QUADRATURE OF THE CIRCLE

BOOK FIVE

THE PARABOLA.

We approach the section explained by this book by way of the said section on right cones in that ancient book of Apollonius, and more recently called the parabola, and to some extent we will be delaying the explanation of the same ; since clearly it is necessary for that section to be completed thoroughly, both for the quadrature of the circle and for other equations regarding circles; on account of which the properties of these are delayed, which are connected with circles and ellipses.

[Translator's note: Throughout this work, Gregorius distinguishes between equidistant and parallel straight lines: when the lines are coplanar, as in this case and other similar cases, there is no distinction, so that here the term parallel is applied rather than equidistant where appropriate.]

ARGUMENT.

This book shall be divided into eight sections in all.
In the first, the parabolic section is deduced from the cone, and its essential and fundamental properties established.
The second considers both discrete as well as continued proportions of lines in parabolas.
The third designates the focus of the section, and the mutual intersection of parabolas or circles.
The fourth considers the properties of parabolas amongst themselves in turn, either of intersecting parabolas, or of tangential circles.
The fifth considers the quadrature of the parabola.
The sixth brings together the segments and the parts of the parabola between them, then the maximum figure of the section to be inscribed.
The seventh shows the origins of various kinds of parabolas which moreover arise from lines, circles, and then from the parabola itself.
The eighth shows the wonderful parallel nature of parabolas with the hyperbola placed between the asymptotes, both in the origin, as well as in the symbolization of the remaining properties.

DEFINITIONS.

I.

The diameter of the parabola is the right line drawn within the parabola, which bisects all of certain parallel lines , and if indeed these are at right angles, it will be called the axis.

Indeed in every parabola the diameters to be parallel to the axis ; and for the section to occur at one point only, as will be shown in its place.

II.

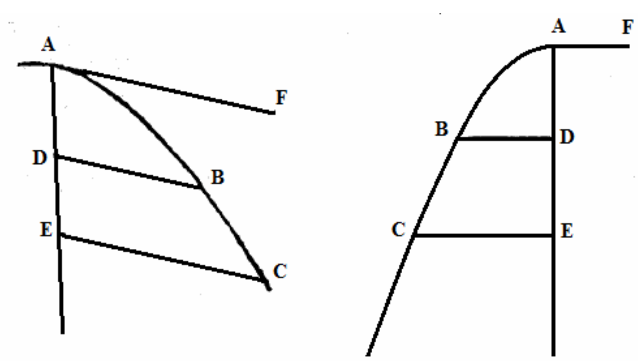
I call the the vertex the point at which the diameter of the section crosses over the perimeter : moreover the point which is common to the axis and to the perimeter of the section is called the vertex of the parabola.

III.

Each and every one of the parallel lines applied to the diameter, and of the bisected parts, is called an ordinate.

IV.

I call the latus rectum [*i.e.* the right side line] to be the ordinate line placed as near as the lines can be applied to the diameter: or the latus rectum is a measure of the magnitude of the length of the ordinate line to the measure of the diameters put in place.



V.

Again that it be peculiar to the parabola before the remaining sections shall vindicate the latus rectum, as the rectangle from the latus rectum and the part of the diameter from the vertex of the same to the point where it may be divided by the ordinate contained by the intercept, always may be placed equal to the square of the line which is called the ordinate of the point.

For example AD shall be the diameter in the case of the parabola ABC, and the latus rectum of that to be the right line AF, truly the ordinates put in place shall be BD, CE: therefore from the ideas of Apollonius, and which we will show also, the square BD shall be equal to the rectangle DAF; and with the rectangle EAF equal to the square CE: and thus likewise it will be shown from the remainder of the ordinates put in place.

That also is required to be observed here, as I have shown in the ellipse, diverse latera recta can be assigned with diverse diameters, when the ordinates of these lines can be said to be put in place ; also different magnitudes can be put in place.

Then there is no necessity for the right sides to be put normally at the ends of their diameters, but from the same therefore able be applied with the angle by which the diameters are intersected by the ordinates.

V.

I call the focus of the parabola the point placed on the axis, situated at a distance from the vertex, which is equal to the fourth part of the latus rectum.

VI.

I call parabolas parallel which for the same axis set up indeed have different apices, but equal latera recta, and concave perimeters in the same direction.

VII.

Parabolas are equal, of which the latera recta serving for the axis are equal.

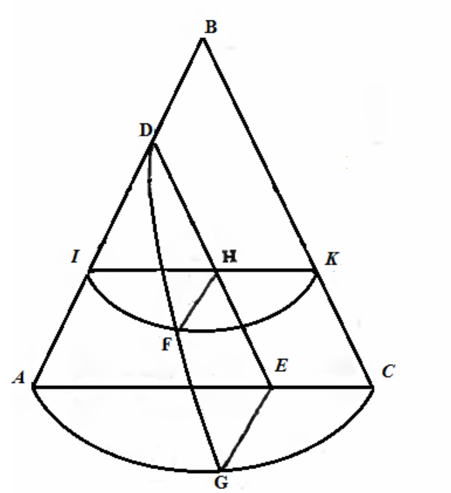
THE PARABOLA : PART ONE

The parabola is drawn from the cone, and set out with the expression of these essential properties and fundamental parts : and indeed in the first place the diameters and the ordinates for these to be put in place, and showing the particular properties of these: and secondly the nature of the latus rectum of these, then the main properties of intersections and tangents shall be designated.

PROPOSITION ONE.

Let the cone ABC be cut by the triangle ABC through the axis, and with ED parallel to the side BC, the section DG may be made through ED, following the right line EG normal to the line AC; moreover through some point H taken on the line ED, the right line IK may be drawn parallel to the diameter AC of the base of the cone: and the plane IFK is drawn through IK, parallel to the plane of the base AGC, crossing the plane DFG along the common intersection FH.

I say the square HF to be to the square EG, as the line HD to the line ED.



Demonstration.

Since the plane IFK shall be parallel to the plane of the base AGC; IFK will be a circle, and FH, EG the common parallel intersections; truly since AC, IK are parallel, and EG is normal to AC, also the right line HF is normal to IK: and hence the square FH is equal to the rectangle IHK : but the square EG, is equal to the rectangle AEC, therefore the square EG is to the square FH, as the rectangle AEC to the rectangle IHK: truly since EH is parallel to KC and thus HK, CE are equal lines in a parallelogram, the rectangle IHK is to the rectangle AEC, as IH to AE, that is as DH to DE; therefore as the square FH to the square EG, thus the right line DH to the right line DE. Q.e.d.

Moreover a section of this kind is called a parabola, the diameter is of which DE, and the ordinates FH, GE put in place for that.

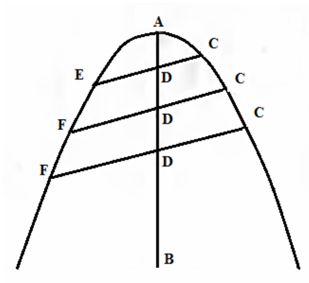
Scholium.

This demonstration is general, and agrees for every cone ; yet concerning that difference, which occurs in the scalene case, while the triangle ABC is not the maximum of these which arise through the axis, the right lines EG, FH *become the ordinates for the diameter* ED *put in place, which will cut the base at an oblique angle, as right line to right line: indeed, since* the *plane* ABC *drawn for the scalene cone through the axis, shall be oblique at the base; and moreover* ED *shall be a line in the plane* ABC, *for that also to be oblique to the base* AGC, *and thus also for the line* EG *which lies in the plane of the base, normal to AC itself in the plane of the base; therefore in the scalene cone, the right lines* EG, FH *put in place for the diameter* DE, *cut the same at oblique angles. Similarly we may show, in the right cone, the ordinate put in place to divide its diameter at right angles.*

Hence again a non trivial difficulty arises, since the parabola DFG *shall be the same in each cone, that is, no matter which diameter* ED *in the scalene must be, some other secondary kind of parabola must be considered in the scalene than that shown in the right cone: or what amounts to the same thing, or in the section of the right cone, some other diameter shall be as the ordinates for that put in place shall cut that at some oblique angle: for by the previous proposition it is allowed for each cone the line* DE *may be shown to be a diameter, as in the right cone for right angles, and for oblique angles in the scalene will cut the ordinate for that in place, yet that does not agree with the discourse from Apollonius, in which several diameters may be assigned to several parabolas of which one right, another oblique will cut the ordinates for these in place: and which may not yet agree with the inclination of the parabolas, some inclination of the line* ED *of the section of the axis may be considered, which the ordinates put in place may cut the diameters at oblique angles : but we will try to satisfy entirely the difficulties associated with this: and we will show the diameters to be given parallel in one and the same parabola, of which one for the right angle, and the other for the oblique angles, will cut the ordinates of these in place, and then no inclination of the parabola to be given.*

Initially the first property of the parabola may be agreed on, from what has been said, if the parallel lines DC *may cut the line* AB *at some angle, and so that the line* AD *shall be to the line* AD; *thus as the square DC will be to the square DC; the points* ACC *to be for the parabola*.

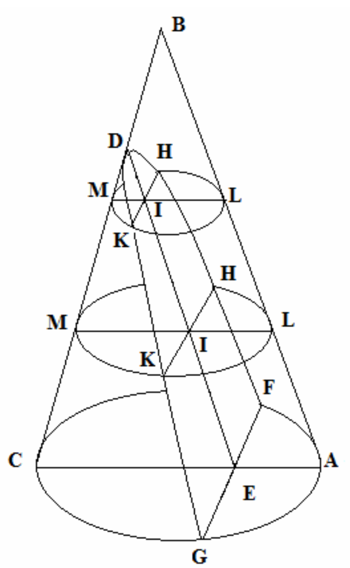
Secondly it is clear, if the ordinate CDE *put in place to* AB, CD *may be put in place parallel to that,* DF *may be added equal to* CD, *the point* F *pertain to the parabola* : *indeed since there shall be* AD *to* AD, *thus the square* CD *to the square* CD, *but the lines* CD *shall be equal to the lines* DE, DF: *also the squares* DE, DF *are proportional to the lines* AD, AD ; *whereby these points which we have called* E and F *partain to the parabola:which I have wished to note here individually and explicitely, thus so that afterwards they shall be required to be assumed.*



PROPOSITION II.

ABC shall be a triangle produced in the plane through the axis ABCG, and on putting DE parallel to the side AB, EG may be drawn normal to the diameter of the base AC, and following that also the right line ED, the plane may be put in place showing the section FDG in the conic surface: moreover some point I may be assumed on ED, through which the right line HK may be put in place in the plane FDG parallel to FG ;

I say HK to be bisected in I.



Demonstration.

The line LM may be drawn parallel to AC, to be put through the point I ; a second line LM may be drawn, the plane LKM being parallel to the plane of the base AGC: therefore LKM is a circle, and HK, FG common parallel intersections , and since by the hypothesis FEG is normal to the diameter AC, and likewise FG is bisected at E; HK also is normal to LM, and is bisected at I. Q.e.d.

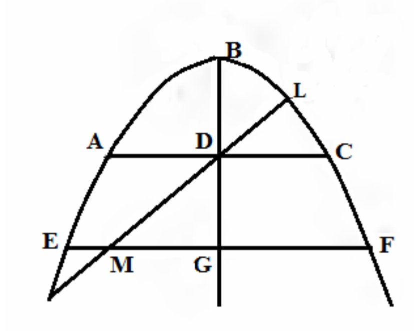
Corollary.

From this proposition it is apparent, in the parabola, that if some right diameter may be bisected, also all the similar parallel diameters shall be bissected, since ED shall be some diameter, and HIK some right line parallel to FG, to be bisected at E.

PROPOSITION III.

To show the diameter of a parabola for a given line cut at two points.

Construction & demonstration.

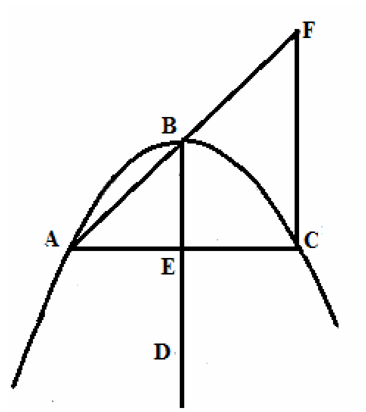


AC shall be the line to be bisected at D and EF put parallel, with which similarly bisected at G, the line BGD shall be drawn through G and D: I say that to be the diameter sought ; if not, LD shall be a diameter, which produced will cut EF at M : therefore since the diameter LD shall bisect AC, also it will be bisected at M, FE itself parallel to AC, but FE is put to be bisected at G: therefore it will bisected at G and M.Which cannot be done; therefore LD can be no other diameter than BD. Therefore we have, &c. Q.e.f.

PROPOSITION IV.

From a given point on the perimeter of the parabola, to put in place the given ordinate of the diameter.

Construction and demonstration.



A shall be a given point on the perimeter of the parabola ABC, and BED shall be the given diameter, for which the ordinate line AEC may be put in place from A, with AB joined and it may be produced to F, so that AB, BF shall be equal, and with FC sent from F parallel to BE, it shall be produced to the parabola at C, and AC shall be joined: it is clear AC to be bisected at E, since AF shall be bisected at B, and FC, BE parallel ; therefore from a give point on the perimeter, etc. Q.e.f.

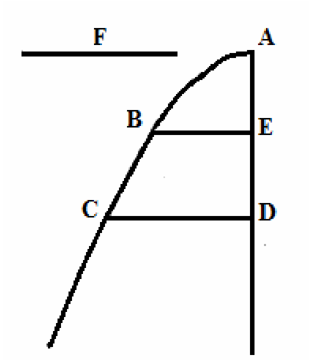
Corollary.

Hence it arises more easily in practise, through a given point on the diameter it shall be required to draw the ordinate line: indeed from some assumed point on the perimeter some ordinate may be put in place, to which through a given point on the diameter is shall be drawn parallel. It is apparent that the ordinate to the diameter can be put in place.

PROPOSITION V.

To place a given ordinate line to a given diameter in a parabola.

Construction & demonstration.

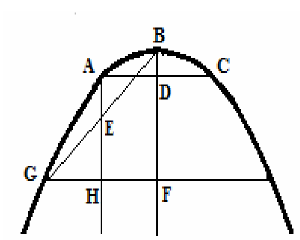


The parabola ABC shall be given ; and on that the diameter AD, to which it shall be required to apply the given ordinate line F ; from some point B assumed on the perimeter of the section, the ordinate line BE for the diameter AD may be drawn, and it shall be become so that the square BE to the square F, thus the line AE to the line AD, and through the point D, DC may be put in place parallel to EB. I say what is required to be done: thus since the part of the diameter AE is to the part of the diameter AD, just as the ordinate square BE, is to the square put in place CD, but from the construction the square BE to the square F, as the line AE to the line AD. Therefore CD is equal to F itself and it is parallel to EB; therefore we have put in place the ordinate line, etc. Q.e.f.

PROPOSITION VI.

The ordinates of two applied lines for two diameters of which one is the axis; with that smaller one to be applied to the axis, to be equal only if the distances from the vertex point were equal.

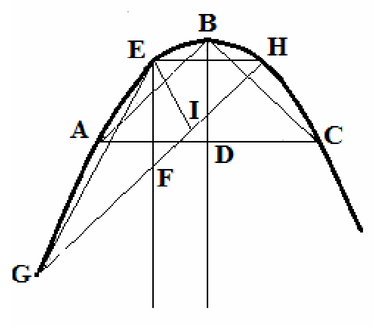
Demonstration.



BD shall be the axis of the parabola ABC axis and AC put to be the ordinate for that : then with AE drawn parallel to BD ; BF shall become four times BD, and FG shall be put parallel to DA, then with FG bisected at H; there may be put HA parallel to BF, and BG may be joined: since BD to BF, from the construction has that ratio of one to four, therefore the square AD to GF, is as one to four; therefore GF is twice AD, that is HF, and BF is twice EH, and hence BG is twice GE; likewise AE is equal to BD, and AC to GF ; from which the ordinate BG is put in place for the diameter AE; but GB is greater than GF, that is AC, therefore the ordinate GB put in place for AE, to be equal to BD, but greater than the right line AC, which is arranged for the ordinate of the axis ; and since ratio preserves the same quadruple of the applied lines, as the parts of the diameters between the vertex and the applied ordinates put in place, hence the truth of the proposition agrees universally for all the applied lines.

Otherwise.

This proposition is demonstrated otherwise by Archimedes. BD, EF shall be diameters of the same height, and indeed BD the axis : and the ordinate lines AC, GH shall be put in place through D and F.



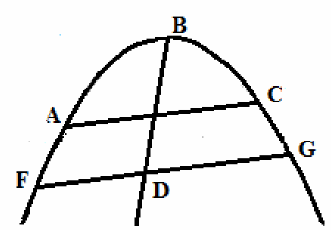
I say the line AC to be smaller than the right line GH, and indeed ABC, GEH may be joined: and the right line EI may be sent from E normal to GH. Therefore since the distances EF, BD, are equal, the triangles GEH, ABC are similar, by Archimedes. Whereby as EI ad BD, thus AC to GH: but BD, that is EF, to be greater than EI (since the angle I in triangle ElF is right): therefore GH will be greater than AC. Q.e.d.

PROPOSITION VII.

To put in place the ordinate for a given point on the diameter of the parabola.

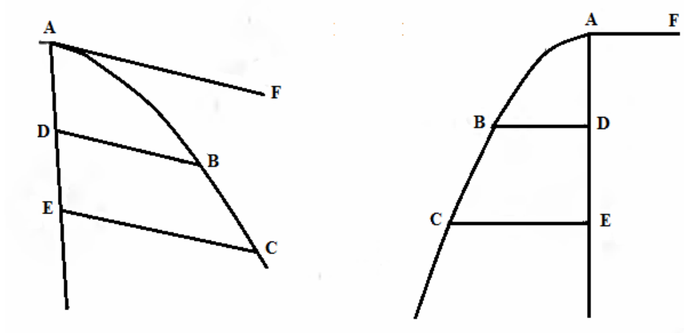
Construction & demonstration.

ABC shall be the given parabola, some diameter of which shall be BD, and on that the point D assumed through which it will be required to locate the ordinate for the diameter BD ; for some A point assumed on the perimeter some line AC shall be drawn, the ordinate for the diameter BD is placed parallel to that through the point D: it is evident FDG to be the ordinate put in place and to be bisected at D. Therefore we have accomplished what was sought.



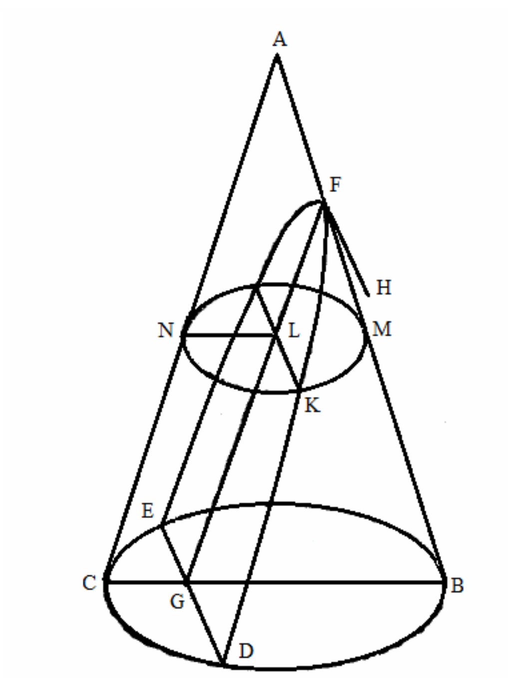
PROPOSITION VIII.

To find the latus rectum of a given diameter of the parabola.



Construction & demonstration.

ABC shall be the parabola, and it shall be required to establish its right line on that diameter AD. Some ordinate BD shall be put in place for the diameter AD, and AD, DB, AF shall become continued proportionals. I say AF to satisfy the demands of the proposition : indeed some other line EC may be put in place parallel to DB therefore so that as AD to AE, thus the square DB to the square EC: moreover as AD to AF, thus the rectangle FAD to the rectangle FAE; therefore so that as the square DB is to the square EC, thus the rectangle FAD is to the rectangle FAE and on permutating so that as the square DB to the rectangle FAD, thus the square EC to the rectangle FAE, but the square BD is equal to the rectangle FAD, because AF, BD, AD, are proportionals; and therefore the square EC is equal to the rectangle FAE, and hence the right line FA of the diameter AD: therefore we have shown, &c. Q.e.f.



Scholium.

It pleases to put in place here the method and the construction, by which Apollonius devised the latus rectum of the parabola in Conics Bk.1 : as well as to show quickly the latus rectum found by us in the preceeding proposition, to be the same as that which Apollonius found by another construction.

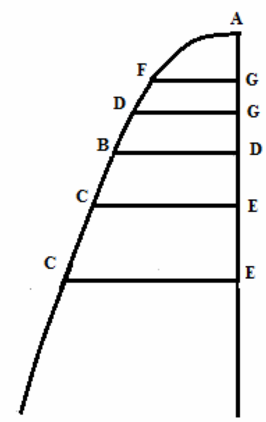
There shall be, he says, the vertex A *of the cone, the circle* B, C *the base*: *and it shall be cut by a plane through the axis which shall make the triangle* ABC ; *and with the base of the cone cut by another plane along the right line* DE *which shall be perpendicular to* BC, *and the line shall make a section on the surface of the cone* DFE, *moreover the diameter of the section* FG *shall be parallel to one of the sides* *of the triangle through the axis : and from the point F of the line* FG, FH *shall be drawn at right angles, and so that the square* BC *shall be to the rectangle* BAC, *thus as the line HF to the line FA* ; *moreover some point K may be taken in the section, and through* K, KL *is drawn, parallel to* DE. *I say the square* KL *to be equal to the rectangle* HFL. *Again he demonstrates the assertion by the most learned and sublime discourse; which since it shall be more difficult for beginners, we have tried to establish more easily from the latus rectum method in the previous proposition, indeed we show with the same figure in place, if* FL, KL, FH, *may become continued proportionals,* FH *to be the latus rectum. Now it remains that I may show, that to be the same as that which Apollonius has devised from the aforesaid construction. With the* MN *line acting through* L *parallel to* BC, *the plane which passes through* LKMN, *parallel to the base plane of the cone, and thus to be a circle: and the square* LK, *that is, from the hypotenuse the rectangle* HFL, *equal to the rectangle* MLN*; whereby the rectangle* MLN *to* LFA *itself, is as* HFL *to* LFA *: but* HFL *is to* LFA *as* HF *to* FA, *therefore as* HF *to* FA, *thus* MLN *to* LFA*: but the ratio* MLM *to* LFA *is composed from the ratio* ML *to* LF, *and from* LN *to* FA; *therefore the proportion* HF *to* FA, *is composed from* ML *to* FL, *and* LN *to* FA: *but* ML *is to* LF, *as* MN *to* NA, *and thus* LN *to* FA, *as* MN *to* MA*; therefore the proportion* HF *to* FA, *is composed from the ratio of the square* BC *to the rectangle* BAC, *therefore* HF *to* FA, *to be as the square* BC *to the rectangle* BAC*: but with the proportion put in place of the square of the line* HF, *to that same line* FA, *since that which the square* BC *has to the rectangle* BAC, HF *will be the line through the Apollonium latus rectum of the parabola ; therefore if* FL, KL, FH *are continued proportionals,* HF *will be the same latus rectum as that which otherwise Apollonius has devised, from which it is apparent each construction to be the same and the one only to be changed into the other; indeed with the proportion put* HF *to* FA, *which is of the square* BC *to the rectangle* BAC; *Apollonious says* HF, KL, FL *to be continued proportionals, and thus we are able with the line* LK *and the rectangle* HFL, *truly with the three* HF, KL, FL *put in place we are able to infer* HF *to be the latus rectum, and all the sides to have the right properties ;truly since with these three put in place continued it follows also* HF *to* FA, *to be as the square* BC *to the rectangle* BAC, *it is apparent* HF *to be the same latus rectum as that which Apollonius proposed.*

Truly, it is apparent from above, for any applied lines thus to become greater in turn, when their diameters are placed further away from the vertex, for the rectangle under the right side will always increase, and with which put in place in order, and it is necessary with that part of the diameter determined before, and the right sides shall be of such a size, of which we will assign the cases in the following propositions.

PROPOSITION IX.

AD shall be the diameter of the parabola ABC equal to the latus rectum, from D the line DB may be put in succession to the diameter AE.

I say the line DB to be equal to AD, and if AD, DB may be equal lines, I say AD to be equal to the latus rectum, which serves for the diameter AD.



Demonstration.

Because the line BD is the ordinate which may be applied to the diameter AD, the square BD will be equal to the rectangle on AD and with the right side, but the line AD is put equal to the right side, therefore the square BD is equal to the square AD, and thus BD, AD are equal lines.

Now the lines AD, BD shall be equal, and a certain ordinate BD put to the diameter AD, I say the line AD to be equal to the right line, since indeed the square BD may be put equal to the square AD, moreover in addition, with the square BD equal to the rectangle on AD and with a right side, the square AD will be equal to the rectangle on AD and with the right side, and thus AD is equal to the right side. Q.e.d.

PROPOSITION X.

With the same in place the ordinate lines CE, FG may be drawn in succession: and indeed CE may fall below BD, truly with the line FG above.

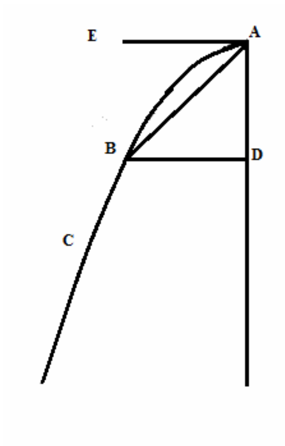
I say AG to GF to be the ratio of a smaller inequality, and AE to EC to be the ratio of a greater inequality.

Demonstration.

Since the line AD may be put equal to the right side, the square FG will be equal to the rectangle GAD, and thus AG, G F, AD to be continued proportionals ; but AD that is BD, shall be greater than FG, and therefore FG is greater than AG. Which was the first part.

Again since the rectangle DAE shall be equal to the square CE, the lines AD, CE, AE will be proportionals : but AD that is BD, is smaller than CE from the previous demonstration; and therefore CE is smaller than the right line AE: and thus the ratio AC to CE is the greater inequality. Q.E.D.

PROPOSITION XI.



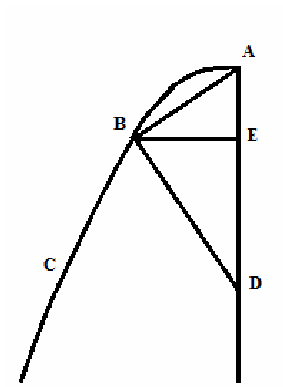
Let AD be some diameter of the parabola ABC, and from A, the ordinate AE to the diameter AD may be put in place; also from A the line AB may be put in place, bisecting the angle EAD, and crossing the parabola again at the point B, from which the ordinate BD to the diameter may be put in place.

I say the line AD to be equal to the latus rectum.

Demonstration.

Since AE, BD are parallel, the angle EAB is equal to the angle ABD; but from the hypothesis the angle EAB is equal to the angle BAD, therefore the angles ABD, BAD are equal: and thus also the lines BD, AD shall be equal: from which AD is equal to the latus rectum. Q.E.D.

PROPOSITION XII.



AD shall be the axis of the parabola ABC, and with the line AB sent from A , again it will cross at the point B of the parabola, from which the ordinate BE may be put in place to the axis; and also the normal BD shall be drawn to AB, crossing the axis at D.

I say DE to be equal to the latus rectum.

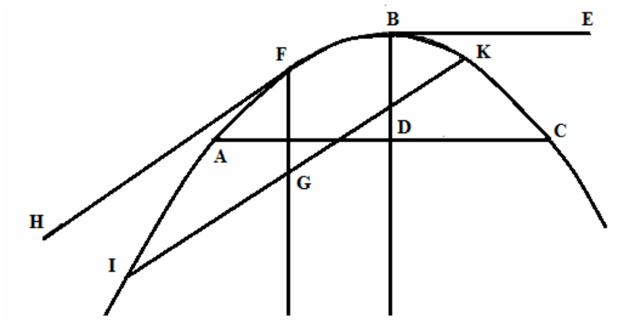
Demonstration.

Since the angle ABD is right, and BE normal to the axis AD, the lengths AE, EB, ED are proportionals: and the square EB equal to the rectangle AED : yet also the rectangle on AE and the latus rectum is equal to the square EB [Prop. VIII], therefore the rectangle AED is equal to the rectangle under AE and the latus rectum; therefore ED is equal to the latus rectum. Q.e.d.

PROPOSITION XIII.

The latus rectum of the axis, is the minimum of the latus rectum lines of the rest of the diameters.

Demonstration.



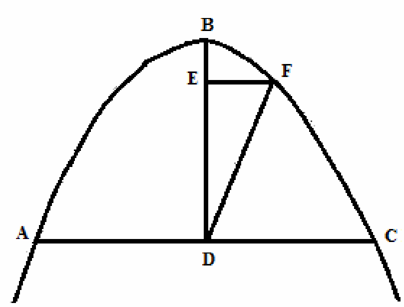
BD shall be the axis of the parabola ABC, and BE shall be its latus rectum, moreover FG shall be some other diameter, and its latus rectum FH may be put in place : I say BE to be smaller than the latus rectum FH: for ADC may be put to be the ordinate for the axes, and with FG taken equal to BD, the ordinate for the diameter FG may be drawn through G, therefore the line IK is greater, therefore IG is greater than AD, and the square IG is greater than the square AD: but the square IG is equal to the rectangle on HF, FG, and the rectangle EBD is equal to the square AD, but therefore the rectangle HFG is greater than the rectangle EBD ; but from the construction BD, FG are equal; therefore the latus rectum FH is greater than the latus rectum BE: therefore the axis latus rectum is the minimum, &c. Q.E.D.

PROPOSITION XIV.

BD shall be the axis of the parabola greater than the latus rectum : and through D the ordinate ADC to the axis put in place: with DE made equal to the latus rectum, and EF may be put in place parallel to AC, and the points D F are joined.
I say the line FD to be equal to the line DC.

Demonstration.

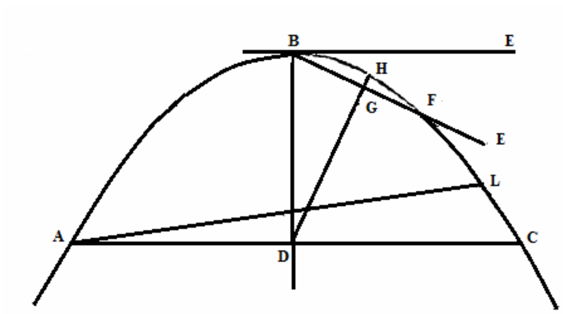
Since ED is equal to the latus rectum, the square FE is equal to the rectangle BED ; therefore with the square DE added, the squares FE, DE are taken together, that is, the square FD, on account of the right angle FED is equal to the rectangle BDE: but the rectangle BDE is equal to the square DC: therefore the squares FD, DC are equal. Q.E.D.



PROPOSITION XV.

Any line acting through the vertex of the diameter and with its ordinate placed parallel, will be a tangent to the section : and on the other hand any line sent from the point of contact required to be a tangent shall be parallel to the diameter.

Demonstration.

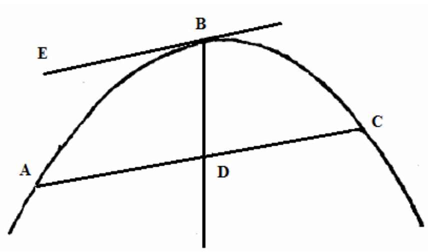


The ordinate ADC shall be put in place for the diameter BD of the parabola ABC: to which BE may be put parallel through the vertex B of the diameter; if indeed it is not a tangent to the parabola, then it may intersect that at F; and BF may be divided into two equal parts at G, the line GD may be put through G and D ; therefore since the line GD shall be put to divide the lines FB, AC, into two equal parts, that will be the diameter for which AC will be the ordinate; but in addition AC also is the ordinate applied to the diameter BD, since it will be cut into two equal parts by that ; therefore the line AC put in place is the ordinate for two diameters, which cannot be done : otherwise indeed AC itself will be bisected by the diameters BD, GD at two different points. Therefore BE cannot cut the parabola, but must be a tangent at B.

Now EB shall be a tangent, and for that to be parallel to AC: I say that ordinate to be applicable to the diameter BD. For if not, AL shall be put to be the ordinate for BD, therefore AL shall be parallel to the tangent BE; and since AC also is parallel to the tangent, AL itself will be parallel to AC, which cannot happen, since the lines cross over each other at A ; therefore AL cannot be put in place for the ordinate BD, otherwise the line AC shall be parallel to the tangent BE. Q.E.D.

PROPOSlTION XVI.

To deduce the tangent through a given point on the perimeter of the parabola.



Construction & *demonstration.*

B shall be a given point on the perimeter of the parabola ABC, it shall be required to put the tangent line through that point : the diameter BD shall be sent from the point B, to which the ordinate AC may be put in place, to which the parallel line BE may be drawn through B : it is evident that to be the tangent ; therefore through the point, &c. That which was required to be done.

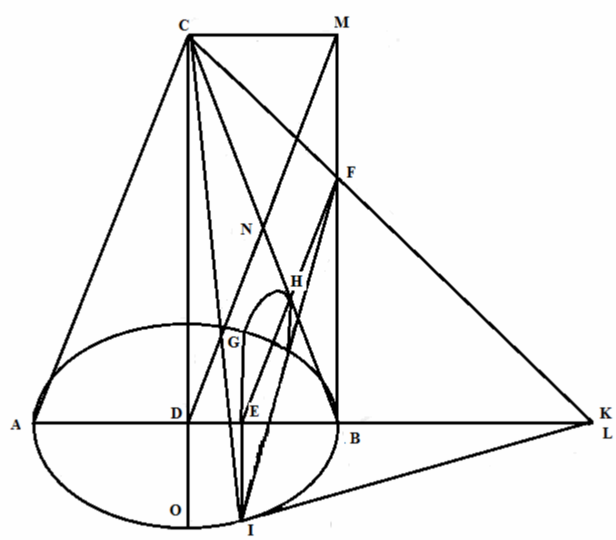
PROPOSlTION XVII.

It shall be agreed the parabola to have a tangent drawn from the diameter : to which a certain ordinate shall be put in place from the point of contact.

I say the diameter intercepted between the applied ordinate and the point at which the tangent meets the diameter from the parabola, to be divided into two equal parts.

Demonstration.

ABC shall be some cone, with a triangular section ABC through the axis, moreover the



diameter of the base AGB shall be AB, on which from some point E (assumed not to be the centre) put in place, so that EH becomes parallel to AC: and EI to be normal to AB: then through HE, EI shall become the section, showing the parabola GHI; and on putting CD for the axis of the cone, the parallelogram CDBM shall be completed, of which the side BM will cross EH producted at F*:* and with CF joined to the diameter AB produced will cross at K: then the tangent of the circle AIB at I then may be put meeting AB at L, which is the same as the point K: now since CK shall be to FK, as CD to FB, that is as MB to FB, that is as DB to ED; also there will be DK to BK, as DB to EB, and on dividing BK to DB, as BE to ED, and on taking DK to DB, as DB to DE ; therefore the proportionals are DE, DB, DK: therefore the tangents put through I agree with the diameter at K: therefore the point K & L is the same; further, the points CE, CI may be joined; therefore since the line CI is on the surface of the cone, the triangles are CEI, CIK, and CIK is a tangent to the cone put on the line CI and finally in the plane CIK with the line IF, will be that tangent at I: since indeed the plane CIK shall be a tangent to the cone, but the line in the same plane shall be FI, and likewise in the plane of the parabola, since the points K and I, shall be the same, it is clear FI to be the tangent of the parabola at I.

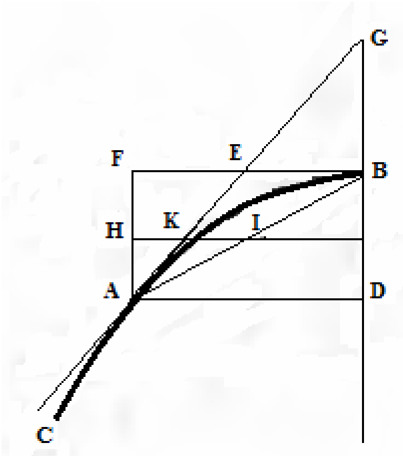
Further since the line CM shall be equidistant and parallel to the semidiameter DB, also AD is equidistant and parallel: from which AC, DM are parallel: and since FE shall be equidistant to AC, EF will be equidistant from DM itself ; but MD is the diameter of the parallelogram DM, bisected at N by the diameter CB, therefore the right line FE at H, also is bisected by the same diameter ; therefore the lines EH, HF are equal. Therefore the tangents to the parabola, &c. Q.E.D.

So that if the assumed point shall be the centre itself, it is clear indeed the demonstration will show in the same manner OM to the tangent of the parabola which is drawn through the lines ND, DO, &c.

PROPOSlTION XVIII.

The right lines AE, BE shall be tangents to the parabola ABC at A and B, of which the diameter is BD, the right lines meeting at E: and EB and indeed AF, parallel to BD, will meet at F.

I say that FB to be bisected at E.



Demonstration.

AE produced shall meet the diameter at G, and the ordinate line AD may be drawn from A ; since AD, EB are parallel, so that as GB to GD, thus EB is to AD, that is FB, but from Prop. XVII, GB is half of GD , and therefore EB also is half of FB. Q.E.D.

Corollary 1.

Hence it is apparent, if with AB joined some part HI may be put parallel to FB, that requiring to be bisected by the tangent AG.

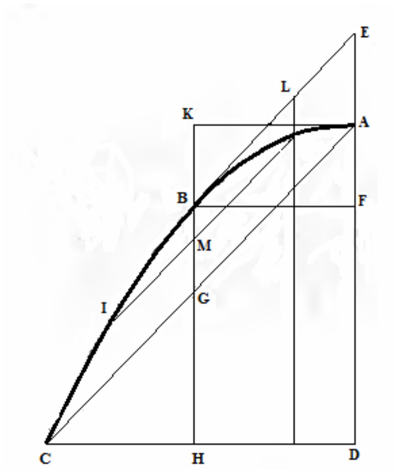
Corollary 2.

It follows secondly the right lines FA, GB, likewise AE, GE to be equal and thus the triangles AFE, GEB to be equal, it is apparent, since FE, EB shall be shown to be equal, and AFE, GEB to be similar triangles on account of GB, FA being parallel.

PROPOSITION XIX.

In the parabola all the diameters are parallel to the axis.

Demonstration.



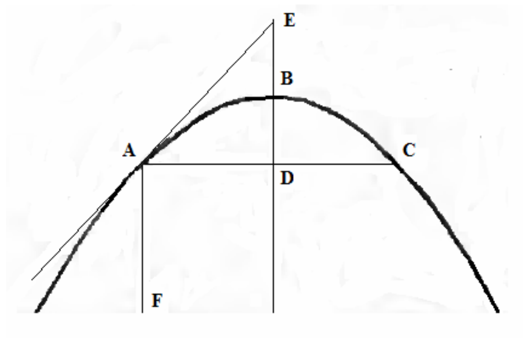
AD shall be the axis of the parabola ABC, for which the tangent BE shall occur at E, and on putting AC equidistant parallel to the tangent BE, the line BG shall be sent from B, equidistant parallel to the axis, and there is put BF, CHD to be ordinates to the axis AD, and BG produced crosses the tangent drawn through A at K: Because BE is the tangent and BF the ordinate to the axis applied to the axis AD, they are equal to the lines EA, FA and since BF is the common height, the triangle EBF is equal to the parallelogram BA and again as AF to AD, thus FB squared to CD squared, and the parallelogram FK to the parallelogram DK: therefore so that as the square FB is to the square CD, thus the parallelogram FK to the parallelogram DK, but as the square FB to the square CD, thus triangle EBA to triangle CAD; therefore as parallelogram FK to parallelogram DK, thus triangle EBF is to triangle ACD*:* and on permutating so that parallelogram FK to triangle BFE, thus parallelogram DK is to triangle ACD: moreover triangle BFE and parallelogram FK are shown to be equal; therefore parallelogram DK and triangle ACD are shown to be equal: and with the common AGHD taken away, the equal triangles AGK, CHG remain. From which since they are similar too, on account of the equidistant lines AK, CH, the sides KG, GH, as well as CG, GA shall be equal: therefore AC is bisected. In the same manner if IK shall be put equidistant parallel to CA, and from K the diameter LK shall be put in place crossing BE at L, it may be shown the line IK to be bisected at M by the line BG: therefore the diameter of the section is BG: whereby since EB may be any tangent, and BH any line equidistant to the axis, it is evident all the diameters to be equidistant parallel to the axis. Q.E.D.

From which it follows all the diameters in the parabola to be parallel.

PROPOSlTION XX.

Every tangent drawn through the end of a ordinate meets its diameter.

Demonstration.

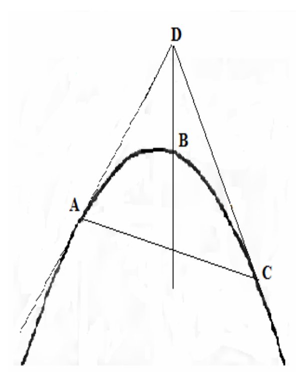


The diameter put in place shall be BD, and the applied ordinae for that AC : I say the tangent acting through A, to cross that diameter BD: for with the diameter AF drawn, it will be parallel to that diameter BD, from which since AE cuts AF once, since it shall be parallel, also the other diameter DB will be intersected on being produced. Q.E.D.

Corollary.

Hence it is evident, whatever the two tangents of the parabola, to meet at some point outside the section: to be apparent from that demonstrated.

PROPOSITION XXI.



Tangents of the applied ordinate acting through the ordinate ends with the same diameter, to meet in one and the same point.

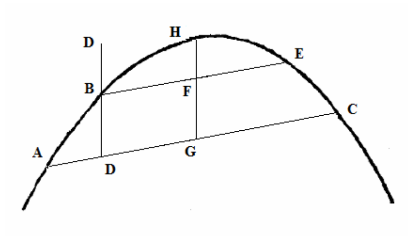
Demonstration.

BD shall be a diameter of the parabola ABC ; with the applied ordinate AC, I say the tangents drawn through A and C, to cross the diameter BD at one and the same point D. The demonstration shows, when the part of the diameter from the section and from the section intersect, the intercepts of the part from the section and from the applied ordinate shall be equal. Therefore the tangents, &c. Q.E.D.

PROPOSITION XXII.

To draw the diameter of the parabola through a given point on the perimeter.

Construction & *demonstration.*

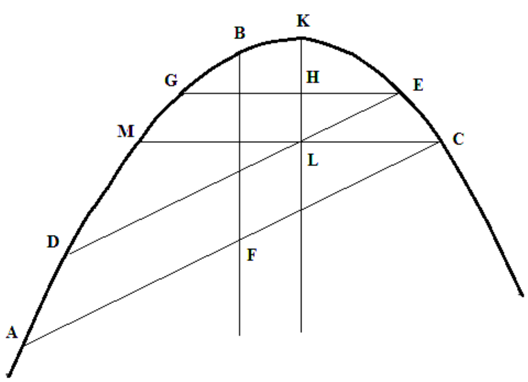


The point B shall be assigned to the perimeter ABC, from which it shall be required to put a diameter in place; with some secant BE drawn, its diameter HF may be shown, to which BD may be put parallel through B: it is apparent that section to be a diameter: therefore from a given point, etc. Q.E.D.

Corollary.

Generally we will use the same method, if from a given point D, either within or outside the section [of the cone], it will be required to put the diameter in place, the construction and demonstration follows from the first proposition.

PROPOSITION XXIII.

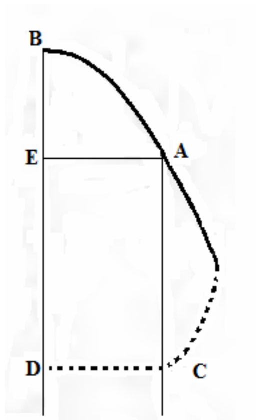


To show the axis of a given parabola.

Construction & demonstration.

ABC shall be the parabola of which it shall be required to find the axis, some two parallel lines may be put in place DE, AC of which the diameter BF may be shown: to which EG and CM may be drawn normally from E and C ; and for one of these, consider EG to be bisected at H, and the diameter through H may be drawn parallel to BF. I say that to be the axis, since indeed it shall be parallel to the diameter BF, that also will be a diameter of the section, and because of EG, CM being parallel one shall cut at right angles, the other bisected and cut at right angles: therefore KL is the axis of the section; therefore we have shown, &c. Q.E.D.

PROPOSITION XXIV.



All the lines in the parabola parallel to the axis will cross the section in one point only.

Demonstration.

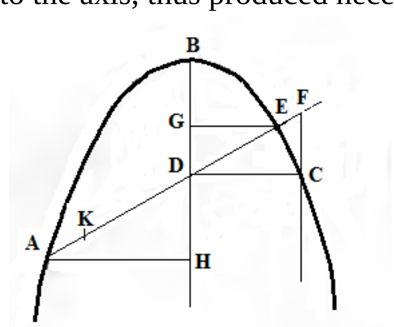
BD may be put for the axis of the parabola ABC; and some other parallel to that I call AC, AC to occur at only one point of the section: but truly, if it may occur again at C : and AE, CD may be put to be the ordinates to the right axis: therefore there will become as the line BE to the line BD, thus the square EA to the square DC, which cannot happen, since the squares AE, CD would be equal to each other, (on account of the parallelogram AD) and BE a smaller right line than BD. Therefore the diameter of the parabola AC occurs only once: Q.E.D.

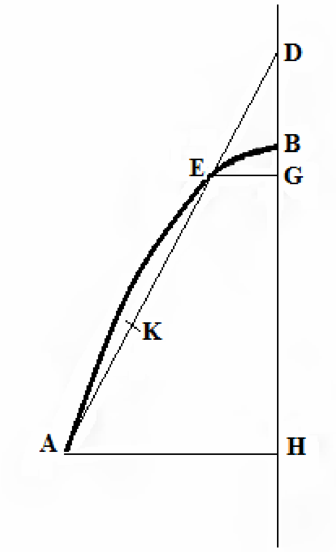
PROPOSITION XXV.

Every line in a parabola, which is not a diameter of the section, intersect at two points.

Demonstration.

BD shall be put to be the axis of the parabola ABC, and KD a line which shall not be a diameter, I say to cross that section : Since KD is not a diameter and shall not be parallel to the axis, thus produced necessarily it will cross that axis at some point D (since it shall be present in the same plane) so that if D were some point within the parabola, the ordinate DC may be put in place from D, and through C the diameter EF ; it will meet that section at one point only ; and since for the same parallel axis produced also will cross KD at some point F, which is outside the section. From which in the first place KDF necessarily will meet the section at some point E. Therefore with the ordinate EG drawn from E, BG, BD, BH will become proportionals, and on putting the ordinate of the line for H, KD itself will cross the section at some point A. Because BG, BD, BH are put to be proportionals, HD to DG, will be as HB to BD, and the square HD to the square DG, thus as the square HB to the square DB, that is, as the line HB to the line BG : but as the square HD to the square DG, thus the square HA quadratum to the square GE: therefore as the line HB to the line BG, thus the square HA to the square GE, from which the point A is for the parabola, and the line DK, a line not parallel to the axis, will cross the section at both sides.





Truly if the line DK shall meet the axis outside the section, it is evident from before the parabola to cross the section at some point E: from which the ordinate line EG may be drawn, BG, BD, BH will become proportionals, and from H the lines HA and EG will be parallel, crossing the line KD at A, therefore since HB to BD, thus BD to BG, and on permutating there will be put in place HD to DG, thus as DB to BG, but as HD to DG: thus AH to EG, therefore as DB to BG, that is by the construction BH to BD, thus as AH to EG. From which the square HA to the square GE, is as the square HB to the square BD, that is (since HB, BD, BG, are continued proportions), as the line HB to the line BG, and hence the point A is for the parabola, and AD cuts the line of the section twice. Q.E.D.

Corollary.

Hence a pretty proposition is established: especially if BG, BD, BH may be put in place in continued proportion and from G the ordinate of the line GE be drawn; and a line may be acting through the points E and D , crossing the line drawn from H : and parallel to GE at A: so that the point A shall be on the parabola. The demonstration will be had from the first part of the proposition.

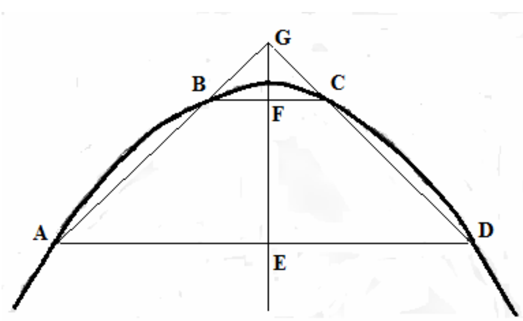
Secondly it follows no line of the parabola to be crossed in more than two points. Indeed since the section of the cone shall be a parabola, and in the cone itself no lines shall occur through more than two points, it is evident nor for any section of the cone, a line to occur through more than two points.

PROPOSITION XXVI.

Any two parallel lines AD, BC may cut the parabola ABC of which the diameter is put to be EF.

I say the lines joining AB, DC cross the diameter at the same point G : and if BC, AD were parallel and AB, CB shall be joined together at G. I say G to be a point in the diameter of the lines BC, AD.

Demonstration.



Because EF is the diameter of the right lines BC, AD, therefore BC, AD are bisected at the points F and E. From which as AE to BF, thus AG is to BG: that is as EG to FG. But as AE to BF, thus ED to FC; Therefore as ED to FC, thus is EG to FG ; that is DG to CG. Therefore the point G is common to the three lines AB, DG, EF. Which was the first part.

Likewise the same is shown if the point G may fall within the parabola, if now AD, BC may be put parallel, and AB, CD may cross each other at some point G, I say G to be a point on the diameter for which BC, AD may be put to be ordinates.

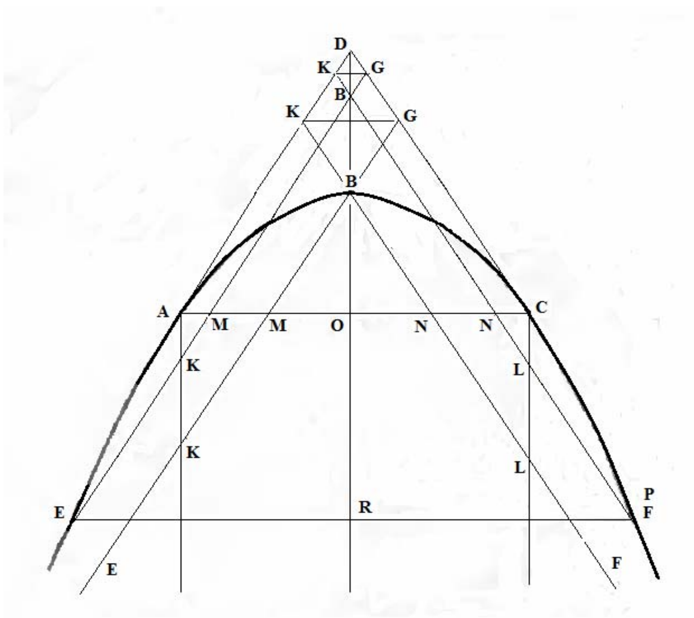
Indeed BC may be bisected at F and the line GE may be sent from G through F : therefore since AD, BC may be put parallel, there will become AE to ED, as BF to FC; but BC is bisected at F, and therefore AD also is bisected at E: from which GE is the diameter of the lines BC, AD &c. Q.E.D.

PROPOSITION XXVII.

AC shall be the ordinate applied to the diameter BD of the parabola ABC, and these tangents AD, CD, acting through A and C, will meet diameter at D: then the point B may be taken, somewhere in the diameter BD: and from B the right lines BE, BF may be put in place, parallel to the ordinates, truly crossing the parabola at E and F.

I say the line EF joined to be parallel to AC, and thus the ordinate to be put in place for the diameter BD.

Demonstration.



The diameters AK, CL may be sent from A and C, the diameters crossing with the right lines BE, BF at L and K: and EB, FB may be produced ; with the tangents crossing at H and G: therefore HD, GB will be a parallelogram, and with HG bisected by the diameter DB, therefore HG, AC are parallel: and since also AD, BE are parallel, HG, AM are equal : similarly since HG, NC, shall be equal, the right lines AM, NC are equal: but AC is bisected at O, therefore the parts remaining MO, ON are equal : whereby as AN to MO, thus CN to NO. But as AM to NO, thus AK to BO; and as CN to NO, thus LC to BO, therefore as AK to BO thus CL to BO, therefore the diameters AK, CL are equal: truly since EB, FB are parallel to the tangents, the parts of these which intercept the parabola, are bisected in K and L. Further the line EP may be put through E parallel to AC, the line crossing the parabola at E and F, the right line BF at P. Therefore since AC is bisected by the diameter BD, and EF by the same will be bisected at R, & ER, RF are equal to each other; truly since AC shall be parallel to HG, also EP, HG will be parallel : and EP also so that HG, bisected by the diameter BD; therefore RP, RF are equal : and the points F, P are one and the same : and F is the common intersection of the lines EF, BF with the parabola: therefore EF, AC, HG are parallel. Q.E.D.

Corollary.

From these it follow in the first place: with the ordinate EF put to the diameter BD as the two tangents AD, CD may be cut at D ; the lines AD, CD drawn from E and F also are parallel to the diameter BD, to cross in one and the same point ; the demonstration is clear from the preceding, of which it is the converse.

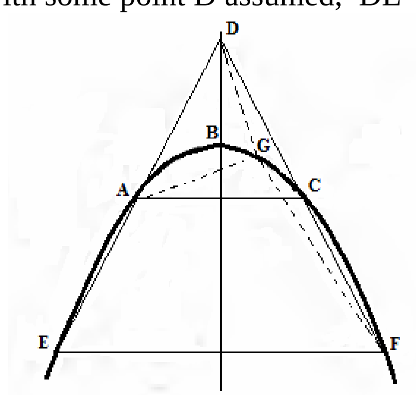
It follows in the second place, with the ordinate AC put for the diameter BD, and with the equal diameters AK, CL dropped from A and C to the diameter BD, so that the right lines through K and L of the ordinates put in place, will cross the diameter DB at one and the same point; it is apparent from the demonstration mentioned before, since the tangents act through A and C, with the ordinates through K and L put parallel, and cross with the diameter BD at one and the same point.

It follows in the third place: the line (QT) to be tangent at the point, at which the right lines (BE, BF) occur parallel to the equidistant tangents, to be equidistant to the right line (EF) with the ends of the lines BE, BF joined together, and thus the lines QT, AC, EF to be parallel, to be evident from the demonstration stated before. [Points and lines not shown on the diagram.]

PROPOSITION XXVIII.

Let BD be the diameter of the parabola ABC, in which with some point D assumed, DE may be dropped cutting the parabolas in the two points A and E and from E the ordinate EF may be put in place, and with FD joined it shall meet the parabola at C.

I say that EF, AC to be parallel lines.

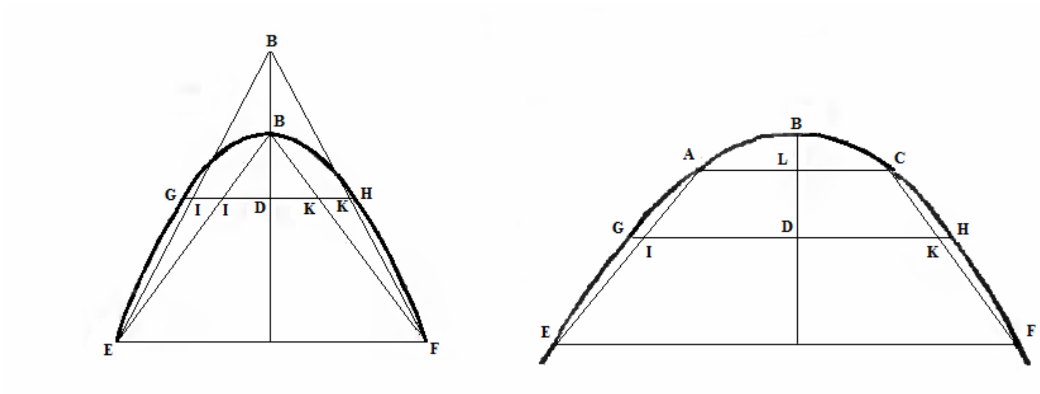


Demonstration.

Indeed if they shall not be parallel, AG may be put equidistant to EF, and from F, FG may be drawn through G, and that will meet the diameter BD at D, but from the construction the right line DF is crossing the section at C ; therefore the right line FGD, is the same as FCD and from which the point G the same as the point C, therefore EF and AC are equidistant. Q.E.D.

PROPOSITION XXIX.

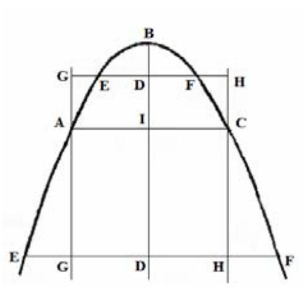
Some lines AC, EF will be equidistant in the parabola, and with AE, CF joined there may be put some other line GH parallel to AC, crossing AE, CF joined at I and K.
I say GI, KH to be lines equal to each other.



Demonstration.

The diameter of the right lines AC, GH shall be put to be LD. Since HG shall be equidistant to the ordinatim of AC put in place, HG will be bisected at D : but ID is equal to DK since there shall be ID to DK, as AL to LC (since EA, FC are lines meeting at the same point of the diameter) and therefore the remainders IG, HK also are equal. Q.E.D.

PROPOSITION XXX.

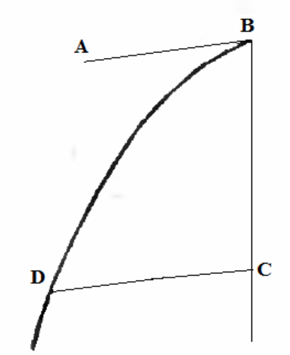


Again the right lines AC, EF will be equidistant in the parabola ABC, and the diameters AG, HC may be put through A and C crossing the line EF at G and H.
I say the lines EG, FH to be equal to each other.

Demonstration.

ID shall be put to be the diameter of the lines AC, EF. Since EF, AC are the ordinate lines put in place for the diameter ID; AC, EF will be bisected at D and I, and also GH is bisected at D, since GH is equal to the right line AC, therefore the remaining EG, HF, are equal to each other. Q.E.D.

PROPOSlTION XXXI.



With the angle ABC given and the point D in that, it will be required to describe the parabola through B and D of which the diameter shall be BC and the tangent AB.

Construction & demonstration.

The line DC may be drawn from D parallel to AB crossing the line BC at C: and there will become so that as BC to DC, thus DC to AB, AB will be the latus rectum of the parabola sought, and hence the parabola has been determined which was desired.

QUADRATURAE CIRCULI

LIBER QUINTUS

DE PARABOLA.

Sectionem hoc libro explanadam aggredimur ab antiquis coni rectanguli sectionem dictam, ab Apollonio & recentioribus parabolam nominatam, atque in eiusdem explicatione nonnihil morosiores erimus; quoniam plane sectio illa ad circuli quadraturam & alias aequationes cum circularibus perficiendas necessaria est, ob admirandas eius proprietates quae cum circularibus & ellipticis plane connexae sunt.

ARGUMENTUM.

Dividetur liber hic in partes omnino octo .

Prima sectionem e cono educit, passionesque illius essentiales & reliquis fundamentales exhibet.

Secunda linearum in parabola proportionem tam continuam quam discretam considerat.

Tertia sectionis focum, & mutuas parabolarum intersectiones Geometrice designat.

Quarta parabolarum, sese mutuo, vel circulum intersectantium contemplatur affectiones.

Quinta parabolam tam convexam quam concavam quadrat.

Sexta, parabolae & segmenta inter se confert, dein maximas sectioni inscribit figuras.

Septima varias exhibet parabolae geneses quae tum ex lineis,circulis, tum ex ipsa oriuntur parabola.

Octava miram exhibet parabolarum parallelarum cum hyperbola inter asymptotos posita, tam in ortu, quam reliquis proprietatibus symbolifactionem.

DEFINITIONES.

I.

Diameter parabolae est recta linea intra parabolam ducta, quae omnes lineas cuidam aequidistantes bifariam dividit, & siquidem ad rectos illas angulos, axis dicetur.

In omni vero parabola diametros axi aequidistare; & sectioni in uno tantum puncto occurrere, suo loco demonstrabitur.

II.

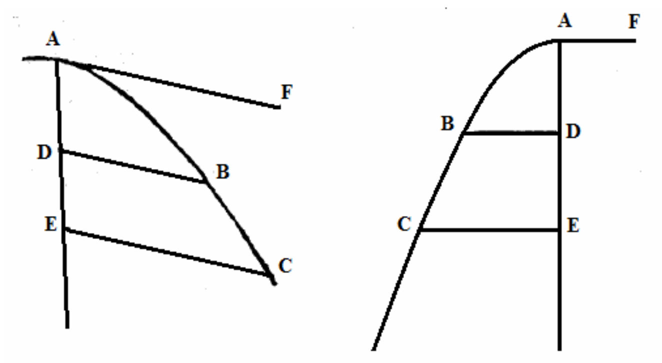
Verticem diametri voco punctum in quo diameter sectionis perimetro occurrit: punctum autem quod axi & sectionis perimetro commune est, vertex dicitur parabolae.

III.

Ordinatim ad diametrum applicari dicitur unaquaeque linearum aequidistantium, ac bifariam divisarum.

IV.

Latus rectum voco lineam iuxta quam possunt ordinatim ad diametrum applicata: sive latus rectum mensura est iuxta quam comparantur potentiae linearum ordinatim ad diametrum positarum.



V.

Porro illud prae reliquis sectionibus peculiare in parabola sibi vindicat latus rectum, quod rectangulum latere recto & parte diametri ab eiusdem vertice, & puncto quo ab ordinatim posita dividitur, intercepta contentum, aequale semper constituat quadrato lineae quae ordinatim poni dicitur.

Sit exempli causa in ABC parabola diameter AD, illiusque latus rectum AF, ordinatim vero positae sint BD, CE: erit igitur ex mente Apollonij, quod & nos quoque demonstrabimus, quadratum BD aequale rectangulo DAF; & EAF rectangulo aequale quadratum CE: & sic de ceteris ordinatim positis idem ostendetur.

Illud quoque hic observandum est, quod & in ellipsi ostendi, diversa diametris singulis assignari latera recta, cum linearum quae ad illas ordinatim poni dicuntur; diversae quoque existant potentiae.

Deinde necessarium non esse latera recta ad extremitates diametrorum suarum; normaliter poni, sed iisdem eo posse applicari angulo quo diametri ab ordinatim positis intersecantur.

V.

Focum parabolae appello, punctum in axe positum, a vertice intervallo dissitum, quod aequale est quartae parti lateris recti.

VI.

Parabolas parallelas voco quae ad eundem axem constitutae diversos quidem habent apices, sed latera recta aequalia, & concavas perimetros versus eandem partem.

VII.

Parabolae aequales sunt, quarum latera recta axibus inservientia sunt aequalia.

PARABOLAE

PARS PRIMA

Parabolam e cono educit, passionesque illius essentiales ac fundamentales exponit : & primo quidem diametros & ordinatim ad illas positas, praecipuasque illarum proprietates exhibet:secundo latus rectum illiusque, naturam, dein secantium ac contingentium primarias designat affectiones.

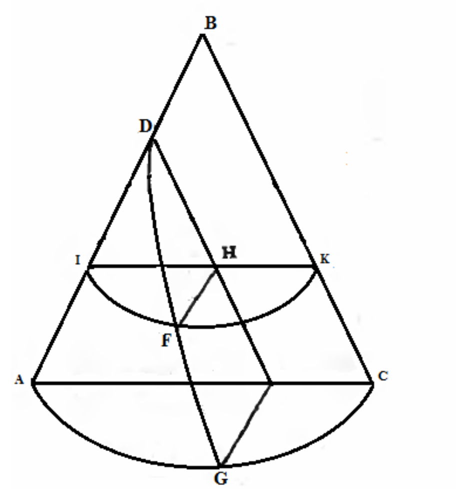
PROPOSITIO PRIMA.

Esto conus ABC sectus triangulo per axem ABC, ductaque ED parallela lateri BC, fiat per ED sectio D G, secundum rectam EG normalem ad lineam AC; ponatur autem per H punctum quodvis in ED lineae assumptum, recta IK aequidistans AC diametro basis coni: & per IK planum ducatur IFK aequidistans plano baseos AGC occurrens plano DFG secundum communem intersectionem FH.

Dico HF quadratum esse ad quadratum EG, ut HP linea est ad lineam ED.

Demonstratio.

Quoniam planum IFK aequidistat plano baseos AGC; circulus erit IFK, & FH, EG communes intersectiones parallelae; quia vero AC, IK aequidistant, & EG normalis est ad AC, recta quoque HF normalis est ad IK: ac proinde FH quadratum rectangulo IHK aequale : sed & EG quadratum, rectangulo AFC aequale est, quadratum igitur EG est ad
quadrarum FH, ut AEC rectangulum ad rectangulum IHK: quia vero EH aequidistat KC adeoque HK, CE lineae in parallelogrammo aequales sunt, rectangulum IHK est ad AEC , ut IH ad AE, id est FH ad DE, quadratum igitur FH ad EG, quadratum igitur FH ad EG, quadratum est ut recta DH ad rectam DE. Quod erat demonstrandum.

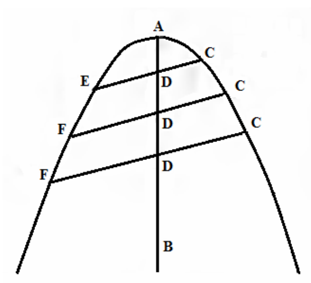


Vocetur autem sectio huiusmodi parabola cuius diameter DE, & ordinatim ad illam positae FH, GE.

Scholion.

Demonstration haec universalis est & omni cono convenit; ca tamen differentia, quod in scaleno dum ABC triangulum non est maximum eorum quae per axem fiunt recta EG, FH *ordinatim ad diametrum* ED *posita, secent illam ad angulos obliquos, in recto autem ad rectos: cum enim planum* ABC *in scaleno per axem ductu, ad basim sit obliquum; sit autem&* ED lineae *in plano* ABC, *illo quoque ad basim* AGC *adeoque & ad lineam* EG *quae in plano basis, ipsi* AC *normalis est,*
obliqua. igitur in scaleno, rectae EG, FH *ordinatim ad diametrum* DB *positae, ad obliquos eandem secant angulos. Similiter ostendemus, in cono recto, ordinatim positas diametrum suam ad rectos dividere.*

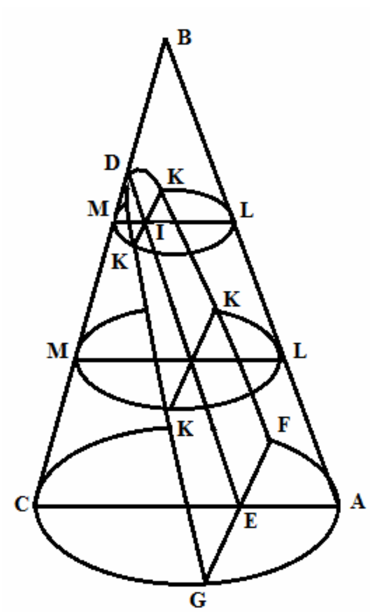
Hinc porro non lemis exsurgit difficutas, cum DFG *parabola in utroque cono sit eadem, id est utrum diametrum* ED *in scaleno, censeri debeat diameter aliqua secundaria parabola qua in cone recto exhibetur: sive quod idem est, an in sectione coni recti, diameter aliqua sit quam ordinatim ad illam positae ad angulos secent obliquos: licet enim per priorem propositionem utroque cono demonstretur* DE *lineam diametrum esse, quam in cono recto ad angulos rectos in scalene ad obliquos secent ordinatim ad illum posita, illud tamen ex discursu Apollonuum non constat, in quavis parabola plures posse assignari diametros quarum unam recte, alteram oblique secent ordinatim ad illam positae: & qua... de parabolis inclinatis necdum constat, posset* ED lineae *quoque inclinatae sectionis axis censeri, quem ordinatim positae ad obliquos diametros secent angulos : sed difficultatibus illis omnino satisfacere conabimur: ostendemusque in una eademque parabola diametros dari aquidistantes, quarum unam ad rectos, ad obliquos alteram, secent ordinatim ad illas positae, ac nullas deinde inclinatus dari parabolas. Ex dictis & prima parabola e proprietate constat primo, si* AB *lineam ad angulum quemcunque aequidistantes secent* DC, *fueritque ut* AD *linea ad lineam* AD, *sic* DC *quadratum* DC; *puncta* ACC *esse ad parabolam*.



Secundo patet si CDE *ordinatim posita ad* AB *aequidistans ponatur* CD, *cui in directum, addatur* DF *aequalis* CD, *punctum* F esse *ad parabolam* : *cum enim sit ut* AD *ad* AD, *sic quadratum* CD *ad* CD *quadratum, ipsis autem* CD *aequentur* DE, DF: *quadrata quoque* DE, DF *proportionalia sunt lineis* AD, AD ; *quare eae qua diximus puncta* E & F *ad parabolam funt:quae singulariter & explicite hic notare volui, eo quod postmodum saepius sint assumenda.*

PROPOSITIO II.

Sit ABC triangulum productum plano per axem ABCG, positaque DE lateri AB aequidistante ducatur EG normalis ad diametrum basis AC; secundum quam, & rectam ED, planum ponatur exhibens in superficie conica sectionem FDG: assumatur autem in ED, punctum quodcunque I per quod in plano FDG recta ponatur HK equidistans FG ;
Dico HK in I bifariam secari.



Demonstratio.

Posita per I punctum, L M parallela AC, ducatur secundum LM, planum LKM aequidistans plano baseos AGC: circulus igitur est LKM & HK, FG communes intersectiones parallelae, & quia FEG ex hypothesi normalis ad diametrum AC, ab eadem in E bifariam est divisa, HK quoque normalis est ad LM, & in I bifariam divisa. Quod erat demonstrandum.

Corollarium.

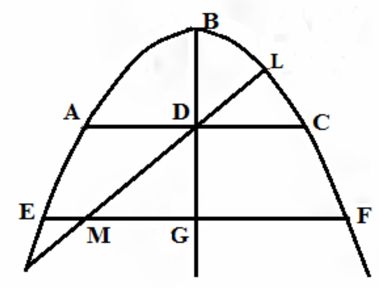
Ex hac propositione patet, in parabola, si diameter rectam quandam bifariam secet, omnes quoque eidem bissectae aequidistantes bifariam secari patet, cum ED diameter sit quaecunquc, & HIK quaevis aequidistantum rectae FG in E bifariam divisae.

PROPOSITIO III.

Data linea, parabolam in duobus punctis secante illius exhibere diametrum.

Constructio & demonstratio.

Divisa AC bifariam in D ponatur EF aequidistans, qua similiter bissecta in G, ducatur per G & D, linea BGD: dico illam diametrum esse quaesitam; si non, sit LD diameter, quae producta secet EF in M : quoniam igitur LD diameter bifariam secat AC, bissecabit e quoque in M, FE ipsi AC aequidistantem, sed FE bissecta ponitur in G: erit igitur in G & M, bissecta linea FE .Quod non potest; non igitur LD diameter est sed BD. Exhibuimus igitur, &c. Quod erat faciendum.

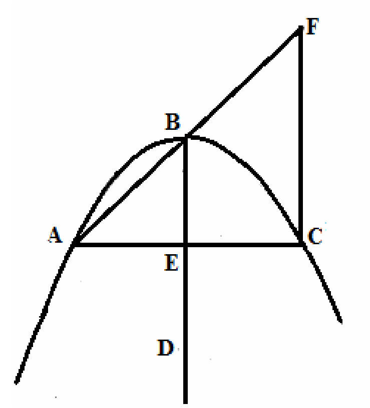


PROPOSITIO IV.

A dato in perimetro parabolae puncto, ad datam diametrum ordinatim ponere.

Constructio & demonstratio.

Sit in perimetro parabolae ABC datum punctum A, & diameter data sit BED ad quam ex A ordinatim ponere lineam AEC, iuncta AB producatur in F, ut AB, BF aequales sint, & ex F demissa FC parallela BE, occurrat parabolae in C, iungaturque AC: patet AC in E bifariam esse divisam, cum AF, bissecta sit in B, & FC, BE aequidistents a dato igitur in perimetro parabolae puncto, &c. Quod erat faciendum.



Corollarium.

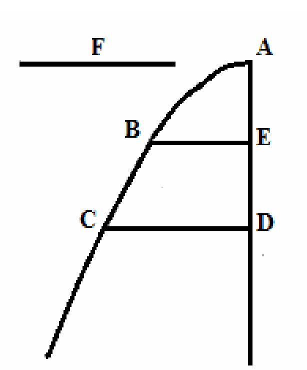
Hinc facilis praxis oritur, per datum in diametro punctum ordinatim ducendi lineam: ex assumpto enim in perimetro quovis puncto ponatur ordinatim quaecunque, cui per datum in diametro punctum aequidistans ducatur. Patet illam ordinatim ad diametrum esse positam.

PROPOSITIO V.

Datam lineam ad datam in parabola diametrum ordinarim ponere.

Constructio & demonstratio.

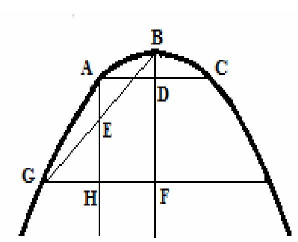
Data sit parabola ABC; & in ea diameter AD, ad quam oporteat datam rectam F ordinatim applicare ; assumpto quovis puncto B in sectionis perimetro, ponatur ex B a linea BE ordinatim ad diametrum AD, fiatque ut BE quadratum ad quadratum F, ita AE ad AD lineam, & per D punctum constituatur DC parallela EB. Dico factum esse quod requiritur: est enim ita pars diametri AE ad AD diametri partem, sicut quadratum ordinatim positae BE, ad quadratum positae CD, sed ex constructione esse BE quadratum ad quadratum F, ut AE linea ad lineam AD. Igitur CD est aequalis ipsi F & est parallela ad EB; datam igitur lineam ordinatim posuimus, &c. Quod erat faciendum.



PROPOSITIO VI.

Ordinatim applicatarum ad duas diametros quarum altera est axis; illa minor est quae ad axem applicatur, modo distantiae a puncto verticis aequales fuerint.

Demonstratio.

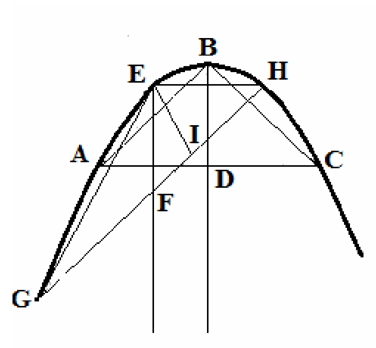


Sit parabolae ABC axis BD & ordinatim ad illum pofita AC: ducta deinde AE parallela BD; fiat BF quadrupla BD, ponaturque FG parallela DA dein FG, bissecta in H ponatur HA aequidistans BF, iungaturque BG: quoniam BD ad BF, eam rationem habet quam unum ad quatuor ex constructione, igitur quadratum AD ad GF, est ut unum ad quatuor; igitur GF linea dupla est AD, hoc est HF & BF dupla est EN; ac proinde BG dupla GE; item AE aequalis ipsi BD, & AC ipsi GF ; unde BG ordinatim posita est ad diametrum AE; est autem GB maior GF, hoc est AC, igitur GB ordinatim posita ad AE aequalem BD; maior est recta AC quae ad axem ordinatim collocata est; & quia eandem rationem seruant quadrata applicatarum, quam partes diametrorum inter verticem & ordinatim applicatas constitutae, hinc universaliter de omnibus applicaris constat veritas propositionis.

Aliter.

Propositio haec aliter per Archimedem demonstrator. Sint BD, EF diametri eiusdem altitudinis, & BD quidem axis : ponanturque per D & F,ordinatim lineaee AC, GH.

Dico AC minorem esse recta GH, iungantur enim ABC, GEH: & ex E recta demittatur EI normalis ad GH. Cum igitur aequales sint distantiae EF, BD, aequalia sunt triangula, GEH, ABC per Archimedem. Quare ut EI ad BD, sic AC ad GH: est autem BD id est EF maior EI (cum angulus I in triangulo ElF rectus sit) maior igitur erit GH quam AC. Quod erat demonstrandum.

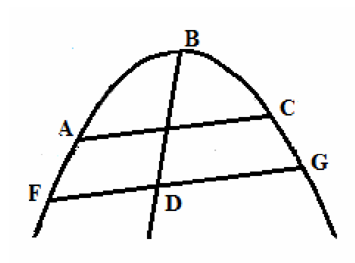


PROPOSITIO VII.

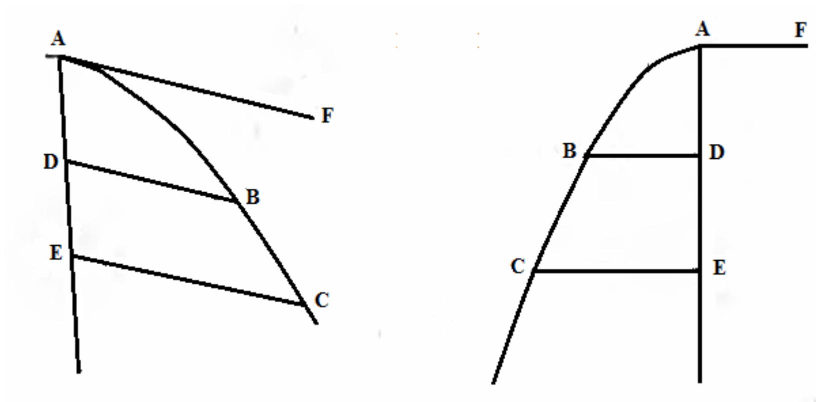
Ad datum punctum in diametro parabolae ordinatim ponere.

Constructio & demonstratio.

Sit data parabola ABC, cuius diameter aliqua BD, & in ea punctum assignarum D per quod oporteat rectam collocare ordinatim ad BD diametrum ; assumpto in perimetro quovis puncto A ducta sit quaevis AC, ordinatim ad diametrum BD cui parallela ponatur per punctum D: patet FDG ordinatim esse positam & in D bifariam divisam. Fecimus igitur quod petebatur.



PROPOSITIO VIII.



Datae diametri in parabola latus rectum exhibere.

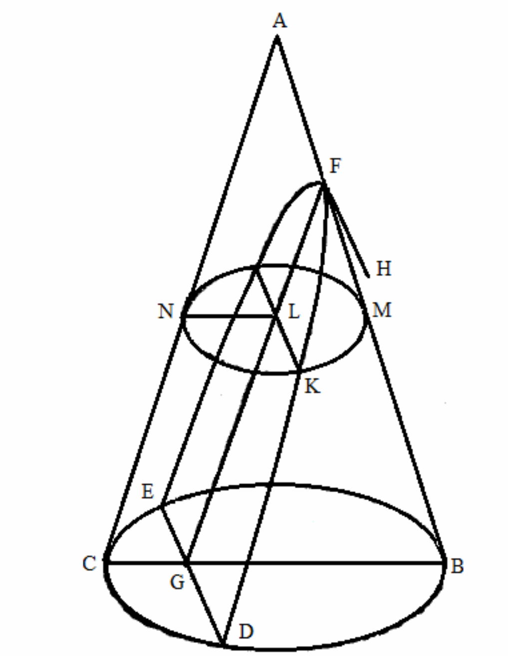
Constructio & demonstratio.

Sit ABC parabola, & in illa diameter AD cuius latus rectum opporteat exhibere, posita sit quaevis BD ordinatim ad diametrum AD, fiantque continuae proportionales AD, DB, AF. Dico AF satisfacere petitioni: applicetur enim quaevis alia EC parallel DB erit igitur ut AD ad AE, sic DB quadratum ad quadratum EC: sed ut AD ad AE, sic FAD rectangulum ad rectangulum FAE; igitur ut quadratum DB est ad quadratum EC, sic FAD rectangulum ad rectanguium FAE & permutando ut DB quadratum, ad rectangulum FAD, sic EC quadratum ad rectangulum FAE, sed BD quadrato aequale est rectangulum FAD, quia AF, BD, AD, proportionales sunt; igitur & EC quadratum aequale est rectangulo FAE, ac proinde FA latus rectum diametri AD: exhibuimus igitur, &c. Quod erat faciendum.

Scholion.

Lubet hic apponere methodum, & constructionem qua Apollonius lib.1. Conicorum latus rectum parabola adinvenit : unaque breviter ostendere latus rectum praecedenti propositione a nobis inventum, idem esse cum eo quod Apollonius alia constructione adinvenit.

Sit, inquit, coni vertex A, *basis, circulus* B, C: *seceturque plano per axem quod sectionem faciat triangulum* ABC ; *secetur & altero plano secante basim coni secundum rectam lineam* DE *quae ad* BC *sit perpendicularis, & faciat sectionem in superficie coni* DFE *lineam, diameter autem sectionis* FG *aequidistans sit uni laterum trianguli per axem: atque a puncto F lineae FG ad rectos angulos ducatur* FH, *& fiat ut quadratum* BC *ad rect angulum* BAC, *ita linea HF ad FA* ; *sumatur autem in sectione punctum quodlibet* K, *& per* K *ducatur* KL, ipsi DE *aequidistans*. *Dico quadratum* KL *rectangulo* HFL *aequale esse. Assertionem porro doctissimo & sublimi discursu demonstrat; qui cum tyronibus difficilior sit, faciliori nos latus rectum methodo praecedenti propositione* *conati sumus expedire, ostendimus enim posita eadem figura, si fiant* FL, KL, FH, *continuae proportionales,* FH *latus rectum esse. Nunc restat ut ostendam, illud idem esse*



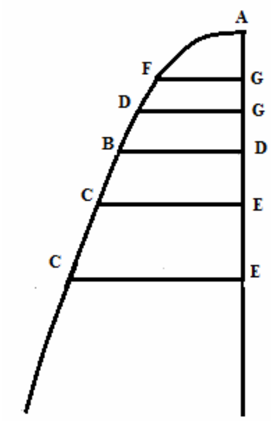
cum eo quod Apollonius constructione antedicta adinvenit. Acta per L linea MN, aequidistante ipsi BC*, erit planum quod transit per* LKMN*, aequidistans plano coni baseos, adeoque circulus: &* LK *quadratum, id est ex hypothesi rectangulum* HFL*, aequale rectanglo* MLN*; quare rectangulum* MLN *ad ipsum* LFA*, est ut* HFL *ad* LFA *: sed* HFL *est ad* LFA *ut* HF *ad* FA*, igitur ut* HF *ad* FA*, sic* MLN *ad* LFA*: sed ratio* MLM *ad ipsum* LFA*, est ut* HFL *ad* LFA *: sed* HFL *est ad* LFA*, ut* HF *ad* FA*, igitur ut* HF *ad* FA*, sic* MLN *ad* LFA : *sed ratio* MLN *ad* LFA *componitur ex ratione* ML *ad* LF*, & ex* LN *ad* FA; *igitur proportio* HF *ad* FA*, componitur ex* ML ad FL, *&* LN *ad* FA: *est autem* ML *ad* LF*, ut* MN *ad* NA*, adeoque &* LN *ad* FA*, ut* MN *ad* MA*; igitur proportio* HF *ad* FA*, componitur ratio quadrati* BC *ad rectangulum* BAC*, igitur* HF *est ad* FA*, ut* BC *quadratum ad rectangulum* BAC*: sed posita proportione quadrati* HF *lineae, ad lineam* FA *eadem, cum ea quam habet* BC *quadratum ad rectangulam* BAC,*erit* HF *linea per Apollonium latus rectum parabolae;igitur si* FL, KL, FH *sunt continuae proportionales, erit* HF *latus rectum idem cum eo quod aliter adinuenit Apollonius, unde patet utramque constructionem in idem incidere & alteram alterius tantum esse conversam; posita enim proportione* H F *ad* FA*, quae est* BC *quadrati ad rectangulum* BAC; *infert Apollonius* HF, KL, FL *continue esse proportionales, adeoque* LK *lineam posse rectangulum* HFL*, nos vero positis tribus continuis* HF, KL, FL *inferimus* HF *latus esse rectum, & omnes lateris recti proprietates habere ;quia vero tribus illis positis continuis sequitur quoque* HF *esse ad* FA*, ut* BC *quadratum ad rectangulum* BAC*, patet* HF *idem latus rectum esse cum eo quod Apollonius proposuit.*

Ex antidictis vero patet ordinatim ad diametrum aliquam applicatas eo maiores esse quo remotiores sunt a vertice suae diametri, nam semper excresit rectangulum sub latere recto, & ordinatim posita, & portio illa diametri antedeterminata necesse est quandoque inter se & lateri recto sint quales, cuius casus sequentibus propositionibus assignabimus.

PROPOSITIO IX.

Sit ABC parabolae diameter AD aequalis lateri recto, ex D ponatur ad diameterum AE ordinatim lineae DB.

Dico DB linea in aequari AD, & si AD, DB linea aequentur, dico AD aequari lateri recto, quod inservit diametro AD.



Demonstratio.

Quoniam BD linea, ordinatim applicatur ad diametrum AD, erit BD quadratum aequale rectangulo super AD & latere recto, sed AD linea aequalis ponitur lateri recto, igitur quadratum BD aequale est quadrato AD, adeoque BD, AD lineae aequales sunt.

Sint iam AD, BD lineae aequales, & BD quidem ordinatim posita ad diametrum AD, dico AD lineam aequari lateri recto, cum enim quadratum BD aequale ponatur quadrato AD, sit autem & BD quadratum aequale rectangulo super AD & latere recto, erit & quadratum AD aequale rectangulo super AD & latere recto, ideoque & AD linea lateri recto aequalis. Quod erat demonstrandum.

PROPOSITIO X.

Iisdem positis ducantur ordinatim lineae CE, FG : & CE quidem cadat infra BD, recta vero, FG supra.

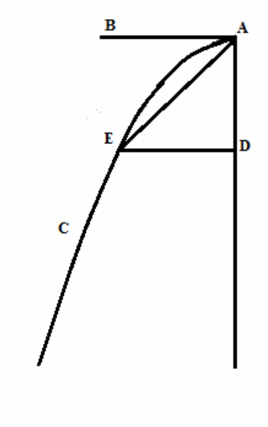
Dico AG ad GF rationem minoris inaequalitatis esse, & AE ad E C maioris.

Demonstratio.

Quoniam AD linea aequalis ponitur lateri recto, erit FG quadratum aequale rectangulo GAD, adeoque AG, G F, AD continuae proportionales ; est autem AD id est BD, maior quam FG,igitur & FG maior est :quam AG. Quod erat primum.

Rursum cum DAE rectangulum aequale sit quadrato CE, proportionales erunt AD, CE, AE lineae: sed AD id est BD, minor est quam CE ex ante demonstratis; igitur & CE minor est recta AE: adeoquc ratio AC ad CE est maioris inaequalitatis. Quod erat demonstrandum.

PROPOSITIO XI.



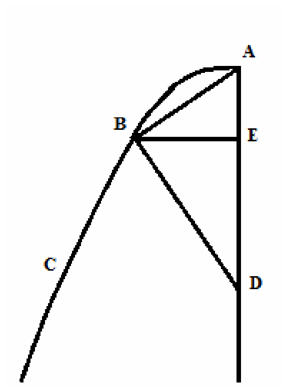
Esto parabolae ABC quaevis diameter AD, & ex A ponatur AE, aequidistans ordinatim ad diametrum AD; ponatur quoque ex A linea AB, dividens bifariam angulum EAD, occurrensque parabolae iterum in B puncto, ex quo ordinatim ad diametrum ponatur BD.

Dico AD lineam aequari lateri recto.

Demonstratio.

Quoniam AE, BD aequidistant, angulus EAB aequatur angulo ABD; sed angulo EAB ex hypothesi aequatur angulus BAD, aequales igitur sunt anguli ABD, BAD: adeoque & lineae BD, AD: unde AD, lateri recto est aequalis. Quod erat demonstrandum.

PROPOSITIO XII.



Sit ABC parabolae axis AD, & ex A demissa linea AB, parabolae iterum occurrat in B puncto ex quo ad axe in ordinatim ponatur BE; ducatur autem & BD normalis ad AB, occurrens axi in D.

Dico DE lineam aequari lateri recto.

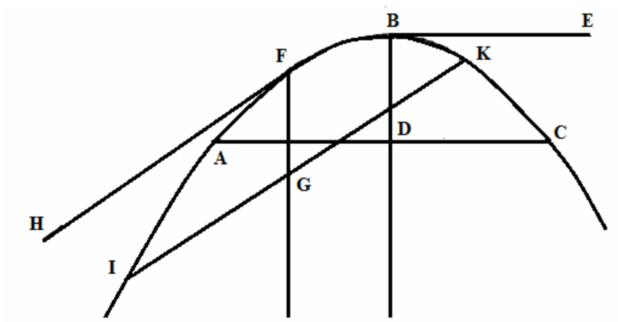
Demonstratio.

Quoniam angulus ABC rectus est, & BE normalis ad axe in AD, proportionales sunt AE, EB, ED & EB quadrato aequale rectangulum AED : sed & EB quadrato aequale est rectangulum super AE & latererecto, rectangulum igitur AED aequale est rectangulo sub AE & latere recto aequalis ergo ED est lateri recto. Quod erat demonstrandum.

PROPOSITIO XIII.

Latus rectum axeos, minimum est laterum rectorum, reliquarum diametrorum.

Demonstratio.



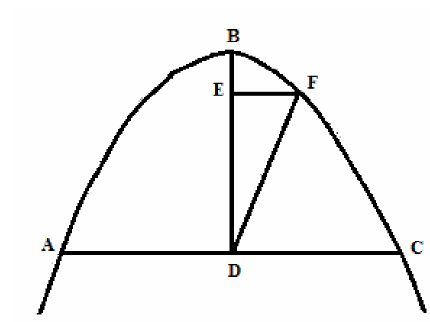
Parabolae ABC axis sit BD, illiusque latus rectum BE, sit autem & alia quaeris
diameter FG, cuius latus rectum ponatur FH: dico BE minus esse latere recto FH: ponatur enim ADC ordinatim ad axem, sumptaque FG aequali BD, ducatur per G ordinatim ad diametrum FG, linea IK maior igitur est IG quam AD, & IG quadratum est maius quadrato AD: sed IG quadratum aequatur rectangulum super HF, FG, EBD rectangulum aequale est quadrato AD, maius igitur est rectangulum HFG, rectangulo EBD aequales autem sunt ex constructionc BD, FG, igitur FH latus rectum maius est latere recto BE: igitur latus rectum axeos minimum est, &c. Quod erat demonstrandum.

PROPOSITIO XIV.

Sit ABC parabolae axis BD maior latere recto : & per D ordinatim ad axem posita ADC: factaque DE aequali lateri recto, ponatur EF et equidistans AC, iunganturque D F.
Dico F *D* lineam aequari lineae DC.

Demonstratio.

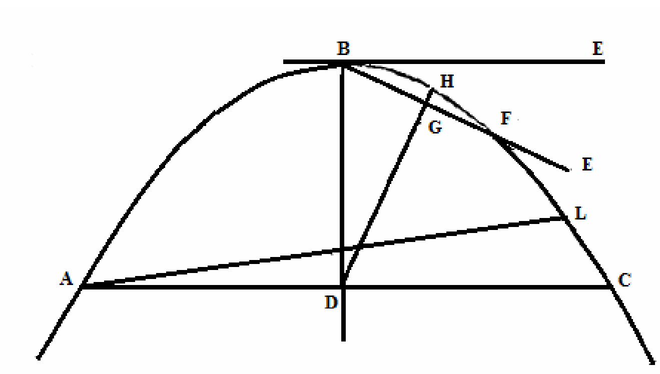
Quoniam ED lateri recto aequalis est, quadratum FE aequatur rectangulo BED ; addito igitur quadrato DE, quadrata FE, DE simul sumpta, id est quadratum FD, ob angulum FED rectum aequale est rectangulo BDE: sed & BDE rectangulo aequale est quadratum DC: aequalia igitur sunt quadrata FD, DC. Quod erat demonstrandum.



PROPOSITIO XV.

Omnis linea per apicem diametri acta, & ordinatim positae aequidistans, sectionem contingit : & contra quae contingenti aequidisat ordinatim posita est ad diametrum ex contactu demissam.

Demonstratio.



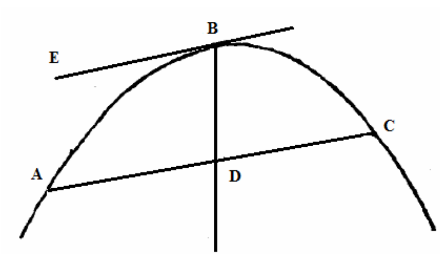
Sit ad ABC parabolae diametrum BD ordinatim posita ADC : cui per B vertice diametri ponatur aequidistans BE; si enim non contingit parabolam, secet illam in F; divisaque BF bifariam in G, ponatur per G & D, linea GD; cum igitur FB, AC, ponatur equidistantes bifariam dividat GD, erit illa diameter ad quam ordinatim posita est AC; sed & AC quoque ordinatim applicata est ad diametrum BD, cum ab illa bifariam secetur ; linea igitur AC ordinatim posita est ad duas diametros, quod fieri non potest : alias enim ipsi AC aequidistantes a diametris BD, GD bifariam in diversis punctis dividerentur. Non igitur BE secat, sed contingit sectionem in B.

Sit iam EB contingens, illique aequidistet AC: dico illam ordinarim esse applicatam ad diametrum BD. Si enim non, ponatur AL ordinatim ad BD, aequidistat igitur AL contingenti BE; & quia A C quoque contingenti parallela est, ipsa AL aequidistat AC, quod fieri non potest, cum in A puncto sese decussent non igitur AL ordinatim posita est ad BD, sed AC linea aequidistans contingenti BE. Quod erat demonstrandum.

PROPOSlTIO XVI.

Per datum in parabolae perimetro punctum, contingentem ducere.

Constructio & *demonstratio.*

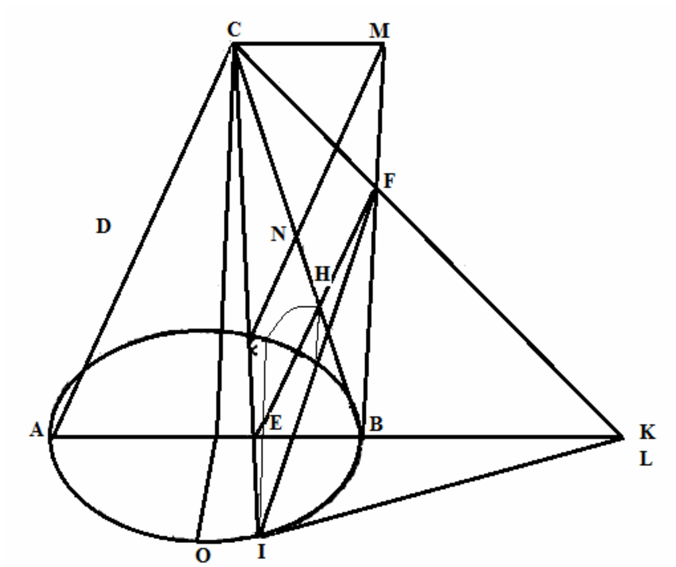


Sit in ABC parabolae perimetro datum punctum B, oporteat per illud contingente ponere: demissa sit ex B diameter BD, ad quam ordinatim ponatur AC, cui aequidistans per B ducatur BE : patet illam esse contingentem; per datum igitur punctum, &c. Quod erat faciendum.

PROPOSlTIO XVII.

Parabolam contingens conveniat cum diametro: ad quam ex puncto contactus ordinatim quaedam posita sit.

Dico diametrum interceptam, inter ordinatim applicatam & punctum in quo contingens



cum diametro concurrit a parabola bifariam dividi.

Demonstratio.

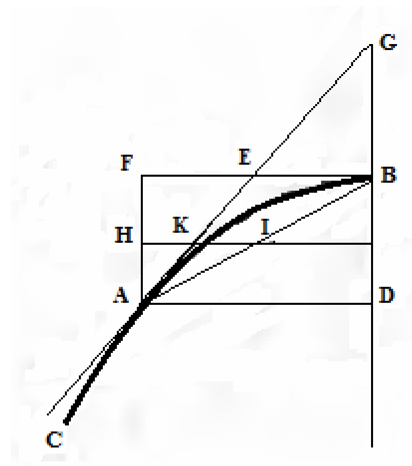
Sit ABC conus quicumque, sectus triangulo per axem ABC, diameter autem baseos AGB, sit AB, in qua assumpto quovis puncto E quod centrum non sit ponatur EH aequidistans AC: & EI normalis ad AB: tum per HE, EI secto, exhibens parabola in GHI; positoque CD, axe coni, perfictatur parallelogrammum CDB, cuius lateri BM occurrat EH producta in F*:* iunctaque CF diametro AB protractae occurrat in K: ponatur dein per I contingens circulum AIB in I, conveniens cum AB in L, puncto quod idem est cum puncto K: cum enim CK, sit ad FK, ut CD ad FB, id est MB ad FB, id est DB ad ED; erit quoque DK ad BK, ut DB ad EB, & dividendo BK ad DB, ut BE ad ED, & componendo DK ad DB, ut DB ad DE ; proportionales igitur DE, DB, DK: igitur contingens per I posita cum diametro convenit in K: idem ergo punctum est K & L; ulterius, iungantur puncta CE, CI; quoniam igitur CI linea in superficie est coni, triangula sunt, CEI, CIK, & planum CIK continget conum in linea CI ponatur tandem in plano CIK linea IF; continget illa parabolam in I: cum enim planum CIK conum contingat, linea autem FI in eadem plano sit, & simul in plano parabolae, cum puncta K & I, in eodem sint, patet FI contingentem esse parabolae in I.

Ulterius cum CM linea aequidistans & parallela sit semidiametro DB, aequidistat quoque & aequalis est AD: unde AC, DM parallelae: & quia FE aequidistat AC, aequidistabit EF ipsi DM*:* est autem MD diameter parallelogrammi DM, in N a diametro CB bifariam divisa, recta igitur FE in H, quoque ab eadem diametro bissecta est aequales ergo sunt lineae EH, HF. Igitur parabolam contingens, &c. Quod erat demonstrandum.

Quod si punctum assumptum ipsum centrum sit, patet demonstratio eodem enim modo ostendetur OM esse contingentem parabolae quae per lineas ND, DO, &c. ducetur.

PROPOSITIO XVIII.

Parabolam ABC cuius diameter BD, contingant in A & B, rectae AE, BE convenientes in E: &: EB quidem AF, aequidistanti BD occurrat in F.
Dico FB in E bifariam esse divisam.



Demonstratio.

Producta AE conveniat cum diametre in G, ponaturque ex A ordinatim lineaeAD ; quoniam AD, EB aequidistant, ut GB ad GD, sic EB est ad AD, it est FB, sed GB dimidia est GD, igitur & EB dimidia quoque est FB. Quod erat demonstrandum.

Corollarium primum.

Hinc paret, si iuncta AB quotvis HI poinatur aequidistances ipsi FB, illas bifariam a contingente AG dividendas.

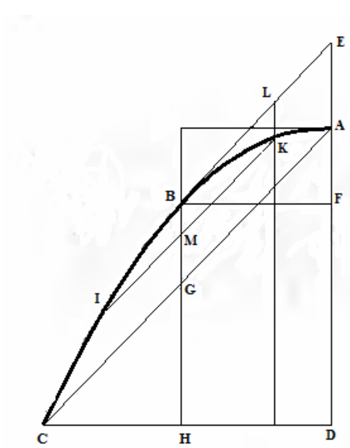
Corollarium secundum.

Sequitur secundo rectas FA, GB, item AE, G E aequales esse adeoque & AFE, GEB triangulia aequalia, patet, cum FE, EB aequales sint ostensae, & AFE, GEB triangula similia ob GB, FA aequidistantes.

PROPOSITIO XIX.

In parabola diametri omnes aequidistant axi.

Demonstratio.



Sit ABC parabolae axis AD, cui in E occurrat contingens BE, positaque AC parallela contingenti BE, demittatur ex B, linea BG aequidist axi, ponitur BF, CHD ordinatim ad axem AD, & BG producta occurrat contingenti per A ductae in K: Quonniam BE cotingens est & BF ordinatim ad axem AD applicata, aequales sunt lineae EA, FA & quia BF communis altitudo est, triangulum EBF, parallelogrammo BA aequale est rursum ut AF ad AD, fic FB quadratum ad quadratum CD, & FK parallelogramum ad parallelogrammum DK: ut quadratum igitur FB ad quadratum CD est, sic FK parallelogrammum ad parallelogrammum D K, sed ut FB quadratum ad quadratum CD, sic EBA triangulum ad triangulum CAD; igitur ut FK parallelogrammum ad parallelogrammum DK, sic EBF triangulum est ad triangulum ACD*:* & permutando ut FK parallelogrammum ad triangulum BFE, sic DK parallelogrammum est ad triangulum ACD: aequalia autem ostensa sunt, triangulum BFE & parallelogrammum FK; aequalia igitur sunt DK parallelogrammum & triangulum ACD: & ablato communi AGHD, aequalia remanent triangula AGK, CHG. Unde cum similia quoque sint ob AK, CH aequidistantes, aequalia sunt latera KG, GH, & CG, GA : diuisa igitur bifariam est AC. Eodem modo si IK ponatur aequidistans CA, & ex K diameter posita sit LK occurrens BE contingenti in L, ostenetur IK in M, bisecari a linea BG: diameter igitur sectionis est BG: quare cum EB contingens sit quaecunque, adeoque & BH quaevis aequidistans axi, patet diametros omnes axi aequidistare. Quod erat demonstrandum.

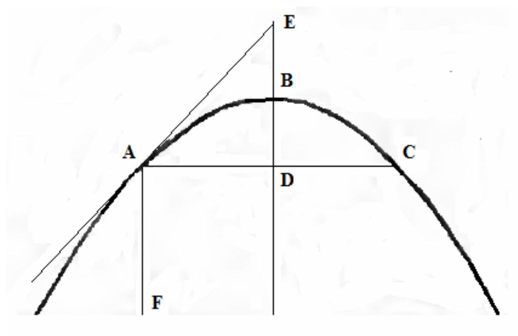
Ex quo sequitur diametros omnes in parabola esse parallelas.

PROPOSlTIO XX.

Omnis contingens per extremitatem ordinatim positae ducta, cum illius diametro convenit.

Demonstratio.

Posita sit diameter BD, & ad illam ordinatim applicata AC: agaturque per A contingens, dico illam BD diametro occurrere *:* ducta enim diametro AF aequidistabit illa BD, unde cum AE secet AF aequidistantium unam, alteram quoque DB, producta intersecabit. Quod erat demonstrandum.



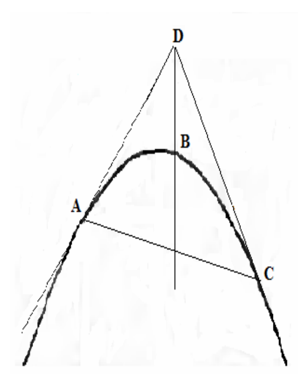
Corollarium.

Hinc patet, quascumque duas contingentes parabolam, convenire in aliquo puncto extra sectionem: patet ex iam demonstratis.

PROPOSITIO XXI.

Contingentes actae per extrema ordidatim applicatae cum eiusdem diametro, in uno eodemque conveniunt puncto.

Demonstratio.

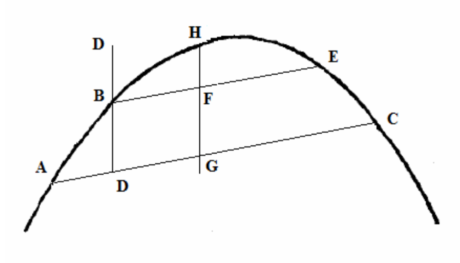


Sit ad ABC parabolae diametrum BD; ordinatim applicata AC, dico contingentes per A & C ductas, diametro BD occurrere in uno eodemque puncto D. Demonstratio patet, cum pars diametri a sectione & contingente intercepta aequalis sit portioni a sectione & ordinatim applicata interceptae. Igitur contingentes, &c. Quod erat demonstrandum.

PROPOSITIO XXII.

Per datum punctum in perimetro parabolae diametrum ducere.

Constructio & *demonstratio.*

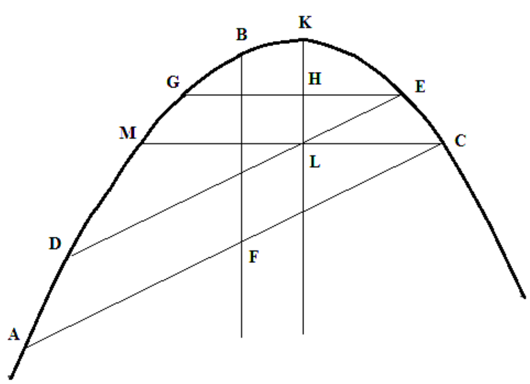


Sit in ABC perimetro assignatum punctum B, ex quo oporteat diametrum ponere ; ducta quavis secante BE, exhibeat ut illius diameter HF, cui per B aequidistans ponatur B D: patet illam sectionis esse diametrum : a dato igitur puncto, etc. Quod erat demonstrandum.

Corollarium.

Eadem omnino praxi utemur, si ex dato, extra vel intra sectionem puncto D, diametrum oporteat ponere, constructio & demonstratio patet ex prima propositione.

PROPOSITIO XXIII.



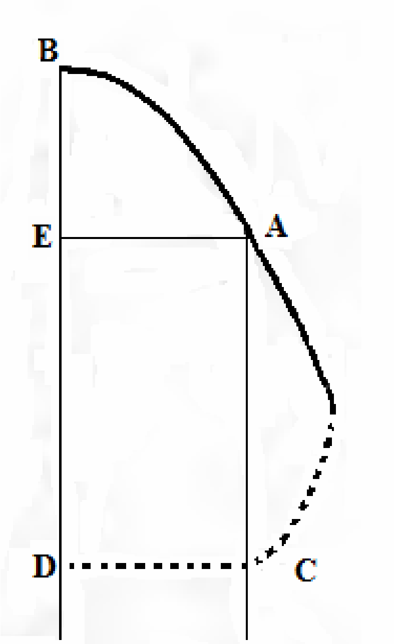
Datae parabolae axem exhibere.

Constructio & demonstrio.

Sit ABC parabola cuius axem oporteat exhibere, ponantur duae quaevis parallelae DE, AC quarum exhibeatur diameter BF: ad quam ex E & C normaliter ducantur EG, CM; alteraque illarum, puta EG bifariam divisa in H ponatur per H aequidistans diametro BF. Dico illam axem esse, quoniam enim diametro BF aequidistat, erat illa quoque diameter sectionis, & quia aequidistantium EG, CM unam ad rectos bisecat angulos, bisecabit & alteram, ad rectos: axis igitur sectionis est KL; exhibuimus ergo, &c. Quod erat faciendum.

PROPOSITIO XXIV.

Omnis linea in parabola axi aequidistans, sectioni in uno tantum puncto occurrit.



Demonstratio.

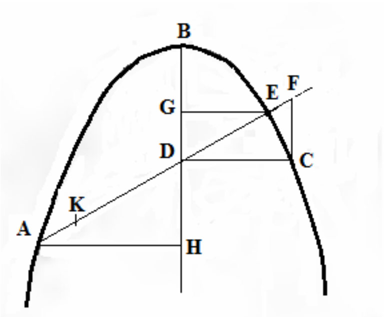
Ponatur ABC parabolae axis BD; & alia quaevis ei aequidistans AC dico AC in uno tantum puncto sectioni occurrere: sin vero, occurrat iterum in C : & ponantur ordinatim ad axem rectae AE, CD: erit igitur ut BE linea ad lineam BD, sic EA quadratum ad quadratum DC, quod fieri non potest, cum AE, CD quadrata aequalia sint
inter se, (ob AD parallelogrammum) & BE linea minor recta BD. Igitur AC diameter, parabolae tantum semel occurrit: Quod fuit demonstrandum.

PROPOSITIO XXV.

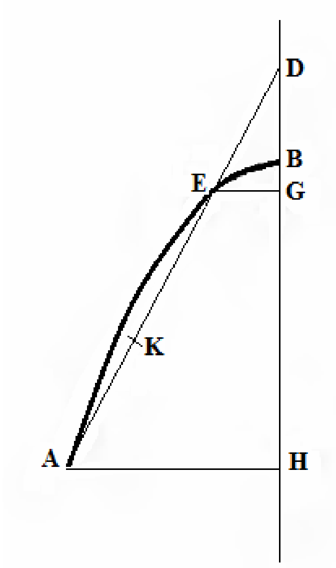
Omnis linea in parabola, quae non est diameter sectioni occurrit in duobus punctis.

Demonstratio.

Ponatur ABC parabolae axis BD, & linea KD quae non sit diameter. Dico illam sectionibis occurrere: Quoniam KD non est diameter adeoque axi non aequidistat,



producta necessario illi occurret in puncto quovis D (cum in eodem plano existat) quod si D punctum fuerit intra parabolam, ponatur ex D ordinatim DC, & per C diameter EF ; occurret illa sectioni in uno tantum puncto *;* & quia eadem aequidistat producta quoque occurret KD productae in F puncto, quod extra sectionem est. Unde KDF prius necessario occurret in puncto quovis E. Ducta igitur ex E ordinatim EG fiant proportionales BG, BD, BH positaque ad H ordinatim lineae, occurrat ipsi KD in puncto quovis A.



Quoniam BG, BD, BH ponuntur proportionales, erit HD ad DG, ut HB ad BD, & HD quadratum ad quadratum DG; ut HB quadratum ad quadratum DB, id est ut HB linea, ad lineam BG : sed ut HD quadratum ad quadratum DG, sic HA quadratum ad quadratum GE: igitur ut HB linea ad lineam BG, sic HA quadratum ad quadratum GE, unde punctum A est ad parabolam, & DK, linea axi non parallela utrimque sectioni occurrit.

Si vero DK linea occurrat axi extra sectionem, patet ante parabola occurrere in puncto quovis E: ex quo ducta ordinatim lineae EG, fiant BG, BD, BH proportionales, & ex H ducata HA parallela EG, occurrens KD lineae in A, quoniam igitur est HB ad BD, ut BD ad BG, erit componendo permutando HD ad DG , ut DB ad BG, est autem ut HD ad DG: sic AH ad EG, ergo ut DB ad BG, id est per constructionem BH ad BD, sic AH ad EG. Unde & HA quadratum ad quadratum GE, est ad HB, quadratum ad quadratum BD, id est (cum HB, BD, BG, sunt continuae), ut HB linea est ad lineam BG, ac proinde punctum A est ad parabolam, & AD linea sectioni bis occurrit. Quod fuit demonstrandum.

Corollarium.

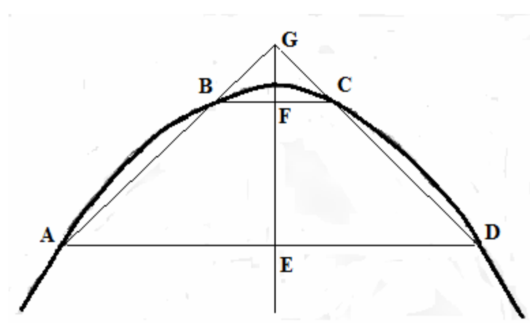
Hinc pulchra educitur propositio: nimirum si BG, BD, BH ponantur continuae & ex G ordinatim lineae GE; agaturque per puncta E & D linea, occurrens rectae ex H ducta: & ipsi GE parallelae in A: quod punctum A sit ad parabolam demonstratio habetur in priori propositione.

Sequitur secundo nullam lineam parabolae in pluribus quam duobus punctis occurrere. Cum enim parabola, sectio sit coni, & ipsi cono nulla linea, in pluribus quam duobus punctis occurrat, patet nec ulli sectioni conicae, lineam in pluribus quam duobus punctis occurrere.

PROPOSITIO XXVI.

Parabolam ABC secent duae quaevis parallelae AD, BC quarum diameter ponatur EF. Dico iunctas AB, DC diametrum in eadem puncto G decussare: & si BC, AD aequidistent iunctaeque AB, CB conveniant in G. Dico G punctum esse in diametro linearum BC, AD.

Demonstratio.



Quoniam EF diameter est rectarum BC, AD, igitur divisae sunt bifariam BC, AD in punctis F & E. Unde ut AE ad BF, sic est AG ad BG: hoc est EG ad FG. Sed ut AE ad BF, ita ED ad FC; Igitur ut ED ad FC, ita est EG ad FG ; hoc est DG ad CG. Igitur punctum G commune est tribus lineis AB, DG, EF. Quod fuit primum.

Idem quoque ostenditur si G punctum cadat intra parabolam, si iam ponantur AD, BC parallelae, & iunctae AB, CD conveniant in puncto quovis G, dico G punctum esse in diametro ad quam BC, AD ordinatim ponuntur.

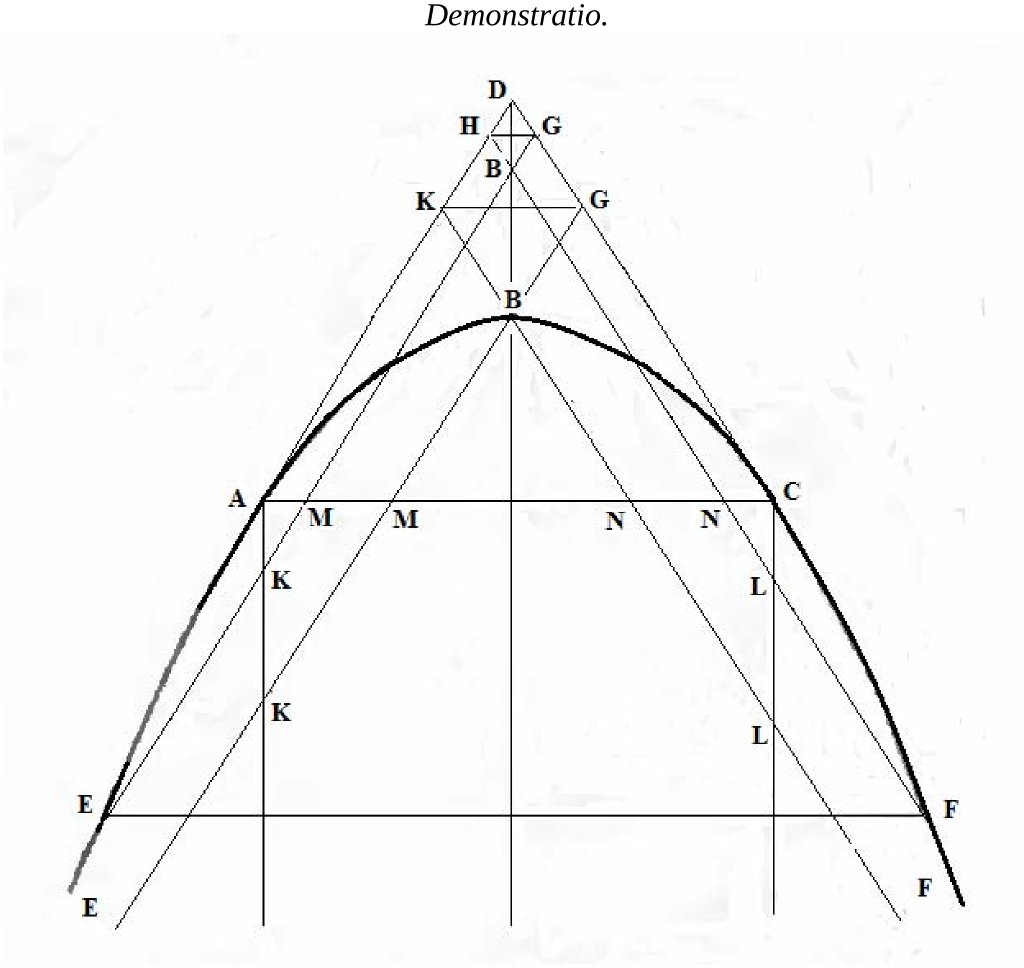
Divisa enim BC bifariam in F demittatur ex G per F linea GE: quoniam igitur ponitur AD, BC aequidistant, erit AE ad ED, ut BF ad FC; sed BC in F bifariam ponitur divisa, igitur & AD in E, quoque divisa est bifariam: unde GE diameter est linearum BC, AD &c. Quod erat demonstrandum.

PROPOSITIO XXVII.

Sit ad ABC parabolam diametrum BD ordinatim applicata AC, aganturque per A & C, contingences AD, CD, convenient illae cum diametro in D: tum punctum sumatur B, quodcunque in diametro BD: & ex B rectae ponantur BE, BF, aequidistantes contingentibus, parabolae vero occurrentes in E & F.

Dico iunctam EF aequidistare AC, adeoque ordinatim esse positam ad diametrum BD.

Demonstratio.



Demittantur ex A & C, diametri AK, CL occurrentes rectis BE, BF in L & K: productaeque EB, FB; contingentibus occurrant in H & G: erit igitur HD, GB parallelogrammum, &c HG bissecta a diametro DB, aequidistant ergo HG, AC: & quia AD, BE quoque parallelae sunt, aequantur HG, AM: similater cum aequentur HG, NC, rectae AM, NC aequales sunt: est autem AC in O, bifariam divisa, aequantur ergo & reliquae MO, ON : quare ut AN ad MO, sic CN ad NO. Sed ut AM ad NO, sic A K ad BO; & ut CN ad NO, sic LC ad BO, igitur ut AK ad BO sic CL ad BO, aequales ergo sunt diametri AK, CL: quia vero EB, FB contingentibus aequidistant, portiones illarum parabolam interceptae, in K & L bifariam sunt divisae. Ulterius ponatur per E aequidistans AC. Linea EP occurrens parabola in E & F, rectae BF in P, quia igitur AC bissecta est a diametro BD, erit & EF ab eadem in R bifariam divisa, & ER, RF aequales inter se; quia vero AC aequidistat HG, aequidistabunt quoque EP, HG : & EP quoque ut HG, a diametro BD bisecta est aequantur igitur RP, RF: & puncta FP unum idemque sunt: estque F communis intersectio linearum EF, BF cum parabola: aequidistant igitur EF, AC, HG. Quod erat demonstrandum.

Corollarium.

Ex his sequitur primo: posita EF ordinatim ad diametrum BD quam in D secent contingentes duae AD, CD; lineas ex E & F ductas ipsis AD, CD aequidistantes diametrum quoque BD, in uno eodemque puncto decussare; patet demonstratio ex praecedenti, cuius conversa est.

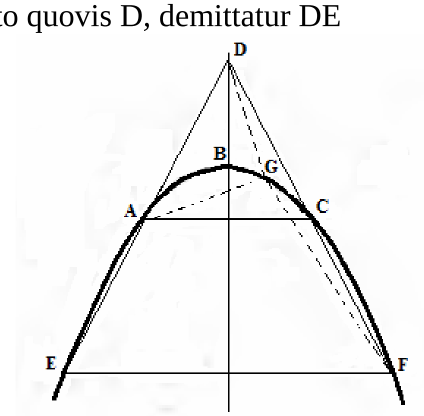
Sequitur secundo: posita AC ordinatim ad diametrum BD demissisque ex A & C aequalibus diametris AK, CL, quod rectae per K & L, ordinatim positae, DB diametro occurrant in uno eodemque puncto; patet ex ante dictis demonstratio, cum tangentes per A & C actae, ordinatim per K & L positis aequidistent, & BD diametro in uno eodemque occurrant puncto.

Sequitur tertio: lineam (QT) coniungentem puncta, in quibus rectae (BE, BF) contingentibus aequidistantes, parallae occurrunt, aequidistare rectae (EF) extrema linearum BE, B F coniungenti, adeoque lineas QT, AC, EF esse parallelus, ex ante dictis demonstratio manifesta est.

PROPOSITIO XXVIII.

Esto ABC parabola diameter BD, in qua assumpto puncto quovis D, demittatur DE secans parabolam in duobus punctis A & E & ex E ordinatim ponatur EF, iunctaque FD occurrat parabolae in C.

Dico EF, AC lineas aequidistare.



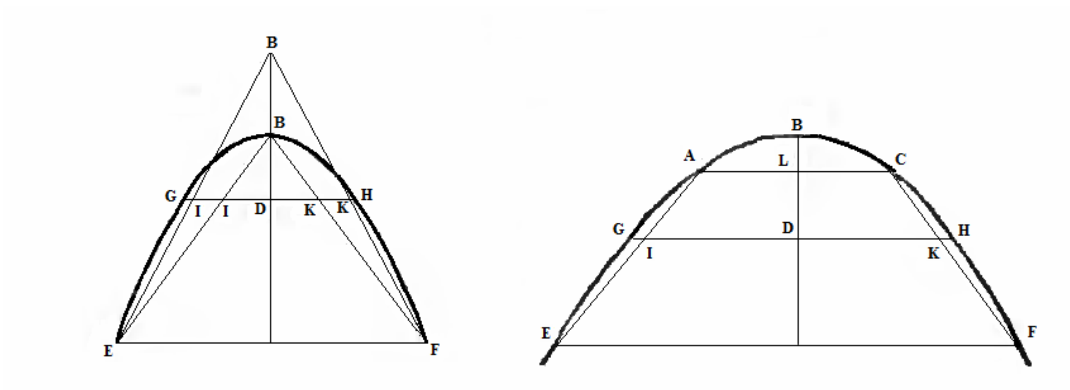
Demonstratio.

Si enim non sint parallelae, ponatur AG aequidistans EF, & ex F per G, ducatur FG, conveniet illa cum diametro BD in D, est autem ex constructione DF linea recta occurrens sectioni in C ; igitur recta FGD, eadem est cum FCD unde & punctum G idem cum C puncto, igitur aequidistant EF & AC. Quod fuit demonstrandum.

PROPOSITIO XXIX.

Aequidistent in parabola quaevis lineae AC, EF, iunctisque AE, CF ponatur alia quavis GH parallela AC, occurrens iunctis AE, CF in I & K.

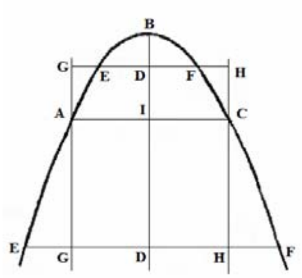
Dico GI, KH lineas esse inter se aequales.



Demonstratio.

Ponatur LD diameter rectarum AC, GH. Quoniam HG aequidistat AC ordinatim positae, erit HG in D bifariam divisa : est autem ID aequalis DK cum sit ID ad DK, ut AL ad LC (quia EA, FC lineae in idem punctum diametri conveniunt) igitur & reliquae IG, HK quoque sunt aequales. Quod erat demonstrandum.

PROPOSITIO XXX.



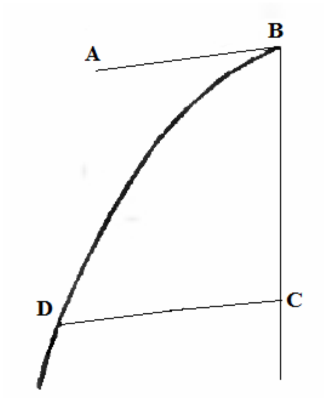
Aequidistet rursum in ABC parabola, rectae AC, EF, ponanturque per A & C diametri AG, HC occurrentes EF lineae in G & H.

Dico EG, FH lineas esse inter se aequales.

Demonstratio.

Ponatur ID diameter linearum AC, EF. Quoniam EF, AC lineae ordinatim positae sunt ad diametrum ID, erunt AC, EF in D & I bifariam divisae, sed & GH in D bifariam est divisa, cum GH aequalis sit rectae AC, igitur reliquae EG, HF, sunt inter se aequales. Quod erat demonstrandum.

PROPOSlTIO XXXI.



Dato angulo ABC & puncto in illo D, oportet per B & D parabolam describere cuius diameter sit BC & AB eandem contingens

Constructio & demonstratio.

Ducatur ex D linea DC parallela AB occurrens BC lineae in C: fiatque ut BC ad DC, sic DC ad AB, erit AB latus rectum parabole quaesitae, ac proinde determinata est parabola quae petebatur.