CHAPTER I

SOUND AND HEARING

1. Since the music proposed by us shall be treated in the manner of a philosphical discipline, in which nothing is allowed to be advanced, unless the precepts and truth of which may be able to be demonstrated from what has preceded, thus foremost, these principles are to be put in place regarding sound and hearing, the former of which concerns the principles on which the essence of music is based, but the goal and end-point of the latter is to embrace the pleasure of listening. For music teaches how to produce and skillfully combine sounds, in order that a pleasing harmony may affect the sense of hearing pleasantly. And thus what is required to be put in place in our investigation of sound, is the nature, production, and the varieties of these; concerning which, it is from physics and mathematics that we shall be able to acquire a sufficient understanding. If truly these particular organs of hearing may be considered, then we will understand the reasons for the perception of sounds. But just how useful these are going to be for the basic fundamentals and confirmation of music, thus it will be evident for each, just how much the charm of sounds may depend on an account of these perceptions, and by which music ought to be explained.

2. All those who have at least perhaps written about this, decide that sound to be present in air, and that as it were to be the vehicle by which it may be carried around from the source. Truly neither can this matter be had otherwise, since nothing except air is present, which surround our ears and may be able to produce the change in these. For whatever it may be objected, the account of hearing perhaps to be compared in the same way as smell and sight, which senses not the air, but are excited by the emissions flowing from the object, yet with the aid of pneumatic pumps, if the source of the sounds may be set up in a place with the air evacuated, thus so that hence it may have no communication with the air, clearly no sound can be perceived, however close you may approach. And immediately the inflow of air is permitted, sound is heard again. From which it follows that the air and its change, that the source of the sound acting produces in that, shall be the true and nearest source of the sound.

3. As truly it may be agreed, that it shall be this change and modification of the air exciting the sense of sound, it will be convenient that the particular cause be examined, by which sound is produced, and the effect on the air arising from that to be examined. On this account, we may care to examine the stretched string, which produces a sound on being struck. But the stroke applied has no other effect apart from the trembling motion in the string, by which between its limits that now travels most quickly on one side then on the other, beyond its rest position. Indeed in thicker strings this motion also is easily observed by the eye, in the more slender even if it may not be discerned, yet there is no doubt that it is present. Besides anyone who touches a ringing bell by hand feels the whole vibration. Truly finally soon it may be shown from the laws of mechanics both the string as well as the bell can be given a trembling motion from nothing other than by a

sharp knock ; and on this account the reckoning about sound will have to be sought in terms of vibrationary motion only.

4. Therefore from the change in the air, which a trembling body produces in that, the sense of sound may be at once aroused and put into effect, it is required to examine how the air may be affected by a vibrating body. Moreover we see the trembling motion to consist in the repetition of a succession of vibrations. From these individual vibrations the air surrounding the trembling body is struck and receives similar vibrations, which in a similar manner are transferred to more distant air particles. Therefore on this account pulses and vibrations are excited in the air all around, and the transportation of any tremblings of the body is carried through the air by that same vibration of the pulses. From which it may be understood the individual particles of the air must be shaken by a similar vibratory motion, as the body itself; yet with this distinction, that the pulsus therefore must become smaller and weaker, as they become more distant from the source, while finally at an exceedingly great distance nothing further may be able to be heard.

5. From these it is understood that besides the pulses carried through the air, the sounding body conveys nothing to the ears ; on account of which it is necessary, that these same pulses in the excited air and in passing through the organ of hearing produce the sense of sound. Truly the sensation is resolved in this manner : Inside the ear cavity an extended membrane is present, called the eardrum from its similarity, which receives the blow of the air and these are advanced further to the auditory nerves ; and in this way it happens, so that while the nerves are affected, a sound may be heard. Therefore sound is nothing other than the perception of successive blows, which eventuate in the particles of the air moving around the organ of hearing, thus so that, whatever source may prevail of this kind to produce the blow in the air, that also shall be applied to the sound to be sent off.

6. The progagation of sound through air is not completed in an instant, but needs to be determined by the time, in which it may be driven forwards through a given distance. But the motion, with which it is progressing, is always the same, and does not depend either on the strength of the sound or on its quality. Truly of all sound, as is evident both from experiments as well as may be gathered from the theoretical computation of the air and from the nature of the pulses, it is progressing in a time of one second through a distance of 1100 Rhenish feet, and in two seconds it travels through 2200 feet, in three seconds 3300, and thus so forth. Also we may observe this tardiness of sounds from day to day ; indeed for the far off distance of a cannon, when it explodes, we hear the sound some little time after the flash, since yet with the cannon standing nearby we may experience each at the same time. For the same reason with thunder where we hear the rumble after having seen the flash, and the repetition of the voice in several places, called echos, which return more slowly after the shout itself .

7. Therefore in whatever manner it may prevail thus to agitate the smallest particles of the air, so that they may receive a trembling motion of this kind, that will produce sound

also. Truly in order that this may be effected not only hard bodies are suitable, but besides these to other ways of producing sound are found ; from which also three kinds of sound arise, if we may regard the causes. The first is of these, which arise from a trembling body, of this kind are the sounds of strings and of bells. Another kind includes those, which are suddenly released from being greatly compressed air to being restored to normality, as the sounds of guns, cannons, thunder and rods vibrating quickly in air. To the third are referred the sounds of instruments, which blown into give a shrill note, as tubes, flutes, etc., the cause of which sound does not depend on the trembling motion of the material, from which the flutes are made, to be examined below.

8. From the first kind requiring to be considered are stretched strings made either from metal or from the intestines of animals, which are acted on to produce sound either by being struck or by being moved by friction. They are struck or pinched also in clavicords, harps and other instruments of this kind; truly they are rubbed in the pandora, in violins with the help of stretched horse hair, by which roughness is induced by rosin. By each way the strings receive a trembling motion; and indeed at first they are distorted from rest and from the natural position, with which done they try to restore the natural state, and actually rush towards that by an accelerated motion. But the great speed, which they have acquired, when they arrive there, they are unable to lose suddenly, nor thus to remain at rest in that state. On account of which these by necessity rush on beyond, and to be returned there in a similar manner; and these oscillations meanwhile will endure, until on account of the resistance clearly vanish.

9. But how many oscillations of this kind a string made in a given time, pulsating or vibrating in some manner, may be defined by calculation from the laws of motion, if the length of the string, its weight, and the force acting may be considered. But the length and weight of the whole string must not be taken, but only of this part, which provide the vibrations and produce the sound and which are usually separated from the whole string by two raised frets. Clearly by these it is impeded, so that less than the whole length of the string may perform vibrations, but so much of that part only, as it pleases. But so that the stretching force may be known, it is arranged especially, with the one end of the string fixed, to suspend a weight from the other, providing the position of the extending force. With these in place if the length of the resonating string shall be *a* thousandth parts of a Rhenish foot and the weight suspended may be had to the weight of the string as *n* to 1, the number of oscillations, that this string may perform in a second, may be expressed by

$$\frac{355}{113}\sqrt{\frac{3166n}{a}},$$

where 113: 355 denotes the ratio of the diameter to the periphery of a circle, 3166 thousandths of a Rhenish foot, the length of the pendulum completing one oscillation per second.

10. These oscillations, as long as they endure, are isochrones or all are completed in equal intervals of time, nor does the magnitude of these disturb this rule, unless perhaps, when the string may be struck excessively hard, from the start the vibrations are more rapid. Evidently the ratio of the strings is the same as of pendulums, of which the oscillations, if they are exceedingly small, are isochronous. So that we may illustrate the rule given in the above paragraph by an example, I have supposed a string of length 1510 thousandth parts of a Rh. ft, which will weigh $6\frac{1}{5}$ grains; I have extended this with a weight of 6 lb. or 46080 grains. With which compared with the preceding paragraph there will be

$$a = 1510$$

and

$$n = 46080 : 6\tfrac{1}{5} = 7432.$$

Whereby the number of vibrations arising in one second will be

$$\tfrac{355}{113}\sqrt{\tfrac{3166\cdot 7432}{1510}} \text{ i.e. } 392.$$

Moreover to this sound the corresponding note is understood in the instrument to be designated *a*.

11. If several stretched strings may be had, it is determined by a simple calculation, what ratio the vibrations of these have between themselves ; it is evident in any string that the number of vibrations produced in a given time to be as $\sqrt{\frac{n}{a}}$ i.e. as *the square root of the stretching weight divided by the weight of the string and by its length.* Therefore if the strings were of the same length, the number of vibrations being produced in the same time will be as *the square roots of the stretching weights divided by the weights of the strings.* If the length and weight of the strings were both equal, the number of vibrations would be as *the square roots of the stretching weights.* And if the stretching weights may be equal, and the strings themselves only differing in length, the number of vibrations will be inversely *as the square roots of the length multiplied by the weights,* i.e. *inversely as the lengths of the strings,* since the weights of the strings are proportional to the lengths.

12. The distinction between bass and treble notes depends on the tardiness and alacrity of the vibrations, and there we say these sounds to be deeper, where fewer vibrations strike the organ of hearing in the same time, and there sharper, where more vibrations of this kind may be perceived in the same time. The truth of this is agreed on from these same experiments; if indeed to the same string successively various weights may be hung on, we perceive sharper sounds emitted from these, if greater weights shall be hung on, but they will be deeper, where the weights are smaller. Moreover, it is certain from the preceding, that greater weights produce faster vibrations. On this account, since in music

especially, it may be seen to discriminate between deeper and sharper sounds, certainly we will measure these same sounds according to the number of the vibrations produced in a certain time, or we will consider the sounds as quantities, the measures of which constitute the number of vibrations produced in a determined time.

13. Truly just as we are unable to conceive either very great or very small things by our senses, thus also with sounds a certain mean value is required ; and all sounds perceived between certain limits are to be put in place, those which go beyond, on account of being either too low or too high may not affect the sense of hearing further. These limits can be determined in a certain manner ; for since the sound *a* shall produce 392 vibrations in a second, the sound designated by the letter *C* meanwhile completes 118 vibrations and the sound $\overset{\equiv}{c}$, 1888. If now we may put the sound to be heard higher and lower by two octaves still barely possible, we will have the two extreme perceptible sounds expressed by the numbers 30 and 7520 ; which interval is large enough and allows a great variety of sounds, certainly which is said to be completed in eight octaves [, as $2^8 = 256$].

14. After the distinction of the deep and sharp sounds it is required the strength and weakness of these. But the strength of this same sound is different for the place of the listener; so that indeed the further the listener stands apart from the pulsating string, there he perceives a weaker sound, since the propagation of pulses as with light always may be weakened by the air. The account of this decrease is, because at greater distances the sound may be spread out more at greater distances; clearly at twice the distance the space, where it is perceived, is four times greater than in the simple one ; on account of which since both there and here the sum of all the beats is equal in magnitude, it follows that the sound at double the distance to be four times weaker. Similarly at triple the distance to be required to be nine time weaker and thus so forth, thus so that the intensity of the sound must decrease as the square of the distances.

15. Thus these conditions are met, if the sound is sent out equally in all directios. But if there were circumstances of this kind, so that more sound may be sent out in one dirction rather than another, also to be perceived there to be stronger, as the following rule may be required. So that if someone may call loudly through a pipe, he who places his ear near the other end of the pipe, hears the sound almost as loud, as if it were the voice heard calling straight from the mouth itself. Similar is the method of the megaphone tube, through which the sound is sent rather into that region in which the tube is directed, which emerges stronger from the same reason. Indeed sounds also are reflected from hard smooth surfaces as light rays, and in this manner the direction of the sound rays, which may thus be called because of the similarity to rays of light, by which it can happen, that many may be put together at the same place.

16. Since a struck string by some oscillation may transmit vibrations through the air, it is necessary, that its motion becomes more relaxed and thus the sound weaker. Certainly this may be observed in vibrating strings; indeed initially the sound is the maximum

intensity, then truly step by step it may become more relaxed, then finally it may stop altogether; yet meanwhile the oscillations remain isochronous and the sounds retain the same depth and acuteness. This intensity from the beginning depends on the striking force on the same string, so that, where this may be greater, there too a stronger sound may be produced. Yet initially, if the blows were exceedingly strong and the string distorted greatly from its natural position, the sound is produced more sharply than later ; and since the oscillations occupy a greater space, not so regular vibrations of the air are impressed ; from which it happens, that sounds both less pleasing and less distinct may be produced.

17. This happens chiefly, if the string is exceedingly loose and not tensed enough ; for then greater amplitudes may be performed in the oscillation, and the sound presented will neither be uniform nor pleasing. On this account it is required for the production of agreeable and equal sounds, that the strings, however many there will be, may be extended by so much weight attached, provided they may not be broken. But the strength of the strings made from the same material is proportional to the thickness [*i.e.* area of cross-section], whereby the maximum breaking weights of the extended strings are as the thicknesses. But the thicknesses of the strings themselves are proportional to the weights of these divided by the lengths, [*i.e.* area of cross-section to be as mass/length] therefore the maximum stretching weights have to be in the direct proportion to the weights of the strings and inversely to the length. That is, if the weight of a string may be called q, the length a and the stretching weight p, p shall be required to be as $\frac{q}{a}$, or $\frac{ap}{q}$ must be of constant magnitude.

18. Moreover where the sounds arise equally strong, in addition it will be required to attend to the striking force as well as the length and weight of the string. Also the place where the string is plucked or struck may be required to be considered; but if we may put all the strings to be struck in the middle, or what amounts to the same, to be struck in similar places, this condition may not enter into the calculation. From this it occurs that, where the striking force shall be greater, there a stronger sound will emerge. Moreover almost all musical instruments are to be made thus, so that all the strings may be struck equally, on account of which we may always apply the same striking force. But then the strength of the sounds depends on the rapidity, with which the particles of the air impinge on the ear from some vibrating string, and this is required to be estimated from the maximum rapidity of the strings [*i.e.* frequency]. Truly this rapidity is proportional to the square root of the weight stretching the string [*i.e.* the tension] divided by its length. Consequently, so that the sounds may become equal to each other, it is necessary, that the stretching weight always shall be in the same ratio to the length of the string.

19. Therefore with the above letters remaining, a [= length], p [= tension], and q [= weight, or mass of string], $\frac{p}{a}$ must be of the same magnitude everywhere. Truly now as before, $\frac{ap}{q}$ has been found necessary to be constant, whereby with this divided by $\frac{p}{a}$,

producing the quotient $\frac{aa}{q}$ which must be constant or $\frac{q}{a}$ to a must be held in the same ratio in all the strings. But $\frac{q}{a}$ is proportional to the density of the string [= density per unit length or line density, which in turn is proportional to the actual density for a string of constant area of cross-section], and thus must be proportional to the density of the string, and similarly also to the same stretched length. But the [frequency of the] sound emitted is as $\sqrt{\frac{p}{aq}}$; in which if in place of p and q the proportionalities a and a^2 may be substituted, the sound will be inversely as the length of the string. Hence on this account the stretching weight, the length, and the weight of the string are all required to be inversely as the sound emitted, or the number of vibrations being completed in a given time. Which rule will find an excellent use in the construction of musical instruments.

20. We have said the sound to be less pleasing, if the string were not stretched enough, therefore so that the excursions made between the vibrations shall be exceedingly great and from these the air may be moved forwards rather as a wind, than the oscillations it is being encouraged to excite. Indeed unless the air may be suddenly struck with a great speed, it receives no simple vibratory motion, such as is required for sound ; but where the string is stretched more, therefore at once after the stroke it has a greater speed. This being agreed on, so that now it is to be noted, the greater vibrations are not to be isochronous with the smaller ones, nor for which likewise does the sound remains the same but shall become deeper little by little. From which it arises easily, that the whole string may not perform oscillations at the same time, but some part of this may arrive quicker to the maximum speed, other parts as well slower from to rest, from which an unequal and rough sound may emerge.

21. As well as these differences of the sounds in music the duration of the sounds also is to be considered. Indeed in many instruments the sounds cannot be prolonged for as long as it pleases, as in these, by which the strings may be excited by striking or pinching. Indeed for these the sounds become weaker little by little and soon cease completely; and on account of this matter of the duration of the sounds not only can it be effected in instruments, in which these sounds can be produced, how the sounds may retain the same force as long as they endure, and for as long as it may please. These are of this kind, the strings of which are rubbed by a plectrum, and other instruments made from flutes, which are driven by wind, such as the wind organ and many others. These same instrument have this prerogative, that all the charm which is present due to the duration of the sounds, may be expressed and produced perfectly. Moreover the duration of the sound is measured from the time elapsed between the beginning and the end.

22. Up to this point we have considered from the first kind of sounds which have an origin in a vibrating body, only the sounds of strings, and likewise we have also we have enumerated and established the primary differences of their sounds. Therefore now, before we may progress to the remaining kinds, other instruments are to be considered also, which produce sounds belonging to this kind. Bells are of this kind, which struck the

whole vibrate and produce sound. Indeed it may be most difficult to determine with the shape and weight of the bell known, what kind of sound it shall be going to give ; yet, if bells were similar and made from the same material, it is readily apparent the sounds to be held in the inverse ratio of the cubes of their weights, thus so that a bell eight time lighter produces a sound in the same time two more are performed and, if it were twenty seven times ligher, the vibrations would arise three times more frequently.

23. In addition musical instruments exist made from elastic rods made from hard wood or metal, by which the sound of bells are imitated. From these, it is certainly easier to be estabished if indeed they have a cylindrical or prismatic form ; for the sounds may be seen to depend only on the length, since any fibre shall be agreed to perform vibrations arising only in the longitudinal direction. But the sounds or vibrations at the time of production will be inversely as the square of the lengths of the rods, if indeed the rods were made from the same material. Indeed the sounds from diverse materials of the prisms depend not only on the ratio of the specific gravity, but also it is necessary to know that ratio of the cohesive and elastic forces, which themselves will have undertaken to determine the sounds themselves from theory.

24. For the second class of sounds I have been concerned with these sounds, which is sent off suddenly either by a remarkable expansion of strongly compressed air, or arising from a stronger force on the air. Of which indeed the latter way is almost similar to the first ; on account of which the rate of vibration of the air cannot go from the original situation, from where it occurred, so that a portion of the air sustaining the shock may itself be compressed, as at first it was left behind, and may expand again. But the compression of the air suddenly expanding by necessity to occupy a more natural space, and therefore it will be forced on the contrary to be contracted again, which also may be done excessively. Therefore from these alternate expansions and contractions, an image of the vibrating body will be produced in the sound it produces in the rest of the surrounding air, and thence in the ear.

25. Although in this way any oscillation in its natural state may be arrive at, yet this cannot happen in the first to be considered, as all its motion may be denied. Indeed from mechanics it is agreed a body in its place of rest with a blow arising cannot remain there, but now by moving to be required to be taken away from. Equally it is with difficulty for a moving body suddenly to be at rest and for one at rest to be suddenly moving; and just as much force is needed for the motion of a body to be removed, as the amount to be giving rise to the same for the body resting. On this account we see neither the oscillations of pendulums, when they have arrived in the vertical situation, can be brought to rest, nor vibrating strings, when they have reached the natural state. Truly the sounds set out generated in this manner can endure only for a short time, unless an echo or some similar resonance which may be present, so that these may be repeated and protracted; for the air motion by diffusing into so many different places by necessity may at once lose its own motion.

[Here Euler may be considering the situation of setting up standing waves in the cannon bore while the ball is being ejected; such patterns could not endure due to the short time intervals involved; any such standing waves would tend towards longer wavelengths as the cannon ball progressed, all else remaining the same.]

26. Therefore all the causes prevail, which now either compress the air or to be compressed naturally to be sent off , thus so that suddenly it may be possible [for the pressure] to relax, so that these also are adapted to the production of sound. On account of which all motions of faster bodies must generate sound; for the air on account of inertia is unable to concede a place freely to bodies, and thus it is compressed by these, which then again in turn induces a vibratory motion in the smallest particles of the air. Hence the origin of the strongest vibrations of rods and of all the faster motions of bodies through the air lead to the production of sound. Nor also must the blowing of the wind and of the hissing sounds be due to other causes; for the forwards part of the air is pushed forwards and compressed by the following posterior part as if by a hard body.

27. Of the sounds, which arise from the sudden expansion of strongly compressed air, the strongest by far without doubt, are those which are heard from gunpowder and thunder. Indeed by various experiments it is agreed that maximally compressed air be present in gunpowder and with heat applied to find a way out, by which it is necessary to produce such a stupendous sound. And for clouds being composed from vapours with many nitrous and sulphurous particles it seems likely, that with these united and exploded so much sound may be able to be produced. But since it shall be difficult to discern from sounds of this kind, how an account of the deep and acute sounds themselves arise in turn, all the sounds pertaining to this kind are not present in music ; on account of which we will pass over investigating, how the smallest particles of the oscillating air are induced .

28. For the third kind of sounds according to the division made of sounds into three kinds, which are generated by wind instruments. The account of which, as more is hidden, thus is required to be investigated with less diligence in a short time. For those who agree the motion of the tube itself set up the vibrations and in this way the sounds of flutes refer to that kind, which is the first by us, I cannot see, how the known properties of flutes may be able to be satisfied. For it is observed cylindrical flutes with equal length also produce like sounds, however great both the cross-sectional area and material may differ between themselves. Therefore how is this may happen, that so very different tubes may vibrate similarly? But the opinion of those, who think only the inner surface may be able to vibrate, is seen to be overturned only by the differences of the material. On account of which the cause of the sounds of this kind must be, that it may depend only on the length of the flute.

29. Although it may suffice to set up our properties while only assessing the properties of flutes, yet, since always a knowledge of the cause may be able to effect the most perfect understanding of any phenomenon, I have used the most effort and diligence, in

order that I might follow the true cause. Moreover in the following manner, by carefully considering the structure of flutes, I have put the following reasoning in place. It is agreed each flute [*i.e*. a wind instrument in general] to be a tube or pipe having one end connected to a mouthpiece, which may receive air from the mouth, or from bellows [air chest] which may be sent into the pipe through a small opening, which opens into the tube of the instrument. But it is required that air expelled through the opening may not rush into the cavity of the tube, but only enter softly and touching the inner surface gently. On account of which the makers of that side tube, which is opposite the small opening, have altered that opening, so that it shall not be in contact with the mouthpiece, and made sharp, so that the air is disturbed by that sharp edge, and may be found on that account to let a weaker stream of air slowly into the tube.

30. But the structure of this mouthpiece is required to be found from experience, since we observe the mouth itself may be imitated by the mouthpiece. For if in the tube without a mouthpiece thus we may blow in air, so that it may glide along an internal surface, likewise sound is produced, as if the tube were equiped with a mouthpiece. And thus an account of the various wind instruments which lack a mouthpiece is to be prepared, so that there the air, must be blown in in some way, just as we see in those called transverse flutes and other similar instruments. But besides, so that with that air entering the pipe may produce a sound, it is required in the first place, that the inner surfaces of the pipe shall be smooth, lest it may impede the slow motion of the air, but then, as the walls of the pipe shall be hard and neither are they going to intrude into the flow of the air, from which also in the third place it is understood for the sides the pipe be required to be properly closed.

31. But these matters, and others, which are required to be observed in the construction of pipes, will be better understood, when we have set out an account of the same, by which sounds are formed in the the pipes. Moreover now it is shown that neither the whole pipe nor only the interior surface is required to generate the vibratory motion. For thus the air entering into that pipe, which now is present in the pipe, by necessity is compressed along the length; with which done, so that this may expand again and then may be narrowed anew, as long as the inflation may last, it may perform oscillations and from these sound may be produced. But now we may see, how from deep or acute sounds this sound according to the laws of mechanics shall be in a ratio to the length of the pipe, from which is may be observed, that this explanation agrees especially well with the phenomina

32. The body, which performs the oscillations and transfers these to the surrounding air, is the air contained in the tube, the quantity of which is known from the length and bore of the tube. Truly the impelling force for the required oscillation is, as we have seen, the inflated air rushing along the internal surface of the tube. But the force of the air present in the tube inducing that pressure, by which it is to be restored from its disturbed to its natural state, and which effects so that it makes the air upon which it acts, to complete a number of oscillations in a given time, is the weight of the atmosphere or

rather the elastic force of the air itself , which is equal to the incumbent pressure of the atmospheric air. This force is to be estimated from a suspended Torricellian barometer, in which quicksilver may be held at a height from 22 as far as 24 inches of Rhenish feet.

33. The account of this column of air, which is present in the tube, is similar to the vibrations made by a tensed string. For this string is itself required to be compared with the air contained in the tube of the pipe ; truly in this case the weight of the atmosphere takes the place of the weight of the extended string, which, even if at this point they may appear to be dissimilar, therefore since the string may be extended by the appended weight, truly the air from the atmosphere may be compressed, yet, if we may consider for that effect, plainly are equivalent between themselves. For since each prevails in the oscillations formed, which arises from a force, that is applied to the subjected body, on being received returns to its natural state. But this is applied either by the compression of the air in the tube or by the extension of the string, will produce the same effect.

34. Therefore since the air in the tube of the pipe may perform oscillations in the same manner as by an extended string, we will be able also to determine the number of oscillations produced in a given time and thus from these to determine the sound itself, which we have treated with vibrating strings. The length *a* of the pipe shall be expressed in scruples of Rh. ft. [1 Rh. ft.= 1000 scruples], the [cross-sectional] area of the pipe *bb,* the specific gravity of air to that of mercury as *m* to *n*, and the height of the mercury in the barometer of *k* similar scruples. Therefore we will have a string of length *a* and with a weight *mabb,* which is extended by a weight equal to the pressure of the atmosphere; truly this is equivalent to a cylinder of mercury, of which the base is *bb, i.e.* to the area of the tube, and the height *k.* On account of which the stretching force is agreed to be *nkbb.* From these the number of oscillations produced per second is found

$$\frac{355}{113}\sqrt{\frac{3166nkbb}{a\cdot mabb}}=\frac{355}{113a}\sqrt{\frac{3166nk}{m}},$$

to which the sound itself is equal, just as we have established how to measure that.

[Recall from Paragraph 9, that the frequency for the stretched string is $\frac{355}{113}\sqrt{\frac{3166n}{a}}$, where 113: 355 denotes the ratio of the diameter to the periphery of a circle, 3166 represents the number of thousandth parts of a Rhenish foot or scruples, for the length of a pendulum completing one oscillation in one second: *i.e.* half the period in modern terms as only a swing from one side to the other side was considered.]

35. Since *m* to *n* always maintain almost the same ratio , and *k* may change a little with the seasons, the sounds of the tubes whether they be cylindrical or prismatic being had in the inverse ratio of the lengths among, thus so that, where the tubes may be shorter, there sharper sounds may be produced, but longer tubes may produce deeper sounds. As that may agree exceedingly well with experience, which will be easily understood, by anyone who may consider the properties of wind instruments mentioned before, to wit, that the

quantity of the sound neither depends on the size of the tube, nor on the material from which the tube has been constructed, but shall depend only on the length. On account of which at this pont there is no doubt, why this exposition of the sounds produced by pipes may not be genuine and demanded by the nature of things.

36. Therefore moreover this exposition may be confirmed more by us, if not only we may consider the ratio of these sounds, but we will investigate also how they may correspond to the sound of a given string and extended by a given weight. For if by experiment it will be agreed the same flute to be in harmony with a given string, as the theory declares, this will be the maximum confirmation. Truly $\frac{n}{m}$, if it may have a maximum value, which happens at the warmest time, to around 12000, but at the coldest time may be taken around 10000. Similarly if the mercury in the barometer will have risen to the maximum step, there is $k = 2460$, but usually at that place with the mercury falling, there is $k = 2260$. Therefore with the barometer and the thermometer standing at the highest levels, the sound of the pipe will be $= \frac{960426}{a}$, and with the same instruments standing at the lowest levels the sound will be $= \frac{840348}{a}$.

37. We take the mean between these, which is $\frac{900000}{a}$ approx., and just that many oscillations will be produced per second on average by the pipe of length *a* in air in an average season. Therefore the pipe which produces 100 vibrations per second, that is 9000 scr., that is 9 Rhenish feet long, and that which produces 118 vibrations and consequently the sound marked *C* on musical instruments, must be 7627 scrup. or a little more than $7\frac{1}{2}$ Rhen. ft. Which also corresponds well enough to experience; for generally a pipe of length 8 ft. is taken for producing the sound *C*, and the difference of half a foot can perhaps be ignored, since thus the same pipe may be able to produce sounds in different seasons which may be in the ratio 840348 to 960426, that is held as 8 to 9, which distinction in such pipes is required to be more than half a foot.

38. And this difference of the sounds of the same pipes with various seasons confirms the truth of our exposition more. Indeed it is always being experienced by musicians, whenever they practice with instruments equipped with strings and likewise with wind instruments, these to be extremely changeable and the strings, with which they shall be consonant with pipes, may sometimes need to have the tension of the strings increased and sometimes must be decreased. And the difference between the most acute note and the deepest note of the same pipe to be around a whole tone, that is the interval between the tones maintaining the ratio 8 to 9. Besides that it is also observed that the pipes are more acute, when the sky is very quiet with much warmth, on the other hand with the gloomiest season taken with the greatest cold the tones of the pipes are deeper. From these also the reason is apparent, whereby initially the pipe will sound deeper, as when now it may be blown energetically; indeed with the same in use and with the inhalation of air, which is present in the pipe, it becomes warm and the sound emerges more acute.

39. The loudness and softness of the sounds produced by pipes by the inflating force of the player, depend then on the ratio, which the width of the pipe holds to its length. Indeed the account of pipes and strings is similar, in that with these the width of the former is compared with the thickness of the latter. On account of which, not any string is suitable for all the sounds required to be produced, but for a given sound a certain thickness certainly is required, thus also a pipe of a given length cannot be made narrower or wider as it pleases, but limits are given, which if they may be transgressed, at that point no sound shall be produced by the pipes. But where more pipes produce similar sounds of equal intensity, it is required that the size of the pipe or the base of the tube to be such that the thickness [i.e. area of cross-section] of the string to be in proportion to the length. From that indeed likewise and for the other case, since it is required for strings, so it follows clearly that the pressure [force] of the atmosphere, which is proportional to the size [i.e. area of cross-section] of the tube, also may have the same ratio to the length of the pipe.

40. Nor truly can the strength of the inflating force be increased of diminished as it pleases. Indeed if pipes may be blown into in an exceedingly faint manner, at this stage no sound is produced, but if it may be blown into stronger than is required, it produces sound, but an octave higher, and if at this point it may be blown into stronger, it will give a sound twelve times and again fifeteen times etc. more acute. So that we may uncover the reason for these higher sounds, it will help to be considering the intensity of the sound to be proportional to the inflation force ; and therefore, as long as the sound maintains the same magnitude, by which the inflation is greater than may be intended, thus greater oscillations of the air to be present in the tube, but it is necessary to understand not more frequent. But the size of the oscillations thus is determined by the size of the tube, so that a certain limit may not be exceeded; whereby if the pipe is inflated stronger, than is required at this level, it will not be able to produce the same sound.

41. But with strings, with which pipes have been agreed to be similar, then both from theory as well as by experiment it can be agreed each half of a stretched string can perform its own oscillations, thus so that from this string not the usual sound may be produced, but one an octave higher may be produced ; that which, if the parts shall be unequal, cannot happen. Similarly if it may be considered divided into three equal parts, perhaps the divided string can be shaken, so that the individual parts themselves, as if separated by small bridges, perform vibrations and a more acute sound than usual, namely the twelfth may be present. The same prevails also for four or more equal parts of a string. But these, as effected and may be confirmed by experiment, has been shown by the celebrated Sauveur in Comment. Acad. Scient. Paris. An. 1701.

[J. SAUVEUR (1653-1716), *Système général des intervalles des sons et son application à tous les systèmes et à tous les instruments de musique*.]

42. Therefore form these it is understood to be able to be adapted to pipes, so that each halves of the pipes themselves may perform the same oscillations and from that may

produce sound an octave higher. In which case, since the oscillations shall be twice as frequent, there will be a need for a stronger inflation force. From which it follows, if the inflation may be increased beyond that determined degree, then the oscillations in this case to be themselves accommodated and the sound to be produced an octave higher. In a similar manner and since here the degree may be given, that the inflation must not exceed, if indeed this may be exceeded, than the three individual parts of the air in the tube themselves begin to oscillate, from which the sound will be sharper by three times or the first twelfth may arise. And if the inflation will be increased, then the sound of the fourth parts oscillating may be heard sharper by two octaves, and thus so forth.

43. And also with these trumpets and bugles, as well as in other instruments that are not held to the same account as pipes, and there the nature depends on a distinctive property, by which the sounds of these may be modified only by the intensity of the inflation. Indeed not all the sounds are able to be produced by these instruments, but only these, which are expressed by the whole numbers 1, 2, 3, 4, 5, 6 etc., and thus in the lowest octave between 1 and 2 they produce no intermediary sound, in the following between 2 and 4 the one intermediary 3, which is the fifth of 2, in the third octave between 4 and 8 the three intermediaries 5, 6, 7 and in the fourth the 7 intermediaries. Truly the structure of these instruments is seen to be of this kind, so that any sound may have a very narrow limits of inflation and thus only a little increase or decrease in the breathing may produce a sound more acute or deeper.

44. Up to this point what has been said about pipes, pertains chiefly to these, of which the tubes have a prismatic or cylindrical form. But whether they may produce such sounds, if the tubes were either diverging or converging or of some other figure, is more difficulat to determine. Yet questions of this kind always can be reduced to strings ; indeed for the figure of any pipe proposed it will be required to consider a similar string to investigate what sound it is going to produce ; with that done, if for the string itself an air column may be put in place and the stretching weight equal to the atmospheric pressure, the sound will be had, that this pipe will return. And if this problem may be solved generally for any form of pipe, a well-known property of prismatic pipes likewise will be apparent, that the above openings produce sound an octave lower on being closed.

45. Other instruments, which may be seen to have some affinity with horns, are trumpets, bugles, etc., which indeed do not produce sound only by blowing, but require blowing jointly with a sound from the mouth, which then are returned wonderfully increased and much stronger, in a similar manner, where magaphones increase voices so much. But better instruments of this kind are recognised from these, which are used in wind organs and for the imitation of these; but these do not resonate by inflation only, but in the mouth piece elastic laminar shapes have been inserted, which are sent into a vibratory motion by the wind and they produce certain weak sound ; but then it advances along the adjoining tube, it acquires so much strength from that, so that it may imitate the sound of a trumpet or bugle.

CAPUT I

DE SONO ET AUDITU

1. Cum musicam nohis propositum sit ad modum philosophicarum disciplinarum pertractare, in quibus nihil, nisi cuius cognitio et veritas ex praecedentibus explicari possit, proferre licet, ante omnia est exponenda doctrina de sonis et auditu, quorum illi materiam, in qua musica versatur, constituunt, hic autem scopum et finem eius, qui est delectatio aurium, complectitur. Docet enim musica varios sonos ita efficere et scite coniungere, ut grata harmonia sensum auditus suaviter afficiant. Quae itaque de sonis exponere institutum nostrum requirit, sunt eorum natura, productio et varietates; quarum rerum sufficiens cognitio ex Physica et Mathesi est petenda. Deinde vero, si cum his praecipua auditus organa considerentur, audiendi rationem ac sonorum perceptionem intelligemus. Quae autem quantam utilitatem allatura sint ad musicae fundamenta stabilienda et confirmanda, cuique ex eo perspicuum erit, quod suavitas sonorum a perceptionis ratione pendent ex eaque debeat explicari.

2. Statuunt omnes, qui hac de re probabilia saltem scripserunt, sonum in aëre consistere huncque eius quasi vehiculum esse, quo a fonte quaqua versus circumferatur. Neque vero aliter res se habere potest, cum nihil nisi aër existat, quod aures nostras circumdet in iisque mutationem efficere possit. Nam quamvis obiiciatur auditus rationem fortasse eodem modo comparatam esse quo olfactus et visus, qui sensus non aëre, sed veris ex obiecto emissis effluviis excitantur, tamen ope antliae pneumaticae demonstratur, si instrumentum sonorum in loco ab aëre vacuo sit constitutum, ita ut cum aëre nullam prorsus habeat communicationem, nullum plane sonum, quantumvis prope accedas, percipi posse. Statim vero, ac aëri ingressus permittitur, sonus iterum auditur. Ex quo consequitur aërem eiusque mutationem, quam instrumentum sonum edens in eo producit, veram esse soni caussam atque proximam.

3. Ut vero constet, quae sit ista aëris mutatio et modificatio sensum soni excitans, considerari conveniet casum particularem, quo sonus producitur, et investigari effectum in aëre ex eo ortum. Hanc ob rem attendamus ad chordam tensam, quae pulsata sonum edit. At pulsu in chorda nihil aliud efficitur nisi motus tremulus, quo ea intra suos terminos nunc cis nunc ultra situm quietis velocissime extravagatur. In crassioribus quidem chordis hic motus etiam oculis facile percipitur, in tenuioribus vero etiamsi cerni nequeat, inesse tamen non dubitandum est. Praeterea qui vel manu campanam sonantem attingit, totam contremiscentem sentiet. Denique vero mox ex Mechanicae legibus ostendetur tam chordam quam campanam praeter motum tremulum a pulsu nil aliud recipere posse; et hanc ob rem statui debebit soni rationem in solo motu tremulo esse qquaerendam.

4. Cum igitur aëris mutatio, quam corpus tremulum in eo producit, sensum soni immediate efficiat et excitet, inquirendum est, quomodo aër a corpore tremulo afficiatur. Videmus autem motum tremulum consistera in successivarum vibrationum repetitione. Hisce singulis vibrationibus aër corpus tremulum ambiens percutitur similesque vibrationes recipit, quas pari modo in ulteriores particulas aëreas transfert. Hacque igitur

ratione istiusmodi pulsus et vibrationes in toto circumfuso aëre excitantur atque ista pulsuum in aërem translatio peragitur qualibet corporis tremuli vibratione. Ex quibus perspicitur singulas aëris particulas simili motu vibratorio contremiscere debere, quo ipsum corpus; hoc tantum discrimine, quod pulsus eo minores et debiliores fiant, quo longius a fonte distent, donec tandem in nimis magna distantia nil amplius percipi possit.

5. Ex his intelligitur praeter pulsus per aërem promotos a corpore sonante ad aures nihil deferri; quamobrem necesse est, ut hi ipsi pulsus in aëre excitati et in organum auditus incurrentes soni sensum producant. Hoc vero modo sensatio absolvitur: Exstat in interna auris cavitate membrana expansa a similitudine tympanum dicta, quae ictus aëris recipit eosque ulterius ad nervos auditorios promovet; hocque fit, ut, dum nervi afficiuntur, sonus sentiatur. Est igitur sonus nihil aliud nisi perceptio ictuum successivorum, qui in particulis aëris, quae circa auditus organum versantur, eveniunt, ita ut, quaecunque res huiusmodi ictus in aëre producere valeat, ea etiam ad sonum edendum sit accommodata.

6. Propagatio soni per aërem non perficitur puncto temporis, sed determinato tempore opus habet, quo per datum spatium propellatur. Motus autem, quo progreditur, est aequabilis et neque a vehementia soni neque eius qualitate pendet. Progreditur vero omnis sonus, ut tam ex experimentis apparet, quam ex computatione theoretica aëris et pulsuum natura colligere licet, tempore minuti secundi per spatium 1100 pedum Rhenanorum, duobusque minutis secundis percurrit 2200 pedes, tribus 3300 et ita porro. Observamus etiam hanc sonorum tarditatem quotidie; longius enim distantis tormenti, cum exploditur, sonitum aliquanto post fulgetrum percipimus, cum tamen tormento propius adstantes utrumque simul sentiamus. Ob similem caussam etiam tonitru demum post fulgur audimus, et vocum repetitiones nonnullis in locis, quae echo dicuntur, tardius ipsum clamorem sequuntur.

7. Quidquid igitur minimas aëris particulas ita commovere valet, ut huiusmodi motum tremulum recipiant, id etiam sonum producet. Ad hoc vero efficiendum non solum corpora dura sunt idonea, sed praeter ea duo alii reperiuntur modi sonos edendi; ex quo etiam tria sonorum genera, si ad caussas respiciatur, nascuntur. Primum est eorum, qui a corpore tremulo oriuntur, cuiusmodi sunt chordarum campanarumque soni. Alterum genus eos comprehendit, qui ab aëre vehementer compressa seseque subito restituente proficiscuntur, ut soni sclopetorum, tormentorum, tonitrui et virgae per aërem celerrime vibratae. Ad tertium referuntur soni instrumentorum, quae inflata tinniunt, ut fistulae, tibiae etc., quorum sonorum caussam non a motu tremulo materiae, ex qua tibiae constant, pendere infra docebitur.

8. Ex primo genere praecipue considerandae sunt chordae tensae sive ex metallo sive ex intestinis animalium confectae, quae vel pulsatione vel attritione ad sonum edendum cientur. Pulsantur et vellicantur quoque in clavicymbalis, catharis aliisque huius generis instrumentis; atteruntur vero in panduris, violinis ope pilorum equinorum tensorum, quibus colophonio scabrities est inducta. Utroque modo chordae motum tremulum

recipiunt; etenim primo ex quiete situque naturali detorquentur, quo facto se in situm naturalem restituere conantur et revera motu accelerato in eum properant. At ingentem celeritatem, quam acquisiverunt, cum eo pervenerunt, subito amittere non possunt, neque ideo in eo statu quiescere. Quamobrem eas ultra excurrere necesse est similique modo eo reverti; atque hae oscillationes tamdiu durabunt, quoad ob resistentiam plane evanescant.

9. Quot autem huiusmodi oscillationes chorda pulsata seu quovis modo tremula facta dato tempore absolvat, ex legibus motus calculo definiri potest, si ad longitudinem chordae eiusque pondus et vim tendentem respiciatur. At longitudo pondusque non sumi debent totius chordae, sed eius solum partis, quae tremula redditur sonumque edit et quae duobus hypomochliis ab integra chorda separari solet. His scilicet impeditur, quominus tota chorda vibrationes perficiat, sed tanta eius solum portio, quanta placet. Quo autem vis tendens cognoscatur, maxime expedit, chordae altero termino fixo, alteri pondus appendere, locum vis tendentis sustinens. His positis si longitudo chordae sonantis sit a partium millesimarum pedis Rhenani pondusque appensum se habeat ad pondus chordae ut n ad 1, erit numerus oscillationum, quem haec chorda minuto secundo absolvit, hic

$$\frac{355}{113}\sqrt{\frac{3166n}{a}},$$

ubi 113: 355 denotat rationem diametri ad peripheriam circuli, 3166 scrupuli praebent longitudinem penduli singulis secundis oscillantis.

10. Oscillationes hae, quoad durant, sunt isochronae seu omnes absolvuntur aequalibus temporis intervallis, neque magnitudo earum hanc regulam turbat, nisi forte, cum chorda nimis vehementer pulsatur, ipso principio vibrationes sunt celeriores. Chordarum scilicet eadem est ratio quae pendulorum, quorum oscillationes, si sunt admodum exiguae, omnes sunt aequitemporaneae. Ut regulam superiori paragrapho datam exemplo illustrarem, sumsi chordam longitudinis 1510 part. milles. ped. Rh., quae ponderabat $6\frac{1}{5}$ gr.; tetendi hanc pondere 6 libr. seu 46080 gran. Quibus cum paragrapho praecedente comparatis erit

$$a = 1510$$

et

$$n = 46080 : 6\tfrac{1}{5} = 7432.$$

Quare numerus minuto sec. editarum vibrationum erit

$$\frac{355}{113}\sqrt{\frac{3166\cdot 7432}{1510}} \text{ i.e. } 392.$$

Huic autem sono congruere deprehendi in instrumento clavem signatam *a*.

11. Si plures habeantur chordae tensae, facile ratio, quam earum vibrationes

inter se habent, determinatur; est scilicet in qualibet chorda numerus vibrationum dato tempore editarum ut $\sqrt{\frac{n}{a}}$ i. e. ut *radix quadrata ex pondere a tendente diviso et per pondus chordae et per eius longitudinem.* Si ergo chordae fuerint eiusdem longitudinis, erunt vibrationum eodem tempore editarum numeri ut *radices quadratae ex ponderibus tendentibus divisis per pondera chordarum.* Si chordae et longitudine et pondere fuerint aequales, erunt vibrationum numeri ut *radices quadratae ex ponderibus tendentibus.* Atque si pondera tendentia sint aequalia et ipsae chordae tantum longitudine differant, erunt vibrationum numeri reciproce ut *radices quadratae ex longitudine ducta in pondus*, i. e. *reciproce ut longitudines chordarum*, quia pondera longitudinibus sunt proportionalia.

12. A tarditate et celeritate vibrationum pendet sonorum distinctio in graves et acutos, eoque sonum graviorem esse dicimus, quo pauciores vibrationes eodem tempore auditus organum feriunt, eoque acutiorem, quo plures eiusmodi vibrationes eodem tempore sentiuntur. Veritas huius ex ipsa experientia constat; si enim eidem chordae successive varia pondera appendantur, sonos ab iis editos acutiores percipimus, si maiora sint pondera appensa, at graviores erunt, quo pondera sunt minora. Certum autem est ex praecedentibus maiora pondera celeriores vibrationes producere. Hanc ob rem, cum in musica praecipue sonorum gravitatis et acuminis discrimen spectetur, ipsos sonos secundum vibrationum certo quodam tempore editarum numerum metiemur, seu sonos ut quantitates considerabimus, quarum mensuras vibrationum determinato tempore editarum numeri constituunt.

13. Quemadmodum vero nostris sentibus res neque nimis magnas neque nimis parvas concipere possumus, ita etiam in sonis quaepiam mediocritas requiritur; sonique omnes sensibiles intra certos terminos erunt constituti, quos qui transgrediuntur, propter nimiam vel gravitatem vel acumen auditus sensum amplius non afficiant. Termini isti quodammodo possunt determinari; cum enim sonus *a* inventus sit edere 392 vibrationes minuto secundo, sonus littera *C* signatus interim 118 absolvit et sonus $\overset{\equiv}{c}$ 1888. Si iam ponamus sonos duabus octavis et acutiores et graviores audiri adhuc vix posse, habebimus extremos perceptibiles sonos numeris 30 et 7520 expressos; quod intervallum satis est amplum et ingentem sonorum variationem admittit, quippe quod octo intervalla octavas dicta complectitur.

14. Post discrimen sonorum gravium et acutorum consideranda est eorum vehementia et debilitas. Est autem vehementia eiusdem soni diversa pro auditoris loco; quo enim longius auditor a chorda pulsata distat, eo debiliorem percipit sonum, cum propagatio pulsuum uti luminis per aërem perpetuo fiat languidior. Ratio huius decrementi est, quod in maioribus distantiis sonus in maius spatium diffundatur; scilicet in dupla distantia spatium, quo est perceptibilis, est quadruplo maius quam in simpla; quamobrem cum ibi aggregatum omnium pulsuum aeque est magnum ac hic, sequitur sonum in dupla distantia esse quadruplo debiliorem. Similiter in tripla distantia noncuplo debiliorem esse

oportet et ita porro, ita ut vehementia soni in duplicata ratione distantiarum decrescere debeat.

15. Haec ita se habent, si sonus quaquaversus se aequaliter expandit. At si eiusmodi fuerint circumstantiae, ut sonus in unam plagam magis propellatur quam in aliam, fortior quoque ibi percipietur, quam iuxta regulam oporteret. Ut si quis per tubum vociferatur, is, qui aurem ad alteram extremitatem tubi admovet, sonum propemodum tam vehementem sentiet, quam si ex ipso ore clamantis vocem excepisset. Similis est ratio tubarum stentoreophonicarum, per quas sonus potius in eam regionem, in quam tuba dirigitur, propellitur quam in aliam ob eamque caussam fortior evadit. Reflectuntur enim etiam soni ut radii luminis a superficie laevi et dura, atque hoc modo radiorum sonorum, quos ad similitudinem radiorum lucidorum ita appellare liceat, directio immutatur, quo fieri potest, ut plures in eundem locum coniiciantur.

16. Cum chorda pulsata quavis oscillatione pulsus per aërem transmittat, necesse est, ut eius motus perpetuo fiat remissior ideoque sonus debilior. Utique observatur hoc in chordis vibrantibus; initio enim sonus est maxime intensus, tum vero pedetemtim fit languidior, donec tandem prorsus cesset; interim tamen oscillationes manent isochronae sonusque nihilominus eundem gravitatis et acuminis gradum retinet. Pendet haec intensitas ipso initio in eadem chorda a vi pulsante, ut, quo maior haec sit, eo fortior quoque prodeat sonus. Initio tamen, si pulsatio fuerit nimis vehemens chordaeque detorsio ex situ naturali nimis magna, sonus acutior editur quam postea; atque cum oscillationes maius spatium occupent, aëri non tam regulares vibrationes imprimuntur; quo fit, ut soni tum minus grati minusque distincti edantur.

17. Evenit hoc potissimum, si chorda nimis est laxa neque satis tensa; tum enim maiores in oscillando redduntur excursiones sonusque neque aequabilis neque gratus existit. Hanc ob caussam ad sonos suaves et aequabiles producendos requiritur, ut chordae, quantum fieri potest, tendantur tantaque pondera appendantur, ut tantum non disrumpantur. Vis autem chordarum ex eadem materia confectarum est crassitiei proportionalis, quare et pondera tendentia chordas ad ruptionem usque sunt ut crassities. Sed chordarum crassities sunt suis ponderibus per longitudinem divisis proportionales, propterea pondera tendentia debebunt esse in chordarum ponderum ratione directa et longitudinum inversa. Id est, si ponatur chordae pondus q, longitudo a pondusque tendens p, oportet sit p ut $\frac{q}{a}$, seu $\frac{ap}{q}$ debet esse constantis magnitudinis.

18. Quo autem soni proveniant aequaliter fortes, oportet praeter longitudinem chordae pondusque tendens attendere ad vim pulsantem. Locus etiam, quo chorda vellicatur vel pulsatur, considerandus esset; sed si ponamus chordas omnes in medio vel, quod eodem redit, in locis similibus impelli, haec conditio in computum non ingredietur. Ex hoc fit, ut, quo maior sit vis pulsans, eo fortior evadat sonus. Soient autem omnia fere instrumenta musica ita esse confecta, ut cunctae chordae aequaliter percutiantur, quamobrem vim pulsantem semper eandem ponemus. Vehementia deinde soni pendet a

celeritate, qua aëris particulae quavis chordae vibratione in aurem impingunt, haecque ex celeritate chordae maxima est aestimanda. Est vero haec celeritas proportionalis radici quadratae ex pondere chordam tendente diviso per longitudinem eius. Consequenter, quo soni fiant aequabiles, necesse est, ut pondus tendens semper sit ut chordae longitudo.

19. Manentibus ergo superioribus litteris *a*, *p* et *q* debet esse $\frac{p}{a}$ ubique eiusdem magnitudinis. Ante vero iam est inventum $\frac{ap}{q}$ constans esse oportere, quare hoc per illud diviso quotus prodiens *aa* debet esse constans seu $\frac{q}{a}$ ad *a* eandem in omnibus chordis tenere rationem. Sed $\frac{q}{a}$ est chordae crassitiei proportionalis, adeoque chordae crassities longitudini proportionalis esse debet, similiterque etiam eidem longitudini pondus tendens. Ipse autem sonus editus est ut $\sqrt{\frac{p}{aq}}$; in quo si loco *p* et *q* proportionalia *a* et a^2 substituantur, erit sonus reciproce ut chordae longitudo. Hanc ob rem et pondus tendens et longitudinem et pondus chordae proportionalia esse oportet reciproce ipsi sono edendo seu numero vibrationum dato tempore absolvendarum. Quae regula in conficiendis instrumentis musicis eximium habebit usum.

20. Diximus sonum minus fore gratum, si chorda non fuerit satis tensa, propterea quod excursiones inter vibrandum factae sint nimis amplae ab iisque aër potius instar venti promoveatur, quam ad oscillationes peragendas incitetur. Nisi enim subito ingenti celeritate aër percutiatur, non facile motum tremulum, qualis ad sonum requiritur, recipit; quo autem magis chorda est tensa, eo maiorem statim post pulsum habet celeritatem. Accedit ad hoc, quod iam est notatum, ampliores vibrationes minoribus non esse isochronas, unde sonus pedetemtim fit gravior neque idem permanet. Deinde facile evenit, ut tota chorda non simul oscillationes absolvat, sed alia eius pars citius, alia tardius tam ad maximam celeritatem quam ad quietem perveniat, ex quo sonus inaequabilis et asper existit.

21. Praeter has sonorum differentias in musica etiam ad durationem sonorum respicitur. In multis quidem instrumentis sonos pro lubitu prolongare non licet, ut in iis, quibus chordae pulsu vel vellicatione excitantur. Namque in his soni pedetemtim fiunt debiliores et mox penitus cessant; et hanc ob rem sonorum durationibus non tantum effici potest, quantum in iis instrumentis, quibus soni, quoad durant, eandem vim retinent et, quamdiu placet, produci possunt. Huiusmodi sunt ea, quorum chordae plectro atteruntur, atque quae tibiis sunt instructa aliisque, quae vento cientur, instrumentis, ut Organum Pneumaticum aliaque plura. Ista prae reliquis hanc habent praerogativam, ut omnis suavitas, quae duratione sonorum existit, perfecte possit exprimi et produci. Mensuratur autem soni duratio ex tempore inter initium et finem interiecto.

22. Hactenus ex primo sonorum genere, qui a corpore tremulo originem habent, sonos tantum chordarum contemplati sumus simulque etiam primarias sonorum differentias

enumeravimus et exposuimus. Nunc igitur, antequam ad reliqua genera progrediamur, alia quoque instrumenta consideranda sunt, quae sonos ad hoc genus pertinentes edunt. Huiusmodi sunt campanae, quae pulsatae totae contremiscunt sonumque edunt. Difficillimum quidem esset ex campanae forma pondereque cognitis, qualem sonum datura sit, determinare; attamen, si campanae fuerint similes et ex eadem materia confectae, facile apparet sonos tenere rationem reciprocam triplicatam ponderum, ita ut campana octuplo levior edat sonum eodem tempore duplo plures oscillationes absolventem et, quae vicies septies fuerit levior, peragat vibrationes triplo frequentiores.

23. Habentur praeterea instrumenta musica baculis elasticis vel ex metallo, quibus campanarum sonos imitantur, vel ex ligno duriore confectis. De his, siquidem formam habent cylindricam vel prismaticam, facilius est certi quidpiam statuere; soni enim tantum a longitudine pendere videntur, cum quaelibet fibra in longitudinem extensa vibrationes seorsim perficere censenda sit. Erunt autem soni seu vibrationum eodem tempore editarum numeri reciproce ut quadrata longitudinum baculorum, siquidem baculi ex eadem materia fuerint fabricati. Ex diversa enim materia constantium prismatum soni non solum a gravitatis specificae ratione pendent, sed etiam cohaesionis et elateris materiae rationem nosse necesse est eum, qui ipsos sonos ex theoria determinare susceperit.

24. Ad secundam sonorum classem eos retuli sonos, qui vel notabili aëris vehementer compressi copia subito dimissa vel validiore aëris percussione oriuntur. Quorum quidem posterior modus priori fere est similis; propter celerrimam enim vibrationem aër e vestigio locum cedere non potest, ex quo fit, ut portio aëris ictum sustinens comprimatur seque, quam primum sibi est relicta, iterum expandat. At aërem compressum derepente se expandentem necesse est maius naturali spatium occupare, et idcirco erit coactus se rursus contrahere, id quod etiam nimium faciet. His igitur alternis contractionibus et expansionibus, corporis tremuli instar, in reliquo aëre pulsus atque in auditus organo sonus producetur.

25. Quanquam hoc modo aër qualibet oscillatione in statum suum naturalem pervenit, tamen in eo prius consistere non potest, quam totum suum motum amiserit. Ex Mechanica enim constat corpus cum impetu in situm suum quietis perveniens in eo permanere non posse, sed motu iam concepto ultra eum transgredi oportere. Aeque est enim difficile corpus motum subito quiescere ac quiescens moveri; atque tanta vi opus est ad corporis motum tollendum, quanta ad eundem producendum. Hanc ob caussam neque pendula oscillantia, cum in situm verticalem pervenerint, quiescere posse videmus neque chordas vibrantes, cum situm naturalem attigerint. Soni vero hoc exposito modo generati brevi tantum tempore durare possunt, nisi echo vel simile quid resonans adsit, quod eos repetat et protrahat; aër enim motum in tam dissita loca diffundendo proprium motum statim amittat necesse est.

26. Omnes igitur caussae, quae aërem vel iam compressum dimittere vel naturalem comprimere, ita ut se subito possit relaxare, valent, eae etiam ad sonum producendum

sunt accommodatae. Quamobrem omnes corporum velociores per aërem motiones sonos generare debent; aër enim propter inertiam corporibus liberrime locum concedere non potest ideoque ab iis comprimitur, qui deinceps se rursus dilatans minimis aëris particulis motum tremulum inducit. Hinc originem ducunt vehementius vibratarum virgarum ·et omnium per aërem celerius motorum corporum soni. Neque etiam ventorum flatuumque soni sibili alii debentur caussae; anterior enim aër ab insequente posteriore aeque ac a corpore duro compellitur atque comprimitur.

27. Sonorum, qui a repentina dimissione aëris vehementer compressi gignuntur, fortissimi procul dubio sunt ii, qui ex pulvere pyrio et tonitruo percipiuntur. Variis enim experimentis constat in pulvere pyrio inesse aërem maxime compressum eique accensione exitum aperiri, unde tam stupendos sonos prodire necesse est. Atque ad nubes constituendas cum vaporibus permultas particulas nitrosas et sulphureas simul ascendere maxime probabile videtur, quae in iis unitae et explosae tantum strepitum edere queant. At cum de huiusmodi sonis difficile sit discernere, quomodo ratione gravitatis et acuminis a se invicem discrepent, omnes ad hoc genus pertinentes soni in musica non sunt recepti; quamobrem oscillationum, quas minimis aëris particulis inducunt, investigationi supersedebimus.

28. Ad tertium sonorum genus pertinent secundum factam initio divisionem soni tibiarum, qui inflatione excitantur. Quorum ratio, ut magis est recondita, ita minori industria quovis tempore est investigata. Nam qui ipsum tubum motum tremulum accipere statuunt atque hoc modo sonos tibiarum ad id genus, quod nohis est primum, referunt, non video, quomodo proprietatibus tibiarum cognitis satisfacere possint. Observatum enim est tibias cylindricas longitudine aequales pares etiam edere sonos, quantumvis tam amplitudine inter se differant quam crassitie atque materia ipsa. Quomodo igitur fieri posset, ut tam diversi tubi similiter contremiscant? Eorum autem sententiam, qui internam tantum superficiem tremulam fieri putant, sola materiei diversitas evertere videtur. Quamobrem caussa horum sonorum eiusmodi esse debet, ut a sola tibiarum longitudine pendeat.

29. Quamvis autem sufficeret ad institutum nostrum proprietates duntaxat tibiarum recensere, tamen, cum caussae cognitio semper cuiusque rei notitiam perfectissimam efficere soleat, operam atque diligentiam adhibui, ut veram caussam consequerer. Sequenti autem modo, tibiarum structura perpensa, ratiocinium institui. Constat cuique tibias esse tubos seu canales altera extremitate peristomium iunctum habentes, quod aërem ex ore vel cista pneumatica recipiat atque per rimam, in quam eius cavitas versus tubum desinit, in tubum emittat. Requiritur autem, ut aër per rimam expulsus non in cavitatem tubi irruat, sed tantum internam superficiem perstringat eique obrepat. Quamobrem artifices illud tubi latus, quod rimae est oppositum, excindunt, ne sit contiguum peristomio, atque acuunt, ut aër in ipsam aciem irruat ab eaque quasi findatur, quo tenuior aëris lamella per tubum prorepat.

30. Huiusmodi autem peristomiorum structuram requiri cum experientia demonstrat, tum ipso ore peristomiis imitandis perspicimus. Nam si in tubum peristomio destitutum ore ita aërem inflamus, ut ad internam superficiem irrepat, perinde sonus editum, ac si peristomio tubus esset instructus. Atque ita est variarum tibiarum peristomiis carentium ratio comparata, ut aër eo, quo expositum est, modo inflari debeat, velut videmus in fistulis transversis vocatis aliisque similibus. Praeterea autem, ut iste aëris in tubum ingressus sonum efficiat, requiritur primo, ut interna tubi superficies sit laevis, ne motus repens aëris impediatur, tum autem, ut tubi latera sint dura neque aëri irruenti cedere queant, ex quo etiam tertio intelligitur tubum ad latera probe clausum esse oportere.

31. Haec autem, aliaque, quae in tibiis construendis observanda sunt, melius cognoscentur, cum ipsam rationem, qua soni in tibiis formantur, exposuerimus. Ostensum autem iam est neque totius tubi neque interioris tantum superficiei motum tremulum generari. Aër enim sic in tubum intrans eum, qui iam in tubo existit, necessario secundum longitudinem comprimit; quo fit, ut is sese iterum expandat tumque denuo coarctetur atque hoc modo, quoad inflatio durat, oscillationes perficiat hisque sonum producat. Videamus nunc autem, quantus gravitate acumineve hic sonus secundum leges mechanicas futurus sit ratione longitudinis tubi, quo, quam egregie haec explicatio cum phaenomenis congruat, perspiciatur.

32. Corpus, quod oscillationes peragit easque in aërem circumfusum transfert, est aër in tubo contentus, cuius quantitas ex tubi longitudine et amplitudine cognoscitur. Vis vero ad oscillandum impellens est, ut vidimus, aër inflatione secundum tubi internam superficiem irruens. At vis aëri in tubo existenti eum nisum inducens, quo ex statu naturali deturbatus se restituere conatur, et quae efficit, ut illum ipsum, quem absolvit, oscillationum dato tempore numerum absolvat, est pondus atmosphaerae seu ipsa illius aëris vis elastica, quae pressioni incumbentis atmosphaerae aëreae est aequalis. Haecque vis existimanda est ex effectu eius, quem in tubo TORRICELLIANO exserit, in quo argentum vivum ad altitudinem a 22 usque ad 24 digitos pedis Rhenani suspensum tenetur.

33. Huius igitur columnae aëreae, quae in tubo inest, oscillantis similis omnino est ratio ei, qua chorda tensa vibrationes conficit. Ipsa enim chorda comparanda est cum aëre in tubo fistulae contento; ponderis vero chordam tendentis hoc casu locum sustinet atmosphaerae pondus, quae, etiamsi prorsus dissimilia videantur, eo quod chorda a pondere appenso extendatur, aër vero ab atmosphaera comprimatur, tamen, si ad effectum respiciamus, plane inter se aequivalent. Nam quod utraque in formandis oscillationibus valet, id provenit a vi, quam corpori subiecto tribuit, se in statum naturalem recipiendi. Haec autem, sive compressione in aërem tubi operetur sive extensione in chordam, eundem producet effectum.

34. Cum igitur aër in tubo fistulae eodem modo oscillationes perficiat quo chorda tensa, poterimus quoque numerum oscillationum dato tempore editarum atque ita ipsum sonum determinare ex iis, quae de chordis vibrantibus tradidimus. Sit tibiae longitudo a

in scrup. ped. Rh. expressa, amplitudo *bb*, gravitas aëris specifica ad eam mercurii ut *m* ad *n* et altitudo mercurii in barometro *k* similium scrupulorum. Habebimus ergo chordam longitudinis *a* ponderisque *mabb*, quae tenditur a pondere aequali pressioni atmosphaerae; haec vero aequivalet cylindro mercurii, cuius basis est *bb,* i. e. amplitudo tubi, et altitudo *k.* Quocirca pondus tendens censendum est *nkbb.* Ex his invenitur oscillationum minuto secundo editarum numerus

$$\frac{355}{113}\sqrt{\frac{3166nkbb}{a\cdot mabb}}=\frac{355}{113a}\sqrt{\frac{3166nk}{m}},$$

cui ipse sonus, quemadmodum eum metiri instituimus, est aequalis.

35. Quia *m* ad *n* propemodum eandem semper tenet rationem atque *k* parum diversis tempestatibus mutatur, erunt soni tibiarum tubos vel cylindricos vel prismaticos habentium inter se reciproce ut longitudines tuborum, ita ut, quo tubi sint breviores, eo soni prodeant acutiores, at longiores tubi sonos graviores reddant. Quod quam egregie cum experientia congruat, quilibet facile intelliget, qui tibiarum proprietates ante commemoratas perpendet, quae huc redibant, ut soni quantitas neque ab amplitudine tubi neque a materie, ex qua tubus sit confectus, sed a sola longitudine pendeat. Quamobrem prorsus non esse dubitandum existimo, quin haec sonorum a tibiis editorum exposita ratio sit genuina et ex ipsa rei natura petita.

36. Eo magis autem haec explicatio nobis confirmabitur, si non solum sonorum horum rationem inspiciamus, sed, quomodo se habeant ad sonum datae chordae datoque pondere tensae, etiam investigabimus. Nam si experientia constiterit eandem tibiam cum data chorda esse consonam, quam theoria declarat, maximum hoc erit firmamentum. Est vero $\frac{n}{m}$, si maximum habet valorem, quod accidit tempore calidissimo, circiter 12000, at frigidissima tempestate deprehenditur 10000. Similiter si mercurius in barometro ad maximum gradum ascenderit, est $k=2460$, at plurimum ibidem mercurio descendante est $k=2260$. Idcirco barometro et thermometro ad maximas altitudines consistentibus erit sonus tibiae $=\frac{960426}{a}$ atque iisdem instrumentis ad minimas altitudines stantibus sonus erit $=\frac{840348}{a}$.

37. Inter hos sumamus medium, quod est $\frac{900387}{a}$ atque tot oscillationes minuto secundo tibia longitudinis *a* in aëre producet tempestate mediocri. Ergo quae tibia 100 vibrationes minuto secundo edit, ea est longa 9000 scr., i.e. 9 pedes Rhenanos, et quae edit 118 vibrationes atque consona est chordae sonum *C* in instrumentis signatum exhibentis, longitudinis esse debet 7627 scrup. seu aliquanto plus quam $7\frac{1}{2}$ ped. Rhenan. Quod etiam satis exacte experientiae respondet; nam vulgo tibia longitudinis 8 ped. assumitur ad sonum *C* edendum, et differentia dimidii pedis penitus est negligenda, eo quod eadem tibia diversis tempestatibus sonos edere queat rationem 840348 ad 960426, i. e. 8 ad 9 tenentes, quod discrimen in tali tibia pluris dimidio pede est aestimandum.

38. Et haec ipsa sonorum diversitas eiusdem tibiae variis tempestatibus veritatem nostrae explicationis magis confirmat. Experiuntur enim perpetuo Musici, quoties instrumentis chordis instructis simul cum pneumaticis utuntur, haec perquam mutabilia esse atque chordas, quo consonae sint cum tibiis, mox intendi moxque remitti debere. Ac differentiam inter sonum acutissimum et gravissimum eiusdem tibiae esse integri toni circiter, quod est intervallum inter sonos rationem 8 ad 9 tenentes. Praeterea id quoque est observatum tum tibias esse acutiores, quando coelum sit maxime serenum cum summo calore, contra turbidissima cum maximo frigore coniuncta tempestate sonos tibiarum esse graviores. Ex his etiam ratio patet, quare tibia initio gravius sonet, quam cum iam strenue sit inflata; ipso enim usu et inhalatione aër, qui in tibia inest, calefit ideoque sonus evadit magis acutus.

39. Vehementia sonorum et debilitas a tibiis editorum cum a vi, qua inflantur, pendet tum a ratione, quam tibiae amplitudo ad longitudinem tenet. Similis enim est ratio tibiarum et chordarum, in iisque amplitudo est comparanda cum crassitie harum. Quemadmodum igitur non quaevis chorda ad omnes sonos edendos est apta, sed ad datum sonum certa quaedam crassities requiritur, ita etiam datae longitudinis tibia non pro lubitu ampla vel augusta potest confici, sed dantur limites, quos si transgrediare, nullum prorsus sonum tibia sit editura. Quo autem plures tibiae sonos edant similes et aeque vehementes, oportet tibiae amplitudinem seu basin tubi sicut chordae crassitiem proportionalem esse longitudini. Ex hoc enim simul et alterum, quod in chordis requiritur, sequitur, ut videlicet pressio atmosphaerae, quae amplitudini est proportionalis, etiam eandem habeat rationem ad longitudinem tibiae.

40. Neque vero vehementia inflatus pro lubitu potest augeri vel minui. Namque si nimis languide tibia infletur, sonum edet prorsus nullum, at fortius, quam par est, inflata non eum, quem debet, edit sonum, sed octava acutiorem, et si adhuc fortius infletur, sonum duodecima porroque decima quinta etc. acutiorem dabit. Ut harum soni ascensionum rationem detegamus, considerari iuvabit soni vim proportionalem esse vi inflatus; et propterea, quamdiu sonus idem quantitate manet, quo magis inflatio intendatur, eo ampliores oscillationes aëris in tubo contenti, non autem frequentiores esse oportere intelligitur. At oscillationum amplitudo tubi amplitudine ita determinatur, ut certum terminum transgredi non possit; quare si tibia fortius infletur, quam ad istum gradum requiritur, eundem sonum edere non poterit.

41. De chordis autem, quibus tibiae similes sunt censendae, tam ex theoria quam experientia constat posse chordae tensae utramque medietatem seorsim suas oscillationes perficere, ita ut ea chorda non sonum solitum, sed octava acutiorem edat; id quod, si partes sint inaequales, fieri non potest. Similiter in tres partes aequales cogitatione saltem divisa chorda ita potest contremiscere, ut singulae partes seorsim, tanquam si ponticulis essent separatae, vibrationes absolvant atque sonum solito acutiorem, nempe duodecimam, exhibeant. Idem etiam valet de quatuor pluribusque partibus chordae

aequalibus. Haec autem, quomodo effici et experimentis confirmari queant, ostendit Cl. D. SAUVEUR in Comment. Acad. Scient. Paris. An. 1701.

42. His igitur ad tibias accommodatis intelligitur fieri posse, ut utraque tibiae medietas seorsim oscillationes perficiat eoque sonum octava acutiorem edat. Quo in casu, cum oscillationes duplo sint frequentiores, maior quoque inflatus vis locum habebit. Ex quo sequitur, si inflatus ultra determinatum illum gradum augeatur, tum oscillationes ad hunc casum se esse accommodaturas sonumque octava acutiorem proditurum. Simili modo cum et hic detur gradus, quem inflatio excedere non debet, si etiam hic transeatur, tum singulae tertiae aëris in tubo contenti partes seorsim oscillare incipient, ex quo sonus triplo acutior seu primi duodecima proveniet. Atque porro si inflatus augebitur, tum quartis partibus oscillantibus sonus duabus octavis acutior audietur, et ita porro.

43. Hisce etiam tubarum buccinarumque, quanquam in ceteris non eam quam tibiae tenent rationem, nititur natura eaque proprietas, qua sola inflationis intensione soni eius moderentur. His enim instrumentis non omnes soni edi possunt, sed ii duntaxat, qui exprimuntur numeris integris 1, 2, 3, 4, 5, 6 etc., sicque in infima octava inter 1 et 2 nullum sonum medium edunt, in sequente inter 2 et 4 unum medium 3, qui est ad 2 quinta, in tertia octava inter 4 et 8 habent tres 5, 6, 7 et in quarta 7 intermedios. Horum vero instrumentorum structura eiusmodi esse videtur, ut quivis sonus valde angustos habeat limites inflationis ideoque parum tantum intenso vel remisso flatu sonus vel acutior vel gravior prodeat.

44. Quae hactenus de tibiis dicta sunt, pertinent potissimum ad eas, quarum tubi habent formam vel prismaticam vel cylindricam. Quales autem sonos edant, si tubi fuerint vel divergentes vel convergentes vel alius cuiusdam figurae, difficilius est determinare. Semper tamen huiusmodi quaestiones ad chordas reduci possunt; figura enim tibiae quacunque proposita oportet chordam similem considerare et, quem sonum sit editura, investigare; quo facto, si ipsa chorda aërea ponatur et pondus tendens aequale vi athmosphaerae, habebitur sonus, quem ea tibia reddet. Atque si hoc problema universaliter solvetur pro quacunque tibiae figura, apparebit simul maxime nota proprietas tibiarum prismaticarum, quae supra apertae sonum octava graviorem edunt.

45. Alia instrumenta, quae cum tibiis aliquam affinitatem habere videntur, sunt tubae, buccinae etc., quae quidem solo inflatu sonum non edunt, sed sonum ex ore cum flatu coniunctum requirunt, quem tum mirifice augent vehementioremque reddunt, simili modo, quo tubae stentoreophonicae voces tantopere augmentant. Melius autem huiusmodi instrumenta cognoscuntur ex iis, quae in organis pneumaticis ad eorum imitationem adhibentur; excitantur; haec autem solo inflatu, sed in peristomio insertae sunt lamellae elasticae, quae a vento immisso motum tremulum recipiunt sonumque debilem quidam edunt; sed dum is per tubum adiunctum progreditur, tantam ab eo vim acquirit, ut sonos tubarum vel buccinarum egregie imitetur.