CHAPTER X

CONCERNING MORE KINDS OF MUSICAL SCALES

1. Now with the above eighteen kinds of scales established, on which both ancient as well as modern music are based, it will not be out of place to pursue some more general kinds of scales, which either may hold a close relation to that treated, or which may be able to be taken into use conveniently for the further perfection of music. Therefore as we have undertaken, as we progress in reviewing the following kinds, we will not present everything in between in order, since there would be an infinite amount of work to be of no use, but we will explain only these, which we have accepted to be in use, which will appear suitable to be put in place.

2. Therefore we will consider the kind [or genus], the exponent of which is $2^m \cdot 3^2 \cdot 5^3$, which it will deserve to be called the *chromatic-enharmonic scale*, since this same exponent shall be composed from the exponents of the chromatic and enharmonic kinds and it shall be the least common divisible of these exponents [or least common multiple of these]. Therefore the octaves of this kind will contain equally three times four or twelve tones, and in the diatonic−chromatic kind, which will arise from just as many divisors of $3^2 \cdot 5^3$, and they will be the following

$$2^{10} : 3^2 \cdot 5^3 : 2^7 \cdot 3^2 : 2^4 \cdot 3 \cdot 5^2 : 2^8 \cdot 5 : 2^5 \cdot 3^2 \cdot 5 : 2^8 \cdot 3 \cdot 5^3 :$$
$$1024 : 1125 : 1152 : 1200 : 1280 : 1440 : 1500 :$$
$$2^9 \cdot 3 : 2^6 \cdot 5^2 : 2^5 \cdot 3^2 \cdot 5^2 : 2^7 \cdot 3 \cdot 5 : 2^4 \cdot 5^3 : 2^{11};$$
$$1536 : 1600 : 1800 : 1920 : 2000 : 2048.$$

3. But the chromatic-enharmonic tones of this kind, in what manner they may be progressing and how great the intervals they may maintain between themselves may be, will be apparent from the following table :

Signs of tones	Tones		Intervals	Names of the intervals
C	$2^8 \cdot 3$	768		
			24:25	Lesser semitone
Cs	$2^5 \cdot 5^2$	800		
			8:9	Major tone
Ds	$2^2 \cdot 3^2 \cdot 5^2$	900		
			15:16	Greater semitone
E	$2^6 \cdot 3 \cdot 5$	960		
			24:25	Lesser semitone
*F**	$2^3 \cdot 5^3$	1000		

			125: 128	Enharmonic diesis
F	2^{10}	1024		
			1024:1125	Major tone less a diesis
*G**	$3^2 \cdot 5^3$	1125		
			125: 128	Enharmonic diesis
G	$2^7 \cdot 3^2$	1152		
			24:25	Lesser semitone
Gs	$2^4 \cdot 3 \cdot 5^2$	1200		
			15:16	Greater semitone
A	$2^8 \cdot 5$	1280		
			8:9	Major tone
H	$2^5 \cdot 3^2 \cdot 5$	1440		
			24:25	Lesser semitone
*c**	$2^2 \cdot 3 \cdot 5^3$	1500		
			125:128	Enharmonic diesis
c	$2^9 \cdot 3$	1536		

4. Therefore the intervals between the adjacent tones are maximally unequal in this kind of scale, evidently majors, semitones and diesis, thus so that a melody composed with this kind may not be transposed to any other kind. And hence therefore more preference will be shown for the diatonic-chromatic kind presented in the preceding chapter, in which nearly all the intervals are perceived to be equal ; and likewise this equality is understood to arise fortuitously and not to be absolutely necessary for harmony being produced, just as indeed is observed for many other scales.

5. Truly three tones are present in this genus, which are not found in the received diatonic−chromatic kind, and these signs are to be noted by the letters with asterisks *F*, G*, c**, since they may approach closely to the tones designated by these letters ; yet indeed from these they are lacking by a diesis. Whereby with such a small difference that it may scarcely be heard, with instruments tuned in the usual manner to the diatonic-chromatic scale also will be able to be produced for a musical work pertaining to the kind $2^m \cdot 3^2 \cdot 5^3$, with the accustomed tones *F, G, c* taken in place of the tones *F*, G*, c**, which errors arise almost beyond the sense of hearing.

6. Certainly the diatonic-chromatic genus will be applied with greater gratitude to musical works of the exponent $2^m \cdot 3^2 \cdot 5^3$, as which is accustomed to happen frequently to musicians, while a melody will have been transferred from a given scale of tones to another one, which often happens, in order that, if the interval before were a minor semitone, in its place either a semitone more or thus a greater limma would be used, which difference at this point provides a major diesis. Besides even if the

instruments may be able to be adapted to the chromatic−enharmonic scale, unless that may agree exactly with the tuning, which still may be scarcely outstanding, a greater agreement will not arise than with similarly tuned instruments.

7. Therefore it is apparent the diatonic-chromatic genus may appear wider, than its exponent $2^m \cdot 3^3 \cdot 5^3$ indicates, since also it may not be inconvenient to be used for musical works provided by the exponent $2^m \cdot 3^2 \cdot 5^3$, from which especially it is seen to be accepted by established kinds of music. But its use in this regard also can be extended to more kinds of compositions, which have been prepared thus, so that the tones from a kind of diatonic−chromatic scale differing from the tones of this kind approach closely, and thus these may be able to be used with care in place of those. Therefore these shall be the kinds of scales, by which a diatonic-chromatic scale can be satisfied, which we will present here further.

8. The exponents of the three old kinds of scales may be coalesced into one, thus so that the *diatonic-enharmonic scale* may be produced, of which the exponent will be $2^m \cdot 3^3 \cdot 5^3$, and in this kind the diatonic, chromatic and enharmonic may be contained together, just as clearly have been corrected by us. Therefore a single octave of this kind will contain 16 tones, without doubt twelve tones of the diatonic−chromatic kind, and 4 new ones besides these, but which are so little different from these, so that they may be omitted without throwing away the sense of harmony, and as we have observed equally from the preceding kind. But the 16 tones of the octave will be the following :

Signs of tones	Tones		Intervals	Names of intervals
C	$2^{10} \cdot 3$	3072		
			24:25	Minor semitone
Cs	$2^7 \cdot 5^2$	3200		
			128:135	Minor limma
*D**	$3^3 \cdot 5^2$	3375		
			125:128	Diesis
D	$2^7 \cdot 3^3$	3456		
			24:25	Minor semitone
Ds	$2^4 \cdot 3^2 \cdot 5^2$	3600		
			15: 16	Semitone major
E	$2^8 \cdot 3 \cdot 5$	3840		
			24:25	Minor semitone
*F**	$2^5 \cdot 5^3$	4000		
			125: 128	Diesis
F	2^{12}	4096		

			125:135	Minor limma
Fs	$2^5 \cdot 3^2 \cdot 5^3$	4320		
			24:25	Minor semitone
*G**	$2^2 \cdot 3^2 \cdot 5^3$	4500		
			125:128	Diesis
G	$2^9 \cdot 3^2$	4608		
			24:25	Minor semitone
Gs	$2^6 \cdot 3 \cdot 5^2$	4800		
			15:16	Semitone major
A	$2^{10} \cdot 5$	5120		
			128:135	Minor limma
B	$2^3 \cdot 3^3 \cdot 5$	5400		
			15:16	Semitone major
H	$2^6 \cdot 3^2 \cdot 5$	5760		
			24:25	Semitone minor
*c**	$2^4 \cdot 3 \cdot 5^2$	6000		
			125:128	Diesis
c	$2^{11} \cdot 3$	6144		

Therefore in place of the exotic tones *D*, F*, G*, c*,* which differ only be a diesis from the primaries *D, F, G, c,* these will be able to be used equally well enough with care.

9. If perhaps for any difference this one, which is the diesis, may be considered greater, it may be considered able to be used rather than the first of the unusual terms, since the diesis shall be the maximum between two minimum intervals, yet this doubtlessly allows an error not greater than a comma. But the exotic tones will differ from the principal kinds by a comma at the most, the exponents of which will be contained in $2^m \cdot 3^n \cdot 5^2$ with the number *n* presented greater than three. But octaves of this kind, if *n* is less than 8, can be seen in the adjoining table.

The exponent of the kind $2^m \cdot 3^7 \cdot 5^2$

Signs of tones	Tones	Logarithms of the tones	Intervals	Names of the intervals
F	2^{15}	15,00000		
			0,07682	Minor limma
Fs	$2^8 \cdot 3^3 \cdot 5$	15,07682		
			0,01791	Comma
*Fs**	$2^4 \cdot 3^7$	15,09473		

			0,05890	Minor semitone
*G**	$2\cdot 3^6\cdot 5^2$	15,15363		
			0,01630	Diaschisma
G	$2^{12}\cdot 3^2$	15,16993		
			0,05889	Minor semitone
Gs	$2^9\cdot 3\cdot 5^2$	15,22882		
			0,01792	Comma
*Gs**	$2^5\cdot 3^5\cdot 5$	15,24674		
			0,07519	Minor semitone with diaschisma
A	$2^{13}\cdot 5$	15,32193		
			0,01792	Comma
*A**	$2^9\cdot 3^4$	15,33985		
			0,05889	Minor semitone
B	$2^6\cdot 3^2\cdot 5^2$	15,39874		
			0,01792	Comma
*B**	$2^2\cdot 3^7\cdot 5$	15,41666		
			0,07519	Minor semitone with diaschisma
H	$2^{10}\cdot 3^2\cdot 5$	15,49185		
			0,01793	Comma
*H**	$2^6\cdot 3^6$	15,50978		
			0,05889	Minor semitone
*c**	$2^3\cdot 3^5\cdot 5^2$	15,56867		
			0,01629	Diaschisma
c	$2^{14}\cdot 3$	15,58496		
			0,05890	Minor semitone
cs	$2^{11}\cdot 5^2$	15,64386		
			0,01792	Comma
*cs**	$2^7\cdot 3^4\cdot 5$	15,66178		
			0,07681	Minor limma
*d**	$3^7\cdot 5^2$	15,73859		
			0,01630	Diaschisma
d	$2^{11}\cdot 3^3$	15,75489		
			0,05889	Minor semitone
ds	$2^8\cdot 3^2\cdot 5^2$	15,81378		
			0,01792	Comma
*ds**	$2^4\cdot 3^6\cdot 5$	15,83170		
			0,07519	Minor semitone with diaschisma
e	$2^{12}\cdot 3\cdot 5$	15,90689		

			0,01792	Comma
e^*	$2^8\cdot 3^5$	15,92481		
			0,05890	Minor semitone
f^*	$2^5\cdot 3^4\cdot 5^2$	15,98371		
			0,01629	Diaschisma
f	2^{16}		16,00000	

Therefore in this kind to the twelve tones of the diatonic-chromatic kind twelve new tones are added, but of which from these the differences are either commas or diaschisma; which since to the listener they may be barely able to be distinguished, with care these new tones may be omitted and in place of these the accustomed ones will be able to be used. And thus the diatonic-chromatic scale is widely known, and the kind is required to be considered equally of which the exponent is $2^m\cdot 3^7\cdot 5^2$.

10. Therefore with the diatonic-chromatic genus gathered together neatly, the exponent of which depending only on $2^m\cdot 3^2\cdot 5^2$, can be used for musical works being required to be expressed, the exponents of which are much more composite and held in the form $2^m\cdot 3^7\cdot 5$. Just as indeed the octave for works of this kind with twice as many the number of tones may be constructed, as the exponent requires, yet on account of such a small difference in the harmony hardly any variation in the may be able to be perceived, whether the complete or incomplete genus may be used. But it is allowed to progress further in a similar manner beyond the power of seven, thus so that the kind of music used today may be taken into service for the general exponent $2^m\cdot 3^n\cdot 5^2$, also however great the number n may be taken.

11. This moreover is itself thus to be understood, and the diatonic-chromatic scale appears to be in service everywhere, as the day to day compositions of musicians over and above testify well enough. Indeed scarcely any present day musical work is found, of which the exponent of the composition may not be more than the exponent of the genus $2^m\cdot 3^3\cdot 5^2$ itself. Yet meanwhile some kinds so music are themselves agreed to be composed, so that the received sounds may not be sufficient, with the matter requiring to be considered with the greatest rigor, but on account of the minimum aberration these notes may be used instead, as with new notes being introduced so that it may be effected for the treatment of more difficult music.

12. But there will be less success, if by increasing the exponent of the 5 genus we may wish to enlarge our diatonic-chromatic scale more. For with the power of 5 increased the tones of this kind above approach customary tones, which may differ by more than a comma and evidently of several diesis from a customary tone, which error, since the diesis shall be about the middle of a semitone, can be used. Yet

meanwhile, so that this may be better understood, I may add an octave of the kind, of which the exponent is $2^m \cdot 3^3 \cdot 5^5$

Signs of tones	Tones	Logarithms of the tones	Intervals	Names of the intervals
F	2^{16}	16,00000		
			0,04260	Semitone less diaschisma
*Fs**	$2^2 \cdot 3^3 \cdot 5^4$	16,04260		
			0,03422	Diesis
Fs	$2^9 \cdot 3^3 \cdot 5$	16,07682		
			0,05889	Minor semitone
*G**	$2^6 \cdot 3^2 \cdot 5^3$	16,13571		
			0,03421	Diesis
G	$2^{13} \cdot 3^2$	16,16992		
			0,02468	Minor semitone less diesis
*Gs**	$2^3 \cdot 3 \cdot 5^5$	16,19460		
			0,03422	Diesis
Gs	$2^{10} \cdot 3 \cdot 5^2$	16,22882		
			0,05889	Minor semitone
*A**	$2^7 \cdot 5^4$	16,28771		
			0,03422	Diesis
A	$2^{14} \cdot 5$	16,32193		
			0,04260	Minor semitone less diaschisma
*B**	$3^3 \cdot 5^5$	16,36453		
			0,03421	Diesis
B	$2^7 \cdot 3^2 \cdot 5^2$	16,39874		
			0,05889	Minor semitone
*H**	$2^4 \cdot 3^2 \cdot 5^4$	16,45763		
			0,03422	Diesis
H	$2^{11} \cdot 3^2 \cdot 5$	16,49185		
			0,05890	Minor semitone
*c**	$2^8 \cdot 3 \cdot 5^3$	16,55075		
			0,03421	Diesis
c	$2^{15} \cdot 3$	16,58496		
			0,02468	Minor semitone less diesis
*cs**	$2^5 \cdot 5^5$	16,60964		
			0,03422	Diesis

cs	$2^{12}\cdot 5^{2}$	16,64386		
			0,07681	Minor limma
*d**	$2^{5}\cdot 3^{3}\cdot 5^{3}$	16,72067		
			0,03421	Diesis
d	$2^{12}\cdot 3^{3}$	16,75488		
			0,02468	Minor semitone less diesis
*ds**	$2^{2}\cdot 3^{2}\cdot 5^{5}$	16,77956		
			0,03422	Diesis
ds	$2^{9}\cdot 3^{2}\cdot 5^{2}$	16,81378		
			0,05889	Minor semitone
*e**	$2^{6}\cdot 3\cdot 5^{4}$	16,87267		
			0,03422	Diesis
e	$2^{13}\cdot 3\cdot 5$	16,90689		
			0,05889	Minor semitone
*f**	$2^{10}\cdot 5^{3}$	16,96578		
			0,03422	Diesis
f	2^{17}	17,00000		

13. Therefore with this kind of tone being added from the start having been inserted alternately between the customary tones, and each of these differs from the principal tone by a diesis; which difference since is perceptible, scarcely can tolerate the omission of all the new tones arising. Besides certain of these tones are closer to the principal ones than those following, for which we have changed the signs ; evidently the tone *Gs** is closer to the tone *G* than the tone *Gs,* thus so that in place of this the tone letter *G* will be agreed to be used; which likewise truly will experience a great difficulty, since the tone *G* might be used in place of the tone *G** , but the different tones *G** and *Gs** cannot be expressed by the same tone. Therefore for such a musical octave rather to be divided up into 24 intervals, so that such a scale is going to have the prerogative, that all the intervals may be almost equal to each other.

14. Moreover it may become generally apparent from this account for twice the number of tones to be present for this new musical scale ; for not only may the scale be able to accommodate tones according to the exponent $2^{m}\cdot 3^{8}\cdot 5^{5}$, but also according to the exponent $2^{m}\cdot 3^{3}\cdot 5^{p}$ with *p* denoting any number greater than five. So that also it would not suffice to be agreed for this general genus $2^{m}\cdot 3^{n}\cdot 5^{p}$ to be satisfied, unless *n* and *p* should be greatly enlarged numbers ; but extremely large numbers are not allowed to be substituted in place of *n* and *p* for this harmony.

15. Therefore for the diatonic-chromatic scale, of which the exponent is $2^m \cdot 3^5 \cdot 5^2$, it cannot be agreed to extend these unaffected harmonies further, as contained in the exponent $2^m \cdot 3^7 \cdot 5^2$ for musical works. For although by the same law it may be able to have the major third greater than the seventh power, yet the rules of these harmonies forbid such works to be composed, of which the exponent may be more composite. On account of which the use of this received scale cannot be agreed to be extended further than to musical works satisfied by the exponent $2^m \cdot 3^7 \cdot 5^2$; nor also are musicians of the present day accustomed to transgress this boundary.

16. But where the scale of the music undertaken, of which the exponent is $2^m \cdot 3^3 \cdot 5^2$, may be satisfied by a more composite exponent $2^m \cdot 3^7 \cdot 5^2$, no matter for which tone or key of the instrument, a tone is produced in two ways, as is understood from the diagram of this kind attached to § 9 ; indeed the keys designated by *H* for the sake of an example will be produced both by the exponent $2^m \cdot 3^2 \cdot 5$ as well as that presented by the exponent $2^m \cdot 3^6$. On account of which we may add the following table, from which it is understood at once, by which the tone in the exponent $2^m \cdot 3^7 \cdot 5^2$ may be expressed by any key, for the first position of *F* denoted by the tone 2^n with n denoting an arbitrary fixed number.

Keys	Primary tones	Secondary tones	Keys	Primary tones	Secondary tones
C	$2^{n-2} \cdot 3$	$2^{n-13} \cdot 3^5 \cdot 5^2$	$\bar{c}$	$2^n \cdot 3$	$2^{n-11} \cdot 3^5 \cdot 5^2$
Cs	$2^{n-5} \cdot 5^2$	$2^{n-9} \cdot 3^4 \cdot 5$	$\overline{cs}$	$2^{n-3} \cdot 5^2$	$2^{n-7} \cdot 3^4 \cdot 5$
D	$2^{n-5} \cdot 3^3$	$2^{n-16} \cdot 3^7 \cdot 5^2$	$\bar{d}$	$2^{n-3} \cdot 3^3$	$2^{n-14} \cdot 3^7 \cdot 5^2$
Ds	$2^{n-3} \cdot 3^2 \cdot 5^2$	$2^{n-12} \cdot 3^6 \cdot 5$	$\overline{ds}$	$2^{n-6} \cdot 3^2 \cdot 5^2$	$2^{n-10} \cdot 3^6 \cdot 5$
E	$2^{n-4} \cdot 3 \cdot 5$	$2^{n-8} \cdot 3^5$	$\bar{e}$	$2^{n-2} \cdot 3 \cdot 5$	$2^{n-6} \cdot 3^5$
F	2^n	$2^{n-11} \cdot 3^4 \cdot 5^2$	$\bar{f}$	2^{n+2}	$2^{n-9} \cdot 3^4 \cdot 5^2$
Fs	$2^{n-7} \cdot 3^3 \cdot 5$	$2^{n-11} \cdot 3^7$	$\overline{fs}$	$2^{n-5} \cdot 3^3 \cdot 5$	$2^{n-9} \cdot 3^7$
G	$2^{n-3} \cdot 3^2$	$2^{n-14} \cdot 3^6 \cdot 5^2$	$\bar{g}$	$2^{n-1} \cdot 3^2$	$2^{n-12} \cdot 3^6 \cdot 5^2$
Gs	$2^{n-6} \cdot 3 \cdot 5^2$	$2^{n-10} \cdot 3^5 \cdot 5$	$\overline{gs}$	$2^{n-4} \cdot 3 \cdot 5^2$	$2^{n-8} \cdot 3^5 \cdot 5$
A	$2^{n-2} \cdot 5$	$2^{n-6} \cdot 3^4$	$\bar{a}$	$2^n \cdot 5$	$2^{n-4} \cdot 3^4$
B	$2^{n-9} \cdot 3^3 \cdot 5^2$	$2^{n-13} \cdot 3^7 \cdot 5$	$\bar{b}$	$2^{n-7} \cdot 3^3 \cdot 5^2$	$2^{n-11} \cdot 3^7 \cdot 5$
H	$2^{n-5} \cdot 3^2 \cdot 5$	$2^{n-9} \cdot 3^6$	$\bar{h}$	$2^{n-3} \cdot 3^2 \cdot 5$	$2^{n-7} \cdot 3^6$
c	$2^{n-1} \cdot 3$	$2^{n-12} \cdot 3^5 \cdot 5$	$\bar{c}$	$2^{n+1} \cdot 3$	$2^{n-10} \cdot 3^5 \cdot 5$
cs	$2^{n-4} \cdot 5^2$	$2^{n-8} \cdot 3^4 \cdot 5$	$\overline{cs}$	$2^{n-2} \cdot 5^2$	$2^{n-6} \cdot 3^4 \cdot 5$
d	$2^{n-4} \cdot 3^3$	$2^{n-15} \cdot 3^7 \cdot 5^2$	$\bar{d}$	$2^{n-2} \cdot 3^3$	$2^{n-13} \cdot 3^7 \cdot 5^2$

ds	$2^{n-7}\cdot 3^2\cdot 5^2$	$2^{n-11}\cdot 3^6\cdot 5$	$\overline{ds}$	$2^{n-5}\cdot 3^2\cdot 5^2$	$2^{n-9}\cdot 3^6\cdot 5$
e	$2^{n-3}\cdot 3\cdot 5$	$2^{n-7}\cdot 3^5$	$\overline{e}$	$2^{n-1}\cdot 3\cdot 5$	$2^{n-5}\cdot 3^5$
f	2^{n+1}	$2^{n-10}\cdot 3^4\cdot 5^2$	$\overline{f}$	2^{n+3}	$2^{n-8}\cdot 3^4\cdot 5^2$
fs	$2^{n-6}\cdot 3^3\cdot 5$	$2^{n-10}\cdot 3^7$	$\overline{fs}$	$2^{n-4}\cdot 3^3\cdot 5$	$2^{n-8}\cdot 3^7$
g	$2^{n-2}\cdot 3^2$	$2^{n-13}\cdot 3^6\cdot 5^2$	$\overline{g}$	$2^{n}\cdot 3^2$	$2^{n-11}\cdot 3^6\cdot 5^2$
gs	$2^{n-5}\cdot 3\cdot 5^2$	$2^{n-9}\cdot 3^5\cdot 5$	$\overline{gs}$	$2^{n-3}\cdot 3\cdot 5^2$	$2^{n-7}\cdot 3^5\cdot 5$
a	$2^{n-1}\cdot 5$	$2^{n-5}\cdot 3^4$	$\overline{a}$	$2^{n+1}\cdot 5$	$2^{n-3}\cdot 3^4$
b	$2^{n-8}\cdot 3^3\cdot 5^2$	$2^{n-12}\cdot 3^7\cdot 5$	$\overline{b}$	$2^{n-6}\cdot 3^3\cdot 5^2$	$2^{n-10}\cdot 3^7\cdot 5$
h	$2^{n-4}\cdot 3^2\cdot 5$	$2^{n-8}\cdot 3^6$	$\overline{h}$	$2^{n-2}\cdot 3^2\cdot 5$	$2^{n-6}\cdot 3^6$
$\overline{c}$	$2^{n}\cdot 3$	$2^{n-15}\cdot 3^7\cdot 5^2$	$\overline{c}$	$2^{n+2}\cdot 3$	$2^{n-13}\cdot 3^7\cdot 5^2$

17. Therefore in this table both the first tones may be shown as well as the second , for which some key is suitable to be used. The first certainly have been derived from the exponent of the kind $2^m\cdot 3^3\cdot 5^2$, to which thence the keys must be tuned most exactly. Truly the tones of the second cannot be produced from the same keys with the greatest rigor ; however since they may differ so little from the first, for these keys requiring to be expressed the first can be used without risk to be added without detracting from the sense of harmony. For even if from the most acute ears may be able to distinguish a comma or diaschisma, by which the tones of the second interval differ from those of the first intervals, yet, since the secondary tones cannot be mixed with the primaries neither in the same consonant nor in two successive consonants, the error also cannot be perceived by the most acute listener. If indeed for the sake of an example the key *F* were used in the first consonants for expressing the tone 2^n, the same key will be able to represent the tone $2^{n-11}\cdot 3^4\cdot 5^2$ to the hundredth part after the first consonant without risk.

18. It is understood also at once from the same table, if the proposed were in a series of numbers either of tones or consonants, by which that series must be expressed by striking keys. But for this to be put into practice the number *n* is required to be taken thus, so that all the numbers proposed may be represented in a table, if indeed the greatest to least may not be taken more than sixteen times. Whereby the number *n* will be required to be defined either from the maximum or minimum of the proposed numbers and with this done the keys for the remaining tones will be put in place easily, if indeed, so that we may put in place, the smallest of the proposed numbers may be present in the least common divisor of the numbers $2^m\cdot 3^7\cdot 5^2$.

19. Therefore all the musical works, to which our diatonic-chromatic scale is adapted, are dealt with in this exponent $2^m\cdot 3^7\cdot 5^2$, thus so that other works of different exponents may not be able to be performed by instruments following this scale. On account of which the exponents of all the musical works must be composed

for these three numbers 2, 3, 5 and only from the powers of these, and neither of any power of five following two nor a power of three can be had above seven; thus as Leibnitz had conclusively made it clear regarding music, even now it may be in place to say it is not usual to count above the fifth.

20. And it may be reasonably difficult in music to introduce another number, for example 7, besides these three numbers, since the consonants of which the exponents of seven may be introduced, since they may be exceedingly harsh and disturb the harmony. For the consonants, in the exponents of which only of seven as well as two may be present, scarcely may be allowed to be present on account of the more favored intervals arising from 3 and 5 being ignored. But with 7 joined with 3 and 5, so that the exponent $2^m \cdot 3 \cdot 5 \cdot 7$ may produce consonants, the consonance brought to bear would be exceedingly complex, so that it would not be pleasant to be heard. Yet we may have a look meanwhile at the tones present in the octave constituting the scale, of which the exponent is $2^m \cdot 3^3 \cdot 5^2 \cdot 7$.

Exponents of the kind $2^m \cdot 3^3 \cdot 5^2 \cdot 7$

Signs of tones	Tones	Logarithms of tones	Intervals	
F	2^{12}	12,00000		
			0,03617	512:525
*Fs**	$2^8 \cdot 3 \cdot 5^2 \cdot 7$	12,03617		
Fs			0,04064	35:36
	$2^5 \cdot 3^3 \cdot 5$	12,07681		
			0,05247	27:28
*G**	27• 5 . 7	12,12928		
			0,04064	35:36
G	$2^9 \cdot 3^2$	12,16992		
			0,03618	512:525
*Gs**	$3^3 \cdot 5^2 \cdot 7$	12,20610		
			0,02272	63:64
Gs	$2^6 \cdot 3 \cdot 5^2$	12,22882		
			0,07039	20:21
*A**	$2^4 \cdot 3^2 \cdot 5 \cdot 7$	12,29921		
			0,02272	63:64
A	$2^{10} \cdot 5$	12,32193		
			0,07039	20:21
*B**	$2^8 \cdot 3 \cdot 7$	12,39232		
			0,00642	224:225
B	$2^3 \cdot 3^3 \cdot 5^2$	12,39874		

			0,05247	27:28
*H**	$2^5 \cdot 5^2 \cdot 7$	12,45121		
			0,04064	35:36
H	$2^7 \cdot 3^2 \cdot 5$	12,49185		
			0,07039	20:21
*c**	$2^5 \cdot 3^3 \cdot 7$	12,56224		
			0,02272	63: 64
c	$2^{11} \cdot 3$	12,58496		
			0,03618	512:525
*cs**	$2^2 \cdot 3^2 \cdot 5^2 \cdot 7$	12,62114		
			0,02272	63:64
cs	$2^8 \cdot 5^2$	12,64386		
			0,07039	20:21
*d**	$2^6 \cdot 3 \cdot 5 \cdot 7$	12,71425		
			0,04064	35 :36
d	$2^8 \cdot 3^3$	12,75489		
			0,05247	27:28
*ds**	210. 7	12,80736		
			0,00642	224:225
ds	$2^5 \cdot 3^2 \cdot 5^2$	12,81378		
			0,07039	20:21
*e**	$2^3 \cdot 3^3 \cdot 5 \cdot 7$	12,88417		
			0,02272	63:64
e 20:.21	$2^9 \cdot 3 \cdot 5$	12,90689		
			0,07039	20:21
*f**	$2^7 \cdot 3^2 \cdot 7$	12,97728		
			0,02272	63: 64
f	2^{13}	13,00000		

CAPUT X

DE ALIIS MAGIS COMPOSITIS GENERIBUS MUSICIS

1. Expositis iam octodecim prioribus generibus, in quibus tam antiqua quam hodierna musica continetur, non incongruum erit genera aliquot magis composita persequi, quae vel ad iam tractata arctam tenent relationem, vel non incommode ad ampliorem musicae perfectionem in usum recipi possent. Non igitur, uti accepimus, in recensendis generibus sequentibus ordine progrediemur omniaque in medium afferemus, quod opus foret infinitum nulliusque utilitatis, sed ea tantum, quae ad institutum idonea videbuntur, explicabimus.

2. Considerabimus ergo genus, cuius exponens est $2^m \cdot 3^2 \cdot 5^3$, quod merito *chromatico-enharmonicum* appellari convenit, cum iste exponens sit compositus ex exponentibus generum chromatici et enharmonici horumque exponentium sit minimus communis dividuus. In huius ergo generis octava continebuntur ter quatuor seu duodecim soni pariter ac in genere diatonico−chromatico, qui orientur ex divisoribus totidem ipsius $3^2 \cdot 5^3$ eruntque sequentes

$$2^{10} : 3^2 \cdot 5^3 : 2^7 \cdot 3^2 : 2^4 \cdot 3 \cdot 5^2 : 2^8 \cdot 5 : 2^5 \cdot 3^2 \cdot 5 : 2^8 \cdot 3 \cdot 5^3 :$$
$$1024 : 1125 : 1152 : 1200 : 1280 : 1440 : 1500 :$$
$$2^9 \cdot 3 : 2^6 \cdot 5^2 : 2^5 \cdot 3^2 \cdot 5^2 : 2^7 \cdot 3 \cdot 5 : 2^4 \cdot 5^3 : 2^{11};$$
$$1536 : 1600 : 1800 : 1920 : 2000 : 2048.$$

3. Soni autem huius generis chromatico-enharmonici, quomodo progrediantur et quanta intervalla inter se teneant, ex tabula sequente apparebit

Signa	Soni		Intervalla	Nomina Intervallorum
C	$2^8 \cdot 3$	768		
			24:25	Hemitonium minus
Cs	$2^5 \cdot 5^2$	800		
			8:9	Tonus maior
Ds	$2^2 \cdot 3^2 \cdot 5^2$	900		
			15:16	Hemitonium maius
E	$2^6 \cdot 3 \cdot 5$	960		
			24:25	Hemitonium minus
*F**	$2^3 \cdot 5^3$	1000		
			125: 128	Diesis enharmonica
F	2^{10}	1024		

			1024:1125	Tonus maior diesi minuta
*G**	$3^2 \cdot 5^3$	1125		
			125: 128	Diesis enharmonica
G	$2^7 \cdot 3^2$	1152		
			24:25	Hemitonium minus
Gs	$2^4 \cdot 3 \cdot 5^2$	1200		
			15:16	Hemitonium maius
A	$2^8 \cdot 5$	1280		
			8:9	Tonus maior
H	$2^5 \cdot 3^2 \cdot 5$	1440		
			24:25	Hemitonium minus
*c**	$2^2 \cdot 3 \cdot 5^3$	1500		
			125:128	Diesis enharmonica
c	$2^9 \cdot 3$	1536		

4. In hoc ergo genere intervalla inter sonos contiguos maxime sunt inaequalia, toni scilicet maiores, hemitonia et dieses, ita ut melodia in hoc genere composita in nullum alium sonum transponi posset. Hincque eo magis praerogativa generis in praecedente capite expositi diatonico-chromatici elucet, in quo intervalla omnia ad sensum fere sunt aequalia; simulque intelligitur hanc aequalitatem fortuito esse natam neque eam ad harmoniam producendam esse absolute necessariam, prout quidem pluribus est visum.

5. Insunt vero in hoc genere tres soni, qui in genere recepto diatonico−chromatico non reperiuntur, eosque signavi litteris *F*, G*, c** asterisco notatia, cum ad sonos in genere consueto his litteris designatos proxime accedant; tantum enim ab iis diesi deficiunt. Quare cum tantilla differentia ab auribus vix percipi queat, instrumentis solito more ad genus diatonico-chromaticum attemperatis etiam non incongrue opera musica ad genus $2^m \cdot 3^2 \cdot 5^3$ pertinentia edi poterunt, sumendis loco sonorum *F*, G*, c** sonis consuetis *F, G, c,* qui error sensui auditus propemodum insensibilis evadit.

6. Maiore certe gratia genus diatonico-chromaticum ad opera musica exponentis $2^m \cdot 3^2 \cdot 5^3$ erit accommodatum, quam quod a Musicis frequenter fieri solet, dum melodiam ex datis sonis compositam ad alios sonos transferunt, quo saepius fit, ut, quod intervallum ante erat hemitonium minus, eius loco hemitonium maius vel adeo limma maius adhibeant, quae differentia adhuc maior diesi existit. Praeterea etiamsi instrumenta ad genus chromatico−enharmonicum accommodata haberentur, nisi ea exactissime assent temperata, quod tamen vix posset praestari, maiorem suavitatem non afferrent quam
instrumenta consueta.

7. Latius ergo patet genus diatonico-chromaticum, quam eius exponens $2^m \cdot 3^3 \cdot 5^3$ declarat, cum etiam non incommode adhiberi queat ad opera musica in exponente $2^m \cdot 3^2 \cdot 5^3$ contenta, ex quo praestantia recepti generis musici non obscure perspicitur. Adhuc autem latius eius usus extenditur etiam ad genera magis composita, quae ita sunt comparata, ut soni a genere diatonico−chromatico discrepantes ad sonos huius generis proxime accedant ideoque hi illorum loco tuto adhiberi queant. Cuiusmodi ergo haec sint genera, quibus genus diatonico-chromaticum satisfacere potest, hic fusius exponemus.

8. Coalescant omnium trium veterum generum exponentes unum, ita ut prodeat genus *diatonico-enharmonicum,* cuius exponens erit $2^m \cdot 3^3 \cdot 5^3$, in hocque genere continentur coniunctim genera diatonicum, chromaticum et enharmonicum, quatenus scilicet a nobis sunt correcta. Huius ergo generis una octava continebit 16 sonos, duodecim nimirum sonos generis diatonico−chromatici et praeter eos 4 novos, qui autem tam parum ab illis sunt diversi, ut sine sensibili harmoniae iactura plane omitti queant, pariter ac de
praecedente genere notavimus. Soni autem 16 unius octavae erunt sequentes:

Signa	Soni		Intervalla	Nomina Intervallorum
C	$2^{10} \cdot 3$	3072		
			24:25	Hemitonium minus
Cs	$2^7 \cdot 5^2$	3200		
			128:135	Limma minus
*D**	$3^3 \cdot 5^2$	3375		
			125:128	Diesis
D	$2^7 \cdot 3^3$	3456		
			24:25	Hemitonium minus
Ds	$2^4 \cdot 3^2 \cdot 5^2$	3600		
			15: 16	Hemitonium maius
E	$2^8 \cdot 3 \cdot 5$	3840		
			24:25	Hemitonium minus
*F**	$2^5 \cdot 5^3$	4000		
			125: 128	Diesis
F	2^{12}	4096		
			125:135	Limma minus
Fs	$2^5 \cdot 3^2 \cdot 5^3$	4320		

			24:25	Hemitonium minus
*G**	$2^2\cdot3^2\cdot5^3$	4500		
			125:128	Diesis
G	$2^9\cdot3^2$	4608		
			24:25	Hemitonium minus
Gs	$2^6\cdot3\cdot5^2$	4800		
			15:16	Hemitonium maius
A	$2^{10}\cdot5$	5120		
			128:135	Limma minus
B	$2^3\cdot3^3\cdot5$	5400		
			15:16	Hemitonium maius
H	$2^6\cdot3^2\cdot5$	5760		
			24:25	Hemitonium minus
*c**	$2^4\cdot3\cdot5^2$	6000		
			125:128	Diesis
c	$2^{11}\cdot3$	6144		

Loco sonorum ergo peregrinorum *D*, F*, G*, c**, qui diesi tantum differunt a primariis *D, F, G, c,* satis tuto hi poterunt usurpari.

9. Si forte cuiquam differentia haec, quae est diesis, maior videatur, quam ut primarios loco peregrinorum adhiberi posse arbitretur, cum diesis sit maximum inter minima intervallum, is tamen admittet sine dubio errorem commate non maiorem. Commate autem ad summum soni peregrini a principalibus differunt in generibus, quorum exponentes continentur in $2^m\cdot3^n\cdot5^2$ existente *n* numero ternario maiore. Huiusmodi autem generum octavas, si *n* est minor quam 8, in adiecta tabula simul conspicere licet.

Generis exponens $2^m\cdot3^7\cdot5^2$

Signa	Soni	Logarithmi Sonorum	Intervalla	Nomina Intervallorum
F	2^{15}	15,00000		
			0,07682	Limma minus
Fs	$2^8\cdot3^3\cdot5$	15,07682		
			0,01791	Comma
*Fs**	$2^4\cdot3^7$	15,09473		
			0,05890	Hemitonium minus
*G**	$2\cdot3^6\cdot5^2$	15,15363		
			0,01630	Diaschisma

G	$2^{12}\cdot 3^{2}$	15,16993		
			0,05889	Hemitonium minus
Gs	$2^{9}\cdot 3\cdot 5^{2}$	15,22882		
			0,01792	Comma
*Gs**	$2^{5}\cdot 3^{5}\cdot 5$	15,24674		
			0,07519	Hemitonium minus cum diaschismate
A	$2^{13}\cdot 5$	15,32193		
			0,01792	Comma
*A**	$2^{9}\cdot 3^{4}$	15,33985		
			0,05889	Hemitonium minus
B	$2^{6}\cdot 3^{2}\cdot 5^{2}$	15,39874	0,01792	Comma
*B**	$2^{2}\cdot 3^{7}\cdot 5$	15,41666		
			0,07519	Hemitonium minus cum diaschismate
H	$2^{10}\cdot 3^{2}\cdot 5$	15,49185		
			0,01793	Comma
*H**	$2^{6}\cdot 3^{6}$	15,50978		
			0,05889	Hemitonium minus
*c**	$2^{3}\cdot 3^{5}\cdot 5^{2}$	15,56867		
			0,01629	Diaschisma
c	$2^{14}\cdot 3$	15,58496		
			0,05890	Hemitonium minus
cs	$2^{11}\cdot 5^{2}$	15,64386		
			0,01792	Comma
*cs**	$2^{7}\cdot 3^{4}\cdot 5$	15,66178		
			0,07681	Limma minus
*d**	$3^{7}\cdot 5^{2}$	15,73859	0,01630	Diaschisma
d	$2^{11}\cdot 3^{3}$	15,75489		
			0,05889	Hemitonium minus
ds	$2^{8}\cdot 3^{2}\cdot 5^{2}$	15,81378		
			0,01792	Comma
*ds**	$2^{4}\cdot 3^{6}\cdot 5$	15,83170		
			0,07519	Hemitonium minus cum diaschismate
e	$2^{12}\cdot 3\cdot 5$	15,90689		
			0,01792	Comma
*e**	$2^{8}\cdot 3^{5}$	15,92481		
			0,05890	Hemitonium minus

*f**	$2^5 \cdot 3^4 \cdot 5^2$	15,98371		
			0,01629	Diaschisma
f	2^{16}		16,00000	

In hoc ergo genere ad duodecim sonos generis diatonico-chromatici duodecim novi soni accedunt, quorum autem ab illis differentiae sunt vel commata vel diaschismata; quae cum auditu vix distingui queant, hi novi soni tuto omitti eorumque loco consueti usurpari poterunt. Genus itaque diatonico-chromaticum aeque late patet, ac censendum est genus, cuius exponens est $2^m \cdot 3^7 \cdot 5^2$

10. Satis igitur concinne genus diatonico-chromaticum, cuius exponens dumtaxat est $2^m \cdot 3^2 \cdot 5^2$, adhiberi potest ad opera musica, quorum exponentes multo magis sunt compositi atque in $2^m \cdot 3^7 \cdot 5$ contenti, exprimenda. Quamvis enim octava pro huiusmodi operibus duplo maiore sonorum numero, prout exponens requirit, instrueretur, tamen ob tantillam differentiam in harmonia vix ulla variatio percipi posset, sive completum sive incompletum genus usurparetur. Simili autem modo ultra septenarium progredi licet, ita ut genus musicum hodie usu receptum inserviat pro generali exponente $2^m \cdot 3^n \cdot 5^2$
quantumvis magnus etiam numerus *n* accipiatur.

11. Hoc autem ita se habere genusque diatonico-chromaticum latissime patere quotidianae. Musicorum compositiones satis superque testantur. Vix enim ullum hodiernum opus musicum reperitur, cuius exponens non magis esset compositus quam exponens ipsius generis $2^m \cdot 3^3 \cdot 5^2$. Interim tamen ipsi quoque musici fateri coguntur, quod summo rigore rem considerando soni recepti non sufficiant, sed ob minimam aberrationem hi soni potius adhibeantur, quam ut novis introducendis sonis musica tractatu difficilior efficeretur.

12. Minus autem feliciter res succedit, si augendo exponentem ipsius 5 genus nostrum diatonico-chromaticum magis amplificare voluerimus. Aucta enim potestate ipsius 5 eiusmodi soni insuper ad sonos consuetos accedunt, qui plus quam commate scilicet diesi plerumque a consuetis discrepant, qui error, cum diesis sit circiter medietas hemitonii, animadverti potest. Interim tamen, quo hoc melius perspiciatur, adiecimus octavam generis, cuius exponens est $2^m \cdot 3^3 \cdot 5^5$

Signa	Soni	Logarithmi Sonorum	Intervalla	Nomina Intervallorum
F	2^{16}	16,00000		
			0,04260	Hemitonium minus demto diaschismate

*Fs**	$2^2 \cdot 3^3 \cdot 5^4$	16,04260		
			0,03422	Diesis
Fs	$2^9 \cdot 3^3 \cdot 5$	16,07682		
			0,05889	Hemitonium minus
*G**	$2^6 \cdot 3^2 \cdot 5^3$	16,13571		
			0,03421	Diesis
G	$2^{13} \cdot 3^2$	16,16992		
			0,02468 1	Hemitonium minus demta diesi
*Gs**	$2^3 \cdot 3 \cdot 5^5$	16,19460		
			0,03422	Diesis
Gs	$2^{10} \cdot 3 \cdot 5^2$	16,22882		
			0,05889	Hemitonium minus
*A**	$2^7 \cdot 5^4$	16,28771		
			0,03422	Diesis
A	$2^{14} \cdot 5$	16,32193		
			0,04260	Hemitonium minus demto diaschismate
*B**	$3^3 \cdot 5^5$	16,36453		
			0,03421	Diesis
B	$2^7 \cdot 3^2 \cdot 5^2$	16,39874		
			0,05889	Hemitonium minus
*H**	$2^4 \cdot 3^2 \cdot 5^4$	16,45763		
			0,03422	Diesis
H	$2^{11} \cdot 3^2 \cdot 5$	16,49185		
			0,05890	Hemitonium minus
*c**	$2^8 \cdot 3 \cdot 5^3$	16,55075		
			0,03421	Diesis
c	$2^{15} \cdot 3$	16,58496		
			0,02468	Hemitonium minus demta diesi
*cs**	$2^5 \cdot 5^5$	16,60964		
			0,03422	Diesis
cs	$2^{12} \cdot 5^2$	16,64386		
			0,07681	Limma minus
*d**	$2^5 \cdot 3^3 \cdot 5^3$	16,72067		
			0,03421	Diesis
d	$2^{12} \cdot 3^3$	16,75488		
			0,02468	Hemitonium minus demta diesi

*ds**	$2^2 \cdot 3^2 \cdot 5^5$	16,77956		
			0,03422	Diesis
ds	$2^9 \cdot 3^2 \cdot 5^2$	16,81378		
			0,05889	Hemitonium minus
*e**	$2^6 \cdot 3 \cdot 5^4$	16,87267		
			0,03422	Diesis
e	$2^{13} \cdot 3 \cdot 5$	16,90689		
			0,05889	Hemitonium minus
*f**	$2^{10} \cdot 5^3$	16,96578		
			0,03422	Diesis
f	2^{17}	17,00000		

13. In hoc igitur genere soni de novo accedentes ad consuetos alternative sunt interserti et eorum quisque a principali suo distat diesi; quae differentia cum non sit insensibilis, omissionem sonorum peregrinorum vix tolerare potest. Praeterea quidam horum sonorum propiores sunt sonis principalibus praecedentibus quam sequentibus, a quibus signa sumus mutuati; sonus scilicet *Gs** propior est sono *G* quam sono *Gs*, ita ut eius loco sonum *G* usurpare potius conveniret; quod vero itidem magnam haberet difficultatem, cum sonus *G* loco soni *G** adhiberi debeat, diversi autem soni *G** et *Gs** non eodem sono exprimi queant. Potius ergo ad talem musicam conveniret octavam in 24 intervalla dividere, quod genus quoque eam habiturum esset praerogativam, ut omnia intervalla inter se fere essent aequalia.

14. Duplicato autem hac ratione numero sonorum hoc novum musicae genus latissime pateret; non solum enim ad genera posset accommodari sub exponente $2^m \cdot 3^8 \cdot 5^5$ contenta, sed etiam sub exponente $2^m \cdot 3^3 \cdot 5^p$ denotante *p* numerum quinario maiorem. Quin etiam sufficeret ad genus universale hoc $2^m \cdot 3^n \cdot 5^p$, id quod satis constat, nisi *n* et *p* sint numeri valde magni; perquam autum magnos numeros loco *n* et *p* substituere ipsa harmonia non permittit.

15. Generi igitur diatonico-chromatico, cuius exponens est $2^m \cdot 3^5 \cdot 5^2$, illaesa harmonia amplior extensio concedi non potest, quam ad opera musica sub exponente $2^m \cdot 3^7 \cdot 5^2$ contenta. Quamvis enim eodem iure ternarius maiorem quam septimam potestatem habere posset, tamen ipsae harmoniae leges vetant talia opera componere, quorum exponens magis esset compositus. Quamobrem usum huius generis recepti latius extendere non conveniet, quam ad opera musica in exponente $2^m \cdot 3^7 \cdot 5^2$ contenta; neque etiam Musici hodierni istum terminum transgredi solent.

16. Quo autem genus musicum receptum, cuius exponens est $2^m \cdot 3^3 \cdot 5^2$, exponenti magis composito $2^m \cdot 3^7 \cdot 5^2$ satisfaciat, cuilibet sono seu clavi instrumentorum duplex sonus affingitur, uti ex schemate huius generis § 9 annexo intelligitur; claves enim verbi gratia *H* signatae tam sonos sub exponente $2^m \cdot 3^2 \cdot 5$ quam sub exponente $2^m \cdot 3^6$ contentas exhibebunt. Quamobrem sequentem tabulam adiecimus, ex qua statim intelligitur, qua clave quilibet sonus in exponente $2^m \cdot 3^7 \cdot 5^2$ contentus debeat exprimi, posito pro
primario ipsius *F* sono 2^n denotante *n* numerum fixum pro arbitrio assumtum.

Claves	Soni primarii	Soni secundarii	Claves	Soni primarii	Soni secundarii
C	$2^{n-2} \cdot 3$	$2^{n-13} \cdot 3^5 \cdot 5^2$	$\overline{c}$	$2^n \cdot 3$	$2^{n-11} \cdot 3^5 \cdot 5^2$
Cs	$2^{n-5} \cdot 5^2$	$2^{n-9} \cdot 3^4 \cdot 5$	$\overline{cs}$	$2^{n-3} \cdot 5^2$	$2^{n-7} \cdot 3^4 \cdot 5$
D	$2^{n-5} \cdot 3^3$	$2^{n-16} \cdot 3^7 \cdot 5^2$	$\overline{d}$	$2^{n-3} \cdot 3^3$	$2^{n-14} \cdot 3^7 \cdot 5^2$
Ds	$2^{n-3} \cdot 3^2 \cdot 5^2$	$2^{n-12} \cdot 3^6 \cdot 5$	$\overline{ds}$	$2^{n-6} \cdot 3^2 \cdot 5^2$	$2^{n-10} \cdot 3^6 \cdot 5$
E	$2^{n-4} \cdot 3 \cdot 5$	$2^{n-8} \cdot 3^5$	$\overline{e}$	$2^{n-2} \cdot 3 \cdot 5$	$2^{n-6} \cdot 3^5$
F	2^n	$2^{n-11} \cdot 3^4 \cdot 5^2$	$\overline{f}$	2^{n+2}	$2^{n-9} \cdot 3^4 \cdot 5^2$
Fs	$2^{n-7} \cdot 3^3 \cdot 5$	$2^{n-11} \cdot 3^7$	$\overline{fs}$	$2^{n-5} \cdot 3^3 \cdot 5$	$2^{n-9} \cdot 3^7$
G	$2^{n-3} \cdot 3^2$	$2^{n-14} \cdot 3^6 \cdot 5^2$	$\overline{g}$	$2^{n-1} \cdot 3^2$	$2^{n-12} \cdot 3^6 \cdot 5^2$
Gs	$2^{n-6} \cdot 3 \cdot 5^2$	$2^{n-10} \cdot 3^5 \cdot 5$	$\overline{gs}$	$2^{n-4} \cdot 3 \cdot 5^2$	$2^{n-8} \cdot 3^5 \cdot 5$
A	$2^{n-2} \cdot 5$	$2^{n-6} \cdot 3^4$	$\overline{a}$	$2^n \cdot 5$	$2^{n-4} \cdot 3^4$
B	$2^{n-9} \cdot 3^3 \cdot 5^2$	$2^{n-13} \cdot 3^7 \cdot 5$	$\overline{b}$	$2^{n-7} \cdot 3^3 \cdot 5^2$	$2^{n-11} \cdot 3^7 \cdot 5$
H	$2^{n-5} \cdot 3^2 \cdot 5$	$2^{n-9} \cdot 3^6$	$\overline{h}$	$2^{n-3} \cdot 3^2 \cdot 5$	$2^{n-7} \cdot 3^6$
c	$2^{n-1} \cdot 3$	$2^{n-12} \cdot 3^5 \cdot 5$	$\overline{c}$	$2^{n+1} \cdot 3$	$2^{n-10} \cdot 3^5 \cdot 5$
cs	$2^{n-4} \cdot 5^2$	$2^{n-8} \cdot 3^4 \cdot 5$	$\overline{cs}$	$2^{n-2} \cdot 5^2$	$2^{n-6} \cdot 3^4 \cdot 5$
d	$2^{n-4} \cdot 3^3$	$2^{n-15} \cdot 3^7 \cdot 5^2$	$\overline{d}$	$2^{n-2} \cdot 3^3$	$2^{n-13} \cdot 3^7 \cdot 5^2$
ds	$2^{n-7} \cdot 3^2 \cdot 5^2$	$2^{n-11} \cdot 3^6 \cdot 5$	$\overline{ds}$	$2^{n-5} \cdot 3^2 \cdot 5^2$	$2^{n-9} \cdot 3^6 \cdot 5$
e	$2^{n-3} \cdot 3 \cdot 5$	$2^{n-7} \cdot 3^5$	$\overline{e}$	$2^{n-1} \cdot 3 \cdot 5$	$2^{n-5} \cdot 3^5$
f	2^{n+1}	$2^{n-10} \cdot 3^4 \cdot 5^2$	$\overline{f}$	2^{n+3}	$2^{n-8} \cdot 3^4 \cdot 5^2$
fs	$2^{n-6} \cdot 3^3 \cdot 5$	$2^{n-10} \cdot 3^7$	$\overline{fs}$	$2^{n-4} \cdot 3^3 \cdot 5$	$2^{n-8} \cdot 3^7$
g	$2^{n-2} \cdot 3^2$	$2^{n-13} \cdot 3^6 \cdot 5^2$	$\overline{g}$	$2^n \cdot 3^2$	$2^{n-11} \cdot 3^6 \cdot 5^2$
gs	$2^{n-5} \cdot 3 \cdot 5^2$	$2^{n-9} \cdot 3^5 \cdot 5$	$\overline{gs}$	$2^{n-3} \cdot 3 \cdot 5^2$	$2^{n-7} \cdot 3^5 \cdot 5$
a	$2^{n-1} \cdot 5$	$2^{n-5} \cdot 3^4$	$\overline{a}$	$2^{n+1} \cdot 5$	$2^{n-3} \cdot 3^4$
b	$2^{n-8} \cdot 3^3 \cdot 5^2$	$2^{n-12} \cdot 3^7 \cdot 5$	$\overline{b}$	$2^{n-6} \cdot 3^3 \cdot 5^2$	$2^{n-10} \cdot 3^7 \cdot 5$
h	$2^{n-4} \cdot 3^2 \cdot 5$	$2^{n-8} \cdot 3^6$	$\overline{h}$	$2^{n-2} \cdot 3^2 \cdot 5$	$2^{n-6} \cdot 3^6$

$\bar{c}$	$2^{n}\cdot 3$	$2^{n-15}\cdot 3^{7}\cdot 5^{2}$	$\bar{c}$	$2^{n+2}\cdot 3$	$2^{n-13}\cdot 3^{7}\cdot 5^{2}$

17. ln hac ergo tabula exhibentur soni tam primarii quam secundarii, ad quos edendos quaelibet clavis est apta. Primarii quidem sunt ipsi soni ex exponente generis $2^{m}\cdot 3^{3}\cdot 5^{2}$ derivati, ad quos proinde claves quam exactissime debent esse adaptatae. Soni vero secundarii summo rigore ab iisdem clavibus edi nequeunt; quia vero tam parum a primariis discrepant, ad eos exprimendos hae claves sine sensibili harmoniae iactura tuto adhiberi possunt. Nam etiamsi ab acutioribus auribus comma seu diaschisma, quibus intervallis soni secundarii a primariis differunt, distingui queat, tamen, quia soni secundarii cum primariis neque in eadem consonantia neque in duarum consonantiarum successione misceri possunt, error etiam ab acutissimo auditu percipi non poterit. Si enim verbi gratia clavis F in prima consonantia ad sonum 2^{n} exprimendum fuerit usurpata, eadem in centesima post primam consonantia tuto sonum $2^{m-11}\cdot 3^{4}\cdot 5^{2}$ repraesentare poterit.

18. Ex hac ergo tabula statim quoque intelligitur, si proposita fuerit in numeris series vel sonorum vel consonantiarum, quibusnam clavibus pulsandis ea series exprimi debeat. Ad hoc autem efficiendum numerum n ita accipi oportet, ut omnes numeri propositi in tabula reperiantur, si quidem maximus minimum non plus quam sedecies comprehendat. Quare numerus n vel ex maximo numerorum propositorum debebit definiri vel ex minimo hocque facto pro reliquis sonis facile debitae claves habebuntur, si quidem, quod ponimus, numerorum propositorum minimus communis dividuus in $2^{m}\cdot 3^{7}\cdot 5^{2}$ contineatur.

19. Omnia ergo opera musica, ad quae genus nostrum diatonico-chromaticum est accommodatum, in hoc exponente $2^{m}\cdot 3^{7}\cdot 5^{2}$ sunt comprehensa, ita ut alia opera diversi exponentis instrumentis secundum hoc genus attemperatis edi nequeant. Quamobrem omnium musicorum operum exponentes ex solis his tribus numeris 2, 3, 5 eorumque potestatibus debent esse compositi neque insuper potestas quinarii secundam nec potestas ternarii septimam superare poterit; adeo ut LEIBNITH effatum omnino locum habeat, cum diceret in musica etiamnum ultra quinarium numerari non solere.

20. Atque sane difficile esset in musicam praeter hos tres numeros alium, puta 7, introducere, cum consonantiae, in quarum exponentes septinarius ingrederetur, nimis dure sonarent harmoniamque turbarent. Consonantiae enim, in quarum exponentibus solus septinarius cum binario inesset, vix essent admittendae ob intervalla suaviora a 3 et 5 orta neglecta. Iuncto autem 7 cum 3 et 5, ut prodiret consonantiae exponens $2^{m}\cdot 3\cdot 5\cdot 7$, consonantia nimis feret composita, ut auditui placere non posset. Interim tamen sonos

in octava constitutos pro genere, cuius exponens est $2^m \cdot 3^3 \cdot 5^2 \cdot 7$, ob oculos ponemus.

Generis exponens $2^m \cdot 3^3 \cdot 5^2 \cdot 7$

Signa Sonorum	Soni	Logarithmi Sonorum	Intervalla	
F	2^{12}	12,00000		
			0,03617	512:525
*Fs**	$2^8 \cdot 3 \cdot 5^2 \cdot 7$	12,03617		
			0,04064	35:36
Fs	$2^5 \cdot 3^3 \cdot 5$	12,07681		
			0,05247	27:28
*G**	27• 5 . 7	12,12928		
			0,04064	35:36
G	$2^9 \cdot 3^2$	12,16992		
			0,03618	512:525
*Gs**	$3^3 \cdot 5^2 \cdot 7$	12,20610		
			0,02272	63:64
Gs	$2^6 \cdot 3 \cdot 5^2$	12,22882		
			0,07039	20:21
*A**	$2^4 \cdot 3^2 \cdot 5 \cdot 7$	12,29921		
			0,02272	63:64
A	$2^{10} \cdot 5$	12,32193		
			0,07039	20:21
*B**	$2^8 \cdot 3 \cdot 7$	12,39232		
			0,00642	224:225
B	$2^3 \cdot 3^3 \cdot 5^2$	12,39874		
			0,05247	27:28

*H**	$2^5 \cdot 5^2 \cdot 7$	12,45121		
			0,04064	35:36
H	$2^7 \cdot 3^2 \cdot 5$	12,49185		
			0,07039	20:21
*c**	$2^5 \cdot 3^3 \cdot 7$	12,56224		
			0,02272	63: 64
c	$2^{11} \cdot 3$	12,58496		
			0,03618	512:525
*cs**	$2^2 \cdot 3^2 \cdot 5^2 \cdot 7$	12,62114		
			0,02272	63:64
cs	$2^8 \cdot 5^2$	12,64386		
			0,07039	20:21
*d**	$2^6 \cdot 3 \cdot 5 \cdot 7$	12,71425		
			0,04064	35 :36
d	$2^8 \cdot 3^3$	12,75489		
			0,05247	27:28
*ds**	210. 7	12,80736		
			0,00642	224:225
ds	$2^5 \cdot 3^2 \cdot 5^2$	12,81378		
			0,07039	20:21
*e**	$2^3 \cdot 3^3 \cdot 5 \cdot 7$	12,88417		
			0,02272	63:64
e 20:.21	$2^9 \cdot 3 \cdot 5$	12,90689		
			0,07039	20:21
*f**	$2^7 \cdot 3^2 \cdot 7$	12,97728		
			0,02272	63: 64
f	2^{13}	13,00000		