CHAPTER VII

CONCERNING THE NAMES GIVEN TO THE VARIOUS INTERVALS

1. With the harmonic rules established generally, which it will be agreed to observe both for the consonants as well as for the composition of these, it is required to be progressing to the various kinds of music, and for these the use of the given precepts to be treated more fully. But before the kinds of music can be conveniently enumerated and set out, the peculiar names given to intervals and established by usage must be explained, so that in the following it may be allowed to examine these matters with these customary tones . But these tones are the names of several musical intervals now established some time ago and thus have been in use for a long time, so that now for the sake both of convenience as well as necessity it shall be required that this discussion may be set forth as a whole.

2. But although these names may be explained in various places, yet the definitions of these are not genuine enough and they are least suitable forms for our presentation. For the intervals, which have come upon their own names themselves from practise and experience, rather than from the nature of the tones which they are accustomed to describe. But we have become accustomed to making use of this method, where intervals are required to be measured by logarithms, and we will continue to express these by logarithms rather than by the ratios of each interval, from which it will be allowed to judge better the size of each interval.

3. But now above, it has been established the interval to be the distance between two tones in the ratio of the depth and height of the pitch of the tones, thus so that, where the difference may be greater between the lower and higher, there also the interval may be said to be greater. Therefore if the tones were equal in pitch, the distance between these would be zero and thus the interval of the tones held in the ratio of equality 1:1 will be zero, as also the logarithm of this ratio is 0. For the intervals, as we have now stated, we may measure by the logarithms of the ratio, which the tones maintain between each other. Moreover we may call this vanishing interval of two tones *unison.*

4. Indeed we will be able to make use of a certain table of logarithms in expressing this ratio of logarithms, in which the logarithm of unity is put to be zero. But it will be especially expedient to make use of a table of this kind, in which the logarithm of two may be put in place to be unity, since the ratio of two occurs most often in expressing the consonants and thus in music with this agreed it may be considered especially the calculation may be made much easier. Behold therefore a table of logarithms of this kind, indeed however many suffices for our purpose:

log. $1 = 0,000000$	log. $5 = 2,321928$
log. $2 = 1,000000$	log. $6 = 2,584962$
log. $3 = 1,584962$	log. $7 = 2,807356$
log. $4 = 2,000000$	log. $8 = 3,000000$.

[For: if $N = 2^x = 10^y$; then $\log_2 N = x = y\log_2 10$

and $\log_{10} N = y = x\log_{10} 2;\quad \therefore\quad x = \frac{\log_{10} N}{\log_{10} 2}$.]

5. After the interval if equal tones, which is called unison, there comes to be considered the interval of the tones held in the twofold ratio 2:1, which is called a *diapason* by the Greek musicians, so that there the interval of any tones, with the other tone being doubled can be considered as little changed, as almost may be had for the same, and therefore in this diapason all the other intervals may be considered to be included.
[e.g. Liddle & Scott : Greek-English lexicon , page 354 ; probably the best source for ancient Greek words and phrases.]

διαπᾱσῶν, ἡ, i.e. ἡ διὰ πασῶν χορδῶν συμφωνία, *the concord of the first and last notes, the octave-scale;*

Truly from Latin this interval is called an *octave*, the ratio of which denominations depend on the diatonic scale of music in general, which we will set out further below. Therefore the measure of this said diapason or octave interval is log.2 − log.1 or log.2, that is 1,000000.

6. In the next position when the ratio of the tones maintained shall be 4:1 the interval shall be 2,000000 and thus twice as great as the octave interval, this interval is accustomed to be called the *disdiapason,* or the double octave. Besides the interval of the tones 8:1, since it is 3,000000 or three times greater than the said octave interval. In a similar manner the interval of the tones 16:1, of which the measure is 4,000000, is called the quadruple octave, the interval of the tones 32:1 the quintuple octave, and thus so on. From which, since the denominations of the greater intervals may be desired from the number of octaves contained in these, the method is evident, why we may have taken the unit for log 2 . For the characteristic of the logarithm expresses some interval which designates how many octaves shall be contained in that interval.

7. *Diapente* [$\delta\iota\alpha\pi\varepsilon\nu\tau\varepsilon$] again from the Greek, or the musical fifth or *quinta* from Latin, may be called the interval of the tones holding the ratio 3: 2 , of which the derivation of the name likewise is assumed from the general diatonic scale.
Therefore the measure of this interval is log.3 − log. 2 = 0,584962 . Therefore this interval is less than the octave interval ; moreover the interval between these cannot be expressed by rational numbers. However, the octave interval to the fifth interval may be expressed approximately by the following ratios

5:3, 7:4, 12:7, 17:10, 29:17, 41:24, 53:31,

which rations have been prepared thus, so that closer ratios cannot be shown with smaller numbers .

8. Again because the measure of the interval of the tones 3:1 is 1,584962, which number is the sum of the measures of the octave and the fifth, this interval is accustomed to be called the octave with the fifth, or the double fifth. In a similar manner the interval of the tones 6:1 the double octave with the fifth or the triple fifth, of which the measure evidently is 2,584962. And in a similar manner the interval of the tones 12:1 may be called the triple octave with the fifth or quadruple fifth, and of the tones 24:1 the quadruple octave with the fifth or quintuple fifth. From which it is seen, if the decimal fraction were ,584962, the interval to be composed from five and just as many octaves, as the characteristic may denote.

9. The *diatessaron* [*διατεσσαρων*] or fourth interval does not differ much from the *diapente* or fifth, which exists between tones maintaining the ratio 4:3, the measure of which therefore is 0,415038. From which it is apparent these two intervals the fifth and the fourth taken together constitute the octave, since the sum of their measures shall be 1,000000, [as logs, or as ratios, 4:3 and 3:2 giving 2:1 ; these ratios hence may be viewed as being complementary]. Again in a similar manner the interval of the tones 8:3, of which the measure is 1,415038, is called the octave with the fourth or the double fourth, and the interval of the tones 16:3, of which the measure is 2,415038, is called the double octave with the fourth, and so on thus.

10. Therefore just as these fifth and fourth intervals, which are smaller than an octave, simply have adopted the names, truly the intervals arising from these with the addition of one or more octaves may be denoted by composite names, thus all the intervals smaller than an octave interval are usually called simple, truly those greater than an octave intervals are accustomed to be called composite. And thus the measures of simple intervals is smaller than one and the characteristic of the measuring logarithms is 0. Truly the logarithms of the composite intervals are greater than one and the characteristics of those are greater than zero. From which it is understood all the simple intervals to be contained within the octave interval, and hence for that reason the octave ratio also may be called diapason.

11. Therefore since the name of composite intervals may be formed from the number of octaves which they contain, and with the name of the excess, which is a simple interval, it will suffice to enumerate the simple intervals, which indeed have been given by the musicians and with the names assigned. So that we may make this more distinct, we will begin with the intervals requiring the minimum examination, which are the *comma [κομμα : small part left over], diesis* [διεσισ : consisting of quarter tones] and the *diaschisma* [half semi-tone] and thus they may be called the minima, since by listening they are scarcely able to be perceived and greater intervals, if to these themselves either

may be added or removed, are not considered to change, thus so that greater intervals may be increased or decreased minimally that may be had for the same. Such a hypothesis can only be allowed as an exercise for the ear, and must be rejected in a perfect harmony.

12. Truly the *comma* is the interval established by the interval of two tones maintaining the ratio 81: 80, thus so that the measure of the comma shall be

$$\log.81-\log.80=0,017920$$

and thus almost 56 comma intervals may fill the interval of an octave. The *diesis* [*sharp*] is the interval of tones maintaining the ratio 128:125 ; therefore its measure is 0,034216. Therefore it is almost twice as large as the *comma* and almost 29 *diesis* are held in the octave. Finally the *diaschisma* is the interval of the tones 2048:2025 and its measure is 0,016296; therefore almost 61 diaschisma fill the octave. Therefore it is agreed the *diaschisma* to be the difference between the *diesis* and the *comma*.

13. These very small intervals do not usually occur in music, nor in turn do such small intervals occur between the tones taken; yet meanwhile the differences of the greater intervals in music are taken to be exceedingly small, so that for these requiring to be expressed there was a need to introduce these minimal intervals. But the minimal intervals, which are actually used in music and are accustomed to express the tones, are the *semitones* both major and minor and *limmata* likewise both major and minor; which intervals, since they may be a little different from each other, for the unskilled may be had as equal to each other and are indicated by the name *semitone* .

14. The major semi-tone is the interval maintaining the ratio of the tones 16:15, therefore its measure is 0,093110. Truly the minor semi-tone is constituted between tones in the ratio 25:24, which ratio surpasses that sharp expressed in the ratio 128:125 ; the measure of the minor semitone therefore will be 0,058894, certainly to which with the measure of the sharp added produces the measure of the semi-tone major. An octave therefore contains almost 10 major semi-tones together with 2 *diesi* or approximately 17 minor semi-tones.

15. The *limma major* [*i.e.* extra major] interval, which agrees with the ratio of the tones 27: 25, exceeds the major semi-tone by a *comma* and therefore its measure is 0,111030. Truly the *limma minor* is the interval maintaining the ratio of the tones 135 :128 and thus also exceeds the *semi-tone minor* by a *comma*, truly with the limma minor taken from the limma major a *diesis* remains. Therefore the measure of the *limma minor* is 0,076814. Therefore approximately 9 *limma major* intervals will constitute an octave, while approximately 13 *limma minor* intervals are required to fill an octave.

16. These four intervals, as we have now said, usually are commonly called semi-tones ; truly also they may be called *second minors,* which name equally with the octave, fifth

and the fourth themselves arises from the general diatonic scale. Truly the complements of these intervals to the octave, which are contained in the ratios of the tones

15:8, 48:25, 50:27 and 256:135,

with the same names of the derivations, are called the *major sevenths*. Thus the measures of these are 0,906890, 0,941106, 0,888970 and 0,923186, which are the largest minor octave intervals which indeed are in use.

17. The semi-tones follow the intervals in order of magnitude, which are accustomed to be indicated by the name of the tone and likewise by the second major. But three kinds of tones are provided, of which the first depending on the ratio 9: 8, is called the *major tone* and the measure of which therefore is 0,169924; therefore 6 major tones of this kind together exceed the octave by more than a comma. The *minor tone* in the ratio 10:9 is less than the major tone by a comma, thus so that its measure shall be 0,152004. For the third tone also refers to the interval of the tones contained in the ratio 256:225, which tone exceeds the diaschism, but truly is less than the diesi. Truly the complements of these tones are called the *minor seventh tones* to the octave.

18. But the tone contained in two semi-tones may be taken in a wider sense. For the major tone is both the sum of the major tone of the semi-tone major and the limma minor as well as the sum from the semi-tone minor and the limma major. Truly the tone minor is the tone from the sum of the major and minor semi-tones. Finally the maximum tone, held in the ratio 256:225, is the sum of the two major semi-tones. The following semi-tones from adjacent intervals arise in a similar manner.

19. Intervals arise from the increase in tone of a semi-tone, to which the name of minor thirds has been given; in whatever way such an interval may deserve this name, by speaking accurately, so that it may contain the ratio of the tones 6:5. Since which intervals, either comma, diaschisma, or diesis may differ from this ratio, this has an agreement with the third minor, which sounds well enough ; that which also is held for the remaining intervals, which are pleasing consonants. The third of the minor complements to the octave is called the *sixth major* held in the ratio 5:3 ; and the measure of the *third minor* therefore is 0,263034 and of the *sixth major* 0,736966.

[Recall from above :

$$\begin{array}{ll} \log.\ 1 = 0,000000 & \log.\ 5 = 2,321928 \\ \log.\ 2 = 1,000000 & \log.\ 6 = 2,584962 \\ \log.\ 3 = 1,584962 & \log.\ 7 = 2,807356 \\ \log.\ 4 = 2,000000 & \log.\ 8 = 3,000000. \end{array}$$

hence $2:1::6:5=5:3$, while $\log(5:3) = 2.321928 - 1.584962 = 0.736966$
while $\log(6:5) = 2.584962 - 2.321828 = 0.263034$.]

20. The third minor semitone barely exceeds the *third major*, that is evident, which constitute a pleasing consonant, and that is the interval of tones maintaining the ratio 5:4. Therefore its measure is 0,321928; therefore this third major is agreed from the major and minor tones together. Truly the complement of the third major to the octave is called the *sixth minor,* which therefore is agreed from the ratio of the sounds maintaining the ratio 8:5, and its measure is 0,678072. The sixth also is called the *hexachord* from the Greek*,* thus so that the *sixth major* may agree with the *hexachord major*, truly the minor with the minor.

21. If to the third major held in the ratio 5:4 there may be added the greater semi-tone 16:15, from these ratios on being compounded the ratio 4:3 will be produced, from which the *diatessaron* or *fourt*h interval may be indicated*.* Truly the complement of this interval to the octave is the *diapente* or *fifth* contained in the ratio 3:2, from which intervals this now it has been performed above. Here it only remains, that we may note the difference between the fifth and fourth to be contained in the major tone 9:8, which difference provided the first idea of the major tone to the ancients.

22. Since now all the intervals differ in turn from each other by a semi-tone in the order of size, except for the fifth and fourth; also a mean sound of the music between the fifth and the fourth may be put in place, which may differ from each other by a semi-tone. But here the sound is called a *triton,* thus since it is consists of three tones, truly also known by other names : the *abundant fourth*, and also the *deficient fifth* or the *false fifth.* But for the four kinds of tritons with various semi-tones four are of a kind , of which the first is held in the ratio 64:45, to be a fourth increased by a major semi-tone. The second kind is the fifth diminished by a major semi-tone, and is held in the ratio 45:32. Truly the third kind in the ratio 18:25 is a fourth, augmented by a minor semi-tone; truly the fourth represented by the ratio 25:36 is a fifth diminished by a minor semi-tone.

23. These simple intervals, or those to be used smaller than the octave, will draw their names from the natural numbers, and to be called the second, third, fourth, fifth, etc., as far as to the octave ; thus also the names of the composite intervals, or those greater than an octave, are put in place in a similar manner. Clearly the octave with two either major or minor may be called the *ninth major* or *minor* ; equally since the octave with three may be called the *tenth*, and the octave with four the *eleventh*, and thus again always with seven being added to the name of the simple interval : thus the *twelfth* is the octave with five added, truly the *fifteenth* is the double octave, from which the names of this kind are understood well enough.

24. So that these intervals together with their names may become apparent at once and both may perceived as well as discerned from each other more easily, is seen in the following table, in which first the names of the simple intervals are set out, then the numerical ratios of the tones, the third the measures of the intervals expressed by logarithms according to this chosen principle ; in the fourth column in addition I have inserted the order of the charm, so that any pleasing intervals, from which it can be

indicated at once, when some intervals are going to be more pleasant to listen to than other intervals.

Name of the Interval	Ratio of the Tones	Measure	Order of the Charm
Diaschisma	2048:2025	0,016296	XXVIII
Comma	81: 80	0,017920	XVII
Diesis	128: 125	0,034216	XX
Semi-tone lesser	25: 24	0,058894	XIV
Limma lesser	135: 128	0,076814	XVIII
Semi-tone greater	16: 15	0,093110	XI
Limma greater	27: 25	0,111030	XV
Tone minor	10: 9	0,152004	X
Tone major	9: 8	0,169924	VIII
Third minor	6: 5	0,263034	VIII
Third major	5: 4	0.321928	VII
Fourth	4: 3	0,415038	V
Tritone	25: 18	0,473932	XIV
	45: 32	0,491852	XIV
	64: 45	0,508148	XV
	36: 25	0,526068	XV
Fifth	3: 2	0,584962	IV
Sixth minor	8: 5	0,678072	VIII
Sixth greater	5: 3	0,736966	VII
Seventh minor	16: 9	0,830076	IX
	9: 5	0,847996	IX
Seventh major	50: 27	0,888970	XVI
	15: 8	0,906890	X
	256: 135	0,923186	XIX
	48: 25	0,941106	XV
Octave	2: 1	1,000000	II

Therefore these intervals are progressing in the order of charm: Octave, Fifth, Fourth, Third major and Sixth major, each Seventh minor, Tone minor and with one Seventh major semitone less than the octave, the Semi-tones and Seventh majors remain.

CAPUT VII

DE VARIORUM INTERVALLORUM

RECEPTIS APPELLATIONIBUS

1. Expositis in genere regulis harmonicis, quas tam in consonantiis quam earum compositione observari convenit, ad varias musicae species est progrediendum pro iisque usus praeceptorum datorum plenius tradendus. Sed antequam commode musicae species enumerari atque exponi possunt, peculiares usuque receptae appellationes debent explicari, quo in posterum more vocibusque consuetis his de rebus tractare liceat. Sunt autem hae voces nomina pluribus intervallis musicis iam pridem imposita atque longo usu iam ita recepta, ut tam commoditatis quam necessitatis gratia omnino necesse sit ea exponere.

2. Quamvis autem haec nomina passim sint explicata, tamen earum definitiones non satis genuinae minimeque ad nostrum institutum idoneae sunt formatae. Intervalla enim, quae propria nomina sunt adepta, ipsa praxi et experientia potius quam ex sonorum natura describi solent. Nos autem ea methodo, qua in intervallis per logarithmos metiendis usi sumus, insistentes tam rationes quam logarithmos proferemus cuique intervallo respondentes, unde melius de quantitate cuiusque intervalli iudicare licebit.

3. Supra autem iam est expositum esse intervallum distantiam inter duos sonos ratione gravitatis et acuminis, ita ut, quo maior sit differentia inter graviorem et acutiorem sonum, eo maius quoque intervallum esse dicatur. Si ergo soni fuerint aequales, distantia inter eos erit nulla ideoque intervallum sonorum rationem aequalitatis 1:1 tenentium erit nullum, uti etiam logarithmus huius rationis est 0. Intervalla enim, ut iam statuimus, per logarithmos rationum, quas soni inter se tenent, metiemur. Vocatur autem hoc intervallum evanescens duorum aequalium sonorum *unisonus.*

4. Possemus quidem in his rationum logarithmis exprimendis quovis logarithmorum canone uti, in quo unitatis logarithmus ponitur cyphra. Maxime autem expediet eiusmodi canonem usurpare, in quo logarithmus binarii collocatur unitas, cum binarius in exprimendis consonantiis saepissime occurrat et in musica maxime respiciatur ideoque hoc pacto · calculus fiat multo facilior. En ergo huiusmodi logarithmorum tabulam, quanta quidem ad institutum nostrum sufficit:

$$\begin{array}{ll} \log.\ 1 = 0,000000 & \log.\ 5 = 2,321928 \\ \log.\ 2 = 1,000000 & \log.\ 6 = 2,584962 \\ \log.\ 3 = 1,584962 & \log.\ 7 = 2,807356 \\ \log.\ 4 = 2,000000 & \log.\ 8 = 3,000000. \end{array}$$

5. Post intervallum sonorum aequalium, quod unisonus appellatur, considerandum venit intervallum sonorum 2:1 rationem duplam tenentium, quod a Graecis Musicis *diapason*

vocatur, eo quod sonorum quorumvis intervallum altero sono duplicando tam parum immutetur, ut fere pro eodem habeatur, atque idcirco in hoc intervallo diapason omnia alia intervalla comprehendi censeantur. A Latinis vero hoc intervallum *octava* nuncupatur, cuius denominationis ratio a genere musico diatonico dicto pendet, quam infra fusius exponemus. Huius ergo intervalli diapason vel octavae dicti mensura est $\log.2 - \log.1$ seu log.2, hoc est 1,000000.

6. Cum deinde sonorum rationem 4:1 tenentium intervallum sit 2,000000 ideoque duplo maius quam intervallum octava, hoc intervallum *disdiapason* atque duplex octava solet appellari. Praeterea intervallum sonorum 8:1, quia est 3,000000 seu triplo maius intervallo octava dicto, triplex vocatur octava. Simili modo intervallum sonorum 16:1, cuius mensura est 4,000000, quadruplex octava vocatur et intervallum sonorum 32:1 quintuplex octava et ita porro. Ex quo, cum denominationes maiorum intervallorum ex numero octavarum in iis contentarum petantur, ratio apparet, cur unitatem pro log. 2 assumserimus. Characteristica enim logarithmi quodvis intervallum exprimentis designat, quot octavae in eo intervallo sint contentae.

7. *Diapente* porro graece seu *quinta* latine vocatur intervallum sonorum rationem 3: 2 tenentium, cuius nominis derivatio itidem ex genere diatonico est desumta. Huius ergo intervalli mensura est $\log 3 - \log 2 = 0,584962$. Minus ergo est hoc intervallum quam intervallum diapason; quam autem inter se haec intervalla teneant rationem numeris exprimi nequit. Proxime autem se habet intervallum diapason ad intervallum diapente in sequentibus rationibus

5:3, 7:4, 12:7, 17:10, 29:17, 41:24, 53:31,

quae rationes ita sunt comparatae, ut minoribus numeris propiores rationes exhiberi nequeant.

8. Quia porro intervalli sonorum 3:1 mensura est 1,584962, qui numerus est summa mensurarum octavae et quintae, hoc intervallum octava cum quinta solet appellari. Simili modo intervallum sonorum 6:1 erit duplex octava cum quinta, quippe cuius mensura est 2,584962. Atque pari modo sonorum 12:1 intervallum vocatur triplex octava cum quinta et sonorum 24:1 quadruplex octava cum quinta. Ex quo perspicitur, si fractio decimalis fuerit ,584962, intervallum esse compositum ex quinta et tot octavis, quot characteristica denotat.

9. Ab intervallo diapente seu quinta dicto non multum discrepat intervallum *diatessaron* seu q*uarta,* quod existit inter sonos rationem 4:3 tenentes, cuius ergo mensura est 0,415038. Unde patet haec duo intervalla quintam et quartam coniuncta octavam constituere, cum summa earum mensurarum sit 1,000000. Simili porro modo intervallum sonorum 8:3, cuius mensura est 1,415038, octava cum quarta, atque intervallum sonorum 16:3, cuius mensura est 2,415038, duplex octava cum quarta appellatur et ita pono.

10. Uti ergo haec intervalla quinta et quarta, quae octava sunt minora, simplicia sunt adepta nomina, intervalla vero ex iis adiectione unius pluriumve octavarum orta nominibus compositis denotantur, ita omnia intervalla minora quam octava intervalla simplicia vocari solent, intervalla vero octava maiora composita. Mensura itaque intervallorum simplicium est minor unitate logarithmorumque ea metientium characteristica est 0. Compositorum vero intervallorum logarithmi maiores sunt unitate seu eorum characteristicae sunt nihilo maiores. Ex quo perspicitur omnia intervalla simplicia intra intervallum octavam esse contenta, hancque ob rationem octava quoque diapason appellatur.

11. Cum igitur intervallorum compositorum appellatio ex numero octavarum, quem continent, et nomine excessus, qui est intervallum simplex, formetur, sufficiet intervalla simplicia, quae quidem a Musicis recepta atque nomina sortita sunt, enumerare. Quod quo distinctius efficiamus, ab intervallis minimis recensendis incipiemus, quae sunt *comma, diesis* et *diaschisma* atque ideo minima appellantur, quia auditu vix percipi possunt atque maiora intervalla, si ipsis vel addantur vel ab ipsis demantur, non immutare censentur, adeo ut intervalla maiora huiusmodi minimis sive aucta sive minuta pro iisdem habeantur. Quod quidem pro crassioribus tantum auribus locum habet, in perfecta harmonia autem omnino non valet.

12. Constituitur vero comma intervallum duorum sonorum rationem 81: 80 tenentium, ita ut commatis mensura sit

$$\log.81 - \log.80 = 0,017920$$

atque ideo fere 56 commata intervallum octavae expleant. Diesis est intervallum sonorum rationem 128:125 tenentium; eius ergo mensura est 0,034216 Est ergo diesis fere duplo maior quam comma atque in octava propemodum 29 dieses continentur. Diaschisma denique est intervallum sonorum 2048:2025 eiusque mensura est 0,016296; diaschismatum ergo 61 propemodum octavam adimplent. Constat igitur esse diaschisma differentiam inter diesin et inter comma.

13. Intervalla haec tam exigua in musica quidem consueta occurrere non solent neque soni tam parum se invicem distantes usurpantur; interim tamen differentiae maiorum intervallorum tam parvae in musica deprehenduntur, ut ad ea exprimenda haec minima intervalla introducere fuerit opus. Intervalla autem minima, quae in musica revera adhibentur et sonis exprimi solent, sunt *hemitonia* tam maiora quam minora atque *limmata* itidem tam maiora quam minora; quae intervalla, cum parum a se invicem distent, ab imperitioribus pro aequalibus habentur nomineque hemitonii indicantur.

14. Hemitonium maius est intervallum sonorum rationem 16:15 tenentium, eius ergo mensura est 0,093110. Hemitonium vero minus constituitur inter sonos 25:24, quae ratio ab illa superatur ratione 128:125 diesin exprimente; erit ergo hemitonii minoris mensura 0,058894, ad quam quippe mensura dieseos addita mensuram hemitonii maioris producit.

Octavam igitur proxime compient decem hemitonia maiora cum duabus diesibus seu 17 hemitonia minora proxime.

15. Limma maius, quod constat sonorum ratione 27: 25, commate excedit hemitonium maius eiusque propterea mensura est 0,111030. Limma vero minus est intervallum sonorum rationem 135 : 128 tenentium ideoque quoque commate excedit hemitonium minus, a limmate vero maiore subtractum relinquit diesin. Mensura ergo limmatis minoris est 0,076814. Novem ergo limmata maiora proxime octavam constituent, limmatum minorum vero ad octavam implendam requiruntur 13.

16. Hae quatuor intervallorum species promiscue, ut iam diximus, hemitonia appellari solent; vocantur vero etiam *secundae minores,* quod nomen aeque ac octava, quinta et quarta ortum suum ex genere diatonico habet. Complementa vero horum intervallorum ad octavam, quae continentur sonorum rationibus

15:8, 48:25, 50:27 et 256:135,

eadem nominis derivatione, *septimae maiores* vocantur. Sunt adeo earum mensurae 0,906890, 0,941106, 0,888970 atque 0,923186, quae sunt maxima octava minora intervalla, quae quidem sunt in usu.

17. Hemitonia quantitatis ordine excipiunt intervalla, quae nomine *toni* itemque *secundae maioris* indicari solent. Tonorum autem tres habentur species, quarum prima, quae ratione 9: 8 constat, *tonus maior* appellatur cuiusque ideo mensura est 0,169924; huiusmodi ergo tonorum sex coniuncti octavam plus quam commate superant. *Tonus minor* ratione 10:9 continetur commateque minor est quam tonus maior, ita ut eius mensura sit 0,152004. Ad tonos tertio quoque refertur intervallum sonis 256:225 contentum, quod tonum maiorem diaschismate, minorem vero diesi superat. Complementa vero horum tonorum ad octavam *septimae minores* vocantur.

18. Tonus autem duo hemitonia lato sensu accepta continet. Est enim tonus maior tam summa ex hemitonio maiore et limmate minore quam summa ex hemitonio minore et limmate maiore. Tonus vero minor est summa ex hemitonio maiore et minore. Tonus denique maximus, ratione 256:225 contentus, est summa duorum hemitoniorum maiorum. Simili modo sequentia intervalla hemitoniis adiiciendis oriuntur.

19. Tonis semitonio auctis oriuntur intervalla, quibus *tertiae minoris* nomen est impositum; quamvis accurate loquendo id tantum intervallum hoc nomen mereatur, quod sonis 6:5 contineatur. Quae intervalla enim vel commate vel diaschismate vel diesi ab hac ratione discrepant, ea congrue pro tertia minore, quae est consonantia satis grata,habentur; id quod etiam de reliquis intervallis, quae suaves sunt consonantiae, est tenendum. Tertiae minoris complementum ad octavam vocatur *sexta maior* ratione 5:3 contenta; tertiaeque minoris propterea mensura est 0,263034 et sextae maioris 0,736966

20. Tertiam minorem hemitonio minore excedit *tertia maior,* ea scilicet, quae gratam consonantiam constituit, illaque est intervallum sonorum rationem 5:4 tenentium. Eius ergo mensura est 0,321928; constat igitur haec tertia maior ex tono maiore et minore coniunctis. Complementum vero tertiae maioris ad octavam vocatur *sexta minor,* quae ergo constat ex sonis rationem 8:5 tenentibus, eiusque mensura est 0,678072. Sexta etiam graece vocatur *hexachordon,* ita ut sexta maior congruat cum hexachordo maiore, minor vero cum minore.

21. Si ad tertiam maiorem ratione 5:4 contentam addatur hemitonium maius 16:15, prodibit his rationibus componendis ratio 4:3, qua intervallum *diatessaron* indicatur seu *quarta.* Huius vero intervalli complementum ad octavam est *diapente* seu *quinta* ratione 3:2 contenta, de quibus intervallis iam supra est actum. Hic superest tantum, ut notemus differentiam inter quintam et quartam esse tonum maiorem ratione 9:8 constantem, quae ipsa differentia veteribus primum ideam toni maioris suppeditavit.

22. Cum iam reliqua intervalla omnia semitoniis a se invicem differant, medium quoque sonum musici inter quintam et quartam collocaverunt, qui ab utroque hemitonio distet. Vocatur autem hic sonus *tritonus,* eo quod ex tribus tonis constet, alias vero etiam *quarta abundans* atque etiam *quinta deficiens* seu *quinta falsa.* Pro quatuor autem variis hemitonii speciebus tritoni quatuor habentur species, quarum prima continetur ratione 64:45 et est quarta cum hemitonio maiore. Secunda species est quinta demto hemitonio maiore et continetur ratione 45:32. Tertia species est quarta cum hemitonio minore, quarta vero est quinta demto hemitonio minore; illa ergo ratione 18:25, haec vero ratione 25:36 continetur, quarum postrema quoque est duplex tertia minor.

23. Uti haec intervalla a numeris sua nomina obtinuerunt et secunda, tertia, quarta, quinta etc. usque ad octavam appellantur, ita etiam similia nomina intervallis compositis seu octava maioribus sunt imposita. Octava scilicet cum secunda sive maiore sive minore *nona* vel maior vel minor vocatur; pariter octava cum tertia *decima* appellatur octavaque cum quarta *undecima* et ita porro septem semper adiiciendis ad nomina intervallorum simplicium: ita *duodecima* est octava cum quinta, *decima quinta* vero est duplex octava, ex quibus huiusmodi nomina satis intelliguntur.

24. Quo haec intervalla quaeque cum suis nominibus uno conspectu appareant faciliusque tam percipiantur quam a se invicem discernantur, sequentem tabulam adiicere visum est, in qua primo nomina intervallorum simplicium sunt collocata, deinde rationes sonorum in numeris, tertio mensurae intervallorum per logarithmos ad hoc institutum electos expressae; in quarta columna praeterea gradus suavitatis adscripsi, quo quaeque intervalla gaudent, ex quibus statim iudicari potest, quanto gratiora auditui alia intervalla aliis sint futura.

Nomina Intervallorum	Ratio Sonorum	Mensura	Gradus Suavitatis
Diaschisma	2048:2025	0,016296	XXVIII
Comma	81: 80	0,017920	XVII
Diesis	128: 125	0,034216	XX
Hemitonium minus	25: 24	0,058894	XIV
Limma minus	135: 128	0,076814	XVIII
Hemitonium maius	16: 15	0,093110	XI
Limma maius	27: 25	0,111030	XV
Tonus minor	10: 9	0,152004	X
Tonus maior	9: 8	0,169924	VIII
Tertia minor	6: 5	0,263034	VIII
Tertia maior	5: 4	0.321928	VII
Quarta	4: 3	0,415038	V
Tritonus	25: 18	0,473932	XIV
	45: 32	0,491852	XIV
	64: 45	0,508148	XV
	36: 25	0,526068	XV
Quinta	3: 2	0,584962	IV
Sexta minor	8: 5	0,678072	VIII
Sexta maior	5: 3	0,736966	VII
Septima minor	16: 9	0,830076	IX
	9: 5	0,847996	IX
Septima maior	50: 27	0,888970	XVI
	15: 8	0,906890	X
	256: 135	0,923186	XIX
	48: 25	0,941106	XV
Octava	2: 1	1,000000	II

Haec ergo intervalla ratione suavitatis ita progrediuntur: Octava, Quinta, Quarta, Tertia maior et Sexta maior, Tonus maior, Tertia minor et Sexta minor, utraque Septima minor, Tonus minor et una Septima maior hemitonio maiore ab octava deficiens, Hemitonia et Septimae maiores reliquae.