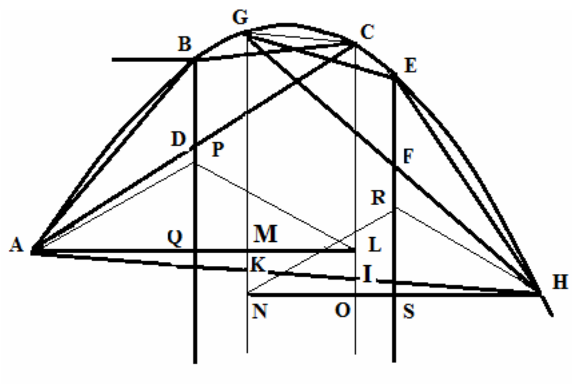
PARABOLA PART FIVE

Squaring the parabola.

PROPOSITION CXXVIII.

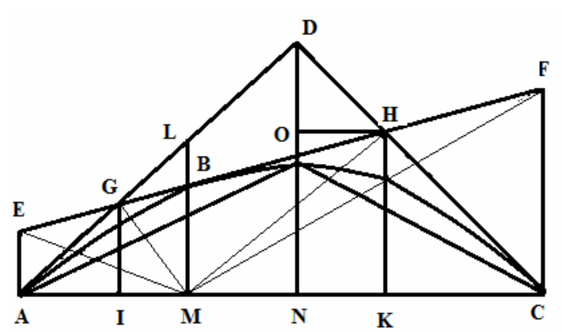
Two equal diameters BD, EF shall cut the parabola ABC: and with the ordinate lines AC, GH put in place through D and F, ABC, GEH shall be joined.
I say the triangles ABC, GEH to be equal to each other.

Demonstration.



With G, C joined ; the line AH shall be put in place, which shall cut the diameters dropped from G and C at K and I : moreover these shall cut the right lines AML, HON orthogonally at L,M, N, O : and indeed AL shall cross the line BD at Q; truly EF shall cross HN itself at S; and with PQ, SR made equal to BD, EF; and APL,HRN shall be joined; therefore since the diameters BD, EF are equal; and the ordinates ADC, GFH put in place for these; CG, AH shall be joined parallel to IK itself: moreover GK, CI are parallel diameters, therefore GCIK is a parallelogram; and GK, CI equal lines, moreover as GK is to IC thus the rectangle AKH is to the rectangle AIH, therefore the rectangles AKH, AIH are equal ; and thus the lines AK, HI are equal; truly as AK to KI, thus AM to ML, and as HI to IK, thus HO to ON, therefore as AM to ML, thus HO to ON: but the lines NO, ML are equal (because AML, HON are at right angles to GM, CL are parallel from the construction, and thus MO is a parallelogram) ; therefore the right lines AM, HO are equal, and thus the whole lengths AL, HN are equal: also PQ, RS are equal by the construction, therefore the triangles APL, NRH : that is ABC and GEH are equal. Q.e.d.
This has been demonstrated otherwise by Archimedes.

PROPOSITION CXXVIX.



Some two equal right lines AD, CD meeting at D shall be tangents to the parabola ABC , and these shall intersect some right line EF at G and H, also being a tangent to the parabola at B, crossing the diameters AE , CF at E and F; moreover with the diameters GI, HK dropped from G

and H, crossing AC at I and K.
I say the right line IK, to be half of AC.

Demonstration.

Since AG, BG shall be tangents to the section, the right line EG is equal to the right line GB : and similarly BH is equal to HF; therefore GH is half or the total length EF; but since AE, CF, GI, HK shall be parallel, AC is divided at I and K, as EF is divided at G and H ; and therefore IK is half of AC. Q.e.d. [The diagram drawn here from the original text is not perfect, as these lengths are not quite equal.]

PROPOSITION CXXX.

With the same put in place, the diameter LM is put in place through B.
I say as GB to BH, thus LB to BM.

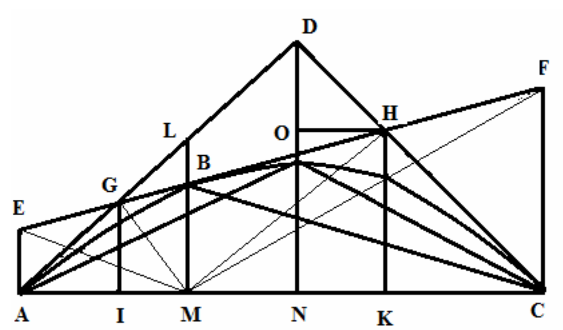
Demonstration.

Because AL is a tangent, LB is to BM, as AM to MC, that is, as EB to BF: but as EB to BF, thus GB is to BH, thus as GB to BH, thus as LB is to BM. Q.e.d.

PROPOSITION CXXXI.

With the same figure remaining, the right lines MG, MH shall be drawn.
I say MDHB to be a parallelogram.

Demonstration.



For as HB shall be to BG, thus MB shall be to BL; by the preceding, and on interchanging HB ad MB, thus GB to BL: but the angles at B contained by proportional sides are equal, therefore the triangles MBH,GBL are similar: and MH parallel to GL: in the same manner if KH may be produced until it shall meet AD, it shall be shown that GM to be parallel to DH: therefore DM is a parallelogram. Q.e.d.

PROPOSITION CXXXII.

With the same figure remaining, AB, BC shall be drawn.
I say the triangle ABC to be equal to the parallelogram MD.

Demonstration.

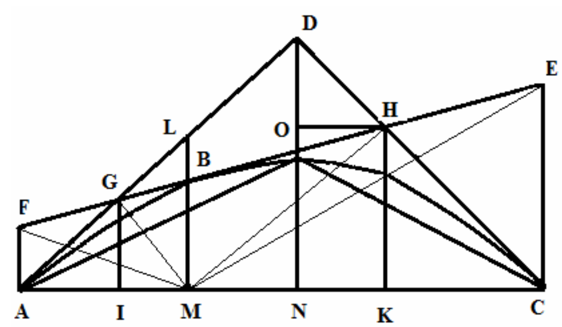
Indeed triangle ABM is equal to the triangle BEM, and likewise triangle BFM is equal to triangle BMC ; therefore the angle EMF is equal to the whole triangle ABC ; but twice the triangle GMH is equal to the triangle EMF, (since the base GH is equal to twice the base EF); therefore the parallelogram MD is equal to the triangle ABC.
Q.e.d.

PROPOSITION CXXXIII.

With the same figure remaining, the diameter DN shall be dropped from D.
I say the lines GI, HK taken together, to be equal to the right line DN.

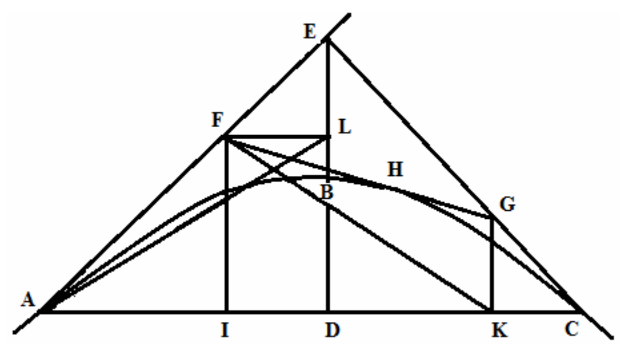
Demonstration.

HO shall be drawn parallel to AC, crossing the line ND at O. Since IM, GI, shall be lines parallel to the lines DO, HO : and moreover GM to be equal and parallel to DH; the triangles IGM, DOH, and thus the sides DO, GI are equal: moreover HK is equal to ON. Therefore GI, HK taken together are equal to the line DN. Q.e.d.



PROPOSITION CXXXIV.

AC shall be the ordinate drawn to the axis BD of the parabola ABC, and the tangents shall meet at E, passing through A and C ; and moreover FG shall be made tangent at H, which shall cut the lines AE, CE at F and G, then the right lines FI, GK shall be dropped, parallel to the axis.

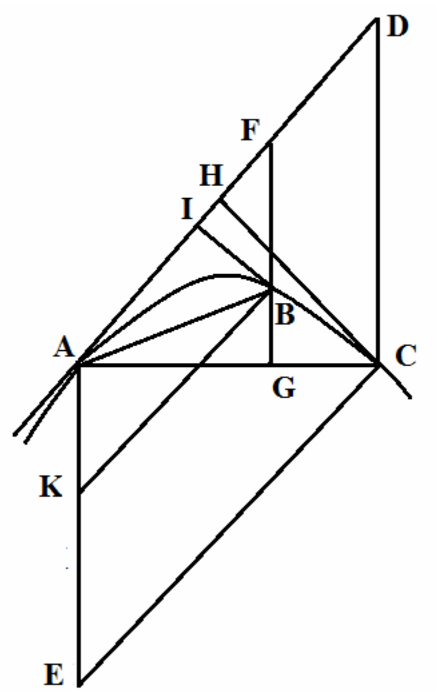


I say the trapezium FIKG, to be equal to the triangle AED.

Demonstration.

FL shall be put parallel to AC, and LA, FG shall be joined. Since FL shall be parallel to AC, and IK is equal to AD, clearly half of AC, the triangles ALD, IFK are equal. Again since in the triangles FKG, AEL, both the bases KG, EL as well as the heights IK, AD shall be equal, also the triangles FKG, AEL are equal to each other : therefore the trapezium FlKG is equal to the triangle AED. Q.e.d.

PROPOSITION CXXXV.



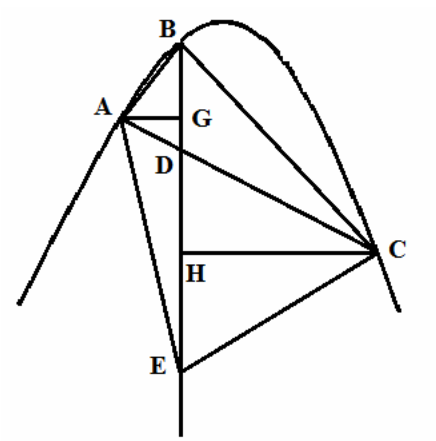
Some right line AC shall be subtended under the parabola ABC, moreover the tangent AD passing through A, meeting the diameter CD at D: also some diameter BG shall be put in place, crossing the line AD at F; and AB shall be joined.

I say the area of the triangle AFB to the area of the triangle ADC to be in the triplicate ratio of the line AF to AD.

Demonstration.

With the diameter AE dropped from A, the lines BK, CE shall be drawn parallel to the tangent AD; and from C and B, the right lines CH, BI normal to AD itself. The ratio of triangle AFB to triangle ADC is composed from the ratio AF to AD, and from the ratio IB ad HC: but as IB to HC, thus FB to CD, that is AK to AE; therefore the ratio of the area of triangle AFB to triangle ADC is composed from the ratio AK to AD, and from the ratio AK to AE: but the ratio AK ad AE, is the square of the ratio KB to EC, that is, AF to AD, (since FK, DE are parallelograms) ; therefore, triangle AFB to triangle ADC has the triplicate ratio of that which AF has to AD. Q.e.d.

PROPOSITION CXXXVI.



Some right line AC shall cut the diameter BD of the parabola at D, and with the ordinate CE put in place, AB, BC shall be joined.

I say the area of triangle ABD to the area of triangle BCE, to be in the ratio of the lines AD to DC.

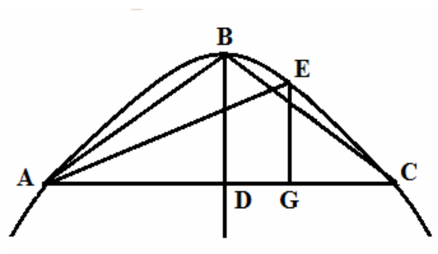
Demonstration.

The lines AG, CH are put in place from A and C normal to the diameter BE ; and AE shall be joined. Thus AD shall be to DC, as BD is to BE: but as AD to DC, thus GD is to DH, that is, AG to CH; therefore so that as BD to BE, thus AG is to CH. But the ratio of the area of triangle ABD to the area of triangle BCE, is composed from the ratio BD to BE, and from AG to CH*;* therefore the ratio of triangle ABD to triangle BCE is the square of the ratio AD to DC. Q.e.d.

PROPOSITION CXXXVII.

To inscribe the maximum triangle for a given terminated parabola.

Construction and demonstration.



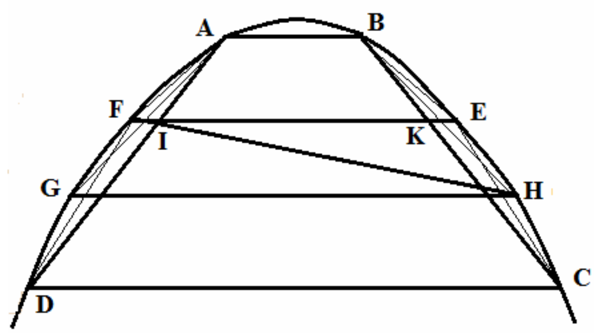
Some right line AC shall subtend the parabola ABC, which the diameter BD bisects at the point D, and AB, CB shall be joined. I say the greatest triangle ABC to be sought. For some other right line GE shall be erected, parallel to the diameter BD: Because the rectangle ADC holds that same ratio to the rectangle AGC, as DB to GE, therefore the right line DB is greater than the right line GE: therefore also triangle ABC is greater than triangle AEC: therefore ABC is the greatest of the triangles. Q.e.d.

PROPOSITION CXXXVIII.

Some two parallel lines AB, DC intersect the parabola ABC: and with BC and AD joined, the maximum triangle AEC shall be inscribed on the segment CB, and EF shall be put in place EF, parallel to AB, and AFD shall be joined.

I say the triangle AFD, to be the maximum of these which are able to be inscribed on the segment AFD; and on the other hand if the triangles AFD, BEC shall be the maximum, I say FE to be parallel to AB.

Demonstration.



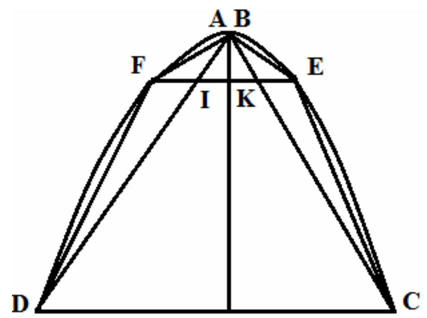
Since AB, CD, FE shall be parallel, the right line FI is equal to the right line KE, and thus the triangles FAI, FID are equal to the triangles, KBE, KEC: therefore if the triangle AFD shall not be a maximum, but some other triangle AGD shall be greater than triangle AFD: and with GH put parallel to AB, BHC may be joined :

so that it will be shown first, the triangle BHC to be equal to triangle AGD: but AGD is greater than triangle AFD, that is as shown, BEC; therefore triangle BHC also is greater than BEC: which is contrary to the hypothesis. Therefore AGD is not the maximum triangle, but AFD instead. Which was the first part.

Now AFD, BEC shall be the greatest triangles, I say the line FE joined to be parallel to AB: truly if this is not the case ; FH shall be put parallel to AB, and BHC shall be joined: therefore the triangle BHC shall be the greatest of these which shall be able to be inscribed in the segment BEC, and thus greater also than triangle BEC, which is absurd: therefore FH shall not be parallel to AB, but FE. Q.e.d.

So that if AB were a tangent to the parabola, the same may be inferred as above, and the same thus has been demonstrated.

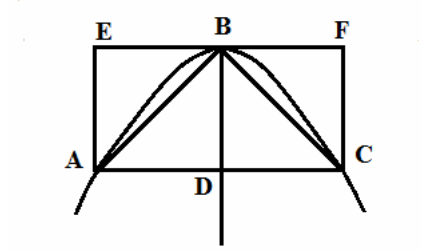
PROPOSITION CXXXIX.



ABC shall be the maximum triangle inscribed in the parabola ABC.

I say that triangle ABC to be greater than half of the parabola ABC.

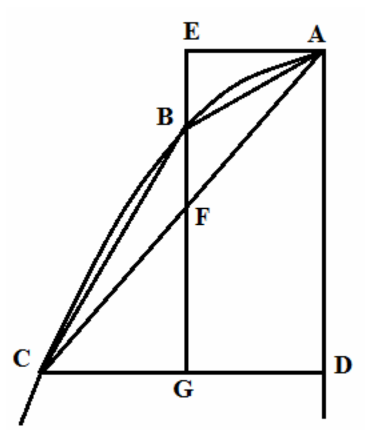
Demonstration.



The rectangle ACF shall be completed ; therefore clearly the parallelogram EC to be greater than the parabola ABC; and therefore the triangle ABC , evidently to be half of the parallelogram EC, but greater than half of the parabola . Q.e.d.

PROPOSITION CXXX.

AD shall be the diameter of the parabola ABC, and with the ordinate CD put in place; and AC joined to be bisected at F, the diameter BF shall be established and AB,CB shall be joined.



I say the triangle ABC to be equal to four times the triangle CAD.

Demonstration.

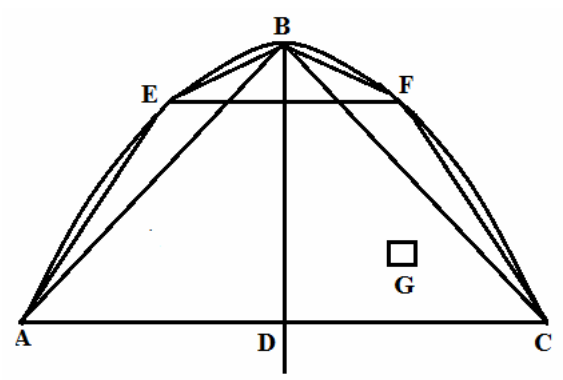
The tangent AE is acting through A, crossing the line BF at E, which produced cuts DC at G, therefore since AE, CD are parallel, so that as CF to FA, thus GF is to FE, moreover AC shall be put bisected at F, and therefore EF is equal to FG and the triangle EAF equal to the triangle

CFG : but triangle EAF is equal to triangle CBA [§.17]: and therefore triangle CFG is equal to triangle ABC, since EB, BF are equal lines: but triangle CAD is four times triangle CFG since AD is twice FG and CD twice CG, and therefore will be four times triangle ABC. Q.e.d.

PROPOSITION CXXXI.

Let ABC be the largest triangle inscribed in parabola ABC: and moreover the largest triangles will be inscribed in the remaining segments : and this shall be done in a repeated manner .

I say the sum of all the triangles to be equal to the parabola ABC.



Demonstration.

If indeed the sum of the triangles shall not be equal to the area of the parabola, and the sum therefore shall be greater or smaller, in the first case the parabola shall be greater than the sum of the series of triangles, and G shall be put to be the excess amount G; therefore since the triangle ABC is the maximum of these, which are able to be inscribed in the parabola, also that will be greater than half the parabola in which it has been inscribed; similarly the two triangles AEB, BFC are greater than half the segments in which they have been inscribed, since which shall be able to be continued without end, and therefore with the amount remaining from the magnitude of the parabola given smaller, and smaller than the magnitude G, therefore there shall be no excess, by which the series of triangles exceeds the area of the parabola: therefore the parabola is greater than the whole sum of the triangles.

Truly, it is evident from the hypothesis, so that neither shall it be smaller than that series of triangles, to be continued always within the parabola, and hence that series however great it may become, with more triangles added, yet it shall remain always a part of the parabola ; therefore since the parabola shall be neither larger nor small than the series of triangles, it is necessary that the series shall be equal to the area of the parabola. Q.e.d.

PROPOSITION CXXXII.

With the same figure in place:

I say the area of the parabola ABC to that of the maximum triangle ABC to be in the same proportion as four to three.

Demonstration.

The maximum triangle ABC is four times the sum of the maximum triangles AEB, BFC which are inscribed by the remaining segments; and these again taken together, four times the remainders of the remaining inscribed segments, and thus by proceeding without end, since those removed will be always be the four triangles, the whole series of triangles, that is the parabola ABC, is to the triangle ABC, the first term of the series, as four to three. Q.e.d.

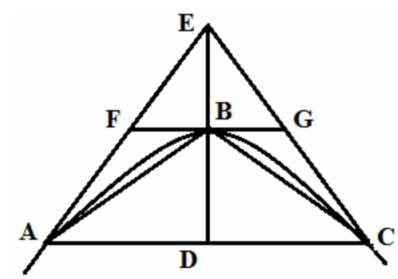
First corollary.

Hence it is evident the maximum triangle ABC to be three times the sum of the remaining segments AEB, BFC. Indeed since the whole parabola, to the maximum inscribed triangle, shall be as four to three; it is clear the triangle itself, to contain three quarters of the parabola; and thus to be three times the remaining parts.

Second Corollary.

It follows the second segments AEB, BFC to be equal to each other: indeed the triangles ABD, BDC are equal to three of the individual singular triangles, and equal to each other.

PROPOSITION CXXXIII.



BD shall be the diameter of the parabola ABC, AC the ordinate line applied, with the tangents acting through A and C, which shall meet the diameter BD at E; the tangent acting through B shall be put in place, which shall cross the lines AE, CE at F and G.
I say the triangle FEG to be greater than half of the concave figure AECBA.

Demonstration.

ABC shall be joined; because AE is a tangent to the parabola, and the ordinates AC is put in place for BD, the lines ED, AE, CE are bisected at the points B, F, G: whereby the triangles EBF, ABF, likewise EBF, EBG, and hence the whole EFG, ABE are equal [in area] ; but triangle AEB is greater than half of the composite figure ABCEA, since the line AB shall fall within the convex parabola, therefore the triangle FEG also is greater than that figure. Q.e.d.

Corollary one.

From what has been demonstrated before it is easy to deduce the triangle FEG to be the maximum of these, which are able to be bought inside the triangle AEC from some other tangent.

Corollary two.

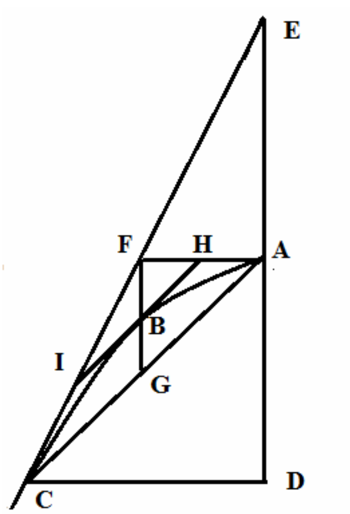
It follows also that the area of triangle ABC, to be twice that of triangle FEG; since ED is twice EB, and AC twice FG.

PROPOSITION CXXXIV.

The right line CE shall be a tangent at C to the parabola ABC, a diameter of which AD meeting the tangent at A, which the line CE shall cross at F, from F a diameter FB is dropped, and the tangent shall be put through B, which shall cross the lines AF, EC at H and I.

I say the area of triangle AEF, to be four times the area of triangle HFI.

Demonstration.



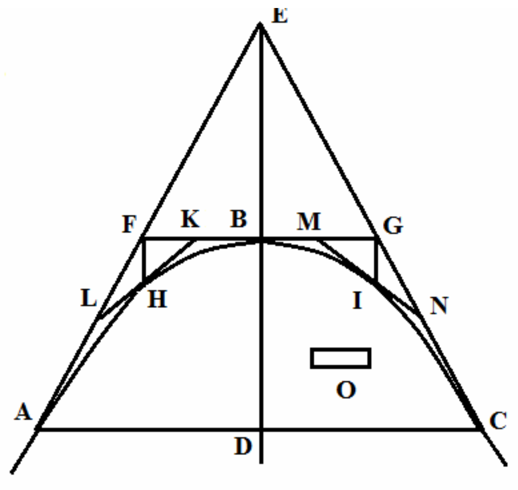
The right line AC shall be drawn, crossing FB produced at G, because EC is a tangent , and CD the ordinate corresponding to the diameter AD, the right lines EA, AD, and thus EF, FC are equal: moreover as EF is to FC, thus AG shall be to GC, (since FG, ED are parallel diameters;) therefore the line AC is bisected at G, and thus parallel to the tangent IH; from which the lines FG, FA also shall be bisected at B & H, and triangle AFC four times triangle FHI: moreover triangle FAE is equal to triangle FAC, because FE, CE are equal lines, and therefore triangle FAE is four times triangle FHI. Q.e.d.

PROPOSITION CXXXV.

BD shall be the diameter of the parabola ABC, AC the applied ordinate, and with the tangents put in place through A and C, which shall meet the diameter BD at E; the tangent shall be drawn through B , which shall cut the lines AE, CE at F and G, then the diameters FH, GI are put in place; and the tangents LK, MN shall be drawn through H and I, and may be continued likewise without end.

I say the curvilinear figure AECBA , to be equal to the total series of triangles.

Demonstration.



Indeed if it shall not be equal ; therefore it is necessary that it shall be either larger or smaller, in the first case the curvilinear figure shall be greater than the series of triangles, in excess by the amount O, half of the concave figure AECBA is greater than the triangle FEG; similarly half the curvilinear figure MGN with which it is inscribed shall be greater than the triangle LFK, and that shall always be the case through that continual removal, there will be left a given smaller curvilinear magnitude AECBA, and therefore the smaller quantity O, therefore O cannot be the excess amount, by which the curvilinear figure AECBA exceeds the series of triangles: therefore neither is that total series greater; similarly it will be shown the figure AECBA also cannot be smaller than the total series of triangles; therefore it is necessary that they shall be equal.

PROPOSITION CXXXVI.

With the same figure remaining:
I say the concave parabola AECB to the triangle FEG, to be in the proportion four to three.

Demonstration.

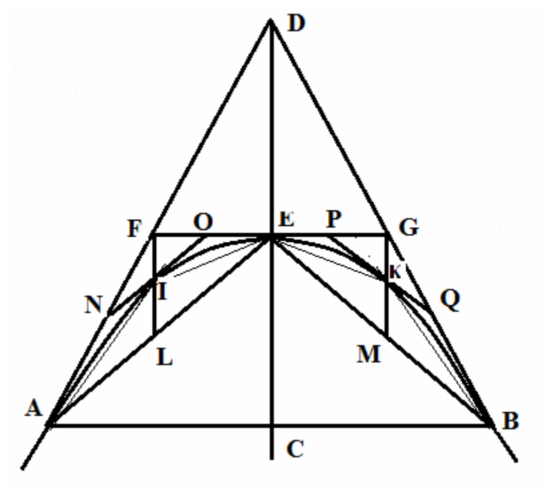
Since the area of the triangle EFG is four times the area of the triangles LFK, MGN, and with these again taken together, to be four times the area of these which are inscribed in the remaining curvilinear figures, and thus by proceeding indefinitely, the quadruples to be performed on the remaining triangular figures, and those to be removed always are of the remaining inscribed triangular figures, the whole series of triangles will become by p. 87 of our book about progressions, that is the ratio of the concave figure AECBA to the triangle FEG, to be as four to three. Q.e.d.

Corollary.

Hence it is evident the area of the triangle FEG to be the three times the area of the remaining shapes AHBFA, CIBGC; indeed the triangle FEG is to the concave figure ABCEA, as three to four: whereby the triangle EFG contains three quarters of the figure ABCEA, and thus is triple of the remaining triangles.

PROPOSITION CXXXVII.

With the same in place :
I say the convex parabola AEB to be double the area of the concave figure AEBDA.



Demonstration.

The maximum triangles AEB, FDG shall be inscribed both on the concave as well as on the convex sides of the parabola; therefore since the parabola AEB is to the triangle AEB as four is to three: moreover the same proportion shall be had by the concave figure AEBD to the triangle FDG, as the triangle AEB to the convex parabola, just as the triangle FDG to the curvilinear figure AEBD : and interchanging, so that triangle AEB shall be to triangle FDG, just as the parabola AEB shall be to the concave figure : but triangle AEB is the double of triangle FDG; and therefore the area of the parabola AEB is twice that of the figure AEBD. Q.e.d.

PROPOSITION CXXXVIII.

To demonstrate the same otherwise.

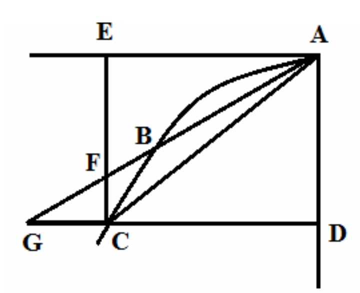
Demonstration.

The greatest triangles AIE, EKB, NFO, PGQ shall be inscribed for the remaining segments, both of the convex as well as for the concave parabola. Because triangle AEB, taken from the parabola is the double of the triangle FDG, taken from the figure ADBEA; and again the triangles AIE, EKB taken from the remaining parabola is the double of the triangles NFO, PGQ, taken from the remaining figure ADBEA, in addition it shall be shown that subtraction shall be in the twofold proportion, either with the term able to be continued in each figure, or the whole series of the greatest triangles inscribed to the parabolas AEB can be continued in each figure, and for that curvilinear figure AEBDA to be equal to the sum of all the greatest triangles, and for parabola AEB to be equal to double the curvilinear figure ADBEA. Q.e.d.

PROPOSITION CXXXIX.

To show a triangle equal to a given parabolic section.

Construction and Demonstration.

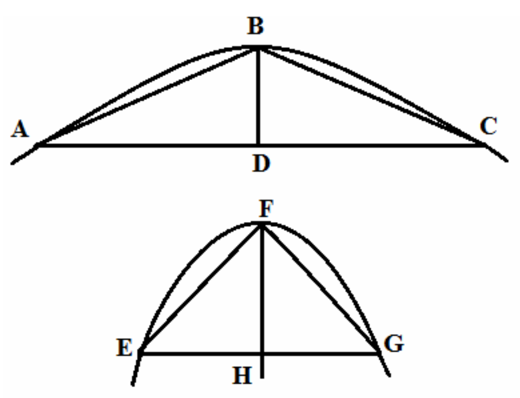


ABC shall be the given section: and with the diameter AD drawn, the ordinate CD shall be put in place, and AE shall be drawn acting through A, parallel to CD itself, crossing the diameter erected from C at E: then EC divided at F so that FC shall be the fourth part of EC; the line AG shall be drawn from A through F, crossing CD at C. I say the triangle GAC to be equal to the given segment ABC. Since AE, CG shall be parallel lines, so that as CF shall be to FE, thus as GC is to EA, but three times CF is FE; and therefore three times GC is EA, equal to CD. And whereby three times the area of triangle GAC is the area of triangle CAD : but the segment ABC is equal to the third part of triangle CAD; and therefore is equal to the triangle GAC : therefore we have shown the triangle to be equal to the given parabolic segment. Q.e.d.

Corollary.

Hence it is evident the triangle GAD to be equal to the parabola ABCD. Thus a problem to be solved in practice, by which the area of a given parabola is desired, to be shown to be equal to a triangle.

PROPOSITION CCXL.



Restricted parabolas can be distinguished amongst themselves on account of the greatest triangles inscribed within them.

Demonstration.

ABC, EFG shall be for restricted parabolas with the greatest inscribed triangles ABC, EFG. I say the parabolas have that that same ratio between themselves as the greatest triangles. Triangle ABC is to the parabola ABC, as three to four: also, triangle EFG is to parabola EFG, as three is to four; therefore as triangle ABC to parabola ABC, thus triangle EFG to parabola EFG, and on interchanging, so that triangle ABC to triangle EFG, thus parabola ABC to parabola EFG. Q.e.d.

Corollary.

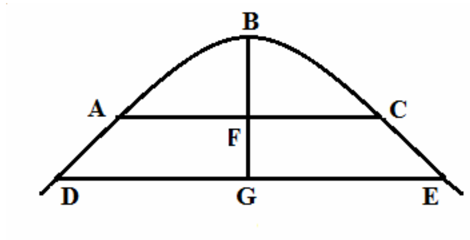
Hence if two parabolas shall have the same or equal chords, these will be to each other as the altitudes ; and if the altitudes were equal, they will be to each other as the bases.

PROPOSITION CCXXLI.

Some two parallel chords AC, DE shall cut the parabola ABC.
I say the parabola ABC to the parabola DBE to be in the threefold ratio AC to DE.

Demonstration.

The diameter BF shall be put in place for which the ordinates shall be AC, DE. Parabola ABC shall have that ratio to parabola BE, which the triangle under AC & BF has to the triangle under DE & BG: but the ratio of the triangle under AC & BF, to the triangle under DE & BG, is the triplicate of the ratio AC to DE, which is composed from the ratio AC to DE, & BF to BG, that is from the square ratio AC to DE; therefore the parabola ABC is to the parabola DBE in triplicate ratio AC ad DE. Q.e.d.

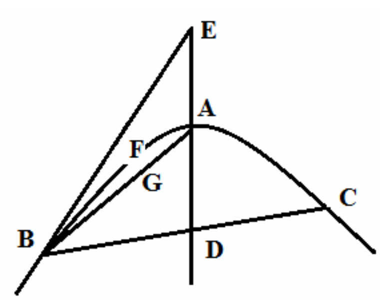


PROPOSITION CCXLII.

The line EB shall be a tangent to the parabola ABC at B, crossing some diameter AE at E, and AB shall be joined.
I say the concave figure BFAEB, to be double the convex figure BFAGB.

Demonstration.

From B, the ordinate BC may be put in place to the diameter AD. Since BE is a tangent, the lines AD, AE shall be equal, and thus the triangles ABD, ABE shall be equal in area : moreover the triangle ABD is three times the magnitude of the segment BFAGB, and therefore the triangle ABE is three times the magnitude of the segment BFAGB; therefore the remaining concave figure BFAEB is twice the convex figure BFAGB. Q.e.d.



PROPOSITION CCXLIII.

AD shall be the diameter of the parabola ABC and DC, GB the ordinates put in place : and with AB joined, AE shall be put through A parallel to DC itself, crossing the diameters at F & E erected from B & C.
I say triangle ABF to be to triangle ACE, or triangle ABG to triangle ACD, thus as the concave figure AHBFA to the concave figure AHCEA.

Demonstration.

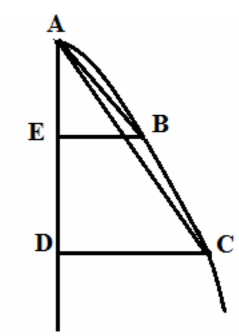
As triangle ABG shall be to triangle ACD, thus segment AHB shall be to the segment ABC, but as segment AHB to segment ABC, thus the figure AHBF to the figure ABC, since AHBF shall be twice segment AHB, and ABCE twice ABC; therefore as triangle ABG to triangle ACD, thus the figure AHBF shall be to the figure ABCE. Q.e.d.

Corollary.

With the same figure put in place it follows that as the parallelogram GF shall be to the parallelogram DE, thus the parabola AGB shall be to the parabola ADC: likewise the convex figure AHBF to the convex figure ABCE.

PROPOSITION CCXXLIV.

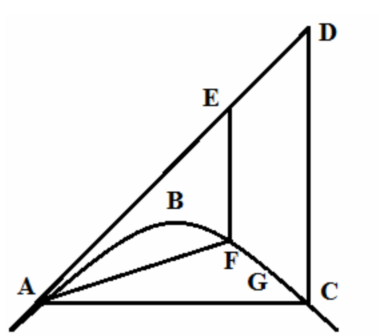
Some lines AB, AC shall cut the parabola ABC, of which the diameter is AD : and the ordinates BE, CD shall be drawn.



I say the parabolic area EBCD, to be four times the area contained by the lines AB, AC and by the parabolic arc BC.

Demonstration.

Since the [area of the] parabola DABC is four times that of the segment ABC, and likewise the parabola EAB is four times the segment AB, the parabola DABC is to the segment ABC as the parabola EAB is to the segment AB; therefore for the parabola DABC to the segment ABC, the whole to the whole shall be as the amount removed EAB to the amount removed AB; therefore the remainder EBCD shall be to the remainder ACBA, as the whole DABC to the whole ABC: whereby the figure EBCD , is four times as great as the figure formed from the lines AC, AB and contained by the parabolic arc BC. Q.e.d.



PROPOSITIO CCXXLV.

Some line AD shall be a tangent to the parabola ABC meeting some diameters DC, FE at D and E, and with AF, AC joined.

I say the concave region EDCGF to be twice as great as the region contained by the part AFGC, and with the lines AF, AC.

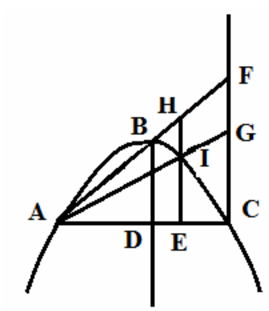
Demonstration.

The concavity ABCDA is twice as great as the part CABC of the parabola: and the concavity ABFEA is the double of the segment ABF; therefore the remainder EDCGF is twice the remainder AFGCA. Q.e.d.

PROPOSITION CCXXLVI.

The line AC shall subtend the parabola ABC, with which divided at D and E, AD, AE, AC shall be in continued proportion, the diameters DB, EI, CF shall be erected, and through the points B and I, the right lines AG, AF are put in place from A cutting the diameter CF at F and G.

I say the region EIGC to the quadrilateral BI to be in the threefold ratio of AC to AE [i.e. ratio of the cubes].



Demonstration.

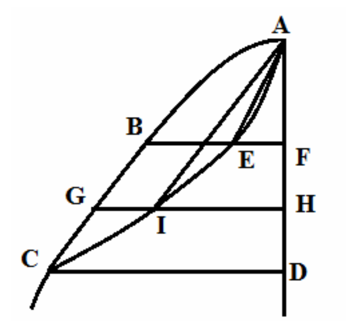
Since AD, AE, AC are placed in continued proportion, so that as CA to EA, thus CE to ED, but as CE to ED, thus CG is to GF; therefore so that as CA shall be to EA, thus CG shall be to GF: from which the area of triangle CAG shall be to the area of triangle GAF, shall be as CE to ED, that is as CA to EA, again as FA to HA, that is, as FG to Hl; but triangle GAF to triangle HAI is in the square ratio FG ad HI, therefore since the ratio of triangle CAG to HAI shall be composed from the ratio of the triangles CAG and GAF, and from GAF to HAI, it is apparent triangle CAG shall be to triangle HAI in cubic ratio FG ad HI : truly since the ratio of quad. EG to quad. BI, is composed from the ratio of the quad. EG to the quad. GH, (i.e. from the ratio of triangle CAG to triangle GAF), and from the ratio of the quads. GH to IB, (i.e. from the ratio of triangle GAF to triangle HAI, since FA, HA, BA shall be in proportion), quad. EG to quad. BI shall be as triangle CAG to triangle HAI; and therefore quad. EG to quad. BI has the threefold ratio AC to AE. Q.e.d.

PROPOSITION CCXXLVII.

AD shall be the diameter of the parabola ABC and with the ordinate CD put in place for that, the parabola AEC shall be described through A and C, which shall have the tangent AD at A, and some ordinates FB, HG shall be put in place cutting the parabola AEC at E and I.

I say the concave figure AFEA, to the figure AHIA, shall be in the square ratio of the parabola BAF to the parabola GAH.

Demonstration.

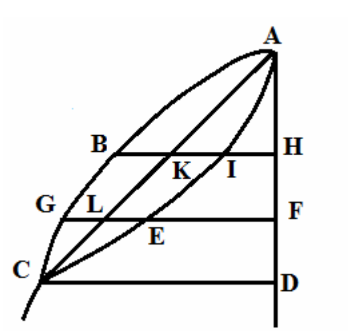


AE, AI shall be joined. The curvilinear figure AFEA has the same ratio to the figure AHIA as the triangle AEF to triangle AIH; but the ratio of triangle AEF to triangle AIH is composed from the ratio AF to AH, that is to the square of the ratio FB to HG, and from the ratio AE to IH, that is to the square of the ratio AF to AH, that is, to the square of the ratio FB to HG; therefore the figure AFEA to the figure AHIA has the sixth power ratio of the line FB to HG; but the parabola BAF to the parabola GAH has the cubic ratio of the line FB to the line HG: therefore the figure AFEA has the square ratio to the figure AHIA, to be in the square ratio of the parabola BAF to the parabola GAH. Q.e.d.

Corollary.

Hence it is evident the concave shape ASE to the concave shape AHI, to be in the cubic ratio AF to AH; for the ratio of triangle AEF to triangle AIH has been composed from AF to AH, and from EF to IH, that is from the square of the ratio AF to AH.

PROPOSITION CCXLVIII.



With the same in place, AH, AF, AD shall be proportional lengths, and AC shall be joined.

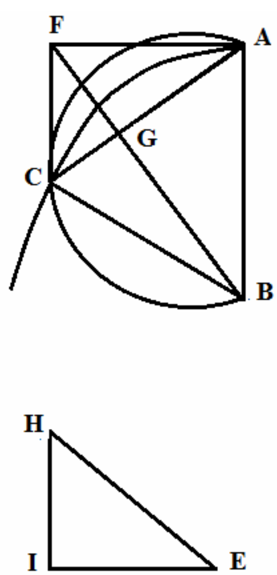
I say the curvilinear shapes IF to ED, to have a sixth power ratio, of which the ratio of the trapezium KF to the trapezium LD is the fourth power.

Demonstration.

Since AH, AF, AD are proportional lines, also the right lines IH, EF, CD, and thus the concave figures AHIA, AFEA, ADCA are in analogous continued proportion. Therefore so that as the figure AHIA shall be to the figure AFEA, thus the curvilinear shape IF shall be to the curvilinear shape ED; but the figure AHIA to the figure AFEA, and therefore the curvilinear shape IF to the curvilinear shape ED has the ratio BH to GF to the sixth power. Again, since AH, AF, AD are proportionals, also the triangles AKH, ALF, ACD are continued in analogous proportion. And thus as triangle AKH is to triangle ALF, thus the trapezium KF is to the trapezium LD: but the triangle AKH to the triangle ALF, is in the square ratio of the line AH to the line AF, that is the fourth power of the ratio HB to FG: and therefore the trapezium KF to the trapezium LD is of the fourth power of the line HB to FG, of which the curvilinear shape IF to the curvilinear shape ED is the sixth power. Q.e.d.

PROPOSITION CCXLIX.

In the parabola ACB, AB shall be the part of the axis equal to the latus rectum, and with AF drawn normal to the axis, some FC may be put parallel to AB, cutting the parabola at C, and AC, BF shall be drawn.



I say these two lines intersect each other normally, and CG, FG, GA, GB to be continued proportionals.

Demonstration.

Since BA shall be equal to the latus rectum, the rectangle BA.FC shall be equal to the square FA. Therefore the three FC, FA, AB are in continued proportion: therefore since the angles CFA, BAF shall be equal, the triangles CFA, FAB are similar, from which the angle FAC is equal to the angle ABF; but the angle FAC together with the angle CAB is equal to a right angle; therefore the angle FBA together with the angle CAB is equal to a right angle, and consequently the angle AGB is right. Consequently since both the angle AFC as well as the angle AGF is right, the three CG, GF, GA are proportionals: truly since the angles AGB, FAB are right, also the lines FG, GA ,GB are proportionals; therefore they shall continue in the same ratio CG, FG, GA, GB. Q.e.d.

PROPOSITION CCL.

Between two given values, to show two related mean values.

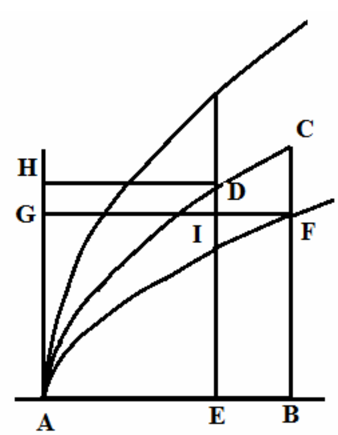
Construction and Demonstration.

Two values HI, IK shall be given, between which it shall be required to show two values, these may be set up at right angles and they shall make a right angled triangle HIK; then the parabola ACB shall be described of which the latus rectum shall be part of the axes AB, upon which the segment of a circle may be established taking the angle ACB equal to IHK, crossing the parabola at C and CF, FA shall be drawn at right angles, thus so that CF shall be parallel to the axis and AC, BF shall be joined. I say what is required has been done, thus the angle at G is shown to be a right angle, and the angle BCA or BCH is equal to the angle K. Therefore the triangle BCG is similar to the angle HIK; therefore since FG, GA shall be the means between CG, GB, also they are found to be the means between HI, IK.

PROPOSITION CCLI.

The two parabolas ABC, AFB shall have a common axis ; and AB shall be equal to the latus rectum of the parabola ABC, and with the ordinates ED, BFC drawn, the area of the parabola ADE shall be equal to the area of the parabola AFB.
I say AE, ED, to be the mean values FB, BA.

Demonstration.



Since, by hypothesis, AB is the latus rectum of the parabola ACB, the rectangle BAE shall be equal to the square ED, therefore as BA to ED, thus ED to AE. Then since the areas of the parabolas are equal, also the areas of the rectangles AED, ABF shall be equal. Therefore as BA shall be to ED, thus reciprocally AE shall be to BF: but since it has been shown that as BA to ED, thus ED to be to AE, therefore as ED to AE, thus AE to BF. Therefore it is clear the four right lines BA, ED, AE, BF to be in continued proportion and hence the means between BA, BF, to be ED, AE. Q.e.d.

PROPOSITION CCLII.

To show the means between any two given values.

Construction and demonstration.

AB, BF, shall be given, from which for the right angle put in place to describe the parabola AC drawn about the axis AB, of which the latus rectum shall be equal to AB itself. Then BF shall meet the parabola AC at C, and about the common axis AB to describe another parabola through A and F. Finally the ordinate DE shall be drawn, making the parabolic segments ADE, AFB equal. I say DE, EA to be the mean values between AB, BF: evident from the preceding demonstration.

PARABOLAE

PARS QUINTA

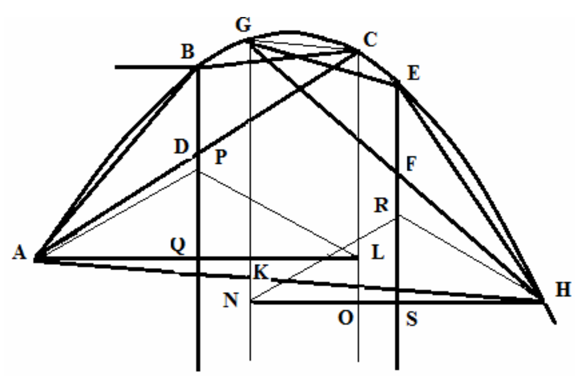
Saepius parabolam quadrat.

PROPOSITIO CXXVIII.

Secent ABC parabolam diametri duae aequales BD, EF: positisque per D & F, ordinatim lineis AC, GH, iungantur ABC, GEH.
Dico ABC, GEH triangula esse inter se aequalia.

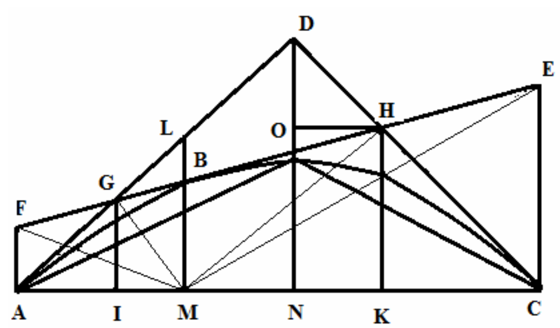
Demonstratio.

Iunctis, G, C; ponatur AH linea, quam in K & I secent demissae ex G & C diametri : illas autem secent ortogonaliter in L,M, N, O rectae AML, HON: & AL quidem occurrat BD lineae in Q. HN vero ipsi EF in S; factisque PQ, SR aequalibus, ipsis BD, EF, iungantur APL,HRN; quoniam igitur aequales sunt diametri BD, EF; &



ADC, GFH ad illas ordinatim positae, iunctae CG, AH sive IK aequidistant: sed & GK, CI diametri parallelae sunt, parallelogrammum igitur est GCI K; & GK, CI lineae aequales, est autem ut GK ad IC sic AKH rectangulum ad rectangulum AIH, rectangula igitur AKH, AIH aequalia sunt ; ideoque &c aequales lineae AK, HI; quia vero est ut AK ad KI, sic AM ad ML, & ut HI ad IK, sic HO ad ON, igitur ut AM ad ML, sic HO ad ON: aequales autem sunt lineae NO, ML (quia AML, HON orthogonales sunt ex constructione, ad GM, GL, aequidistantes, adeoque MO parallelogrammum est) rectae igitur AM, HO, adeoque totae AL, HN aequales sunt aequales sunt: sed & PQ, RS per constructionem aequales sunt, triangula igitur APL, NRH id est ABC; GEH aequalia sunt. Quod erat demonstrandum.
Est haec Archimidis, aliter demonstrata.

PROPOSITIO CXXVIX.



Parabolam ABC, contingant duae quaevis AD, CD convenientes in D, secetque illas in G & H recta quaedam EF, contingens quoque parabolam in B occurrens AE , CF diameteris in E & F; demittantur autem ex G & H diametri GI, HK, occurrentes AC in I & K.
Dico rectam IK, dimidium esse AC.

Demonstratio.

Quoniam AG, BG sectionem contingant, recta EG aequalis est GB : similiter & BH aequalis HF, GH igitur dimidium est totius EF; sed cum AE, CF, GI, HK aequidistent, erit AC in I &;K, divisa, ut EF divisa est in G & H ; igitur & IK dimidium est AC. Quod erat demonstrandum.

PROPOSITIO CXXX.

Iisdem positis, ponatur per B diameter LM.

Dico esse ut GB ad BH, sic LB ad BM.

Demonstratio.

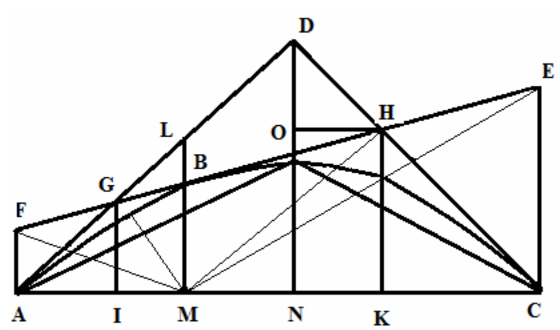
Quoniam AL est contingens, LB est ad BM, ut AM ad MC, hoc est EB ad BF: sed ut EB ad BF, sic GB est ad BH, igitur ut GB ad BH, sic LB est ad BM. Quod erat demonstrandum.

PROPOSITIO CXXXI.

Eadem manente figura, ducantur rectae MG, MH.
Dico MD esse parallelogrammum.

Demonstratio.

Est enim ut HB ad BG, sic MB ad BL; per praecedentem & pcrmutando HB ad MB, sic GB ad BL: sunt autem anguli ad B lateribus proportionalibus contenti aequales, triangula igitur MBH,GBL similia sunt: & MHd parallela GL: eodem modo si KH producatur donec cum AD conveniat, oftenditur GM aequidistare ipsi DH: parallelogrammum igitur est DM. Quod fuit demonstrandum.



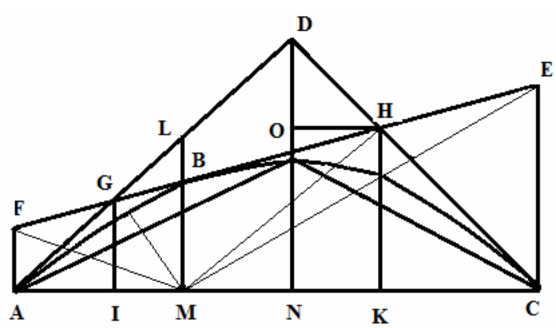
PROPOSITIO CXXXII.

Eadem manente figura, ducantur AB, BC.
Dico triangulum ABC aequale esse parallelogrammo MD.

Demonstratio.

Est enim BEM triangulum triangulo ABM aequale similiter triangulum BFM aequale BMC triangulo; igitur totum angulum EMF toti triangulo ABC est aequale; est autem EMF triangulum duplum trianguli GMH, (quia basis EF dupla est baseos GH.) igitur MD parallelogrammo aequale est triangulum ABC.
Quod fuit demonstandum.

PROPOSITIO CXXXIII.

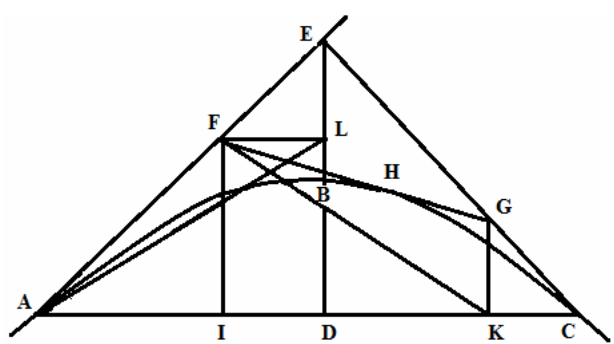


Eadem manente figura, demittatur ex D diameter DN.
Dico lineas GI, H K simul sumptas, aequari rectae DN.

Demonstratio.

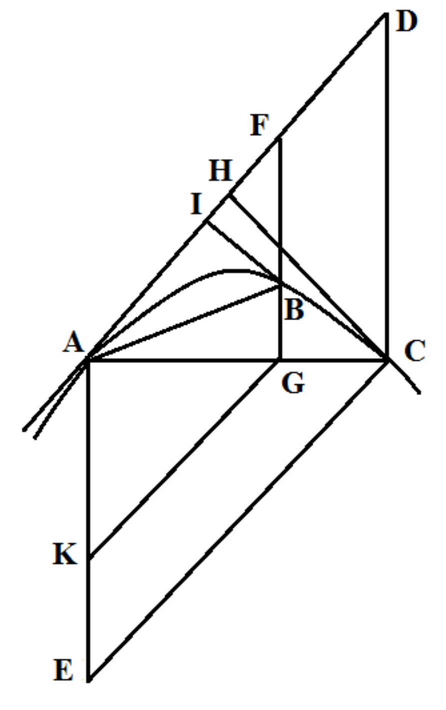
Ducatur HO parallela AC, occurrens ND lineae in O. Quoniam IM, GI, aequidistant lineis DO,HO : est autem & GM aequalis & parallela DH; triangula IGM, DOH, adeoque & latera DO, GI aequalia sunt: est autem HK aequalis ON. Igitur GI, HK simul sumptae, sunt aequales lineae DN. Quod fuit demonstandum.

PROPOSITIO CXXXIV.



Sit ad ABC parabolae axem BD ordinatim ducta AC, actaeque per A & C, contingentes conveniant in E; ponatur autem & FG contingens in B, quae AC, CE lineas secet in F & G, tum rectae demittantur FI, GK, axi aequidistantes.

Dico trapezium FJKG, aequale esse triangulo AED.



Demonstratio.

Ponatur FL parallela AC, iunganturque LA , FK. Quoniam FL aequidistat AC, & IK est aequalis AD, dimidio scilicet AC, triangula ALD, IFK aequalia sunt. Rursum cum in triangulis FKG, AEL, tam bases KG, EL quam altitudincs IK, AD aequales sint, triangula quoque FKG, AEL sunt inter se aequalia: trapezium igitur FlKG aequale est triangulo AED. Quod erat demonstrandum.

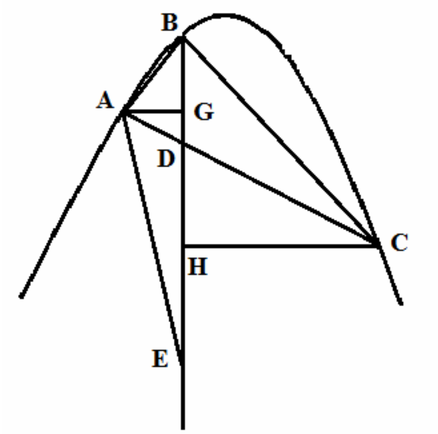
PROPOSITIO CXXXV.

Parabolam ABC subtendat AC quaecunque, ponatur autem per A contingens AD, coveniens cum CD diametro in D: ponatur quoque diameter quaevis BG, occurres AD lineae in F; iunganturque AB.
Dico triangulum AFB ad triangulum ADC triplicatam habere rationem lineae AF ad AD.

Demonstratio.

Demissa ex A diametro AE, ducantur lineae BK, CE parallelae contingenti AD; & ex C, & B rectae CH, BI normales ipsi AD; ratio AFB trianguli ad triangulum ADC composita est ex ratione AF ad AD, & ex ratione IB ad HC: sed ut IB ad HC, sic FB ad CD, id est AK ad AE; igitur ratio trianguli AFB ad ADC triangulum composita est ex ratio AK ad AD, & ex ratione AK ad AE: est autem ratio AK ad AE, duplicata rationis KB ad EC, id est: AF ad AD, (quia FK, DE parallelogramma sunt) igitur triangulum AFB ad ADC, triangulum; triplicatam habet rationem eius, quam habet AF ad AD. Quod erat demonstandum.

PROPOSITIO CXXXVI.



Parabolae ABC diametrum BD, secet in D recta quaevis AC, positaque ordinatim CE, iungantur AB, B C.
Dico triangulum ABD ad BCE, triangulum habere rationem lineae AD ad DC.

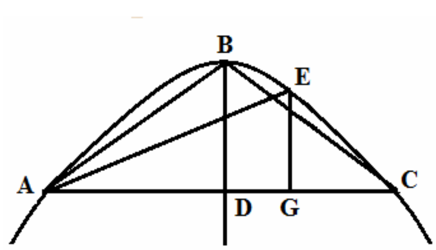
Demonstratio.

Ponantur ex A & C lineae AG, CH normales ad diametrum BE; iunganturque AE. Ut AD ad DC, sic BD est ad BE:sed ut AD est ad DC, sic GD est ad DH, hoc est AG ad CH; igitur ut BD ad BE, sic AG est ad CH. Est autem ratio trianguli ABD ad BCE triangulum, composita ex ratione BD ad BE, & AG ad CH*;* igitur ratio trianguli ABD ad BCE triangulum duplicata est rationis AD ad DC.Quod erat demonstrandum

PROPOSITIO CXXXVII.

Datae parabolae terminatae maximum inscribere triangulum.

Constructio & demonstratio.



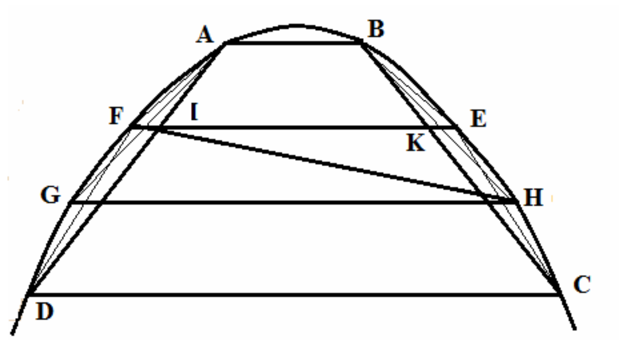
Parabolam ABC subtendat quaevis AC, qua divisa bifariam in D, ponatur diameter BD, & iungantur AB, CB. Dico triangulum ABC esse quaesitum. Erigatur enim quavis alia GE, parallela BD diametro: Quoniam ADC rectangulum ad AGC rectangulum eam rationem obtiner, quam DB a GE, igitur recta DB, maior est recta GE: ergo etiam triangulum ABC, maius triangulo AEC: igitur maximum est triangulorum ABC. Quod demonstrare oportuit.

PROPOSITIO CXXXVIII.

Parabolam ABC intersecent duae quaevis parallelae AB, DC: iunctisque BC, AD, segmento CB triangulum inscribatur maximum AEC, ponaturque EF, aequidistans AB, & iungantur AFD; .

Dico AFD triangulum, illorum esse maximum quae AFD segmento inscribi possunt; & contra si triangula AFD, BEC fuerint maxima, dico FE aequidistare AB.

Demonstratio.

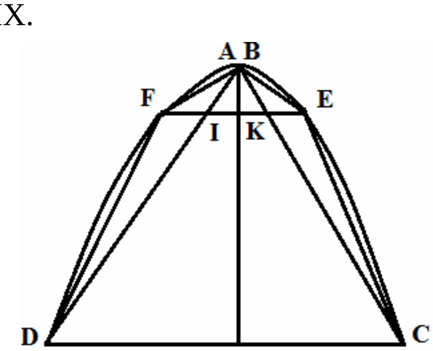


Quoniam AB, CD, FE aequidistant, recta FI aequatur KE, adeoque triangula FAI, FID aequalia sunt triangula FAI, FID aequalia sunt triangulis, KBE, KEC: si igitur AFD triangulum non sit maximum, si aliud AGD, maius triangulo AFD: positaque GH parallela AB, iungantur BHC: ostendetur ut prius, triangulum BHC, aequari triangulo AGD: sed AGD maius est triangula AFD, id est ut ostendi, BEC, triangulum igitur BHC maius quoque est triangulo BEC: quod est contra hypothesim. non igitur AGD triangulum maximum est , sed AFD. Quod erat primum.

Sint iam AFD, BEC triangula maxima, dico iunctam FE aequidistare AB: sin vero ; ponatur FH aequidistans AB, iunganturquc BHC: triangulum igitur BHC maximum est eorum quae BEC segmento inscribi possunt, adeoque & maius BEC triangulo, quod absurdum: non igitur FH aequidistat AB, sed FE. Quod erat demonstrandum.

Quod si AB contingat parabolam, eadem inferri possunt quae prius, eademque prorsus est demonstratio.

PROPOSITIO CXXXIX.

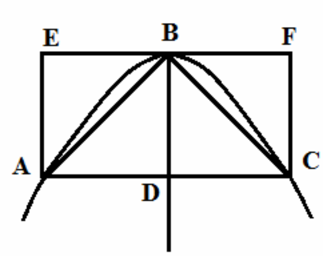


Esto ABC parabolae inscriptum triangulum maximum ABC.

Dico illud maius esse dimidio parabolae ABC.

Demonstratio.

Perficiatur rectangulum ACF ; manifestum igitur est EC parallelogrammum maius esse parabola ABC; igitur & ABC triangulum, dîmidium scilicet

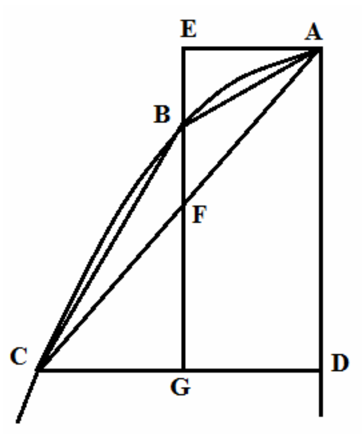


parallelogrammi EC maius dimidio parabolae A BC. Quod erat demonstrandum.

PROPOSITIO CXXX.

Sit ad ABC parabolae diametrum AD, positas ordinatim CD; iunctaque AC divisam F bifariam, ponatur diameter BF iunganturque AB,CB.
Dico CAD triangulum quadruplum esse trianguli ABC.

Demonstratio.



Agatur per A contingens AE, occurrens BF lineae in E, quae producta secet DC in G, quoniam igitur aequidistant AE,CD, ut CF ad FA, sic GF est ad FE, ponitur autem AC in F bifariam divisa, igitur & EF aequalis est FG & EAF triangulum aequale triangula CFG : sed EAF triangula aequale est triangulum CBA: igitur CFG triangulum aequale est triangula ABC, quia EB, BF lineae aequales sunt:est autem CAD quadruplum trianguli CFG quia AD dupla est FG & CD dupla CG, igitur & quadruplum erit trianguli ABC. Quod erat demonstrandum.

PROPOSITIO CXXXI.

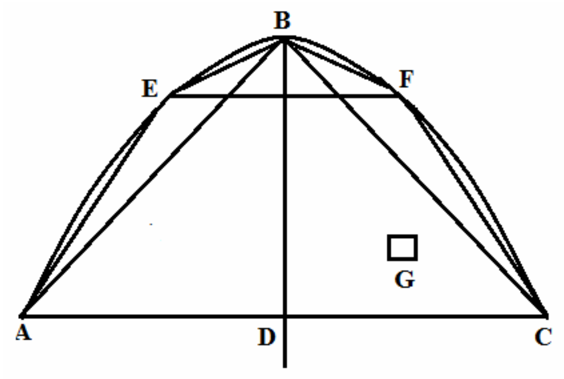
Esto ABC parabolae inscriptum triangulum maximum ABC: inscribantur autem & residuis segmentis triangula maxima: & hoc semper fiat.

Dico toti triangulorum seriei aequalem esse parabolam ABC.

Demonstratio.

Si enim non sit aequalis, maior igitur est vel minor, sit primum parabola maior tota triangulorum serie, & excessus ponatur quantitas G; quoniam igitur triangulum ABC maximum est illorum, quae parabolae illd erit dimidio parabolae cui inscriptum est; similiter triangula duo AEB, BFC maiora sunt dimidiis segmentorum quibus inscribuntur, quod cum sine termino continuari possit, relinquetur ex parabola quantitas, data minor, ergo & minor quantitate G, ergo illa excessus non est, quo parabola triangulorum seriem excedit: ergo parabola maior non est tota triangulorum serie.

Quod vero neque minor illa sit, manifestum est; cum triangulorum series, ex hypothesi semperintra parabolam continuetur, ac proinde series illa quantumcunque aucta, plurium triangulorum addditione, semper tamen pars maneat parabolae; cum igitur serie triangulorum, nec maior, nec minor sit parabola, aequalis ut sit necesse est. Quod erat demonstrandum.



PROPOSITIO CXXXII.

Eadem positâ figurâ:
Dico ABC parabolam ad triangularum maximum ABC eam habere proportionem quam quatuor ad tria.

Demonstratio.

Triangulum maximum ABC, quadruplum est triangulorum maximorum AEB, BFC quae residuis inscribuntur segmentis; & illa rursum simul sumpta, quadrupla triangularum residuis segmentis inscriptorum, atque ita sine termino procedendo, cum ablata semper quadrupla sint triangulorum, quae residuis inscribuntur segmentis, tota trianguloram series, id est parabola ABC, est ad triangulû ABC, primum seriei terminum, ut quatuor ad tria. Quod erat demonstrandum.

Corollarium primum.

Hinc manifestum est triangulum maximum ABC triplum esse residuorum segmentorum AEB, BFC. cum enim tota parabola, ad triangulum maximum inscriptum, sit ut quatuor ad tria; patet ipsum triangulum,tres quartas continere parabolae ; adeoque & residuorum esse triplum,

Corollarium secundum.

Sequitur secundo segmenta AEB, BFC esse inter se aequalia: triangula enim ABD, BDC singula singulorum tripla sunt, & inter se aequalia.

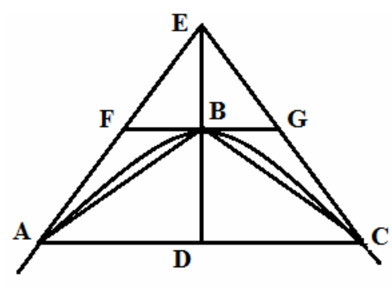
PROPOSITIO CXXXIII.

Sit ad ABC parabolae diametrum BD, ordinatim applicata AC, actisque per A & C contingentibus, quae cum diametro BD conveniant in E, ponatur per B contingens, quae AE, CE lineis occurrat in F & G.

Dico FEG triangulum, maius esse dimidio figurae concave AECBA.

Demonstratio.

Iungantur ABC. quoniam AE parabolam contingit, & AC ordinatim ponitur ad BD, ED, AE, CE, in B, F, G punctis bissectae sunt: quare EBF, AB F triangula, item EBF, EBG, ac proinde tota EFG, ABE aequalia sunt ; sed AEB triangulum maius est dimidio figurae mixtilineae ABCEA, cum AB latus cadat intra parabolam convexam, triangulum igatur FEG illo maius quoque est. Quod erat demonstrandum.



Corollarium primum.

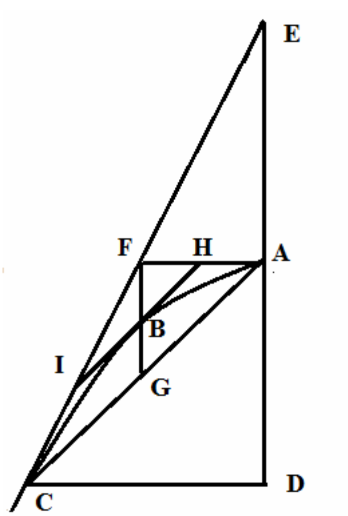
Ex antè demonstratis facile deducitur triangulum FEG, maximum esse illorum, quae intra triangulum AEC ab alia quavis contingente auferri possunt.

Corollarium secundum.

Sequitur quoque, ABC triangulum, duplum esse trianguli FEG; est enim ED dupla EB. & AC dupla FG.

PROPOSITIO CXXXIV.

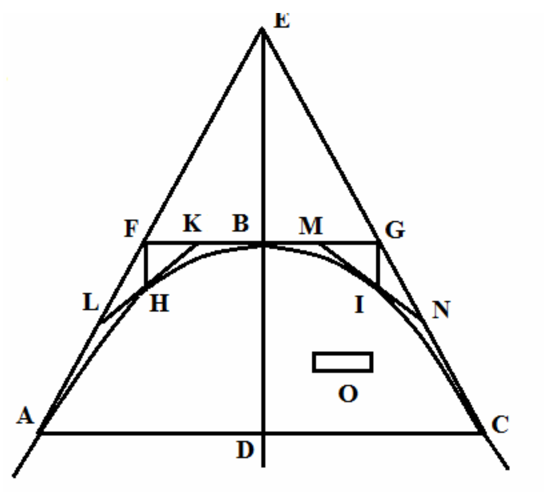
Parabolam ABC cuius diameter AD contingat in C recta CE conveniens cum diametro in E, actaque per A contingente, quae CE lineae occurrat in F, demittatur ex F diameter FB, & per B ponatur contingens, quae AF, EC lineis occurrat in H & I.
Dico triangulum AEF, quadruplum esse triangula HFI.



Demonstratio.

Ponatur recta AC, occurrens FB productae in G, quoniam EC contingens est , & CD ordinatim applicata ad diametrum AD, rectae EA, AD, adeoque & EF, FC aequales sunt: est autem ut EF ad FC, sic AG ad GC, (cum FG, ED diametri aequidistent;) linea igitur AC in G bisecta est, adeoque IH contingenti parallela; unde FG, FA lineae in B & H, bifariam quoque sint divisae, & AFC triangulum quadruplum trianguli FHI: est autem FAE aequale triangule FAC, quia FE, CE lineae aequales sunt, igitur & FAE, quadruplum est trianguli FHI. Quod erat demonstrandum.

PROPOSITIO CXXXV.



Sit ad ABC parabolae diametrum BD, ordinatim applicata AC, positisque per A & C,
contingentibus quae diametro BD occurrant in E; ducatur per B , contingens, quae AE, CE lineas secet in F & G, diametri deinde ponantur FH, GI; & per H & I, contingentes LK, MN atque idem sine termino continuetur.

Dico figuram mixtilineam AEC, BA , aequalem esse toti triangulorom seriei.

Demonstratio.

Si enim non sit aequalis; maior igitur vel minor ut sit necesse est, sit primum figura mixtilinea maior triangulorum serie, excessu quantitatis O, triangulum FEG maius est dimidio figurae concavae AECBA; similiter triangula LFK, MGN maiora sunt dimidiis mixtilineorum quibus inscribuntur, & id semper sit; igitur per ablationem illam continuatam, relinquetur mixtilineo AECBA quantitas data minor, ergo & minor quantitate O, ergo O excessus non est, quo mixtilineum AECBA excedit triangulorum seriem: igitur nec illud serie tota maius est; similiter ostendetur AECBA figuram minorem quoque non esse tota triangulorum serie aequalis igitur ut sit necesse est.

PROPOSITIO CXXXVI.

Eadem manente figurae:
Dico parabolam concavam AECB ad triangulum FEG, eam proportionem habere quam quatuor ad tria.

Demonstratio.

Quoniam EFG triangulum quadruplum est triangulorum LFK, MGN, & illa rursum simul sumpta, quadrupla illorum quae residuis figuris mixtilineis inscribuntur, & ita sine termino procedendo, ablata semper quadrupla sunt triangulorum residuis figuris inscriptoru, erit per 87.libri nostri de progressionibus tota triangulorum series id est concavum AECBA, ad FEG triangulum, ut quatuor ad tria.
Quod erat demonstrandum.

Corollarium.

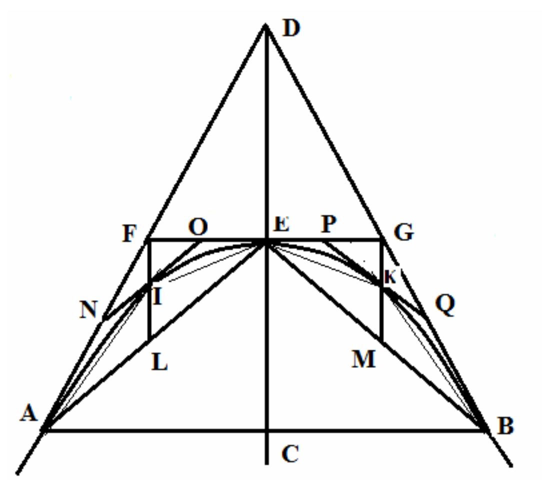
Hinc patet FEG triangulum triplum esse residuorum AHBFA, CIBGC, est enim FEG triangulum ad figuram concavam ABCEA, ut tria ad quatuor: quare triangulum EFG tres quartas continet figurae ABCEA, adeoque residuorum triplum est.

PROPOSITIO CXXXVII.

Iisdem positis :
Dico AEB parabolam convexam duplam esse figurae concavae AEBDA.

Demonstratio.

Inscribantur tam concavae quam conuexae parabolae, triangula maxima AEB, FDG, quoniam igitur AEB parabola est ad triangulum AEB ut quatuor ad tria: eandem autem habeat proportionem figura concava AEBD, ad triangulum FDG, erit ut



triangulum AEB ad parabolam convexam, sic FDG triangulum ad figuram mixtilineam AEBD : & permutando ut AEB triangulum ad triangulum FDG, sic parabola AEB ad figuram concavam : sed AEB triangulum duplum est trianguli FDG; igitur & parabola AEB dupla est figurae AEBD. Quod erat demonstrandum

PROPOSITIO CXXXVIII.

Idem aliter demonstrare.

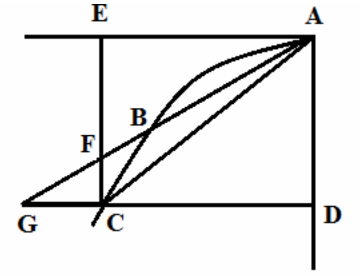
Demonstratio.

Inscribantur segmentis residuis tam parabolae convexae, quam concavae, triangula maxima AIE, EKB, NFO, PGQ. Quoniam triangulum AEB, ablatum ex parabola duplum est trianguli FDG, ablati ex figura ADBEA; & iterum triangula AIE, EKB ablata ex residuo parabolae dupla triangulorum NFO, PGQ, ablatorum ex residuo figurae ADBEA, insuper ostensum sit ablationem illam in proportione dupla, sive termino inutraque figura posse continuari, sive totam triangulorum maximorum seriem parabolae AEB inscriptorum, illi aequari & figuram mixtilineam AEBDA aequari toti triangularum maximorum seriei figurae illi inscriptorum, parabola AEB, dupla est figurae mixtilineae ADBEA. Quod erat demonstrandum.

PROPOSITIO CXXXIX.

Dato segmento parabolico triangulum aequale exhibere.

Constructio & demonstratio.



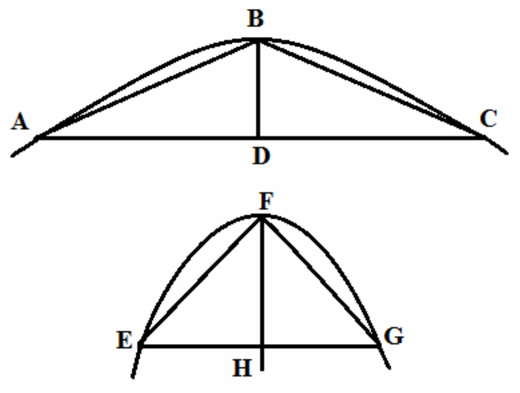
Sit ABC segmentum datum: ductaque diametro AD ponatur ordinatim CD, agaturque per A ipsi CD, aequidistans AE, occurrens erecta ex C diametro in E: tum EC divisa in F ut FC quarta pars sit EC, ducatur ex A per F, linea AG, occurrens CD in C. Dico GAC triangulum aequale esse dato segmento ABC. Quoniam AE, CG lineae aequidistant, ut CF ad FE, sic GC, tertia pars est EA hoc est CD. Quare & GAC triangulum tertia pars est trianguli

CAD : est autem ABC segmentum aequale tertia parti trianguli CAD; igitur & GAC triangulo est aequale : dato igitur segmento parabolico aequale triangulum exhibuimus; quod erat imperatum.

Corollarium.

Hinc patet triangulum GAD aequale esse parabolae ABCD. adeoque eadem praxi solui problema quo petitur datae parabolae, triangulum aequale exhiberi.

PROPOSITIO CXXL.



Parabolae terminatae eam inter se sortiuntur rationem quam triangula maxima illis inscripta.

Demonstratio.

Sint ABC, EFG parabolis terminatis triangula maxima inscripta ABC, EFG. Dico parabolas illam inter se habere rationem qua triangula maxima. Triangulum ABC est ad ABC, parabolam ut tria ad quatuor : trianguli, quoque EFG est ad parabolam EFG,
ut tria ad quatuor; igitur ut ABC triangulum ad parabolam ABC, sic EFG triangulum ad parabolam EFG, & permutando ut ABC triangulum ad triangulum EFG, sic ABC parabol ad parabolam EFG. Quod erat demonstrandum.

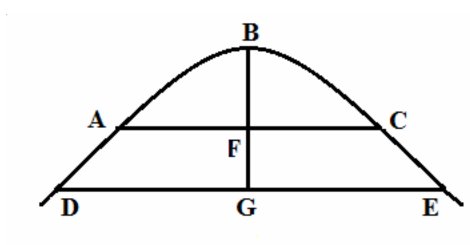
Corollarium.

Hinc si duae parabolae habeant eandem vel aequalem subtensam, erunt illae inter se ut altitudines ; & si altitudines fuerint aequales, erunt inter se ut bases.

PROPOSITIO CXXLI.

Parabolam ABC secent duae quaevis parallelae AC, DE.
Dico ABC parabolam ad DBE parabolam esse in triplicata ratione AC ad DE.

Demonstratio.



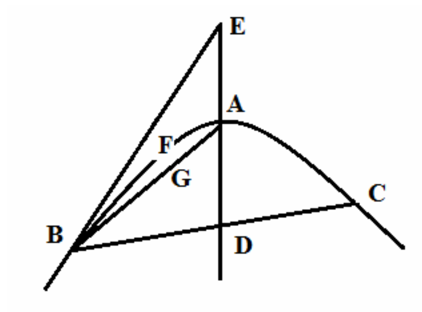
Ponatur diameter BF ad quam ordinatim positae sint AC, DE. Parabola ABC ad BE parabolam eam habet rationem, quam triangulum sub AC & BF ad triangulum sub DE & BG: sed ratio trianguli sub AC & BF, ad triangulum sub DE & BG, est triplicata rationis AC ad DE, quia composita ex ratione AC ad DF, & BF ad BG, hoc est ex duplicata ratione AC ad DE; igitur ABC parabola est ad parabolam DBE in triplicata ratione AC ad DE. Quod fuit demonstrandum.

PROPOSITIO CXXLII.

Parabolam ABC contingat in B linea EB, conveniens cum diametro quacunque AE in E, iunganturque AB.
Dico figuram concavam BFAEB, duplam esse convexae BFAGB.

Demonstratio.

Ponatur ex B, ordinatim AC ad diametrum AD. Quoniam BE est cotingens, erit AD, AE lineae aequales, adeoque ABD, ABE triangula aequalia: est autem ABD triangulum triplum segmenti BFAGB, igitur & triangulum ABE triplum est segmenti BFAGB; residua igitur figura concava BFAEB dupla est convexae BFAGB. Quod erat demonstrandum.

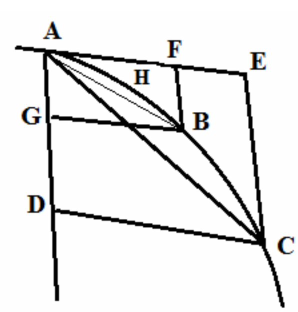


PROPOSITIO CXXLIII.

Sint ad ABC parabolae diametrum AD, ordinatim positae DC, GB: iunctisque AB, AC ponatur per A aequidistan ipsi ipsi DC, occurrens erectis ex B & C, diametris in F & E.
Dico esse ut ABF triangulum ad triangulum ACE sive ABG ad ACD triangulum, sic AHBFA figuram concavam ad figuram AHCEA.

Demonstratio.

Ut ABG triangulum ad triangulum ACD, sic AHB segmentum ad segmentum ABC, sed ut AHB ad ABC segmentum, sic AHBF figura ad figuram ABC, cum AHBF



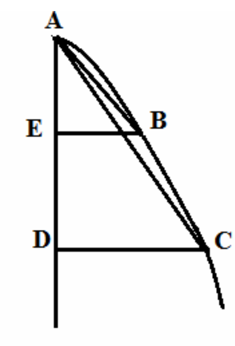
duplum sit segmenti AHB, & ABCE duplum ABC; igitur ut triangulum ABG ad ACD triangulum, sic AHBF figura ad figuram ABCE. Quod erat demonstrandum.

Corollarium.

Eadem posita figura sequitur esse ut GF parallelogrammum ad paralldogrammum DE, sic AGB parabolam ad parabolam ADC: item convexum AHBF ad convexum ABCE.

PROPOSITIO CXXLIV.

Parabolam ABC cuius diameter AD secent utcunque lineae AB, AC: ducanturque ordinatim BE, CD.

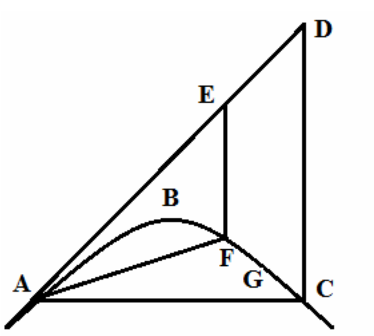


Dico spatium parabolicum EBCD, quadruplum esse spatii CABC lineis AB, AC & parabolica BC contenti.

Demonstratio.

Quoniam parabola DABC quadrupla est segmenti ABC & EAB parabola quadrupla segmenti AB, parabola DABC est ad segmentum ABC ut EAB parabola ad segmentum AB; igitur cum parabola DABC ad ABC, totum ad totum sit ut EAB ablatum ad ablatum AB, erit reliquum EBCD, ad reliquum ACBA, ut DABC totum ad totum ABC: quare EBCD figura, quadrupla est figurae lineis AC, AB & parabolica BC contentae. Quod erat demonstrandum.

PROPOSITIO CXXLV.



Contingat ABC parabolam linea quaecunque AD conveniens cum diametris quibusvis DC, FE in D & E, iunganturque AF, AC.

Dico concavum EDCGF duplum esse partis AFGC, lineis AF, AC contentae.

Demonstratio.

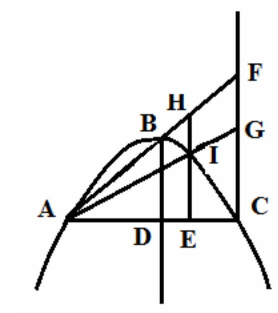
Concavum ABCDA duplum est parabolae CABC: & concavum ABFEA duplum est segmenti ABF; igitur residuum EDCGF duplum est residui AFGCA.

Quod erat demonstrandum.

PROPOSITIO CXXLVI.

Parabolam ABC subtendat linea AC, qua divisa in D & E, AD, AE, AC continuè proportionales sint, erigantur diametri DB, EI, CF. & per B & I, puncta ex A rectae ponantur AG, AF secantes CF diametrum in F & G.

Dico EIGC spatium ad quadrilaterum BI triplicatam habere rationem AC ad AE.



Demonstratio.

Quoniam AD, AE, AC ponuntur continuè proportionales, ut CA ad EA, sic CE ad ED, sed ut CE ad ED, sic CG est ad GF; igitur ut CA ad EA, sic CG ad GF: unde triangulum CAG ad GAF triangulum , est ut CE ad ED, id est ut CA ad EA, id est FA ad HA, id est FG ad Hl; est autem GAF triangulum ad triangulum HAI in duplicata ratione FG ad HI, igitur cum ratio trianguli CAG ad HAI componatur ex ratione triangulii CAG ad GAF, & ex GAF ad HAI, patet CAG triangulum esse ad triangulum HAI in triplicata ratione FG ad HI : quia vero ratio quadrilateri EG ad BI quadrilaterum, componitur ex ratione EG ad GH quadrilaterum , (id est ex ratione trianguli CAG ad triangulum GAF), & ex ratione GH ad IB, quadrilaterum , (id est: ex ratione trianguli GAF ad HAI triangulum, cum FA, HA, BA proportionalem sint) erit EG quadrilaterum ad quadrilaterum BI ut CAG triangulum ad triangulum HAI; igitur & EG quadrilaterum ad BI quadrilaterum rationem habet triplicatam AC ad AE. Quod erat demonstrandum.

PROPOSITIO CXXLVII.

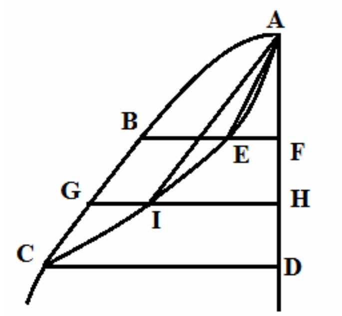
Esto ABC parabolae diameter AD positaque ad illam ordinatim CD, describatur per A & C, parabola AEC, quam in A contingat AD, ponanturque ordinatim quaevis FB, HG secantes AEC parabolam in E & I.

Dico figuram concavam AFEA, ad figuram concavam AHIA, duplicatam habere rationem parabolae BAF ad parabolam GAH.

Demonstratio.

Iungantur AE, AI. Figura mixtilinea AFEA ad figuram AHIA eam habet rationem quam

AEF triangulum ad triangulum AIH; ratio autem trianguli AEF ad triangulum AIH composita est ex ratione AF ad AH , hoc est duplicata rationis FB ad HG, & ex ratione AE ad IH, hoc est duplicata rationis AF ad A, id est quadruplicata rationis FB ad HG; igitur AFEA figura ad figuram AHIA sextuplicatam habet rationem lineae FB ad HG; sed BAF parabola ad GAH parabolam triplicatam habet rationem FB lineae ad lineam HG: igitur figura AFEA ad figuram AHIA, duplicatam habet rationem parabolae BAF ad parabolam GAH. Quod erat demonstrandum.



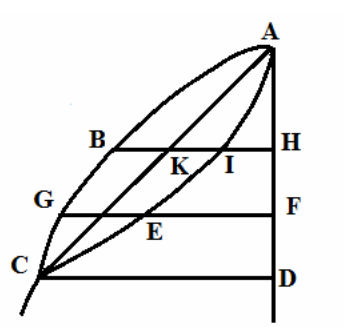
Corollarium.

Hinc patet concavum ASE ad concavum AHI, triplicatam habere rationem AF ad AH; nam AEF triangulum ad triangulum AIH rationem habet compositam est AF ad AH, & EFad IH, id est ex duplicata rationis AF ad AH.

PROPOSITIO CXXLVIII.

Iisdem positis sint AH, AF, AD proportionales, iunganturque AC.
Dico mistilineum IF ad ED mistilineum, rationem habere sextuplicatam, cuius ratio trapezii KF ad LD, trapezium est quadruplicata.

Demonstratio.



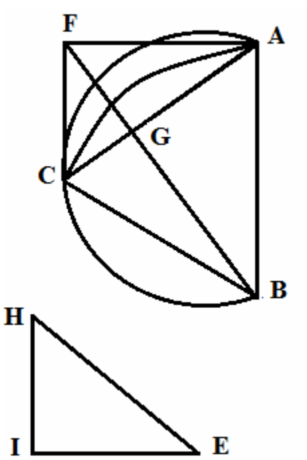
Quoniam AH, AF, AD lineae proportionales sunt, rectae quoque IH, EF, CD, adeoque figurae concavae AHIA,

AFEA, ADCA in continua sunt analogia. Igitur ut AHIA figura ad figuram AFEA, sic IF mistilineum est ad mistlineum ED; sed AHIA figurae ad figuram AFEA, sextuplicatam habet rationem BH ad GF, igitur & IF mistlineum ad mistlineum ED sextuplicatam habet rationem BH ad GF. Rursum, quia AH, AF, AD proportionales sunt, triangula quoque AKH, ALF, ACD in continua sunt analogia. Adeoque ut AKH triangulum est ad triangulum ALF, sic KF trapezium est ad trapezium LD: sed AKH triangulum ad triangulum ALF, duplicatam habet rationem lineae AH ad AF, hoc est quadruplicatam rationis HB ad FG:
igitur & KF trapezium ad trapezium LD quadruplicatam habet rationem lineae HB ad FG, cuius IF mistilineum ad mistilineum ED habet sexcuplicatam. Quod erat demonstrandum.

PROPOSITIO CXXLIX.

In parabola ACB fit AB pars axeos aequalis lateri recto ductaq; A F ad axem normali, ponatur quaeuis FC parallela AB, secans parabolam in C, & ducantur AC, BF.
Dico has duas sese orthogonaliter intersecare & CG, FG , GA , GS continue esse proportionales.

Demonstratio.



Cum BA aequalis sit lateri recto, erit rectangulum BAFC aequale quadrato FA. Igitur sunt tres in continua analogia FC, FA, AB: ergo cum anguli CFA, BAF aequales sint, similia sunt triangula CFA, FAB, unde angulus FAC aequalis angulo ABF; est autem angulus FAC una cum CAB, recto aequalis; igitur etiam angulus FBA una cum angulo CAB recto est aequalis, & consequenter angulus AGB rectus est. Ulterius cum tam angulus AFC quam AGF rectus est, tres CG,GF,GA proportionales sunt: quia vero anguli AGB, FAB recti sunt, lineae quoque FG, GA ,GB proportionales sunt; eandem igitur continuant rationem CG, FG, GA, GB. Quod erat demonstrandum.

PROPOSITIO CCL.

Inter duas datas, duas medias exhibere organice.

Constructio & demonstratio.

Sint duae datae HI, IK inter quas duas medias oporteat exhibere, constituantur hae ad angulos rectos & conficiant triangulum orthogonum HIK; deinde describatur parabola ACB cuius latus rectum sit AB pars axeos, super quo segmentum circuli constituatur capiens angulum ACB aequalem IHK, occurrens parabolam in

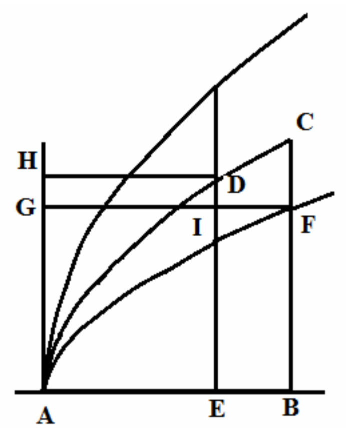
C & ducantur CF, FA ad angulos rectos, ita ut CF sit aequidistans axi & iungantur AC, BF. Dico factum quod requiritur, nam ostensum est angulos ad G rectos esse , estque angulus BCA seu BCH aequalis angulo K. Ergo BCG triangulum simile triangulo HIK; igitur cum FG, GA mediae sint inter CG, GB, etiam in HI, IK inventae erunt mediae.

PROPOSITIO CCLI.

Habeant duae parabolae ABC, AFB communem axem ; sitque AB linea lateri recto aequalis parabolae ABC, ductisque ordinatim BFC, ED, sit ADE parabola aequalis AFB. Dico AE, ED, medias esse inter FB, BA.

Demonstratio.

Quoniam AB ex hypothesi est latus rectum parabolae ACB, rectangulum BAE aequatur quadrato ED, igitur ut BA ad ED, sic ED ad AE. Deinde quia parabolae aequales sunt aequantur etiam rectangula AED, ABF. Ergo ut BA ad ED, sic reciproce AE ad BF: sed cum ostendi ut BA ad ED, sic ED esse ad AE, ergo ut ED ad AE, sic AE ad BF. liquet igitur quatuor rectas BA, ED, AE, BF esse in continua analogia & proindc inter BA, BF, medias esse ED, AE. Quod erat demonstrandum.



PROPOSITIO CCLII.

Inter duas datas, duas medias exhibere.

Constructio & demonstratio.

Datae sint AB, BF, quibus ad angulum rectum dispositis describe parabolam AC circa axem AB, cuius rectum latus aequale sit ipsi AB. occurrat deinde BF, parabola AC in C, & circa communem axem AB aliam describe parabolam per A & F. Demum ducatur ordinatîm DE, faciens segmenta parabolica ADE, AFB aequalia. Dico DE, EA esse medias inter AB, BF: demonstratio ex preaecedenti manifesta est.