CHAPTER VI

CONCERNING SERIES OF CONCORDS

1. Just as it has been made abundantly clear in the two preceding chapters, it may be required to consider both concords as well as successions of two concords, so that they may present a gratifying harmony to the ears. But these two items generally are not sufficient for the production of an agreeable musical work. As indeed many more concords and successions of concords may be perceived with pleasure, besides what is required by tradition, so that also the order may arise, which is present in the concords and in the sequences themselves, may be grasped by the mind and from that the intended aim, evidently the agreeableness of the music.

2. Indeed just as although the consonants by their own charm may not be able to produce any harmony alone without the agreement of the adjoining concords, thus so that also the account of several in succession is to be compared, so that, even if some of these shall be put in place next to each other by prescribed rules, yet, unless some exceptional precepts may be observed, they may produce an unbearable din to the ears. On account of which, we will establish several rules in this chapter which it will be required to observe concerning the union of several concords.

3. That part of the music, which instructs how several concords thus to be joined together, so that they may constitute agreeable singing, accustomed to be called generally a *simple composition ;* for the name of any musical compilation is usually called a composition. Therefore for a simple composition, which is the foundation of all the remaining compositions, it is required to be known before everything, in which the charm of several successive concords or may consist of a whole melody. Then from this principle rules are to be deduced, which it is required to observe in simple composition.

4. But the foundation of the charm, which can be present in several of the successive concords, generally is similar to these foundations, by which the charm both of the concords as well as of two in succession has been shown to be constructed. On account of which for the harmony of several concords it is required to be perceived themselves to be in sequence, so that the order may be recognised in the sounds and concords, both of the individual parts as well as with all those presented together.

5. Therefore just as the harmony or charm of each of the concords as well as of two in succession is perceived, if the exponent of the individual and of all the sounds, which is recognised both in one as well as in each of the concords present, thus it is seen the harmony of several concords themselves in sequence to be readily accepted, if the exposition of all the sounds may be understood, which may be considered to constitute this series. From which it is understood, that the charm of several concords may

themselves be understood to be required to be in sequence, so that the exponent of all the sounds and concords may be understood from these compositions.

6. But the exponent of all the sounds, from which all the concords themselves agree to be in sequence, is the least common divisor of the numbers representing the sounds. On account of which the series of concords proposed from the number, which is the smallest common divisor of all the sounds occurring in these, will be able to be defined with the help of the table shown and the rules treated, so that from the whole step of the facility the series of concords may be understood without difficulty. And from the order of the charm, which the tables or rules show, it will be able to understand, how from the charm and hearing any proposed series of concords may be going to be accepted.

7. Therefore since the exponent of the series of concords, from which a judgement must be made concerning the harmony, shall be the smallest common divisor of all the numbers representing the individual sounds occurring, it is evident that divisible number to become by the exponents both of the simple concords as well as of a succession of any two. On this account if the exponent of the whole series concords were known, it is necessary, that also both the individual concords as well as the successions of two may be perceived ; and on this account consequently the whole common link may be obtained.

8. Therefore it is understood from the exponent of several concords, if that either were known before now or at last could be perceived from some concords, how such sounds and concords may be able to occur. And thus this exponent determines the bounds or limits of the needs of music, or way of the music as it is accustomed to be called by musicians, and it includes all the suitable sounds and excludes the incongruous ones. And this limitation may be called also the musical mode, thus so that the musical mode shall be certain collections of sounds, which on their own, on being suitably arranged, agree to be used in musical works, and in general no others are allowed to be used.

9. Therefore since the musical mode may be determined by the exponent of all the sounds, which constitute the mode, hence we will call this exponent *the exponent of the mode*. Whereby if the complete concords may be represented, of which the exponent shall be of the same kind, in which all the sounds will belong, which will be able to be used in this manner. Therefore it is to be understood at once that it will be permitted to judge , whether in the proposed musical work the mode may be used or whether it depends on another mode; which occurs, if sounds may be used which are not contained in the exponent of the mode.

10. But because we have said that a fault lies beyond the mode, yet that is required to be understood with this restriction, that is to be true only as long as that mode may be kept. For it is entirely permissible and is accustomed to happen with the maximum charm, that the mode may become changed over from one mode to another ; and that not only in the same musical work, but also in the same part of that. And from that change or

succession of modes the same precepts are required to be held, which are treated by the succession of concords.

11. Therefore just as we have attributed the exponent of any of its own concords and likewise any of the two concords in succession, thus also any part or period of a musical work in which it may be used in the same way, will have its own exponent determined and similarly of two periods of this kind in succession. Finally truly the exponent of a whole musical work may include all the previous exponents or entirely all the sounds, which were used in all the parts.

12. Therefore, so that a musical work may please, it is required, so that initially the exponents of all the individual concords may be perceived ; then, so that the exponents of two successive concords may be recognised; in the third place, so that the exponents of two individual periods may come to mind; fourthly, so that the exponents of two successive periods or changes of mode may be perceived ; finally fifthly, so that of all the periods, that is of the whole musical work, the exponent may be understood. Therefore anyone who understands all this, finally understands that musical work perfectly and can judge that correctly.

13. Without doubt, why the recognition of some such a musical work cannot be achieved without so much difficulty, indeed also the strengths of the human intellect being overcome for a long time on account of the exponent of the whole musical work as well as the number of these in the composition, so that the mind may not be able to understand everything. But however great this fear of difficulty may seem, yet consideration of the mode may raise the mind, ad this understanding is acquired in stages. So that even the exponent of two successive concords may be perceived without difficulty from the perceived exponents of the concords, even if it may be a very hard composition and by itself the mode may be scarcely able to be recognised, thus also by the successive recognition of the simpler exponents this same comprehension of greater compositions thus follows without difficulty.

14. For in whatever way the understanding must be come upon of the exponent from the succession of two concords, not arising from the exponent itself, or the order of the charm which it has, but arising from the order of the successions. Thus also the exponent of the mode or of a period known from the exponents both of the concords as well as from a succession of concords is more easily returned. And this understanding itself of the exponents of the modes is as if guided by the exponents of the successive known modes. From which finally it is evident the recognition of the exponent of the whole musical work emerges easily enough.

15. Therefore so that the musical work may be heard with pleasure, it will be necessary, that the exponents of two successive concords may not be a much greater composition than the exponents of their concords; then, so that the exponents of the modes may not exceed much the exponents of the successive exponents ; finally, so that the exponent of

the whole musical work may surpass these exponents with the facility of being perceived in pairs. For in this same perception with the recognition progressing from the more simple towards the more complex truly it may turn towards the charm and pleasure, which the listener can experience from the music ; just as has been shown abundantly in the second chapter on the true principles of harmony.

16. Therefore from this is evident enough, how a musical work may be required to be prepared, so that it may be pleasing to the ears of intelligent listeners, likewise truly also it is understood a musical work of any kind of music such as we require, which is contrary to this precept to be displeasing to the listeners must be a sin. On this account again it is readily apparent, imperfect musical works of this kind may be acceptable to less intelligent listeners; certainly which happens, when imperfections and faults brought together contrary to the precepts of harmony are not observed, yet meanwhile certain not incongruous matters are perceived and appreciated.

17. Therefore since the exponent of several concords shall be the exponent of all the sounds required to constitute these concords, this will be the least common divisor of all the individual sounds being represented. But it will be more fitting to be found from the exponents of the concords taken together with the indices, in a similar manner, which we have demonstrated how to find the exponent of a succession in the preceding chapter. For the same precepts, which have been treated for two concords, also prevail for three or more. Evidently the exponent of a series of several concords is nothing other than the least common divisor of the exponent of all the individual concords.

18. First we will consider several successive simple sounds produced together, of which the mutual relation shall be expressed by the following numbers $a:b:c:d:e$, and we may seek the exponent of this series of sounds. But since a simple sound shall be a concord of the first order and its exponent, unless it may be compared with others, shall be unity, the letters $a:b:c:d:e$ will denote the indices of the simple sounds, certainly which contain the relation, as these sounds may be considered as if to hold concords among themselves. Therefore according to the manner of representing the concords, these sounds may be expressed thus: $1(a):1(b):1(c):1(d):1(e)$.

19. But the exponent of this simple series of sounds is the same as that which may be agreed on from the exponent of the concord made from these sounds. But the exponent of this simple series of sounds is the same exponent, which becomes the exponent of the concords agreed from these simple sounds. Truly the exponent of the concord $a:b:c:d:e$ is the least common divisor of the numbers $a:b:c:d:e$ which we may put to be *D.* On account of which with some exponent *D* considered for the image of the concords for these successive sounds $1(a):1(b):1(c):1(d):1(e)$, that is, the least common divisor of the indicated *a, b, c, d, e,* since all these exponents themselves shall be 1. And for the order of the charm, to which the number *D* is referred, it must be judged, as the pleasure which shall arise on hearing that series of sounds.

20. Now A, B, C, D, E shall be the exponents of the successive concords established and $a:b:c:d:e$ the respective indices of these, which express the relation, which the bases of these concords maintain between each other, thus so that this series of concords may be represented in this manner $A(a):B(b):C(c):D(d):E(e)$. In which series we may put the indices $a:b:c:d:e$ to be numbers prime between themselves, thus so that besides unity they may have no other common divisor. For if they may have a common divisor, they must be divided by that initially, as the exponent of the series may be found.

21. Moreover the sounds contained in the concords $A(a)$ are the divisors of the individual exponent A multiplied by a; whereby the least common divisor of these will be Aa. In a similar manner least common divisors of the established concords of the sounds $B(b):C(c):D(d):E(e)$ will be Bb, Cc, Dd, Ee. On account of which of all the sounds contained in these successive concords the least common divisor will be the least common divisor of the numbers Bb, Cc, Dd, Ee. And this least common divisor will be the exponent itself of the proposed series of concords, which is sought.

22. For the sake of an example the following sequences of concords shall be proposed:

$$\begin{array}{c} 8:12:16:24:32:48, \\ 8:12:20:24:40:60, \\ 9:12:18:27:36:54, \\ 10:15:20:30:45:60, \\ 9:15:30:36:45:60. \end{array}$$

Therefore each of these sounds may be divided by the greatest common divisor, and the least common multiple of the quotients may be found; and here the exponent of the concord will be the least common multiple, and the greatest common divisor will be the index. With which done these concords will be expressed thus:
$24(4):30(4):36(3):36(5):60(3);$ From which the exponent of the series of these concords will be found $=4320$ $[=72\cdot 60 = l.c.m.\ of\ 24, 30, 36, 60.]$, which number belongs to the XVI order.
[*i.e.*

$$\begin{array}{l} 2^3 : 2^2\cdot 3 : 2^4 : 2^3\cdot 3 : 2^5 : 2^4\cdot 3; \quad g.c.d. = 4;\ l.c.m.\ of\ 2:3:4:6:8:12 = 24; \\ 2^3 : 2^2\cdot 3 : 2^2\cdot 5 : 2^3\cdot 3 : 2^3\cdot 5 : 2^2\cdot 3\cdot 5; \quad g.c.d. = 4;\ l.c.m.\ of\ 2:3:5:6:10:15 = 30; \\ 3^2 : 2^2\cdot 3 : 3^2\cdot 2 : 3^3 : 2^2\cdot 3^2 : 3^3\cdot 2; \quad g.c.d. = 3;\ l.c.m.\ of\ 3:4:6:9:12:18 = 36; \\ 2\cdot 5 : 3\cdot 5 : 2^2\cdot 5 : 2\cdot 3\cdot 5 : 3^2\cdot 5 : 2^2\cdot 3\cdot 5; \quad g.c.d. = 5;\ l.c.m.\ of\ 2:3:4:6:9:12 = 36; \\ 3^2 : 3\cdot 5 : 2\cdot 3\cdot 5 : 2^2\cdot 3^2 : 3^2\cdot 5 : 2^2\cdot 3\cdot 5; \quad g.c.d. = 3;\ l.c.m.\ of\ 3:5:10:12:15:20 = 60, \end{array}$$

from which the above follows.
The manner of extracting the least common multiple is shown in the following example:
$2:3:4:6:8:12 :: 2 \to 2\cdot 3 \to 2^2\cdot 3 \to 2^3\cdot 3 \to 2^3\cdot 3 = 24$, which can be generalised readily . Thus, given the first approximation to the *l.c.m* to be the first number written as a product of prime numbers to various powers, the second approximation becomes the second term multiplied by the first term, only if the second term contains new primes

written similarly, otherwise common prime factors are ignored if they are of equal or lesser powers than those from the first or previous term, etc.]

23. Therefore it is understood both from the treatment of the rules as well as from the example presented, how from any proposed series of concords the exponent of these may be found, from which the mutual harmony of these concords may be allowed to be judged. Evidently the exponent if any concord must be multiplied by its own index and of everything found in this manner, the smallest common divisor of these must be investigated [not to be confused with the greatest common divisor used initially to simplify the ratios; as we have indicated previously, this is Euler's way of describing the least common multiple of these numbers, which the French version of the work uses] ; and this will be the exponent of the series of concords proposed.

24. If two or more series of concords of the whole musical work being composed may be joined together, the exponents of which may be found by using these precepts, clearly *M*, *N*, *P*, *Q* etc., initially it is required to be shown clearly, whether any one of these exponents may designate the same sound or different sounds. For in this case the ratio, which the sounds of the individual series maintain between themselves, which may be denoted by unity, is required to be denoted by the smallest numbers, which numbers, I may put to be *m, n, p, q* etc., will be the indices of the exponents joined together, thus so that these series joined in this manner by the exponents and indices may be expressed thus :

$$M(m):N(n):P(p):Q(q):\text{etc.}$$

25. Therefore since a musical mode may be expressed by a series of concords of this kind, likewise it is understood, how it will be required to judge concerning the transfer from one mode to another and in the joining together of several modes. Evidently if the successive modes may be joined by exponents and indices expressed thus

$$M(m):N(n):P(p):Q(q):\text{etc.},$$

the exponent from that the nature and character of the whole musical work will be had from these manners of composition, if the minimum common divider of the numbers $Mm,\ Nn,\ Pp,\ Qq$ etc. may be sought; for this will be the exponent of the whole musical work proposed.

26. Therefore so that the correct judgement may be brought to a proposed musical work, in the first place the individual concords are to be considered and their exponents to be investigated. In the second place the succession of some two concords may be considered. Thirdly it may be agreed to consider together several concords, in which the same mode may be retained. Fourthly, the succession of two modes or the transition from one mode into another is required to be carried out by inspection. Finally the composition of all the modes together in a musical work is required to be examined. Just as all these

things may be required to be executed one at a time with the aid of exponents, which has been established in a very satisfactory manner.

27. Therefore it remains in this chapter so that we may be allowed to show, in what way a whole series of concords thence may be required to composed a musical work, which may show a pleasing harmony to the listener. In which business thus we will keep returning, so that we may elicit the exponents of the individual concords either from the exponent of a given mode or from a series of concords. Therefore since an extremely large number of exponents may be able to be taken and from any of these innumerable series of concords may be able to be deduced, this same science appears to be an extremely vast field and always not only with new works, but also it will require to be augmented with new ways.

28. Indeed in this time, in which the study of music has been carried to such a level of perfection, certainly is worthy of admiration, which all of those skilled in music shall be occupied in the composition of new works, but of the number of modes, which is small enough and for a long time up to the present now undertaken without any desire to be increased. The cause of this concern may be seen, so that the true principles of harmony up to this point were unknown on account of the musicians lacking in the study of music theory and to be influenced only by experience and usage.

29. Since the exponent of the series of concords shall be the least common divisor of the exponents of the individual concords multiplied by their indices, these will make all the divisors from the series of exponents of concords from the exponents and indices of the individual concords. Whereby if the exponent of the series of concords shall be given, for example *M,* for the concords themselves taken requiring to be found, as many divisors of *M* as it may have pleased, which shall be *Aa, Bb, Cc, Dd* etc. From these found $A(a):B(b):C(c):D(d):\text{etc.}$ will represent the series of concords, of which the exponent will be the given number *M.*

30. But from these divisors taken this is required to be considered, that these exponents will remove all the exponents considered in *M,* that is, so that a least common divisor may not be had less than *M.* Which if it may occur, from the start at once some concords may be arranged, the exponents of which may deplete the given number *M* ; and with this convenience agreed on , so that from some concords heard from the beginning, the exponent of the whole series of concords may be understood at once and with that known it may be able to judge more easily the harmony of the whole series . But concerning these more will be forthcoming below.

CAPUT VI

DE SERIEBUS CONSONANTIARUM

1. Quemadmodum tam consonantias quam duarum consonantiarum successiones comparatas esse oporteat, ut auribus gratam harmoniam offerant, tam in duobus praecedentibus capitibus abunde est explicatum. Hae autem duae res omnino non sufficiunt ad opus musicum suave producendum. Nam quo plures consonantiae consonantiarumque successiones cum voluptate percipiantur, praeter tradita requiritur, ut etiam ordo, qui in omnibus consonantiis sese in sequentibus inest, animo comprehendatur atque ex eo intentus scopus, scilicet suavitas, oriatur

2. Sicuti enim consonantiae solae etsi per se suavissimae sine ratione coniunctae nullam harmoniam efficiunt, ita etiam plurium successionum ratio est comparata, ut, etiamsi earum quaeque iuxta leges praescriptas sit instituta, tamen, nisi praecepta peculiaria observentur, auribus maxime ingratus strepitus excitetur. Quamobrem quas leges circa coniunctionem plurium consonantiarum observari oporteat, hoc capite exponemus.

3. Ea musicae pars, quae plures consonantias ita inter se iungere docet, ut suavem concentum, constituant, vocari vulgo solet *compositio simplex;* compositionis enim voce intelligi solet operis cuiusque musici confectio. Ad compositionem simplicem ergo, quae fundamentum est omnium reliquarum compositionum, absolvendam ante omnia nosse oportet, in quo suavitas plurium consonantiarum successivarum seu integri concentus consistat. Deinde ex hoc principio regulae sunt deducendae, quas in compositione simplici observari oportet.

4. Fundamentum autem suavitatis, quae in plurium consonantiarum successione inesse potest, omnino simile est iis fundamentis, quibus suavitas tam consonantiarum quam binarum successionum constare est demonstrata. Quamobrem ad harmoniam plurium consonantiarum sese insequentium percipiendam requiritur, ut ordo, qui in singulis partibus, hoc est in sonis et consonantiis tam singulis quam omnibus coniunctis inest, cognoscatur.

5. Quemadmodum igitur tam cuiusque consonantiae quam binarum successionis harmonia seu suavitas percipitur, si exponens singulorum et omnium sonorum, qui tam in una quam utraque consonantia insunt, cognoscitur, ita facile perspicitur harmoniam plurium sese insequentium consonantiarum apprehendi, si exponens omnium sonorum, qui hanc seriem consonantiarum constituant, concipiatur. Ex quo intelligitur, quo suavitas plurium consonantiarum sese insequentium percipiatur, requiri, ut exponens omnium sonorum et consonantiarum ex iis compositarum cognoscatur.

6. Exponens autem omnium sonorum, ex quibus omnes consonantiae sese insequentes constant, est minimus dividuus numerorum sonos repraesentantium. Quocirca proposita consonantiarum serie ex numero, qui est minimus communis dividuus omnium sonorum in iis occurrentium, ope tabulae exhibitae atque regularum traditarum definiri poterit, quo facilitatis gradu integra consonantiarum series apprehendatur. Atque ex gradu suavitatis, quem vel tabula vel regulae monstrant, intelligi poterit, quam suavis audituique accepta futura sit quaecunque proposita consonantiarum series.

7. Cum igitur exponens seriei consonantiarum, ex quo de harmonia iudicium ferri debet, sit minimus communis dividuus omnium numerorum sonos singulos occurrentes repraesentantium, perspicuum est illum numerum divisibilem fore per exponentes tam simplicium consonantiarum quam successionum binarum quarumque. Quamobrem si cognitus fuerit exponens totius consonantiarum seriei, necesse est, ut etiam tam singulae consonantiae quam binarum successiones percipiantur; atque hac ratione consequenter universus nexus apprehendetur.

8. Ex exponente ergo seriei plurium consonantiarum intelligitur, si is vel ante iam fuerit cognitus vel ex aliquot consonantiis demum perceptus, quales soni qualesque consonantiae occurrere queant. Determinat itaque iste exponens limites seu ambitum, uti a Musicis vocari solet, operis musici et comprehendit omnes sonos convenientes incongruosque excludit. Haecque limitatio etiam modus musicus appellatur, ita ut modus musicus sit certorum sonorum congeries, quos solos in concinnando opere musico adhibere convenit, praeterque eos alios introducere omnino non licet.

9. Cum igitur modus musicus per exponentem omnium sonorum, qui modum constituunt, determinetur, hune exponentem posthac *exponentem modi* vocabimus. Quare si consonantia completa repraesentetur, cuius exponens sit hic ipse exponens modi, in hac consonantia omnes inerunt soni, qui in hoc modo usurpari poterunt. Intellecto ergo hoc exponente statim iudicare licet, utrum in proposito opere musico modus sit servatus an vero vitium contra modum sit commissum; id quod accidit, si soni adhibeantur in exponente modi non contenti.

10. Quod autem vitium esse diximus extra modum excurrere, id tantum cum hac restrictione est intelligendum, quamdiu iste modus teneatur. Omnino enim permissum est et cum maxima venustate fieri solet, ut modus immutetur atque ex alio modo in alium fiat transitus; idque non solum in eodem opere musico, sed etiam in eadem eius parte. Atque de hac modorum mutatione seu successione eadem praecepta sunt tenenda, quae de successione consonantiarum sunt tradita.

11. Quemadmodum igitur cuivis consonantiae suum tribuimus exponentem itemque cuivis binarum consonantiarum successioni, ita etiam quaelibet operis musici portio seu periodus, in qua idem servatur modus, suum determinatum habebit exponentem similiterque duarum huiusmodi periodorum successio. Tandem vero integri musici operis

exponens complectetur omnes priores exponentes seu omnes omnino sonos, qui in omnibus partibus erant adhibiti.

12. Quo ergo opus musicum placeat, requiritur, ut primo singularum consonantiarum exponentes percipiantur; deinde, ut binarum consonantiarum successionum exponentes cognoscantur; tertio, ut singularum periodorum exponentes animadvertantur; quarto, ut successionum binarum periodorum exponentes seu modorum mutationes percipiantur; quinto denique, ut omnium periodorum, hoc est totius operis musici, exponens intelligatur. Qui ergo haec omnia perspicit, is demum opus musicum perfecte cognoscit de eoque recte iudicare potest.

13. Non dubito, quin talis cognitio operis musici summopere difficilis, imo etiam vires humani intellectus longe superans videatur propter exponentem totius operis musici tam compositum numerum, ut animo comprehendi omnino nequeat. Sed quantopere haec apprehensio difficilis videatur, tamen mirum in modum sublevatur intellectus, dum ista perceptio per gradus acquiritur. Uti enim exponens successionis duarum consonantiarum non difficulter percipitur perceptis exponentibus consonantiarum, etiamsi sit valde compositus et per se vix cognosci posset, ita etiam cognitis successive simplicioribus exponentibus hoc ipso apprehensio magis compositorum non adeo difficulter consequitur.

14. Nam quemadmodum perceptio exponentis successionis duarum consonantiarum non ex ipso exponente seu gradu suavitatis, quem habet, debet aestimari, sed ex ordine successionis, ita etiam exponens modi seu unius periodi cognitis exponentibus tam consonantiarum quam successionum facilior redditur. Atque haec ipsa exponentium modorum apprehensio quasi manuducit ad exponentes successionum modorum cognoscendos. Quibus denique perspectis cognitio exponentis totius operis musici satis facilis evadit.

15. Quo igitur opus musicum cum voluptate audiatur, oportet, ut exponentes successionum duarum consonantiarum non multo sint magis compositi quam ipsarum consonantiarum exponentes; deinde, ut exponentes modorum non multum excedant exponentes successionum; denique, ut exponens totius operis musici illos exponentes facilitate percipiendi parum superet. In ista enim perceptione et a simplicioribus ad magis composita progrediente cognitione versatur vera suavitas et voluptas, quam auditus ex musica haurire potest; quemadmodum in capite secundo ex genuinis harmoniae principiis abunde est demonstratum.

16. Ex his igitur satis perspicitur, quomodo opus musicum comparatum esse oporteat, ut auditoribus intelligentibus placeat, simul vero etiam intelligitur opera musica, in quibus contra haec praecepta est peccatum, huiusmodi, quales requirimus, auditoribus displicere debere. Quomodo porro istiusmodi opera musica imperfecta auditoribus minus intelligentibus accepta esse queant, facile quoque apparet; quippe quod fit, quando imperfectiones et vitia contra harmoniae praecepta commissa non advertunt, interim tamen quaedam non incongrue posita attendunt et percipiunt.

17. Cum igitur exponens plurium consonantiarum sit exponens omnium sonorum illas consonantias constituentium, erit is minimus communis dividuus numerorum singulos sonos repraesentantium. Commodius autem ex exponentibus consonantiarum cum indicibus coniunctis poterit inveniri, simili modo, quo in capite praecedente docuimus exponentem successionis invenire. Eadem enim praecepta, quae pro duabus consonantiis sunt tradita, valent quoque pro tribus pluribusque. Exponens scilicet seriei plurium consonantiarum nil aliud est nisi minimus communis dividuus exponentium singularum consonantiarum.

18. Consideremus primo plures sonos simplices successive editos, quorum mutua relatio expressa sit sequentibus numeris $a:b:c:d:e$, quaeramusque exponentem seriei huius sonorum. Cum autem sonus simplex sit consonantia primi gradus eiusque exponens, nisi cum aliis comparetur, sit unitas, denotabunt litterae $a:b:c:d:e$ indices istorum sonorum simplicium, quippe quae relationem continent, quam hi soni tanquam consonantiae considerati inter se tenent. Ad modum igitur consonantiarum hi soni ita debebunt exprimi: $1(a):1(b):1(c):1(d):1(e)$.

19. Huius autem seriei simplicium sonorum idem est exponens, qui foret exponens consonantiae ex iis sonis constantis. Consonantiae vero $a:b:c:d:e$ exponens est minimus communis dividuus numerorum $a:b:c:d:e$ quem ponamus esse *D.* Quamobrem his sonis successivis ad instar consonantiarum spectatis erit seriei consonantiarum harum $1(a):1(b):1(c):1(d):1(e)$. exponens quoque *D*, hoc est minimus communis dividuus indicum *a, b, c, d, e,* cum ipsi exponentes omnes sint 1. Atque ex gradu suavitatis, ad quem numerus *D* refertur, iudicari debet, quam grata futura sit auditui ista sonorum series.

20. Sint nunc A, B, C, D, E exponentes consonantiarum successive positarum atque $a:b:c:d:e$ earum respectivi indices, qui relationem exprimunt, quam earum consonantiarum bases inter se tenent, ita ut haec consonantiarum series hoc modo sit repraesentanda $A(a):B(b):C(c):D(d):E(e)$. In qua serie ponimus indices $a:b:c:d:e$ inter se esse numeros primos, ita ut praeter unitatem alium non habeant communem divisorem. Si enim haberent divisorem communem, per eum ante essent dividendi, quam exponens seriei quaereretur.

21. Soni autem in consonantia $A(a)$ contenti sunt divisores exponentis *A* singuli per *a* multiplicati; quare eorum minimus communis dividuus erit *A a.* Simili modo sonorum consonantias $B(b):C(c):D(d):E(e)$. constituentium minimi communes dividui erunt Bb, Cc, Dd, Ee. Quamobrem omnium sonorum in his consonantiis successivis contentorum minimus communis dividuus erit minimus communis dividuus numerorum Bb, Cc, Dd, Ee. Hicque minimus communis dividuus erit ipse exponens propositae consonantiarum seriei, qui quaeritur.

22. Sint exempli gratia consonantiae sequentes propositae:

$$\begin{array}{l} 8 : 12 : 16 : 24 : 32 : 48, \\ 8 : 12 : 20 : 24 : 40 : 60, \\ 9 : 12 : 18 : 27 : 36 : 54, \\ 10 : 15 : 20 : 30 : 45 : 60, \\ 9 : 15 : 30 : 36 : 45 : 60. \end{array}$$

Huius igitur cuiusque soni per maximum communem divisorem dividantur quotorumque quaeratur minimus communis dividuus; eritque hic exponens consonantiae, maximus communis divisor vero index. Quo facto hae consonantiae ita exprimentur $24(4):30(4):36(3):36(5):60(3);$ ex quibus exponens seriei harum consonantiarum reperietur $=4320$, qui numerus ad gradum XVI refertur.

23. Intelligitur ergo tam ex traditis regulis quam ex allato exemplo, quomodo quacunque proposita consonantiarum serie inveniri oporteat exponentem earum, ex quo de harmonia illarum consonantiarum mutua iudicare liceat. Scilicet exponens cuiusvis consonantiae multiplicari debet per suum indicem omniumque hoc modo inventorum productorum minimus communis dividuus investigari; eritque hic exponens seriei consonantiarum propositae.

24. Si duae pluresve consonantiarum series ad integrum opus musicum componendum iungantur, quarum exponentes per haec tradita praecepta iam sint inventi, scilicet *M*, *N*, *P*, *Q* etc., primodispiciendum est, utrum unitas cuiusvis horum exponentium eundem sonum an diversos designet. Hoc enim casu ratio, quam soni singularum serierum, qui unitate denotantur, inter se tenent, minimis numeris est denotanda, qui numeri, quos ponam esse *m*, *n*, *p*, *q* etc., erunt indices exponentibus iungendi, ita ut illae series iungendae hoc modo per exponentes et indices sint exprimendae:

$$M(m):N(n):P(p):Q(q):\text{etc.}$$

25. Cum igitur huiusmodi consonantiarum series exponente expressa sit modus musicus, intelligitur, quomodo de transitu ex uno modo in alium itemque de coniunctione plurium modorum iudicandum sit. Scilicet si modi successive coniuncti sint per exponentes et indices ita expressi

$$M(m):N(n):P(p):Q(q):\text{etc.},$$

exponens ex eoque natura et indoles totius operis musici ex illis modis compositi habebitur, si minimus communis dividuus numerorum $Mm,\ Nn,\ Pp,\ Qq$ etc. quaeratur; hic enim erit exponens totius operis musici propositi.

26. Quo ergo de proposito opere musico rectum iudicium ferri queat, primo singulae consonantiae sunt perpendendae earumque exponentes investigandi. Secundo binarum quarumque consonantiarum successiones considerentur. Tertio plures consonantias,

quibus modus continetur, coniunctim contemplari conveniet. Quarto inspicienda est successio duorum modorum seu transitus ex uno modo in alium. Quinto denique omnium modorum in opere musico iunctorum compositio est inquirenda. Quae singula quomodo ope exponentium exequi oporteat, satis superque est expositum.

27. Superest ergo, ut in hoc capite, quantum adhuc licet, monstremus, quomodo consonantiarum seriem indeque integrum opus musicum confici oporteat, quod auditui gratam harmoniam exhibeat. In quo negotio ita versabimur, ut ex dato modi seu seriei consonantiarum exponente singularum consonantiarum exponentes eruamus. Cum igitur perquam magnus exponentium numerus accipi atque ex quolibet eorum innumerabiles consonantiarum series deduci queant, ista scientia latissime patet atque perpetua non solum novis operibus, sed etiam novis modis augeri poterit.

28. Hoc quidem tempore, quo musicae studium ad tantum perfectionis gradum est evectum, admiratione utique est dignum, quod omnes musicae periti tantum in componendis novis operibus sint occupati, modorum autem numerum, qui satis est parvus et a longo abhinc tempore iam receptus, augere omnino non curent. Cuius rei caussa esse videtur, quod vera harmoniae principia adhuc fuerint incognita atque ob horum defectum musicae studium sola experientia et consuetudine sit excultum.

29. Cum exponens seriei consonantiarum sit minimus communis dividuus exponentium singularum consonantiarum per indices suos multiplicatorum, erunt haec facta ex exponentibus et indicibus singularum consonantiarum omnia divisores exponentis seriei consonantiarum. Quare si exponens seriei consonantiarum sit datus, puta *M,* ad consonantias ipsas inveniendas sumantur, quot libuerit, divisores ipsius *M,* qui sint *Aa, Bb, Cc, Dd* etc. His inventis repraesentabunt $A(a):B(b):C(c):D(d):\text{etc.}$ seriem consonantiarum, cuius exponens erit datus numerus *M.*

30. His autem divisoribus sumendis hoc est advertendum, ut ii exponentem propositum *M* exhauriant, hoc est, ut minorem non habeant minimum communem dividuum, quam est *M.* Quod obtinebitur, si statim ab initio aliquot consonantiae collocentur, quarum exponentes datum numerum *M* exhauriant; hocque pacto et hoc habebitur commodum, quod statim ab initio auditis aliquot consonantiis totius consonantiarum seriei exponens percipiatur ex eoque cognito facilius de harmonia totius seriei iudicari queat. De his autem plura infra tradentur.