CHAPTER XIV

CONCERNING THE INTERCHANGE OF MODES AND SYSTEMS

1. However often interchanges may be varied which may occur in a single system, yet, if the same system may be retained for a long time, by necessity it may appear to become a thing of contempt rather than of pleasure. Indeed since music may require both variety as well as pleasure in the notes and consonances, the piece being heard is required to be changed. Therefore just as the exponent of the system being heard may be represented by the composition in the preceding chapter, thus, when this now is understood well enough, it must be able to be changed into another system.

2. Moreover the change can be made in several ways ; for initially the system will allow only various changes, with the mode of each species remaining unchanged. Then the change may become more perceptible, if it may be able to pass over into another kind of mode ; changes of this kind can be deduced abundantly from the above table of the modes. Truly besides thus also several of the individual species and systems of this kind admit variations not shown in the given table, which arise, if the indices may be combined with the exponents ; from which the maximum variety in the music may be introduced.

3. For just as the comparison of the diverse consonances may be established amongst themselves not only by the exponents, but also by the indices, thus the same mode with diverse indices requiring to be added together also may put diverse forms in place, which have not been expressed in the tables in the above chapters, where unity always maintains the place of the index. Here therefore, where we have established diverse modes and diverse systems to be prepared, and transitions among themselves from these into others to be put in place, we will attach an index of each mode and system.

4. But so that it may be understood, how a composition may be able to be made in as system, the exponent of which is joined with the index, we will begin with the indices which are powers of two. Therefore $\mathrm{E}\left(2^{n}\right)$ shall be the exponent of a system, for which $\mathrm{F}=2^{m}$; it is evident the composition for the E can happen and that then the n value of the octave must be returned more sharp. But as this shall be troubled by several inconveniences, the composition may be made in the system of the exponent E for the value $\mathrm{F}=2^{m-n}$; which will pertain also to the proposed system.

5. But if the index were not a power of two, but some other number p, the composition into the system, of which the exponent is E(p), for the case $\mathrm{F}=2^{m}$ will be done by composing in a system of the exponent E and then the individual tones by raising by the interval $1:p$. But since in this manner generally the notes may be raised exceedingly high, the power of two may be taken approximately to be p, which shall be 2^{k}, and thus

the composition may become in the system of exponents E(2*k*) according to the first case, so that from the product the whole composition may be transposed into the interval $2^k : p$. Therefore here on this account following the precepts of the preceding chapter in any system, of which the exponent is associated with an index, a musical composition will be able to be formed.

6. Therefore if a musical work may be composed from several parts, and each of which may be referred to a special system, then first of all the exponent of the whole musical work is required to be considered, which is the least common divisor of all the exponents of the system, which are used. Thus with this exponent assumed as it pleases, for this system the exponents of these may be deduced in turn, in a similar manner, where before the exponents had been derived from the exponent of the individual consonances of the system.

7. But whatever exponent chosen, by which the whole musical work may be contained, likewise too it will be required be determined, by which the note F may be indicated, which must remain unchanged in all the systems. Nor still only in that system, in which F may be designated by the power of two, are they found in such a musical work, but besides that also in all these, in which the value of F is smaller. But this happens on account of the indices joining with the exponents of the system, which, if they were equal, are reduced to a system, in which the smaller powers of two express the tone F ; just as it is understood from the account treated before of being placed into systems, of which the exponents have been joined by indices.

8. But before these systems may be defined, which are contained within an exponent in works of music, it will be agreed to enumerate the modes held in that exponent. Not only truly are the modes themselves to be examined required to reconsidered, by which the exponents may be shown, but also the individual variations of the same mode, which are indicated by the indices. Again species will be derived from these modes, which likewise on account of the value of F, for any composition of which given, provide systems to be established, just as now is being undertaken.

9. Truly the modes, if the simpler ones may be excluded, are to be expressed by the two exponents $2^n \cdot 3^3 \cdot 5$ and $2^n \cdot 3^2 \cdot 5^2$; now that mode, of which the exponent is $2^n \cdot 3^3 \cdot 5^2$, is considered to be composed from these two placed together. The former of these modes, $2^n \cdot 3^3 \cdot 5$, is called the *major mode* by musicians, truly the latter $2^n \cdot 3^2 \cdot 5^2$, the *minor mode*; and these are used by musician almost exclusively in their works. Moreover each of these modes includes several variations on themselves being added to the indices, which have been given special denominations by musicians, which can be seen from the table added below.

Major Modes

$2^n \cdot 3^3 \cdot 5(2^m)$	C Major
$2^n \cdot 3^3 \cdot 5(2^m \cdot 3)$	G Major
$2^n \cdot 3^3 \cdot 5(2^m \cdot 5)$	E Major
$2^n \cdot 3^3 \cdot 5(2^m \cdot 3^2)$	D Major
$2^n \cdot 3^3 \cdot 5(2^m \cdot 3 \cdot 5)$	H Major
$2^n \cdot 3^3 \cdot 5(2^m \cdot 3^3)$	A Major
$2^n \cdot 3^3 \cdot 5(2^m \cdot 3^2 \cdot 5)$	F*s* Major
$2^n \cdot 3^3 \cdot 5(2^m \cdot 3^4)$	E Major
$2^n \cdot 3^3 \cdot 5(2^m \cdot 3^3 \cdot 5)$	C*s* Major
$2^n \cdot 3^3 \cdot 5(2^m \cdot 3^4 \cdot 5)$	G*s* Major

Minor Modes

$2^n \cdot 3^2 \cdot 5^2(2^m)$	A Minor
$2^n \cdot 3^2 \cdot 5^2(2^m \cdot 3)$	E Minor
$2^n \cdot 3^2 \cdot 5^2(2^m \cdot 3^2)$	H Minor
$2^n \cdot 3^2 \cdot 5^2(2^m \cdot 3^3)$	F*s* Minor
$2^n \cdot 3^2 \cdot 5^2(2^m \cdot 3^4)$	C*s* Minor
$2^n \cdot 3^3 \cdot 5^2(2^m \cdot 3^5)$	G*s* Minor

10. Here we have reviewed only the variations of these modes, which are held by the exponent $2^n \cdot 3^7 \cdot 5^2$, for which we have observed now to be received to be used conveniently enough in the diatonic-chromatic genus, without any notable detriment to the harmony. Thus we will attribute these names to the variations of the modes themselves, since generally systems of each of these modes include these notes themselves which are agreed by musicians to constitute the *ambitus* of the modes named. Thus anyone who may consider general systems of the mode $2^n \cdot 3^3 \cdot 5(2^m)$ set out in the table, will seize upon the ambitus of the C Major mode so called by musicians and equally to contain the mode $2^n \cdot 3^2 \cdot 5^2(2^m)$ to agree with the ambitus of the mode A Minor.

11. Therefore so that it may be apparent, which exponents of this kind of two of these variations of the modes may be found in any musical work, which can be taken to be expressed for a whole musical work, we will consider which exponent $2^n \cdot 3^7 \cdot 5^2$ of the diatonic-chromatic kind can be taken in a wider sense, but must not exceed what we have now shown above. And thus $2^n \cdot 3^3 \cdot 5^2$ will be the simplest exponent, from which musical works will be able to be composed, in which certain variations of the modes are present; and hence it includes the four modes :

Modes	
$2^n \cdot 3^3 \cdot 5(2^m)$	C Major
$2^n \cdot 3^3 \cdot 5(2^m \cdot 5)$	E Major
$2^n \cdot 3^2 \cdot 5^2(2^m)$	A Minor
$2^n \cdot 3^2 \cdot 5^2(2^m \cdot 3)$	E Minor.

Truly the species of all these modes and of their variations will be produced, if in place of n and m successive individual numbers may be substituted, which sum $m+n$ cannot be returned greater than k.

12. Now the main variety in musical works of this kind can be had by interchanging the systems amongst themselves, so that scarcely a work may be seen to be worth the effort of more composite exponents. For besides, so that not only a sufficient variety may be contained in this exponent, but also with all the works of this kind received in the first place is in agreement with the diatonic-chromatic genus without any aberration, and it happens in accordance with more composite works. Also the permutation of these modes is used frequently by today's musicians, in the solemn works of which they have gone from the Major E mode to the E Minor, and with this in turn into C Major and that in turn to A Minor.

13. Because this kind of musical works, which is the simplest to use, and thus which deserves to be considered the most perfect, is followed by this, the exponent of which is $2^k \cdot 3^4 \cdot 5^2$, in which all the permutations of the modes and systems are dealt with, which indeed are accustomed to be used generally by musicians, thus so that almost all the musical works may be contained in this exponent, if clearly they may be transposed by the due mode. Indeed no one who wishes to examine musical works according to this standard, will consider the modes to be interchanged among themselves, but may bring together the mutual relation of these modes, just as with the mutual relation of the modes discussed here.

14. Moreover this exponent $2^k \cdot 3^4 \cdot 5^2$ is included in the following seven variations of the major and minor modes between themselves :

	Modes
$2^n \cdot 3^3 \cdot 5(2^m)$	C Major
$2^n \cdot 3^3 \cdot 5(2^m \cdot 3)$	G Major
$2^n \cdot 3^3 \cdot 5(2^m \cdot 5)$	E Major
$2^n \cdot 3^3 \cdot 5(2^m \cdot 3 \cdot 5)$	H Major
$2^n \cdot 3^2 \cdot 5^2(2^m)$	A Minor
$2^n \cdot 3^2 \cdot 5^2(2^m \cdot 3)$	E Minor
$2^n \cdot 3^2 \cdot 5^2(2^m \cdot 3^2)$	H Minor

Now anyone who may consider, how great a supply of species and systems may be contained in these modes, the most useful variation in this kind not only will be admired, but also it will be known other permutations of the modes not even to be used by musicians, thus so that it would be superfluous to consider more composite exponents.

15. But from the enumeration of these various modes and systems, which it is permitted to use in the composition of a whole musical work, the setting out is required, by which the most convenient modes may be interchanged amongst themselves and how the change from one mode to another may be performed. Indeed just as in the same mode all the consonances pertaining to that being shared generally will not be allowed to be taken together, but only these, which are related to each other and make pleasing successions, thus in the same manner in a composition of the various modes the changes between themselves must be pleasing.

16. Hence it is understood in the following thus two modes in turn are required to be prepared, so that they may have one or more consonances in common. For when such a consonance, which is common to each mode arises, then the first mode is able to finish conveniently, truly the latter will be able to begin, with this agreed on without a leap or intolerable delay being experienced. Also besides, with a pause introduced, or at the end of a principal part of the work, a new mode or theme can begin; for then the pause may be considered to fill a place in the common part of the consonance.

17. Therefore since the harmonic triades, which are contained in the exponent $2^n \cdot 3 \cdot 5$, shall be undertaken mainly by musicians, of which they are agreed in successive musical works, it is required to be seen, since they may have common consonances of this kind, and nevertheless, so that it may be observed, into which modes it may be able to move

from a given mode. But for the sake of brevity we may ignore the powers of two in this inquiry, both in the exponents as well as in the indices, since from these only the species can be varied.

$2^n \cdot 3^3 \cdot 5(2^m)$ C Major

Harmonic triades

$3\cdot 5(1) : 3\cdot 5(3) : 3\cdot 5(3^2)$

$2^n \cdot 3^3 \cdot 5(2^m \cdot 3)$ G Major

Harmonic triades

$3\cdot 5(3) : 3\cdot 5(3^2) : 3\cdot 5(3^3)$

$2^n \cdot 3^3 \cdot 5(2^m \cdot 5)$ E Major

Harmonic triades

$3\cdot 5(5) : 3\cdot 5(3\cdot 5) : 3\cdot 5(3^2 \cdot 5)$

$2^n \cdot 3^3 \cdot 5(2^m \cdot 3\cdot 5)$ H Major

Harmonic triades

$3\cdot 5(3\cdot 5) : 3\cdot 5\ (3^2 \cdot 5) : 3\cdot 5\ (3^3 \cdot 5)$

$2^n \cdot 3^2 \cdot 5^2(2^m)$ A Minor

Harmonic triades

$3\cdot 5(1) : 3\cdot 5(3) : 3\cdot 5(5) : 3\cdot 5(3\cdot 5)$

$2^n \cdot 3^2 \cdot 5^2(2^m \cdot 3)$ E Minor

Harmonic triades

$3\cdot 5(3) : 3\cdot 5(3^2) : 3\cdot 5\ (3\cdot 5) : 3\cdot 5(3^2 \cdot 5)$

$2^n \cdot 3^2 \cdot 5^2(2^m \cdot 3^2)$ H Minor

Harmonic triades

$3\cdot 5(3^2) : 3\cdot 5(3^3) : 3\cdot 5\ (3^2 \cdot 5) : 3\cdot 5(3^3 \cdot 5)$

18. Between these coupled together it will be apparent initially to be easy to pass over from the mode C Major to the mode G Major, and in turn, since these two may have triades in common, evidently $3\cdot 5(3)$ and $3\cdot 5(3^2)$; in the second place neither can a transition be given in turn from the mode C Major into the mode E Major, nor into H Major, since no common consonances shall be present. In the third place there will be

also an easy transition from the mode C Major into the mode A Minor, since each have the two common consonances $3\cdot 5(1)$ et $3\cdot 5(3)$. Fourthly equally easy will be the transition from the mode C Major into E Minor, since also they have the two common triades $3\cdot 5(3)$ and $3\cdot 5(3^2)$. Fifthly it is understood the transition from the mode C Major into H Minor to be more difficult, since only a single common consonance, namely $3\cdot 5(3^2)$, may intervene between these.

19. Similarly, since with regard to the mode G Major, in the first place it is seen neither a transfer to the mode E Major nor to H Major to be given, on account of no common consonance; secondly the transition is difficult from the mode G Major to A Minor, since each has the single common consonance $3\cdot 5(3)$. But the third transition may emerge easily from the mode G Major to E and to H Minor on account of each having two common consonances. Again the mode E Major has the easy transition into the mode H Major, equally too into the modes A and E Minor; since there are two common consonances in all places; truly the transition will be difficult from the mode E Major into the mode H Minor on account of the single common consonance.

20. But the transition from the mode H Major to the mode A Minor is exceedingly difficult both on account of a single common consonance, but also on account of very diverse systems, an account of which will be set out further soon. But a transition from the mode H Major into the modes E and H Minor will be easier on account of two common consonances. Again the transition from the mode A Minor into E Minor is easier, truly none into the mode H Minor; and finally an easy transition will be had from the mode E Minor into H Minor.

Truly all these may be had at a glance shown in this table:

	C Maj.	G Maj.	E Maj.	H Maj.	A Min.	E Min.	H Min.
C Maj.	–	easy	none	easy	none	easy	difficult
G Maj.	easy	–	none	none	difficult	easy	easy
E Maj.	none	none	–	easy	easy	easy	difficult
H Maj.	none	none	easy	–	difficult	easy	easy
A Min.	easy	difficult	easy	difficult	–	easy	none
E Min.	easy	easy	easy	easy	easy	–	easy
H Min.	difficult	easy	difficult	easy	easy	easy	–

Therefore it is observed a transition from the mode E Minor into all the remaining modes to be easy.

21. Hence moreover so much is understood, how many variations of the consonances of the same kind may have two common modes, from which indeed with enough care a judgement can be made for the transition from one mode into another. Truly if it may

happen, so that two modes, even if they may have common kinds of consonances, yet they do not allow common species, then the above judgement will have to cease. On this account not only the modes in general, as we have made here, but the species and systems of these are required to be considered, from which it may be apparent, whether each of these same consonances may have a place. And with this done it may be concluded finally, what kind of transitions may be allowed and how.

22. Anyone who will consider it worthwhile to unite everything from this account with these works of the musicians of today, will come upon a greater similarity there, so that he will devote more in a comparison of the studies. There is no doubt why our theory of music may not provide the occasion for expert artists to advance this science with the aid of the true theory, even now unknown, to be carried forwards to a greater level of perfection.

FINIS

CAPUT XIV

DE MODORUM ET SYSTEMATUM PERMUTATIONE

1. Quantumvis etiam multiplex sit varietas, quae in unico systemate locum habet, tamen, si idem systema diutius retineatur, fastidium potius quam delectationem pariat necesse est. Cum enim musica tam varietatem quam suavitatem in sonis et consonantiis requirat, saepius obiectum auditus permutandum est. Quemadmodum igitur per compositionem in capite praecedente traditam exponens systematis auditui repraesentatur, ita, cum is iam satis fuerit perspectus, ad aliud systema transitus fieri debebit.

2. Mutatio autem haec plurimis modis fieri potest; primo enim systema solum varias mutationes admittit, manentibus modo eiusque specie invariatis. Deinde sensibilior fiet mutatio, si in aliam speciem modi vel alium etiam modum transitus fiat; cuiusmodi mutationes ex superiori tabula modorum et systematum abunde colligi possunt. Praeterea vero ipsi modi atque adeo etiam singulae eorum species et systemata plures admittunt variationes in tabula data non exhibitas, quae oriuntur, si indices cum exponentibus coniungantur; unde maxima varietas in musicam inducitur.

3. Quemadmodum enim diversarum consonantiarum comparatio inter se non per solos exponentes, sed etiam per indices instituitur, ita etiam idem modus diversis indicibus adiungendis diversas formas induit, quae in tabula superioris capitis non sunt expressae,

ubi perpetuo unitas indicuum locum tenet. Hic igitur, ubi diversos modos diversaque systemata inter se comparare atque transitiones ex aliis in alia exponere instituimus, ad exponentem cuiusque modi et systematis indicem annectemus.

4. Quo autem intelligatur, quomodo compositio in systemate, cuius exponens cum indice est coniunctus, fieri debeat, ab indicibus, qui sunt binarii potestates, ordiemur. Sit igitur $\mathrm{E}\left(2^{n}\right)$ exponens systematis, pro quo est $\mathrm{F}=2^{m}$; manifestum est compositionem pro exponente E fieri posse eamque tum *n* octavis acutiorem reddi debere. Hoc autem cum pluribus incommodis sit obnoxium, compositio fiat in systemate exponentis E pro valore $\mathrm{F}=2^{m-n}$; quae pariter ad propositum systema pertinebit.

5. Si autem index non fuerit potestas binarii, sed quivis alius numerus *p,* compositio in systemate, cuius exponens est E*(p),* pro casu $\mathrm{F}=2^{m}$ fiet componendo in systemate exponentis E tumque singulos sonos intervallo 1 : *p* elevando. Cum autem hoc modo plerumque ad sonos nimis acutos perveniatur, sumatur potentia binarii ipsi *p* proxima, quae sit 2^{k}, atque compositio fiat in systemate exponentis E(2*k*) secundum casum priorem, quo facto tota compositio transponatur intervallo $2^{k}:p$. Hac itaque ratione secundum praecepta praecedentis capitis in quolibet systemate, cuius exponens cum indice est coniunctus, compositio musica formari poterit.

6. Si igitur opus musicum ex pluribus partibus constet, quarum quaeque ad peculiare systema referatur, tum ante omnia exponens totius operis musici est considerandus, qui est minimus communis dividuus omnium exponentium systematum, quae usurpantur. Ex hoc itaque exponente pro lubitu assumto ipsa systemata eorumque exponentes vicissim deducentur, pari modo, quo ante ex exponente systematis singularum consonantiarum exponentes sunt derivati.

7. Electo autem pro arbitrio exponente, quo integrum opus musicum componendum contineatur, simul quoque potestatem binarii determinatam esse oportet, qua sonus F indicatur, quaeque in omnibus systematibus invariata manere debet. Neque tamen ideo ea systemata sola, in quibus F eadem binarii potestate designatur, in tali opere musico locum inveniunt, sed praeter ea etiam omnia illa, in quibus valor ipsius F est minor. Accidit autem hoc propter indices cum exponentibus systematum coniunctos, qui, si pares fuerint, ad systemata reducuntur, in quibus minores binarii potestates sonum F exprimunt; quemadmodum ex ante tradita ratione componendi in systematibus, quorum exponentes cum indicibus sunt coniuncti, intelligitur.

8. Antequam autem ipsa systemata, quae in operis musici exponente continentur, definiantur, modos in eo exponente contentos enumerari convenit. Non solum vero ipsi modi in se spectati, quatenus exponentibus exhibentur, sunt recensendi, sed singulae etiam eiusdem modi variationes, quae per indices indicantur. Ex modis porro

derivabuntur species, quae simul ob valorem ipsius F datum systemata praebent, pro quorum quolibet compositio, prout iam est praeceptum, instituenda est.

9. Modi vero, si simpliciores excipiantur, praecipue sunt duo exponentibus $2^n \cdot 3^3 \cdot 5$ et $2^n \cdot 3^2 \cdot 5^2$ expressi; nam ille modus, cuius exponens est $2^n \cdot 3^3 \cdot 5^2$, ex his duobus compositus est censendus. Horum modorum prior $2^n \cdot 3^3 \cdot 5$ a Musicis *modus durus,* posterior vero $2^n \cdot 3^2 \cdot 5^2$ *modus mollis* appellatur; hisce fere solis Musici in suis operibus utuntur. Uterque autem horum modorum plures variationes indicibus adiungendis complectitur, quae a Musicis peculiares denominationes obtinuerunt, quas ex subiuncta tabella videre licet.

Modi duri

$2^n \cdot 3^3 \cdot 5(2^m)$	Modus C durus
$2^n \cdot 3^3 \cdot 5(2^m \cdot 3)$	Modus G durus
$2^n \cdot 3^3 \cdot 5(2^m \cdot 5)$	Modus E durus
$2^n \cdot 3^3 \cdot 5(2^m \cdot 3^2)$	Modus D durus
$2^n \cdot 3^3 \cdot 5(2^m \cdot 3 \cdot 5)$	Modus H durus
$2^n \cdot 3^3 \cdot 5(2^m \cdot 3^3)$	Modus A durus
$2^n \cdot 3^3 \cdot 5(2^m \cdot 3^2 \cdot 5)$	Modus F*s* durus
$2^n \cdot 3^3 \cdot 5(2^m \cdot 3^4)$	Modus E durus
$2^n \cdot 3^3 \cdot 5(2^m \cdot 3^3 \cdot 5)$	Modus C*s* durus
$2^n \cdot 3^3 \cdot 5(2^m \cdot 3^4 \cdot 5)$	Modus G*s* durus

Modi molles

$2^n \cdot 3^2 \cdot 5^2(2^m)$	Modus A mollis
$2^n \cdot 3^2 \cdot 5^2(2^m \cdot 3)$	Modus E mollis
$2^n \cdot 3^2 \cdot 5^2(2^m \cdot 3^2)$	Modus H mollis
$2^n \cdot 3^2 \cdot 5^2(2^m \cdot 3^3)$	Modus F*s* mollis
$2^n \cdot 3^2 \cdot 5^2(2^m \cdot 3^4)$	Modus C*s* mollis
$2^n \cdot 3^3 \cdot 5^2(2^m \cdot 3^5)$	Modus G*s* mollis

10. Hic eas tantum modorum variationes recensuimus, quae in exponente $2^n \cdot 3^7 \cdot 5^2$ continentur, ad quem genus diatonico-chromaticum nunc usu receptum satis commode et sine notabili harmoniae detrimento adhiberi posse adnotavimus. Ideo autem haec nomina

istis modorum variationibus tribuimus, quia pleraque cuiusque horum modorum systemata eos ipsos sonos complectuntur qui a Musicis ambitus modorum nominatorum constituere censentur. Ita qui modi $2^n \cdot 3^3 \cdot 5(2^m)$ pleraque systemata in tabula exposita contemplatur, deprehendet iis ambitum modi C duri a Musicis ita vocati contineri pariterque modum $2^n \cdot 3^2 \cdot 5^2(2^m)$ cum ambitu modi A mollis congruere.

11. Quo igitur appareat, cuiusmodi binorum horum modorum variationes in quolibet opere musico locum inveniant, exponentes, qui ad integra opera musica exprimenda accipi possunt, consideremus, quos exponentem $2^n \cdot 3^7 \cdot 5^2$ generis diatonico-chromatici latiori sensu accepti non superare debere iam supra ostendimus. Erit itaque $2^n \cdot 3^3 \cdot 5^2$ simplicissimus exponens, ex quo opera musica, in quibus quidem modorum variationes insunt, componi possunt; hincque sequentes quatuor modos in se complectitur:

$2^n \cdot 3^3 \cdot 5(2^m)$	Modus C durus
$2^n \cdot 3^3 \cdot 5(2^m \cdot 5)$	Modus E durus
$2^n \cdot 3^2 \cdot 5^2(2^m)$	Modus A mollis
$2^n \cdot 3^2 \cdot 5^2(2^m \cdot 3)$	Modus E mollis.

Species vero omnes horum modorum eorumque variationum prodibunt, si loco *n* et *m* successive singuli numeri integri substituantur, quae aggregatum $m+n$ non maius reddant quam *k*.

12. In huius ergo generis operibus musicis iam summa varietas in permutandis systematibus inter se locum habere potest, ut vix opus esse videatur opera musica magis compositorum exponentium requirere. Praeterquam enim, quod sufficiens varietas in hoc exponente contineatur, omnibus etiam huiusmodi operibus genus diatonico-chromaticum receptum apprime congruit sine ulla aberratione, secus ac contingit in operibus magis compositis. A Musicis etiam hodiernis horum modorum permutatio frequenter adhibetur, in quorum operibus solennes sunt transitus ex modo E duro in E mollem ex hocque in C durum et A mollem et vicissim.

13. Hoc genus operum musicorum, quod, uti est simplicissimum, ita perfectissimum spectari meretur, sequitur hoc, cuius exponens est $2^k \cdot 3^4 \cdot 5^2$, in quo omnes modorum et systematum permutationes comprehenduntur, quae quidem a Musicis plerumque adhiberi

soient, ita ut in hoc exponente fere omnia opera musica contineantur, si scilicet debito modo transponantur. Non enim, qui opera musica ad hanc normam examinare cupit, ipsos modos per se permutatos consideret, sed eorum relationem mutuam, quam cum mutua relatione modorum hic exhibitorum conferat.

14. Complectitur autem iste exponens $2^k \cdot 3^4 \cdot 5^2$ in se sequentes septem modorum duri et mollis variationes:

$2^n \cdot 3^3 \cdot 5(2^m)$	Modus C durus
$2^n \cdot 3^3 \cdot 5(2^m \cdot 3)$	Modus G durus
$2^n \cdot 3^3 \cdot 5(2^m \cdot 5)$	Modus E durus
$2^n \cdot 3^3 \cdot 5(2^m \cdot 3 \cdot 5)$	Modus H durus
$2^n \cdot 3^2 \cdot 5^2(2^m)$	Modus A mollis
$2^n \cdot 3^2 \cdot 5^2(2^m \cdot 3)$	Modus E mollis
$2^n \cdot 3^2 \cdot 5^2(2^m \cdot 3^2)$	Modus H mollis

Qui nunc contempletur, quanta specierum et systematum copia in his modis contineatur, summam varietatem in hoc genere non solum admirabitur, sed etiam agnoscet alias modorum permutationes a Musicis nequidem usurpari, ita ut superfluum foret exponentes magis compositos considerare.

15. Enumeratis autem variis modis et systematibus, quibus in componendo integra opere musico uti licet, exponendum est, quinam modi commodissime inter se permutentur et quomodo transitus ex uno modo in alium fieri debeat. Quemadmodum enim in eodem modo non licet omnes consonantias eo pertinentes promiscue inter se coniungere, sed eas tantum, quae sibi sunt affines atque successiones gratas efficiant, ita simili modo in compositione variorum modorum transitus inter ipsos gratus esse debet.

16. Hinc intelligitur binos modos se invicem subsequentes ita esse oportere comparatos, ut unam pluresve consonantias inter se habeant communes. Quando enim ad talem consonantiam, quae utrique modo communis est, pervenitur, tum commode prior modus finiri, posterior vero inchoari poterit, neque saltus seu lacuna intolerabilis hoc pacto sentietur. Praeterea etiam pausa interposita vel principali operis parte finita novus modus incipi potest; tum enim pausa consonantiae communis locum implere censetur.

17. Cum igitur triades harmonicae, quae exponente $2^n \cdot 3 \cdot 5$ continentur, a Musicis sint potissimum receptae, quarum successione opera musica constant, videndum est, quinam modi communes habeant eiusmodi consonantias, quinamque minus, quo perspiciatur, in quosnam modos ex modo dato transitus fieri queat. Negligemus autem in hac disquisitione brevitatis gratia binarii potestates, tam in exponentibus quam indicibus, quia iis tantum species variantur.

$2^n \cdot 3^3 \cdot 5(2^m)$ Modus C durus

Triades harmonicae

$3\cdot 5(1):3\cdot 5(3):3\cdot 5(3^2)$

$2^n \cdot 3^3 \cdot 5(2^m \cdot 3)$ Modus G durus

Triades harmonicae

$3\cdot 5(3):3\cdot 5\left(3^2\right):3\cdot 5(3^3)$

$2^n \cdot 3^3 \cdot 5(2^m \cdot 5)$ Modus E durus

Triades harmonicae

$3\cdot 5(5):3\cdot 5(3\cdot 5):3\cdot 5(3^2 \cdot 5)$

$2^n \cdot 3^3 \cdot 5(2^m \cdot 3 \cdot 5)$ Modus H durus

Triades harmonicae

$3\cdot 5(3\cdot 5):3\cdot 5\ (3^2 \cdot 5):3\cdot 5\ (3^3 \cdot 5)$

$2^n \cdot 3^2 \cdot 5^2(2^m)$ Modus A mollis

Triades harmonicae

$3\cdot 5(1):3\cdot 5(3):3\cdot 5(5):3\cdot 5(3\cdot 5)$

$2^n \cdot 3^2 \cdot 5^2(2^m \cdot 3)$ Modus E mollis

Triades harmonicae

$3\cdot 5(3):3\cdot 5(3^2):3\cdot 5\ (3\cdot 5):3\cdot 5(3^2 \cdot 5)$

$2^n \cdot 3^2 \cdot 5^2(2^m \cdot 3^2)$ Modus H mollis

Triades harmonicae

$3\cdot 5\left(3^2\right):3\cdot 5(3^3):3\cdot 5\ \left(3^2 \cdot 5\right):3\cdot 5(3^3 \cdot 5)$

18. His inter se comparatis patebit primo ex modo C duro facile esse in modum G durum transire atque vicissim, cum duas habeant triades communes, scilicet $3\cdot 5(3)$ et $3\cdot 5(3^2)$; secundo ex modo C duro neque in modum E durum neque H durum transitum dari neque vicissim, cum nulla adsit consonantia communis. Tertio facilis erit quoque transitus ex modo C duro in modum A mollem, quia duae consonantiae $3\cdot 5(1)$ et $3\cdot 5(3)$ utrique sunt communes. Quarto aeque facilis erit transitus ex modo C duro in E mollem, quia etiam duae triades $3\cdot 5(3)$ et $3\cdot 5(3^2)$ ipsis sunt communes. Quinto intelligitur

transitum ex modo C duro in H mollem difficiliorem esse, cum unica tantum consonantia communis, nempe $3\cdot 5(3^2)$, inter eos intercedat.

19. Similiter, quod ad modum G durum attinet, perspicitur primo ex eo neque in modum E durum neque H durum transitum dari, ob nullam consonantiam communem; secundo difficilem esse transitum ex modo G duro in A mollem, ob unicam consonantiam $3\cdot 5(3)$ utrique communem. At tertio transitus facilis evadet ex modo G duro in E et H molles ob duas utrinque consonantias communes. Modus porro E durus facilem habet transitum in modum H durum, pariter quoque modos A et E molles; quia ubique duae consonantiae sunt communes; difficilis vero erit transitus ex modo E duro in modum H mollem propter unicam consonantiam communem.

20. Ex modo autem H duro difficilis admodum est transitus in modum A mollem tam ob unicam consonantiam communem, quam ob systemata nimis diversa, quorum ratio mox fusius exponetur. At in modos E et H molles facilius ex modo H duro transibitur ob duas consonantias communes. Porro facilis est transitus ex modo A molli in E mollem, nullus vero in modum H mollem; facilis denique habebitur transitus ex modo E molli in H mollem.
Haec vero omnia uno conspectu in tabula hac repraesentantur:

	C dur.	G dur.	E dur.	H dur.	A moll.	E moll.	H moll.
C dur.	–	facilis	nullus	facilis	nullus	facilis	difficilis
G dur.	facilis	–	nullus	nullus	difficilis	facilis	facilis
E dur.	nullus	nullus	–	facilis	facilis	facilis	difficilis
H dur.	nullus	nullus	facilis	–	difficilis	facilis	facilis
A moll.	facilis	difficilis	facilis	difficilis	–	facilis	nullus
E moll.	facilis	facilis	facilis	facilis	facilis	–	facilis
H moll.	difficilis	facilis	difficilis	facilis	nullus	facilis	–

Perspicuum ergo est ex modo E molli in omnes reliquos transitum esse facilem.

21. Hinc autem tantum intelligitur, quotnam eiusdem generis consonantiarum variationes bini modi habeant communes, unde quidem satis tuto iudicium de transitu ex alio modo in alium formari potest. Verum si accidat, ut duo modi, etiamsi consonantiarum genera habeant communia, tamen species communes non admittant, tum superius iudicium cessare debebit. Hanc ob rem non solum modi in genere, ut hic fecimus, sed ipsorum species et systemata sunt consideranda, quo pateat, utrum in iis consonantiae eaedem locum habeant. Hocque facto demum concludatur, quales transitus admittantur et quomodo.

22. Qui haec omnia cum Musicorum · hodiernorum ratione componendi ipsorumque operibus conferre dignabitur, eo maiorem congruentiam deprehendet, quo plus studii in

comparationem impendet. Quamobrem non dubito quin haec nostra de musica theoria expertis artificibus occasionem sit praebitura hanc scientiam ope verae theoriae etiamnum ignoratae ad maiorem perfectionis gradum evehendi.

FINIS