CHAPTER IV.
Concerning the central forces by which moving bodies may be kept in orbits, and the motion of the apses.

DEFINITION.

187. ABE shall be some *stationary* orbit, to which the figure *abe* is said to be that of a similar and equal *rotating* orbit, because both figures are directed to move around the point C, and indeed they are understood to be moving according to some central force ; yet on that account, so that the angle AC*a*, which its axis *ae* will describe in some time, shall be in a given ratio to the angle *a*C*b* subtended, which the arc of the curve of the moving body *ab* has described by advancing for the same time. The motion of the axis *ae* is said to be the motion of the apses. [Thus, for the earth rotating around the sun, the apses, here the perihelion and aphelion, slowly change with time ; similarly, perigee and apogee for the moon; the formula for the force responsible for this kind of motion is required to be found.]

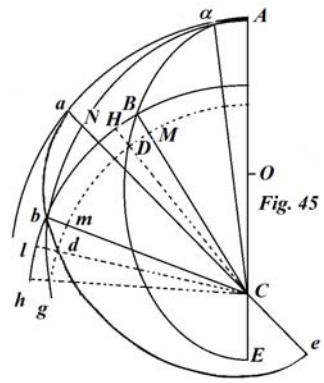


Fig. 45

COROLLARY I.

188. Because the angle *a*CA is put in the given ratio to the angle *b*C*a* everywhere, also by adding the angle *b*CA will be in a given ratio to the angle *b*C*a*, or with the circle *b*B described through the point *b* of the orbit, cutting the fixed orbit at B. Following the illustrious Newton, we may nominate the ratio of the angle BCA to the angle *b*CA equal to the given ratio F to G.

[Thus, both orbits obey the same law of attraction for the radial motion, while the body in the rotating orbit has a greater angular speed than that in the fixed orbit, given in the ratio F:G, relative to the fixed circle for measuring the angles.]

COROLLARY II.

189. Since the mobile body *a*, describing *abe* by moving around the rotating orbit, shall have a two-fold motion, evidently that, by which it may advance in its own moveable orbit, also by the motion of the orbit itself, from which twin motion it is apparent the composite motion is going to be along the curved line AN*b*. Therefore it is the same, as if the mobile body were moving along some immoveable curve AN*b*, instead of advancing along the moveable orbit *abe*, and the centripetal or the central force at the point *b* of the curve AN*b* in short being the same as that required, so that the moving body shall be able to proceed on the curve *abe* without departing from the curve *ab*, while restricting the motion to the same plane *abea*. Therefore for determination of the central force, which shall retain the body always on the perimeter of the moveable orbit, an expression for such a central force is sought for the individual points of the fixed curve AN*b* with the twofold resulting motion, clearly from the motion of the orbit and from the motion of the

body on the orbit ; as that which the Celebrated Varignon published in the *Actis. Acad. Scient. Paris*. 1705 for the 5th Dec. Truly, because the whole matter can be elegantly derived from the theory of centrifugal forces, it will be a pleasure to show a little of that. [At this point Hermann switches from Newton's description in terms of centripetal forces to Huygens' description in terms of centrifugal forces, as used by Varignon. This indicates a return to Descartes vortices as an explanation of gravity, as the whole plane is considered to rotate, and the body experiences a force directed away from the centre of attraction; this of course is contrary to Newton, but we need to go along with it for the present as it offered an alternative though incorrect physical explanation at the time. In fact, the business of equating centrifugal and centripetal forces was a well-known trap for the unwary, still perpetuated in some older physics texts, which shows a lack of understanding of what Newton postulated. However, by considering the centrifugal force term $\frac{mv^2}{r}$ not as a force *per se*, but rather as the analytical form of Newton's second law adopted for motion due to a central attraction such as gravity, the reasoning is made sound.]

PROPOSITION XXXI. THEOREM.

190. Fig. 45. *The excess of central force required for that to happen, so that some body may be able to move in an orbit* abe *rotating about the point* C, *is to be added to the central force, by which the same body may be acted on, to be carried in the fixed orbit* ABE *at an equal distance* BC *or* b*C*, *by a force equal to the excess over the centrifugal force at the point b, in the orbit of the moving body described by the arclet bh, in addition to the centrifugal force of the moving body attached to the same extremity* B *of the transported radius* CB, *and trying to describe the arclet* BH *in that same time, in which the point b may complete its own arclet bh.*

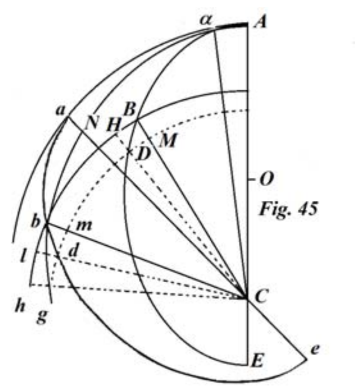


Fig. 45

Because the body is required to move in a rotating orbit, from the rotation or circular motion of the plane *abe*, it acquires a force trying to recede from the centre C, and accordingly from the curve *ab* itself, and it is necessary that a certain force exerted directly to the point C may diminish a force of this kind, according to that, so that the body may rotate in the moveable orbit, but may not wander beyond that; and this force is required to be exactly equal to that centrifugal force arising from the motion of the place alone, otherwise the body will not be detained in its orbit, if the action of the centrifugal force were greater or less : but the centrifugal force arising from the motion of the plane *abe* is greater than the centrifugal force of the point *b*, arising from the motion of the transported radius *b* struggling to describe the [elemental] angle *bCh*, in addition to the centrifugal force acting at the point B in the stationary orbit ABE, with the centrifugal force describing the arclet BH ; for the motion of the angle *bCh* not only involves the motion of the angle BCH of the radius BC carried around in the stationary orbit at the same time as the other *bCh* ; but also the motion of the plane of the orbit itself, and therefore the centrifugal force, which results from the motion of the point *b* carried by the radius *b*C, describing the angle *b*C*h*, not only contains within itself the centrifugal force of the point B carried by the radius of the stationary orbit BC , describing the angle BCH, but also the centrifugal force, which arises from the motion of the plane *abe*, and thus this centrifugal force is equivalent to the excess of the centrifugal force of the point *b*, above the centrifugal force of the point B on the fixed orbit ; and the same centrifugal force arising from the motion of the plane *abe* is equal to the centrifugal force, by which it must be reduced; and this reducing action is equal to the excess of the central action at the point of the moving orbit above the centripetal force at the point B in the stationary orbit with the other *b* equal to the other. Therefore this excess of the centripetal force is equal to the excess of the centrifugal force at *b* and B. Q.E.D.

PROPOSITION XXXII. THEOREM.

191. *With the same in place, the excess of the force at the point b of the orbit, above the centripetal force at the point B in the stationary orbit at the equivalent point, will be to the centripetal force at the point* A *of the stationary orbit in the ratio composed from the ratio* $\mathrm{F}^2-\mathrm{G}^2$ *to* G^2 *and from the ratio of the volume* $\mathrm{AC}^2.\mathrm{AO}$ *to the volume* BC^3; *with the radius of the circle* AO *present of the same curvature as the curve* ABE *at* A*, or the radius of the circle of curvature at* A.

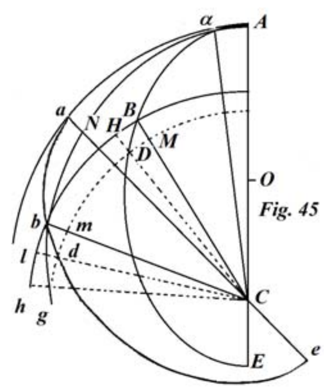


Fig. 45

For the sake of brevity we will designate the velocities of the circulating radii *bC*, BC, AC in Fig. 45. by *ub*, *u*B, *nA;* and the centrifugal forces of the points *b*, B, A of the rotations by *cfb*, *cf*B, *cf*A, and finally the centripetal forces at the same points by *cpb*, *cp*B, *cp*A.

I. Now, because the velocities of the radii C*b*, GB are as the angles described by these themselves in the same time, there will be

$ub : uB = \text{ang.}b\mathrm{C}h : \text{ang.BCH}\ (\S.188.) = \mathrm{F}:\mathrm{G}$, therefore $(ub)^2 : (u\mathrm{B})^2 = \mathrm{F}^2 : \mathrm{G}^2$. And in equal circles the centrifugal forces are in the square ratio of the speeds, therefore $cfb : cfB = \mathrm{F}^2 : G^2$, & on being divided, $cfb - cfB : cfB = \mathrm{F}^2 - G^2 : G^2$.

II. Again since in the fixed orbits in equal times equal areas BCD or BCH, and ACα are described, the velocities of the radii BC, AC will be inversely proportional with these radii, that is :

$u\mathrm{B} : u\mathrm{A} = \mathrm{AC}:\mathrm{BC}$, & $(u\mathrm{B})^2 : (u\mathrm{A})^2 = \mathrm{AC}^2 : \mathrm{BC}^2$ and (from §.183. eq. 1.) there will be

$(u\mathrm{B})^2 = \mathrm{BC}.cf\mathrm{B}$, & $(u\mathrm{A})^2 = \mathrm{AC}.cf\mathrm{A}$; [*i.e.* $\frac{v^2}{r}\times r = v^2$.] Therefore

$\mathrm{BC}.cf\mathrm{B} : \mathrm{AC}.cf\mathrm{A} = \mathrm{AC}^2 : \mathrm{BC}^2$, or also on multiplying this equation by AC:BC, as shown,

there will be $AC.BC.cfB : AC.BC.cfA = cfB : cfA = AC^3 : BC^3$ [*], truly (in no.1.) there was $cfb - cfB : cfB = F^2 - G^2 : G^2$ [**], therefore from the ratio composed and the equation, there becomes [*i.e.* * by **] $cfb - cfB : cfA = \left(F^2 - G^2\right).AC^3 : G^2.BC^3$. Again because (from §.183) $(uA)^2 = AC.cfA$, & (from §.154), $(uA)^2 = AO.cpA$, with AO the radius of curvature of the circle ABE at the point A, there will be $AC.cfA = AO.cpA$, and thus $cfA : cpA = AO : AC$; therefore from the equation anew,
$cfb - cfB : cpA = \left(F^2 - G^2\right).AC^2.AO : G^2.BC^3$; and by the preceding Prop., there is $cfb - cfB = cpb - cpB$, therefore the excess of the centripetal force at *b* to the force at B, is to the centripetal force at A, in a composite ratio, from the ratio $F^2 - G^2$ to G^2 and in the ratio of the solid $AC^2.AO$ to the solid BC^3. Q.E.D.

COROLLARY I.

192. And thus at any point *b* of the moving orbit *abe*, the central force will be as $cpB, + (F^2 - G^2).AC^2.AO.cpA : G^2.BC^3$.

COROLLARY II.

193. Hence if the orbit ABE, *abe*, were elliptical, thus so that $cpB : cpA = AC^2 : BC^2$, and thus $cpB = AC^2.cpA : BC^2$, there will be :

$cpb = AC^2.cpA : BC^2, + \left(F^2 - G^2\right).AC^2.AO.cpA : G^2.BC^3$; [Equations written in this manner mean, of course: $cpb = \frac{AC^2.cpA}{BC^2} + \frac{\left(F^2 - G^2\right).AC^2.AO.cpA}{G^2.BC^3}$.]

SCHOLIUM.

194. Now with the general law of the central force found on moveable orbits, the problem, by which the motion of the apse *ae* may be determined, can be found from the given law of the ratio F to G of the centripetal forces, with a solution soon being established without difficulty by the method. The way followed by the Cel. Newton in the solution of this same problem is both ingenious and extremely elegant, supposing the orbits to be almost circles, he has described each more broadly in *Sect. IX. Lib.* I. *Princ. Phil. Nat. Math.* [see *e.g.*, the translation presented on this website] and later illustrated by David Gregory in a more verbose commentary in his *Astronomiae Physicae & Geometricae Elementa* Lib. IV. Sect. II. The praiseworthy method of Newton consists in the comparison of the terms of some infinite series with the homologous terms in the rule for determining the centripetal force in a revolving orbit, and that may be seen to demand, for individual new examples, new series and requiring a new calculation to be

put in place. But what, if I may have found an easy way, from which the same may be able to be obtained without the aid of any infinite series, though much longer, since it presents a general method, yet it shall disclose the law of the centripetal force , the ratio of F to G ?

195. Generally the centripetal force at the point *b* of the rotating orbit shall be, as $\mathrm{P}:\mathrm{BC}^3$, and there will be (from §.193.)
$\mathrm{P}=\mathrm{AC}^2.\mathrm{BC}^2.cp\mathrm{A},+\left(\mathrm{F}^2-\mathrm{G}^2\right).\mathrm{AC}^2.\mathrm{AO}.cp\mathrm{A}:\mathrm{G}^2,\&,$ if the decrease of the infinitesimal magnitude P may be called Q.*bm* ; the formula determining the centripetal force at the point *d* of the rotating orbit, will be :

$$\mathrm{P},-\mathrm{Q}.bm=\mathrm{AC}^2.d\mathrm{C}.cp\mathrm{A},+\left(\mathrm{F}^2-\mathrm{G}^2\right).\mathrm{AC}^2.\mathrm{AO}.cpA:\mathrm{G}^2$$

$$=\mathrm{AC}^2.\mathrm{BC}.cp\mathrm{A}-\mathrm{AC}^2.bm.cp\mathrm{A},+\left(\mathrm{F}^2-\mathrm{G}^2\right).\ \mathrm{AC}^2.\mathrm{AO}.cp\mathrm{A}:\mathrm{G}^2$$

which taken from the first leaves
$\mathrm{Q}bm=\mathrm{AC}^2.bm.cp\mathrm{A},\mathrm{vel}\ \mathrm{Q}=\mathrm{AC}^3.cp\mathrm{A}$; which value substituted into the first equation of this article, produces : $\mathrm{P}=\mathrm{QB.BC}+\left(\mathrm{F}^2-\mathrm{G}^2\right).\mathrm{Q.AO}:\mathrm{G}^2$, from which there may be elicited : $\mathrm{F}:\mathrm{G}=\sqrt{\left(\mathrm{P}-\mathrm{Q.BC}+\mathrm{Q.AO}:\mathrm{Q.AO}\right)}.$ Truly, because the ellipse ABE which, as we may suppose with Newton, approaches to the shape of a circle, thus BC, AC & AO themselves are treated as equal, for with half the parameter of the ellipse, or AO, taken from half the diameter of the circle AC or BC at which it is defined, it will not differ ; and therefore the proceeding ratio will change into this simple form
$\mathrm{F}:\mathrm{G}=\sqrt{P}:\sqrt{\mathrm{Q.AO}}.$ Which is the general rule, which was being sought above and which was promised above, for the orbit of the rotating ellipse.

196: It pleases to show the above rule by a single example, which will be put in place of everything shown by us, and that we will borrow from the illustrious Newton. Therefore the centripetal force at the point *b* of the rotating orbit shall be as the magnitude $\frac{az^m+bz^n}{z^3}$, where *z* indicates *b*C or BC in fig. 45, and hence there will be
$\mathrm{P}=az^m+bz^n,\&\ \mathrm{Q}.bm=\mathrm{Q}.dz=amz^{m-1}dz+bnz^{n-1}dz$, that is
$\mathrm{Q}=az^{m-1}+bz^{n-1}$, hence $\mathrm{F}:\mathrm{G}\left(=\sqrt{\mathrm{P}}:\sqrt{\mathrm{Q.AO}}\right)=\sqrt{\left(az^m+bz^n:amz^{m-1}\mathrm{AO}+bn^{z-1}\mathrm{AO}\right)}.$
Truly, because just as it has been noted in the preceding article, z or $b\mathrm{C}=\mathrm{AC}=\mathrm{AO}$, with Newton that can be put equal to one, and there is found $\mathrm{F}:\mathrm{G}=\sqrt{(a+b:am+bn)}$ in a straight forwards manner as Newton did in example 3 after Prop. 45. *Lib. I. Princ. Phil. Nat.* Truly if there were $\mathrm{P}=az^m-bz^n$, we may find equally as the most praiseworthy man did : $\mathrm{F}:\mathrm{G}=\sqrt{(a-b:am-bn)}$.

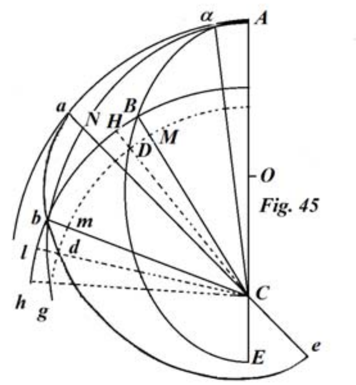
Finally if the central force were as $\mathrm{P}:\mathrm{BC}^3$, and thus $\mathrm{P}=\mathrm{BC}^{m+1}$, from the motion of the apses, or from the ratio F:G, the index *m* of the power BC^m may be found. For in this case there is found $Q = \overline{m+3}BC^{m+2}$; from which if $AO = AC = BC = 1$, there will be $F : G = \sqrt{(1 : m+3)}$ and thus $m = \left(G^2 - 3F^2\right) : F^2$.

[An introduction to this challenging problem can be found at present in Wikipedia : 'Newton's Theorem of Rotating Orbits ' ; however, no mention is made here of Hermann's work, nor of the contemporary sources cited therein.]

CAPUT IV.
De Solicitationibus centralibus quibus corpora in orbibus mobilibus detinentur, & de motu Apsidum.

DEFINITIO.

187. Sit ABE quilibet orbis *immobilis,* cui figura abe similis & aequalis dicatur orbis *mobilis,* quia circa punctum C ad quod solicitationes centrales diriguntur, reapse moveri intelligendus est ; ea tamen ratione ut angulus AC*a*, quem axis ejus *ae* aliquo tempore descripsit, sit ad angulum *a*C*b*, quem arcus curvae mobilis *ab* a mobili in curva incedente eodem hoc tempore descriptus subtendit, in ratio data. Axis *ae* motus, dicitur motus Apsidum.



COROLLARIUM I.
188. Quia angulus *a*CA ponitur ubique ad angulum *bCa* in data ratione, erit etiam componendo angulus *bCA* ad angulum *bCa,* vel descripto per orbis mobilis punctum *b* circulo *b*B immobilem orbem secante in puncto B; angulus *b*CA ad angulum BCA in data ratione. Cum Illustri Newtono nominemus rationem anguli BCA ad angulum *b*CA aequalem data F ad G.

COROLLARIUM II.

189. Cum mobile *a,* orbem mobilem *abe* circumeundo describens, duplicem habeat motum, scilicet eum, quo in suo orbe mobili incedit, tum etiam motum ipsius orbis, ex gemino hoc motu liquet compositum iri motum secundum lineam curvam *ANb.* Idcirco idem est, si mobile in curva immobili AN*b* movetur, quam si in orbe mobili *abe* incederet,& vis centripeta aut solicitatio centralis in curvae Anb puncto *b* requisita prorsus eadem erit cum ea qua opus est ut mobile in curva mobili *abe* semper incedere possit absque eo, ut unquam a curva *ab* recedat, durante motu plani ejusdem *abea.* Propterea ad determinationem solicitationis centralis, quae corpus semper in perimetro orbitae mobilis retinere possit, tantum quaerenda esset expressio solicitationum centralium pro singulis curvae immotae *ANb* ex duplici motu resultantis, scilicet ex motu orbitae & ex motu corporis in orbita, punctis; ut Cel. Varignon id egit in Actis. Acad. Scient. Paris. 1705 ad d. 5 Dec. Verum, quia tota res non ineleganter ex theoria conatuum centrifugerum derivari potest, id paucis ostendere libet.

PROPOSITIO XXXI. THEOREMA.

190 Fig. 45. *Excessus soliciiationis centralis requisitae ad id ut corpus aliquod in orbita mobili circa punctum* C, *ut* abe *revolvi queat, supra solicitationem centralem, qua idem corpus in orbe immobili* ABE *delatum in pari distantia* BC; *vel* bC *urgetur, aequatur excessui conatus centrifugi puncti* b, *in orbe mobili arculum* bh *describentis, supra conatum centrifugum mobilis cujusdam extremitati* B *radii vectoris* CB *annexi usque arculum* BH *describere nitentis eo tempusculo, quo punctum* b *suum arculum* bh *conficeret.*

Quia corpus in orbita mobili movendum a rotatione seu motu circulari plani *abe* acquirit conatum recedendi a centro C, atque ad eo a curva ipsa *ab*, necessum est, ut quaedam solicitatio ad punctum C directa conatum ejusmodi excussorium retundat, ad id ut corpus in orbita mobili gyrari queat, nec extra eam vagetur; hancque solicitationem praecise aequalem esse oportet conatui illi centrifugo a solo plani motu orto, alioqui corpus in orbita mobili non detineretur; si solicitatio conatu centrifugo major vel minor esset: sed conatus centrifugtis a motu plani *abe* oriundus est excessus conatus centrifugi puncti *b,* a motu radii vectoris *b* angulum *bCh* describere molientis, supra conatum centrifugum puncti B in orbita quiescenti ABE, arculum BH describere conantis; nam motus angularis *bCh* non solum involvit motum angularem BCH radii vectoris BC in orbita quiescenti alteri *bCh* contemporaneum ; sed etiam motum ipsius orbitae plani, ac propterea conatus centrifugus, qui resultat a motu puncti *b* in ratio vectore *b*C, angulum *b*C*h* describente, non solum in se continet conatum centrifugum puncti B in radio vectore BC orbitae quiescentis, angulum BCH describente, sed etiam conatum centrifugum, qui provenit a motu plani *abe,* adeoque hic conatus aequivalet excessui conatus centrifugi puncti *b,* supra conatum centrifugum puncti B in orbita immobili; atqui idem conatus centrifugtis a motu plani *abe* ortus aequalis est solicitationi centrali, a qua retundi debet; & haec solicitatio coercens conatum centrifugum aequatur excessui solicitationis centralis in puncto orbitae mobilis supra solicitationem centripetam in orbitae immobilis puncto B cum altero *b* aeque alto. Ergo hic excessus solicitationum centripetaum aequatur excessu conatuum centrifugorum in *b* & B.
Quod erat demonstrandum.

PROPOSITIO. XXXII. THEOREMA.

191. *ltsdem positis, excessus solicitationis in orbitae mobilis puncto* b, *supra solicitationem centripetam in orbitae quiestentis puncto* B *aequealto, erit ad soliciiationem centripetam iu puncto* A *orbitae immobilis in composita ratione ex ratione* F^2-G^2 *ad* G^2 *& ratione solidi* $AC^2.AO$ *ad solidum* BC^2; *existente* AO *radio circuli ejusdem curvitatis cum curva* ABE *in* A, *seu radio circuli curvam osculantis in* A.

Compendii gratia velocitates circulationis radiorum *bC,* BC, AC Fig. 45. dcsignabimus respective per *ub*, *u*B, *nA;* conatusque centrifuges punctorum b, B, A ; gyrantium per *cfb.*, *cf*B, *cf*A, ac denique solicitationes centripetas in iisdem punctis per, *cpb*, *cp*B, *cp*A.
Jam I.quia velocitates radiorum Cb, GB sunt ut anguli eodem tempore ab ipsis descripti, erit $ub:uB=\text{ang.}bCh:\text{ang.BCH}(\S.\ 188.)=F:G,$ ergo $(ub)^2:(u\text{B})^2=F^2:G^2.$ Atqui in circulis aequalibus conatus centrifugi sunt in duplicata ratione velocitatum, ergo $cfb:cfB=F^2:G^2$, & dividendo $cfb-cfB:cfB=F^2-G^2:G^2$.

II. Porro cum in orbita quiescenti aequalibus temporibus aequales areolae BCD vel BCH, & ACα describantur, velocitates radiorum BC, AC hisce radiis erunt reciproce proportionales, hoc est
$u\text{B}:u\text{A}=\text{AC}:\text{BC},\ \ \&\ \ (u\text{B})^2:(u\text{A})^2=\text{AC}^2:\text{BC}^2$ atque (§.183. aequ.1.) est $(u\text{B})^2=\text{BC.}cf\text{B},\ \ \&\ \ (u\text{A})^2=\text{AC.}cf\text{A}.$ Ergo $\text{BC.}cf\text{B}:\text{AC.}cf\text{A}=\text{AC}^2:\text{BC}^2$, vel etiam ductis in hac ultima analogia antecedentibus in AC & consequentibus in BC, eritque $\text{AC. BC.}cf\text{B}:\text{AC.BC.}cf\text{A}=cf\text{B}:cf\text{A}=\text{AC}^3:\text{BC}^3$, verum (num.1.) erat $cfb-cf\text{B}:cf\text{B}=F^2-G^2:G^2$, ergo per rationem compositionem & ex aequo fiet $cfb-cf\text{B}:cf\text{A}=(F^2-G^2).\text{AC}^3:G^2.\text{BC}^3$. Porro quia (§.183) $(u\text{A})^2=\text{AC.}cf\text{A},\ \ \&\ \ (\S.154)(u\text{A})^2=\text{AO.}cp\text{A}$, existente AO radio circuli curvam ABE in puncto A osculantis, erit $\text{AC.}cf\text{A}=\text{AO.}cp\text{A}$, atque adeo $cf\text{A}:cp\text{A}=\text{AO}:\text{AC}$; ergo denuo ex aequo $cfb-cf\text{B}:cp\text{A}=(F^2-G^2).\text{AC}^2.\text{AO}:G^2.\text{BC}^2$; atqui per Prop. praec. est $cfb\ -cf\text{B}=cpb-cp\text{B}$, ergo excessus solicitationis centripetae in *b* solicitationem in B, est ad solicitationem centripetam in A; in composita ratione, ex ratione F^2-G^2 ad G^2 & ratione solidi $\text{AC}^2.\text{AO}$ ad solidum BC^3. Quod erat demonstrandum.

COROLLARIUM I.

192. Adeoque in quolibet puncto b orbitae mobilis *abe*, solicitatio centralis erit ut $cp\text{B},+\ (F^2\text{-}G^2).\text{AC}^2.\text{AO.}cp\text{A}:G^2.\text{BC}^3$.

COROLLARIUM II.

193. Hinc si orbita ABE, *abe*, fuerit elliptica, ita ut $cp\text{B} : cp\text{A} = \text{AC}^2 : \text{BC}^2$, atque adeo $cp\text{B} = \text{AC}^2.cp\text{A} : \text{BC}^2$, erit

$$cpb = \text{AC}^2.cp\text{A} : \text{BC}^2, \ + \ \left(\text{F}^2 - \text{G}^2\right).\text{AC}^2.\text{AO}.cp\text{A} : \text{G}^2.\text{BC}^3$$

SCHOLION.

194. Inventa jam generali lege solicitationum centralium in orbitis mobilibus, problema, inveniendi ex data lege solicitationum centripetarum rationem F ad G, qua motus apsidis *ae* determinatur, solutu difficile non erit per methodum mox exponendam. Ingeniosa est via & oppido elegans quam Cel. Newtonus sequutus est in solutione ejusdem Problematis, supponens orbitas propemodum circulares, quamque fusius explicat Sect.IX. *Lib.* I. *Princ. Phil. Nat. Math.* & postea prolixo commentario illustravit Dav. Gregorius in suis Astronomiae Physicae & Geometricae Elementis Lib. IV. Sect. II. Laudata Newtoni methodus consistit in comparatione terminorum alicujus seriei infinitae cum homologis terminis in Canone pro determinatione solicitationum centripetarum in orbita mobili, eaque pro singulis novis exemplis novas series novumque calculum subducendum deposcere videtur. Sed quid, si modum facillimum aperuero, quo idem absque ullo serierum infinitarum auxilio obtineri queat, imo longe plura, quandoquidem praebet canonem generalem, quaecunque solicitationis centripetae sit lex, rationem F ad G manifestantem ?

195. Sit generaliter solicitatio centripeta in orbitae mobilis puncto b, ut $\text{P} : \text{BC}^3$, eritque (§.193.) $\text{P} = \text{AC}^2.\text{BC}^2.cp\text{A}, + \left(\text{F}^2 - \text{G}^2\right).\text{AC}^2.\text{AO}.cp\text{A} : \text{G}^2, \&,$ si decrementum infinitesimum magnitudinis P dicatur Q.*bm* ; formula determinans solicitationem centripetam in orbitae mobilis puncto *d*, erit;

$$\text{P}, -\text{Q}.bm = \text{AC}^2.d\text{C}.cp\text{A}, + \left(\text{F}^2 - \text{G}^2\right).\text{AC}^2.\text{AO}.cpA : \text{G}^2$$

$$= \text{AC}^2.\text{BC}.cp\text{A} - \text{AC}^2.bm.cp\text{A}, + \left(\text{F}^2 - \text{G}^2\right).\ \text{AC}^2.\text{AO}.cp\text{A} : \text{G}^2$$

quae ex priore subducta relinquet

$\text{Q}bm = \text{AC}^2.bm.cp\text{A}$, vel $\text{Q} = \text{AC}^2.cp\text{A}$ *;* qui valor in prima hujus articuli aequatione substitutus, efficiet $\text{P} = \text{QB}.\text{BC} + \left(\text{F}^2 - \text{G}^2\right).\text{Q}.\text{AO} : \text{G}^2$, ex qua elicietur

$\text{F} : \text{G} = \sqrt{\left(\text{P} - \text{Q}.\text{BC} + \text{Q}.\text{AO} : \text{Q}.\text{AO}\right)}.$ Verum, quia ellipsin ABE ad formam circularem quam proxime accedere cum Newtono supponimus, ideo ipsae BC, AC & AO tanquam aequales tractandae sunt, nam semissis parametri ellipseos, seu AO, a semidiametro AC vel BC circuli in quem definit, non differet ; ac propterea praecedens analogia abit in hanc simpliccm $\text{F} : \text{G} = \sqrt{P} : \sqrt{\text{Q}.\text{AO}}.$ Qui est Canon generalis, qui quaerebatur atque promittebatur supra, pro orbita mobili elliptica.

196: Regulam praecedentem unico exemplo illustrare libet, quod nobis omnium instar erit, idque ab Illustri Newtono mutuabimur. Sit ergo solicitatio centripeta in orbitae mobilis puncto *b*, ut quantitas $\frac{az^m+bz^n}{z^3}$, ubi *z* significat *b*C vel BC in fig.45, eritque proinde $\mathrm{P} = az^m + bz^n$, & $\mathrm{Q}.bm = \mathrm{Q}.dz = amz^{m-1}dz + bnz^{n-1}dz$, hoc est $\mathrm{Q} = az^{m-1} + bz^{n-1}$, hinc $\mathrm{F}:\mathrm{G}\left(=\sqrt{\mathrm{P}}:\sqrt{\mathrm{Q.AO}}\right) = \sqrt{\left(az^m + bz^n : amz^{m-1}\mathrm{AO} + bn^{z-1}\mathrm{AO}\right)}$. Verum, quia juxta monita in articulo praecedenti, z vel $b\mathrm{C} = \mathrm{AC} = \mathrm{AO}$, cum Newtono aequales unitati poni possunt, reperitur $\mathrm{F}:\mathrm{G} = \sqrt{(a+b:am+bn)}$ prorsus ut habet Newtonus in exemplo 3 post Prop. 45. *Lib. I. Princ. Phil.Nat.* Sin vero fuisset $\mathrm{P} = az^m - bz^n$, invenissemus pariter ut Laudatiss. Vir, $\mathrm{F}:\mathrm{G} = \sqrt{(a-b:am-bn)}$.

Denique si solicitatio centralis $\mathrm{P}:\mathrm{BC}^3$ fuerit ut, atque adeo $\mathrm{P} = \mathrm{BC}^{m+1}$, ex motu apsidum, seu ex ratione F:G, invenietur index *m* potestatis BC^m. Nam reperietur hoc casu $\mathrm{Q} = \overline{m+3}\mathrm{BC}^{m+2}$; unde si $\mathrm{AO} = \mathrm{AC} = \mathrm{BC} = 1$, erit $\mathrm{F}:\mathrm{G} = \sqrt{(1:m+3)}$ atque adeo $m = \left(\mathrm{G}^2 - 3\mathrm{F}^2\right):\mathrm{F}^2$.