# CHAPTER V

## THE SUCCESSION OF TWO CONSONANTS

1. Just as in the preceding chapter and above we have shown how several sounds may be able to be brought together, so that sounding them at the same time they may affect the sense of hearing with a pleasing harmony. Therefore it will be required in order, in this chapter, that we may investigate, how two sounds or two concords [consonants] of this kind may be perceived to be agreeable, which follow each other in turn and sounding in succession. Indeed it does not suffice only for the success of the charm that each concord arising shall be pleasing, but also they must have a certain mutual affection, where also the succession of the sounds itself may sooth the ears and be pleasing to the sense of hearing.

2. But by the general rules treated in chapter II, from which all charm is effected, it is agreed to consist of two pleasing successive concords, if an order may be perceived, which the simple parts or individual sounds maintain between each other. Therefore accordingly it being required to know, how easily a succession of two concords may be recognised by the ear, the single sounds of each concord is required to be expressed by numbers and the least common multiple of which is to be found. Which sought in the table of the orders of charm there it is shown, how many are required to be discerned for the proposed succession required to be perceived.

3. Therefore both these concords of the succession must be considered as sounding at the same time and the exponent of this composite concord will be indicated as the charm, and that succession of concords shall be easily perceived. Indeed the exponent of this composite concord is the least common multiple of all the sounds, which are present in each concord. Moreover from this the charm is required to be judged from the smallest common multiple of a succession of concords. On this account this same number will be the exponent from our succession, thus so that the exponent of two successive concords shall be the least common multiple of all the sounds in each of the included concords.

4. It is understood from this principle that sounds which may please simultaneously, the same also must please produced successively. But there is some difference in the order of the charm [or accord], where the two concords may be heard at the same time or successively. For two concords, which themselves are heard in sequence certainly are pleasing, likewise those produced at the same time may affect the ear a little harder. Thus two sounds holding the ratio 8:9 produced simultaneously may be heard less gently, yet the same sounding successively may be heard with much more pleasure.

5. Indeed just as the simplest concord of three sounds is composed from more notes than the simplest concord of two sound, thus, so that it may depend on several concords, also it will contain more, even if it shall be the simplest of its kind. Yet not only does this

not stand in the way of appreciating its charm, but also more is perceived from multi-sound concords than from simple sounds or from concords containing only two sounds. For more can be present in many sounds, which the order may contain, and by which they may increase the perceived pleasure. Thus nor yet is that a reason for multiplying the sound of concords exceedingly, lest such a variety of multiples arriving at the ear at the same time may confound rather delight the senses.

6. But in the succession of two concords, from the same their very nature it is required that the exponents be more composite than that of the individual concords from which they are formed. And this is not an inconvenience for on account of the agreeability, the concords in succession can themselves be grouped together, without the pleasure arising from the sounds being diminished. For just as with multi-sound concords the exponent may be more composite here without the pleasure of the listener being diminished, because that will still come about, if the individual concords may be constructed from fewer sounds, thus a succession of composite exponents is more likely to be allowed than the exponents of concords all sounding together, and without any loss in the pleasure of listening.

7. Yet meanwhile it cannot be denied, where the simpler were the exponents of the concords, there also that same succession and order which is present in that, to be easier to be perceived. For the rules, which we have handled above concerned with easing perception, appear to be most widely apparent and not to suffer from any exception. But if we wish to use successions of exceedingly simple concords, the varieties which give the most joy to music must be completely removed. For the concords will be required to be much more simple and all to become almost equal to each other. From which it is understood also if it is allowed to use more simple composite exponents of the successions and these of such a kind may designate simple concords which disturb all the harmony.

8. So that two consecutive concords may be perceived to be heard with pleasure, it is required, that in the first place each concord by itself may please and then also these in succession shall be pleasing to be heard. That is indicated by the exponents of the concords, as shown in the previous chapter. Truly this can be understood for the following exponent as well. Thus a judgement is required to be established, so that several successive orders of pleasurable listening may be attributed to the concords themselves, since an exponent of this can be put in place greater than the exponent of these individual concords.

9. For the exponent of a succession of two concords to be defined, it is not sufficient to have considered each concord, but it is necessary, that also the relation of the sounds may be considered, which is expressed in each by these same numbers. For the same concord can be shown in an infinite number of ways, just as the sounds constituting that may be taken higher or lower in pitch, provided they may hold the same ratio between

each other. But in the succession of two concords besides the concords themselves there must be attended to the order maintained, by which each is expressed. This can be done most conveniently from a comparison of the bases, which correspond to each of the concords ; for if these may refer to different sounds, the exponent of the succeeding exponents will not be the least common multiple of the exponent of the concords, but an account of some base also is required to be taken into the calculation.

10. Therefore if a given sound may be taken as the base, not only the octave sounds 1 and 2 may constitute an octave, but also 2 and 4 or 3 and 6 and generally $a$ and $2a$ will show the same concord, of which the exponent is 2. Indeed if the nature of this concord may be understood correctly from the exponent 2, the multiplier $a$ is ignored ; truly an account of the number $a$ is required to be had, if other concords may be taken together. Indeed, for example, this concord of the sounds $2b$ and $3b$ may follow, which is a musical fifth and has the exponent 6, and from the exponents 2 and 6 alone the exponent of the succeeding cannot be deduced, but the ratio of the numbers $a$ and $b$ will be required to know as well, since the exponent of the succession shall be the least common multiple of the numbers $a$, $2a$, $2b$ et $3b$.

11. For just as the exponent of any simple sound is 1, truly the numbers expressing the same of several sounds of this kind must be considered, thus also in the comparison of several concords as well as the exponents of these also the relation of these is required to be inspected. Because of this, since the concords considered amongst may be expressed from the base unity, in the comparison of several concords a number is required to be attributed to each base, with the sound of this agreeing with the ratio of all the sounds. From which it is seen in the comparison of several concords any must be expressed by a double number, truly with the first by its exponent and then by the index, by which the base may be set out with respect to the remaining bases.

12. We will always add the index to the exponent of the concord, but enclosed by brackets, in order that it may be distinguished from the exponent : such that 6(2), where 6 is the exponent of the concord, which therefore agrees by having this relation for the sounds 1 : 2 : 3 : 6 ; truly the index 2 placed following is referring to another concord, and shows the base of this concord, with regard to the other, which is 1, must be 2 by this relation, *i.e.* an octave higher. On account of which the sounds for the following ratio must be had for the exponent expressed by the numbers 2 : 4 : 6 : 12.

13. Just as the same concords can be expressed by an infinitude of numbers, only the may hold the same ratio between themselves, and the exponent of the concords 2 : 3, 4 : 6, 6 : 9 etc. is the same, even if the sounds themselves shall be different, thus the index of the concords may determine, how many concords proposed shall be able to be established from this infinitude of numbers; as that is required for the comparison of several concords being established. Moreover it appears the numbers, which arise from the exponent, are required to be multiplied by the index ; for in this manner the base of

the concord shall be equal to the index, and all the sounds hold the same relation among themselves.

14. From these also it is clear, how both the exponent as well as the index of the concord may be able to be found from the sounds, with the given numbers expressed by constants. For the exponent is found, whereby all the numbers may be divided by the greatest common divisor, and the least common multiple of the quotients is sought according to how many there are present. Truly the index will be that greatest common divisor itself, by which the proposed numbers are able to be divided. Thus the index of the concord 3 : 6 : 9 : 15 will be 3 and the exponent 30, or the least common multiple of the numbers 1 : 2 : 3 : 5. Therefore we will express this concord in this manner 30(3).

15. Let the exponent be *A* and *a* the index of some concord, truly the multiples of *A* shall be $1,\ \alpha,\ \beta,\ \gamma,\ \delta$ etc.; the sounds of this concord will have this ratio between each other $1:\alpha:\beta:\gamma:\delta:\text{etc.}$, of which numbers the least common multiple is *A.* But by inserting the index *a* the sounds of the concord *A(a)* must be expressed by the following numbers

$$a:\alpha a:\beta a:\gamma a:\delta a:\text{etc.},$$

of which the least common multiple of the numbers will be *Aa* on account of the greatest common divisor *a.* Truly in estimating the pleasure of this concord the number *a* is ignored and the charm may be judged from the exponent *A* alone.

16. But this concord *B(b)* may follow the concord *A(a)* , the multiples of which exponent *B* shall be $1,\ \eta,\ \theta,\ i,\ \chi$ etc., and these numbers may express the ratios of the sounds

$$b:\eta b:\theta b:ib:\chi b:\text{etc.}$$

Therefore since the agreement of the successive sounds may be reduced to the agreement of the concord from each successive sound, the exponent of the succession will be the least common multiple of the numbers

$$a:\alpha a:\beta a:\gamma a:\delta a:b:\eta b:\theta b:ib:\chi b;$$

for these sounds will be had, if both the concords may be heard at the same time. Truly because the least common multiple of the sounds $a:\alpha a:\beta a:\gamma a:\delta a$ is $Aa$, truly $Bb$ of the remaining ratios $b:\eta b:\theta b:ib:\chi b$, thus the exponent of the succession of concords will be the least common multiple of the numbers *Aa* and *Bb* .

17. Moreover since the agreement of the concords may be judged incorrectly from the least common multiple of the numbers expressing the sounds, if these numbers were not

prime, but were to have had a common multiple, the same also is required to be extended to a succession of two concords. Whereby if the numbers

$$a:\alpha a:\beta a:\gamma a:\delta a:b:\eta b:\theta b:ib:\chi b;$$

may have a common multiple, initially the individual numbers must be divided by that common multiple and the quotients of these substituted in their place. Truly this cannot eventuate, unless the indices *a* and *b* were composite numbers between themselves. On this account, as often as the indices of two concords have a common multiple, it is required initially to divide the indices by that, so that the exponent of the succession may be found.

18. Therefore the indices *a* and *b* of the concords *A(a)* and *B(b)* shall be prime relative to each other ; the minimum exponent of the succession of these concords will be the least common multiple of the numbers *Aa* and *Bb.* Towards finding this it is necessary that initially the greatest common divisor is sought, which shall be *D.* So that this may become known either of these numbers may be divided by *D* and the quotient multiplied by the other ; and the product *ABab: D* will become the least common multiple sought of the numbers *Aa* et *Bb* and likewise the exponent of the succession of concords proposed, from which the pleasant listening of the succession may become known.

19. Because *a* and *b* may be put to be relatively prime, the numbers *Aa* and *Bb* themselves will have a common multiple, if either *A* and *B*, *A* and *b,* or *B* and *a* were composite numbers. But where several multiples of this kind may be found, there the greatest common divisor of the numbers *Aa* and *Bb* will be greater also. But where this common multiple will be greater, there the least common multiple will be smaller, and therefore the succession of pleasant concords will be greater. For since the exponent of the successions shall be *ABab:D,* so that the greater the greatest common divisor *D* becomes, there the simpler will be the quotient *ABab:D* and for which a simpler order of charm will pertain.

20. *A* shall be a number pertaining to the degree of charm *p*, *B* to the degree *q, a* to the degree *r* and *b* to the degree *s;* truly *t* shall be degree of the agreement of the greatest common divisor *D* . With these in place the number *ABab:D* will be referred to the degree

$$p+q+r+s-t-2$$

as can be gathered from principles treated above. Therefore with the numbers given *A, B, a, b* et *D* the degree of the accord can become known, to which the succession of the concords *A(a)* and *B(b)* will pertain, evidently the degree $p+q+r+s-t-2$. So that the smaller this number may become, there the succession will be more in accord.

21. For the sake of an example the concord 120 (2) is constructed from the sounds

$$2 : 4 : 6 : 8 : 10 : 12 : 16,$$

may be followed by the concord 60 (3) agreeing from the sounds

$$3 : 6 : 9 : 12 : 15,$$

and of which the first is of the tenth order, the latter of the ninth. Therefore the succession must be judged from the least common multiple of the numbers 240 and 180, of which the greatest common divisor is 60 belonging to the ninth order. Therefore since there shall be $A = 120,\ a = 2,\ B = 60,\ b = 3$ and $D = 60$, there will be [the degrees of charm] $p = 10,\ q = 9,\ r = 2,\ s = 3$ and $t = 9$ and thus

$$p + q + r + s - t - 2 = 13.$$

Whereby the exponent of the succession is of the 13th degree, and the charm of the succession is of this order.

[Recall the Euler formula for the order of the charm from Sect. 31 of Ch. 2 : if $n = p_1^{a_1} p_2^{a_2} \cdots p_r^{a_r}$, where the $p_i$ are distinct primes and the powers are greater than or equal to one. Thus, the order or degree is given by the formula :

$$E(n) = 1 + \sum_{k=1}^{r} a_k (p_k - 1);$$

thus $120 = 2^3 \cdot 3^1 \cdot 5^1$, $60 = 2^2 \cdot 3^1 \cdot 5^1$,

$$p = 1\cdot 3 + 2\cdot 1 + 4\cdot 1 + 1 = 10\ ;\ q = 1\cdot 2 + 2\cdot 1 + 4\cdot 1 + 1 = 9\text{; etc.]}$$

22. If the exponents of each concord may be given, thus the indices will be able to be determined, so that the most agreeable ones may emerge in succession. Let $M$ be the least common multiple of the exponents $A$ and $B$ ; it is evident the exponent of the succession $ABab{:}D$ either is equal to $M$ itself or to be greater than that by some multiple ; for it cannot be smaller. Therefore the most agreeable concord in the succeeding order will be truly if $ABab{:}D$ were equal to $M$ itself, truly less agreeable concords will be had in the succeeding ones, if $ABab{:}D$ were equal either to $2M$, $3M$, $4M$ etc. Whereby on putting $ABab = nDM$ the indices $a$ and $b$ therefore return the succeeding values of the agreeability or charm, where the value of the number $n$ will be smaller.

23. We will call the succession to be of the *first order*, if the least common multiple of the numbers $Aa$ and $Bb$ were equal to $M$ itself, or equal to the least common multiple of

the numbers *A* and *B.* Truly we will call the succession to be of the *second order*, the exponent of which is $2M$. Again a succession will be of the *third order* for us, if the exponent of which is 3*M* or 4*M,* because the numbers 3 and 4 pertain to the third or fourth order of the charm. [These latter two numbers actually both pertain to the third order, from the above formula.] And generally that succession, of which the exponent is *nM,* will be of the same order, of which the order of the charm is the number *n.* Truly here it is required to beware, lest the orders of the successions be confused with the orders of the charm ; for we will call the succession to be of the *first order*, from which a simpler succession cannot be given from the same remaining exponents of the concords, even if the succession may refer to an order of much greater charm.

24. Therefore it is evident the orders of the succession of the concords *A* and *B* to be of the first order, if *a* and *b* were unity; for the least common multiple of the numbers *A.*1 and *B.*1 is *M.* Yet besides it can happen, that succession of concords $A(a)$ and *B*(*b*) may be of the first order, even if $a$ may not be $=b$. This occurs, if *b* into *Bb* may have an equal or smaller number of dimensions than into *A* and likewise *a* in *Aa* may have a smaller number of dimensions than *B.* For if this were the case, *M* also will be the least common multiple of the numbers *Aa* and *Bb.*

25. The greatest common divisor of the exponents *A* and *B* shall be *d* and $A=dE$ and $B=dF$ ; *E* and *F* will be numbers relatively prime to each other. Besides $e$ shall be a multiple of *E* and $f$ a multiple or *F*; the succession of concords *dE*(*f*) and *dF*(*e*) will be of ordered primes. For the smallest common multiple of the numbers *dEf* and *dFe* is *dEF,* just as *A* and *B* or *dE* and *dF* are of the numbers. So that if we may suppose $A=15$ and $B=18$, there is $d=3$, $E=5$ and $F=6$. Whereby *e* can be either 1 or 5 and $f$ either 1, 2, 3 or 6. Therefore the succession will be of the first order, either if *A*(*a*) is 15(1), 15(2), 15(3) or 15(6), or truly represented by the following concords if $B(b)$ shall be either 18(1) or 18(5).

26. From these it is easily seen, whatever indices it may be required to assume, so that the exponent of the succession may become 2*M* or 2*dEF,* in which case the succession is of the second order. And in a similar manner it will be able from the determination of the indices, that the exponent of the succession may become *ndEF* or that succession of the given orders, which can happen in a number of ways, which may be difficult and time wasting to enumerate. If the exponents of the concords are 15 and 18, the succession is of the second order, if the first concord were either 15(1) and 15(3) and the other either 18(2) or 18(10), likewise if the first were either 15(4) or 15(12) with the other present being either 18(1) or 18(5).

27. If the exponents of the concords shall be equal or $B=A$, a single succession of the first order will be had, if there is $ab=1$, which therefore will be *A*(1) and *A*(1). Therefore two successions will be of the second order *A*(1): *A*(2) and *A*(2): *A*(1), of which the

exponent is $2A$. There will be four successions of the third order, evidently $A(1):A(3)$ and $A(1):A(4)$ and the inverse of these. There will be six successions of the fourth order, evidently $A(1):A(6),\ A(2):A(3),\ A(1):A(8)$ and the three inverses of these. And any succession of this kind will be of an order, of which the degree of charm is the product indicated.

28. If the exponent of one of the concords were twice the other exponent or $B=2A$, there will be these two successions of the first order : $A(1):2A(1)$ and $2A(1):A(2)$; for of these the exponent is $2A$, the same as those of the exponents $A$ and $2A$. The succession of the second order is $4A$; therefore such will be the successions $A(1):2A(2),\ A(4):2A(1)$ and the inverse of these. In a similar manner the successions of each order will be found, if there were $B=3A$, and generally, if $B=nA$; from which simpler successions will be able to be found easily, which are able to be used.

29. Therefore if the exponents of the concords were equal to each other, the successions of the orders of the first, second, third as far as to the sixth order will be the following, with Roman numerals denoting the orders of the succession and $A$, $A$ the exponents of each concord : [These can be established by considering the example in Sect. 21 above.]

I. $A(1):A(1)$.

II. $A(2):A(1)$.

III. $A(3):A(1),\ A(4):A(1)$.

IV. $A(6):A(1),\ A(3):A(2),\ A(8):A(1)$.

V. $A(5):A(1),\ A(9):A(1),\ A(12):A(1),\ A(4):A(3),\ A(16):A(1)$.

VI. $A(10):A(1),\ A(5):A(2),\ A(18):A(1),\ A(9):A(2),\ A(24):A(1),\ A(8):A(3),\ A(32):A(1)$.

Truly if the exponents of the concords were $2A$ and $A$, the successions of these first and following orders will be had:

I. $2A(1):A(1),\ 2A(1):A(2)$.

II. $2A(1):A(4),\ 2A(2):A(1)$.

III. $2A(1):A(6), 2A(1):A(3),\ 2A(3):A(1),\ 2A(3):A(2),\ 2A(1):A(8),\ 2A(4):\ A(1)$.

IV. $2A(1):A(12),\ 2A(2):A(3),\ 2A(3):A(4),\ 2A(1):A(16),\ 2A(8):A(1)$.

V. $2A(1):A(12),\ 2A(1):A(5),\ 2A(5):A(1),\ 2A(5):A(2),\ 2A(1):\ A(18),$
$2A(1):A(9),\ \ 2A(9):A(1),\ \ 2A(9):A(2),\ 2A(1):A(24),\ 2A(3):A(8),$
$2A(4):A(3),\ 2A(1):A(32),\ 2A(16):A(1)$.

If the exponents of the concords were these $A$ and $3A$, the successions of the following orders will be :

I. $3A(1):A(1),\ 3A(1):A(3).$

II. $3A(1):A(6),\ 3A(1):A(2),\ 3A(2):A(1),\quad 3A(2):A(3).$

III. $3A(1):A(9),\ 3A(3):A(1),\ 3A(1):A(12),\ 3A(1):A(4),\ 3A(4):A(1),\ 3A(4):A(3).$

IV. $3A(1):A(18),\ 3A(3):A(2),\ 3A(2):A(9),\ 3A(1):A(24),\ 3A(1):A(8),$
$3A(8):A(1),\quad 3A(8):A(3).$

If the exponents were $A$ and $4A$, the successions will be :

I. $4A(1):A(1),\ 4A(1):\ A(2),\ 4A(1):A(4).$

II. $4A(1):A(8),\ 4A(2):A(1).$

III. $4A(1):A(12),\ 4A(1):A(6),$
$4A(1):A(3),\ 4A(3):\ A(1),\ 4A(3):A(2),\quad 4A(3):A(4),\ 4A(1):\ A(16),\ 4A(4):A(1).$

IV. $4A(1):A(24), 4A(2):A(3),\ 4A(3):A(8),\ 4A(6):A(1),\quad 4A(1):A(32),\ 4A(8):A(1).$

If the exponents were $A$ and $6A$, the successions will be :

I. $6A(1):A(1),\ 6A(1):A(2),\ 6A(1):A(3),\ 6A(1):A(6).$

II. $6A(1):A(12),\ 6A(1):A(4),\ 6A(2):A(1),\ 6A(2):A(3).$

III. $6A(1):A(18),\ 6A(1):A(9),\ 6A(3):A(1),\ 6A(3):A(2),$
$6A(1):A(24),\ 6A(1):A(8),\ 6A(4):A(1),\ 6A(4):A(3).$

If the exponents were $2A$ and $3A$, the successions will be :

I. $3A.(1):2A(1),\ 3A(2):2A(1),\ 3A(1):2A(3),\ 3A(2):2A(3).$

II. $3A(1):2A(2),\quad 3A(1):2A(6),\ 3A(4):2A(1),\ 3A(4):2A(3).$

III. $3A(1):2A(9),\ 3A(3):2A(1),\quad 3A(6):2A(1),\ 3A(2):2A(9),\ 2A(1):2A(12),\ 3A(1):2A(4),$
$3A(8):2A(1),\ 3A(8):2A(3).$

If the exponents were $A$ and $8A$, the successions will be :

I. $8A(1):A(1),\ 8A(1):A(2),\ 8A(1):A(4),\ 8A(1):A(8).$

II. $8A(1):A(16),\ 8A(2):A(1).$

III. $8A(1):\ A(24),\ 8A(1):A(12),\ 8A(1):A(6),\ 8A(1):A(3),\ 8A(3):A(1),$
$8A(3):A(2),\ 8A(3):A(4),\ 8A(3):A(8),\ 8A(1):A(32),\ 8A(4):A(1).$

If the exponents were $A$ and $5A$, the successions will be :

I. $5A(1)$: $A(1)$, $5A(1)$: $A(5)$.
II. $5A(1)$: $A(l)$, $5A(1)$: $A(2)$, $5A(2)$: $A(1)$, $5A(2)$: $A(5)$.

If the exponents were $A$ and $9A$, the successions will be :

I. $9A(1):A(1),\ 9A(1):A(3),\ 9A(1):A(9).$

II. $9A(1):A(18),\ 9A(1):A(6),\ 9A(1):A(2),\ 9A(2):A(1),\ 9A(2):A(3),\ 9A(2):A(9).$

If the exponents were $A$ and $12A$, the successions will be :

I. $12A(1):A(1),\quad 12A(1):A(2),\ 12A(1):\ A(3),\ 12A(1):A(4),\ 12A(1):A(6),\ 12A(1):A(12).$

II. $12A(1):A(24),\ 12A(1):A(8),\ 12A(2):A(1),\ 12A(2):A(3).$

If the exponents were $3A$ and $4A$, the successions will be :

I. $4A(1):3A(1),\ 4A(1):3A(2),\ 4A(1):3A(4),\ 4A(3):3A(1),\ 4A(3):3A(2),\ 4A(3):3A(4).$

II. $4A(1):3A(8),\ 4A(2):3A(1),\ 4A(3):3A(8),\ 4A(6):3A(1).$

If the exponents were $A$ and $16A$, the successions will be :

I. $16A(1):A(1),\ 16A(1):A(2),\ 16A(1):A(4),\ 16A(1):A(8),\ 16A(1):A(16).$

II. $16A(1):A(32),\ 16A(2):A(1).$

30. Therefore from these it is understood well enough, how from the succession of two given concords both the exponent of the succession as well as the order may be able to be defined; with which matters known it will be easy to judge, the degree of the agreement which the ear will find in this succession. As well as for any proposed concord others also will be assigned of any kind, which may constitute a succession of a given order following the first, either of the first, second, third, etc.; and that will be able to be put in place in a number of outstanding ways, just as both from the precepts treated as well as may be apparent added together from the table.

31. Also it is understood to be able to produce a successions of two concords in several general ways, of which the exponent of the succession shall be the same. So that which may be made clearer, the exponent of the succession may be given, which shall be *E;* each two multiples of this may be taken $M$ and $N$, of which the least common multiple shall be *E*. These multiples again may be resolved into two factors, thus so that there shall be $M = Aa$ and $N = Bb$, of which $a$ and $b$ shall be prime between themselves. From these found this succession of concords may be put in place $A(a):B(b)$ and $E$ will be the exponent of this succession.

# CAPUT V

## DE CONSONANTIARUM SUCCESSIONE

1. Quemadmodum sonos plures comparatos esse oporteat, ut simul sonantes auditus sensum grata harmonia afficiant, in capite praecedente satis superque docuimus. Hoc igitur capite ordo requirit, ut investigemus, cuiusmodi esse debeant duo soni vel duae consonantiae, quae se invicem sequentes atque successive sonantes suaves sint perceptu. Non enim ad suavitatem successionis sufficit, ut utraque consonantia seorsim sit grata, sed praeterea quandam affectionem mutuam habere debent, quo etiam ipsa successio aures permulceat sensuique auditus placeat.

2. Per generales autem regulas capite II traditas, quibus omnis suavitas efficitur, constat duarum consonantiarum successionem placere, si ordo, quemtenent utriusque partes simplices seu soni singuli inter se, percipiatur. Ad cognoscendum igitur, quam facile duarum consonantiarum successio animo comprehendatur, singulos sonos utriusque consonantiae debitis numeris exprimi oportet horumque numerorum minimum communem dividuum investigari. Qui in tabula graduum suavitatis quaesitus ostendet, quantum perspicacitatis requiratur ad successionem propositam percipiendam.

3. Ambae igitur consonantiae successionis tanquam simul sonantes considerari debebunt huiusque consonantiae compositae exponens declarabit, quam suavis et perceptu facilis sit ipsa consonantiarum successio. Exponens enim istius consonantiae compositae est minimus communis dividuus omnium sonorum, qui in utraque consonantia continentur. Ex hoc autem minimo communi dividuo de successionis consonantiarum suavitate est iudicandum. Hanc ob rem iste numerus nobis erit successionis exponens, ita ut exponens successionis duarum consonantiarum sit minimus communis dividuus omnium sonorum in utraque consonantia contentorum.

4. Ex hoc principio intelligitur, qui soni simul sonantes placeant, eosdem etiam successive editos placere debere. In ipso autem gradu suavitatis, quo duae consonantiae vel simul vel successive sonantes percipiuntur, aliquid interest. Duae enim consonantiae, quae sese insequentes auditui admodum sunt gratae, aliquanto durius aures afficient simul editae. Sic duo soni rationem 8:9 tenantes simul pulsi minus placide accipiuntur, iidem tamen successive sonantes cum multo maiore voluptate audiuntur.

5. Quemadmodum enim simplicissima consonantia trisona magis est composita quam simplicissima bisona, ita, ex quo pluribus sonis constet consonantia, magis etiam erit composita, etiamsi sit simplicissima in suo genere. Hoc tamen non obstante suavitas non solum eadem, sed etiam maior percipitur ex consonantiis multisonis quam ex sono simplici vel consonantiis duobus tantum sonis constantibus. Plura enim inesse possunt in pluribus sonis, quae ordinem contineant, quaeque percepta suavitatem augent. Neque

tamen ideo nimis multiplicare licet sonos consonantiarum, ne tot variae multiplicesque perceptiones simul ad auditum pervenientes sensum potius confundant quam delectent.

6. Sed in successionibus duarum consonantiarum ipsa vel natura requirit, ut exponentes sint magis compositi quam singularum consonantiarum. Et hanc ob rem suavitati non obest consonantias sese sequentes collocare, quae simul sonantes minus placerent. Sicut enim in multisonis consonantiis exponens magis compositus suavitatem non minuit, id quod tamen eveniret, si consonantia ex paucioribus sonis constaret, ita successionum exponentes magis licet esse compositos quam exponentes consonantiarum sine ullo suavitatis detrimento.

7. Interim tamen negari non potest, quo simplicior fuerit successionis duarum consonantiarum exponens, eo facilius etiam ipsam successionem et ordinem, qui in ea inest, percipi. Regulae enim, quas supra de perceptionis facilitate tradidimus, latissime patent neque obnoxiae sunt ulli exceptioni. Sed si nimis simplices successiones adhibere voluerimus, varietas, qua maxime gaudet musica, penitus tolleretur. Multo enim magis simplices esse oporteret consonantias omnesque fere inter se similes. Ex quo intelligitur etiam magis compositos exponentes successionum adhiberi licere eosque eiusmodi, qui, si simplices consonantias designarent, omnem harmoniam turbarent.

8. Quo duae consonantiae successive sonantes cum suavitate percipiantur, oportet, ut primo utraque consonantia per se placeat et deinde etiam ipsa successio auditui sit grata. Illud declarant exponentes consonantiarum, ut in praecedente capite est ostensum. Hoc vero intelligi potest ex successionis exponente. Iudicium vero ita est instituendum, ut plures suavitatis gradus successioni tribuantur quam ipsis consonantiis, quia eius exponens magis quam harum potest esse compositus.

9. Ad exponentem successionis duarum consonantiarum definiendum non sufficit utramque consonantiam in se considerasse, sed necesse est, ut etiam relatio sonorum, qui in his consonantiis per eosdem numeros exprimuntur, spectetur. Eadem enim consonantia infinitis modis potest exhiberi, prout soni eam constituantes vel acutiores vel graviores accipiuntur, dummodo inter se praescriptam teneant rationem. At in successione duarum consonantiarum praeter ipsas consonantias attendi debet ad tenoris gradum, quo utraque exprimitur. Hoc commodissime fiet comparandis basibus, quae utrique consonantiae respondent; hae enim si ad diversos sonos referantur, successionis exponens non erit minimus communis dividuus exponentium consonantiarum, sed ratio basium quoque in computum est ducenda.

10. Si igitur datus sonus tanquam basis accipiatur, non solum soni 1 et 2 diapason constituent, sed etiam 2 et 4 vel 3 et 6 vel generaliter $a$ et $2a$ eandem consonantiam, cuius exponens est 2, exhibebunt. Huius quidem consonantiae, si in se spectetur, natura ex exponente 2 recte cognoscitur et multiplicator $a$ negligitur; verum si cum aliis consonantiis coniungatur, huius numeri $a$ est ratio habenda. Sequatur enim hanc

consonantia sonorum *2b* et 3b, quae est diapente et exponentem habet 6, atque ex solis exponentibus 2 et 6 successionis exponens non potest deduci, sed praeterea rationem numerorum *a* et *b* nosse oportebit, cum successionis exponens sit minimus communis dividuus numerorum *a,* 2 *a,* 2*b* et 3*b*.

11. Quemadmodum enim cuiusvis simplicis soni exponens est 1, in comparatione vero plurium huiusmodi sonorum numeri eorum relationem exprimentes considerari debent, ita etiam in comparatione plurium consonantiarum praeter earum exponentes etiam ipsarum relatio est inspicienda. Hanc ob rem cum consonantiae in se spectatae basis unitate exprimatur, in comparatione plurium consonantiarum cuiusque basi is tribuendus est numerus, qui illius sono ratione omnium sonorum competit. Ex quo perspicitur in comparatione plurium consonantiarum quamlibet duplici numero exprimi debere, primo nempe exponente suo et deinde indice, quo basis respectu reliquarum basium exponitur.

12. Indicem consonantiae exponenti semper adiungemus, sed uncinulis inclusum, ut ab exponente distingui queat: sicut 6(2), ubi 6 est exponens consonantiae, quae ergo ex sonis hanc relationem 1 : 2 : 3 : 6 habentibus constat; index vero 2 ad aliam consonantiam puta sequentem est referendus et ostendit basin huius consonantiae, quae in se spectata est 1, ista relatione esse debere 2. Quamobrem soni huius consonantiae ratione ad sequentem habita exponi debent numeris 2 : 4 : 6 : 12.

13. Quemadmodum eadem consonantia infinitis numeris exprimi potest, modo ii eandem inter se rationem teneant, et consonantiarum 2 : 3, 4 : 6, 6 : 9 etc. idem est exponens, etiamsi ipsi soni sint diversi, sic index consonantiae determinat, quibus ex his infinitis numeris consonantia proposita sit exponenda; id quod ad comparationem plurium consonantiarum instituendam requiritur. Apparet autem numeros, qui ex exponente resultant, singulos per indicem esse multiplicandos; hoc enim modo basis consonantiae fit indici aequalis et omnes soni eandem relationem inter se retinent.

14. Ex his etiam apparet, quomodo consonantiae ex sonis per datas numeros expressis constantis tam exponens quam index inveniri queat. Exponens enim invenitur, dum omnes numeri per maximum communem multipleem dividuntur et quotorum minimus communis dividuus quaeritur. Index vero erit ille ipse maximus communis multiple, per quem propositi numeri dividi possunt. Sic consonantiae 3 : 6 : 9 : 15 index erit 3 et exponens 30 seu minimus dividuus numerorum 1 : 2 : 3 : 5. Hanc igitur consonantiam hoc modo exprimemus 30 (3).

15. Sit consonantiae cuiusque exponens *A* et index $\alpha$, ipsius *A* vero multiplees $1,\ \alpha,\ \beta,\ \gamma,\ \delta$ etc.; habebunt soni huius consonantiae hanc rationem $1:\alpha:\beta:\gamma:\delta:$ etc., quorum numerorum minimus communis dividuus est *A.* Sed adiecto indice *a* soni consonantiae *A(a)* sequentibus numeris exprimi debebunt

$$a:\alpha a:\beta a:\gamma a:\delta a:\text{etc.},$$

quorum numerorum minimus communis dividuus erit *Aa* ob maximum communem multipleem *a.* In suavitate vero ipsius consonantiae aestimanda numerus *a* negligitur et suavitas ex solo exponente *A* aestimatur.

16. Sequatur autem consonantiam *A(a)* haec *B(b),* cuius exponentis *B* multiplees sint $1,\ \eta,\ \theta,\ i,\ \chi$ etc., numeri autem sonos exprimentes hi

$$b:\eta b:\theta b:ib:\chi b:\text{etc.}$$

Cum igitur successionis suavitas reducta sit ad consonantiae ex utraque compositae suavitatem, successionis exponens erit minimus communis dividuus numerorum

$$a:\alpha a:\beta a:\gamma a:\delta a:b:\eta b:\theta b:ib:\chi b;$$

hi enim soni haberentur, si ambae consonantiae simul audirentur. Quia vero numerorum $a:\alpha a:\beta a:\gamma a:\delta a$ minimus communis dividuus est *Aa,* reliquorum vero $b:\eta b:\theta b:ib:\chi b$ hic *Bb,* erit successionis exponens minimus communis dividuus numerorum *Aa* et *Bb.*

17. Cum autem consonantiae suavitas ex minimo communi dividuo numerorum sonos exprimentium perperam iudicetur, si illi numeri non fuerint primi, sed multipleem communem habuerint, idem quoque in successione duarum consonantiarum est tenendum. Quare si numeri

$$a:\alpha a:\beta a:\gamma a:\delta a:b:\eta b:\theta b:ib:\chi b;$$

habeant communem multipleem, per eum singuli ante omnia debent dividi et quoti eorum loco substitui. Hoc vero evenire non potest, nisi indices *a* et *b* fuerint numeri inter se compositi. Hanc ob rem, quoties indices duarum consonantiarum communem multipleem habent, per hunc ante indices dividi oportet, quam exponens successionis quaeratur.

18. Sint igitur consonantiarum *A(a)* et *B(b)* indices *a* et *b* numeri inter se primi; erit successionis harum consonantiarum exponens minimus communis dividuus numerorum *Aa* et *Bb.* Ad hunc inveniendum necesse est, ut ante quaeratur maximus communis multiple, qui sit *D.* Quo cognito alteruter numerus per *D* dividatur quotusque per alterum numerum multiplicetur; eritque factum *ABab: D* minimus communis dividuus numerorum *Aa* et *Bb* atque simul exponens successionis consonantiarum propositarum, ex quo suavitas successionis innotescet.

19. Quia *a* et *b* ponuntur numeri inter se primi, ipsi numeri *Aa* et *Bb* communem multipleem habebunt, si vel *A* et *B* vel *A* et *b* vel *B* et *a* fuerint numeri compositi. At quo

plures inveniantur huiusmodi multiplees, eo maior erit maximus communis multiple numerorum *Aa* et *Bb.* Sed quo magis erit compositus maximus iste communis multiple, eo minor erit minimus communis dividuus et propterea eo suavior consonantiarum successio. Cum enim exponens successionis sit *ABab:D*, quo maior erit maximus communis multiple *D*, eo simplicior erit quotus *ABab:D* ad simplicioremque suavitatis gradum pertinebit.

20. Sit *A* numerus ad suavitatis gradum *p* pertinens, *B* ad gradum *q*, *a* ad gradum *r* et *b* ad gradum *s;* maximus vero communis multiple *D* sit gradus *t.* His positis numerus *ABab:D* ad gradum

$$p+q+r+s-t-2$$

referetur, quemadmodum ex supra traditis colligi licet. Datis ergo numeris *A*, *B*, *a*, *b* et *D* innotescet gradus suavitatis, ad quem successio consonantiarum *A(a)* et *B(b)* pertinebit, scilicet gradus $p+q+r+s-t-2$. Qui numerus quo minor erit, eo suavior successio esse debebit.

21. Exempli causa consonantiam 120 (2) constantem ex sonis

$$2 : 4 : 6 : 8 : 10 : 12 : 16$$

sequatur consonantia 60 (3) constans ex sonis

$$3 : 6 : 9 : 12 : 15,$$

quarum illa est gradus decimi, haec gradus noni. Successio ergo ex minimo communi dividuo numerorum 240 et 180 iudicari debet, quorum maximus communis multiple est 60 ad gradum nonum pertinens. Cum igitur sit $A=120,\ a=2,\ B=\ 60,\ b=3$ et $D=60$, erit $p=10,\ q=9,\ r=2,\ s=3$ et $t=9$ ideoque

$$p+q+r+s-t-2=13.$$

Quare successionis exponens est gradus 13, cuius gradus est suavitas successionis.

22. Si dentur utriusque consonantiae exponentes, indices ita determinari poterunt, ut successio quam suavissima evadat. Sit exponentium *A* et *B* minimus communis dividuus *M;* manifestum est exponentem successionis *ABab*:*D* vel aequalem esse ipsi *M* vel eo maiorem; minor enim esse non potest. Suavissima ergo erit successio, si *ABab*:*D* aequalis fuerit ipsi *M,* minorem vero suavitatis gradum successio habebit, si *ABab*:*D* aequalis fuerit vel *2M* vel *3M* vel 4*M* etc. Quare posito $ABab=nDM$ indices *a* et *b* eo suaviorem reddent successionem, quo minor erit numerus *n.*

23. Successionem *ordinis primi* vocabimus, si minimus communis dividuus numerorum *Aa* et *Bb* fuerit aequalis ipsi *M* seu minimo communi dividuo numerorum *A* et *B.* Successionem *ordinis secundi* vero vocabimus, cuius exponens est 2*M*. Porro successio *ordinis tertii* nobis erit, cuius exponens est vel *3M* vel *4M,* quia numeri 3 et 4 ad gradum tertium suavitatis pertinent. Atque generaliter ea successio, cuius exponens est *nM,* eiusdem erit ordinis, cuius gradus suavitatis est numerus *n.* Hic vero cavendum est, ne ordines successionum cum gradibus suavitatis confundantur; successionem enim *ordinis primi* vocamus, qua simplicior manentibus iisdem consonantiarum exponentibus dari nequit, etiamsi ipsa successio ad multo ulteriorem suavitatis gradum referatur.

24. Perspicuum est igitur consonantiarum *A* et *B* successionem fore ordinis primi, si *a* et *b* sint unitates; numerorum enim *A*1 et *B*1 minimus communis dividuus est *M.* Fieri tamen praeterea potest, ut successio consonantiarum $A(a)$ et $B(b)$ sit ordinis primi, etiamsi $a$ non sit $=b$. Evenit hoc, si *b* in *Bb* vel aequalem vel minorem habeat dimensionum numerum quam in *A* atque simul *a* in *Aa* aequalem vel minorem dimensionum numerum quam in *B.* Hoc enim si fuerit, erit *M* quoque minimus communis dividuus numerorum *Aa* et *Bb.*

25. Sit exponentium *A* et *B* maximus communis multiple *d* atque $A=dE$ et $B=dF$ *;* erunt *E* et *F* numeri inter se primi. Sit praeterea *e* multiple ipsius *E* et *f* multiple ipsius *F;* erit consonantiarum *dE*(*f*) et *dF*(*e*) successio ordinis primi. Nam numerorum *dEf* et *dFe* minimus communis dividuus est *dEF*, idem qui ipsorum numerorum *A* et *B* seu *dE* et *dF.* Ut si sit $A=15$ et $B=18,$ est $d=3,$ $E=5$ et $F=6$. Quare poterit esse *e* vel 1 vel 5 et *f* vel 1 vel 2 vel 3 vel 6. Successio ergo erit ordinis primi, si *A*(*a*) est vel 15(1), 15(2), 15(3) vel 15(6), sequens vero consonantia $B(b)$ vel 18 (1) vel 18 (5).

26. Ex his porro facile apparet, quales indices assumi oporteat, ut successionis exponens fiat *2M* seu *2dEF*, quo casu successio est ordinis secundi. Similique modo effici poterit determinandis indicibus, ut exponens successionis fiat *ndEF* seu ipsa successio dati ordinis, id quod pluribus modis fieri poterit, quos enumerare difficile et supervacaneum esset. Si exponentes consonantiarum sunt 15 et 18, successio est ordinis secundi, si prior consonantia fuerit vel 15(1) vel 15(3) et altera vel 18(2) vel 18(10), item si prior fuerit vel 15(4) vel 15(12) existente altera vel 18 (1) vel 18 (5).

27. Si exponentes consonantiarum sint aequales seu $B=A$, unica successio habebitur ordinis primi, si est $ab=1$, quae ergo erit *A*(1) et *A*(1). Ordinis secundi vero erunt duae successiones *A* (1): *A* (2) et *A* (2): *A* (1), quarum exponens est 2*A.* Ordinis tertii quatuor erunt successiones, nempe $A(1):A.(3)$ et $A(1):A(4)$ harumque inversae. Ordinis quarti sex erunt successiones, scilicet $A(1):A(6),$ $A(2):A(3),$ $A(1):A(8)$ atque harum tres inversae. Atque huiusmodi successio quaelibet eius erit ordinis, cuius gradus suavitatis est factum indicum.

28. Si exponens alterius consonantiae fuerit duplum alterius exponentis seu $B = 2A$, ordinis primi erunt duae successiones hae: $A(1):2A(1)$ et $2A(1):A(2)$; horum enim exponens est $2A$, idem qui ipsorum exponentium $A$ et $2A$. Successionum ordinis secundi exponens est $4A$; tales ergo successiones erunt $A(1):2A(2),\ A(4):2A(1)$ harumque inversae. Simili modo successiones cuiusque ordinis reperientur, si fuerit $B = 3A$, et generaliter, si $B = nA$; ex quibus successiones simpliciores, quae usum habere possunt, facile reperiri poterunt.

29. Si ergo exponentes consonantiarum inter se fuerint aequales, successiones ordinis primi, secundi, tertii usque ad sextum ordinem erunt sequentes, denotantibus numeris Romanis ordines successionum et $A$, $A$ exponentes utriusque consonantiae:

I. $A(1):A(1)$.
II. $A(2):A(1)$.
III. $A(3):A(1),\ A(4):A(1)$.
IV. $A(6):A(1),\ A(3):A(2),\ A(8):A(1)$.
V. $A(5):A(1),\ A(9):A(1),\ A(12):A(1),\ A(4):A(3),\ A(16):A(1)$.
VI. $A(10):A(1),\ A(5):A(2),\ A(18):A(1),\ A(9):A(2),\ A(24):A(1),\ A(8):A(3),\ A(32):A(1)$.

Si vero exponentes consonantiarum fuerint $2A$ et $A$, habebuntur successiones ordinis primi et sequentium istae:

I. $2A(1):A(1),\ 2A(1):A(2)$.
II. $2A(1):A(4),\ 2A(2):A(1)$.
III. $2A(1):A(6), 2A(1):A(3),\ 2A(3):A(1),\ 2A(3):A(2),\ 2A(1):A(8),\ 2A(4):\ A(1)$.
IV. $2A(1):A(12),\ 2A(2):A(3),\ 2A(3):A(4),\ 2A(1):A(16),\ 2A(8):A(1)$.
V. $2A(1):A(12),\ 2A(1):A(5),\ 2A(5):A(1),\ 2A(5):A(2),\ 2A(1):\ A(18)$,
$2A(1):A(9),\ 2A(9):A(1),\ 2A(9):A(2),\ 2A(1):A(24),\ 2A(3):A(8)$,
$2A(4):A(3),\ 2A(1):A(32),\ 2A(16):A(1)$.

Si consonantiarum sese insequentium exponentes fuerint $A$ et $3A$, erunt successiones secundum ordines sequentes:

I. $3A(1):A(1),\ 3A(1):A(3).$

II. $3A(1):A(6),\ 3A(1):A(2),\ 3A(2):A(1),\quad 3A(2):A(3).$

III. $3A(1):A(9),\ 3A(3):A(1),\ 3A(1):A(12),\ 3A(1):A(4),\ 3A(4):A(1),\ 3A(4):A(3).$

IV. $3A(1):A(18),\ 3A(3):A(2),\ 3A(2):A(9),\ 3A(1):A(24),\ 3A(1):A(8),$
$3A(8):A(1),\quad 3A(8):A(3).$

Si exponentes fuerint $A$ et $4A$, erunt successiones:

I. $4A(1):A(1),\ 4A(1):A(2),\ 4A(1):A(4).$

II. $4A(1):A(8),\ 4A(2):A(1).$

III. $4A(1):A(12),\ 4A(1):A(6),\ 4A(1):A(3),\ 4A(3):A(1),\ 4A(3):A(2),\ 4A(3):A(4),\ 4A(1):A(16),\ 4A(4):$

IV. $4A(1):A(24), 4A(2):A(3),\ 4A(3):A(8),\ 4A(6):A(1),\ 4A(1):A(32),\ 4A(8):A(1).$

Si exponentes fuerint $A$ et $6A$, erunt successiones:

I. $6A(1):A(1),\ 6A(1):A(2),\ 6A(1):A(3),\ 6A(1):A(6).$

II. $6A(1):A(12),\ 6A(1):A(4),\ 6A(2):A(1),\ 6A(2):A(3).$

III. $6A(1):A(18),\ 6A(1):A(9),\ 6A(3):A(1),\ 6A(3):A(2),\ 6A(1):A(24),\ 6A(1):A(8),\ 6A(4):A(1),\ 6A(4):A($

Si exponentes fuerint $2A$ et $3A$, erunt successiones:

I. $3A.(1):2A(1),\ 3A(2):2A(1),\ 3A(1):2A(3),\ 3A(2):2A(3).$

II. $3A(1):2A(2),\ 3A(1):2A(6),\ 3A(4):2A(1),\ 3A(4):2A(3).$

III. $3A(1):2A(9),\ 3A(3):2A(1),\ 3A(6):2A(1),\ 3A(2):2A(9),\ 2A(1):2A(12),\ 3A(1):2A(4),\ 3A(8):2A(1),\ 3$

Si exponentes fuerint $A$ et $8A$, erunt successiones:

I. $8A(1):A(1),\ 8A(1):A(2),\ 8A(1):A(4),\ 8A(1):A(8).$

Il. $8A(1):A(16),\ 8A(2):A(1).$

III. $8A(1):A(24),\ 8A.(1):A(12),\ 8A(1):A(6),\ 8A(1):A(3),\ 8A(3):A(1),$
$8A(3):A(2),\ 8A(3):A(4),\ 8A(3):A(8),\ 8A(1):A(32),\ 8A(4):A(1).$

Si exponentes fuerint $A$ et $5A$, erunt successiones:

I. $5A(1)$: $A(1)$, $5A(1)$: $A(5)$.
II. $5A(1)$: $A(l)$, $5A(1)$: $A(2)$, $5A(2)$: $A(1)$, $5A(2)$: $A(5)$.

Si exponentes fuerint $A$ et $9A$, erunt successiones:

I. $9A(1):A(1),\ 9A(1):A(3),\ 9A(1):A(9).$
II. $9A(1):A(18),\ 9A(1):A(6),\ 9A(1):A(2),\ 9A(2):A(1),\ 9A(2):A(3),\ 9A(2):A(9).$

Si exponentes fuerint $A$ et $12A$, erunt successiones:

I. $12A(1):A(1),\ 12A(1):A(2),\ 12A(1):A(3),\ 12A(1):A(4),\ 12A(1):A(6),\ 12A(1):A(12).$
II. $12A(1):A(24),\ 12A(1):A(8),\ 12A(2):A(1),\ 12A(2):A(3).$

Si exponentes fuerint $3A$ et $4A$, erunt successiones:

I. $4A(1):3A(1),\ 4A(1):3A(2),\ 4A(1):3A(4),\ 4A(3):3A(1),\ 4A(3):3A(2),\ 4A(3):3A(4).$
II. $4A(1):3A(8),\ 4A(2):3A(1),\ 4A(3):3A(8),\ 4A(6):3A(1).$

Si exponentes fuerint $A$ et $16A$, erunt successiones:

I. $16A(1):A.(1),\ 16A(1):A(2),\ 16A(1):A(4),\ 16A(1):A(8),\ 16A(1):A(16).$
II. $16A(1):A(32),\ 16A(2):A(1).$

30. Ex his igitur satis intelligitur, quemadmodum data duarum consonantiarum successione tum exponens successionis tum etiam ordo possit definiri; ex quibus rebus cognitis facile erit iudicare, quo suavitatis gradu proposita consonantiarum successio auditui accepta sit futura. Praeterea proposita quacunque consonantia alia datae quoque speciei assignari poterit, quae illam sequens constituat successionem dati ordinis vel primi vel secundi vel tertii etc.; idque plerumque pluribus modis praestari poterit, quemadmodum cum ex traditis praeceptis tum ex tabula adiecta fuse apparet.

31. Intelligitur etiam ex dictis plurimis plerumque modis successiones duarum consonantiarum produci posse, quarum idem sit exponens successionis. Quod ut clarius percipiatur, datus sit exponens successionis, qui sit *E;* huius sumantur duo quique multiplees *M* et *N*, quorum minimus communis dividuus sit *E*. Hi multiplees porro in duo factores resolvantur, ita ut sit $M = Aa$ et $N = Bb$, quorum *a* et *b* sint inter se numeri primi. His inventis constituatur ista consonantiarum successio $A(a):B(b)$ eritque huius successionis exponens *E*.