

[37]
PARS TERTIA.

De Rectangulorum inter se proportione.

PROPOSITIO LV.

Si AB linea, divisa fuerit utcumque in C & D.
 Dico ACB, CDB rectangula, aequalia esse rectangulis BDA, DCA.

Demonstratio.

Quadratum AB, aequale est BA quadrato; sed AB^2 quadratum aequatur quadratis AC, CD, DB, una cum rectangulis ACB, CDB, bis sumptis, & BA quadratum aequatur quadratis BD, CD, CA, una cum rectangulis BDA, DCA bis sumptis. Igitur ablatuis communibus quadratis AC, CD, DB, remanent ACB, CDB rectangula, aequalia rectangulis BDA, DCA. a. 4 Secundi.

Corollarium.

Propositio haec quoque vera est, si AB linea, utcumque & quotcumque punctis dividatur. Eademque est methodus progrediendi, & demonstrandi, qua in propositione usi sumus.

[37]
PART THREE.

Concerning rectangles in proportion.

BOOK I. §3.

PROPOSITON 55.

IF the line AB is divided in some way by the points C and D, then I assert that the sum of the rectangles ACB and CDB is equal to the sum of the rectangles BDA and DCA.



Prop.55. Fig.1

Demonstration.

The square AB is equal to the square BA; but the square AB^2 is equal to the sum of the squares AC, CD, and DB together with twice the rectangles ACB and CDB, and the square BA is equal to the sum of the squares BD, CD, and CA together with twice the rectangles BDA and DCA. Therefore with the common squares AC, CD, and DB taken away, there remains the sum of the rectangles ACB and CDB equal to the sum of the rectangles BDA and DCA. a. 4 Secundi.

[Thus, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2.(ab + bc + ca) = a^2 + b^2 + c^2 + 2.bc + 2a.(b + c) = a^2 + b^2 + c^2 + 2.ab + 2c.(a + b)$, giving $bc + a.(b + c) = ab + c.(a + b)$: contiguous elements can be added.]

Corollary.

This proposition is also true if the line AB is divided in any manner by any number of points. We are to proceed and demonstrate the proposition by the same method as we have used in this proposition.

PROPOSITIO LVI.

Si fuerit AB ad BC, sic AD ad DF, & BC linea aequalis EF.
 Dico ABCE, rectangulam aequale esse rectangulo CBD.

Demonstratio.

Quoniam per constructionem est, ut AB ad BC, sic AD ad DF, erit permutando, invertendo, ut AD ad AB, sic DF ad FE, id est ad BC : & dividendo, ut AB ad BD, sic FE ad DE, id est BC ad DE; quare ABDE rectangulum, aequale est rectangulo CBD. Quod erat demonstrandum. a. 4 Secundi.

BOOK I.§3.

PROPOSITON 56.

IF the ratio AB to BC is equal to AD to DF, and the lines BC and EF are equal, then I assert that the rectangle ABDE is equal to the rectangle CBD .



Prop.56. Fig.1

Demonstration.

Since by construction, AB to BC is in the same ratio as AD to DF, thus by inverting and interchanging, AD to AB is as DF to FE, that is BC: and on division, as AB to BD, thus FE to DE, that is BC to DE; whereby rect. ABDE is equal to rect. CBD. Q.e.d.

[Thus, $AB/BC = AD/DF$, and hence $AD/AB = DF/BC = DF/FE$.

Hence, $AD/AB - 1 = DF/FE - 1$, or $BD/AB = DE/FE = DE/BC$: giving $AB.DE = CB.BD$ as required.]

PROPOSITIO LVII.

Si fuerit AB recta divisa in C & D ut AC DB, lineae sint inter se aequales;
 Dico CB quadrarum aequari quadrato AC una cum rectangulo ABCD.

Demonstratio.

Quadratum AB, aequale est quadratis AC, CB, una cum ACB rectangulo bis sumpto; sed AC quadratum una cum rectangulo ACB aequali est

[38]

rectangulo CAB, igitur quadratum AB, aequale est quadrato CB, una cum rectangulis ACB, CAB. Rursum AB quadratum, aequale est rectangulis ABCD, CAB, DBA : igitur quadratum CB, una cum rectangulis ACB, CAB, aequale est rectangulis ABCD, CAB, DBA. quare dempto communi rectangulo CAB; aequalia remanent CB quadratum, una cum rectangulo ACB, rectangulis ABCD, DBA. id est rectangulis ABCD, BDA, una cum quadrato DB : ablatis igitur aequalibus rectangulis BDA, ACB, manet CB quadratum, aequale quadrato DB, id est AC, una cum rectangulo ABCD. Quod fuit demonstrandum.

BOOK I.§3.

PROPOSITON 57.

IF the line AB is divided by the points C and D in order that the lines AC and DB are equal, then I assert that the square CB is equal to the sum of the square AC and the rectangle ABCE.



Prop.57. Fig.1

Demonstration.

The square AB is equal to the sum of the squares AC and CB together with twice the rect. AC.CB. But the square AC together with the rect. AC.CB is equal to rect. CA.AB, and hence the square AB is equal to the square CB together with the sum of the rectangles AC.CB and CA.AB. Again, the square AB is equal to the sum of the rectangles ABCD, CA.AB, and DB.BA. Therefore the square CB together with the rectangles AC.CB and CA.AB is equal to the rectangles ABCD, CA.AB, and DB.BA. Whereby with the common rect. CA.AB taken away, there remains the square CB with the rect. AC.CB equal to the sum of the rectangles ABCD and DB.BA, that is to the rectangles ABCD and BD.DA together with the square DB : with the equal rectangles BD.DA and AC.CB taken away, there remains the square CB equal to the square DB or AC together with the rect. ABCD. Q.e.d.

[Thus, $AB^2 = AC^2 + CB^2 + 2.AC.CB$. But $AC^2 + AC.CB = CA.AB$;
 hence $AB^2 = CB^2 + AC.CB + CA.AB$.

Again, $AB^2 = AB.CD + CA.AB + AB.BD$;

hence $CB^2 + AC.CB + CA.AB = AB.CD + CA.AB + AB.BD = AB.CD + BD^2 + AD.DB$;

and as $AC.CB = AD.DB$, there remains $CB^2 = AB.CD + BD^2$. (This can be more easily found from the difference of the squares of CB and BD.)]

PROPOSITIO LVIII.

Si fuerit AD linea, divisa in B & C ut AB CD, lineae sint aequales, sumatur autem inter B & C, punctum quodvis E,
 Dico AED rectangulum, aequale esse rectangulis CEA, EBA, una cum quadrato AB.

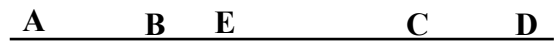
Demonstratio.

Rectangulum AED, aequale est rectangulis a ABEC, BEC, BECD, una cum quadrato AB; sed iisdem, aequalia sunt rectangula AECD ABE, una cum quadrato AB; (quia AEC, b aequatur rectangulis ABEC, BEC) igitur AED rectangulum, aequale est rectangulis CEA, EBA, una cum quadrato AB. Quod fuit demonstrandum. a 1. Secundi ;b 2 ibid.

BOOK I.§3.

PROPOSITON 58.

IF the line AD is divided by the points B and C in order that the lines AB and CD are equal, also some other point E is taken between B and C, then I assert



Prop.58. Fig.1

that the rectangle AED is equal to the sum of the rectangles CEA and EBA, together with the square AB.

Demonstration.

The rect. AE.ED is equal to the sum of the rectangles ^a AB.EC, BE.EC, BE.CD, together with the square AB; but this is also equal to the sum of the rectangles AE.EC, AB.BE, together with the square AB; (since rect. AE.EC ^b is equal to the sum of the rectangles AB.EC and BE.EC). Therefore the rect. AE.ED is equal to the sum of the rectangles CE.EA and EB.BA together with the square AB. Q.e.d. a 1. Secundi ;b 2 ibid.

[Thus, AE.ED = (AB + BE).(EC + CD) = AB.EC + BE.EC + BE.CD + AB² = AE.EC + AB.BE + AB², as AE.EC ^b = AB.EC + EB.BA, as the underlined rectangles are equal. Hence, AE.ED = AE.EC + AB.BE + AB² as required.]

PROPOSITIO LIX.

Si fuerit AC linea, utcunque divisa in D, B, & E ;
 Dico rectangula ADC, AEC, DBE ; aequalia esse rectangulis ABC, ADB, BEC,
 ADEC.

Demonstratio.

Rectangulum ADC, aequale est rectangulis ^c ADB, ADBE, ADEC: & AEC rectangulum ^d aequale est rectangulis CEB, CEBC, CEDA; quare addito rectangulo DBE, erunt ADCL AEC, DBE rectangula, aequalia rectangulis ADB, ADBE, ADEC, CEB, CEBC, CEDA, DVBE. Sed & ABC rectangulum aequale est rectangulis DBE, ADBE, CEBC, CEDA; additis igitur rectangulis ADB, BEC, ADEC erunt ABC, ADB, BEC, ADEC rectangula, aequalis rectangulis ADB, ADBE, ADEC, CEB, CEBC, CEDA, DBE: sed & iisdem rectangulis, ostensa sunt aequalis, rectangula ADC, AEC, DBE; Igitur rectangula ADC, AEC, DBE , aequalia sunt rectangulis ABC, ADB, BEC, ADEC. Quod fuit demonstrandum. c ibid ;d ibid.

BOOK I.§3.

PROPOSITON 59.

IF the line AC is divided by the points D, B and E in some manner, then I assert that the sum of the rectangles ADC, AEC, and DBE is equal to the sum of the rectangles ABC, ADB, BEC, and ADEC.



Prop.59. Fig.1

Demonstration.

The rect. AD.DC is equal to the sum of the rectangles ^c AD.DB, AD.BE, AD.EC; the rect. AE.EC ^d is equal to the sum of the rectangles CE.EB, CE.BD, and CE.DA. Whereby, with the rect. DB.BE added, the sum of the rectangles AD.DC, AE.EC, and DB.BE is equal to the sum of the rectangles AD.DB, AD.BE, AD.EC, CE.EB, CE.BD, CE.DA, and DB.BE . But rect. AB.BC is equal to the sum of the rectangles DB.BE, AD.BE, CE.BD, and CE.DA; therefore with the rectangles AD.DB, BE.EC, AD.EC added, the sum of the rectangles AB.BC, AD.DB, BE.EC, AD.EC is equal to the sum of the rectangles AD.DB, AD.BE, AD.EC, CE.EB, CE.BD, CE.DA, and DB.BE. But the same rectangles have been shown to be equal to the sum of the rectangles AD.DC, AE.EC, and DB.BE. Therefore the sum of the rectangles AD.DC, AE.EC, and DB.BE is equal to the sum of the rectangles AB.BC, AD.DB, BE.EC, and AD.EC. Q.e.d. c ibid ;d ibid.

[Thus, AD.DC = AD.DB + AD.BE + AD.EC .

The rect. AE.EC = CE.EB + CE.BD + CE.DA

Whence, on adding: AD.DC + AE.EC + DB.BE = AD.DB + AD.BE + AD.EC + CE.EB + CE.BD + CE.DA + DB.BE . But rect. AB.BC = DB.BE + AD.BE + CE.BD + CE.DA;

hence: AD.DC + AE.EC + DB.BE = AD.DB + AD.BE + AD.EC + CE.EB + CE.BD + CE.DA + DB.BE = AD.DB + AB.BC + AD.EC + CE.EB.

Hence, AD.DC + AE.EC + DB.BE = AD.DB + AB.BC + AD.EC + CE.EB.

Using the algebraic notation introduced in Prop. 1, where $AD = a$, etc. this result amounts to:
 $a(d + b + e) + e(a + d + b) + db = (a + d)(b + e) + ad + be + ae.$]

PROPOSITIO LX.

Si fuerit AB linea, divisa in quinque partes aequales, punctis C, D, E, F & ei quaevis in directum adijciatur GA ;
 Dico GD quadratum, aequalia esse quadrato AG, una cum rectangulo GCB.

[39]

Demonstratio.

Quadratum GD, aequale est quadratis ^a AG, AD una cum rectangulum GAD bis sumpto : sed AD quadratum aequale est quadratis AC, CD una cum rectangulo ACD bis sumpto ^b; id est una cum quadratis DE, EF, & GAD rectangulum bis sumptum, aequalia est rectangulis GACE, GAEB, id est rectangulo GACB (ob CE, EB lineas aequales lineae CB) igitur quadratum GD, aequale est quadratis AG, AC, CD, DE, EF, una cum rectangulo GA, CB. Rursum rectangulum GCB aequale est rectangulis GACB, ACB. sed ACB rectangulum aequale est rectangulis ACD, ACDE, ACEF, ACFB, id est quadratis AC, CD, DE, EF, una cum rectangulo GACB; Igitur addito quadrato AG, erit GCB rectangulum, una cum quadrato AG, aequale quadrato GD. Quod erat demonstrandum. a 4. Secundi ; b Per eandem.

BOOK I.§3.

PROPOSITON 60.

IF the line AB is divided by the points C, D, E and F into five equal parts adjoined in some manner along that line GA, then I assert that the square is equal to the sum of the square AG and the rectangle GCB.



Prop.60. Fig.1

[39]

Demonstration.

The square GD is equal to the sum of the squares ^a AG and AD together with twice the rect. GA.AD ; but the square AD is equal to the sum of the squares AC and CD together with twice the rect. AC.CD ^b, that is with the squares DE and EF; and twice the rect. GAD is equal to the sum of the rectangles GACE and GAEB, that is to the rect. GACB (as the lines CE and EB are equal to the line CB). Therefore the square GD is equal to the sum of the squares AG, AC, CD, DE, and EF, together with the rect. GA.CB. Again, the rect. GCB is equal to the sum of the rectangles GA.CB and AC.CB . But the rect. ACB is equal to the sum of the rectangles AC.CD, AC.DE, AC.EF and AC.FB, that is to the sum of the squares AC, CD, DE, and EF and the rectangle GA.CB. Therefore the sum of the rect. GC.CB and the square AG is equal to the square GD. Q.e.d. a 4. Secundi ; b Per eandem.

$$[GD^2 = AG^2 + AD^2 + 2.GA.AD; AD^2 = AC^2 + CD^2 + 2.AC.CD = AC^2 + CD^2 + DE^2 + EF^2;$$

$$2.GA.AD = GA.CE + GA.EB = GA.CB; \text{ therefore}$$

$$GD^2 = AG^2 + AC^2 + CD^2 + DE^2 + EF^2 + GA.CB. \text{ Again,}$$

$$GC.CB = GA.CB + AC.CB ;$$

$$\text{but } AC.CB = AC.CD + AC.DE + AC.EF + AC.FB = AC^2 + CD^2 + DE^2 + EF^2 ; \text{ hence}$$

$$GC.CB = GA.CB + AC^2 + CD^2 + DE^2 + EF^2 ; \text{ and therefore}$$

$$GD^2 = AG^2 + GC.CB$$

Using the algebraic notation introduced in Prop. 1, where $GA = g$, $AC = a$, etc. this result amounts to:
 $(g + 2a)^2 = g^2 + 4a.(g + a).$]

PROPOSITIO LXI.

Sicut AB in quoties partes utcunque in C, D, E, &c.
 Dico quadrato AC, CD, DE, EB, simul cum rectangulis ACB, CDB, DEB bis sumptis, aequari quadrato totius AB.

Demonstratio.

Quadratum enim AB, aequatur quadratis AC, CB, & rectangulo ^c ACB bis sumpto : eodem modo quadratum CB aequale est quadratis CD, DB, & rectangulo CDB bis sumpto; denique & quadratum DB aequale est quadratis DE, EB, & rectangulo DEB bis sumpto. collectis igitur in unum quadratis AC, CD, DE, EB, & rectangulis ACB, CDB, bis sumptis; exsurget quantitas, quadrato AB aequalis
 Quod erat demonstrandum. c 4. Secundi .

Corollarium.

Hinc colligere licet; quod de duabus rectis lineis, quomodocumque divisis etiam secundum dissimillimas rationes, iudicium ferre debeamus. Ex discursu enim posito in demonstratione huius propositionis constat i quadrata partium cuiuscumque lineae, cum rectangulis bis simul sumptis; quae sub partibus fiunt, secundum tenorem proportione contentum, aequalia esse quadrato totius; unde eam propositionem habere necessarium est quadrata partium ratios, simul cum rectangulis bis sumptis, ad quadrata omnia partium alterius, cum rectangulis suis bis sumptis, quam ipsam de quadrata totarum inter se obtinent. Quod admiratione non caret, cum una quantitatam, in paucissimas partes possit dividi, plaera vero in quam plurimas.

Hoc etiam quod subiungam, ignorantibus Geometriam maxime, videbitur parum credibile; si datum numerum verbi gratia 100 quis iubeatur, secum iacitus in plures pro libitu partes partiri, deinde singularum partium quadrata, in unam summam collecta, seponat; quae summae, ex multiplicationibus partium inter se, secundum sensum propositione contenum, coniuncta, certum quaedam numerum sibi computarit. Alter vero Geometrie gnarus, sponsione cum eo facta, certet se divinaturum etiam eum numerum, quem supputatione facta, in codicillis conscripserit. Ut res etiam ryrorum captui magis accommodetur: eam fusius nonnihil deducam.

BOOK I.§3.

PROPOSITON 61.

Let the line AB be cut a number of times by the points C, D, E, etc in some manner adjoined in some manner. I assert that the sum of the squares AC, CD, DE, EB taken together with twice the sum of the rectangles ACB, CDB, and DEB is equal to the square of the whole length AB.



Prop.61. Fig.1

Demonstration.

The square AB is indeed equal to the sum of the squares ^c AC and CB together with twice the rect. AC.CB; in the same manner the square CB is equal to the sum of the squares CD and DB together with twice the rect. CD.DB, and finally the square DB is equal to the sum of the squares DE and EB and twice the rect. DE.EB. Therefore with the squares AC, CD, DE, EB collected together and with the rectangles ACB, CDB taken twice, there quantities arise equal to the given square AB. Q.e.d. c 4. Secundi.

$$[AB^2 = AC^2 + CB^2 + 2.AC.CB; CB^2 = CD^2 + DB^2 + 2.CD.DB; DB^2 = DE^2 + EB^2 + 2.DE.EB; therefore AB^2 = AC^2 + CD^2 + DE^2 + EB^2 + 2.AC.CB + 2.CD.DB + 2.DE.EB.]$$

Using the algebraic notation introduced in Prop. 1, where AC = a, etc. this result amounts to:
 $(a + c + d + e)^2 = a^2 + c^2 + d^2 + e^2 + 2a(c + d + e) + 2c(d + e) + 2de.]$

Corollary.

Hence we are allowed to sum what is true for two lines, and we ought to make some comments concerning lines divided in any manner into unlike ratios. Indeed in the course of the demonstration of this proposition the position has been taken : find the sum of the squares of the arbitrary ratios of the parts of the line considered, together with twice the sum of the rectangles, and following the same course with the method applied to the sum of the squares and rectangles of any further subdivisions. The sum is equal to the square of the whole length. Thus, to have this proposition true, it is necessary to have the sum of the squares of the ratios of one subdivision, together with twice the sum of the rectangles, equal to the sum of the squares of some other subdivision with their rectangles taken twice, in order that each gives the square of the total length. For it is not without our admiration that a quantity can be divided into its smallest parts possible[Note: the microfilm I am using has some illegible words at the end of this section].

This section which I am going to add as well, with complete disregard for geometry, will make it seem a little more credible ; if a given number, say 100, is taken to be set out in several parts for argument's sake, then the squares of the individual parts are collected together in a single sum and set aside; which sums, by multiplication of the parts among themselves, following the argument of the proposition in contention, are added together and a certain number computed. The correct number is otherwise indeed known from geometry, and one might make a wager based on that fact that the same number is indeed written in your notebook from computation .

[40]

Ponatur numerus 100 expositus; divisus in quotuis partes; verbe gratia in AC, CD, DB. sitque AC partium 20 : CD, partium 30: residuum igitur partium 50. iubetur singulas partes multiplicare per seipsas; prodacent hae multiplicationes tres summam; quarum prima 400; secunda 900. tertia 2500. partes continebit. Ulterius iubetur AC, multiplicare per numerum CB. hoc est 20. per 80. & emergit summa 1600. quae bis sumpta, excrescit ad summam 3200. Denique hoc facto, etiam numerum CD ducere in DB, & exsurget summa 1500. quae bis sumpta efficit numerum 3000. tandem colligit hae producta in unam massam, cuius summa est 10000. quam Geometra discursu propositionis iam positae, sola multiplicatione numeri 100. per 100. factam illico manifestam habebit; scilicet 10000.

	100		
A	20	30	50
	C	D	
	400	900	2500
	1600	1500	
	3200	3000	
		400	
		900	
		2500	
		3000	
		3200	
		10000	

The number 100 is set out divided into a number of parts; for argument's sake into the parts AC, CD, and DB. AC shall be 20 of the whole, CD 30, and the remainder DB 50. The individual parts are selected and squared, and these multiplications produce three sums, the first of which will contain 400, the second 900, and the third 2500 parts. Of the others to be selected is AC times by CB, that is 20 by 80, giving rise to the sum 1600, which taken twice is increased to the sum 3200. Briefly according to this, the number CD is taken with DB, and the sum 1500 arises, which taken twice gives rise to the number 3000. Finally collect these numbers into a single term, the sum of which is 10000. Since by the geometrical discourse of the proposition thus established, one need only multiply the number 100 by itself, which will of course show to be 10000 immediately.

PROPOSITIO LXII.

Si fuerit AB linea, divisa utcunque punctis CD.
 Dico rectangulum sub AB, & composita ex ACDB ; una cum rectangulo sub CD,
 & composita ex ACDB; equalia esse rectangulis BCA, DBC, ADB, CAD, simul
 sumptis.

Demonstratio.

Rectangulum super AB, & composita ex AC, DB, aequale^a est rectangulis CAB, DBA; id est rectangulis
 ACB, ADB, una cum quadratis AC, DB ; id est aequale rectangulis CDB, ACDB, DCA, BDCA, una cum
 quadratis AC, DB. Rursum rectangulum super CD, & composita ex ACDB, aequale est rectangulis ACD,
 CDB: igitur rectangulum super AB, & composita ex ACDB, unu cum rectangulo super CD, & composita
 ex ACDB, aequale est rectangulis, CDB, ACDB, DCA, BDCA, ACD, CDB, una cum quadratis AC, DB.
 Iterum rectangulum BCA, aequale est rectangulis DCA, BDCA; item DBC rectangulum, aequale est
 rectangulo BDC, una cum quadrato DB; item ADB rectangulum ,aequale est rectangulis CDB, ACDB;
 denique rectangulumCAD, aequale est rectangulo ACD, una cum quadrato AC; igitur rectangula BCA,
 DBC, ADB, CAD, aequalia sunt rectangulo sub AB, & composita ex ACDB, una cum rectangulo sub CD,
 & composita ex ACDB. Quod erat demonstrandum. a 1. Secundi .

BOOK I.§3.

PROPOSITON 62.

If the line AB were cut in some
 points CD. I assert that the
 rectangle under AB, and
 composed from ACDB is equal to
 the sum of the rectangles BCA, DBC, ADB and CAD.



Prop.62. Fig.1

Demonstration.

The rectangle on AB, and composed from AC and DB^a is equal to the rectangles CAB and DBA, that is
 equal to the sum of the rectangles ACB and ADB together with the squares AC and DB. This is equal to the
 sum of the rectangles CDB, ACDB, DCA, BDCA, with the squares AC and DB. Again, the rectangle on
 CD, and composed from AC and DB, is equal to the sum of the rectangles ACD and CDB. Therefore the
 rectangle on AB, and composed from ACDB, together with the rectangle on CD, and composed from
 ACDB, is equal to the sum of the rectangles CDB, ACDB, DCA, BDCA, ACD, CDB, together with the
 squares AC and DB. Again the rectangle BCA is equal to the sum of the rectangles DCA and BDCA;
 likewise the rectangle DBC is equal to the rectangle BDC and the square DB; and likewise the rectangle
 ADB is equal to the rectangles CDB and ACDB. Finally the rectangle CAD is equal to the rectangle ACD,
 with the square AC; therefore the sum of the rectangles BCA, DBC, ADB, and CAD is equal to the
 rectangle under AB, and composed from ACDB, together with the rectangle under CD, composed from
 ACDB.

Q.e.d. c 4. Secundi.

[rect. on AB from AC and DB = CA.AB + DB.BA = CA.CB + AD.DB + AC² + DB² = CD.DB + AC.DB +
 DC.CA + BD.CA + AC² + DB² ;

Again, rect. on CD from AC and DB = AC.CD + CD.DB. Hence the sum of these rectangles is

= CD.DB + AC.DB + DC.CA + BD.CA + AC.CD + CD.DB + AC² + DB² ;

Now rect. BC.CA = DC.CA + BD.CA ; rect. DB.BC = BD.DC + DB² ; rect. AD.DB = CD.DB + AC.DB;

and rect. CA.AD = AC.CD + AC² . Hence the sum of these rectangles:

BC.CA + DB.BC + AD.DB + CA.AD = rect. on AB from AC and DB + rect. on CD from AC and DB as
 required.

In algebraic terms:

$$a.(c + d) + d.(c + d) + d.(a + c) + a.(a + c) = (a + d)(c + d) + (a + d)(a + c)$$

= (a + d)(a + 2.c + d) = (a + c + d).(a + d) + c.(a + d), where the first and the last terms are those in the
 theorem.]

PROPOSITIO LXIII.

Rectum AB, divisam utcumque in CD, iterum in E dividere, ut AED rectangulum, aequale sit rectangulo BEC

[41]

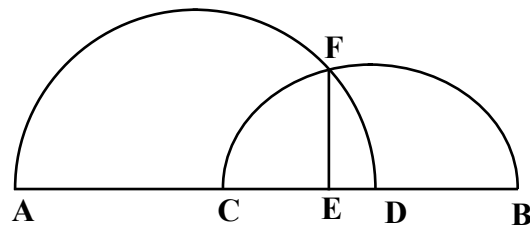
Constructio & Demonstratio.

Super AD, CB ut diametris, circuli describantur, AFD, DFC qui occurrant sibi mutuo in F: tum ex F recta demittatur FE, normalis ad lineam AB. Dico punctum E, satisfacere peritioni patet; cum tam AED, quam BEC recangulum aequale, ^a sit quadrato FE. igitur lineam AB, utcumque in C & D divisam, iterum secuimus in E; &c. . Quod erat faciendum a 33. Terti .

BOOK I.§3.

PROPOSITON 63.

The line AB is divided in some way by the points C and D, and for a second time by E, in order that the rectangle AE.ED is equal to the rectangle BE.EC.



Prop.63. Fig.1

Construction & Demonstration.

Upon AB CD as diameters, the circles AFD and DFC are described which cut each other in F. Then from F the line FE is sent normal to the line AB. I assert that the point E satisfies the requirement: since the rect. AE.ED as rect. BE.EC are equal to the square FE ^a. Therefore the line AB, divided somehow by C and D, we have cut in E, etc. Q.e.d. a 35. Tertii.
 [AE.ED = CE.EB = EF² .]

PROPOSITIO LXIV.

Lineam AB divisam utcumque in C, D, iterum dividere in E , ut CEA rectangulum, aequale sit rectangulo DEB.

Constructio & Demonstratio.

Describantur super AC, DB lineis , ut diametris, circuli AIC, BHD: quos in H & I contingit linea HI; quae bifariam in G divisa, demittatur ex G linea GE normalis ad rectam AB. Dico punctum E, esse quod queritur. secetur BE linea in F, ut EF quadratum sit aequale rectangulo DEB. & per F, ex G ducatur recta GN, occurrens circulo DHB in M, & N. & ex G ducatur altera GK, occurrens circulo AIC in L & K, facta constructione ut prius. Quoniam per constructionem, EF quadratum, aequale ponitur rectangulo DEB; & HG recta est tangens, erit FG quadraum, aequale ^b rectangulo MGN; sed FG quadratum aequale est quadratis EG, EF. Igitur & MGN rectangulum aequale est quadratis, EG, EF: id est quadrato EG, una cum rectangulo DEB; eodem modo ostendetur LGK rectangulum , aequari quadrato EG, una cum rectangulo, AEC. Unde, cum aequalia sint rectangula LGK, MGN, dempto communi quadrato EG, erunt AEC, DEB rectangula, inter se aequalia. Divisimus igitur lineam AB in E, &c. Quod erat faciendum. ^b Pappi L7. Pr.159.

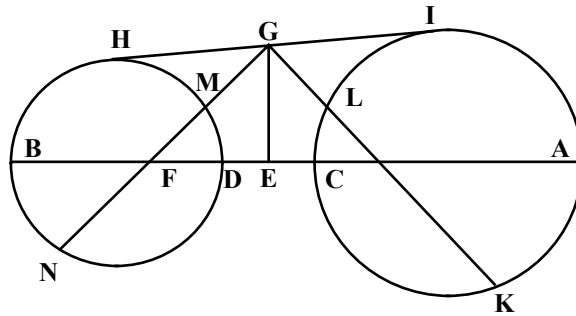
BOOK I.§3.

PROPOSITON 64.

The line AB is divided in some way by the points C and D, and for a second time by E, in order that the rectangle EC.EA is equal to the rectangle ED.EB.

Construction & Demonstration.

The circles AIC and BHD are described on the lines AC and DB as diameters, and which the line HI touches the circles in H and I. HI is bisected in G, and from G the line GE is sent normal to the line AB. I assert that E is the point sought. The line BE is cut in F, in order that the square EF is equal to the rectangle DE.EB. From G through F the line GN is drawn, cutting the circle DHB in M and N. From G another line GK is drawn, crossing the circle AIC in L and K, according to the same construction as the first. Since by construction, the square EF is put equal to the DE.EB; and since the line HG is a tangent, the square FG is equal^b to the rectangle MG.GN; but the square FG is equal to the sum of the squares EG, EF. Therefore the rectangle MG.GN is equal to the sum of the squares EG and EF: that is to the square EG, together with the rectangle DE.EB. In the same way is can be shown that the rectangle LG.GK is equal to the square EG, together with the rectangle AE.EC. Hence, since the rectangles LG.GK and MG.GN are equal with the common square EG taken away, the rectangles AE.EC and DE.EB are equal to each other. Therefore we can divide the line AB in E, &c. Quod erat faciendum.^b Pappi L7. Pr.159.



Prop.64. Fig.1

PROPOSITIO LXV.

Data sint duae lineae A & BF. oportet BF lineae, quandam FC adjicere, ut quadratum A ad BCF rectangulum datam habeat rationem D ad E.

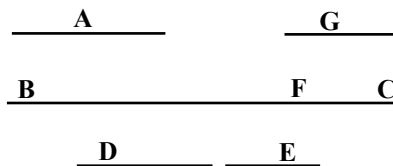
Constructio & Demonstratio.

Fiat ut D ad E, sic A quadratum, ad quadratum G; dein FB lineae quaedam adiungatur FC, ut BC, G, FC, tres sint in continua analogia: quid fiet si data differentia extremarum BF, & media G. inveniuntur extremae FC, CB per Andersonium & alios. Dico factum esse quod petitur. Quoniam BC, G, FC lineae sunt continuae proportionales, erit CF rectangulum, aequale quadrato G: igitur quadratum A est ad rectangulum BCF ut A quadratum, ad quadratum G; id est per constructionem ut D ad E. Igitur rectae BF quandam addidimus, &c. Quod erat faciendum.

BOOK I.§3.

PROPOSITON 65.

Two lines are given A and BF, it is required to adjoin some line FC to the line BF, in order that the ratio of the square of A to the rectangle BC.CF is in the given ratio D to E.



Prop.65. Fig.1

Construction & Demonstration.

Thus the ratio of the square A to the square G shall be made in the ratio D to E; then the line FB is added to some line FC, in order that BC, G, and FC are three lines in analogous continued proportion, which can be done if the difference of the extremes of the ratio BF and the mean G are given. A method for finding the extremes of the ratio, FC and CB, has been found by Anderson et al. I assert that what was asked has been done. Since the lines BC, G, and FC are lines in continuous proportion, the rect. BC.CF is equal to the square G. Therefore the square A is to the rect. BC.CF

[42]

as the square A is to the square G; that is by construction as D to E. Therefore the line BF, &c. Q.e.d.

[$A^2 : G^2 = D : E$; $G^2 = BC.CF$; hence $A^2 : G^2 = A^2 : BC.CF = D : E$.

To explore the method would take Gregorius too far out of his way : Alexander Anderson, who came from Aberdeen (King's College library still has his original publications dating from 1615), during his sojourn in Paris, had become acquainted with Vieta's work, and went on to extend and edit Vieta's posthumous papers. In the brief work *Pro Zetetico Apollonii Redivivi*, Anderson investigated the solution of various proportionalities involving cubic equations in a primitive pre-algebraic form that originated with Vieta . Others including Snell were later to work on this problem. A modern reference for the ideas behind this work, which stemmed originally from Apollonius, is p.190 - 191, *A History of Greek Mathematics*, vol. ii, Sir Thomas Heath, Dover 1981.]

PROPOSITIO LXVI.

Datae lineae AB, inaequaliter in C divisae, quendam BD adiicere, ut BDA
 rectangulum , aequale sit quadrato CD.

Constructio & Demonstratio.

Fiat AE aequalis CB. & fiat quadrato CB aequale rectangulum ECBD. Dico factum esse quid petitur. Sunt enim per constructionem continuae DB, BC, CE, & quia AE aequalis est CB, erunt & BD,DC,DA^a continuae proportionales ; unde CD quadrato aequale est rctangulum BDA. addidimus igitur rectam, &c. Quod erat faciendum. a 5. Huius.

BOOK I.§3.

PROPOSITON 66.

For the given line AB, unequally divided in C, some line BD is to be added on to
 AB, in order that the rectangle BD.DA shall be equal to the square CD.

Construction & Demonstration.

The line AE is made equal to CB. And the square CB is made equal to the rectangle EC.BD. I assert that what was sought has been done. For indeed from the construction the lines DB, BC, and CE, are in continued proportion. Since AE is equal to CB: BD, DC, and DA^a are continued proportionals; hence the square CD is equal to the rectangle BDA. We can therefore the line, &c. Quod erat faciendum. a 5. Huius.



Prop.66. Fig.1

[We are given $BC/CE = DB/BC$; then $BC/DB = CE/BC$ and $\underline{CD/DB} = AC/BC$, giving $CD/AC = DB/BC$, from which by addition once more, we have $\underline{AD/AC} = \underline{CD/BC}$, and $\underline{AD/CD} = AC/BC = \underline{CD/DB}$; i. e. $AD.DB = CD^2$ as required. Geometrically, we jump from ratio to ratio, starting from the r.h.s. of the diagram with BC/BD]

PROPOSITIO LXVII.

Si fuerit A ad B, ut C ad D, & E ad F, ut G ad H. Dico AH rectangulum, ad rectangulum BG, eam habere rationem, quam habet CF rectangulum, ad rectangulum DE.

Demonstratio.

Ratio rectanguli AH, ad BG rectangulum, est composita ex ratione A ad B, & ex H ad G. Sed etiam ratio CF rectanguli, ad rectangulum DE, composita est ex ratione C ad D, id est per constructionem A ad B; & ex F ad E, id est H ad G. Igitur rectangulum AH, ad rectangulum BG, eam habet rationem, quam CF rectangulum, ad rectangulum DE. Quod erat demonstrandum. b 23. Sexti.

BOOK I.§3.

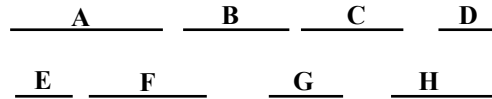
PROPOSITON 67.

If the ratio A to B were as C to D, and E to F as G to H, then I say that the rectangle AH to the rectangle BG has the same ratio as the rectangle CF to the rectangle DE.

Demonstration.

The ratio of the rect. AH to the rect. BG, is composed of the ratio A to B, and from H to G. But the same ratio of the rect. CF, to rect. DE, is composed from the ratio C to D, that is by construction A to B; and from F to E, that is H to G. Therefore the rect. AH to the rect. BG has the same ratio as the rect. CF to the rect. DE. Q. e. d. b 23. Sexti.

[AH/BG = A/B . H/G ; likewise, the corresponding ratio CF/DE =C/D . F/E, from which the result follows.]



Prop.67. Fig.1

PROPOSITIO LXVIII.

Si A, B, C, D, lineae fuerint in continua ratione, sint autem & alia quator E, F, G, H in continua analogia; Dico AH rectangulum, ad rectangulum DE, rationem habere triplicatum eius, quam habet BG rectangulum, ad rectangulum CF.

Demonstratio.

Ratio rectanguli AH, ad rectangulum DE, ^c composita est ex ratione A ad D, id est , ex triplicata ^d ratione B ad C, quia A, B, C, D continuae sunt proportiionales & ex H ad E, id est ex triplicata ratione G ad F; sed BG rectangulum, ad rectangulum CF, rationem habet compositam ^e ex B ad C, & ex G ad F; igitur rectangulum AH, ad rectangulum DE, rationem habet triplicatam , eius quam habet DE rectangulum, ad rectangulum CF. Quod erat demonstrandum. c 23. Ibid ; d 10 *Defin.* Quinti; e 23 Sexti.

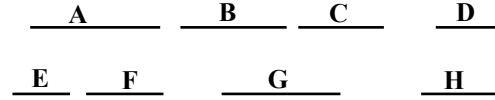
BOOK I.§3.

PROPOSITON 68.

If A, B, C and D, were four lines in a continued ratio, and also E, F, G, and H were another four lines in continued proportion similarly, then I say that the rectangle AH to the rectangle DE has the triplicate ratio of that which the rectangle BG has to the rectangle CF.

Demonstration.

The ratio of the rect. AH to the rect. ED^c, is composed of the ratio A to D, that is from the triplicate ratio B to C, as A, B, C, D are continued proportionals and from H to E, that is from the triplicate ratio G to F; but rect. BG, to rect. CF, has a ratio composed^c from B to C, and from G to F; therefore rect. AH, to rect. DE, has the triplicate ratio, of that which rect. CF has to rect. DE. Q.e.d. c 23. Ibid ; d 10 *Defin.* Quinti; e 23 Sexti.



Prop.68. Fig.1

[A/B = B/C = C/D = r and E/F = F/G = G/H = s are given. Then AH/ED = A/D . H/E = r³/s³; the ratio BG/CF = B/C . G/F = r/s, hence AH/DE = A/D . H/E = r³/s³ = (BG/CF)³ = (CF/DE)³, as CF/DE = C/D. F/E = r/s.]

PROPOSITIO LXIX (LXVII).

Si AB, AC, AD, lineae fuerint continuae, quibus aequales fiant AE, AF, AG, in directum;
 Dico DCG rectangulum, ad rectangulum CFB, esse ut DA ad BA.

Demonstratio.

Centro A, intervallis AC, AD, semicirculi describantur DHG, CKF; erectaque ex C normali CH, ducatur recta AH, occurrens circulo CKF in K, iunganturque BK, ut AC ad AD, sic AK ad AH, sed ut AC ad AD sic AB est ad AC per constructionem; igitur ut AB ad AC, sic AK ad AH ; adeoque BK, HC lineae sunt parallelae, & BK recta, normalis ad lineam AC : quare DCG rectangulum, est ad rectangulum CBF, ut HC quadratum, ad quadratum KB, id est in duplicata ratione lineae HC ad KB, id est AC ad AB, id est ut AD, linea ad lineam AB. Quod erat demonstrandum.

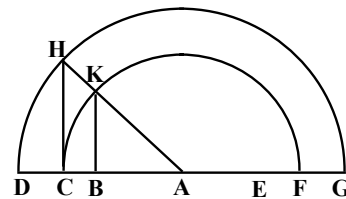
BOOK I.§3.

PROPOSITON 69 (67).

If AB, AC, AD were lines in a continued ratio, from which the equal lines AE, AF, AG, are made in order. I say that the rectangle DCG is to the rectangle CFB as the square DA is to the square BA.

Demonstration.

With centre A, semi-circles DHG and CKF are described with radii AC, AD; and a normal CH is erected from C, and the line AH is drawn, crossing the circle CKF in K, and the points BK are joined. As AC is to AD, thus AK is to AH, but as AC to AD thus by construction AB is to AC; therefore as AB to AC, thus AK to AH ; and hence the lines BK and HC are parallel and the line BK, normal to the line AC : whereby the rect. DCG is to the rect. CBF, as the square HC is to the square KB, that is in



Prop.69. Fig.1

duplicate ratio of the line HC to KB, that is AC to AB, that is as the line AD to the line AB. Q.e.d.

[$AC/AD = AK/AH$; and $AC/AD = AB/AC$, hence $AB/AC = AK/AH$ and $AC/AB = AH/AK$. Hence, $DC.CG/CB.BF = CH^2/KB^2 = AC^2/AB^2 = AD^2/AB^2$ as required. Note: Prop. 69 is wrongly called Prop. 67 again, and so on to the end of the first book.]

PROPOSITIO LXX (LXVIII).

Si fuerint quotunque lineae continuae proportionales AB, CD, DB, EB, FB, GB. Dico rectangula ABDE, CBDE, CBEF, DBEF, DBFG, EBFG in continua esse analogia; & quidem in ratione AB ad CB.

Demonstratio.

Rectangulum enim ABDE ad rectangulum CBDE, est ut AB^a linea, ad lineam CB; & CBDE rectagulum ad rectangulum CBEF, est ut^b DE ad EF, id est, ut AB ad CB; Rursum rectangulum CBEF, ad rectangulum DBEF; est ut CB ad DB, id est AB ad CB. quia AB, CB, DB, &c. ponuntur continuae, & DBEF rectangulum, ad rectangulum DBFG, est ut EF ad FG, id est iterum AB ad CB; & sic de ceteris. Igitur rectangula ABDE, CBDE, CBEF, &c. in continua sunt analogia, & quidem in ratione AB ad CB. Quod erat demonstrandum. a 1. *Sexti*; b 2. *Ibid*.

BOOK I.§3.

PROPOSITON 70 (68).

If there are a number of lines AB, CD, DB, EB, FB, G, B in continued proportion, then I assert that the rectangles AB.DE, CB.DE, CB.EF, DB.EF, DB.FG, EF.GB are similarly in continued proportion, and indeed in the ratio AB to CB.

Demonstration.

Indeed the ratio of rect.AB.DE is to rect. CB.DE as the line AB^a is to the line CB; and the rect. CB.DE to the rect. CB.EF, is as^b DE to EF, that is, as AB to CB; Again the rect. CB.EF to the rect. DB.EF is as CB to DB, that is AB to CB, since AB, CB, DB, &c. are placed in continued proportion, and the rect. DBEF to the rect. DB.FG, is as EF to FG, that is again as AB to CB; and so on for the others. Therefore the rectangles AB.DE, CB.DE, CB.EF, etc. are in continued proportion in analogy, and indeed in the ratio AB to CB. Q.e.d. . a 1. *Sexti*; b 2. *Ibid*.



Prop.70. Fig.1

PROPOSITIO LXXI (LXIX).

Si fuerint tres ordines continuae proportionalium A, B, C, D. E, F, G, H. I, J, K, L, M. & AF rectangulo aequale fiat quadratum N: & R quadratum aequale rectangulo EK; sit autem & IF rectangulo, aequale quadratum O, & EB rectangulo, quadratum S. dein & rectangulo CH, aequale quadratum P; & T quadratum , aequale rectangulo GM; denique rectangulo LH, aequale quadratum Q, & GD rectangulo, quadratum V.

Dico quadratum N esse ad quadratum P, ut est quadratum S, ad quadraum V: & R quadratum, ad quadratum T, ut O quadratum ad quadratum Q.

Demonstratio.

Quadratum enim N ad quadratum P, id est per constructionem, AF rectangulum, ad rectangulum CH rationem habet^a compositam, ex A ad C, id est per hypothesim ex duplicata ratione A ad B; & ex F ad H, id est duplicata ratione E ad F; sed ratio quadrati S ad quadratum V, id est per hypothesim rectanguli EB, ad rectangulum GD, etiam componitur ex ratione B ad D, id est duplicata ratione A ad B, & ratione E ad G, id est duplicata ratione E ad F, igitur ut quadratum N ad quadratum P: sic quadratum S ad quadratum V. Eodem modo ostenditur quadratum R, ad quadratum T esse, ut quadratum O, ad quadratum Q. Quod erat demonstrandum. a 23. *Sexti.*

BOOK I. §3.

PROPOSITON 71 (69).

If there are three series of continued proportions A, B, C, D. E, F, G, H. I, J, K, L, M, and the square N is made equal to the rect. AF: and the square R made equal to the rect. EK; and also the square O is equal to the rect. IF, and the square S to the rect. EB, the square P to the rect. CH, the square T equal to the rect. GM, the square Q equal to the rect. LH, and the square V equal to the rect. GD.

I assert that square N is to square P, as square S is to square V: and square R to square T, as square O to square Q.

Demonstration.

A. B. C. D.
 E. F. G. H.
 I. K. L. M.
 N. P. S. V.
 R. T. O. Q.

Prop.71. Fig.1

For square N to the square P, that is by construction as rect. AF to rect. CH has the composition ratio^a from A to C, that is by hypothesis from the duplicate ratio A to B; and from F to H, that is the duplicate ratio E to F; but the ratio of square S to square V, that is by hypothesis of rect. EB to rect. GD, also put together from the ratio B to D, that is the duplicate ratio of A to B, and with the ratio E to G, that is the duplicate ratio E to F. Therefore as square N to square P: thus square S to square V. In the same way it can be shown that square R to square T to be as square O to square Q.

Q.e.d. a 23. *Sexti.*

[In modern terms:

$A = a ; B = ar ; C = ar^2 ; D = ar^3 . E = e ; F = es ; G = es^2 ; H = es^3 ; I = i ; K = it ; L = it^2 ; M = it^3 .$
 $N^2 = A.F = aes ; R^2 = E.K = eit ; O^2 = I.F = ies ; S^2 = E.B = ear ; P^2 = C.H = aer^2s^3 ; T^2 = G.M = eis^2t^3 ;$
 $Q^2 = L.H = eis^3t^2 ; V^2 = G.D = aer^3s^2 .$
 $N^2/P^2 = aes/aer^2s^3 = 1/r^2s^2 = A/C . F/H = (A/B)^2 . (E/F)^2 ; S^2/V^2 = ear/aer^3s^2 = 1/r^2s^2 = E/G . B/D =$
 $(A/B)^2 . (E/F)^2 ,$ from which the result follows.]

PROPOSITIO LXXII (LXX).

Sit AC linea divisa inaequaliter in B, oportet utrimque rectas aequales adijcere AD, CE, ut ABC rectangulum, ad rectangulum DBE, datam habeat rationem, R ad S.

Constructio & Demonstratio.

Descripto super AC, ut diametro, semicirculo AFC, erigatur ex B, normalis BF; & fiat ut R linea ad S. lineam, ita BF quadratum, ad quadratum BG, tum centro communi H, intervallo HG, describatur semicirculus DGE occurrens AC lineae productae in D & E. Dico AD, CE, lineas satisfacere petitioni. Rectangulum enim ABC, aequale est quadrato^b FB, & GB quadrato, aequale est rectangulum DBE: igitur rectangulum ABC, est ad rectangulum DBE, ut FB quadratum, ad quadratum BG, id est linea R ad lineam S per constructionem. datae igitur lineae AC rectas aequales vtrisque adiecimus; &c. Quod erat demonstrandum.

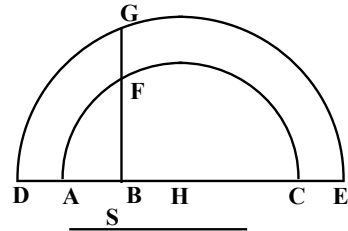
BOOK I.§3.

PROPOSITON 72 (70).

Let AC be a line divided unequally in B, it is required to add equal lines AD, CE, in order that the rectangle ABC to the rectangle DBE shall have the given ratio R to S.

Demonstration.

With a semi-circle AFC described on AC as diameter, a normal BF is erected from B; the square BF shall be made to the square BG in the ratio of the line R to the line S. Then with the common centre H, with radius HG, the semi-circle DGE is described cutting the line AC produced in D and E. I say that the lines AD and CE satisfy the requirements. Indeed the rect. ABC is equal to the square^b FB, and the square GB is equal to the rect. DBE: therefore the rect. ABC is to the rect. DBE. as the square FB to the square BG, that is to the line R to the line S by construction. Therefore we may add to the line AC equally on both sides, etc. Q.e.d. b 35. *Tertii.*



Prop.72. Fig.1

Dato quadrato A, & linea DF, utcunque in E divisi, exhibere rectam, quae divisa secundum rationem DE ad EF, exhibeat quadratum sub tota, una cum rectangulo sub segmentis, ad quadratum A, in data ratione B ad C.

Constructio & Demonstratio.

Inventa M, media inter B & C, fiat ut quadratum B ad quadratum M; ita DF quadratum una cum rectangulo DEF, ad quadratum G, dein ut G linea, ad DF lineam. sic latus quadrati, A fiat ad quandam IK; quae ita secetur in L, ut DF est divisi in E. Dico IK lineam esse quaesitam. Quoniam est ut G linea, ad lineam DE, sic latus quadrati A ad rectum IK; erit invertendo, permutando, DF ad IK, ut G ad latus quadrati A. Rersum cum IK, DF lineae proportionaliter sint

[45]

divisae, erit ut IK ad DF sic LK ad EF. Est autem ratio rectanguli ILK ad rectangulum DEF, a composita ex ratione IL ad DE, & LK ad EF, id est ex duplicata ratione IK ad DF, item quadratum IK ad quadratum DF, duplicatam habet rationem, eius quam habet IK linea, ad lineam DF; igitur, erit ILK rectangulum una cum quadrato IK ad rectangulum DEF una cum quadrato DF. in duplicata ratione IK ad DF. & quia est ut DF ad G sic IK ad latus quadrati A, erit ut rectangulum DEF una cum quadrato DF, ad quadratum G, sic ILK rectangulum una cum quadrato IK, ad quadratum A. sed per constructionem est DEF rectangulum una cum quadrato DF, ad quadratum G, ut quadratum B ad quadratum M, id est ut linea B ad lineam C, igitur rectangulum ILK una cum quadrato IK, est ad quadratum A, ut B ad C. Unde dato quadrato A, &c. Quod erat faciendum.

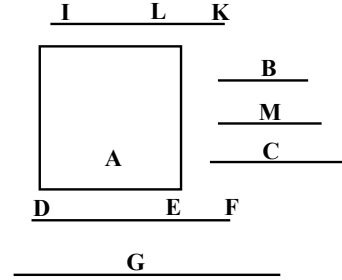
BOOK I.§3.

PROPOSITON 73 (71).

Given the square A, and a line DF divided by E in some manner, to find a line which has been divided according to the ratio DE to EF, and which shows the ratio: sum of the square under the total length and the rectangle under the segments to the square A that is equal to the given ratio B to C.

Demonstration.

Find the mean between B and C, and thus the square B to the square M is made in the ratio of the sum of the square DF and the rect. DEF to the square G. Then as the line G is to the line DF, thus the side of the square A shall be made to some length IK; which thus is cut in L, as DF had been divided by E. I assert that the line IK is that sought. Since the line G is to the line DE, thus as the side of the square A to the line IK. By inverting and interchanging, the ratio of DF to IK, is as G to the side of the square A. Again, as the lines IK and DF will be divided proportionally



Prop.73. Fig.1

as IK to DF thus LK to EF. But also the ratio of the rectangle ILK to the rectangle DEF, ^a put together from the ratio IL to DE, and LK to EF, that is from the square of the ratio IK to DF, likewise the square IK to the square DF, has the square ratio, of that which the line IK has to the line DF. Therefore the rect. ILK added to the square IK will be to the rect. DEF added to the square DF in the squared ratio IK to DF. And because DF is to G thus as IK is to the side of the square A: as the rect. DEF with the square DF, to the square G, thus the rect. ILK with the square IK, to the square A. But from the construction the ratio of the rect. DEF with the square DF, to the square G, shall be as the square B to the square M, that is as the line B to the line C, therefore the rect. ILK with the square IK, is to the square A, as B to C. Hence, from the given square A, &c. Q. e. f. Q.e.d. a 23. *Sexti.*

[$M^2 = B.C$; then $B^2/M^2 = (DF^2 + DE.EF)/G^2$ and $G/DF = a/IK$, where we have set $a^2 = A$,
 $IL/LK = DE/EF$ and $IL/DE = LK/EF$. ILK is the required line.
 For, as $DF/IK = G/a$, and as $IK/LK = DF/EF$; $IK/DF = LK/EF$. Also, $rect. ILK / rect. DEF = IL.LK/DE.EF = IL/DE . LK/EF = (LK/EF)^2 = (IK/DF)^2 (= a^2/G^2)$. Hence, $IL.LK/IK^2 = DE.EF/DF^2$
 and $(IL.LK + IK^2)/(DE.EF + DF^2) = (IK/DF)^2 = a^2/G^2$.
 Thus, $(IL.LK + IK^2)/a^2 = (DE.EF + DF^2)/G^2 = B^2/M^2 = B/C$ as required.]

PROPOSITIO LXXIV (LXXII).

Esto AB linea divisa utcunque in C, oportet eam augere recta BD, ut ACB rectangulum, ad rectangulum ADB, datam habeat rationem E ad F.

Constructio & Demonstratio.

Fiat ut E ad F, sic ACB rectangulum ad G quadratum; deinde data media G, & excessu extremarum AB, inveniantur extremae AD, DB per Andersonium & alios. Dico factum esse quod petitur : est enim ut E ad F, sic ACB rectangulum, ad quadratum G, sed quadrato G, per constructionem aequale est rectangulum ADB, igitur ut E ad F, sic ACB rectangulum, ad rectangulum ADB; datae igitur lineae AB, quantum adieciimus, &c. Quod erat faciendum.

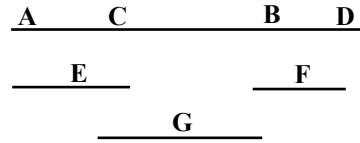
BOOK I.§3.

PROPOSITON 74 (72).

Let the line AB be divided in C in some manner. It is required to add the line BD to this line, in order that the rectangle ACB to the rectangle ADB shall have the given ratio E to F.

Construction & Demonstration .

Thus the ratio of the rect. ACB to the square G shall be made in the ratio E to F ; then from the given mean G, and from the excess of the extremes AB, the extremes of the ratio AD, DB can be found from the method of Anderson and others. I say that what was desired had been accomplished : for indeed as E is to F, thus the rect. ACB is to the square G. But the square G by construction is equal to the rect. ADB; therefore as E to F, thus the rect. ACB to the rect. ADB; therefore for the given line AB, we may add some line, &c. Q.e.f.



Prop.74. Fig.1

[$AC.CB/G^2 = E/F$; but $AD.DB = G^2$ also, from the construction of Anderson et al. Hence, $AC.CB = AD.DB$ as required.]

PROPOSITIO LXXV (LXXIII).

Datam rectam divisam in C utcunque, iterum in D secare , ut DAB rectangulum ad rectangulum DCB, datam habeat ratiomem E ad F.

Constructio & Demonstratio.

Fiat ut E ad F, sic AB linea ad lineam G; dein AB secetur in D, ut AD sit ad DC, sicut CB est ad G. Dico factum esse quod petitur.

Ratio DAB rectanguli, ad rectangulum DCB, composita est ex ratione AB ad CB, & AD ad DC, id est per constructionem CB ad G. sed etiam ratio b AB ad G, id est E ad F, componitur ex ratione AB ad CB, & CB ad G, igitur rectangulum DAB ad rectangulum DCB, eam habet rationem quam AB linea ad G, id est E ad F; rectam igitur AB in D secuimus, &c. Quod erat faciendum. b *Defin. 5 Sexti.*

BOOK I.§3.

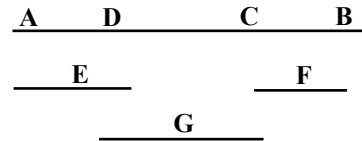
PROPOSITON 75 (73).

The given line AB is cut by some point C, and again in D, in order that the rectangle DAB to the rectangle DCB shall have the given ratio E to F.

Construction & Demonstration .

Thus the line AB to the line G shall be made in the ratio E to F; then AB is cut in D, in order that AD shall be to DC, thus as CB is to G. I say that the task is done.

For the ratio of rect. DAB to rect. DCB, which has been composed from the ratio AB to CB, and AD to DC, that is by construction as CB to G. But also the ratio ^b AB to G, that is E to F, is made from the ratio AB to CB, and CB to G; therefore the rect. DAB to the rect. DCB, has that ratio which the line AB has to G, that is E to F; therefore we have cut the line AB in D, &c. Quod erat faciendum. b *Defin. 5 Sexti.*



Prop.75. Fig.1

&c. Q.e.f.

[$DA.AB/DC.CB = AB/CB . AD/DC = AB/CB . CB/G = AB/G = E/F$, as required.]

PROPOSITIO LXXVI (LXXIV).

Rectam AB divisam utcunque in C , iterum secare in D, ut ABD rectangulum ad quadratum CD, datam habeat rationem E ad F.

Constructio & Demonstratio.

Fiat ut E ad F, sic AB ad G; dein AB secetur in D, ut DB, DC & G sunt lineae continuae. Dico factum esse quod iubetur.

[46]

Ratio enim rectanguli ABD, ad quadratum CD, componitur ex ratione AB ad CD, & DB ad CD, id est ex ratione CD ad G. sed & ratio AB ad G (id est E ad F per constructionem) composita est ex ratine^a AB ad CD, & ex CD ad G, igitur ABD rectangulum ad quadratum CD, eam habet rationem , quam E ad F. divisimus ergo AB lineam in D, &c. Quod erat faciendum. a *Defin. 5 Sexti.*

BOOK I.§3.

PROPOSITON 76 (74).

The line AB is divided by some point in C, and again in D, in order that the rectangle ABD to the square CD has the given ratio E to F.

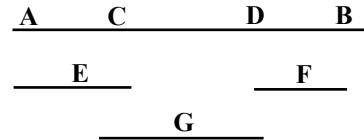
Construction & Demonstration .

Thus the line AB to the line G shall be made in the ratio E to F; then AB is cut in D, in order that DB, DC, and G are lines in continued proportion. I say that what was ordered has been done.

For the ratio of rect. ABD to the square CD, which is composed from the ratio AB to CD, and DB to CD, that is from the ratio CD to G. But also the ratio AB to G, (that is E to F by construction), is made from the ratio^a AB to CD, and CB to G; therefore the rect. ABD to the sq. CD, has the ratio which the line AB has to G, that is E to F; therefore we have cut the line AB in D, &c. Quod erat faciendum. b *Defin. 5 Sexti.*

&c. Q.e.f.

[AB.BD/CD.CD = AB/CD. BD/CD = AB/CD . CD/G = AB/G = E/F, as required.]



Prop.76. Fig.1

PROPOSITIO LXXVII (LXXV).

Lineam AB divisam utcunque in C & D , iterum dividere in E, ut AEC rectangulum ad rectangulum EBD, datam habeat rationem F ad G.

Constructio & Demonstratio.

Fiat ut AC ad DB, sic F ad H; & AE ad EB, ut H ad G. Dico factum esse quod iubetur. Ratio enim lineae F ad G, ^b composita est ex, F ad H, & H ad G . sed ratio EAC rectanguli ad rectangulum EBD composita est ex ratione^c AC ad DB, id est per constructionem F ad H, & ex AE ad EB, id est H ad G. igitur ut F ad G, sic EAC rectangulum , ad rectangulum EBD. divisimus igitur lineam AB in E, &c. Quod erat faciendum. b *Defin. 5 Sexti*; c 23 *Sexti.*

BOOK I.§3.

PROPOSITON 77 (75).

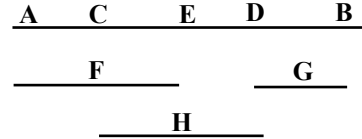
The line AB is divided by some points in C and D, is again to be divided in E in order that the rectangle AEC to the rectangle EBD has the given ratio F to G.

Construction & Demonstration .

Thus the line AC to the line DB shall be made in the ratio F to H; and AE to EB as H to G. I say that what was ordered has been done.

For the ratio of rect. EAC to the rect. EBD has been made from the ratios F to H and H to G; but the ratio of the rect. EAC to the rect. EBD has been made from the ratio that is by construction F to H^c, and from AE to EB, that is H to G. Therefore as F to G, thus rect. EAC to rect. EBD. We have divided the line AB in E, &c. Quod erat faciendum. b *Defin. 5 Sexti* ; c 23 *Sexti*.

[EA.AC/EB.BD = AE/EB . AC/BD = H/G . F/H = F/G, as required.]



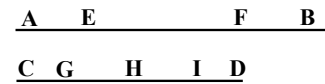
Prop.77. Fig.1

PROPOSITIO LXXVIII (LXXVI).

Sint AB, CD, divisae quomodocunque ; Dico quadrata partium AB, una cum rectangulis AEB, EFB bis sumptis, ad quadrata partium lineae CD, una cum rectangulis CGD, GHD, HID bis sumptis, eam habere rationem quam AB quadratum ad quadratum CD.

Constructio & Demonstratio.

Demonstratum est AB^d quadratum aequare quadratis partium lineae AB una cum rectangulis AEB, EFB, bis sumptis. quemadmodum etiam de quadratis partium CD eiusque rectangulis CGD, GHD, HID bis sumptis: patet ergo ea inter se illam obtinere rationem quae inter quadrata AB, CD reperitur. d 61 *Huius*.



Prop.78. Fig.1

BOOK I.§3.

PROPOSITON 78 (76).

The lines AB and CD are divided in some manner. I say that the ratio of the square of part of the line AB taken together with twice the sum of the rectangles AEB and EFB to the square of part of the line CD taken together with twice the sum of the rectangles CGD, GHD, and HID, is the same as the ratio of the square AB to the square CD.

Construction & Demonstration .

It has been shown that the square AB^d is equal to the squares of the parts of the line AB together with twice the sum of the rectangles AEB and EFB, as also concerning the squares of the parts of CD and the sum of the rectangles of these CGD, GHD and HID taken twice. It is therefore apparent that the same ratio is obtained between these which is found between the squares AB and CD. d 61 *Huius*.

PROPOSITIO LXXIX (LXXVII).

Datarum duarum alteram ita secare, ut sectae partes cum insecta, in continua sint analogia.

Propositionem hanc demonstratam invenies in libro nostro de progressionibus geometrici props- 36 duplici methodo : luber alia tamen praxi hic eandem expedire.

Constructio & Demonstratio.

Sint AB, BC lineae quarum alteram BC, ita oporteat partire in D. ut sint in continua ratione, AB, BD, DC: divisam AB bifariam in E, fiat rectangulo super datis ABC contento, una cum quadrato dimidiaie, EB aequale quadratum ED, cadet punctum D, inter C & B, cum EC quadratum sit aequale ^a quadrato BE una cum rectangulo ACB, quod maius est rectangulo ABC: adeoque & EC quadratum maius quadrato ED. Dico itaque peractum quod postulatur: constituto enim super AB semicirculo AFB, ducatur tangens FD, iungaturque FE ad centrum; erit itaque quadratum DE, quadrato DF, hoc est b rectangulo BDA: una cum quadrato EB, hoc est quadrato EF aequale. Adeoque rectangulum ADB, una cum quadrato EB, ipsi ABC cum eodem quadrat aequabitur, ablato igitur communi quadrato BE remanet ADB rectangulum, aequale ABC rectangulo, quare AD est ad AB, ut est BC ad DB, & ut AB ad BD, ita DB ad DC; sunt igitur CD, DB, BBS in continua ratione; igitur datarum duarum alteram ita secuimus, &c, Quod erat faciendum.

a 6 Secundi.

BOOK I.§3.

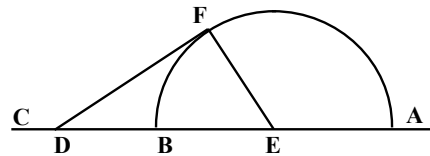
PROPOSITON 79 (77).

To divide the second part of a given section thus, in order that the first part of the section with the cut parts of the second section are in continued proportion.

You will find the demonstration of this Proposition in our book on geometric progressions - proposition 36 - by a second method : however it is pleasing to give the same here in another way to be expedient .

Constructione & Demonstration.

The section shall be the lines AB and BC, and it is required to divide the line BC in D in order that the lines AB, BD and DC are in a ratio of continued proportion. The line AB is bisected in E, with the given rect. contained upon ABC together with the square of the half EB made equal to the square ED ; as the point D falls between C and B, the square EC is equal to the ^a square BE, together with the rect. ACB, which is larger than the rect. ABC: and also the square EC is larger than the square ED. I say that what was demanded has been accomplished: Indeed with the semi-circle AFB set up on AB, the tangent FD is drawn, and FE is joined to the centre of the circle; and thus the square DE is equal to the square DF, that is to the rect. BDA ^b, together with the square EB which is equal to the square EF. Thus the rect. ADB, together with the square EB, will be equal to the rect. ABC with the same square, therefore with the common square BE taken away there remains the rect. ADB equal to the rect. ABC, whereby AD is to AB, as BC is to DB, and as AB to BD, thus DB to DC; therefore the lines CD, DB, BA are in a continued ratio; therefore of the two given lines, we have thus cut the other, &c, Quod erat faciendum. a 6 Secundi.



Prop.79. Fig.1

[$EC^2 = EB^2 + AC.CB$, for $EC^2 = (EB + BC)^2 = EB^2 + (BC^2 + 2.EB.BC) = EB^2 + BC.(BC + AB) = BE^2 + CA.CB$;
 $ED^2 = EB^2 + BD.DA$, for $ED^2 = (EB + BD)^2 = EB^2 + (BD^2 + 2.EB.BD) = EB^2 + BD.(BD + AB) = BE^2 + DB.DA$;
 Again, $DE^2 = DF^2 + FE^2 = DB.DA + EB^2$. Thus rect. ADB (or $BD.DA$) + $EB^2 = AB.BC + EB^2$ by construction, and hence $BD.DA = AB.BC$ giving $AD/AB = BC/DB$ and hence $AB/BD = BD/DC$, as $(AD - AB)/AB = (BC - DB)/DB$ gives $DB/AB = BC/AD = CD/DB$, from which $DB^2 = AB.CD$ as required.]

PROPOSITIO LXXX (LXXVIII).

Datarum duarum alteram ita partiri, ut rectangulum sub indivisa & altera parte divisae, ad quadratum residuae, datam habeat rationem.

Constructio & Demonstratio.

Propositionem quam prius particularem solvimus in ratione aequalitatis, conabimur quoque uniuersaliter solvere.

Sint igitur, AB, BC lineae, oporteatque dividere CD, in E, ut rectangulum AB DE, ad quadratum CE, rationem habeat datam H, ad I. Fiat ut H ad I, sic AB ad FG: dein CD dividatur in E, ut ED, EC, FG sint continuae per praecedente. Dico factum esse quid petitur. erit enim rectangulum super ABED, ad quadraum CE, ut idem ABED rectangulum, ad rectangulum FG ED, id est ut AB, ad FG, id est per constructionem ut H ad I. Divisimus igitur lineam CD, &c. Quod erat faciendum.

Huius Propositionis aliam inuenies demonstrationem in libro nostro de progressionibus Propos. 37.

BOOK I.§3.

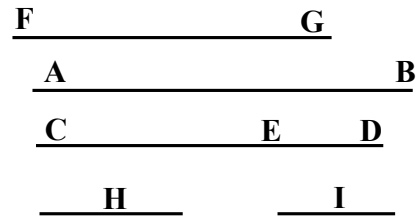
PROPOSITON 80 (78).

One of two given lines to be divided in order that the ratio of the rectangle formed by the whole first line and one part of the other line to the square of the remaining part of the other line have a given ratio.

Constructio & Demonstratio.

The particular proposition that we solved before with an equal ratio we will try to solve uniuersally too.

The lines shall be AB and CD, and it is required to divide the line CD in E in order that the rectangle AB.DE to the square CE shall have the given ratio H to I. Thus AB to FG shall be made in the ratio H to I: then CD is divided in E, in order that ED, EC, FG shall be in continued proportion by the preceding proposition. I say that the required task has been performed. Indeed the ratio of the rect. on AB.ED to the square CE, is the same as the rect. AB.ED to the rect. FG.ED, that is as AB to FG, that is by construction as H to I. Therefore we have divided the line CD, &c, Q. e. f.
 We have found another demonstration of this Proposition in our book 'Concerning Progressions', Prop. 37.



Prop.80. Fig.1

[$AB/FG = H/I$, then CD is divided by E such that $ED/EC = EC/FG$;
 The ratio $AB.ED/CE^2 = AB/CE$. $ED/CE = AB/CE$. $EC/FG = AB/FG = H/I$, as required.]

PROPOSITIO LXXXI (LXXIX).

Datum AB sectam in C & D, ita secare in E puncto , inter C & D constituto , ut rectangulum AED, ad CEB rectangulum, obtineat rationem quadrati F, ad G quadratum.

[48]

Constructio & Demonstratio.

Dividatur AD in H ; & CB in I bifariam : & recta HI divisa sit in E, ut sit HE ad EI, sicut est F ad G. Dico rectangulum AED, ad CEB rectangulum habere rationem quadrati F, ad G , quadratum. .
 Huius rei invenies demonstrationem reperies libro de parabola. Ex cuius proprietatibus est eruta.

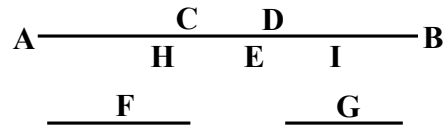
BOOK I.§3.

PROPOSITON 81 (79).

Given the line AB cut in C and D, thus is cut in the point E placed between C and D, in order that the ratio of the rectangle AED to the rectangle CEB equal to the ratio F to G holds.

Constructione & Demonstration.

The line AD is divided in H, and CB is bisected in I.
 The line HI is divided by E, in order that HE to EI shall be as F to G. I say that the ratio of the rect. AED to the rect. CEB, has the ratio of the square F to the square G.
 You will find the demonstration of this Proposition in our book about the parabola, from the properties of which it has been taken.



Prop.81. Fig.1

PROPOSITIO LXXXII (LXXX).

Datum iterum AB, sectam utcunque in C & D, denuo partiri in E puncto , ut rectangulum AEC, ad BED rectangulum, datam rationem contineat quadrati F, ad G.

Constructio & Demonstratio.

Dividantur ut prius rectae AD, CB, in H ; & I punctis bifariam : quo facto dividatur HI in E, secundum proportionem G, ad F. Dico rectangulum AEC, ad BED rectangulum, datam habere rationem quadrati F, ad G, quadratum. .
 Huius quoque demonstrationem invenies eodem libro de parabola.

BOOK I.§3.

PROPOSITON 82 (80).

Given again the line AB cut somehow in C and D, to be cut anew in the point E, in order that the ratio of the rectangle AEC to the rectangle BED equal to the ratio F to G holds.

Construction & Demonstration.

The lines AD and CB are divided as previously in H and I as the points of bisection: with which accomplished HI is divided by E, in the proportion G to F. I say that the ratio of the rect. AEC to the rect. BED, has the ratio of the square F to the square G.



Prop.82. Fig.1

You will find the demonstration of this Proposition too in our book about the parabola.

PROPOSITIO LXXXIII (LXXXI).

Datum denuo AB, divisam utcunque in C & D, iterum dividere in E, ut rectangulum ACE, ad BDE rectangulum, datam obtineat rationem quadrati F ad G.

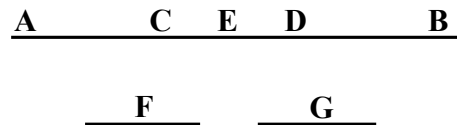
Constructio & Demonstratio.

Fiat ut AC ad DB; ita HI, ad KL; & ut F ad G, ita IM ad KN, tandem fiant in continuata analogia HI, ML, OI; & similiter KL, NK, PK: denique dividatur CD in E, secundum rationem IO ad KP; Dico factum quid requiritur. Rectangulum ACE, ad BDE, habet rationem compositum ex AC, ad DB, hoc est HI, ad KL, & ex ratione CE ad ED, hoc est OI, ad KP. Igitur rectangulum ACE, ad BDE eandem obtinet rationem, quam rectangulam, HIO ad LKP; sed HIO ad LKP, eandem habet rationem, quam quadratum IM ad KN, (ex constructione enim sint tres in continua analogia tam IO, IM, IH, quam KP, KN, KL) hoc est quam quadratum F, ad G, quadratum; Igitur rectangulum ACE, ad BDE, eandem rationem continet, quam F quadratum, ad G quadratum. Perfecimus igitur quod imperatum fuit. *a 23 Sexti.*

BOOK I.§3.

PROPOSITON 83 (81).

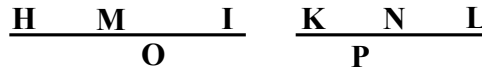
Given anew the line AB divided somehow in C and D, to be cut again in the point E, in order that the ratio of the rectangles ACE to BDE to the ratio of the squares F to G shall hold.



Prop. 83. Fig.1

Construction & Demonstration.

The ratio AC to DB shall be made as HI to KL;
 and thus IM to KN as F to G. However, HI, MI,
 and OI are made in continuous analogous
 proportion ; and similarly KL, KN, KP: and then
 CD is divided in E, following the ration IO to KP. I
 say that what was required has been done. Rect.



Prop. 83. Fig.2

ACE to rect. BDE^a has a ratio composed from AC to DB, that is HI to KL, and from the ratio CE to ED, that is OI to KP. Therefore the rect. ACE to rect. BDE contains the same ratio as rect. HIO to rect. LKP; but HIO to LKP, has the same ratio as the square LM to the square IN, (from the construction there are indeed three lines in continuous analogous proportion as IO, IM, IH, so KP, KN, KL) that is as the square F to the square G. Therefore the rect. ACE to the rect. BDE, has the same ratio as the square F to the square G. We have completed what was required. *a 23 Sexti.*

[AC.CE/BD.DE = AC/DB . CE/DE = HI/KL . OI/KP = HI.IO/KL.KP = IM²/KN², (as HI/MI = MI/OI and KL/KN = KN/KP) = F²/G².

PROPOSITIO LXXXIV (LXXXII).

Data basi, aggregato laterum, & altitudine trianguli; exhibere triangulum.

[49]

Constructio & Demonstratio.

Dato laterum aggregato, aequalis ponatur AB quae bifariam divisam in C. fiat DE aequalis basi trianguli, bifariam divisae in G, & altitudini aequalis ponatur F; ex lateribus AG, CB,DE, fiat triangulum DHE: nam AC, CB simulsumptae maiores sunt DE. erit igitur DHE isosceles : deinde fiat ut HC quadratum, ad quadratum F, sic ACB rectangulum, ad rectangulum AKB. & erigatur KG aequalis ipsi F, parallela HC, iunganturque DG, GE. Dico DGE triangulum esse quaesitum.
 demonstrationem huius invenies libro de ellipsi Propos. 121.

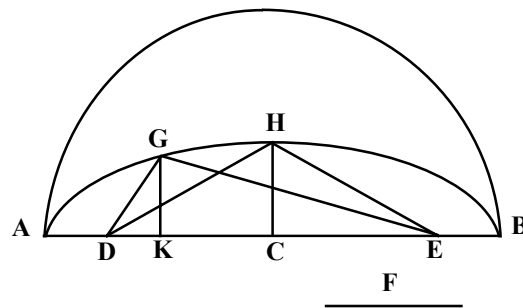
BOOK I.§3.

PROPOSITON 84 (82).

For a given base, with the sum of the sides, and with the altitude of the triangle given, to exhibit the triangle.

Construction & Demonstration.

The line AB is put equal to the given sum of the sides which is bisected in C. DE is made equal to the base of the triangle, which is bisected in C, and F is set equal to the altitude. From the lines AG, CB, and DE, the triangle DHE is made: for the sum of AC and CB is greater than DE. The triangle DHE will therefore be isosceles: then the ratio of the squares HC to F shall be made as the ratio of the rectangles ACB to AKB. KG is erected equal to F parallel to HC, and DG and GE are joined. I say that DGE is the triangle sought.



Prop.84. Fig.1

You will find the demonstration of this Proposition in the book of the ellipse, Prop. 121.

PROPOSITIO LXXXV (LXXXIII).

Datam AB sectam in C, dividere in D, ut quadratum AD, aequale sit CDB
 rectangulo.

Constructio & Demonstratio.

Divisa CB in E bifariam, ponatur ex E normalis EG, aequalis ipsi AE : iunctisque AG, describatur per C, B, G puncta circulus CBG, occurrrens EG lineae in F : divisamque FG bifariam in I, agatur per I recta IH parallela ipsi CB, occurrrens AG lineae in H puncto; ex quo normalis erigatur HD, occurrrens CB lineae in D. Dico factum esse quod petitur : cum enim HI parallela sit ipsi AE, poniturque AE, EG lineae aequales, erunt & HI, IG quoque inter se aequales, & H communis intersectionis linearum HI, HG cum circulo. Unde & HD eundem contingitur in H : estque CDB^a rectangulum aequale quadrato HD, id est AD, Divisimus igitur AB lineam, &c. Quod erat faciendum. a 36 Tertii.

BOOK I. §3.

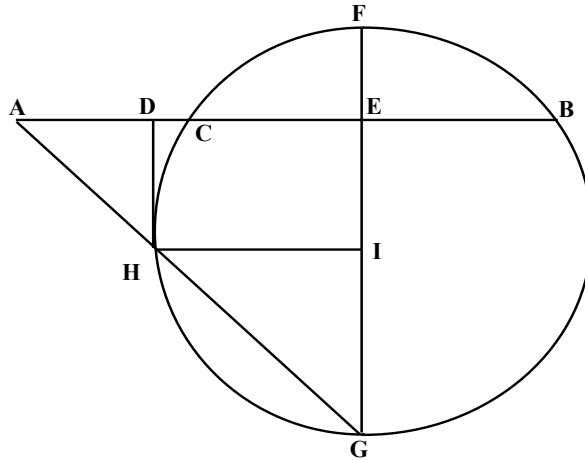
PROPOSITON 85 (83).

Given the line AB cut in C, that is to be divided in D so that the square AD shall
 be equal to the rectangle CDB.

Construction & Demonstration.

The line CB is bisected in E, a normal line EG is drawn from E equal to the line AE : the points A and G are joined, and the circle CBG is drawn through the points C, B, and G, crossing the line EG in F. FG is bisected in I, and the line IH is sent through I parallel to CB, crossing the line AG in the point H; from which the normal HD is erected, crossing the line CB in D. I say that what was sought has been done.

For indeed HI shall be parallel to AE, and the lines AE and EG are made equal to each other; the lines HI and IG will also be equal to each other, and the common point of intersection of the lines HI and HG with the circle shall be H. Hence HD is a tangent to the circle at H : and the rect. CDB^a shall be equal to the square HD, that is AD. We have therefore divided the line AB, etc. Q.e.f. a 36 Tertii.



Prop.85. Fig.1

PROPOSITIO LXXXVI (LXXXIV).

Datam AB sectam in C, denuo partiri in D, ut quadratum AD, ad rectangulum
 BDC, datam obtineat rationem.

Constructio & Demonstratio.

Data sit ratio E ad F, & fiat rectangulum aliquod GIK, quod ad rectangulum LIM rationem eandem cuius
 ratone E ad F contineat : hoc facto ponatur quaedam IS, orthogonalis ad GM, & inventa IN media inter KI,

IG; dividatur KG bifariam in O, & erigatur OT parallela IS, donec concurrat cum TS, quae aequidistet GM, in T : & iungatur TN, quae SI productae occurrat in P. Deinde inventa media IQ inter LI, & IM, ducatur recta PQ, occurrens TS protractae in R.

[50]

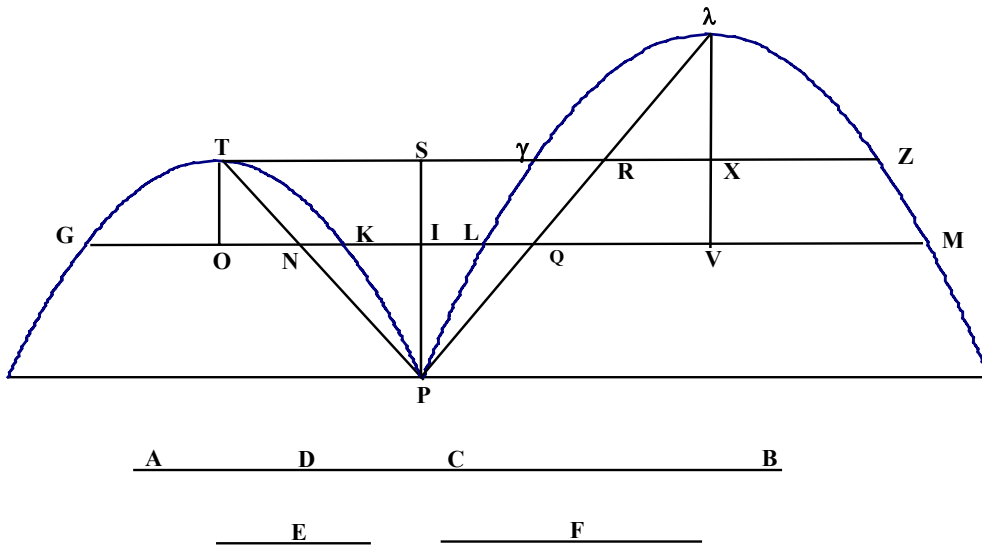
tandem dividatur ; M bifariam in V, & erigatur VX aequidistant TO, concurrens cum PQ in λ ; & fiat ut $V\lambda$ quadratum, ad quadratum $X\lambda$, ita quadratum VL, ad $X\lambda$ quadratum : Dico TZ lineam divisam esse in S, γ secundum rationem postulatam : unde si dividatur AB in D, & C, ut est divisa TZ in S, & γ : habebitur ratio quadrati AD, ad CDB rectangulum, in ratione E ad F. quod fuit postulatum. Vltius non pergo in demonstratione huius rei, cum non sit huius loci; sed eam reperies libro de parabola Geometrica tractatam, & perfectam.

Libri primi finis.

BOOK I.§3.

PROPOSITON 86 (84).

Given the line AB cut in C, to be divided in D anew, so that the square AD to the rectangle BDC shall be in equal to a given ratio.



Construction & Demonstration.

The ratio of two lines E to F shall be given, and some rect. GIK shall hold the same ratio E to F to another rect. LIM. Following this, some line IS is placed orthogonal to GM, and the mean IN found between KI and IG. KG is bisected in O, and the line OT is erected parallel to IS, then the points T and S are joined together to give a line TS equidistant from GM meeting OT in T. The line TN is drawn which crosses SI produced in P. Following this the mean IQ is found between LI and IM, and the line PQ is drawn cutting TS produced in R. At last LM is bisected in V, and the line VX erected equidistant to TO, crossing the line PQ in λ . The square $V\lambda$ to the square $X\lambda$ is thus made in the ratio of the square VL to the square $X\lambda$. I say that the line TZ has been divided by S and γ in the ratio required. Thus if AB is divided by D and C as TZ is divided by S and γ , then the ratio of the square AD to the rect. CDB will have the ratio E to F, which was required. I will not go on further with the demonstration of this, as this is not the place, and you can find this propostion worked out geometrically in the book on the parabola.

End of Book One.