

CHAPTER XI

CONSONANCES

IN THE DIATONIC-CHROMATIC GENUS

1. It has been shown clearly in the preceding chapter, §16, which tones may be present in the diatonic-chromatic genus, where not only have the notes been defined which the keys of the instruments themselves may indicate, but also the secondary tones, which the same keys are able to produce conveniently enough. Now therefore we may progress to the consonances and we will set out the genus, for which the diatonic-chromatic genus may be adapted to express consonances, and besides, by which keys it may be agreed to represent whichever consonance.
2. When two tones either may be raised or lowered by an octave, truly the tones may be had an octave apart or differ by an octave, even if they are not for the same tones, yet may be considered as similar tones on account of the ratio of the consonances, of which the exponents unless they may differ by a power of two, may not be agreed to be considered as similar tones. Therefore a collection of consonances of this kind will be called by the special name of the consonances. Thus for example  $2^m \cdot 3 \cdot 5$  establishes a certain class of consonances, and by substituting definite numbers in place of  $m$  particular consonances will be produced establishing this kind.
3. Consequently therefore we may express the consonances of this kind in the form  $2^m \cdot A$ , in which  $m$  may indicate an indefinite number and  $A$  truly is a definite odd number. Moreover these consonances to be included comprehensively by this kind will be determined from these exponents  $A, 2A, 2^2A, 2^3A, 2^4A$  etc. Indeed the consonances constituting these tones will be expressed by the same letters and will be present only in different octaves, by which the tones of these consonances will differ from each other in turn ; which difference will not change the nature of the consonances very much.
4. Yet meanwhile these consonances contained within a single kind cannot be considered to be completely the same ; for they certainly differ on account of the charm, by which each is perceived by the listener. Thus if the consonance of the exponent  $A$  may pertain to the degree of agreeability  $n$ , then the consonance  $2A$  will refer to the degree  $n+1$ , the consonance  $2^2A$  to the degree  $n+2$ , the consonance  $2^3A$  to the degree  $n+3$  etc. On account of which the simplest consonance of the same kind and the easiest to be perceived will be that which has the exponent  $A$ , following that in the order of agreeability will be  $2A$ , then truly  $2^2A$  and thus so forth.
5. Whereby therefore the greater the number becomes to be substituted in place of  $m$  in the exponent of the consonance of the kind  $2^m \cdot A$ , the greater the consonance to be

composed from that the more difficult to be perceived by the hearer. Therefore since our faculty of perception may not be extended beyond a certain limit, a limit in the order of pleasantness is required to be established, beyond which more consonances should not be allowed to be produced. But such a limit cannot be established other than by experience ; truly it is agreed by musicians more composite consonances very rarely to be accustomed to be used, than which may pertain to the level XII, and if from such they may be used, thus seen not requiring approval. Therefore if this limit may be put in place by us, which higher consonances shall be forbidden and to be dismissed from harmony.

6. Therefore so that now we will enumerate and present the consonances, which are to be found in our diatonic-chromatic genus, for exponents are required to be accepted from these of this kind, which will be contained in the exponent of the genus  $2^m \cdot 3^3 \cdot 5^2$ . Even if also this genus may be satisfied by the exponent  $2^m \cdot 3^7 \cdot 5^2$ , yet on account of the reason established above the consonances may not be able to be used, which may not be present in  $2^m \cdot 3^3 \cdot 5^2$ . Therefore we will have the twelve following kinds of consonances :

I. $2^m$	V. $2^m \cdot 3 \cdot 5$	IX. $2^m \cdot 3 \cdot 5^2$
II. $2^m \cdot 3$	VI. $2^m \cdot 5^2$	X. $2^m \cdot 3^3 \cdot 5$
III. $2^m \cdot 5$	VII. $2^m \cdot 3^3$	XI. $2^m \cdot 3^2 \cdot 5^2$
IV. $2^m \cdot 3^2$	VIII. $2^m \cdot 3^2 \cdot 5$	XII. $2^m \cdot 3^3 \cdot 5^2$ .

7. Indeed these kinds of consonances, if indices may be added to the exponents above, and they are able to occur in several forms. Indeed whatever the exponent of the genus  $2^m \cdot A$ ,  $B$  will be able to be determined by some index, so that the kind may be expressed in this manner  $2^m A(B)$ , provided  $2^m AB$  were a divisor of  $2^m \cdot 3^7 \cdot 5^2$ , if indeed it may be conceded for the diatonic-chromatic to be extended this far. But since the base of each consonance shall denote a single tone by unity, there will be  $2^m A(B)$  consonances of the base  $B$ ; thus so that, in whatever way the index  $B$  may be varied, the consonances expressed by  $2^m A(B)$  only differ in the determination by the bases.

[Thus base 3 has 4 possibilities, while base 5 has 3 possibilities : yet only those may be chosen for which  $2^m A(B)$  is a divisor of  $2^m \cdot 3^7 \cdot 5^2$  ]

8. But since here only consonances shall be treated amongst themselves by us, truly with these indices that may not be changed, here we will ignore the indices or rather we may put one for the index. For the consonances described in this manner will be easy to transform into some index, by substituting in place of the tone designated by unity, the tone expressed by the index and in place of the others remaining to the base with the same separation of the intervals. Therefore since the letter  $F$  may give 1 tone or some

whole number of octaves distant from the tone  $F$ , in this chapter the base will be either the tone  $F$  or some octave lower than  $F$ .

9. Therefore in all the consonances which we will represent here, the tone or the note  $F$  will be indicated either by unity or by some power of two, just as the circumstances will demand. For it is seen to show all the consonances within three octave intervals, thus so that all the tones either lower than  $F$  or higher than  $\bar{\bar{f}}$  we may be going to ignore. Therefore since following this tradition we may seldom be able to show complete consonances, but only the first, second, fourth, etc. will denote the key  $F$ , from which we may obtain all the forms, by which certain consonances are able to be prepared within the interval of the three prescribed intervals.

10. For these notes requiring to be expressed we will make use of two orders of pentagrams [*i.e.* the staff or staves], the first of which is made in the descant clef [or treble key, indicating  $C$ ], and the other in the bass clef [indicating  $F$ ], and in which we will thus represent these consonances in the customary manner, so that all the notes may be contained between these pentagrams. And this also is the reason, why we shall be unable to have sounds either lower than  $F$  or higher than  $\bar{\bar{f}}$ . Nor truly also can a wider space be assumed on account of the other tones being required to be substituted in the latter in place of  $F$ , lest more consonances successively greater than four intervals of the octave may be required.

11. Therefore for this reason, we have copied out the consonances of each kind in the usual manner according to the order of pleasing of the music notes. Indeed above we have added the exponent of the described consonances, between the pentagrams we have added the order of the pleasantness and below we have added the numbers, by which in each consonance the tone  $F$  may be indicated. Besides the consonances in the first part of this table we have produced consonances as far as to order XII as they are used the most often; yet below we have continued as far as to the consonances for order XV, which actually may be taken for dissonances. And indeed several kinds may not be allowed to continue to that stage on account of the exceedingly narrow interval, in which the more composite consonances may be able to be represented. Thus the consonance of the first kind  $2^3$  cannot be shown within the interval of the three octaves and much less the following sequences of consonances, on account of which these also have been omitted.

12. Therefore this table begins with the unison or most simple tone, which certainly is the simplest of the consonances. This is followed by the said octave consonance, of which the two tones constitute that octave distant in turn by the interval of an octave, and from this follows the most simple consonance, which is perceived the most easily and for which on being produced two strings alone can be tuned most easily by hearing only. The third consonance consists of three tones and its tones in turn are an octave apart and thus produce a pleasing harmony. And these are the consonances of the first kind, more of which lie outside the interval of the three octaves.

*Species I*                      *Species II*                      *Species III*

1   2   2<sup>2</sup>                      3   2·3   2<sup>2</sup>·3   2<sup>3</sup>·3                      5   2·5   2<sup>2</sup>·5                      2<sup>3</sup>·5   2<sup>4</sup>·5

I   II   III                      III   IV   V   VI                      V   VI                      VII   VIII   IX

1   1   1                      1   1   1   2   2                      1   1   2                      1   2   2<sup>2</sup>   2   2<sup>2</sup>   2<sup>2</sup>

13. The second kind includes these consonances, in which besides the octave interval the fifth and fourth intervals occur. Which indeed pertains to the fifth, it is apparent that to be the simplest produced, thus so that the octave with the fifth not only itself provides more pleasure to the listener than the simple fifth, but also may be used for tuning instruments with greater success. Evidently with the tone *F* established from that it will be much easier to form the tone  $\bar{c}$  rather than *c*. On account of which it may be wished to tune musical instruments by hearing not the simple fifth, but the octave it may form with the fifth, from which considerable aid may be perceived. The remaining consonances of this kind occur frequently and certainly have been appreciated by the listener.

*Species IV*

2·3<sup>3</sup>   2<sup>2</sup>·3<sup>2</sup>   2<sup>3</sup>·3<sup>2</sup>                      2<sup>4</sup>·3<sup>2</sup>   2<sup>5</sup>·3<sup>2</sup>

VI                      VII                      VIII                      IX                      X

2   2   2<sup>2</sup>   2   2<sup>2</sup>   2<sup>3</sup>                      2<sup>3</sup>·2<sup>3</sup>   2<sup>3</sup>

14. The simplest consonance of the third kind is the twofold octave with the major third, which interval is much more pleasant to be heard rather than either the simple major third or the octave with a major third. On this account the business for musical instruments being well tuned will be arranged for two octaves with major thirds to be formed, rather than from simple major thirds; or if the tones may appear to be exceedingly far apart, octaves with major thirds according to this plan may perhaps be able to be used. Therefore with these aids in the tuning of musical instruments according to the rules treated above may be agreed to be of the greatest use, from which the prescribed operation thus will be easier and rendered more exact.

15. Therefore these are the three most simple kinds, in the first of which there is only a single tone, in the remaining only two tones occur, certainly if one or more octaves differing from each other by an octave may be considered as the same; and on this account rarely to be used unless in diaphonic compositions [*i.e.* music played by two parts] on account of so great a simplification. Truly the following kinds include a more copious number of tones, so that they may be suited for polyphonic music. The fourth genus is of this kind, in the consonances of which the three tones *F*, *C* and *G* are found ; but more often musicians use this kind, when they add to the bass either a fifth with a second, or a seventh with a fourth; which consonances indeed are accustomed to be called dissonances by musicians, not so much because there is less charm, but for which the following kind with the three first parts are accustomed to be called consonances.

16. Therefore the fifth species follows, which may supply both more consonances, as well as several dissonances to the music. Such consonances are chiefly the two, which are evident from the start of this kind of series, of which the first may be constructed from the tones *F*, *A*, *C*, or otherwise from the tones *A*, *C*, *E*. And these two consonances, in whatever order the tones may be arranged, are accustomed to be called *harmonic triades*. But the *Triades* are called *principal* if the tones were disposed thus, so that to the lowest may differ from one remaining either by a major or minor third, and truly by a fifth from the other. Therefore from these principal triades smaller principal triades may arise, if the tones may be placed in another order.

*Species V*

The diagram shows two staves of music, treble and bass clef, with a brace on the left. Above the treble staff are six groups of notes, each with an interval ratio above it: 3 · 5, 2 · 3 · 5, 2<sup>2</sup> · 3 · 5, 2<sup>3</sup> · 3 · 5, 2<sup>4</sup> · 3 · 5, and 2<sup>5</sup> · 3 · 5. Below the bass staff are six groups of notes, each with an interval ratio below it: 1, 2, 1, 2, 2<sup>2</sup>, 2, 2<sup>2</sup>, 2<sup>3</sup>, 2, 2<sup>2</sup>, 2<sup>3</sup>, 2<sup>2</sup>, 2<sup>3</sup>, and 2<sup>3</sup>. The groups are labeled VII, VIII, IX, X, XI, and XII between the staves.

17. Again these harmonic triades truly now may be called major or minor [*i.e.* *hard* or *soft* in the original], depending on whether a major or minor third is joined with the fifth; therefore the triad *F*, *A*, *C*, is hard, truly *A*, *C*, *E* soft. Therefore how the pleasantness of each triad of the tones may be required to be set out, is clearly seen from the table, likewise from which it is apparent, how much charm may be relinquished, if the tones may be set out in another order. From which the most suitable consonance or *chord*, as it is accustomed to be called by musicians, several will be treated in the manner expressed below.

18. Besides these two triades this same fifth species contains several dissonances, thus so called by musicians, which may be seen from each part of the table. For musicians in composing works are accustomed to treat triades only for the consonances to be used, both hard as well as soft, and to fill the maximum part of the work with these ; truly all the remaining consonants, which they only combine with these, they treat as secondaries and give the name of dissonances, since sometimes they may have just as much charm as triades, indeed whenever these are accustomed to be performed.

19. The consonances of the sixth species are exceedingly harsh, since the most simple, which can be expressed within the interval of the three octaves, must rise to the eleventh order; therefore it is hardly ever used by musicians and it is agreed that these are to be used most rarely. So that the seventh as well as the eighth consonances are more tolerable and can be intermingles most agreeably with simpler consonances. Truly the ninth and tenth species on account of excessive harshness are not to be used except with the greatest circumspection. Of the remaining two species indeed there cannot be shown consonances, which will not exceed the twelfth order; therefore the consonances of these kinds or rather the dissonances are to be sought in another part of the table.



one or more octaves added and for this reason, if perhaps circumstances may prevent the simple tone from being used, the upper tone used must be as close as possible to the bass.

22. If a major third were prescribed, its place is not the simple tone, but it is fitting to be increased by two octaves ; truly on the other hand a minor *e* is more pleasing to be heard, if either the simple tone may be taken or perhaps minimally removed from the bass. Again the sixth both major as well as minor are more pleasing, where they take smaller distances from the bass. In a similar manner the minor seventh is required to be close to the bass or the simple tone more distant ; truly the major seventh, where it is at a greater distance from the base, thus will be more pleasing. It is agreed for the second major in a major tone to be maximally distant from the bass, truly that, which may be present in the minor tone, must be minimally distinct from the bass. In a similar manner the second minor, where it may be closer to the bass, thus will be more pleasant. And finally the third, where is taken further from the bass, there will have its charm disturbed less.

23. Therefore these rules are required to be observed, if a single tone must be added to the bass, which indeed hardly ever arises in use ; yet meanwhile these rules equally retain their use, if several tones must be added to the bass; indeed for any number they are required to be observed to prevail the same, even if only a single tone may be present. But just as the tones, if several were prescribed to the base, must express the greatest degree of pleasantness, here this will be able to be seen from the table added below, which has been formed from the above table only with some deeper tones rejected, so that each low tone of the bass may be present.

24. But for this to be expressed distinctly there was a need for three sets of five lined staves, in the lowest of which the bass notes alone may be represented with superscribed numbers, as is accustomed to be done with the bass continuo, or in the general bass ; the two remaining staves truly contain whole consonances, by which the numbers ascribed to the bass may be expressed most conveniently and pleasantly. Here indeed we have made use of an empty scale, but it will be easy by the transposition of this table used to be adapted to any other scale and other tones. As before we may distinguish the order of the agreeability and also we have noted the species, to which each consonance pertains. Finally this table consists of two parts, in the first of which the consonances are enumerated as far as to the tenth kind, in the latter of the two remaining the kinds of consonances have been enumerated.

Chapter 11 of Euler's E33:  
 TENTAMEN NOVAE THEORIAE.....  
 Translated from Latin by Ian Bruce; 4/28/2019.  
 Free download at 17centurymaths.com.

The image displays musical notation for Euler's species VIII, consisting of two staves. The notation is organized into three groups corresponding to Species VI, VII, and VIII. Each group contains three chords, labeled XIII, XIV, and XV. The chords are represented by notes on a staff with a treble clef and a key signature of one sharp (F#). Below each chord, its prime factorization is given. The bottom staff shows the same chords with their prime factorizations written below them.

Species	Chord Label	Prime Factorization
Species VI	XIII	$2^4 \cdot 5^2$
	XIV	$2^5 \cdot 5^2$
	XV	$2^6 \cdot 5^2$
Species VII	XIII	$2^6 \cdot 3^3$
	XIII	$2^4 \cdot 3^2 \cdot 5$
	XIV	$2^5 \cdot 3^2 \cdot 5$
Species VIII	XV	$2^6 \cdot 3^2 \cdot 5$
	XIV	$2^3$
	XV	$2^3$

*Species IX* *Species X*

$2^2 \cdot 3 \cdot 5^2$   $2^3 \cdot 3 \cdot 5^2$   $2^4 \cdot 3 \cdot 5^2$   $2^2 \cdot 3^3 \cdot 5$   $2^3 \cdot 3^3 \cdot 5$   $2^4 \cdot 3^3 \cdot 5$

XIII XIV XV XIII XIV XV

$2^2$   $2^3$   $2^2$   $2^3$   $2^2$   $2^3$   $2^4$   $2^5$   $2^2$   $2^3$   $2^4$   $2^2$   $2^3$   $2^4$   $2^5$   $2^2$   $2^3$   $2^4$   $2^5$   $2^6$

  

*Species XI* *Species XII*

$3^2 \cdot 5^2$   $2 \cdot 3^2 \cdot 5^2$   $2^2 \cdot 3^2 \cdot 5^2$   $3^3 \cdot 5^2$

XIII XIV XV XV

$2^2$   $2^3$   $2^2$   $2^3$   $2^4$   $2^2$   $2^3$   $2^4$   $2^5$   $2^2$   $2^3$   $2^4$

  

*Species I* *Species II* *Species III* *Species IV*

II III V V VIII VI VII IX

8 5 4 3 6 5 5 7

2 4 4

*Species V*                      *Species VI*                      *Species VII*

<i>Species V</i>					<i>Species VI</i>				<i>Species VII</i>					
VII	VIII	IX	X	XI	XI	XII	XIV	IX	X	XI	XII			
	7	6	6	6				6	5	7	7			
5	6	5	5	4	5	6	4	6 <sup>b</sup>	6	6	5	4	5	4
3	3	3	3	3	3	4	2	3	3	4 <sup>b</sup>	2	2	4	3

*Species VIII*

<i>Species VIII</i>																		
IX	X			XI			XII			XIII			XIV	XV				
							7			7	7	7	6	7				
6	5	7	7	6	5	5	6	5	4	6	4	5	4	5	5	4	6	5 <sup>b</sup>
5	3	5	5	3	4	3	5	4	3	5	3	4	3	3	4	3	4	4
3	2	3	2	2	3	2	3	3	2	3	2	3	2	2	3	2	2	2





CAPUT XI

DE CONSONANTIIS  
IN GENERE DIATONICO-CHROMATICO

1. Quinam soni insint in genere diatonico-chromatico, in capite praecedente § 16 clare est ostensum, in quo loco non solum soni sunt definiti, quos claves instrumentorum per se significant, sed etiam secundarii soni, quos eadem claves satis commode repraesentare possunt. Nunc igitur ad consonantias progrediemur et exponemus, ad quas consonantias exprimendas genus diatonico-chromaticum sit aptum, praetereaue, quibus clavibus quamque consonantiam repraesentari conveniat.

2. Cum binarios sonos octava vel elevet vel deprimat, soni vero octava vel octavis differentes, etsi non pro iisdem, tamen pro similibus habeantur, eandem ob rationem consonantias, quarum exponentes nonnisi potestate binarii differunt, pro similibus haberi conveniet. Huiusmodi igitur consonantiarum similium congeries nomine speciei consonantiarum appellabitur. Ita verbi gratia  $2^m \cdot 3 \cdot 5$  exponit speciem quandam consonantiarum ac substituendis loco  $m$  numeris definitis prodibunt singulae consonantiae hanc speciem constituentes.

3. Species igitur consonantiarum huiusmodi formis  $2^m \cdot A$  posthac exprimemus, in quibus  $m$  numerum indefinitum,  $A$  vero definitum imparem significat. Ipsae autem consonantiae sub hac specie comprehensae determinabuntur his exponentibus  $A, 2A, 2^2A, 2^3A, 2^4A$  etc. Soni enim has consonantias constituentes in singulis iisdem exprimentur litteris et differentia tantum in octavis consistet, quibus soni harum consonantiarum a se invicem discrepabunt; quae differentia naturam consonantiae non multum immutabit.

4. Interim tamen hae consonantiae sub una specie contentae non penitus pro iisdem sunt habendae; differunt enim utique ratione suavitatis, qua quaeque auditu percipitur. Ita si consonantia exponentis  $A$  ad gradum suavitatis  $n$  pertineat, tum consonantia  $2A$  ad gradum  $n+1$ , consonantia  $2^2A$  ad gradum  $n+2$ , consonantia  $2^3A$  ad gradum  $n+3$  etc. referetur. Quamobrem consonantiarum eiusdem speciei simplicissima et perceptu facillima erit, quae exponentem habet  $A$ , eam ordine suavitatis sequetur consonantia  $2A$ , hanc vero  $2^2A$  et ita porro.

5. Quo maior ergo in exponente speciei consonantiarum  $2^m \cdot A$  loco  $m$  numerus substituitur, eo magis consonantia fit composita audituque perceptu difficilior. Cum igitur nostra facultas percipiendi non ultra datum gradum extendatur, terminus in gradibus suavitatis est figendus, ultra quem consonantias magis compositas reddere non liceat. Talis autem terminus nisi per experientiam constitui non potest; constat vero a

Musicis consonantias magis compositas usurpari rarissime solere, quam quae ad gradum XII pertineant, et si talibus utantur, ideo non probandum esse videtur. Sit igitur nobis iste terminus constitutus, quem consonantiae superantes sint illicitae atque ex harmonia exterminandae.

6. Quo igitur consonantias, quae in genere nostro diatonico-chromatico locum inveniunt, enumeremus et exponamus, pro iis eiusmodi exponentes sunt accipiendi, qui in exponente generis  $2^m \cdot 3^3 \cdot 5^2$  contineantur. Etiam si enim hoc genus quoque exponenti  $2^m \cdot 3^7 \cdot 5^2$  satisfaciat, tamen ob allatam causam consonantiae adhiberi nequeunt, quae in  $2^m \cdot 3^3 \cdot 5^2$  non contineantur. Habebimus ergo sequentes duodecim consonantiarum species:

I. $2^m$	V. $2^m \cdot 3 \cdot 5$	IX. $2^m \cdot 3 \cdot 5^2$
II. $2^m \cdot 3$	VI. $2^m \cdot 5^2$	X. $2^m \cdot 3^3 \cdot 5$
III. $2^m \cdot 5$	VII. $2^m \cdot 3^3$	XI. $2^m \cdot 3^2 \cdot 5^2$
IV. $2^m \cdot 3^2$	VIII. $2^m \cdot 3^2 \cdot 5$	XII. $2^m \cdot 3^3 \cdot 5^2$ .

7. Hae quidem species consonantiarum, si ad exponentes insuper indices adiungantur, pluribus formis occurrere possunt. Quivis enim speciei exponens  $2^m \cdot A$  indice quocumque  $B$  poterit determinari, ut species hoc modo exprimat  $2^m A(B)$ , dummodo  $2^m AB$  fuerit divisor ipsius  $2^m \cdot 3^7 \cdot 5^2$ , si quidem generi diatonico-chromatico haec latior extensio concedatur. Cum autem basis cuiusque consonantiae sit sonus unitate denotatus, erit consonantiae  $2^m A(B)$  basis  $B$ ; ita ut, quomodocumque varietur index  $B$ , consonantiae per  $2^m A(B)$  expressae tantummodo ratione basium discrepent.

8. Cum autem hic nobis tantum propositum sit consonantias in se spectatas tractare, eae vero indicibus non immutentur, indices hic negligeros seu potius pro indice unitatem sumemus. Consonantia enim hoc modo descripta facile ad quemvis indicem poterit transformari, substituendo loco soni unitate designati sonum indice expressum et loco reliquorum alios a basi iisdem intervallis distantes. Cum igitur 1 sonum det littera  $F$  signandum seu aliquot integris octavis a sono  $F$  distantem, basis in hoc capite perpetuo erit sonus vel  $F$  vel aliquot octavis gravior quam  $F$ .

9. In omnibus igitur consonantiis, quas hic repraesentabimus, sonus seu clavis  $F$  nobis vel unitate vel binario vel potestate binarii indicabitur, prout circumstantiae postulabunt. Consonantias enim omnes intra trium octavarum intervallum exhibere visum est, ita ut sonos vel graviore quam  $F$  vel acutiores quam  $T$  simus neglecturi. Cum igitur secundum hoc institutum raro consonantias completas exhibere queamus, modo primam modo secundam modo quartam etc. clavem  $F$  denotabit, quo omnes formas, quibus quaeque

consonantia intra praescriptum trium octavarum intervallum comparera potest, obtineamus.

10. Ad sonos hos exprimendos utemur binis pentagrammatis ordinariis, quorum alterum discanti, alterum bassi clave est instructum, in hisque consonantias more consueto ita repraesentabimus, ut omnes notae inter haec pentagrammata contineantur. Haecque etiam est ratio, cur sonos neque graviores quam  $F$  neque acutiores quam  $\bar{f}$  simus adhibituri.

Neque vero etiam amplius spatium assumi potest propter alios sonos in posterum loco  $\bar{f}$  substituendos, ne plures consonantiae successivae maius quam quatuor octavarum intervallum requirerent.

11. Hac igitur ratione cuiusque speciei consonantias secundum ordinem suavitatis notis musicis more consueto descripsimus. Supra quidem exponentem consonantiarum descriptorum, inter pentagrammata vero gradum suavitatis atque infra numeros adiunximus, quibus in quaque consonantia sonus  $F$  indicatur. Praeterea consonantias in priore parte huius tabulae ad gradum XII tantum produximus tanquam saepius in usum receptas; infra tamen consonantias ad XV. gradum usque continuavimus, quae revera pro dissonantiis sunt habendae. Plerasque quidem species non eousque continuare licuit ob intervallum nimis angustum, in quo consonantiae magis compositae repraesentari possent. Sic primae speciei consonantia  $2^3$  intra intervallum trium octavarum exhiberi non potest multoque minus sequentes consonantiae, quamobrem eae quoque sunt omissae.

12. Incipit ergo haec tabula ab unisono seu sono simplici, qui utique est consonantiarum simplicissima. Hunc sequitur consonantia octava dicta, cuius duo soni eam constituentes intervallo octavae a se invicem distant haecque est post unisonum simplicissima consonantia, quae facillime percipitur et ad quam edendam duae chordae solo auditu facile temperari possunt. Tertia consonantia est trisona eiusque soni octavis a se invicem distant ideoque gratam harmoniam conficiunt. Atque hae sunt consonantiae speciei primae, quarum plures intra intervallum trium octavarum non cadunt.

13. Secunda species complectitur eas consonantias, in quibus praeter octavam intervalla quinta et quarta occurrunt. Quod quidem ad quintam attinet, patet eam simplicissimam reddi, si octava augeatur, ita ut octava cum quinta non solum gratius se auribus offerat quam simplex quinta, sed etiam ad temperanda instrumenta feliciori cum successu adhibeatur. Fixo scilicet sono  $F$  ex eo multo facilius erit sonum  $\bar{c}$  formare quam  $c$ . Quamobrem qui instrumenta musica solo auditu temperare voluerit, non simplices quintas, sed octavas cum quintis efformet, unde non parvi momenti percipiet subsidium. Reliquae huius speciei consonantiae frequenter occurrunt audituique admodum sunt acceptae.

14. Tertiae speciei simplicissima consonantia est duplex octava cum tertia maiore, quod intervallum auditui multo suavius est quam vel simplex tertia maior vel octava cum tertia

maiore. Hanc ob rem ad bene temperanda instrumenta musica magis expediet duplices octavas cum tertiis maioribus formare quam simplices tertias maiores; seu si soni nimis videantur remoti, octavae cum tertiis maioribus saltem ad hoc adhiberi poterunt. His igitur auxiliis in temperandis instrumentis musicis secundum regulas supra traditas maxime uti conveniet, quibus operatio praescripta eo faciliior et exactior reddetur.

15. Hae igitur sunt tres simplicissimae species, in quarum prima unicus tantum sonus, in reliquis duo solum occurrunt, si quidem soni una vel pluribus octavis a se invicem discrepantes pro iisdem habeantur; atque hanc ob rem nisi in diphoniis ob tantam simplicitatem raro adhiberi solent. Sequentes vero species maiorem sonorum copiam complectuntur, ut in polyphoniis etiam commode locum habere queant. Huiusmodi est species quarta, in cuius consonantiis tres soni *F*, *C* et *G* reperiuntur; saepius autem Musici hac specie utuntur, quando ad bassum vel quintam cum secunda vel septimam cum quarta adiungunt; quae quidem consonantiae a Musicis dissonantiae appellari solent, non tam eo, quod minus sint suaves, quam quod speciem sequentem cum tribus prioribus solam consonantias appellare consueverint.

16. Sequitur ergo species quinta, quae tam omnes consonantias magis compositas, quam plures dissonantias musicis suppeditat. Tales consonantiae sunt potissimum duae, quae statim ab initio huius speciei conspiciuntur, quarum prima ex sonis *F*, *A*, *C*, altera vero ex sonis *A*, *C*, *E* constat. Haeque duae consonantiae, quocumque ordine soni collocentur, *triades harmonicae* vocari solent. *Triades autem principales* appellantur, si soni ita fuerint dispositi, ut ad infimum reliquorum alter tertia sive maiore sive minore distet, alter vero quinta. Ex iisdem igitur triadibus principalibus minus principales oriuntur, si soni alio ordine disponantur.

17. Trias porro harmonica *dura* vocatur, in qua tertia maior cum quinta est coniuncta, *mollis* vero, in qua tertia minor cum quinta coniungitur; dura igitur est trias *F*, *A*, *C*, mollis vero *A*, *C*, *E*. Harum ergo triadum, quomodo utraque suavissime sonis sit exprimenda, ex tabula clare perspicitur, ex qua simul patet, quantum suavitati decedat, si soni alio ordine disponantur. De aptissimo autem quamque consonantiam seu *accortum*, prout a Musicis vocari solet, exprimendi modo infra plura tradentur.

18. Praeter has duas triades haec eadem species quinta continet plures dissonantias a Musicis ita vocatas, quas ex utraque parte tabulae videre licet. Solent enim musici in componendis operibus tantum triadibus tam dura quam molli pro consonantiis uti iisque maximam operum partem implere; reliquas vero consonantias omnes, quas illis tantum intermiscunt, tanquam secundarias tractant nomineque dissonantiarum appellant, quamvis saepius tantundem vel etiam plus suavitatis habeant quam triades, prout quidem hae efferri solent.

19. Speciei sextae consonantiae sunt admodum durae, cum simplicissima, quae intra intervallum trium octavarum exprimi potest, ad gradum undecimum ascendat; rarissime

igitur a musicis adhibetur raroque ea uti convenit. Septimae speciei ut et octavae consonantiae sunt magis tolerabiles et magna cum gratia consonantiis simplicioribus intermisceri possunt. Nona vero et decima species ob nimiam ruditatem nonnisi cum summa circumspectione usurpari possunt. Residuarum duarum specierum. ne consonantia quidem exhiberi potest, quae gradum duodecimum non transcenderet; earum igitur specierum consonantiae seu potius dissonantiae in altera tabulae parte sunt quaerendae.

20. Hinc utiles regulae deduci possunt pro basso continuo, quam fieri potest, suavissime efferendo, in quo posito consonantiae edendae sono gravissima numeris adscriptis indicari solet, cuiusmodi soni acutiores cum eo simul sint edendi. Hi autem soni per numeros ab intervallorum nominibus receptis petitos indicantur, ita ut 6 denotet sextam, 7 septimam etc. esse cum basso coniungendam. Non autem hi numeri simplicia tantum intervalla denotant, sed una pluribusve octavis aucta, prout occasio postulat; atque sollertiae musici relinquitur, utrum intervallis simplicibus an compositis uti expediat.

21. Ut igitur huiusmodi regulas tradamus, incipiemus a simplicibus intervallis, quibus ad bassum unicus sonus adiungi debet. Ac primo quidem, si octava fuerit signata, suavius erit simplicem octavam adiungere quam vel duplicem vel triplicem. Si quinta tam perfecta quam imperfecta (imperfectae enim quintae in hoc negotio pro perfectis haberi solent) adiungi iubeatur, non simplicem sed octavam cum quinta adhibere conveniet. Quarta contra simplex suavior erit auditui quam una pluribusve octavis aucta et hanc ob rem, si forte circumstantiae prohibeant simplici uti, tam parum, quam fieri potest, a basso remota adhiberi debet.

22. Si tertia maior fuerit praecepta, eius loco non simplicem, sed duabus octavis auctam adhibere decet; tertia vero minor e contrario auditui est gratior, si simplex capiatur vel saltem a basso quam minime remota. Sextae porro tam maiores quam minores sunt suaviores, quo minus a basso distantes capiuntur. Simili modo septima minor basso proxima seu simplex remotioribus est praeferenda; septima vero maior, quo maiore a basso intervallo distat, eo erit gratior. Secunda maior tono maiore constans a basso maxime, ea vero, quae tono minore continetur, a basso minime distare debet. Pari modo secunda minor, quo basso propior capitur, eo erit suavior. Tritonus denique, quo longius a basso accipitur, eo minus suavitatum turbabit.

23. Hae ergo regulae sunt observandae, si unicus sonus ad bassum adiungi debet, quod quidem rarissime usu venit; interim tamen hae regulae usum suum aequae retinent, si plures soni cum basso debent coniungi; de quolibet enim eadem valent, quae, si solus adesset, observanda forent. Quomodo autem soni, si plures numeri basso fuerint inscripti, suavissime exprimi debeant, ex tabula hic adiecta videre licebit, quae ex priore est formata reiectis tantum aliquot sonis gravissimis, ut quivis sonus bassi locum obtineat.

24. Ad haec autem distincte exprimenda opus erat tribus pentagrammatis, in quorum infimo solae bassi notae cum numeris suprascriptis, uti in basso continuo seu generali fieri solet, repraesentantur; duo reliqua pentagrammata vero continent integram consonantiam, qua numeri basso adscripti commodissime et suavissime exprimuntur. Scala hic quidem usi sumus vacua, sed facile erit per transpositionem huius tabulae usum ad quamvis aliam scalam sonosque alios accommodare. Distinguimus ut ante gradus suavitatis atque etiam species, ad quam quaeque consonantia pertinet, notavimus. Duabus denique haec tabula quoque constat partibus, in quarum priore consonantiae usque ad speciem decimam, in posteriore vero duarum reliquarum specierum consonantiae sunt enumeratae.