ad ejusdem celeritatem a corpore M ipsi impressam, sicut $PN.QN$ ad RN^2, liquet hanc celeritatem illa perpetuo majorem esse. Imo haec velocitas mobili B a corpore M collata major erit ea, quam idem B a corpore A celeritate AD ipsum impulisset, nam loco corporis X intelligi potest quod vis corpus; atque adeo unum ipsi A aequale, quod ab impellente A totam celeritatem (§217.) CD accipiet, adeo ut idem sit, sive mobile A cum sua celeritate AD immediate impingat in quiescens B, sive id impellat ope intermedii ipsi A aequali, cui totam suam celeritatem contulerit; unde cum generalitcr oftensum sit, interventu cujusvis corporis X, diversi ab M, minorem corpori B velocitatem collatum iri quam a medio proportionali M inter extrema A, & B ; sequitur necessario etiam corpus A alteri B minorem daturum esse celeritatem quam M, etiamsi immediate in id impingat. Atque hoc corollarium continet demonstrationem facilem Propositionis penultimae tractatus *Hugenii* de Motu corporum ex Percussione.

COROLLARIUM III.

234 Hinc primum ex corporibus continue proportionalibus ultimo majorem celeritatem dabit per corpora quotcunque interposita continue proportionalia, quam si eidem quiescenti immediate occurrisset.

SCHOLION.

Fig. 54. 235. Ex praecedentibus jam facile elicietur regula determinadi velocitatem, quam ultimum corpus quiescens ex serie quotcunque continue proportionalium accipiet transmisso motu a primo in secundum, tertium, quartum &c. Primum corpus dicatur A, ejusque celeritas ab initio AD, ultimum V : dividatur celeritas *initialis* AD in puncto C, in reciproca ratione secundi corporis ad primum, vel, quod idem est, in ratione cujuslibet corporis in serie proportionalium ad corpus, quod illud in eadem serie proxime praecedit, ita ut AC ad CD sit in hac eadem ratione, deinde ratio totius AD ad duplam CD continuetur in tot terminis uno minore quot sunt seriei corporum proportionalium, quorum ultimus sit Q, dico hanc Q exponere celeritatem ultimo corpori V collatam ab omnibus praecedentibus. Adeoque si numerus terminorum fuerit $n+1$, celeritas initialis AD primi corporis erit ad Q celeritatem ultimo U acquisitam, sicut AD^n ad $(2CD)^n$, hinc habebitur $Q=(2CD)^n : AD^{n-1}=(2CD)^n$, sumta scilicet AD instar unitatis, quod hoc loco fieri licet.

236. *Exempl.* Sint centum corpora in continua ratione dupla, motusque primum incipiat a maximo, quaeritur Q seu celeritas minimo V conferenda. Hoc casu erunt $AC=\frac{1}{3}$, et $CD=\frac{2}{3}$, atque adeo $(2.CD)^n=\left(\frac{4}{3}\right)^{99}$; est enim $n=99$. Igitur habemus $Q=\left(\frac{4}{3}\right)^{99}$, & Log-us $Q=99.\text{log-um}\frac{4}{3}$; atqui log-us $(4:3)=0.1249388$ qui Logarithmus ductus in 99 dat $99\cdot\text{Log.}(4:3)=12.3689412=\text{Log.Q}$, ergo $Q=2338520732310$ quam proxime, nam hic numerus log-mo illi, cujus characteristica

est 12 proxime convenit; atque adeo celerias, quam corpus centesimum accipere debet, est ad celeritatem primi *A* ut 2338520732310 ad 1 proxime.

Sin vero motus a minimo incipiat, erit hujus celeritas ab initio ad velocitatem, quam centesimo & maximo dabit interpositis 98 reliquis continue proportionalibus, sicut numerus 27103713483146067 ad unitatem proxime. Hugenius in postrema propositione tractatus supra (§.133.) laudati numeros pro hisce casibus nostris minores exhibet, transscriptorum forte aut calculi lapsu.

PROPOSITIO XLIII. THEOREMA.

237. *Centrum gravitatis duorum corporum sphaericorum elasticorum sibi imvicem oblique occurrentium, eadem directione & celeritate feretur post conflictum, quibus ferebatur ante concursum.* Fig. 55.

Globi elastici P, Q sibi mutuo occurrant in D celeritatibus propriis PD, QD, & post occursum resiliant celeritatibus propriis D*p*, *Dq*, ita ut hae lineae eadem tempore aequabili motu absolvantur, quo rectae PD, QD ante synodum; sintque puncta M & *m* centra gravitatis globorum in P & Q, item in *p* , *q* existentium; probandum est rectas MD & D*m* tum aequales, tum etiam in directum positas esse; quandoquidem MD exponit velocitatem & directionem centri gravitatis M ante conflictum, *Dm* vero velocitatem & directionem centri communis gravitatis post collisionem. Sit D punctum contactus globorum, & BA recta jungens centra eorundem, quae, ubi globi se mutuo contingunt, per punctum contactus transibunt; ex centris Globorum P, Q demissae sint ad AB perpendiculares PA, QB, & ex M recta MC reliquis QB, AP parallela, quibus positis, manifestum est ex art. 29, motui juxta PD aequipollere laterales PA & AD, & motui QD laterales QB & BD. Jam quia motus juxta PA & QB paralleli sunt, globi quatenus motibus hisce agitantur, in se mutuo agere non possunt, sed soli motus juxta AD & BD, utpote directe contrarii, in casu conflictus considerandi veniunt reliquis tantisper sepositis & postea iterum resumendis. Porro quia M est centrum gravitatis globorum in P & Q existentium, etiam C eorundem globorum in A & B positorum erit centrum gravitatis; adeoque factis $\mathrm{D}c=\mathrm{CD}$, nec non $ca=\mathrm{CA}$ & $cb=\mathrm{CB}$, erit (§. 225.) D*a* celeritas propria globi A, & D*b* celeritas alterius B, post collisionem in indefinita linea *ab*. Porro per *a* & *b* ductae sint rectae $ap=\mathrm{AP}$, & $bq=\mathrm{BQ}$ ipsi AB normales, quo posito & quia in ipso conflictu motus juxta PA & QB, utpote paralleli, nullam mutationem subire potuerunt, hi motus post impulsum illibati sunt conservandi; atque adeo post congressum corpus P a puncto D recedet motu D*p* resultante ex lateralibus D*a* & *ap,* alterumque Q motu D*q* nascente ex lateralibus D*b* & *bq.* Verum Q quia (constr.) $ca=\mathrm{CA};\ cb=\mathrm{CB};\ ap=\mathrm{AP};$ & $bq=\mathrm{BQ}$, figura *abqp* similis & aequalis erit figurae alteri ABQP , atque adeo $cm=\mathrm{CM}$: unde cum (constr.) etiam sit $\mathrm{D}c=\mathrm{CD}$ triangulum *Dcm* simile & aequale erit triangulo DEM, atque adeo lineae MD & *Dm,*in directum positae erunt, hinc centrum gravitatis globorum *P,* Q post conflictum eadem celeritate & directione qui ante, progredietur. Quod erat demonstrandum.