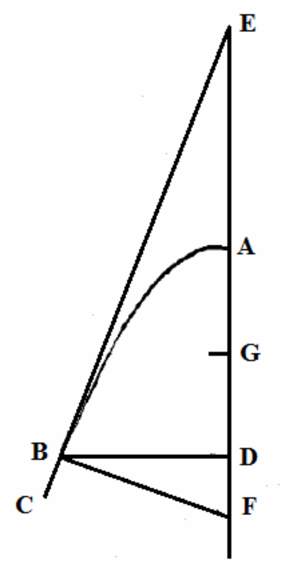
PARABOLA

PART THREE

Part 3 assigns the focus of the parabola geometrically, and the mutual intersection of parabolas.

PROPOSITION CXIV.



Some line BE shall be a tangent at B to the parabola ABC, the axis of which AD, crossing the axis at E : and with the ordinate BD put in place, BF shall be put normal to BE, crossing the axis at F.
I say the line DF to be equal to half the latus rectum which lies on the axis.

Demonstration.

AG shall be taken equal to the latus rectum ; since BD is put as the ordinate to the axis, the square BD is equal to the rectangle DAG: truly since the angle EBF is right, and the square BD is equal to the rectangle EDF: therefore the rectangles EDF, DAG are equal to each other, and as ED it to AD, thus AG is to DF: but ED is twice AD, since EB shall be the tangent; and therefore AG is the double of the right line DF: and therefore DF is equal to half the latus rectum. Q.e.d.

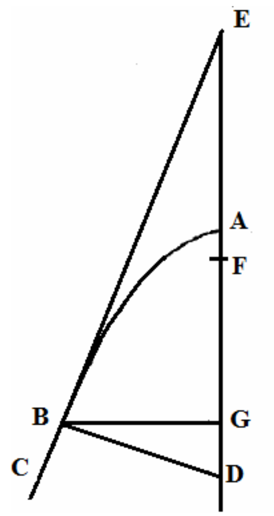
PROPOSITION CXV.

With the same figure remaining : AG shall be the latus rectum and half of that shall be equal to DF: moreover the ordinate DB may be put in place, and the tangent BE, and BF shall be joined.
I say the angle EBF to be right.

Demonstration.

Since EB is a tangent, and with the ordinate BD put in place, ED will be double the right line AD: and since AG is put to be double DF, the rectangle DAG will be equal to the rectangle EDF: but the rectangle DAG is equal to the square BD, and therefore the square DB is equal to the rectangle EDF, from which since DB shall be put normal to EF, it is apparent the angle EBF to be right. Q.e.d.

PROPOSITION CXVI.



The line BE shall be a tangent to the parabola ABC, of which AD is the axis, meeting the axis at E: and on putting the right line BD normal to EB, ED shall be bisected at F.
I say FA to be the fourth part of the latus rectum.

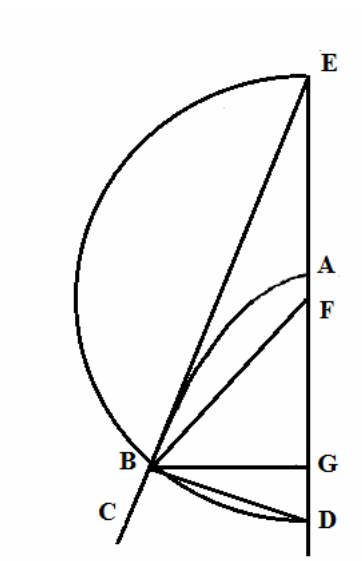
Demonstration.

BG shall be put as the ordinate to the axis ; therefore since ED shall be bisected at F, and EG at A (on account of the tangent EB) so that as ED to EF, thus EG to EA, and thus as the remainder AF to the remainder GD, so that the whole ED is to the half of that therefore as AF is to the half of GD: that is, as the fourth part of the latus rectum; since GD shall be half the latus rectum. Q.e.d.

PROPOSITION CXVII.

BE shall be some tangent to the parabola ABC, of which AD is the axis, and the angle BEA shall be made equal to the angle EBF.
I say the line FA to be equal to the fourth part of the latus rectum; and if FA shall be the fourth part of the latus rectum, I say the lines FB, FE and consequently the angles BEF, EBF to be equal to each other.

Demonstration.

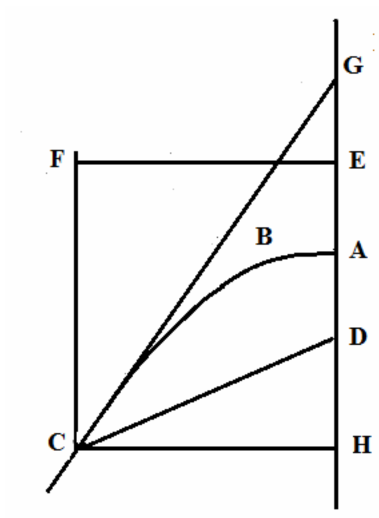


The ordinate line BG [of the latus rectum] shall be put in place : and the normal BD to the tangent EB. Because the angles EBF, BEF are equal from the hypothesis, also the lines FB, FE are equal, and in addition the angle EBD shall be right; if with centre F, a circle shall be described with the radius FE, that will pass through B and D [*elementa*]. Whereby EF, FD shall be equal lines, and thus FA is equal to the fourth part of the latus rectum [§.116 above] . Which was the first part.
Since again the angle RBD shall be put right, and with the ordinate BG applied to the axis AD; DG will be equal to half the latus rectum [§.114 above], and thus equal to AF doubled: whereby as EA to EG; thus AF to GD, and on adding so that as EF shall be to ED, as AF to GD: whereby ED is the double of EF, and the circle described with centre F and radius FE, will pass through B and D; and the lines FB, FE as well as the angles FBE, FEB will be made equal to each other. Q.e.d.

PROPOSITION CXVIII.

Let the fourth part of the axis AD of the parabola ABC be equal to the latus rectum: and with AE made equal to AD, the line DC may be drawn from D crossing the parabola at C; and from C the diameter CF shall be erected (which shall be normal to the axis AD), crossing EF at F.

I say the lines DC, FC to be equal to each other.



Demonstration.

CG shall be the tangent acting through C, meeting the axis at G; to which the ordinate CH is put in place. Since CG is the tangent, the right lines AG, AH are equal [§.117 above]; therefore with the equal lines AD, AE added, the totals GD, EH are equal ; but the right line DC is equal to GD, and therefore DC is equal to the line HE, that is, to CF. Q.e.d.

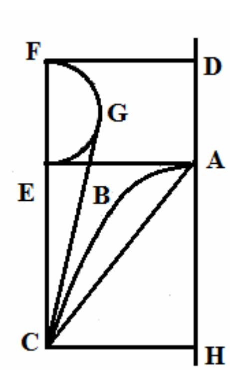
PROPOSITION CXIX.

AD shall be the latus rectum of the parabola ABC produced beyond the parabolic section, AE shall be the tangent at A to the parabola, and with the secant AC sent from A, the diameter CF shall be erected from C, intersecting the tangent AE at E, and DF at F, which shall be parallel to AE, so that with the semicircle FGE constructed on EF as diameter, the line CG shall be drawn from C, which shall be a tangent to the semicircle at G.

I say the lines AC, CG to be equal to each other.

Demonstration.

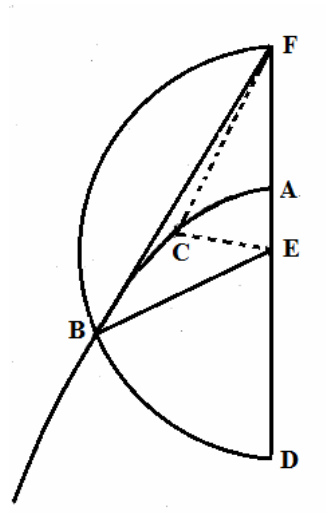
The ordinate CH shall be put in place from C; the square AC is equal to the squares AH, HC: but the square HC is equal to the rectangle HAD (since AD by the hypothesis is equal to the latus rectum) ; therefore the square AC is equal to the square AH together with the rectangle HAD, that is to the rectangle AHD; that is to the rectangle ECF; but with the rectangle ECF equal to the square CG, and therefore to the square AC ; and thus the lines CG, CA are equal to each other. Q.e.d.



PROPOSITION CXX.

Let AD be the axis of the parabola ABC : and the line AE equal to the fourth part of the latus rectum; a circle is described with centre E, and with some radius EF, intersecting the axis at F and D, and the parabola at B: and FB shall be joined.

I say that FB shall be a tangent to the parabola at the point B.



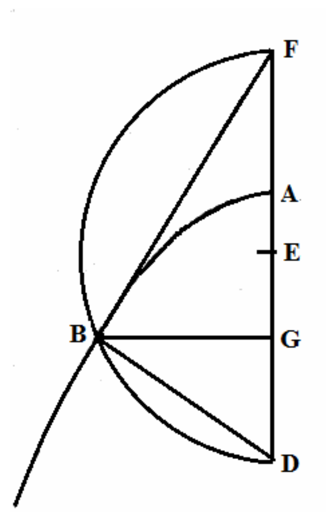
Demonstration.

If indeed it shall not be a tangent ; the tangent FC shall be drawn from F: that shall lie between B and A, or beyond B. Initially it shall fall between B and A, and BE, CE shall be joined. Because PC is the tangent line, and AE the fourth part of the latus rectum, the line CE is equal to the right line FE [§.117], that is, EB. Which is absurd; therefore the tangent EC cannot lie between A and B; similarly it can be shown FC cannot lie beyond B. Whereby FB alone is the tangent to the parabola. Q.e.d.

PROPOSITION CXXI.

AD shall be the axis of the parabola ABC, and AE the fourth part of the latus rectum : a circle is described with centre E and with some radius EF, crossing with the axis at F and D, and moreover with the parabola at B, and from B the ordinate BG is drawn to the axis.

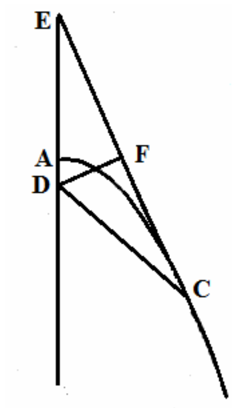
I say the line DG, to be half of the latus rectum.



Demonstration.

The points FB, BD shall be joined; therefore the angle FBD is a right angle in the semicircle; but by the preceding, FB is a tangent, therefore DG is half the latus rectum [§.117]. Q.e.d.

PROPOSITION CXXII.



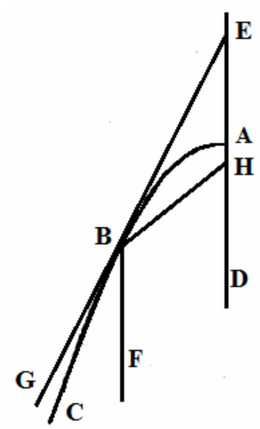
Some line CE shall be a tangent to the parabola ABC at C, meeting the axis AD of the parabola at E,. And with CE bisected at F, the normal FD to the tangent EC crossing the axis at D.

I say the line AD, to be the fourth part of the latus rectum.

Demonstration.

The points C D shall be joined. Because FD is normal to the tangent EC, the angles EFD, CFD are equal; moreover from the hypothesis the right lines EF, FC are equal, and FD common, therefore the triangles EDF, CDF, are equal and the sides ED, CD are equal: and whereby the angles CED, ECD are equal; and the line AD is equal to the fourth part of the latus rectum [§.114]. Q.e.d.

PROPOSITION CXXIII.



Some right line BE crossing the axis at E shall be a tangent to the parabola ABC, of which AD shall be the axis, and with the line BF dropped from B parallel to the axis, the angle FBG shall become equal to the angle EBH.

I say the line AH to be equal to the fourth part of the latus rectum : and if AH were the fourth part of the latus rectum, and BF parallel to the axis, I say the angles EBH, FBG to be equal.

Demonstration.

Because FB shall be parallel to ED, the angle AEB is equal to the angle FBG, that is to the angle HBE: from which the line e AH is equal to the fourth part of the latus rectum [§.114]. Which was the first part.

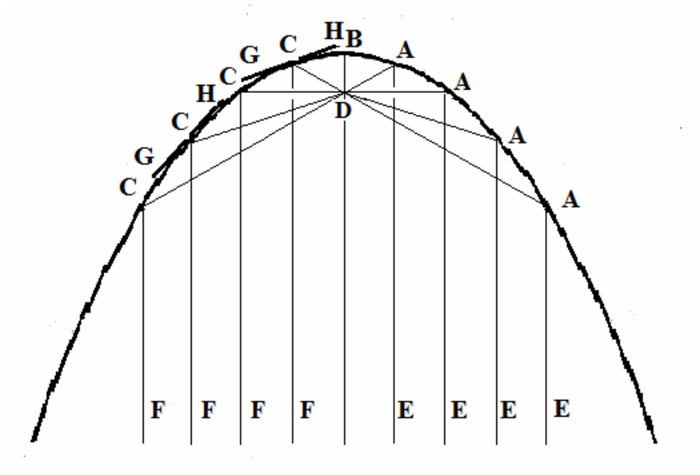
Again since the line AH shall be the fourth part of the latus rectum, the angle HBE is equal to the angle AEB, that is FBG, since FB, ED are parallel; moreover the point H is called the focus of the parabola. Q.e.d.

PROPOSITION CXXIV.

To show the focus of a given parabola.

Construction & Demonstration.

BD shall be the axis of the given parabola ABC, to which some number of parallel lines AE, CF may be put in place: but the tangent GH acts through C, and with the right line CD drawn from C, the angle HCD shall become equal to the angle FCG: by the previous proposition it shall be agreed to call the point D the focus of the parabola, and thus BD to be the fourth part of the latus rectum, and because the line CF is any line parallel to the axis, if for the remaining lines also the angle of incidence shall become equal to the angle of reflection, all the reflected rays will come together at the point D which shall be designated as the fourth part of the latus rectum; therefore we have shown the focus of the given parabola.



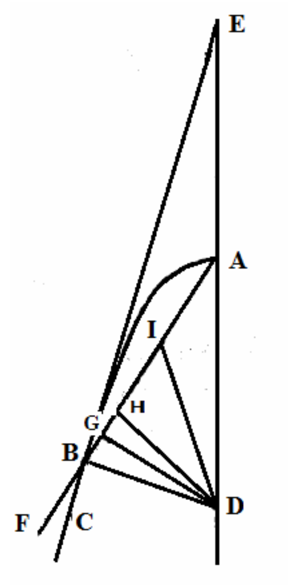
Again we consider the focal point D, because combustion may happen at that point, and the reflected rays gathered together there, may produce an intense heat; since truly the point D (which is determined from the fourth part of the latus rectum from the vertex assumed initially) allows no width: so that the parabolic form shall become the most suitable of all, for the rays from the sun being received and reflected; and produces the effect much more strongly than the remaining figures.

But so that in the proposition we may assume all the rays CF, AE to be parallel to the axis, that likewise we have made in the consideration of the same tendencies on being concerned with Optics, Sundials, and by falling straight down like a priest's gown, and thus far so that the foundations and beginnings concerning everything has a place (if a few modern writers may be excepted), not because on account of the rays of the sun being parallel, but because of the immense distance between us and the sun, the rays shall strike in the mirror as if indeed they were parallel, thus so that in turn no divergence shall be perceived: and from a plane mirror if two lines may be drawn towards the sun, at however small an angle you incline the angles in turn, these will agree much as before, by which they shall be able to arrive at the solar disc: and if in turn with the physical angle taken from the centre of the sun, at however small an angle you may draw the two rays, their reflections will not fall on the surface of the mirror, but on each side at an great distance from the mirror, and the reasoning for this being, with the immense distance of the sun from the earth: the sun's rays therefore strike the mirror truly as if they were assumed to be parallel. With which in place it will be able to be shown easily, the focus in the circle and in all the conic sections (as well as in the parabola) to be allowed a certain place, and thus the parabolic figure to be the most outstanding for combustion: but another time and place for the consideration of these others, if God will give me life.

PROPOSITION CXXV.

Some right line BE shall be a tangent to the parabola ABC at B, the axis of which it shall intersect at E: and with the line BD drawn normal to the tangent from B, which shall cross the axis at D: if the line AD were greater than half the latus rectum.

I say the right line BD to be smaller than AD.



Demonstration.

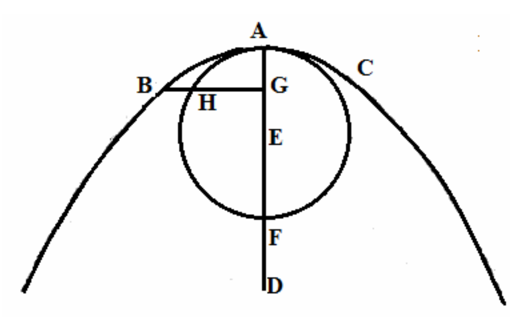
The line ABF shall be drawn from A through B : and from D the right line DG normal to AF. Since the angle EBD is put to be right, the angle ABD is smaller than right, and thus the angle FBD is obtuse ; whereby DG lies between A and B and therefore BG shall become equal to GH, and HD may be joined : therefore since HG, GD shall be equal to the right lines BG, GD, and with a right angle contained by them; the line DH is equal to BD, and the angle GHD smaller than a right angle. Again since AI, ID shall be equal to the lines HI, ID, the triangles AID, HID are equal [*i.e.* in area] ; whereby AD, subtending the obtuse angle AID, is greater than the right line HD, that is, greater than BD. Q.e.d.

PROPOSITION CXXVI.

AD shall be the axis of the parabola ABC : and with AE taken which shall not be greater than half the latus rectum, the circle AHF shall be described with centre E and with the radius AE.

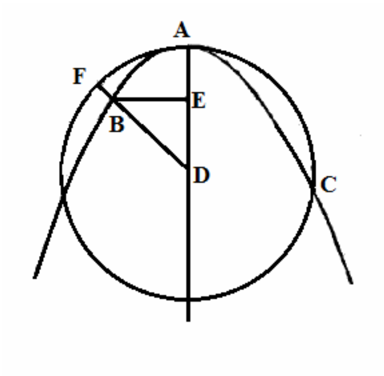
I say the circle AHF to be a tangent within the parabola at A.

Demonstration.



The line AD shall be made equal to the latus rectum, and some ordinate GB shall be drawn intersecting the circle at H. Because AE is supposed not to be greater than half the latus rectum, the point F will lie either above D, or at D itself, and thus the rectangle AGF is smaller than the rectangle GAD itself ; and therefore the square HG is smaller than the square BG, so that H will lie within the parabola. It may be shown in the same manner that all the remaining points besides A, lie within the parabola: therefore the circle AHF touches the parabola ABC at the point A within the parabola. Q.e.d.

PROPOSITIO CXXVII.



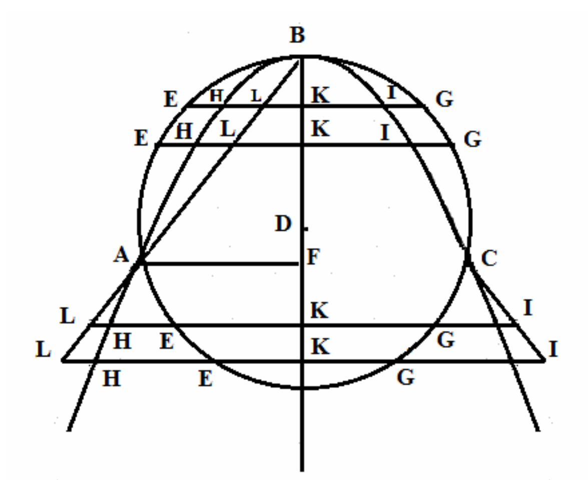
The axis AD shall be greater than half the latus rectum of the parabola ABC: and the circle AFG shall be described with centre D and radius AD.
I say that circle to intersect the parabola.

Demonstration.

The line AE shall be made equal to half the latus rectum and the ordinate EB shall be put in place through E : and with the line DBF drawn from D through B intersecting the circle at F and the parabola at B. Because AD is greater than half the latus rectum, the right line DB is smaller than AD, that is DF. Therefore the point F falls outside the parabola. Whereby the circle AFG shall intersect the parabola. Q.e.d.

PROPOSITION CXXVIII

Let the axis BD be greater than half the latus rectum of the parabola ABC ; and with the centre D, the circle AEB shall be described with the radius DB, intersecting the parabola at A; and with the ordinate line AF drawn from A, some lines EG are drawn parallel to AF, crossing the circle at E and G, the parabola at H and I, and the axis at K.
I say the rectangle EHG to be to the rectangle EHG, as the rectangle FKB is to the rectangle FKB.



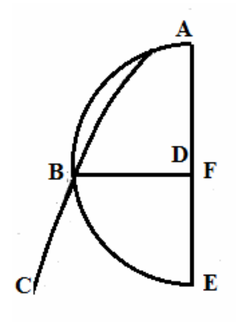
Demonstration.

AB joined shall cross the line EG at L. Because the line EG in the circle AEB, shall intersect the diameter BD at right angles at K, the right line EG is bisected at K, and also HI in the parabola ABC, also is bisected at K ; therefore the right lines EH, IG are equal to each other. Again as the rectangle ALB is to the rectangle ALB, thus the rectangle ELG is to the rectangle ELG, likewise the rectangle HLI is to the rectangle HLI: but the rectangle ELG is equal to the rectangles HLI, EHG, and therefore the rectangle EHG is to the rectangle EHG, as the rectangle ALB is to the rectangle ALB; that is as the rectangle FKB to the rectangle FKB. Q.e.d.

PROPOSITION CXXIX.

Let the axis AD of the parabola ABC be greater than half the latus rectum : and the circle ABE, with centre D and radius AD, shall be described intersecting the parabola at B and the axis at E; and the ordinate line BF shall be drawn from B to the axis.

I say the FE to be equal to the latus rectum.



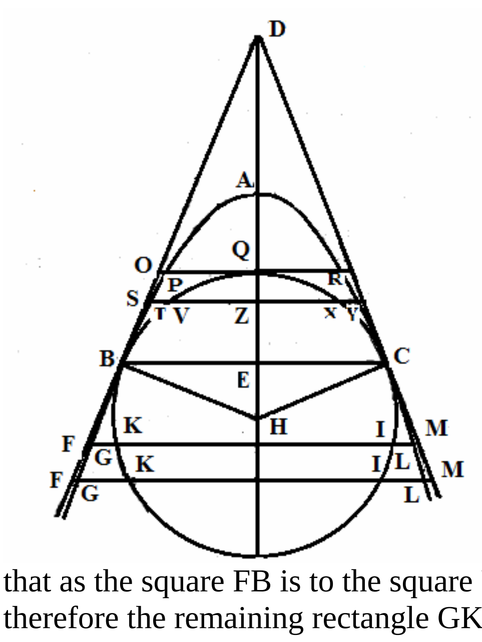
Demonstration.

Since the ordinate line BF has been drawn for the axis, the square FB is equal to the rectangle on FA with the latus rectum: also the square FB is equal to the rectangle AFE : therefore the rectangle AFE is equal to that which shall be upon AF and the latus rectum; whereby the line FE is equal to the latus rectum. Q.e.d.

PROPOSITIO CXXX.

The line BD shall be a tangent at B to the parabola ABC, of which the axis is AE, intersecting the axis at D, moreover the line BH may be put in place at B, normal to the tangent, crossing the axis at H: and the circle BQC shall be described with centre H and radius HB: and some number of normals IK to the axis may be put in place, crossing the parabola at GL, the tangent BD at FF: and by intercepting the circle within the parabola at I and K.

I say the rectangle GKI to be to the rectangle GKI, as the square FB to the square FB



Demonstration.

BC shall be put parallel to FI, crossing the parabola at C; and DC shall be joined, crossing FL at M. Because the ordinate BC to the axis DE is put in place ; the right line BC is bisected at E, and the lines BE, EC are equal, so that DC [§.117] will be a tangent to the parabola at C, truly since B is a point on the perimeter of the circle, C also is a point on the perimeter of the circle, and because the angle DBH is right, also the angle DCH is right, and DC to be a tangent to the circle ; therefore as the square FB to the square FB, thus the rectangle FKM to the rectangle FKM: but since FB, CM also are tangents to the parabola, so that as the square FB is to the square FB, thus the rectangle GFL is to the rectangle GFL, therefore the remaining rectangle GKI is to the rectangle GKI, as the square FB is to the square FB. Q.e.d.

PROPOSITION CXXXI.

With the same figure remaining, if the line AH shall be greater than half the latus rectum. I say the circle BQC inside the parabola to be in contact at two points.

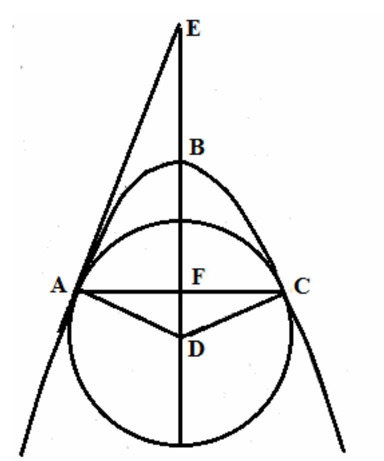
Demonstration.

The line OPR shall be drawn through Q parallel to BC, and some other STVXY parallel to the same; because AH is greater than half the latus rectum, the line AH is greater than the line BH; and the circle BQC falls below A; thence so that as the square OB is to the square SB, thus the rectangle POR is to the rectangle TSX: but also as the square OB to the square SB, thus the square OQ to the rectangle VSX, therefore as the rectangle POR to the rectangle TSY, thus the square OQ to the rectangle VSX, and on interchanging, so that as the square OQ to the rectangle POR, thus the rectangle VSX to the rectangle TSY: but the square OQ is greater than the rectangle POR, and therefore the rectangle VSX is greater than the rectangle TSY; further, the rectangle TSY together with the square TZ, is equal to the square SZ; and also the square SZ is equal to the rectangle VSZ together with the square VZ, therefore the square VZ together with the rectangle VSX to be greater than the rectangle TSY; therefore the square VZ is smaller than the square TZ: because the point V lies within the parabola: similarly it may be shown that all the remaining points of the perimeter of the circle BQC to lie within the parabola besides B and C: therefore the circle BQC lies within the parabola, with the two points B and C being tangential.

PROPOSITION CXXXII.

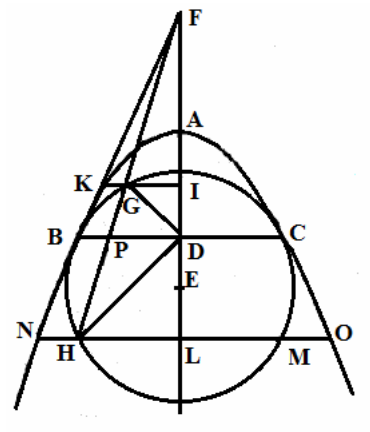
From a given point on the axis to describe a circle which shall lie within the parabola with two tangential points.

Construction & Demonstration.



BD shall be the axis of the parabola ABC, and at the point D within which it will be required to describe a circle with the centre D, which will touch the parabola at two points : moreover it will be required for the line BD to be greater than half the latus rectum, DF shall become equal to half the latus rectum, and the right line FAC shall be drawn, the ordinate to the axis : and the points D, A shall be joined: then the circle shall be described with centre D, and with the radius DA ; I say the circle to be tangential to the parabola at the two points within the parabola ; indeed the tangent line AE shall act through A meeting the axis at E. Because the line AE shall be a tangent at A, and FD is equal to half the latus rectum, the angle EAD is right [§.115]; but from the hypothesis BD is greater than half the latus rectum; therefore by the preceding, the circle with centre D, with the radius DA will be a tangent to the parabola at the two interior points : therefore from a given point D on the axis we will describe the circle, etc.... Q.e.d.

PROPOSITION CXXXIII.



The circle HBG shall be tangential to the parabola ABC, of which the axis shall be AD, at the two interior points B and C, and the tangent to the parabola at B, which shall cross the axis at F, shall be drawn from F, the line FH, cutting the circle at G and H, then through G and H, the normals KGI, NHL may be drawn, cutting the parabolas at K and N, truly the axis at I and L.
I say as HL to GI, thus NL to KI.

Demonstration.

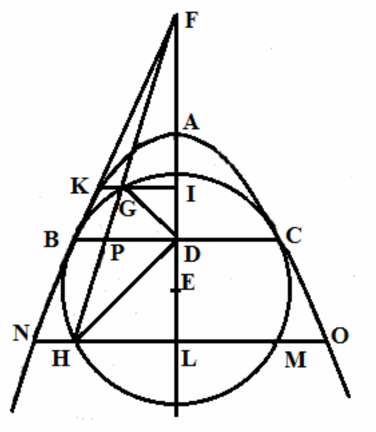
Because the circle BGC is tangential to the parabola at B and C, and the line FB drawn through B shall be a tangent to the parabola at B, the same line BF is also a tangent to the circle at B; therefore as HF is to FG, thus HP is to PG [§30. *The Circle*]: and as LF to FI, thus LD to DI; but as LF is to FI, thus LH is to IG; therefore LD is to DI, as LH to IG; truly since LF has been divided in the extreme and mean ratio of the proportionals at D and I, and FD bisected at A: the proportionals are AI, AD, AL; from which LA to AI, has the twofold ratio of LA to DA, that is LD to DI; but also the ratio LA to AI, to be in the twofold ratio of LN to KI, therefore as NL is to KI, thus LD is to DI, *i.e.* LH to GI. Q.e.d.

Proposition CXXXIV.

With the same in place:
I say the right lines KI, GD; likewise NL, HD to be equal to each other.

Demonstration.

It has been shown the right lines AI, AD, AL to be in continued proportion, from which also IK, BD, LN will be proportionals; but also GD, BD, HD are continued proportions; therefore the lines GD, KI , and NL, DH are equal to each other; for the rectangle GDH is equal to the rectangle KINL, and in addition it is shown GE to DH to have the same ratio as ID to DL, that is KI ad NI. Q.e.d.



Proposition CXXXV.

With the same in place:
I say the square DL to be equal to the rectangle NHO.

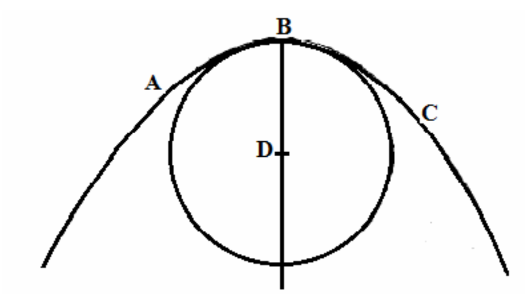
Demonstration.

The square HD is equal to the squares HL, LD: but [from above] the square HD is equal to the square NL, therefore the square NL is equal to the squares HL, LD; but also the square NL is equal to the square HL and the rectangle NHO, therefore with the common square HL removed, the rectangle NHO will be equal to the square DL. Q.e.d.

PROPOSITION CXXXVI.

To assign the point on the axis of a given parabola for which the circle described in side the parabola shall be the maximum of these, which shall be tangential to the parabola.

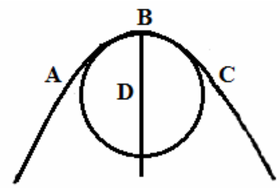
Construction & Demonstration.



BD shall be the axis of the parabola ABC, it is required to assign on that the point D, so that the centre of the circle described from that point shall be the greatest of these which touch the parabola at only one point, the line DB shall be made equal to half the latus rectum: and with the centre D, the circle shall be described with the radius BD ; I say that to be the proportion to satisfy the proposition since DB is put equal to half the latus rectum, the circle described with the radius DB, is tangential to the parabola at B, which truly will be the maximum of the tangential circles; because from that it will be apparent all the circles which have a centre beyond D, will intersect the parabola ; hence these are the smaller circles, which truly have a centre between D and B, and which always have a radius smaller than DB : therefore the circle described by the radius DB, is the greatest of the tangential circles; therefore we have shown, etc. Q.e.d.

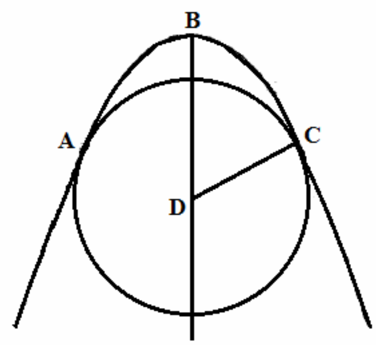
PROPOSITION CXXXVII.

From a given point on the axis of the parabola, to draw the line to the periphery, the shortest of all these which can be drawn from the same point.



Construction & Demonstration.

BD shall be the axis of the parabola ABC, and D some given point on that : it is required to draw from D the shortest line of these, which can be drawn from the same point D to the perimeter of the parabola.



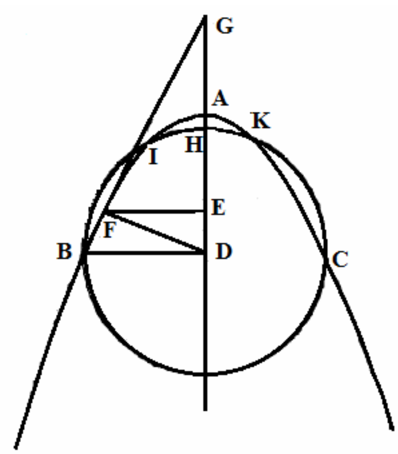
In the first case, the right line BD shall not be greater than half the latus rectum. I say BD to be the line sought : for the circle is described with centre D and with the radius DB, from the preceding this will be touching the interior of the parabola at the point B only : therefore all the remaining lines from D drawn to the periphery are greater than the line BD.

In the second case, BD shall be greater than half the latus rectum: the circle with centre D shall be described touching the parabola ABC at the two points A, C, and with the right line DC drawn : that will be the minimum of these (as is evident) which can be drawn from D to the periphery of the parabola: therefore from the given point D, we shall have drawn the line, &c. Q.e.d.

PROPOSITION CXXXVIII.

AD shall be the axis of the parabola ABC, greater than the latus rectum ; and from D the ordinate line DB shall be drawn, and the circle BHC shall be described with centre D and with radius DB.

I say that circle to intersect the parabola at four points.



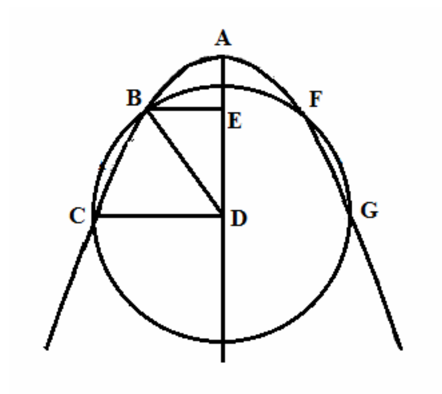
Demonstration.

ED shall be taken equal to half the latus rectum ; and with the ordinate EF put in place, the tangent FG shall be acting through F, meeting the axis at G. Because FG is the tangent, and ED equal to half the latus rectum, the angle DFG is right [§.115], truly since the line AD is greater than half the latus rectum, FD is the minimum of these lines [§.137], which can be drawn from D to the periphery: and thus BD shall be the smallest ordinate in place; therefore the circle described with centre D and with the radius DB shall fall beyond F, and along some other part beyond the parabola. Again since DB shall be smaller than the right line AD, (since DA shall be greater than the latus rectum,) the point A shall lie beyond the circle BHC, and the following part of that within the parabola: And whereby it will intersect the parabola at some point other than A. Similarly it may be shown the circle BHC, to cross the other part of the parabola towards AC at some point other than C; therefore the circle with centre D, described with the radius DB, shall cut the parabola at four points. Q.e.d.

PROPOSITION CXXXIX.

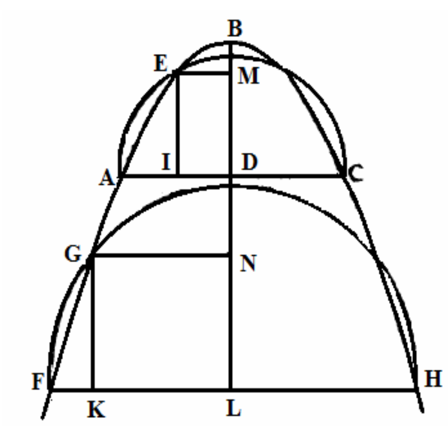
AD shall be the axis of the parabola ABC, greater than the latus rectum; and with DC put as the ordinate for that ; the circle CBF shall be described with centre D and radius DC; that will cross the circle at the four points C, B, F,G, and from B, the point of intersection, the ordinate BE to the axis shall be drawn:

I say the line ED shall be equal to the latus rectum.



Demonstration.

The points BD shall be joined. Because BD is equal to DC, and the ordinates EB, CD, are put in place normal to the axis, the line ED is equal to the latus rectum. Q.e.d. [Converse of §.14.]



PROPOSITION CXL.

If the semicircles AEC, FGH will have cut the parabola ABC, the axis of which is BD, in just as many ways so that the individual semicircles of which will cross the parabola at four individual points; moreover, the intersections of the lines EI, GK dropped from E and G, will be normal to the lines AC, FH which are the ordinates put to the axis.

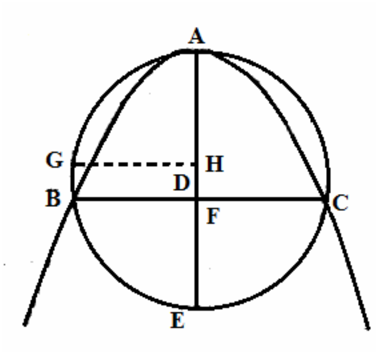
I say the lines EI, GK to be equal.

Demonstration.

Indeed the ordinates EM, GN shall be drawn : from the preceding both MD as well as NL to be equal to the latus rectum of the axis ; and therefore are equal to each other. Q.e.d.

PROPOSITION CXLI.

The axis AD of the parabola ABC shall be greater than half the latus rectum AD : and with the centre D and radius DA, the circle ABC shall be described, crossing the axis at E: it is required to show the points of intersection B and C.

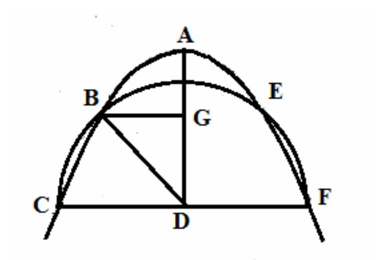


Construction & demonstration.

The line EF shall be assumed equal to the latus rectum, and the right line FBC shall act through F as ordinate to the axis: I say the circle ABC to cross the parabola at B and C, with FC put in place for the ordinate to the axis ; the square of this is put equal to the rectangle AFE, since FE is assumed equal to the latus rectum: but the rectangle AFE in the circle ABE, also is equal to the square FB: therefore the point B pertains both to the parabola ABC, and to the circle ABE, the same discussion shall be applied to the point C. Therefore we have set out what was ordered.

PROPOSITION CXLII.

Let the axis AD of the parabola ABC be greater than the latus rectum: and with the ordinate line CDF acting through D, the circle CBE shall be described with centre D and with the radius CD, the circle shall be described, it will be required to show that will meet the parabola at four points.



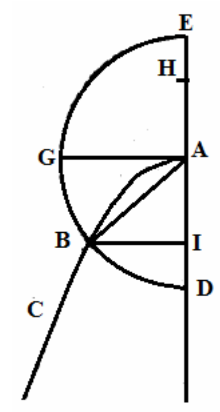
Construction & demonstration

DG shall be assumed equal to the latus rectum, and the ordinate shall be put equal to GB: I say B to be one point of the intersection, BD shall be joined. Because BG, CD may be put to be the ordinates to the axis, and GD is equal to the latus rectum, the line CD is equal to BD : therefore the circle described with centre D, radius DC, will pass through B; it is shown in the same way, the same circle to pass through E: so that indeed it shall evident it shall pass through C and F; therefore we have shown the four points of the intersection. Q.e.d.

PROPOSITION CXLIII.

A shall be the apex of the parabola ABC, and with centre A, the circle EGB shall be described with some radius AE ; it is required to show the intersection of the point B with the parabola.

Construction & Demonstration.

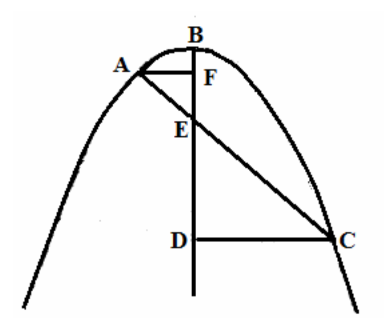


With the axis AD in place, AH shall be taken equal to the latus rectum, and with the tangent AG through A which shall intersect the circle at G, the square AG shall become equal to the rectangle HIA; and the ordinate IB shall be drawn through the point I: I say the point B, to be that which is required; AB shall be joined. Since the applied line IB is the ordinate for the axis, and the line AH is equal to the latus rectum, the square AB is equal to the rectangle HIA, but from the construction the rectangle HIA also is equal to the square AG ; therefore the squares AG, AB are equal. Therefore the circle with centre A and with the radius AG described, passes through B; therefore we have shown the point of intersection B. Q.e.d.

PROPOSITION CXLIV.

BD shall be the diameter of the parabola ABC which some right line AE shall cut, meeting the parabola at A: it is required to find C, the other point of intersection.

Construction & Demonstration.

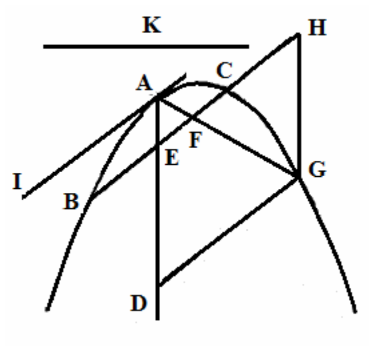


From A, the right line AF shall be drawn ordinate to the diameter ; and BF,BE, BD shall become proportionals: and the ordinate DC shall be drawn through D, crossing the line AE at the point C: therefore since BF,BE, BD are proportionals ; and AF, DC the ordinates put in place, the point C pertains to the parabola [*Cor*. §.23]: also C is a point on the right line AE: therefore C is the common point of intersection of the line AE with the parabola, therefore we have shown, etc. Q.e.d.

PROPOSITION CXLV.

AD shall be the diameter of the parabola ABC : and BC the ordinate put in place for that, moreover some line AF shall be drawn from A, cutting the line BC at F: which produced will cross the parabola at some point G, it will be required to show that point.

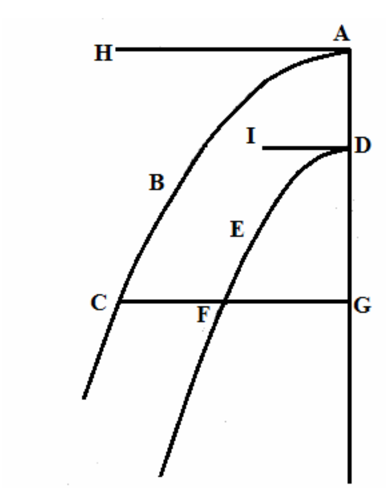
Construction & Demonstration.



The diagram is produced: and the ordinate GD put in place, GH is erected parallel to the diameter AD, crossing BC at H. Moreover AI shall be the latus rectum. Since GH shall be parallel to the diameter AD, the rectangle FEH, that is the rectangle FEGD, is equal to the square CE, that is, BE. Also the square BE is equal to the rectangle IAE; therefore the rectangles EAI, FEGD are equal , whereby so that as FE is to AE, thus AI is to GD: and indeed GD shall become equal to the right line K: therefore K will be the given line. Again since the ordinate GD put in place shall be to the diameter AD, as AI to GD, that is as K, thus K is to AD : and therefore the ordinate line DG shall be drawn from D to become that which was sought.

PROPOSITION CXLVI.

The parabolas ABC, DEF shall be constituted from the same AG with the different apices A and D ; and indeed the upper parabola ABC shall have a greater latus rectum AH than the latus rectum ID of the lower parabola DEF.



I say these sections produced indefinitely never meet.

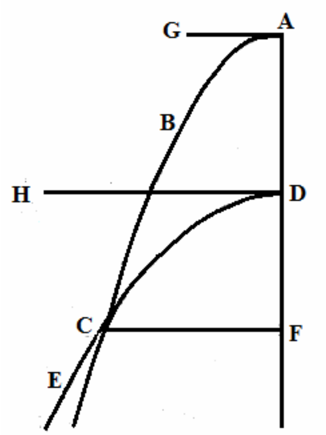
Demonstration.

Some line CG shall be put in place for the ordinate of the axis AG, crossing the parabolas ABC at C and DEF at F. Therefore the square FG is equal to the rectangle IDG, and the square CG is equal to the rectangle HAG: but the rectangle IDG is smaller than the rectangle HAG, (since the latus rectum ID put in place itself to be smaller than HA and DG itself smaller than AG), and therefore the square FG is smaller than the square CG: and the point F falls between C and G, and since the same is shown concerning any point of the parabola DEF, it is evident these sections produced indefinitely never come together. Q.e.d.

PROPOSITION CXLVII.

The two parabolas ABC, DCE shall be set up on the same axis, and the latus rectum of the parabola ABC shall be smaller than the latus rectum of the parabola DCE, nor shall the vertex A be common.
I say these parabolas meet, and moreover it will be required to assign the point of concurrence.

Construction & Demonstration.

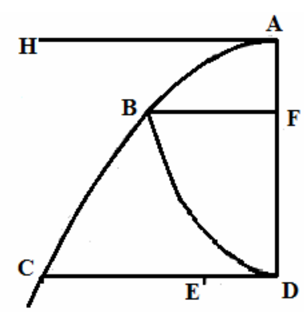


AG shall be the latus rectum of the parabola ABC, and DH the latus rectum of the parabola DCE, a certain line DF may be added to the line AD, so that as AF shall be to FD, just as HD is to AG; and from F the ordinate FC shall be put in place crossing the parabola DCE at C: I say that to be the point of intersection.
Since thus there is HD to AG, as AF to FD , the rectangle HDF shall be equal to the rectangle GAF; but the rectangle HDF is equal to the square FC; and therefore the rectangle GAF also is equal to the square FC: from which the point C is common to the parabolas ABC, DCE. Therefore we have shown the point of concurrence.

PROPOSITION CXLVIII.

The parabolas ABC, DBF shall be had on the common axis AD, and they shall have the opposite vertices A, D.
Moreover, it shall be required to show the common points of intersection of these.

Construction & Demonstration.

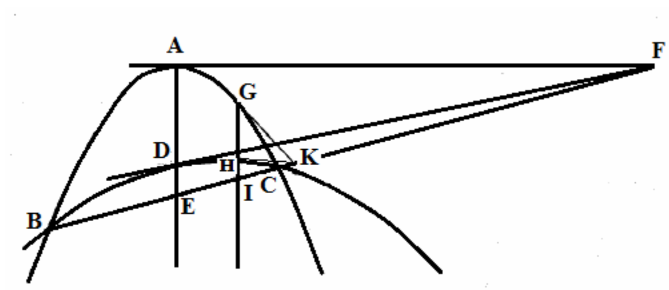


AH shall be the latus rectum of the parabola ABC, and ED the latus rectum of the parabola DB ; and AD shall be cut at F, so that the rectangle HAF shall be equal to the rectangle FDE: and FB shall be the ordinate through F, crossing the parabola ABC at B. I say B to be the point of the intersection, since indeed FB drawn in the parabola ABC shall be the ordinate to the axis AD, shall be equal to the rectangle FAH. But the rectangle FAH is put equal to the rectangle FDE; therefore the square FB also is equal to the rectangle FDE, whereby FB is the applied ordinate for the axis FD, in the parabola DB : and thus so that B is a point common point of each parabola. Therefore we have shown, &c. Q.e.d.

PROPOSITION CXLIX.

ABC, BDC shall be two unequal parabolas, having the common right line AE, which indeed shall become the axis of the parabola ABC, truly a diameter of the parabola BDC; and the parabolas shall intersect each other at the points B & C, it will be required to show these points.

Construction and Demonstration.



What is required shall be done: with the tangent AF acting through A, the line through B and C shall be drawn, that will meet AF at some point F: for if it shall not meet, it shall be parallel to AF : therefore CB will be the ordinate put in place for the axis AE in the parabola ABC; and because it is bisected at E; in the parabola BDC also the ordinate will be drawn, and for that to be at right angles to the diameter DE. Therefore DE would be the axis of the parabola BDC : which is contrary to the hypothesis.

Therefore CB shall be put to meet AF at some point F, from F the line FD shall be the tangent to the parabola BDC at some point D. Because the line AF is a tangent to the parabola ABC, the lines FB, FE, FC are proportionals [§.81], also the proportionals are the same in the parabola BDC; therefore DF is the tangent at the point D, in which DE is a diameter.

Again some other diameter GH may be drawn crossing the parabolas at G and H, and the line BC at I, and with the tangent GK acting through G, which since it shall meet BF at K, from K the line KH shall be drawn, tangent to the parabola BDC at some point H. Therefore since the line GK is a tangent to the parabola ABC, the lines KB, KI, KC are in continued proportion : and since the same are proportional in the parabola BDC, therefore the point H of the tangent HK, on the diameter HI therefore shall be combined with the tangents acting through the given points A and D, which shall meet at some point F; and with some diameter drawn GH; the tangents are acting through G and H, which also meet at K: therefore the points F, K, E, I are given: whereby the points B, C are given: which we have shown through resolution to be on the line FK ; therefore we have shown the intersections of the two parabolas CBC, BDC .Q.e.d.

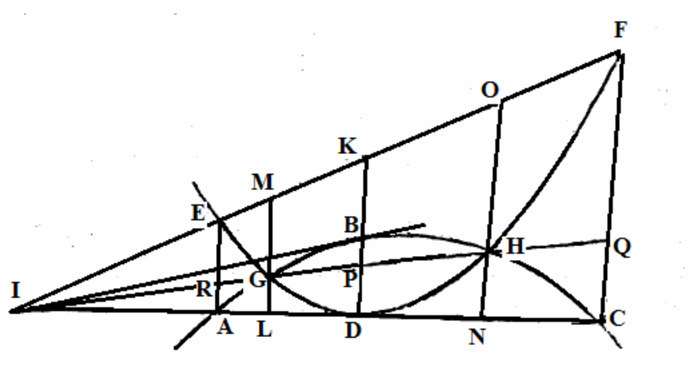
PROPOSITION CL.

The two parabolas ABC, EDF shall intersect each other, placed inversely to each other, having parallel axes : it will be required to show the points of intersection G and H.

Construction and Demonstration.

DB shall be the axis of the parabola EDF, and what is asked shall be done: and with the line AC acting through D which shall be a tangent to the parabola EDF at D; the right line GH acting through the points of intersection G and H, which cross over the line AC at some point L: if indeed it shall not meet this line AC, the line shall be parallel to AC; therefore in this case, for the right line GH in the parabola EDF, the ordinate put in place is for the axis DB, and thus shall cut the diameter BD in the parabola ABC at right angles; and since GH shall be bisected by the line BD, BD is the right axis of the parabola ABC. Which is contrary to the hypothesis.

Therefore HG is not parallel to AC, but produced with that will meet at some point I, therefore with the right line IB drawn from that point, being a tangent to the parabola ABC at some point B, since the right line ID is a tangent, the lines IG, IP, IH shall be continued proportionals in the parabola EDF: and thus the same also are proportionals in the parabola ABC, therefore the point of contact B of the line IB is the half of BD.



Again a diameter AE shall be erected from A, crossing a diameter of the parabola EDF at E: and the right line IG at R; and the line IF shall be drawn from I through E, crossing the parabola EDF at F, and with the points joined the line CF shall arise. Because AE, KD, are parallel right lines, so that as AI is to DI, thus EI is to KI: but as AI is to DI, thus AD is to DC (since IA, ID, IC are proportionals), and as EI shall be to KI, thus EK is to KF, since EI, KI, FI are proportionals; therefore so that as AD is to DC, thus EK is to KF, and therefore the line FC shall be parallel to KD: with the diameters LM, NO put in place the line GH may be produced further through G and H, until FC may be crossed at Q. Therefore so that the rectangle ADC will be to the rectangle ANC, thus as the line BD to the line HN: but as the rectangle ADC shall be to the rectangle ANC, (that is, as the rectangle EKF to the rectangle EOF,) thus also the line KD is to the line HO; therefore as

BD to HN, thus KD is to OH; and on interchanging and inverting, so that as KD to BD, thus OH is to HN, that is FQ to QC, that is MG to GL, that is ER to RA.

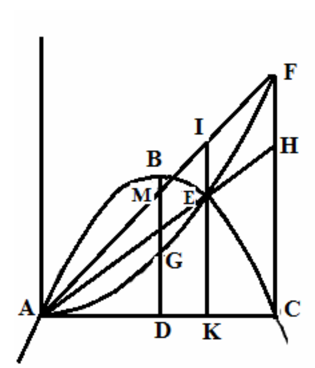
Therefore on adding together, with the axis of the parabola DB found, since the points D and B shall be given, with the tangent BI acting through B, likewise the tangent to the parabola EDF acting through the point D, cutting ABC, in A and C: that will meet with BI (as shown) at some point I : therefore the points I, A, C are given ; therefore the diameter AE, CR shall be erected from A and C, crossing the diameters of the parabola at the points E and F: and the line drawn from I through E will cross that right line CF at F, (as has been shown) and the line BD at K; and from which some point K also has been given ; therefore will become as KD to BD, thus ER to RA, or FQ to QC, and the right line IR or IQ shall be drawn, it is apparent from the resolution, the points of intersection B and C to be present there. Therefore we have shown, etc. Q.e.d.

PROPOSITION CLI.

BD shall be the axis of the parabola ABC, and with the ordinate AC to that put in place, truly the parabola AEF shall be described through A of which the vertex shall be A, and the tangent line AC; also the parabola AEF shall cross the parabola ABC at E, it will be required to show the point of intersection E.

Construction & Demonstration.

What is asked shall be done: and the diameter CH erected from C diameter shall meet the parabola AEF at F ; and FA shall be drawn, meeting the line BD at M: then the right lines AEH, IEK shall be put through E; and indeed AE shall meet FC in H; truly IK will be parallel to the axis BD, therefore so that as the rectangle ADC shall be to the rectangle AKC, thus the line BD to the line EK: but as the rectangle ADC is to the rectangle AKC, that is, as the rectangle AMF to the rectangle AIF, thus the line MG to the line IE: therefore as BD is to EK, thus MG is to IE; and on interchanging, so that as BD to MG, thus EK is to IE, that is, as CH is to FH.

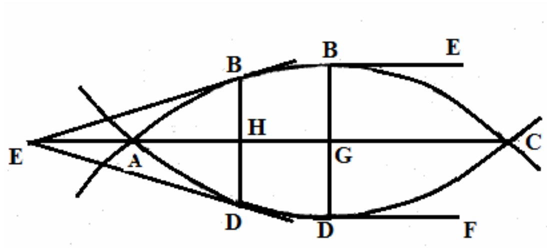


Therefore by the composition since the lines AC, BD shall be given with the points B D, C G, the diameter CF shall be erected from C, crossing the parabola AEF at F, F also will be a given point: thereupon the right line FA shall be drawn crossing the line BG at M, and M will be a given point; and hence if there may become so that as BD shall be to MG, thus as CH shall be to HF, and with the right line AH drawn, it becomes clear from the resolution, the point E to be on the line AH; therefore we have shown, etc. Q.e.d.

PROPOSITIO CLII.

The two parabolas ABC, ADC placed inversely to each other will intersect at the points A and C, having the common diameters BD : and one of the points C shall be given C: it will be required to show the other point.

Construction and Demonstration.



The tangents BE, DF shall be acting through the points B and D; which at first shall be parallel; and BE shall be drawn parallel to CA, meeting the parabola ABC at A, and the right line BD at G: therefore the line AC will be the ordinate in the parabola ABC, the ordinate put in place for the diameter BD, and thus bisected at G. But also AC shall be parallel to the tangent DF; and therefore AC is the ordinate drawn in parabola ADC for BD, and bisected at G: therefore the point A is common to each parabola.

Secondly the tangents acting through B and D concur at E : and the right line EC shall be drawn, meeting the diameter BD at H, and the parabola ABC at A, therefore EA, EH, EC will be proportionals; but also the same lines are proportionals in the parabola ADC, and H, G, C are the given points ; and therefore the point A is given, and that is common to the parabola ADC, therefore we have shown the other point meeting point A, which was postulated.

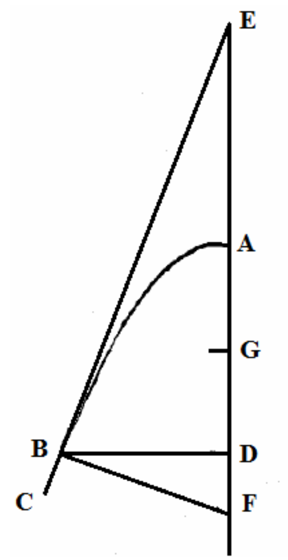
PARABOLAE

PARS TERTIA

Sectionis focum et mutuas parabolarum intersectiones geometrice designat.

PROPOSITIO CXIV.

Parabolam ABC cuius axis AD, contingat in B quaevis BE occurrens axi in E : positaque ordinatim BD, ponatur BF normalis ad BE, occurrens axi in F.
Dico DF lineam aequari dimidio lateris recti quod axi inservit.



Demonstratio.

Sumatur AG aequalis lateri recto. quoniam BD ordinatim ponitur ad axem, quadratum BD aequale est rectangulo DAG: quia vero angulus EB F rectus est, erit & quadrato BD aequale rectangulum EDF. igitur EDF, DAG rectangula sunt inter se aequalia,& ut ED ad AD, sic AG est ad DF: est autem ED dupla ipsius AD, cum EB, fit contingens; igitur & AG dupla est rectae DF: adeoque DF aequalis dimidio lateris recti. Quod erat demonstrandum.

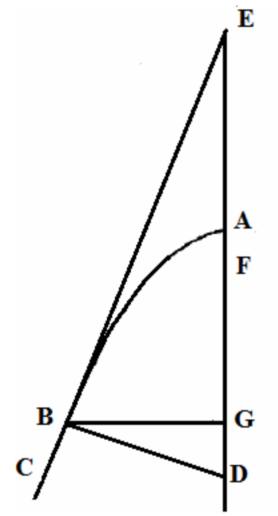
PROPOSITIO CXV.

Eadem manente figura : sit AG latus rectum & illius dimidio aequetur DF: ponatur autem ordinatim DB, & BE contingens, iunganturque BF.
Dico angulum EBF rectum esse.

Demonstratio.

Cum EB fit contingens, & BD ordinatim posita, erit ED dupla rectae AD: &quia AG dupla ponitur DF, erit DAG rectangulii aequale rectangulo ED F: sed DAG rectangulo aequale est quadratum BD, igitur & quadratum DB aequale est rectangulo EDF, unde cum DB normalis ponatur ipsi EF, patet angulum EBF esse rectum. Q.e.d.

PROPOSITIO CXVI.



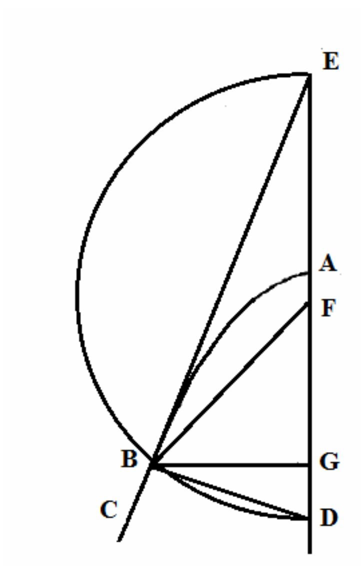
Parabolam ABC cuius axis AD, contingat linea BE,conveniens cum axe in E: positaqua BD ad EB rectam normaliter, dividatur ED bifariam in F.

Dico FA quartam partem esse lateris recti.

Demonstratio.

Ponatur BG ordinatim ad axem; cum igitur ED in F bifariam fit divisa, & EG in in A (ob EB cotingentem) vt ED ad EF, sic EG ad EA, unde & residuum AF ad residuum GD, ut totum ED est ad dimidium sui EF:A F igitur dimidium est GD: hoc est quarta pars lateris recti; cum GD lateris recti fit. Quod erat demonstrandum.

PROPOSITIO CXVII.



Parabolam ABC cuius axis AD, contingat recta quaevis BE, & angulo BEA aequalis fiat angulus EBF.

Dico FA lineam aequalem esse quartae parti lateris recti; & si FA sit quarta pars lateris recti, dico lineas FB, FE; & consequenter angulos BEF, EBF esse inter se aequales.

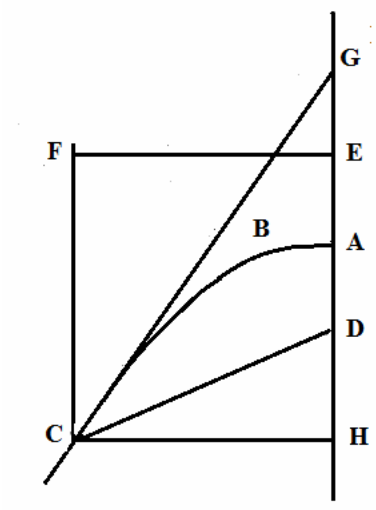
Demonstratio.

Ponatur ordinatim linea BG: & BD normalis ad contingentem EB. Quoniam anguli EBF, BEF ex hypothest aequales sunt,erunt &.FB, FE lineae quoque aequales, unde cum & angulus EBD rectus fit; si centro F, intervallo FE circulus describatur, transibit is per B & D. Quare & EF, FD linea aequales, adeoque FA aequalis quartae parti lateris recti. Quod erat primum.

Rursum cum angulus RBD ponatur rectus, & BG ordinatim applicata ad axem AD; erit DG aequalis dimidio lateris recti, adeoquae dupla AF: quare EA ad EG; ut AF ad GD, & componendo ut EF ad ED, sit AF ad GD: quare ED dupla est ipsius EF, & circulus centro F intervallo FE descriptus, transibit per B & D; eruntque lineae FB, FE & anguli FBE, FEB inter se aequales: Q.e.d.

PROPOSITIO CXVIII.

Esto ABC parabolae axis AD aequalis quartae parti lateris recti: factaque AE aequali AD, ducatur ex D linea DC occurrens parabolae in C; & ex C erigatur diameter CF, occurrens EF (quae normalis sit ad axem AD) in F.



Dico DC; FC lineas esse inter se aequales.

Demonstratio.

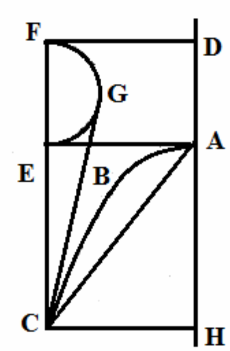
Agatur per C contingens CG,conveniens cum axe in G; ad quem ordinatim ponatur CH. Quoniam CG est contingens, rectae AG, AH aequales sunt; additis igitur aequalibus AD, AE totae GD, EH aequales sunt; sed DC recta est aequalis GD, igitur & DC aequatur lineae HE, id est CF. Q.e.d.

PROPOSITIO CXIX.

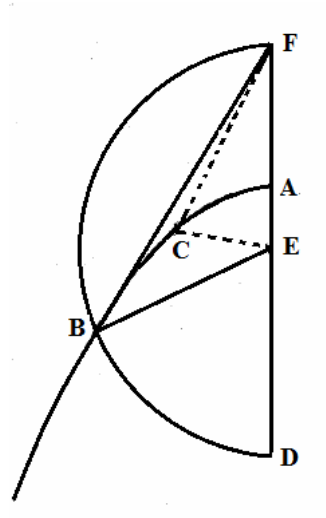
Parabolam ABC cuius axis AD productus extra sectionem, sit aequalis lateri recto, contingat in A linea AE, demissaque ex A secante AC, erigatur ex C diameter CF, occurrens AE contingenti in E, & DF quae parallela AE in F, factoque super EF ut diametro, semicirculo FGE, ducatur ex C, linea CG quae semicirculum contingat in G. Dico AC, CG lineas esse inter se aequales.

Demonstratio.

Ponatur ex C ordinatim CH; quadratum AC aequale est quadratis AH, HC: sed HC quadratum aequale est rectangulo HAD (quia AD per hypothesim est aequalis lateri recto) quadratum igitur AC aequale est quadrato AH una cum rectangulo HAD, id est rectangulo AHD; id est rectangulo ECF; est autem & ECF rectangulo aequale
quadratum CG, quadrata igitur AC ; CG adeoque & lineeae inter se aequantur. Q.e.d.



PROPOSITIO CXX.

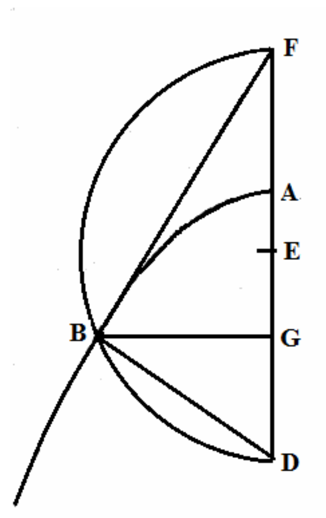


Esto ABC parabolae axis AD: & AE linea aequalis quartae parti lateris recti: centro E, intervallo quovis EF circulus describatur occurrens axi in F & D, parabolae vero in B: iunganturque FB.
Dico FB rectam contingerc parabolam in puncto B.

Demonstratio.

Si enim non contingat ; ponatur ex F contingens FC: cadet illa inter B & A; vel ultra B. Cadat primo inter B & A, iunganturque BE, CE. Quoniam PC linea est contingens, & AE quarta pars lateris recti; linea CE aequalis est rectae FE, hoc est EB. Quod absurdum; igitur EC contingens non caderc ultra B. Quare FB sola parabolam contingit. Q.e.d. '

PROPOSITIO CXXI.

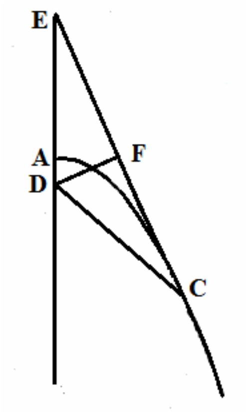


Esto ABC parabolae axis AD, & AE linea quarta pars lateris recti: centro E intervallo quovis EF circulus describatur, occurrens axi in F & D, parabolae autem in B, & ex B recta ducatur BG ordinatim ad axem.
Dico DG lineam, dimidium esse lateris recti.

Demonstratio.

Iungantur puncta FB, BD erit igitur angulus FBD in semicirculo rectus ; est autem per praecedentem FB contingens, DG igitur dimidium est lateris recti. Q.e.d.

PROPOSITIO CXXII.



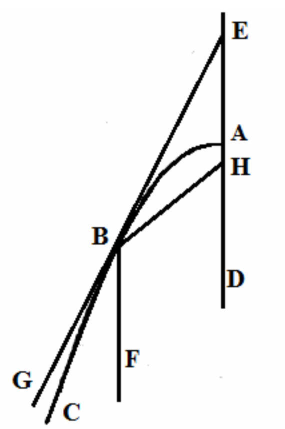
Parabolam ABC cuius axis AD contingat in C linea quaevis CE, conveniens cum axe in E. divisaque bifariam CE in F, ducatur FD normalis ad EC contingentem occurrens axi in D.

Dico AD lineam, quarta esse partem lateris recti.

Demonstratio.

Iungantur puncta CD. Quoniam FD normalis est ad tangentem EC, anguli EFD, CFD aequales sunt; sunt autem & rectae EF, FC ex hypothest aequales, & FD communis,
triangula igitur EDF, CDF, & ED, CD latera aequalia sunt: quarc & anguli CED, ECD aequantur; & AD linea est aequalis quartae parti lateris recti. Q.e.d.

PROPOSITIO CXXIII.



Parabolam ABC cuiusaxis AD, contingat in B recta quaevis BE conveniens cum axe in E, demissaque ex B linea BF parallela axi, fiat angulo FBG, aequalis angulus EBH.

Dico AH lineam aequalem esse quartae parti lateris recti : & si AH fuerit quaerta pars lateris recti, & BF, parallela axi, dico angulos EBH, FBG esse aequales.

Demonstratio.

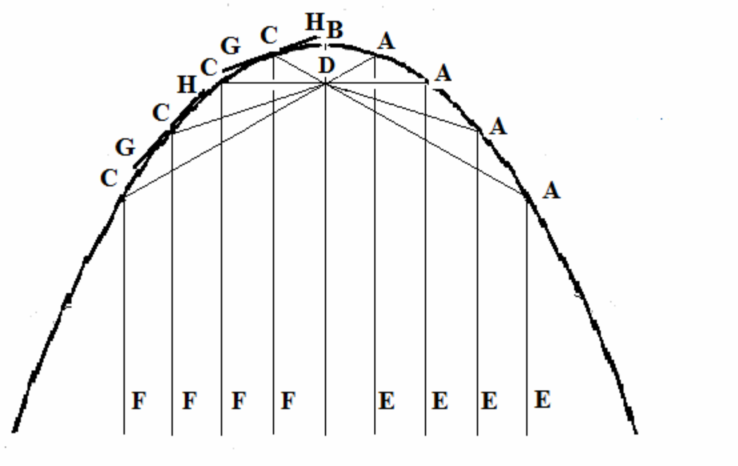
Quoniam FB aequidistat ED, angulus AEB aequalis,est angulo FBG hoc est angulo HBE: unde AH linea aequalis est quartae parti lateris recti. Quod erat primum.
Rursum cum AH linea fit quarta pars lateris recti, angulus HBE aequalis est angulo AEB , hoc est FBG, cum FB, ED aequidistent; vocetur autem punctum H focus parabolae. Q.e.d.

PROPOSITIO CXXIV.

Datae parabolae focum exhibere.

Constructio & Demonstratio.

Sit datae parabolae ABC, axis BD, cui quotvis ponantur aequidistantes, AE, CF: agatur autem per C contingens GH, ductaque ex C recta CD, fiat angulo FCG, aequalis angulus HCD: constat per praecedentem propositionem punctum D, esse focum parabolae, adeoque BD quartam esse partem lateris recti, & quoniam CF linea est quaecunque parallela axi, si in reliquis quoque angulo incidentia aequalis fiat angulus reflexionis, radii reflexi omnest convenient in D puncto quod quartam lateris recti partem designat; datae igitur parabolae focum exhibuimus. quartam lateris recti partem designat; datae igitur parabolae focum exhibuimus.



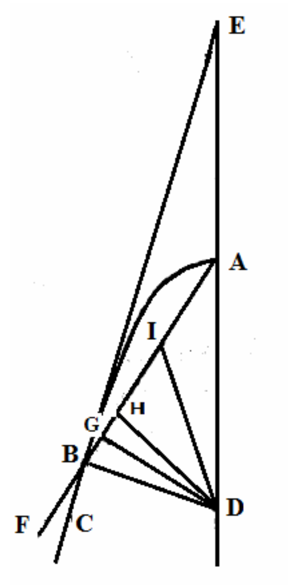
Porro focum indigitamus punctum D, quod in illo fiat combustio, & radii reflexi praecise in illo coeuntes, intensissimum producant calorem; quia vero punctum D (quod quartam lateris recti partem determinat a vertice initio sumpto) llatitudinem nullam admittit: fit ut forma parabolica, omnium sit aptissima radiis solis recipiendis & reflectendis; multoque vehementiores, quam reliaque figurae, causet effectus.

Quod autem in propositione radios CF, AE omnes parallelos axi assumamus, id eadem de causa fecimus qua idem Optici, Gnomonici, & Statici in poderius deorsum tendentibus, assummere coguntur, & hactenus ut fundamentum ac principium ab omnibus (si Neothericos paucos excepias)habitum est, non quod a parte rei solares radii paralleli sint, sed quod ob immensum nos inter & solem spatium, radii in speculum incidant ac si ver paralleli essent, sic ut nulla ad invicem inflexio perceptibilis sit: & si e planeo speculi duas lineas versus solem eductas, quantumvis minimo angulo ad invicem inclines, convenient illae multo ante, quam ad solarem discum perveniere possint: & si vicissum ex solis centro angulo physico, quantumvis minimo, duos educas radios, non cadent illi in superficeim speculi, sed immenso utrimque intervallo a specula decident, ratio utriusque est, immensa solis a terra distantia: radii igitur solares in speculum incidentes ut vere paralleli assumuntur. quo posito demonstrari facile posset, focum in circulo & omni sectione coni (praeterquam in parabola)latitudinem admittere, adeoque figuram parabolicam ad comburendum omnium esse praestantissimam: sed de his alio tempore & loco, si Deus vitam dederit.

PROPOSITIO CXXV.

Parabolam ABC cuius axis AD contingat in B recta quaevis BE conveniens cum axe in E: ductaque ex B linea BD normalis ad contingentem, quae axi occurrat in D: si AD linea fuerit maior dimidio lateris recti.

Dico rectam BD minor esse AD.



Demonstratio.

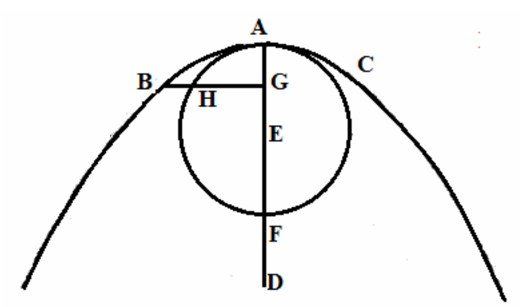
Ducatur ex A per B, linea ABF : & ex D recta DG normalis ad AF. quoniam angulus EBD rectus ponitur, angulus ABD recto minor est, adeoque FBD angulus obtusus; quare DG cadet inter A & B fiat ergo BG aequalis GH, iunganturque HD: cum igitur HG, GD linea aequales sint rectis BG, GD, & anguli illis contenti recti; linea DH aequalis est BD, & angulus GHD recto minor. Rursum cum AI, ID linea aequentur lineis HI, ID. triangula AID, HID aequalia sunt .quare AD, subtendens angulum obtusum AID, maior est recta HD, hoc est BD. Q.e.d.

PROPOSITIO CXXVI.

Esto ABC parabolae axis AD: sumptaque AE quae non sit maior dimidio lateris recti, centro E intervallo AE circulus describatur AHF.

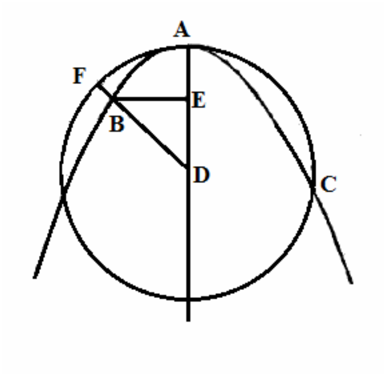
Dico circulum AHF contingere intus parabolam in A.

Demonstratio.



Fiat AD linea aequalis lateri recto, & ordinatim ducatur quaevis GB occurrens circulo in H. Quoniam AE supponitur non maior dimidio lateris recti, cadet F punctum supra, vel in ipsum D, adeoque AGF rectangulum minus est rectangulo GAD ; igitur & HG quadratum, minus est quadrato BG punctum igitur H cadet intra parabolam. eodem modo demonstrantur reliqua omnia circuli puncta, praeter A cadere intra parabola: circulus igitur AHF, parabolam ABC interius in A contingit. Q.e.d.

PROPOSITIO CXXVII.



Esto ABC parabolae axis AD maior dimidio lateris recti: centroque D, intervallo AD circulus describatur AFG.
Dico circulum illum intersecare parabolam.

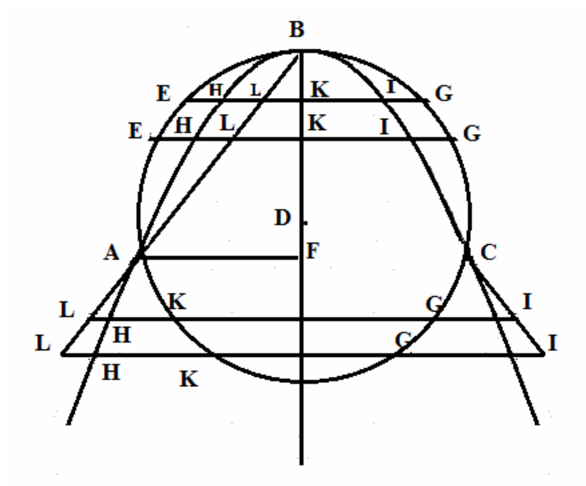
Demonstratio.

Fiat AE linea aequalis dimidio lateris recti & per E recta ponatur ordinatim EB: ducaturque ex D per B , linea DBF occurres circulo in F& parabole in B. Quoniam AD maior est dimidio lateris recti, recta DB minor est AD hoc est DF. punctum igitur F cadet extra parabolam. quare circulus AFG parabolam secat.

Q.e.d.

PROPOSITIO CXXVIII

Esto ABC parabolae axis BD maior dimidio lateris recti; centroque D, intervallo DB circulus describatur AEB. occurrens parabolae in A; ductaque ex A ordinatim linea AF, ducantur quotvis EG parallelae AF, occurrentes circulo in E & G, parabolae in H & I, axi in K.
Dico EHG rectangulum esse ad rectangulum EHG, ut FKB rectangulum est ad rectangulum FKB.



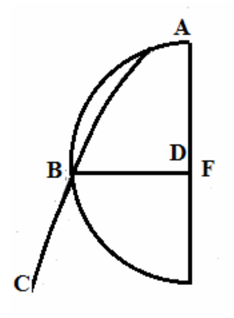
Demonstratio.

Iuncta AB occurrat EG lineis in L. Quoniam EG linea in circulo AEB, ad angulos rectos in K secat diametrum BD, recta EG in K, divisa est bifariam, sed & HI in parabola ABC, quoque in K divisa est bifariam ; rectae igitur EH, IG inter se aequales sunt. Rursum ut ALB rectangulum ad rectangulum AIB, sic ELG rectangulum ad rectangulum EIG, item HLI rectangulum ad rectangulum HLI: sed ELG rectangulum aequale est rectangulis HLI, EHG, igitur & EHG rectangulum est ad retangulum EHG, ut ALB rectangulum ad rectangulum ALB; id est FKB rectangulum, ad rectangulum FKB. Q.e.d.

PROPOSITIO CXXIX.

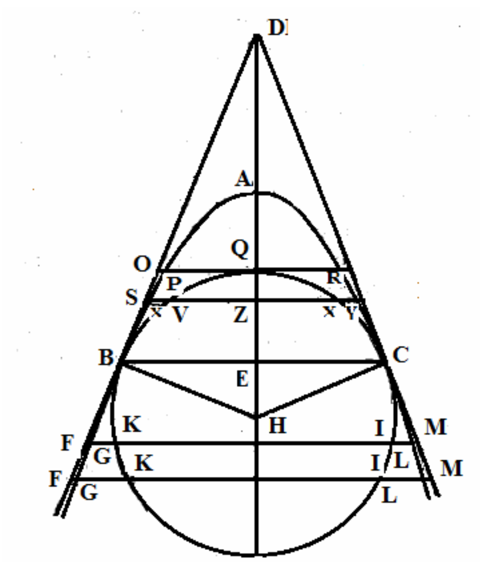
Esto ABC parabolae axis AD maior dimidio lateris recti : centroque D intervallo AD, circulus describatur ABE: occurrat is parabolae in B & axi in E. ducaturquc ex B, linea BF, ordinatim ad axem.

Dico lineam FE aequalem esse lateri recto.



Demonstratio.

Quoniam BF ordinatim ducta est ad axem quadratum FB aequale est rectangulo super FA & latere recto : sed & FB quadratum quoque aequalc est rectangulo AFE : rectangulum igitur AFE aequale est ei quod sit super AF & latere recto. quare FE linea lateri recto est aequalis . Q.e.d.



PROPOSITIO CXXX.

Parabolam ABC cuius axis AE contingat in B linea BD , occurrens axi in D, ponatur autem ex B linea BH normalis ad contingentem, occurrens axi in H: centroque & intervallo HB circulus describatur BQC: ponaturque ad axem normales quotcunque IK. occurrentes parabolae in GL, contingenti BD in FF: & circulo intra parabolam intercepto in I & K.

Dico GKI rectangulum esse ad rectangulum GKI, ut quadratum FB ad quadratum FB

Demonstratio.

Ponatur BC aequidistans FI, occurrens parabolae in C; iunctaque DC occurrat FL rectis in M. Quoniam BC ordinatim ponitur ad axem DE; recta BC in E bisecta est, & BE, EC linea aequales sunt, unde DC parabolam in C continget, quia vero B punctum in perimetro circuli est, punctum quoque C in circuli est perimetro, & quia DBH angulus rectus est, angulus quoque DCH rectus est, angulus quoque DCH rectus est, & DC linea circulum contingit ; igitur ut FB quadratum ad quadratum FB, sic FKM rectangulum ad rectangulum FKM: sed cum FB, CM lineae parabolam quoque contingant, ut FB quadratum ad quadratum FB sic GFL rectangulum est ad rectangulum GFL, residuum igitur rectangulum GKI ad GKI rectangulum, est ut quadratum FB ad quadratum FB. Q.e.d.

PROPOSITIO CXXXI.

Eadem manente figura si AH linea maior sit dimidio lateris recti. Dico circulum BQC parabolam intus in duobus punctis contingere.

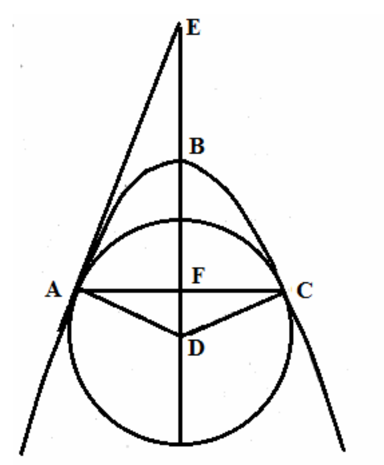
Demonstratio.

Ponatur per Q linea OPR aeqidistans BC, & altera quaevis STVXY eidem parallela, quia AH maior est dimidio lateris recti, AH linea maior est BH; & circulus BQC cadit infra A; deinde ut OB quadratum ad quadratum SB, sic POR rectangulum ad rectangulum TSX: sed est quoque ut quadratum OB ad quadratum SB, sic quadratum OQ ad rectangulum VSX, igitur POR rectangulum ad rectangulum TSY, ut quadratum OQ ad rectangulum VSX, & permutando convertendo ut quadratum OQ ad rectangulum POR, VSX rectangulum ad rectangulum TSY: sed OQ quadratum maius est rectangulo POR, igitur & rectanguulum maius est rectangulo TSY; ulterius, rectangulum TSY una cum quadrato TZ, aequale est quadrato SZ; est autem & quadrato SZ aequale rectangulum VSZ una cum quadrato VZ, quadratum igitur VZ una cum rectangulo VSX rectangulum maius esse rectangulo TSY; quadratum igitur VZ minus est quadrat TZ: quia punctum V cadit intra parabolam: similiter ostendentur reliqua omnia puncta perimetri circularis BQC cadere intra parabolam, praeter B & C: circulus igitur BQC parabolam intus in duobus contingit punctis.

PROPOSITIO CXXXII.

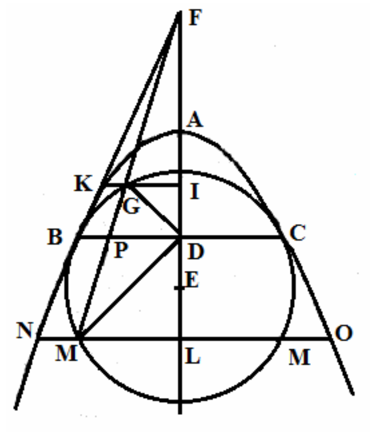
A dato in axe puncto circulum describere qui parabolam intus in duobus punctis contingat.

Constructio & Demonstratio.



Sit ABC parabolae axis BD, & in eo punctum datum D oportet centro D circulum describere, qui parabolam interius contingat describere, qui parabolam interius contingat in duobus punctis: oportet autem BD lineam maiorem esse dimidio lateris recti, fiat DF aequalis dimidio lateris recti, fiat DF aequalis dimidio lateris recti: & per F recta agatur FAC, ordinatim ad axem: iunganturque puncta D, A: tum centro D intervallo DA circulus describatur; dico illum contingere interius parabolam in duobus punctis ; agatur enim per A cotingens AE conveniens cum axe in E. Quoniam linea AE parabolam contingit in A, & FD aequalis est dimidio lateris recti, angulus EAD rectus est; est autem ex hypothest BD maior dimidio lateris recti; ergo per praecedentem circulus centro D, intervallo DA descriptus parabolam continget interius in duobus punctis : a dato igitur in axe puncto D circulum descripsimus, &c. Quod erat faciendum.

PROPOSITIO CXXXIII.



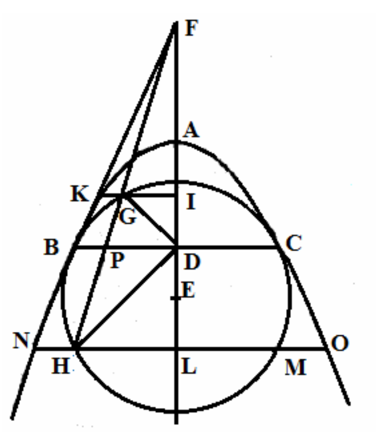
Parabolam ABC cuius axis AD contingat interius in duobus punctis B & C circulus HBG actaque per B contingente parabolam in B, quae axi occurrat in F, ducatur ex F, line linea FH, secans circulum in G &. H, Dein per G & H, normales ducantur KGI, NHL occurrentes parabolae in K & N, axi vero in I & L.

Dico esse ut HL ad GI, sic NL ad KI.

Demonstratio.

Quoniam circulus BGC parabolam intus contingit in B & C, & FB linea per B ducta parabolam in B contingit, eadem BF circulum quoque contingit in B; igitur ut HF ad FG, sic HP est ad PG: & ut LF ad HF ad FI, sic LD ad DI; IG, sed ut LF ad FI, sic LH ad IG; igitur LD est ad DI, ut LH ad IG. quia vero LF in D & 1, extrema & media ratione proportionali divisa est, & FD bisecta in A, proportionales sunt AI, AD, AL; unde LA ad AI, duplicata habet rationem LA ad DA, id est LD ad DI: sed ratio quoque LA ad AI, duplicata rationis LN ad KI, igitur NL est ad KI, ut LD ad DI, id est LH ad GI. Q.e.d.

PROPOSlTIO CXXXIV.



Iisdem positis:

Dico rectas KI, GD; item NL, HD esse inter se aequales.

Demonstratio.

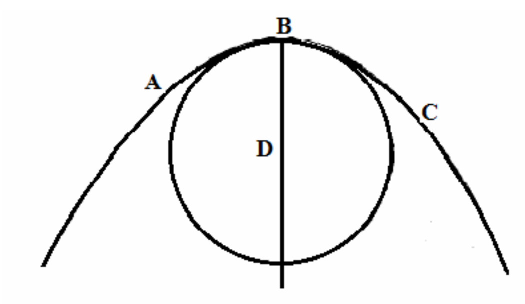
Ostensum est rectas AI, AD, AL in continua esse analogia, unde quoque proportionales erit IK, BD, LN; sed etiam sunt continuae GD, BD, HD; igitur GD, KI lineae, & NL, DH sunt inter se aequales; nam rectangulum GDH, rectangulo KINL est aequale, & ostensum insuper est GE ad DH, eandem habere rationem quam ID ad DL, hoc est KI ad NI. Q.e.d.

PROPOSlTIO CXXXV.

Iisdem positis:
Dico quadratum DL aequari rectangulo NHO.

Demonstratio.

Quadratum HD est aequale quadratis HL, LD: est autem quadrato aequale quadratum NL, igitur NL quadratum est aequale quadratis HL, LD; sed etiam quadratum NL aequalc est aequale quadrato HL & rectangulo NHO, dempto ergo communi quadrato HL, erit NHO rcctangulum aequale quadrato DL. Q.e.d.



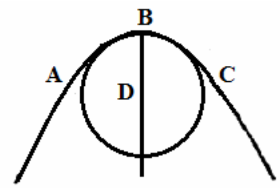
PROPOSITIO CXXXVI.

IN datae parabolae axe punctum assignare quo centra circulus describatur maximus illorum, qui parabola intus in uno tantum contingunt puncto.

Constructio & Demonstratio.

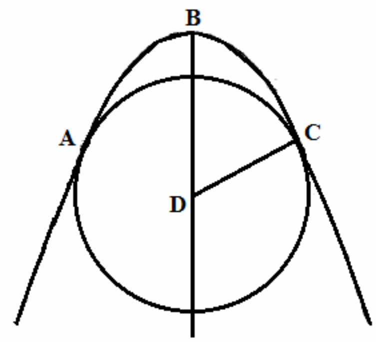
Esto ABC parabolae axis BD,oportet in illo punctum assignare D, quo centra circulus describatur maximus eorum qui parabolam in uno tantum puncto contingunt, fiat DB linea aequalis,dimidio lateris recti: centroque D, intervallo BD circulus describatur; dico illum satisfacere proportioni quoniam DB aequalis ponitur dimidio lateris recti, circulus radio DB descriptus, intus parbolam in B contingit. quod vero contingentium circulorum maximus sit, ex illa patet quod circuli omnes qui centrum habent ultra D, parabolam intersecent; quorum vero centrum inter D & B, cadit semidiametrum semper minorem habeant semidiametero DB, ac proinde illi circuli minores sunt: circulus igitur radio DB descriptus, contingentium circulorum maximus est; exhibuimus igitur, & c. Q.e.d.

PROPOSITIO CXXXVII.



Ex dato in axe parabolae puncto, lineam ad peripheriam ducere, brevissimam illarum quae ex eadem puncta duci possunt.

Constructio & Demonstratio.



Esto ABC parabolae axis BD, & in eo punctum datum D: oportet ex D puncto lineam ducere brevissimam illarum, quae ex eodem puncto ad parabolae perimetrum educi possunt.

Primo, recta BD non sit maior dimidio lateris recti. dico BD lineam esse quaesitam: describatur enim centro D intervallo DB circulus; continget is per praecedentem parabolam interius in punctosolo B: igitur reliquae omnes lineae ex D ad peripheriam ductae maiores sunt linea BD.

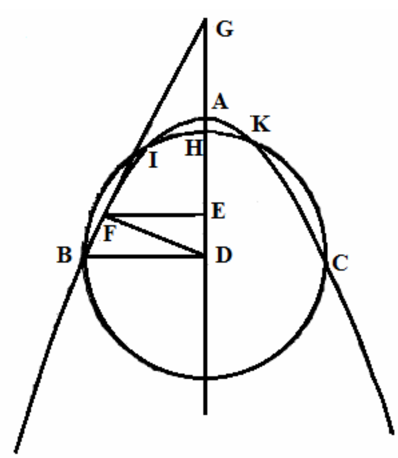
Secundo BD maior sit dimidio lateris recti: centro D circulus describatur cotingens ABC parabolam in duobus punctis A, C. ducaturque recta DC: erit illa minima (ut patet) illarum que ex D duci poterunt ad peripheriam parabolae: ex dato igitur puncto D, lineam duximus, &c. Q.e.d.

PROPOSITIO CXXXVIII.

Sit ABC parabolae axis AD maior latere recto; ductaque ex D ordinatim linea DB, centro D,intervallo DB circulus describatur BHC.

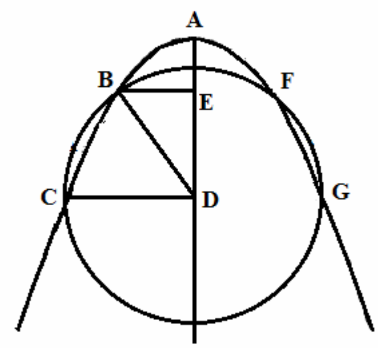
Dico illum interfecarc parabolam in quatuor punctis.

Demonstratio.



Sumatur ED aequalis dimidio lateris recti; positaque ordinatim EF, agatur per F contingens FG, conveniens cum axe in G. Quoniam FG est contingens, & ED aequalis dimedio lateris recti, angulus DFG rectus est, quia vero AD linea maior est dimidio lateris recti, FD minima est earum, quae ex D ad peripheriam duci possunt: adeoque & minor BD ordinatim in posita; circulus igitur centro D intervaIlo DB , descriptus cadet ultra F, & secundum aliquam sui partem extra parabolam. Rursum cum DB, minor sit recta AD, (cum DA maior sit latere recto,) cadet circulus BHC infra punctum A: & secundum aliquam sui partem intra parabolam: Quare & in alio puncto A parabolam inter secabit. Similiter ostendatur circulum BHC, versus partem AC occurrere parabolae in alio puncto quam in C; circulus igitur centro D, intervallo DB descriptus, parabolam secat in quatuor punctis. Q.e.d.

PROPOSITIO CXXXIX.

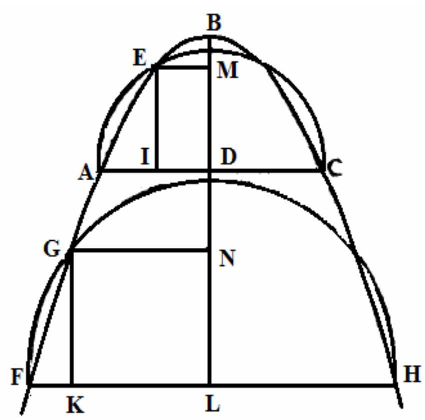


Sit ABC parabolae axis AD maior latere recto; positaque ad illum ordinatim DC; centro D intervallo DC circulus dcscribatur CBF; occurret ille parabolae in quatuor punctis C, B, F,G, & ex B puncto intersectionis ducatur BE ordinatim ad axem:
Dico ED lineam aequalc esse lateri recto.

Demonstratio.

Iungantur BD. Quoniam BD est aequalis DC, & E B, CD, ordinatim ponuntur ad axem, ED linea aequalis lateri recto est. Q.e.d.

PROPOSITIO CXL.



Si parabolam ABC cuius axis BD, secuerint quotcunque semicirculi AEC, FGH quorum singuli occurrunt parabolae in quatuor punctis; demissae autem ex E & G, intersectionum punctis rectae fuerint EI, GK normales ad lineas AC, FH quae ordinatim positae sunt ad axem.
Dico EI, GK lineas esse aequales.

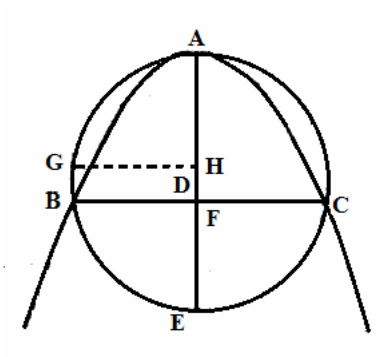
Demonstratio.

Ducatur enim ordinatim EM, GN : erit per praecedentem tam MD quam NL aequalis lateri recto axeos ; igitur & aequales inter se. Q.e.d.

PROPOSITIO CXLI.

Esto ABC parabolae axis AD maior dimidio lateris recti : centroque D intervallo DA, circulus describatur ABC, occurrens axi in E: oportet exhibere puncta intersectionum B & C.

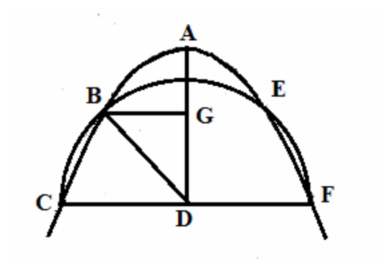
Constructio & demonstratione



Sumatur EF linea aequalis lateri recto, & per F recta agatur FBC ordinatim ad axem: dico circulum ABC occurrere parabolae in B & C cum FC ordinatim posita sit ad axem ; quadratum illius aequatur AFE rectangulo, quia FE lateri recto assumitur aequalis: sed rectangulo AFE in circulo ABE, aequale quoque est quadratum FB: igitur B punctum pertinet & ad parabolam ABC, & ad circulum ABE idem discursus aptetur puncta C. praEstotimus ergo quod imperatum fuit.

PROPOSITIO CXLII.

Esto ABC parabolae axis AD maior latere recto: actaque per D ordinatim linea CDF, centra D intervallo CD, circulus describatur CBE, occurretis parabolae in quatuor punctis: oportet illa cxhibere.



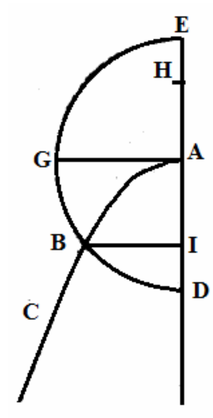
Constructio & demonstratione

Sumatur DG aequalis lateri recto , ponaturque ordinatim GB: dico B punctum unum esse intersectionis, iungatur BD. Quoniam BG, CD ordinatim ponantur ad axem, & GD aequalis lateri recto est, linea CD aequalis est BD : circulus igitur centro D, intervallo DC descriptus, transibit per B. eodem modo ostenditur, eundem circulum transiere per E: quod vero per C & F, transire, manifestum est; exhibuimus igitur puncta quatuor intersectionis. Quod erat faciendum.

PROPOSITIO CXLIII.

Esto ABC parabolae apex A, centroque A, intervallo quovis AE circulus describatur EGB; oportet exhibere B punctum intersectionis cum parabola.

Constructio & demonstratione.



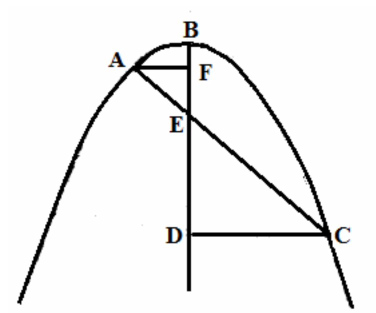
Posito axe AD sumatur AH aequalis lateri rccto, positaquc per A contingente AG quae circulo occurrat in G, quadrato AG fiat aequale rectangulum HIA; & per I ordinatim ducatur IB: dico punctum B, esse id quod quaeritur. iungatur AB. Quoniam IB applicata est ad axem, & AH linea aequalis lateri recto, quadrato AB, aequale est rectangulum HIA, sed HIA rectangulum quoque aequale est quadrato AG per constructionem; igitur AG, AB quadrata aequalia sunt. igitur circulus centro A intervallo AG descriptus, transit per B, exhibuimus igitur punctum intersectionis B. Quod erat postulatum.

PROPOSITIO CXLIV.

Esto ABC parabolae diameter BD quam secet utcunque recta quaevis AE, occurrens parabolae in A: oportet exhibere C, punctum aliud intersectionis.

Constructio & demonstratione.

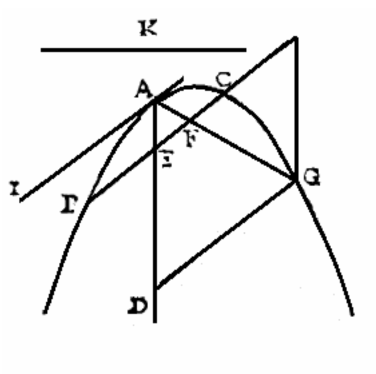
Ducatur ex A linea AF ordinatim ad diametrum ; fiantque proportionales BF,BE, BD: & per D, ordinatim ducatur DC, occurrens AE linaeae in C puncto: quoniam igitur BF,BE, BD proportionales sunt ; & AF, DC ordinatim positae, punctum C est ad parabolam : est autem & C punctum in recta AE: igitur C communis intersectio est AE lineae cum parabola, exhibuimus igitur, &c. Quod erat faciendum.



PROPOSITIO CXLV.

ESto ABC parabolae diameter AD : & ordinatim ad illam posita BC, ducatur autem ex A quaevis AF, secans BC lineam in F: quae producta occurret parabolae in puncto quovis G. oportet illud exhibere.

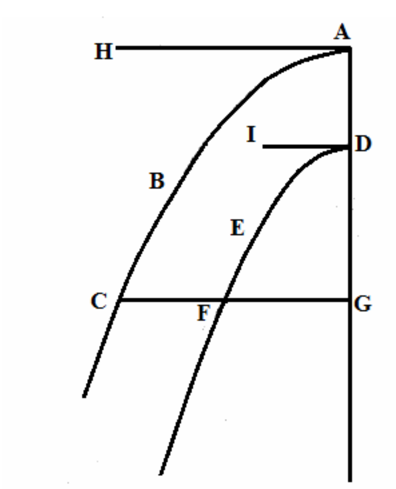
Constructio & demonstratione.



Factum sit: positaque ordinatim GD, erigatur GH parallela diametro AD, occurrens BC in H. Sit autem AI latus rectum. Quoniam GH aequidistat AD diametro, FEH rectangulum, hoc est rectangulum FEGD, aequale est quadrato CE, hoc est BE. sed & quadrato BE aequale est rectangulum IAE; rectangula igitur EAI, FEGD aequalia sunt , quare ut FE ad AE, sic AI est ad GD: & GD quidem fiat aequalis recta K. erit igitur K linea data. Rursum cum GD ordinatim posita sit ad diametrum AD, ut AI ad GD, hoc est ad K, sic K est ad AD : igitur & AD, ducaturque ex D ordinatim linea DG, constat factum esse quod petebatur.

PROPOSITIO CXLVI.

Habeant ABC, DEF parabolae ex eundem axem AG constitutae apices diversos A, D, ; & ABC quidem superior, habeat AH latus rectum maius latere recto ID parabolae DEF.



Dico sectiones illas in infinitum productas nusquam convenire.

Demonstratione.

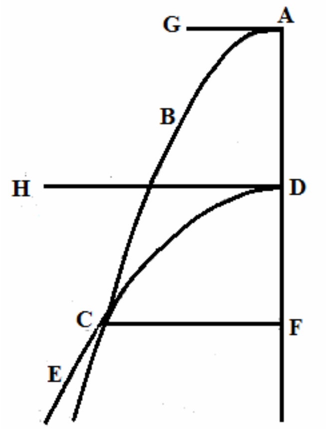
Ponatur ordinatim ad axem AG, quaecunque linea CG, occurrens ABC parabolae in C, & DEF in F. erit igitur FG quadratum aequale rectangulo IDG, & CG quadratum aequale rectangulo HAG: sed IDG rectangulum minus est rectanguo HAG, (quia ID latus rectum minus ponitur ipso HA & DG minor ipsa AG) igitur & quadratum FG minus est quadrato CG: & F punctum cadit inter C & G. & quia idem de quovis puncto parabolae DEF ostenditur, patet sectiones illas in infinitum productas nusquam conveniere. Quod erat demonstrandum.

.

PROPOSITIO CXLVII.

Sint duae parabolae ABC, DCEE ad eundem axem constitutae, & ABC parabolae latus rectum, minus sit latere recto parabolae DCE, nec vertex A communis sit.
Dico parabolas illas concurrere. oportet autem punctum punctum concursus assignare.

Constructio & demonstratione.

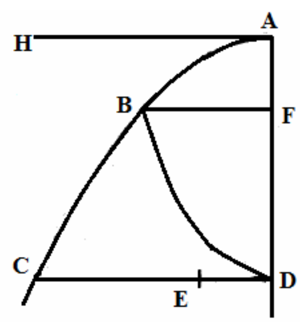


Sit AG latus rectum parabolae ABC, & DH latus rectum parabolae DCE , lineae AD adiiciatur quaedam DF, ut AF sit ad FD, sicut HD est ad AG. & ex F ordinatim ponatur FC occurrens parabolae DCE in C: dico illud esse punctum intersectionis.
Quoniam est ut HD ad AG , sic AF ad FD , erit HDF rectangulo, aequale rectangulum GAF. sed HDF rectangulo aequale est quadratum FC; igitur & GAF rectangulo aequale quoque est quadratum FC. unde punctum C est ad parabolas ABC, DCE. exhibuimus ergo punctum concursus.

PROPOSITIO CXLVIII.

Habeant parabolae ABC, DBF commune in axem AD, & vertices oppositos A, D.
Oporteat autem earum intersectiom puncta exhibere.

Constructio & demonstratione.

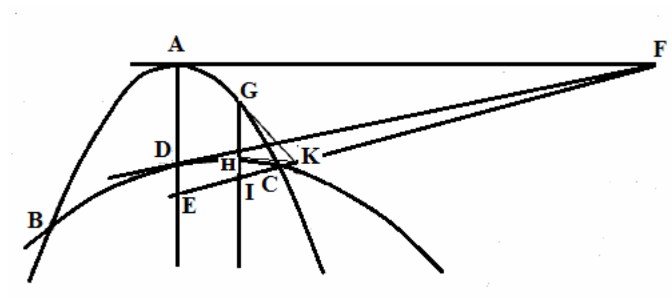


Sit ABC parabolae latus rectum AH,& DB parobolae latus rectum ED ; seceturque AD in F, ut HAF rectangulo aequale sit rectangulum FDE: & per F ordinatim ponatur FB, occurrens parabolae ABC in B.
Dico B punctum esse intersectionis, cum enim FB in ABC parabola ordinatim ducta sit ad axem AD, FAH rectangulum aequale est quadrato FB. Sed FAH rectangulum aequale ponitur rectanglo FDE; quadratum igitur FB aequale quoque est rectangulo FDE, quare FB ordinatim applicata est ad FD axem, in parabola DB : adeoque punctum B ut eique parabolae est commune. Exhibuimus igitur, &c. Quod erat faciendem.

PROPOSITIO CXLIX.

Sint duae parabolae ABC, BDC inaequales, habentes communem rectam AE, quae quidem axis fit parabolae ABC, diameter vero BDC; secentque se in B & C punctis, oportet illa exhibere.

Constructio & demonstratione.



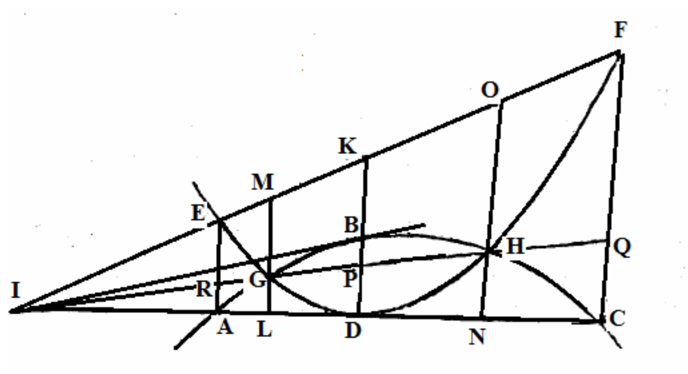
Factum sit quod petitur: acta per A contingente AF, ducatur linea per B & C, conveniet illa cum AF in puncto quovis F: si enim non conveniat, aequidistet AF : erit igitur CB, ordinatim posita ad axem AE in parabola ABC, & quia in E bissecta est; in BDC parabola quoque ordinatim erit ducta, ipsique DE diametro ad angulos rectos. igitur DE axis est parabolae BDC : quod est contra hypothesi conturret ergo CB, cum AF in puncta quovis F ponatur ergo ex F linea SD contingens parabolam BDC in puncto quovis D. Quoniam AF est contingem in parabola ABC, lineae FB, FE, FC proportionales sunt, sed & in parabola BDC eaedem sunt proportionales; punctum igitur D contingen, is DF, in diametro est DE. Rursum ducatur diameter quaevis alia GH occurrens parabolis in G & H, & BC linea in I, actaque per G contingene GK, quae cum BF conveniat in K, ducatur ex K linea KH, contingens parabolam in BDC in puncto quovis H. Quoniam igitur GK linea contingit parabola ABC, rectae KB, KI, KC continuae proportionales sunt: sed & eadem quoque sunt proportionales in parabola BDC, igitur H punctum contingenis, HK, in diametro est HI. igitur componendo cum A, D puncta data sint, agantur per A & D, contingences, quae convenient in puncto quodam F; ductaque diametro quavis GH; agantur per G & H, contingentes quae etiam convenient in K: igitur data sunt puncta F, K, E, I: quare & data erunt puncta B, C: quae in FK linea esse per resolutionem ostendimus ; igitur exhibuimus duarum parabolarum CBC, BDC intersectiones.Quod erat praestandum.

PROPOSITIO CL.

Intersecent sese parabolae duae ABC, EDF inverse positae, habentes axes parallelos: oportet G & H, puncta intersectionum exhibere.

Constructio & demonstratione.

Sit EDF parabolae axis DB, & factum sit quod petitur: actaque per D lineae AC quae EDF parabolam contingat in D, ponatur per G & H, puncta intersectionem recta GH, quae conveniet cum AC in puncto quovis L si enim non conveniat, aequidistet AC; recta igitur GH in parabola EDF, ordinatim posita est ad axem DB, adeoque & diametrum BD in parabola ABC ad angulos secat rectos; & quia GH a BD, linea bissecta est, recta BD axis est parabolae ABC. Quod est contra hypothesim; igitur HG non aequidistat AC, sed producta cum illa convenit in puncto quovis I, ducatur ergo ex recta IB, contingens parabolam ABC in puncto quodam B, quoniam ID recta est contingens, lineae IG, IP, IH continuae sunt proportionales in parabola EDF: sed & eadem quoque sunt proportionales in parabola ABC, punctum igitur contactus B lineae IB est in dimetro BD.



Rursum erigatur ex A diameter AB occurrens EDF parabolae in E: & IG rectae in R; ducaturque per E ex I, linea IF, occurrens parabolae ED F in F, & iungantur puncta CF. Quoniam AE, KD, rectae parallelae sunt, ut AI ad DI, sic EI est as KI: sed ut AI ad DI, sic AD est ad DC (quia IA, ID, lC proportionales sunt) & ut EI ad KI, sic EK est ad KF, quia EI, KI; FI proportionales sunt; igitur ut AD est ad DC, sic EK est ad KF, adeoque FC linea aequidistat KD: ulterius per G & H, positis diametris LM, NO producatur GH linea donec FC occurrat in Q. Erit igitur ut ADC rectangulum ad rectangulum ANC, sic BD linae ad lineam HN: sed ut ADC rectangulum ad rectangulum ANC, (hoc est ut EKF ad rectangulum EOF,) sic KD linea quoque est ad lineam HO; igitur ut BD ad HN, sic KD est ad OH; & permutando invertendo ut KD ad BD, sic OH est ad HN, hoc est FQ ad QC, hoc est MG ad GL, hoc est ER ad RA.

Componendo igitur, invento axe DB parabolae EDF, cum data sint puncta D & B, acta per B contingente BI, agatur per D quoque contingens parabolam EDF in puncto D, secans ABC, in A & C: conveniet illa cum BI(ut ostenti) in puncto quovis I : data igitur sunt puncta I, A, C erigantur ergo ex A & C, diametri AE, CF occurrentes parabolae

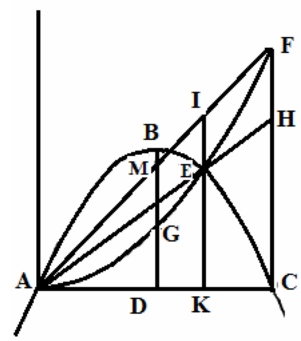
EDF, in punctis E & F: ductaque ex I per E linea occurret illa rectae CF in F, (ut ostensum est) & BD lineae in K. unde & K punctum quoque datum est fiat igitur ut KD ad BD, sic ER ad RA, vel FQ ad QC, ducaturque recta IR vel IQ:_patet per resolutionem, in illa esse puncta intersectionum B & C. igitur exhibuimus, &c. Quod erat faciendum.

PROPOSITIO CLI.

Esto ABC parabolae axis BD, & ordinatim ad illum posita AC, per A vero describatur parabola AEF cuius vertex sit A, & AC linea contingens; occurrat quoque AEF parabola, parabolae ABC in E, oportet E punctum intersectionis exhibere.

Constructio & demonstratione.

Factum sit quod petitur: erectaque ex C diameter CH occurrat parabolae AEF in F ; ducaturque FA, occurrens BD lineae in M: dein per E rectae ponantur AEH, IEK; & AE quidem occurrat FC in H,I K vero aequidistet axi BD erit igitur ut ADC rectangulum ad rcctangulum AKC, sic BD linea ad lineam EK: sed ut ADC rectangulum ad rectangulum
AKC hoc est ut AMF rectangulum ad rectangulum AIF, sic MG linea ad lineam IE: igitur ut BD ad EK, sic EK est ad IE, hoc est CH est ad FH.

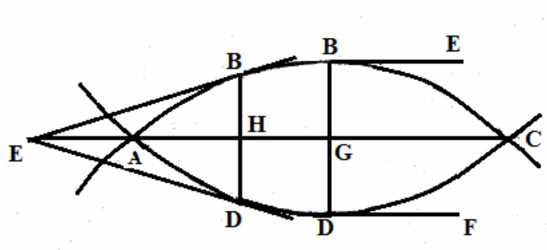


Igitur per compositionem cum AC, BD lineae & puncta BD, CG data sint, erigatur ex C diameter CF, occurrens AEF parabolae in F, erit F quoque punctum datum: ducatur dein recta FA occurrens BG lineae in M, erit & M punctum datum; ac proinde si fiat ut BD, ad MG, sic CH ad HF, ducaturque recta AH, patet per resolutionem; E punctum esse in linea AH. exhibuimus ergo &c, Quod erat faciendum.

PROPOSITIO CLII.

Intersecent se duae parabolae ABC, ADC inverse positae, habentes communes diametros BD, in punctis A & C: datumque sit unum punctorum C: oportet alterum exhibere

Constructio & demonstratione.



Agantur per B & D puncta contingentes BE, DF; quae primo aequidistant; ducaturque ipsi BE parallel CA, occurrens ABC parabolae in A, & BD recto in G: erit igitur AC linea in parabola ABC, ordinatim posita ad diametrum BD, adeoque in G divisa bifariam. Sed etiam AC aequidistat DF contingenti; igitur & AC in parabola ADC ad BD, ordinatim ducta est, & in G divisa bifariam: igitur A punctum utrique parabolae est commune.

Secundo concurrant in E contingentes per B & D actae: ducaturque recta EC, occurrens BD diametro in H, & ABC parabolae in A, erunt igitur proportionales EA, EH, EC; sed eadem quoque lineae sunt proportionales in parabola ADC, & H, G, C puncta data sunt ; igitur & A punctum datum est, estque illud ADC parabolae commune, exhibuimus ergo alterum punctum occursus A, quod fieri postulabatur.