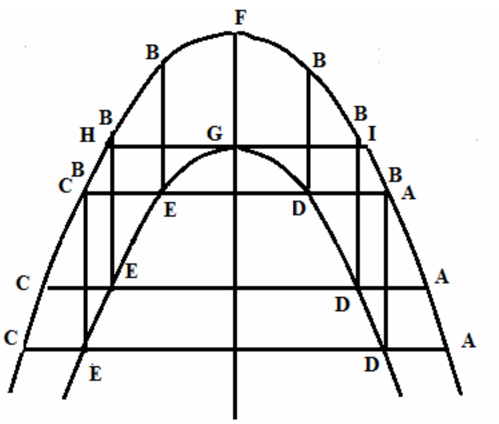
PARABOLA

PART EIGHT

The wonderful parallel symbolism between the asymptotes of the parabola and the hyperbola will be established.

PROPOSITION CCCXLIII.



FG shall be the axis of the parabola ABC and HGI the ordinate for that: and with HI drawn parallel to CA, the squares HG shall become equal to the rectangles CEA, CDA. I say the points E, G, D to be equivalent to points of the parabola ABC.

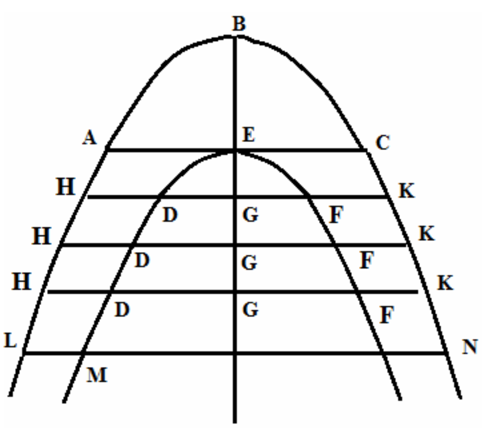
Demonstration.

The diameters EB, DB shall be erected from D and & E. Because HI, CA shall be parallel, and the square HG, that is the rectangle HGI are equal to the rectangle CEA, or CDA, also the diameters FG, BE, BD are equal: and whereby the points E, G, D belong to a parabola, equal to the parabola ABC.

PROPOSITION CCCXLIV.

The same shall be in place as before:
I say these parabolas produced indefinitely always approach each other, yet never intersect.

Demonstration.



Indeed, since by the preceding, also the right sides of these equal parabolas ABC, DEF, whereby produced indefinitely at no point will they concur: which I show thus that although they approach each other in turn more closely; indeed the ordinates HK, LN, & LN shall be put in place for the diameter BG in the parabola ABC, as the ordinate shall be more removed from the vertex B, since by hypothesis the HK are equal in the rectangles HDK, LMN. Whereby as HD is to LM, as MN to DK; but MN is greater than DK, since LN is greater than

HK (as it is placed more distant from the vertex B,) therefore the right line HD also is greater than LM.

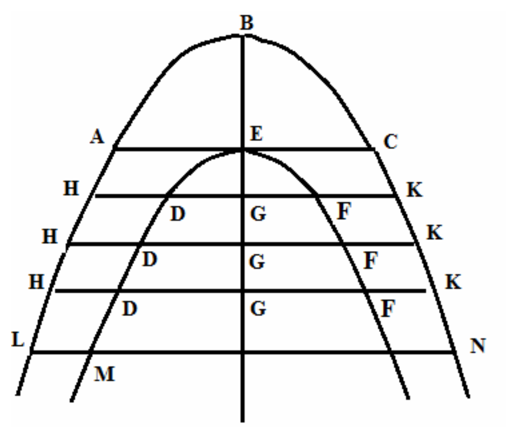
Likewise if any line shall be placed more distant from the vertex B, it shall be assumed to be parallel [to the corresponding line in the other curve], LM will be shown to be greater than any line put parallel to itself within its position ; therefore the lines in turn always agree more as they approach the asymptotes of the parabolas ; therefore they are the symptotes of the parabolas ABC, DEF.

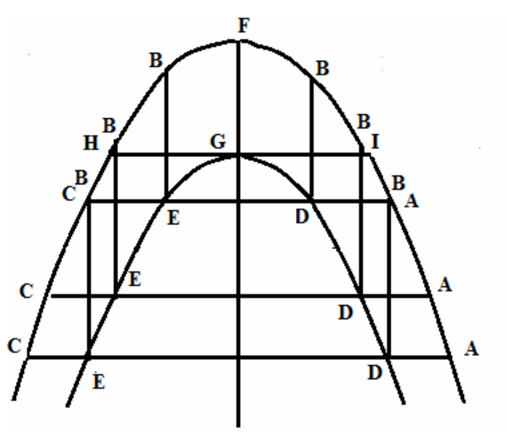
PROPOSITION CCCXLV.

With the same figure put in place : it shall be proposed to describe the given parallel line of the parabolas or the given asymptote of the given proposition.

Construction & demonstration.

The parabola ABC shall be given: in that the diameter BE shall be put in place as well as the ordinate AEC to be the applied line, and with AC put in place parallel to HK; to that diameter BE: HK shall be cut at D and F, so that both the rectangles HDK as well as HFK, shall be equal to the square AE: it is agreed from the preceding demonstation , the points DEF to belong to a parabola parallel to the parabola ABC ; therefore we will have done what was demanded.





PROPOSITION CCCXLVI.

With the figure assumed of proposition 343, ABC, DEF shall be parallel or asymptotic parabolas, and some equal distances EC DA shall be put in place.

I say the rectangles CEA, to be equal to the corresponding rectangles CDA.

Demonstration.

The diameters DB, EB shall be erected from D and E; therefore since they are parallel to the diameters of ABC, DGE, they are equal to the diameters BE, DB that shall arise from that same construction; but since the ratio of the diameters DB, BE is maintained, the rectangles CEA, CDA also hold the same ratio; therefore the rectangles CEA are equal to each other, as are the rectangles CDA.

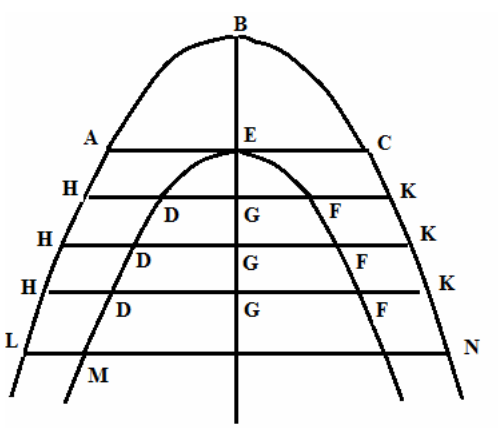
Corollary.

From what has been said it follows the right lines CE, DA to be equal to each other. The demonstration is evident, since the rectangles CEA, CDA shall be equal.

PROPOSITION CCCXLVII.

All the diameters of parallel parabolas are common.

Demonstration.



The figure of the parallel parabolas assumed shall be ABC, DEF of proposition 344, and some diameter BG in the parabola ABC shall be put in place ; moreover the applied ordinates for that shall be HK, crossing the parabola DEF at D and F. I say the lines FD to be bisected at G: indeed since HK by the ordinate construction are put in place to the diameter BG, the right lines HK are bisected at G: but HD, FK have been shown to be equal; therefore the remainders DG, GF also are equal, and thus for the applied ordinate BG, the common diameter of each parabola is therefore BG.

PROPOSITION CCCXLVIII.

The intercept of the tangent of parallel parabolas shall be bisected at the point of contact.

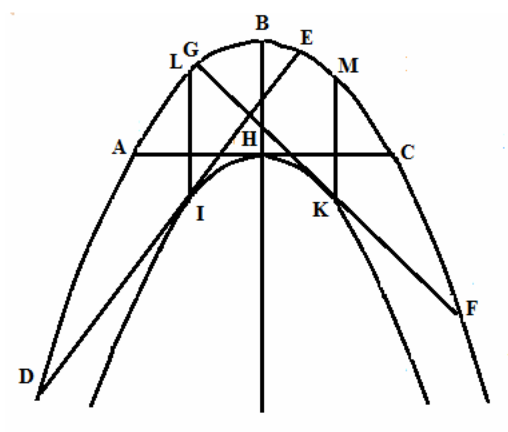
Demonstratio.

The figure of Prop. 347 is assumed. Some line shall be put in place through the point E in the perimeter of the parabola DEF assumped to be the tangent AC ; I say AC to be bissected at E, with the diameter BG put in place through E, some line DFis put in place , parallel to AC ; crossng the parabola ABC at H and K; therefore since FD, shall be parallel to the tangent AC ; the right line FD is bisected at G ; but HD, FK are equal HD, FK: therefore the whole line HK is bisected at G: whereby AC parallel to HK, shall be bisected by the same diameter through E.

PROPOSITION CCCXLIX.

The triangles which are made from the tangents, from the intercepts with parallel parabolas, and with the diameters drawn through the points of contact, are similar to each other.

Demonstration.



ABC, HIK shall be parallel parabolas : and the tangents AC, DE, FG to the parabola IHK shall be put in place : moreover the diameters BH, LI, MK are put in place through the points of contact H, I, K, and it is understood the lines FMG, CBA, DLE to be joined : I say the triangles FMG, CBA, DLE to be equal to each other: indeed since the parabolas are parallel, the diameters LI, BH, MK are equal : moreover, the tangens AC, DE, FG are bisected at H, I, K; therefore the triangules ABC, FMG, DLE are equal.

[This proposition is incorrect. I.B.]

PROPOSITION CCCL.

With the same figure put in place: in parabola ABC the three lines AC, DE, FG, are taken from equal segments, moreover AC, DE, FG bisected at H, I, K.
I say the points H, I, K belong to the parabola parallel to ABC .

Demonstration.

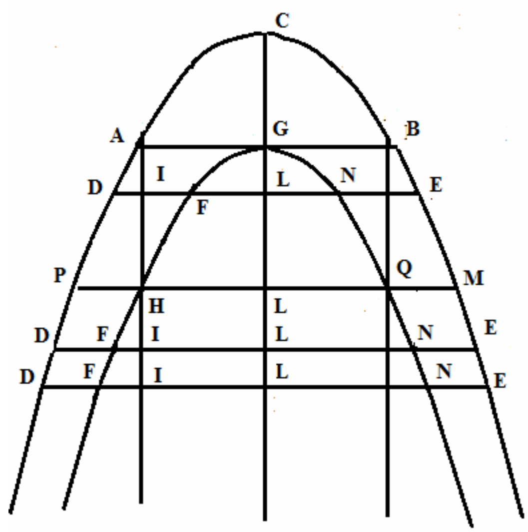
The diameters HB, MK, LI shall be erected from the points H, I, K: since the segments DAE, ABC, GMF are equal, also the triangles DLE, ABC, FMG, are equal : and whereby the diameters LI, BH, MK are equal, and the points H, I, K belong to the parabola parallel to the parabola ABC.

PROPOSITION CCCLI.

ABC, FHG shall be parallel parabolas, moreover the right line AB shall be a tangent to teh parabola FHG at G; and DE shall be put parallel to AB.
I say the square AG to be equal to the singular rectangle DFE.

Demonstration.

AH shall be put parallel to the diameter CG, and crossing the parabola FHG at the point H, through which the right line PHQ is put parallel to AB : so that as CG is to AH, thus the rectangle AGB, that is the square AG, (for the tangent AB is bisected at G) is to the rectangle PHM: moreover the diameters CG, AH are equal; and therefore the rectangle PHM is equal to the square AG. But the rectangles PHM, DFE are equal, therefore the square AG shall be equal to the individual rectangles DFE.



PROPOSITION CCCLII.

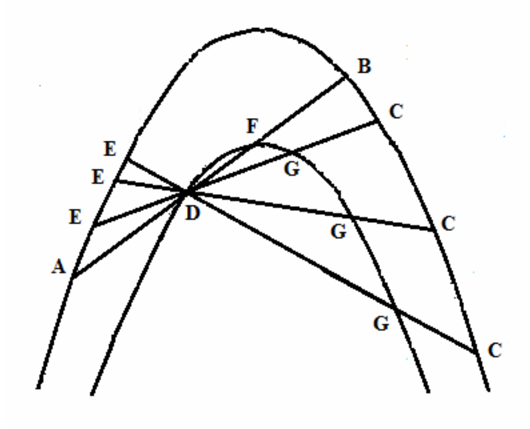
With the same in place: the right line AH shall cross the line DE at I:
I say the rectangles DIE to be equal to the squares FL.

Demonstration.

Since the line DE shall be bisected at L, and not bisected at F, the squares LD are equal to the squares LF together with the rectangles DFE: by the same reason the squares LD are equal to the squares LI together with the rectangles DIE : therefore the squares DFE together with the squares FL are equal to the squares LI likewise with the rectangles DIE: but they have been shown to be equal to the rectangle DFE, with the squares LI that is AG, therefore the remaining rectangles DIE are equal to the squares FL.

PROPOSITION CCCLIII.

With some point D assumed within parabola ABC some lines AB, EC shall be put in place through D: moreover the lines AD, ED shall be equal to the lines BF, CG.
I say the points D, F, G to belong to the parabola parallel to the parabola ABC.

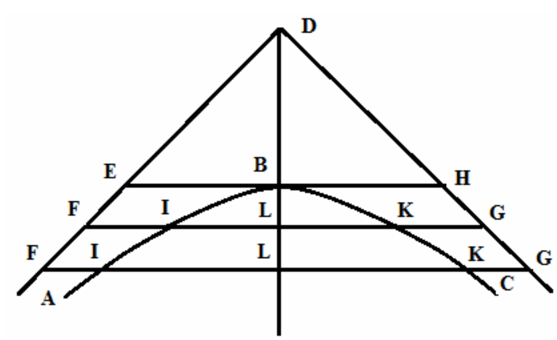


The demonstration is evident from teh Corollary of this Prop., where the demonstration has been put in place of the parallel parabolas ABC, DFG with the right lines ED, GC, likewise AD, FB to be equal to each other.

Application of parallel parabolas to the hyperbola put in place between the asymptotes.

Application of this proposition 343.

PROPOSITION CCCLIV.



The line EH shall subtend the angle EDH, with which bisected at B, EH shall be put in place, parallel to FG, which shall be cut at I, so that the rectangles FIG shall be equal to the square EB.
I say BII to belong to the same hyperbola.

The demonstration is to be found in our book on the hyperbola, proposition 14.

Application of this proposition 344.

PROPOSITION CCCLV.

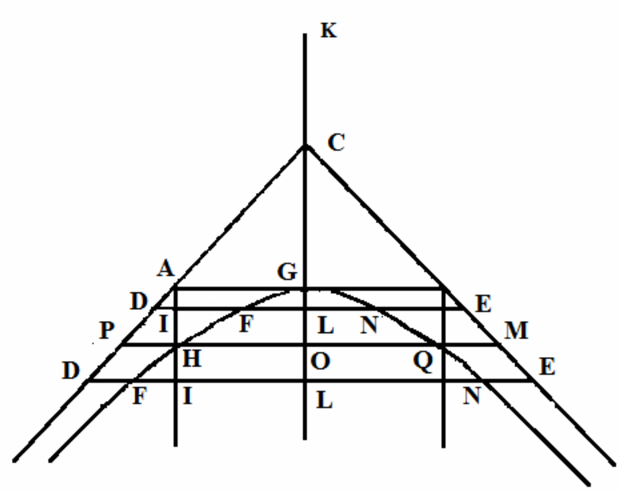
With the same figure in place.
I say the lines ED, DH to be produced indefinitely, always to approach closer to the hyperbola, but never to meet.

The demonstration is had in our book on the hyperbola, prop.15.

Application of the propositions 346, 351 *of this chapter.*

PROPOSITION CCCLVI.

With the right line KG put in place at the vertex G of the diameter of the hyperbola FGN put in place between the asymptotes AC, CR, the diameter KG shall be bisected, and indeed AB shall be parallel to DFE.
I say the rectangles DFE to be equal as amongst themselves and to the rectangles DNE, as well as to the square AG.
Where the demonstration is found in the proposed book on the hyperbola.



Corollary.

From these it follow also, the lines DF, NE to be equal to each other.

Application to proposition 352 of this.,

PROPOSITION CCCLVII.

With the same in place:
I say the rectangles DIE to be equal to the squares LF.
Demonstration: see book on the hyperbola, prop.17.

Application of proposition 348.

PROPOSITION CCCLVIII.

Every tangent of the hyperbola and with the asymptotes agreeing at the point of contact shall be bisected at the point of contact.
Demonstration: see book on the hyperbola, prop.29.

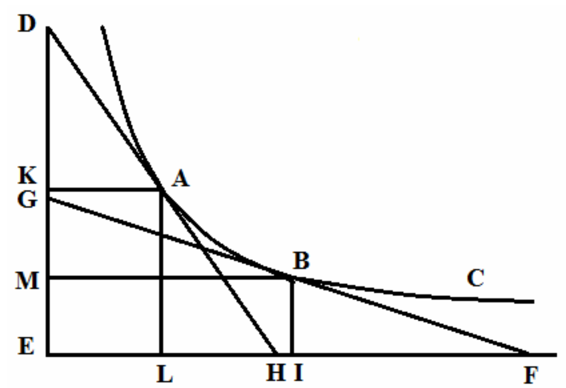
Application of proposition 349.

PROPOSITION CCCLIX.

The two tangent lines DAH, FBG which shall be put in place on the hyperbola ABC between the asymptotos ED, EF shall constitute the triangles HED, FEG.
I say these to be equal to each other.

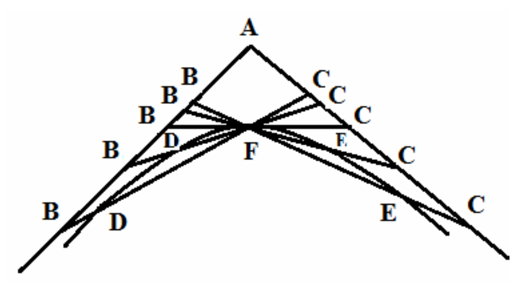
Demonstration: see book on the hyperbola, second part.

Application of proposition 353 of this part.



PROPOSITION CCCLX.

Some point F is assumed within the angle BAC, through which the right lines BFC shall be put in place, pertaining to each side of the angle at C and B: and the lines BF shall be equal to the lines CF, and in turn the lines EC equal to the lines DB:
I say the points EFD to lie on a hyperbola of which the asymptotes shall be BA, AC.
Demonstration: see the final part of the hyperbola.

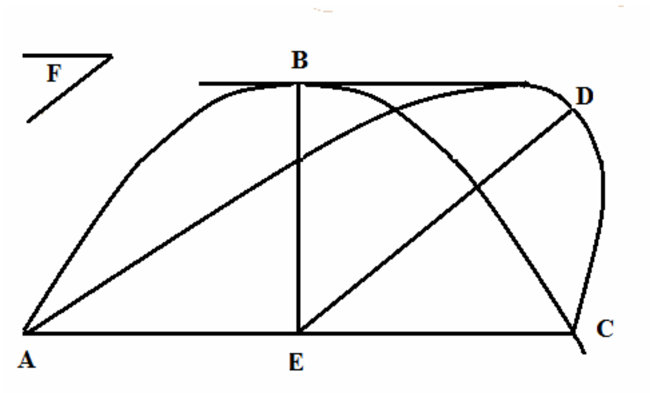


PROPOSITION CCCLXI.

To show the inclination to a given right parabola ABC, of which the ordinates put in place shall make a given angle to the diameter.

I call a parabola to be inclined, a whole parabola which has the ordinate lines put in place at an oblique angle to the axis : again it will be agreed from the following proposition, no parabolas to be given from their natural inclination.

Construction and demonstration.

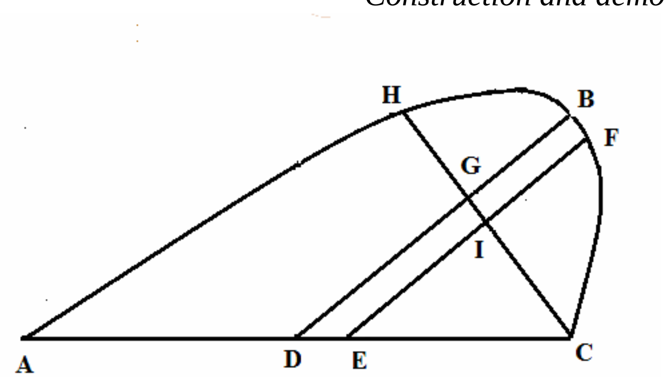


BE shall be the axis of the parabola ABC, and the ordinate AC put in place for that: and with the tangent through B put in place, the line ED shall be drawn through that, which shall make the angle EDB equal to the given angle F and the parabola ADE shall described, having the diameter ED, I say that to be put in place to be satisfactory since indeed each parabola shall have the common chord AC, and the same height, these shalll be agreed to be equal to each other ; but because the parabola ADE shall be inclined, it will be agreed the line AC and the diameter DE parallel to that angle, to cut each other at an oblique angle.

PROPOSITION CCCLXII.

To show the axis of the given inclination of a given parabola.

Construction and demonstration.



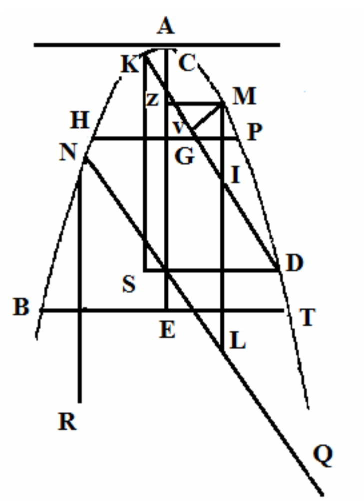
BD shall be the diameter of the inclined parabola ABC, and the ordinate for that AC : moreover the line CH shall be drawn from C normal to BD. And CH shall be bisected at I, the right line IF through I shall be put parallel to the right line BD : I say IF to be the axis of the inclined parabolae, since indeed FE shall be parallel to the diameter BD, the right line FE also is a section, since truly the line HC is bisected and cut at right angles, it shall be agreed FE to be the axis of the parabola ABC: therefore we have shown the axis of the inclined parabola.

Hence it is apparent in short that the inclination of the parabola provides nothing further to be given concerning the nature of the parabola which shall be different from the nature of the erect parabola: indeed that and the primary express the common understanding of the parabola, so that the ordinate shall be applied to the axis, and cut the same at right angles; that which also I may wish to understand with regard to the ellipse and hyperbola: indeed likewise in every section of the cone, whether it were an hyperbola or an ellipset, the ordinate is shown applied to the axis, and the same to be divided at right angles : from which nothing at all to be given concerning the nature of conic sections from their inclination, or which may be understood to be differ from what may be had from the right nature of the constructon.

PROPOSITION CCCLXIII.

To assign the diameter in a given parabola, to which a given line may serve for the latus rectum, in a smaller way that may not be present for the latus rectum of a given parabola.

Construction and demonstration.

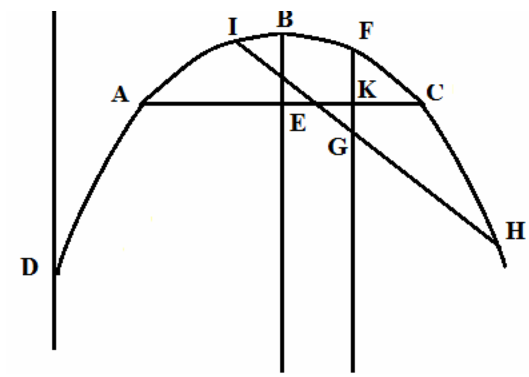


The line A and the parabola BCD shall be given, it shall be required to show the diameter which the latus rectum shall have to be equal to the given line A; so that what is desired shall be done, and ML shall be a diameter, for which the line A shall serve as the latus rectum. The axis CG of the parabola BCD shall be found, equal to the latus rectum: moreover the line MI shall become equal to the line CG, and the individual lines ML, CE equal to the line A: likewise the ordinates of these lines through G and I and may be put in place through L and E, for the diameters of these lines HP, KD, BT, NQ; therefore since MI, CG are equal lines, the segments KMD, HCP shall be equal : therefore put in place from D, the line DS to become normal to the diameter KS dropped from K, the right lines HP, SD are equal. Likewise RQ, BT shall be shown to be equal. Again since ML shall be put equal to its right line A, and NL the ordinate to ML, the right lines NL, ML are equal; since truly MI is equal to CG, and as CG is given as ML, that is NL, also the right line KI is given, since ML shall be to MI, as the square NL to the square KI: and because the lines CG,CE also are given, and thus HP, BT, and also SD, RQ (which will have been shown equal to these) are given: therefore since the angles KSD, NRQ, shall be right, also given are the triangles KSD, NSQ, and the angles SKD, RNQ that is KIM, NLM on adding, and therefore with the given right line ML equal to A, and with the points of the division I abd L ; will be applied at I and L, of the given cubes KD, NQ in the given angles KIM, NLM; moreover the right lines KD, NQ shall be bisected at I and L. then the points of the parabola shall be described through K, M, D ; that will pass through N and Q, since by resolution it shall be shown MI to M to be, as the square KI to the square NL : Then the axis CG of the parabola KMD shall be found: and the diameter ML shall be transferred into the given parabola, and may be put in place from the axis of the same, with the interval ZM, normal to the axis : it is apparent by the resolution, with the position found to be the diameter for which the given A shall serve as the latus rectum.

PROPOSITION CCCLXV.

To apply a given line to the parabola ; which shall remove a segment equal to the given line. Moreover it will be required that the line D, not to be smaller than the line AC.

Construction and demonstration.



ABC shall be the given segment and D the given line : it shall be required to apply the line D to the parabola, so that a segment may be removed, equal to that given: what is required shall be done, and the line IH equal to D, the segment HFI shall be taken away, equal to the given ABC : AC, HI shall be bisected at the points E and G, through which the diameters EB, FG shall be put in place; therefore since the segments ABC, HFI are equal, the right lines BE, FG also are equal [§.218 & §.260]; therefore BE and IG are given , half of HI or D now shall make BE, IG, FK to be in proportion; therefore now also FK is given; therefore the diameter is found from the preceding, of which FK is the latus rectum and shall be put into the parabola ABC, and with FG equal to BE the ordinate line HI is put in place through G: it is evident by the rcsolution and construction, HI to be the right line D, and HFI the segment from the given ABC therefore the line, etc to given. Q.e.d.

Scholium.

I have resolved to give two other parts, which are concerned with the parabola, to be adjoined to the present book; but since I shall direct one's attention this book, where I have pursued the properties of the Parabolic section, nevertheless it has grown into a *great mass, which shall be on a par with the remaining sections of the cone, in which I have carred over these parts to the other books; that with the greatest matter of the argument removed, which would put a strain on such books to explain.*

The first part puts together an account of the symbolic relationships and similitudes, which lie between a parabola and a spiral figure; indeed there is a wonderful likeness between these; nor do I intend to depart from my opinion, where planely it is Archimedes who has persuaded me to arrive at that understanding which he has left for us, whereby through the production of spiral figures a right line is produced, which shall be equal to the circumference of a circle. Besides these the parabola has many other properties in common with the spiral of Archimedes, not only along the first circular course, but along some number shown; which you will be able to consider in its place.

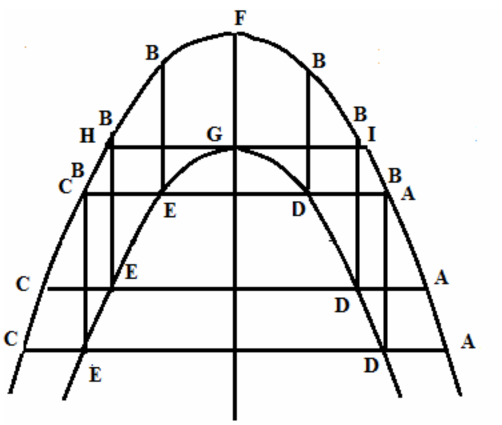
The second part which was being considered here to be concerned with virtual parabolas, the nomenclature of which I have taken from the properties of these which thus are similar to these properties of real parabolas, in order that they may not disagree with the same, except for the aspect as I have said thus; for they are equal to the followig surfaces considered, and a second ordinate applied to the diameter; yet in this respect they differ, because the diameter lines of these shall be parabolic; moreover in parabolas which are conic sectionst, the axes or diameters are right lines. Moreover we conclude the treatment of this material in a book concerned with the drawing of plane figures in the plane, there because the use of virtual parabolas may be considered for equations requiring to be formed for the bodies, which are parts of cylinders, having a concave or convex surface. But more concerning these in its place.

PARABOLAE

PARS OCTAVA

Miram exhibet parabolarum parallelarum, cum hyperbola inter asymptotos constituta, symbolisationem.

PROPOSITIO CCCXLIII.



Sit ABC parabolae axis FG & ordinatim ad illum applicata HGI : ductisque HI, parallelis CA, quadrato HG aequalia fiant rectangula CEA, CDA. Dico E, G, D puncta esse ad parabolam aequalem parabolae ABC.

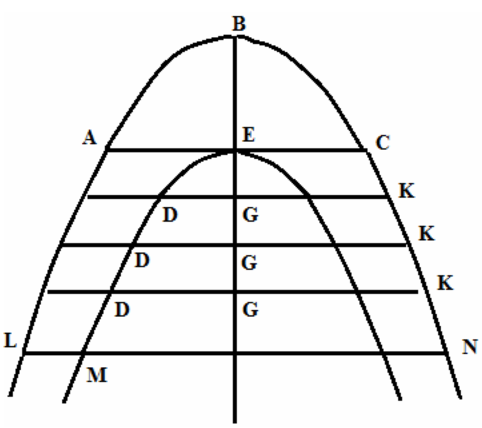
Demonstratio.

Erigantur ex D & E, diametri EB, DB. Quoniam aquidistat HI, CA, & HG quadrato id est HGI rectangulo aequalia sunt rectangula CEA, sint CDA, diametri a quoque FG, BE, BD aequales sunt: quare & puncta E, G, D ad parabolam, aequalem parabolae ABC.

PROPOSITIO CCCXLIV.

Eadem posita sint quaepriùs :
Dico parabolas illas in infinitum productas magis semper ad invicem accedere, nusquam tamen occurrere.

Demonstratio.



Cum enim per praeedentem aequales sint parabolae ABC; DEP, latera recta quoque illarum aequalia sunt, quare nusquam, in infinitum productae concurrent: quod vero magis semper ad invicem accedant sic ostendo; ponantur ad BG diametrum in parabola ABC, ordinatim HK, LN; & LN quidem remotior sit a vertice B, quam HK aequalia igitur sunt rectangula HDK, LMN ex hypothesi. Quarc HD est ad LM, ut MN ad DK; sed MN maior est DK, quia LN, maior est HK (utpote remotior a vertice B,) recta igitur HD quoque maior est LM.

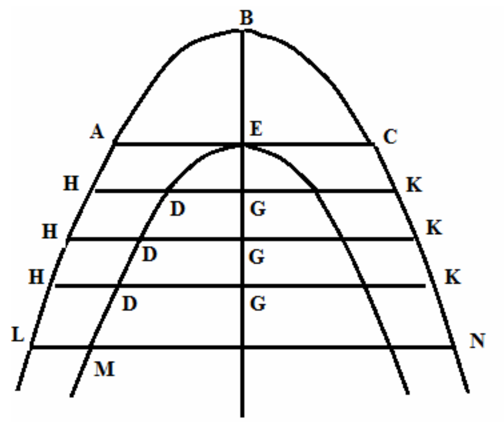
Similiter si remotior quaevis a vertice B, parallela assumatur, ostendetur LM maiorem esse quavis sibi aequidistante infra se posita; magis igitur semper ad invicem accedunt parabolae ABC; DEF: asymptoticae igitur sunt parabolae ABC, DEF.

PROPOSITIO CCCXLV.

Eadem posita figura: propositum sit datae parabolae parallelam sive asymptoticam describere.

Constructio & demonstratio.

Sunt ABC parabola datae ponatur in illa diameter BE ad quam ordinatim applicetur AEC positisque AC parallelis HK: secentur HK in D & F, ut tam HDK quam HFK, rectangula aequalia sint quadrato AE: constat ex ante demonstratis, DEF puncta esse ad parabolam parallelam parabolae ABC ; fecimus igitur quod fuit postulatum.



PROPOSITIO CCCXLVI.

Assumpta figura propositionis 343, sint ABC, DEF parabolae parallelae seu asymptotice, & quaevis ponantur aequidistantes ECDA.

Dico rectangula CEA, inter se, uti & CDA rectangulis aequarei.

Demonstratio.

Erigantur ex D &E diametri DB, EB; quoniam igitur parallelae sunt diametri ABC , DGE, aequales sunt diametri BE, DB uti ex ipso ortu constat; sed quam rationem diametri DB, BE servant, eandem quoque continent rectangula CEA, CDA; aequalia igitur sunt rectangula CEA inter, uti & rectangulis CDA.

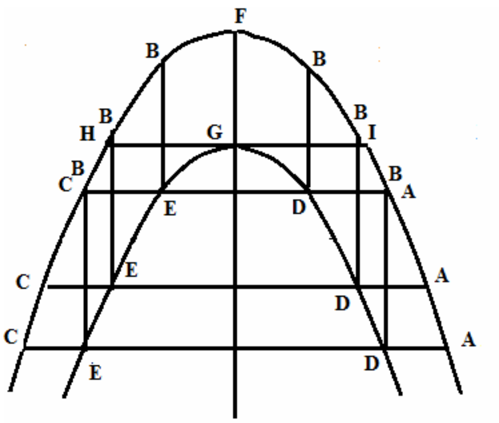
Corollarium.

Ex dictis sequitur rectas CE, DA esse inter se aequales. Demonstratio patet, cum CEA. CDA rectangula aequalia sint.

PROPOSITIO CCCXLVII.

Parallelarum parabolarum diametri omnes communes sunt.

Demonstratio.

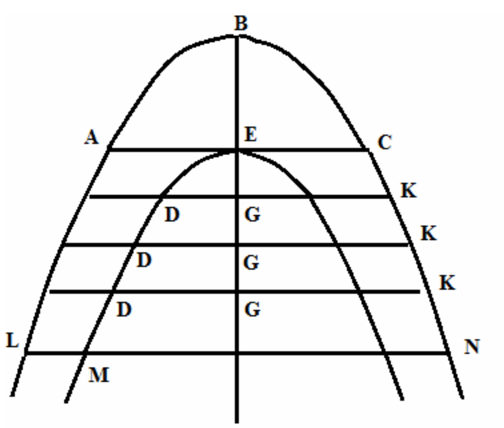


Assumatur figura propositionis 344 sint ABC, DEF parabolae parallelae, & quaevis ponatur diameter BG in parabola ABC; sint autem ad illam applicatae ordinatis HK, occurrentes parabolae DEF in D & F. Dico FD lineas in G bifariam dividi : cum enim HK per constructionem ordinatim ponantur ad BG, diametrum, rectae HK in G bissectae sunt : sunt autem aequales ostensae, HD, FK; residuae igitur DG, GF quoque aequales sunt, adeoque ad BG ordinatim applicatae, communis igitur est BG diameter utrique parabolae.

PROPOSITIO CCCXLVIII.

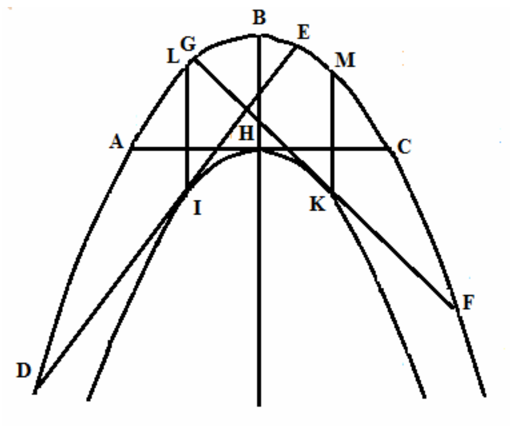
Contingens parabolis parallelis intercepta in concactu bissecatur.

Demonstratio.



Ponatur per E punctum quodvis in perimetro parabolae DEF assumptum contingens AC ; dico AC in E bissecari , posita per E, diametro BG, ponatur quaevis DF, aequidistans AC ; occurrens parabolac ABC in H & K; quoniam igitur FD, aequidistat contingeni AC ; recta FD in G, bissecta est; sunt autem aequales HD, FK: tota igitur HK in G bissecta est: quare & AC aequidistans HK, ab eadem diametro in E bissecatur .

PROPOSITIO CCCXLIX.



Triangula quae fiunt a contingentibus, parabolis parallelis interceptis, & diametris per contactuum puncta ductis, inter se aequalia sunt.

Demonstratio.

Sint ABC, HIK parabolae parallele: & IHK parabolam contingant AC, DE, FG: ponantur autem per H, I, K, contactus diametri BH, LI, MK, intelliganturque iungi FMG, CBA, DLE : dico triangula FMG, CBA, DLE inter se esse aequalia: quoniam enim parabolae parallela sunt, diametri LI, BH, MK aequales sunt : sunt autem AC, DE, FG contingentes in H, I, K bifariam divisae; aequalia igitur sunt triangul ABC, FMG, DLE.

PROPOSITIO CCCL.

Eadem posita figura : sint in ABC parabola lineae tres AC, DE, FG, aequalia auferentes segmentae secentur autem AC, DE, FG bifariam in H, I, K.
Dico puncta H, I, K esse ad parabolam parallelam parabolae ABC .

Demonstratio.

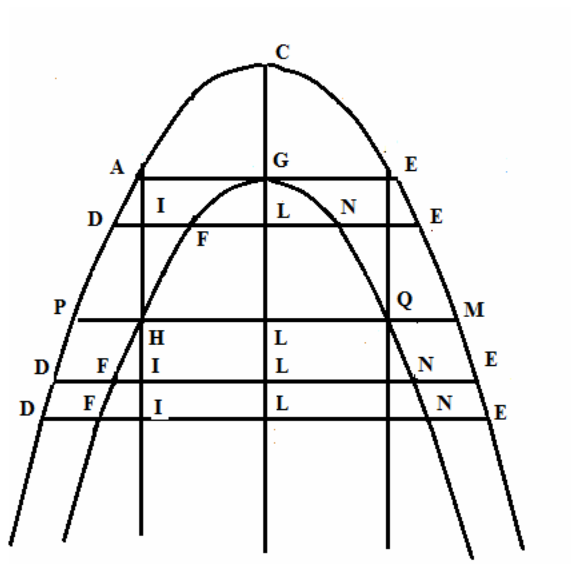
Erigantur ex H, I, K punctis diametri HB, MK, LI: quoniam aequalia sunt segmenta DAE, ABC, GMF, triangula quoque DLE, ABC, FMG, aequalia sunt : quare & diametri LI, BH, MK aequales sunt, & puncta H, I, K ad parabolam parallelam ABC

PROPOSITIO CCCLI.

Sint ABC, FHG parabolae: parallelae, recta autem AB contingat in G parabolam FHG; ponanturque DE aequidistantes AB.
Dico AG quadrato, aequari rectangula singula DFE.

Demonstratio.

Ponatur AH aequidistans diametro CG, occurrensque FHG parabolae in H puncto per quod recta ponatur PHQ parallela AB : ut CG ad AH, sic AGB rectangulum id est quadratum AG, (est enim AB contingens in G bissecta) ad rectangulum PHM: aequales autem sunt diametri CG, AH,igitur & PHM rectangulum aequalc est quadrato AG. Sed PHM, DFE rectangula aequalia sunt, quadrato igitur AG aequalia sunt rectangula singula DFE.



PROPOSITIO CCCLII.

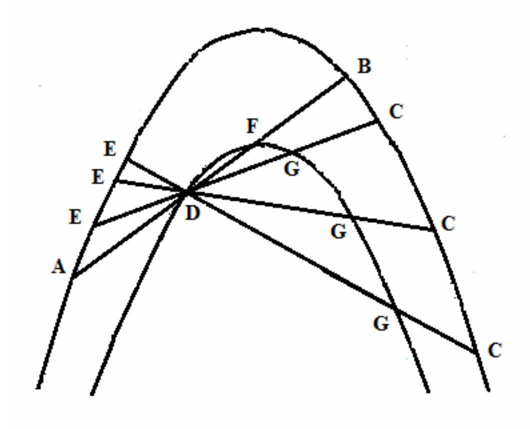
Iisdem positis: recta AH occurrat DE lineis in I:
Dico DIE rectangula aequari quadratis FL.

Demonstratio.

QVoniam DE lineae in L divisae sunt bifariam, & non bifariam in F, quadrata LD aequalia sunt quadratis LF una cum rectangulis DFE: eadem de causa quadrata LD aequalia sunt quadratis LI una cum rectangulis DIE : rectangula igitur DFE una cum quadratis FL aequalia sunt quadratis LI simul cum rectangulis DIE: aequalia autem ostensa sunt rectangulo DFE, quadratis LI id est AG, residua igitur rectangula DIE residuis quadratis FL aequalia sunt.

PROPOSITIO CCCLIII.

Intra parabolam ABC assumpto quovis puncto D ponantur per D lineae quocunque AB, EC: fiant autem AD, EC lineis aequales BF, CG.
Dico D, F, G puncta esse ad parabolam parallelam parabolae ABC.

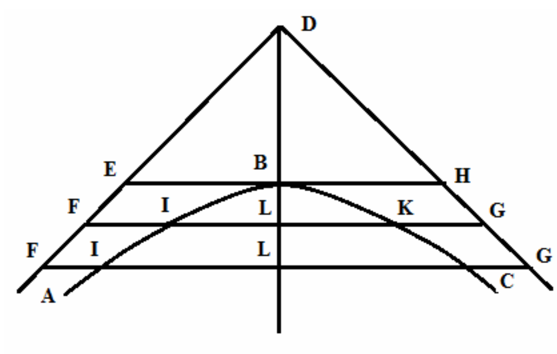


Demonstratio manifesta est ex Coroll. huius, ubi demonstratum est positis parabolis parallelis ABC, DFG rectas ED, GC, item AD, FB esse inter se aequales,

Applicatio parabolarum pârallelarum ad hyperbolam inter asymptotos positarum.

Applicatio propositionis 343 huius.

PROPOSITIO CCCLIV.



Angulum EDH subtendat linea EH, qua bifariam divisa in B, ponantur EH, aequidistantes FG, quae in I secentur, ut FIG rectangula, aequalia sint quadrato EB.
Dico BII ad eandem esse hyperbolam.

Demonstratio habetur in libro nostro de hyperbola propositione 14.

Applicatio propositionis 344 huius.

PROPOSITIO CCCLV.

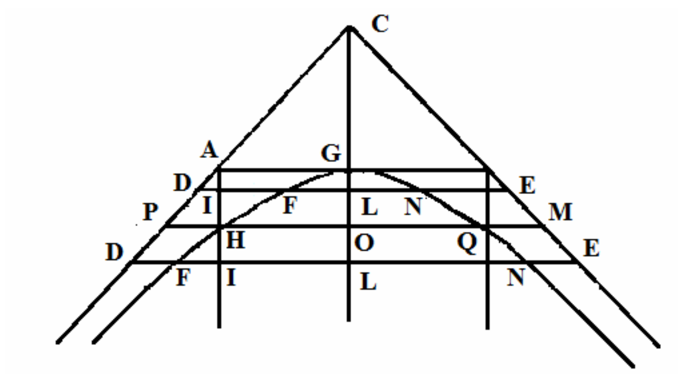
Eadem posita figura.
Dico ED, DH lineas in infinitum productas, magis semper ac magis ad hyperbolam accedere, nusquam autem convenire.

Demonstratio habetur in lib. nostro de hyperbola propos.15.

Applicatio propositionum 346, 351 *huius.*

PROPOSITIO CCCLVI.

Ab recta inter asymptotos AC, CR hyperbolae FGN constituta in G vertice diametri KG divisa sit bifariam, & AB quidem aequidistent DFE.
Dico DFA rectangula aequari inter se uti & rectangulis DNE, sive quadrato AG.
Demonstrationem unde in lib. de hyperbola propos.



Corollarium.

Ex his quoque sequitur, lineas DF, NE esse inter se aequales.

Applicatio propos.352 huius,

PROPOSITIO CCCLVII.

Iisdem positis:
Dico DIE rectangula aequari quadratis LF.
Demonstrationem vid.lib.de hyp.prop *17.*

Applicatio propositionis 348.

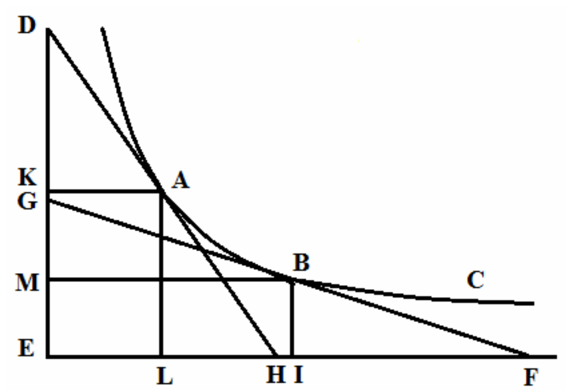
PROPOSITIO CCCLVIII.

Omnis contingens hyperbolam & cum asymptotis conveniens in puncto contactus bifariam secatur.
Demonstrationem vid.lib.de hyperb.29.

Applicatio propositionis 349.

PROPOSITIO CCCLIX.

Hyperbolam ABC inter asymptotos ED, EF constitutam contingant duae lineae DAH, FBG quae triangula constituant HED, FEG.
Dico illa esse inter se aequalia.

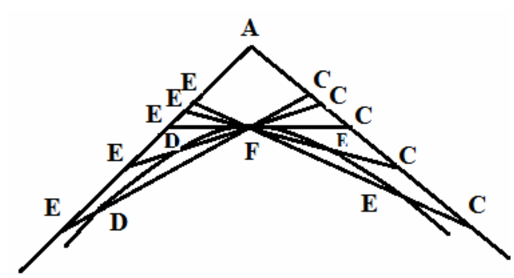


Demonstrationem vide in libro de hyperbola,parte secunda,

Applicatio propositionis 353.huius.

PROPOSITIO CCCLX.

Intra angulum BAC punctum assumatur quodvis F, per quod rectae ponantur BFC, pertinentes ad utrumque anguli latus in C & B: fiantque BF lineis aequales CE, & vicissim CF aequales DB:
Dico puncta EFD esse ad hyperbolam cuius asymptoti sint BA, AC.
Demonstrationem vide in hyperbola parte, ultima.

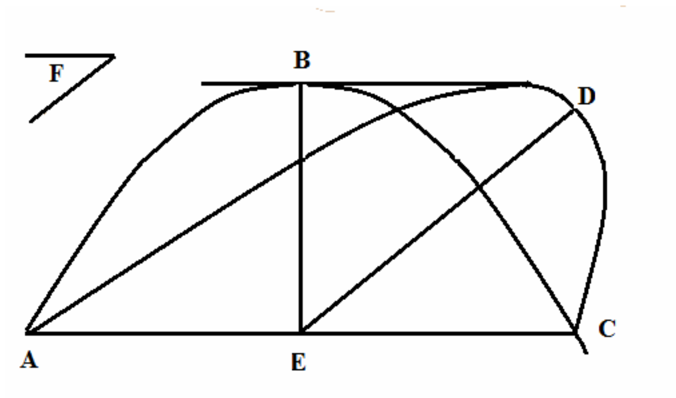


PROPOSITIO CCCLXI.

Datae parabolae rectae ABC, aequalem exhibere inclinatam, cuius ordinatim ad diametrum positae datum angulum constituant.

Parabolam inclinata voco, omnem parabolam quae lineas habet ordinatim ad diametros positas ad angulos obliquos: porro nullas dari parabolas ex natura sua inclinatas, ex sequenti propositione constabit.

Constructio & demonstratio.

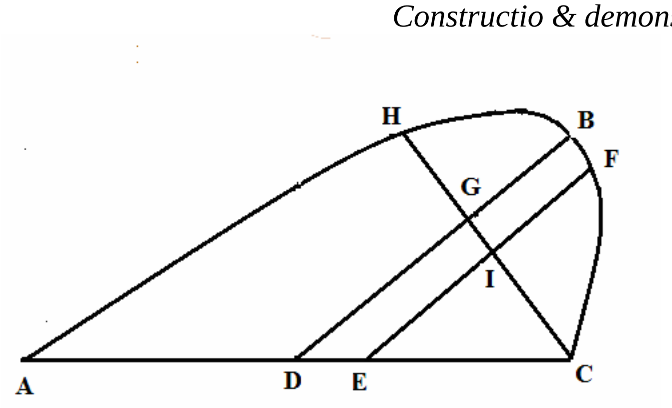


Sit ABC parabolae axis BE, & ordinatim ad illum posita AC: positaque per B contingente, ducatur ad illam linea ED, quae angulum EDB aequalem faciat dato F & describatur parabola ADE, habens ED diametrum, dico illam satisfacere petitioni cum enim utraque parabola , communem habeat subtensam AC, & eandem altitudinem, constat illas inter se esse aequales; quod autem ADE parabola sit inclinata, ex eo patet AC linea & illi aequidistans, diametrum DE ad angulos secent obliquos.

PROPOSITIO CCCLXII.

Datae parabolae inclinatae axem exhibere.

Constructio & demonstratio.



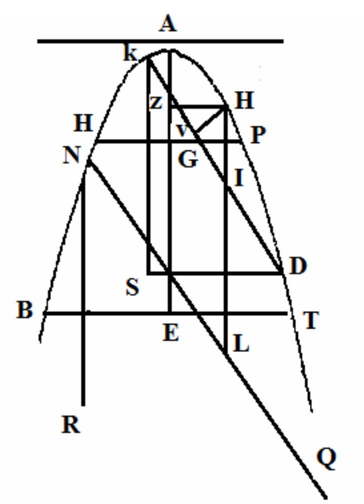
Sit ABC parabolae inclinatae diameter BD, & ad illam ordinatim posita AC : ducatur autem ex C linea CH normalis ad BD. Divisaque CH bifariam in I, ponatur per I recta IF aequidistans BD : dico IF axem esse parabolae inclinatae, cum enim FS aequidistet BD diametro, recta FE quoque sectionis est, quia vero lineam HC, bifariam & ad rectos secat angulos, constat FE axem esse parabolae ABC: exhibuimus igitur datae parabolae inclinatae axem.

Hinc patet nullas prorsus inclinatus dari parabolae quae diverso sint naturae a rectis: utrique enim essentialis illa & primaria passio communis, quod ordinatim ad axem applicatae, eundem ad angulos secent rectos; id quod etiam intelligi velim de hyperbola & ellipsi: similiter enim in omni sectione coni, sive hyperbola, sive ellipsis fuerit, ostenditur, ordinatim ad axes applicatas, eosdem ad angulos rectos dividere: unde nullas omnino dari coni sectiones natura sua inclinatus, sive quae diversam a rectis habeant naturam constat.

PROPOSITIO CCCLXIII.

In data parabola diametrum assignare, cui data linea inserviat pro latere recto, modo minor ea non existat latere recto axeos datae parabolae.

Constructio & demonstratio.



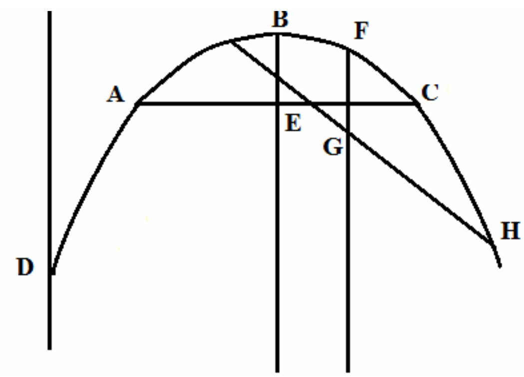
Data sit linea A & parabola BCD, oporteat exhibere diametrum quae latus rectum habeat aequale data lineae A; factum sit quod petitur; & ML sit diameter, cui A linea inserviat pro latere recto. Inveniatur CG axis parabolae BCD, aequalis lateri recto: lineae autem CG aequalis fiat MI, & ML, CE lineae singulae aequales rectae A: ponanturque per L, & E, item G & I ordinatim ad diametros suas lineae HP, KD, BT, NQ; quoniam igitur MI, CG lineae aequales sunt, segmenta KMD, HCP aequantur : posita igitur ex D, linea DS normali ad demissam ex K diametrum KS, rectae HP, SD aequales sunt. Similiter aequales oftendentur RQ, BT. Rursum cum ML aequalis ponatur lateri suo recto A & NL ordinatim ad ML, rectae NL, ML aequales sunt; quia vero MI aequalis est CG, & CG data est uti & ML hoc est NL, recta quoque KI data est, cum sit ML ad MI, ut NL quadratum ad quadratum KI: & quia datae quoque sunt lineae CG, CE adeoque & HP, BT, rectae quoque SD, RQ (quae illis ostensae fuerit aequales) datae sunt: igitur cum anguli KSD, NRQ, recti sint, data quoque sunt triangula KSD, NSQ, & anguli SKD, RNQ id est KIM, NLM componendo igitur data ML recta aequli A, & punctis divisionum

I & L ; applicentur ad I & L, datae cubeae KD, NQ in datis angulus KIM, NLM rectae autem KD, NQ bissectae sint in I & L. tum per K, M, D, puncta parabola describatur ; transibit illa per N & Q, cum per resolutionem ostensum sit esse MI ad ML ut KI quadratum ad quadratum NL : Inveniatur deinde parabolae KMD axis CG: & ML diameter transferatur in parabolam datam, ponaturque ab axe eiusdem, intervallo ZM, normalis ad axem : patet per resolutionem, positione inventam esse diametrum cui data A serviat pro latere recto.

PROPOSITIO CCCLXV.

Data lineam applicare ad parabolam ; quae segmemtum auferat dato aequale. Oportet autem lineam D, non minorem esse linea AC.

Constructio & demonstratio.



Sit ABC segmentam datum & data linea D: oporteat rectam D applicare ad parabolam, ut segmentum auferat, dato aequale: factum sit quod petitur, & IH linea aequalis D, segmentum auferat HFI, dato ABC aequale : bisecentur AC, HI in E & G, punctis, per quae diametri ponantur EB, FG. quoniam igitur segmenta ABC, HFI aequalia sunt , rectae BE, FG quoque sunt aequales; datae igitur sunt BE & IG, dimidia HI sive D fiant iam BE, IG, FK proportionales. data ergo etiam est FK inveniatur igitur per praecedentem diameter, cuius PK latus rectum est & ponatur in parabola ABC, factaque FG aequali BE ponatur per G ordinatim linea HI: paret per rcsolutionem & constructionem, HI lineam rectae D, & HFI segmentum dato ABC aequale esse datam igitur lineam, & c. Quod erat faciendum.

Scholion.

Destinaveram duas alias Partes, quae parabolam concernunt, praesenti libro adiungere; sed cum advertam librum hunc, quo Parabolicae sectionis proprietates prosecuti sumus , in molem nimis magnam excrescere, quam ut libris reliquis de coni sectionibus par sit, in alios libros eas partes transtuli; exigente id maxime argumento materiae, quam tales libri explanare intendunt.

Pars autem prima symboli rationes ac similitudines complectitur, quae sunt inter parabolam ac spiralem figuram; mira enim est inter has conformitas; neque a mea sententia recedereaudeo, quo mihi plane persuasum est Archimedem in eam notitiam pervenisse quame nobis reliquit, qua per contngentem spiralis figurae linea rectam exhibet, quae circuli circumferentia sit aequalis. habet praetereae parabola multas alias propietates, cum spirali Archimedea communes, non solum secundum primam circulationem, sed secundum quotconque numero exhibitas; quas fusius considerare poteris suo loco.

Secunda pars quae huc spectabat agit de parabolis virtualibus, quarum nomenclaturam desumpsi a proprietatibus illarum, quae ita similes sunt proprietatibus verarum parabolarum, ut ad iisdem non discrepent, nisi solo aspectu ut ita dicam; nam aequales sunt secundum superficies consideratae , & secundum ordinatim applicatas ad diametrum; in hoc tamen differunt, quod earum diametri lineae sint parabolicae; in parabolis autem quae sectiones conicae sunt, axes vel diametri rectae sunt lineae. Coniecimus autem tractatum uius materiae in librum de ductibus planorum in plana, eo quod usus virtualium parabolarum, spectet ad aequationes formandas cum corporibus , quae partes Cylindricae sunt, concavam habentes superficiem, vel convexam. Sed de his plura suis locis.