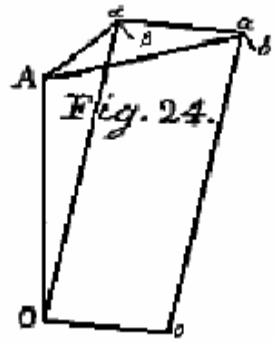
Chapter VI

Concerning the Relative Motion of Bodies Acted on by Forces in General.

THEOREM 6

238. If a corpuscle *A* is acted on by any forces, the motion of this particle with respect to [or relative to] the point *O*, which is itself carried uniformly in a straight line, is to be determined for the same forces acting (Fig. 24).



DEMONSTRATION

In an element of time *dt*, the corpuscle *A* is carried along on account of the innate motion through an interval *Aa*, moreover on account of the forces acting it is turned away through the element *ab*, thus in order that *ab* is the effect produced on the body by the forces acting on *A* in the element of time *dt*. But meanwhile the point *O* progresses through an element of distance *Oo*, thus so that in the elapsed element of time *dt* this point is now at *o*, which before was at *O*, moreover the corpuscle is now at *b*, which before was at *A*. Now from *O* there is drawn $O\alpha$ equal and parallel to *oa* itself, and likewise $\alpha\beta$ equal and parallel to *ab*; and with respect to the point *O*, the corpuscle is seen to arrive at β [*b* in the O.O.] from *A* in the same element of time *dt*, which motion thus the corpuscle itself has, and if on account of the initial motion it should describe the interval $A\alpha$ and likewise from α it is turned away through the element $\alpha\beta$. Clearly if the corpuscle is acted on by no forces and it is moved uniformly along the straight line *Aa*, the relative motion will also be uniform along the straight line $A\alpha$, as we have shown above [recall that the speed of *O* has to be subtracted (as a vector) to give the relative speed w.r.t. *O*]. But now on account of the forces acting on the absolute motion, the interval *ab* is produced, but in the relative motion the interval $\alpha\beta$, which since it is equal and parallel to *ab*, the same relative motion and absolute motion are generated by the same forces. Hence if the point *O* is carried along a straight line uniformly, the motion of the corpuscle *A* with respect to this, from whatever forces are acting, is thus had, and as if the point *O* were at rest, and the corpuscle acted on by the same forces.

COROLLARY 1

239. Hence if we are familiar with the forces acting on the corpuscle A, from these precepts presented before, not only the absolute motion of this, but also the relative motion with respect to the point *O* can be defined, which is progressing uniformly along a straight line.

COROLLARY 2

240. And it follows that the same differentio-differential formulas are determined both for absolute and relative motion; yet the distinction is discerned in the integration, since each case duly has to be accommodated with the initial starting conditions.

COROLLARIUM 3

241. Therefore if the point *O*, with respect to which the motion is to be measured, remains at rest or is moving uniformly along a straight line, then the inquiry into the motion is obtained in the same way. Clearly as the effect of inertia is not changed in this case, and thus the effect of the forces remains the same.

EXPLANATION 1

242. While the [reference] point and the corpuscle are considered to be transferred from *O* to *o* and from *b* to β respectively, it has to be the case that β with respect to *O* and *b* with respect to *o* maintain the same situation, because, as *O* and *o* are considered as points, the matter is not at all seen to be determined unless, as we have agreed above, the distance *ob* and the relative position $O\beta$ are maintained. [This means that in an absolute frame, the corpuscle moves from *A* to *b* according to its initial speed, while the applied forces produces the small change *ab* in addition; if the initial reference point also moves from *O* to *o* in the same time *dt*, with some other constant speed, then the net result as viewed from *O* or *o* is the same, as the constant relative speed vector *Oo* is added to each.] Now one may take the absolute directions as fixed, thus in order that $O\beta$ is not only equal to *ob* but also to be established along the same straight direction, which comes about if $O\beta$ it taken equal and parallel to *ob*. It comes back to the same thing, if following the first instructions received, in place of the point *O* an extended body is assumed, in which it is agreed to consider three or four fixed points ; moreover this body *O* then, with respect to which this other motion is to be reckoned, thus is agreed to be moving along *Oo*, in order that the individual points are carried along with equal speeds in directions parallel between themselves. For then, the situation which the corpuscle *b* maintains with respect to the four points assumed

in the body *o*, also applies to the corpuscle at β with respect to the four points assumed in the body *o*; thus the corpuscle holds the same situation translated to β with respect to the four points, relative to the body at *O*. With these noted it is clear that the body is in absolute motion, since it is being transferred from *A* to *b*, while the point is progressing from *O* to *o*, to agree with the relative motion, in which it is transferred from A to β. Because although here only the element of time *dt* has been shown, since the same can be shown in a similar way for all the elements of time, we can affirm correctly in general that the whole relative motion here defined corresponds to the absolute motion.

SCHOLIUM

243. This which has been treated and will again be treated concerning the motion of a corpuscle *A* with respect to a point *O*, otherwise and chiefly in astronomy are accustomed to be proposed under the title of *apparent motion*. For clearly any point *O*, with respect of which the motion of the corpuscle *A* is considered, acts as a spectator, and thus the question is proposed, how the motion of the corpuscle appears to this spectator. For the spectator, in whatever way the point *O* is moving, which is the station of this, is not thought to be aware of his own motion, thus in order that he may observe himself to remain constantly in the same place *O*. Whereby now a corpuscle may be seen at *A*, and in the lapse of an element of time *dt* at β, and the corpuscle will appear to have been translated meanwhile from *A* to β, yet since in actual fact it has arrived at *b* from *A* ; hence the translation from *A* to β is called the *apparent motion*. In the case therefore of our theorem the spectator is assumed to be moving forwards along a straight line and we have shown the apparent motion of the corpuscle *A* to follow from these mechanical principles defined, if the body is put in place to be acted on by the same forces, which actually act on it. Without doubt the same differentio-differential formulas express the apparent motion as well as the true motion ; but they must be integrated for the apparent motion thus, so that there is agreement between the true motion initially or at some given time. Hence at last the whole distinction reveals itself in the integration.

EXPLANATION 2

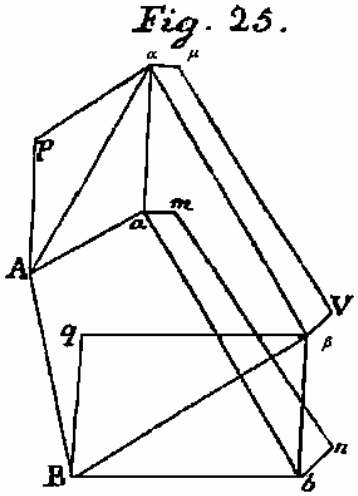
244. Therefore the forces disturbing the relative motion must be equal to these which affect the absolute motion, since we take the effects or the elements *ab* and $\alpha\beta$ to be equal. And this equality of the forces is easily observed in the calculation, if they should pertain to the kind of absolute forces, which act in the same way on the corpuscle either moving and at rest; but if the corpuscle *A* is not acted on by forces of this kind, but which depend on the speed of this corpuscle, and the resistance of fluids is a force of this kind, then the magnitudes of these forces is to be sought from the true speed of the corpuscle that it has in absolute motion, and the same is to be put to use in the relative motion. [Thus, a calculation is effected that essentially uses the absolute

speed always, to arrive at the true forces, and annulling the fictitious effect introduced by the relative motion, as Euler explains at some length.] Just as if the corpuscle *A* were moving in the fluid, the resistance or the force, that is suffered from that, depends on the absolute speed of the corpuscle in which the element *Aa* has been traversed, and the same force must be introduced in the calculation for the relative motion ; and a significant error may be made, if we wish to define the resistance from the speed of the relative motion, in which the element $A\alpha$ is completed. In order that we may avoid this error we have to consider the fluid itself, in as much as for the relative motion as if it is carried by a motion equal and opposite to that, to which the point *O* is moving ; for then the fluid by this aforesaid motion affects the relative motion of the corpuscle progressing along $A\alpha$, and the resting fluid affects the corpuscle only by the absolute motion along the length $A\alpha$. Moreover always, as often as the relative motion is sought, not only the corpuscle *A*, but as if the whole interval with all the bodies, which are able to act in that, has to be considered to be moving with a motion equal and opposite to that which the point *O* has, since by this pretended motion the point *O* is reduced to rest.

THEOREM 7

245. If two bodies *A* and *B* are moving in some manner under the action of some forces, and at the same instant an additional equal motion is impressed on these along the same direction, then the motion between themselves is conserved.

DEMONSTRATION



The right line *Aa* expresses the motion of the body *A* or it is the interval described by that in the element of time *dt* (Fig. 25) ; and in a similar manner the body *B* has such a speed, by which in the same element of time *dt* it describes the interval *Bb*; moreover from the forces acting the one is deflected from *a* to *m*, now the other from *b* to *n*, thus so that now in the elapsed time *dt* the right line *mn* refers to the relative situation, which before was being referred to by the right line *AB*. But suddenly at the beginning of the time element *dt* each body has an equal motion impressed along the same direction, from which alone the body *A* is transferred to *p* and *B* to *q* in the element of time *dt*, thus in order that the right lines *Ap* and *Bq* are about to be equal and parallel. Moreover with that motion now in place agreed upon, if the parallelograms $Aa\alpha p$ and $Bb\beta q$ are completed, the diagonals $A\alpha$ and $B\beta$ refer to the

interval which the bodies are able to complete on account of their own motion in the time *dt*. Now on account of the equal and parallel right lines $a\alpha$ and $b\beta$ also *ab* and $\alpha\beta$ are equal and parallel, thus in order that the relative situation $\alpha\beta$ after the new motion impressed agrees with that relative motion in place *ab*. Again there is taken $\alpha\mu$ equal and parallel to *am*, and $\beta\nu$ equal and parallel to *bn*, and since μ and ν are now in place of the bodies, with the agreed upon forces acting, then also $\mu\nu$ is equal and parallel to *mn*. Whereby with the same forces acting remaining the impressed motion changes nothing in the situation and in the relative motion of both bodies.

COROLLARY 1

246. This also is apparent for more bodies ; for however many there should be, if at the same time equal and parallel individual motions are impressed, then the relative motion does not change between themselves, even for whatever individual forces acting.

COROLLARY 2

247. This new impressed motion is responded to in the same way, and as if the whole interval with the bodies is abruptly changed in direction uniformly by that new motion. For the composition of the motion here put in place agrees with the translation of the interval.

SCHOLION 1

248. Here the discussion has not only been about the impression by forces concerning true motion, which certainly cannot happen without a notable sudden force, as for motion for bodies that we only consider in the mind. Indeed neither, which have been treated in this chapter, are true changes made in the motion to be referred to, since our custom here is any kind of absolute motion to be reduced to relative motion, thus in order that the formulas only show relative motion, with the absolute motion clearly not allowed to be changed. And hence also this can thus be demonstrated from the preceding one : besides the bodies *A* and *B* there is considered the point *O*, which follows the direction *Oo* parallel to that, along which the new motion is impressed on the bodies, which is uniformly with the same speed, thus in order that in the element of time *dt* the element of distance to be transversed $Oo = Ap = Bq$ parallel to these, but in the opposite direction. Therefore since we have shown the relative motion of the bodies *A* and *B* with respect to the point *O* with the same forces acting, and the absolute motion to be determined, it is hence evident that the relative motion can be obtained, if a motion equal and contrary to that can be impressed, to which the point *O* is moving. Moreover in this way the point *O* is reduced to rest, but with that motion impressed on the bodies *A* and *B* along *Ap* and *Bq*; and since these retain the same

motion with respect to the point *O*, they also keep the same relative motion between each other.

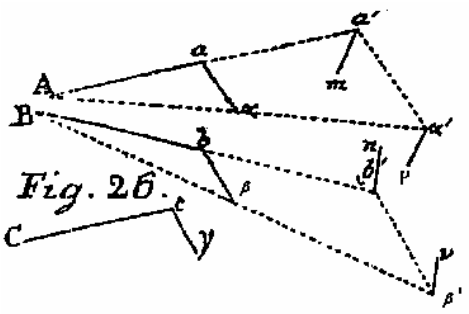
SCHOLION 2

249. The question concerning any relative or apparent motion is reduced to this being determined, that first there it has to be defined just how much motion can be impressed on the bodies even in the imagination, then it should be understood to be acted on by forces of such a kind besides these, by which the bodies are actually urged, so that, if the absolute motion is still to be presented and to be expressed by the formulas presented above, from these the relative motion described is to be produced. For it is apparent so that with the motion in place as always with the forces acting, a change of this kind can be considered, so that the motion produced in this way agrees with the change in the relative motion. Hence this whole business is summed up by two changes, the one inserted into the motion, the other made in the forces acting, but each is only an imaginary change ; thus no difficulty can arise from these, just as for the bodies *A* and *B*, motions in addition to those along *Ap* and *Bq* must be impressed by which the bodies are now carried forwards. For it is sufficient to have declared this impression of forces thus to be understood, in order that the body *A* with a speed along the side *Aa*, then if the above speed *Ap* is attributed, then it is agreed to be progressing along the diagonal $A\alpha$ from the motion expressed. Clearly this impressed motion or rather added on has conformed with the rules given above about the resolution of the motion along two or three sides, which also is established in the mind only. Such impressed motion is thus usually referred to, so that the whole interval with the bodies contained in that can be considered to have suddenly been changed by a certain motion. And indeed in the first theorem we have seen, that if a point, with respect to which it has been necessary to reckon the motion, is progressing uniformly in a fixed direction, that for the relative motion to be defined then nothing has to be changed in the forces acting, but only the motion in place has thus to be changed, as an equal and opposite motion to that impressed above, by which that point is moving.

THEOREM 8

250. If the corpuscles *A*, *B* and *C* are moving in some manner under the action of some forces along parallel directions, and these in addition are acted on with the forces themselves in proportion to the masses, then the relative motion of these in place is not disturbed.

DEMONSTRATION



Now the bodies that were at *A, B, C,* which now, on account of the initial motion as well as the forces acting, can arrive at *a, b, c,* in the element of time *dt*, from which points now in place the relative motion is defined (Fig. 26). Moreover we can consider besides these forces, that these are acted on by forces along lines parallel to the directions $a\alpha, b\beta,$ and $c\gamma$, which are in proportion to the masses of these, and these bodies are now not found at *a, b,* and *c,* but at α,β,γ thus so that the intervals $a\alpha, b\beta, c\gamma$ are equal and parallel to each other; and it is apparent that the relative situation of the points α,β,γ to be the same between themselves and of the points *a, b, c,* where they shall be in the future, if these new forces had not been added.

COROLLARY 1

251. Hence if any corpuscles *A, B, C* , in addition to the forces present, by which they are actually urged, are acted upon by forces themselves in proportion to the masses along directions parallel to each other, then at some time they will hold the same position relative to each other, if these new forces were absent.

COROLLARY 2

252. Therefore the relative motion of the planets and of the sun are not changed between each other, if in turn these bodies in addition to the forces by which action they are to be disturbed, are considered to be acted on by new forces in proportion to the masses, then they are to be impelled along directions parallel to each other.

COROLLARY 3

253. If these added forces are thus taken, in order that the one acting on the body A, is equal and opposite to that, by which action it is disturbed, then the motion of this is unchanged; because if we are to consider individual moments, then the body A remains in its own state both moving forwards uniformly along a direction.

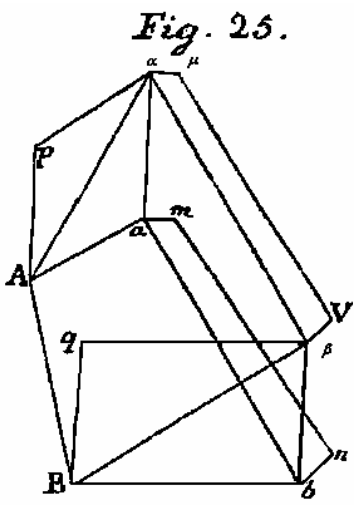
EXPLANATION

254. Hence it is possible for doubt to arise, or, even if the points α and β keep the same situation between themselves as with the points a and b, hence, is it not possible for another relative situation to be produced ? To disprove this we may set aside the first forces by which these corpuscles are indeed acted on, and also with the distant forces added set aside, then in the following time element the corpuscles arrive at a' and b', in order that $aa' = Aa$ and $bb' = Bb$; but if these forces are admitted for the previous time interval dt, then the bodies arrive at α' and β', in order that $\alpha\alpha' = A\alpha$ and $\beta\beta' = B\beta$, and thus $b'\beta'$ is equal and parallel to $a'\alpha'$, thus in order that the relative position of the points α', β' is the same as of the points a', b'. Indeed here it might be rightly objected that the elements $\alpha\alpha'$ and $\beta\beta'$ have been wrongly assumed to be equal to $A\alpha$ and $B\beta$, since on account of the action of the forces are changed, but since each change is similar, nevertheless the elements $\alpha\alpha'$ and $\beta\beta'$ remain equal and parallel between each other, that which is sufficient, even if they are not precisely twice of the same $a\alpha$ and $b\beta$. Moreover whatever forces act on both during the same time interval dt, first A equally from α' and from a' is distorted, and then B equally from β' and from b'; and thus also, whether the new forces proportional to the masses are added or not, thus the same relative situation is conserved. For we put A to be transferred from a' to m by the appropriate forces, and likewise to be transferred from α' to μ, in order that $a'm$ is equal and parallel to $\alpha'\mu$; in a similar manner, if the corpuscle B is transferred by the appropriate forces from b' to n, then the same is transferred from β' to ν, in order that $\beta'\nu$ is equal and parallel to $b'n$. Therefore since the points μ and ν hold the same relative situation as the points m and n, it is also apparent in the succession of the time that the relative situation is not changed by the addition of the above forces.

PROBLEM 19

255. If the corpuscle B is moved in some manner by the forces acting, to determine the relative motion of this with respect to the corpuscle A, since that also has been moved in some manner by the forces acting (Fig. 25).

SOLUTION



Initially each of the bodies has an impressed motion equal and opposite to that, by which the corpuscle *B* is then carried, and in the first instant at least the corpuscle *B* is reduced to rest; but both bodies advance likewise by the relative motion between them, and if such a common motion were not impressed on these ; in fact also, since this change is only made in the initial state, each following motion is expressed by the same formulas. Now the corpuscle *B*, as far as it has been subjected to the action of the forces, then indeed has moved ; now if that is acted on continuously by the above forces with these equal and opposite, we can consider, in order that the effect of these is cancelled, and that *B* continues to remain at rest. Whereby, lest the relative motion is disturbed, we can also consider similar forces to be applied to the corpuscle *A* in the individual elements of time, which are opposite to these, on which the corpuscle *B* is acted, and these forces are to each other as the mass *A* to the mass *B*. In this manner the corpuscle *B* clearly is returned to rest, with the motion of the other *A* relative to this unchanged, and hence this is the motion of A according to this respective motion, of such a kind that is agreed to be apparent to the spectator at *B*. Therefore the relative motion is to be determined from calculation, and the corpuscle A is considered to be acted on by two kinds of forces; clearly in the first place by these real forces which are acting : then the forces which are acting on the body *B*, either increased or decreased in the ratio of the masses *B* to *A* and understood to be applied along directions opposite to the body *A* above. From these forces the motion of the body *A*, as if it should be absolute, can be determined from the previous conditions and so the relative motion sought of *A* can be obtained.

COROLLARY 1

256. Therefore if in the elapsed time *t* the corpuscle *A* is acted on by a force equal to *P*, and the corpuscle *B* now by a force equal to *Q*, hence there is taken a force equal to $\frac{AQ}{B}$, which is applied to the above corpuscle *A* in a direction opposite to that, in which the force *Q* acts on the corpuscle *B*. [Thus, the accelerations are equal.]

COROLLARY 2

257. But if from these forces applied to the corpuscle *A* for a certain time the differentio-differential formulas of this motion can be collected together, the integration according to the initial state which is considered as known, has to be accommodated, then clearly the constants advanced in the integration are determined from this state.

SCHOLIUM 1

258. With the help of this rule the motion of the moon, such as considered from the centre of the earth, is accustomed to be defined ; although indeed heavenly bodies on account of their great size might hence seem to be excluded, yet below it will be shown that these likewise are to be moving as if the masses of these were gathered together at the centre of gravity, thus in order that a [point] image of the points can be considered. Therefore it is not sufficient to know the forces continually acting on the moon, in order that the apparent motion of the moon can be defined, but also the forces have to be sought diligently, to the action of which the earth itself is subjected. These forces then must be considered to be applied, and required to be diminished in the ratio of the mass of the earth to the mass of the moon and these in addition of the moon in the opposite direction to these acting on the earth ; and from these forces acting jointly the motion of the moon must be determined, such as to agree with what is evident from the centre of the earth. In a like manner if the centre of the sun is not at rest and the motion of the planets with respect to the centre of the sun to be defined, all the forces, which act on the sun, must be transferred to the planets in the manner set out above. Thus it is apparent that the use of this problem is to be the greatest in the whole of theoretical astronomy ; indeed also thus in the investigation of other motions, where in most cases it is expedient to know the relative motion, too much help is offered.

SCHOLIUM 2

259. And these are, in which these, which I have explained in the above books concerned with the motion of points, have been considered to be partially made clear and partially to be supplemented, where indeed not only the principles of motion are seen to be set out clearer and to be confirmed, but also I have assisted in the application of these to certain non trivial cases and I have rendered the reduction of these to easier absolute measures. Now moreover also the principles of relative motion, almost carelessly set out in these books, here I have set out to explain more carefully, since that also has the most outstanding use in what follows. And thus I progress to these parts of mechanics, which clearly I have not touched on in these books, and in the first place indeed rigid bodies occur, the shape of which is incapable

of change, of which it is necessary to explain the motion, both when they have been left to themselves as when they are acted upon by various forces. Then at last one may wish to pursue these investigations of the motion of flexible bodies, both of elastic materials and of fluids ; also of the direction that must be referred to from the coming together of many bodies, and of the innate qualities of these arising. If we are able to assess these different areas carefully, then we will understand the most important fields of mechanics uncovered by our studies, the culture of which may promise the greatest harvest.

CAPUT VI

DE MOTU RESPECTU CORPUSCULORUM A VIRIBUS QUIBUSCUNQUE ACTORUM

THEOREMA 6

238. Si corpusculum *A* a viribus quibuscunque sollicitetur, eius motus respectu puncti *O*, quod uniformiter in directum fertur, per easdem vires determinabitur (Fig. 24)

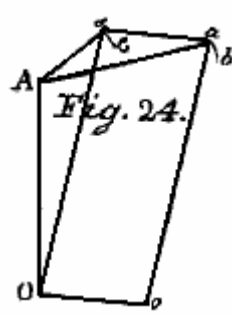


Fig. 24.

DEMONSTRATIO

Tempusculo *dt* feratur corpusculum *A* ob motum insitum per spatium *Aa*, ob vires autem sollicitates detorqueatur per spatiolum *ab*, ita ut *ab* sit effectus virium tempusculo *dt* in corpusculo *A* productus. Interea autem punctum *O* progrediatur per spatium *Oo*, ita ut elapso tempusculo *dt* hoc punctum sit in *o*, cum ante fuisset in *O*, corpusculum autem in *b*, cum ante fuisset in *A*. Iam ex *O* ducatur $O\alpha$ ipsi *oa* aequalis et parallela, itemque $\alpha\beta$ aequalis et parallel ipsi *ab*; atque respectu puncti *O* corpusculum videbitur ex *A* in *b* pervenisse eodem tempusculo *dt*, qui motus ita se habebit, ac si ob motum insitum descripsisset spatium $A\alpha$ simulque ex α detorqueretur per spatiolum $\alpha\beta$. Scilicet si corpusculm a nullis viribus sollicitaretur ac per *Aa* aequabiliter in directum moveretur, etiam motus respectivus foret aequabilis rectilineus per $A\alpha$ aequabiliter in directum moveretur, uti supra ostendimus. Nunc autem ob vires sollicitantes in motu absoluto producitur spatiolum *ab*, in respectivo autem spatiolum $\alpha\beta$, quod cum illi sit parallalum et aequale, motus respectivus *ab* iisdem viribus turbatur ac motus absolutus. Hinc si punctum *O* uniformiter in directum feratur, eius respectu motus corpusculi *A*, a quibuscunque viribus sollicitetur, perinde se habebit, ac si punctum *O* quiesceret corpusculumque ab iisdem viribus sollicitaretur.

COROLLARIUM 1

239. Si ergo vires noverimus, quibus corpusculum A sollicitatur, ex iis per praecepta ante tradita non solum eius motum absolutum, sed etiam respectivum ad punctum O, quod uniformiter in directum progreditur, relatum definire valemus.

COROLLARIUM 2

240. Atque adeo eaedem formulae differentio-differentiales tam motum absolutum quam respectivum determinabunt; discrimen tantum in integratione cernetur, quae utroque casu rite ad statum initialem est accommodanta.

COROLLARIUM 3

241. Sive ergo punctum O, cuius respectu motus aestimatur, quiescat sive moveatur uniformiter in directum, investigatio motus perinde se habet. Scilicet uti effectus inertiae hoc casu non mutatur, ita etiam effectus virium idem manet.

EXPLICATIO 1

242. Dum punctum et corpusculum ex *o* et *b* in *O* et β mente transferuntur, efficiendum est, ut β respectu *O* eundem situm teneat ac *b* repectu *o*, quod, cum *O* et *o* ut puncta specentur, rem minime determinare videtur, quandoquidem, ut supra innuimus, sola distantia *ob* et $O\beta$ situm respectivum contineret. Verum stabilito iam spatio absoluto plagas seu directiones fixas assumere licet, ita ut $O\beta$ non solum ipsi *ob* aequalis sed etiam in eandem plagam directa statui debeat, id quod evenit, si $O\beta$ ipsi *ob* aequalis ac parallela accipitur. Res eodem redit, si secundum prima praecepta loco puncti *O* corpus extensum assumatur, in quo tria vel quatuor puncta fixa concipere liceat; tum autem hoc corpus *O*, cuius respectu motus alterius aestimatur, ita secundum *Oo* moveri est censendum, ut singula eius puncta paribus celeritatibus secundum directiones inter se parallelas ferantur. Tum enim, quem situm tenuerit corpusculum *b* respectu quatuor punctorum in corpore *o* assumtorum, eundem situm tenebit corpusculum in β translatum respectu eorundem quatuor punctorum, dum corpus adhuc est in *O*. His notatis manifestum est motum corpusculi absolutum, quo ex *A* in *b* transfertur, dum punctum *O* in *o* progreditur, convenire cum motu respectivo, quo ex A in β transfertur. Quod etsi hic tantum de temporis elemento *dt* est ostensum, quoniam idem de omnibus temporis elementis simili modo ostenditur, recte affirmamus in genere totum motum respectivum hic definitum motui absoluto respondere.

SCHOLION

243. Quae hic de motu respectivo corpusculi *A* respectu puncti *O* sunt tradita ac porro tradentur, alias et potissimum in Astronomia sub titula *motus apparentis* proponi solent. In puncto scilicet *O*, cuius respectu motus corpusculi *A* aestimatur, spectator constituitur, et quaestio ita proponitur, quomodo huic spectatori motus corpusculi sit appariturus. Nam spectator, quomodocunque punctum *O*, quod est eius statio, moveatur, motum suum non sentire censetur, ita ut se constanter in eodem loco *O* persistere arbitretur. Quare cum nunc vidisset corpusculum in *A*, elapso autem

tempusculo *dt* in β, corpusculum ipsi interea ex *A* in β translatum videbitur, cum tamen revera ex *A* in *b* pervenerit; dicitur ergo translato ex *A* in β *motus apparens*. In casu ergo nostri Theorematis spectator uniformiter in directum promoveri assumitur atque demonstravimus motum apparentem corpusculi *A* per praecepta Mechanica definitum iri, si corpusculum ab iisdem viribus, quae actu in id agunt, sollicitari statuatur. Eadem nimirum formulae differentio-differentiales tam motum apparentem quam motum verum expriment; pro motu autem apparente ita integrari debent, ut initio vel aliquo tempore dato cum motu apparente conveniant. Totum ergo discrimen demum in integratione se exerit.

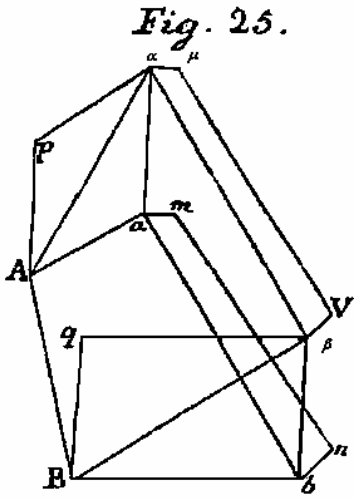
EXPLICATIO 2

244. Vires motum respectivum turbantes propteria illis, quae motum absolutum afficiebant, aequales esse debent, quia effectus seu spatiola *ab* et $\alpha\beta$ aequalia deprehendimus. Atque haec virium aequalitas in calculo facile observatur, si ad genus virium absolutarum pertineant, quae perinde in corpusculum motum agunt atque quiescens; sin autem corpusculum *A* ab eiusmodi viribus sollicitetur, quae ab eius celeritate pendent, cuiusmodi est resistentia fluidorum, quantitas earum virium ex celeritate corpusculi vera, quam in motu absoluto habet, est petenda eademque in motu respectivo adhibenda. Veluti si corpusculum *A* in fluido moveretur, resistentia seu vis, quam ab eo patitur, pendebit ab eius celeritate absoluta, qua spatiolum *Aa* percurrit, eademque vis in calculum pro motu respectivo introduci debet; atque insignis error committeretur, si resistentiam ex celeritate motus respectivi, qua spatiolum $A\alpha$ conficitur, definire vellemus. Quem errorem ut evitemus, ipsum fluidum, quatenus absolute quiescit, pro motu respectivo, quasi motu aequali et opposito ei, quo punctum *O* movetur, ferretur, contemplari debemus; tum enim fluidum hoc motu praeditum aequae afficiet corpusculum motu respectivo per $A\alpha$ progrediens, atque fluidum quiescens afficit corpusculum motu absoluto per $A\alpha$ latum. Perpetuo autem, quoties de motu respectivo quaestio est, non solum corpusculum *A*, sed totum quasi spatium cum omnibus corporibus, quae in id agere queant, motu aequali et contrario ei, quem punctum *O* habet, moveri est concipiendum, quandoquidem hoc motu ficto puncto *O* ad quietem redigitur.

THEOREMA 7

245. Si duo corpora *A* et *B* utcunque moveantur a viribus quibuscunque sollicitata iisque eodem momento insuper motus aequales secundum eandem directionem imprimantur, motum inter se eundem conservabunt.

DEMONSTRATIO



Exprimat recta *Aa* motum corporis *A* seu sit spatium ab eo tempusculo *dt* descriptum (Fig. 25) ; similique modo corpus *B* tantam habeat celeritatem, qua eodem tempuculo *dt* describeret spatium *Bb*; a viribus sollicitantibus autem illud ex *a* in *m*, hoc vero ex *b* in *n* deflectatur, ita ut nunc elapso tempore *dt* recta *mn* referat situm relativum, qui ante recta *AB* referebatur. Incipiente autem tempusculo *dt* subito utrique corpori motus aequalis secundum eandem directionem imprimatur, quo solo corpus *A* in *p* et *B* in *q* tempusculo *dt* transferretur, ita ut rectae *Ap* et *Bq* futurae sint aequales ac parallelae. Accedente autem motu iam insito, si parallelogramme $Aa\alpha p$ et $Bb\beta q$ compleantur, diagonales $A\alpha$ et $B\beta$ spatia referent, quae corpora ob utrumque motum tempusculo *dt* essent percursura. Iam ob rectas $a\alpha$ et $b\beta$ aequales et parallelas etiam *ab* et $\alpha\beta$ erunt aequales et parallelae, ita ut situs relativus $\alpha\beta$ post novum motum impressum conveniat cum situ relativo *ab*. Capiatur porro $\alpha\mu$ aequalis et parallela ipsi *am*, et $\beta\nu$ aequalis et parallela ipsi *bn*, et cum μ et ν nunc sint loco corporum, accedentibus viribus sollicitantibus, erit quoque $\mu\nu$ aequalis et parallela ipsi *mn*. Quare manentibus iisdem viribus sollicitantibus motus impressus nihil mutat in situ et motu relativo amborum corporum.

COROLLARIUM 1

246. Hoc etiam ad plura patet corpora; quotcunque enim fuerint, si singulis simul motus aequales et paralleli imprimantur, motus eorum relativus inter se non mutabitur, a quibuscunque etiam viribus singula sollicitentur.

COROLLARIUM 2

247. Motus hic de novo impressus eodem redit, ac si totum spatium cum corporibus motu illo novo abriperetur uniformiter in directum. Compositio enim motus hic adhibita cum translatione spatii convenit.

SCHOLIUM 1

248. Hic non tam de vera motus impressione sermo est, quae utique sine notabili concussione fieri non posset, quam de motu, quem corporibus mente tantum imprimi concipimus. Neque enim, quae in isto capite traduntur, ad veras mutationes in motu factas sunt referenda, cum institutum nostrum hic sit motus quoscunque absolutos ad respectivos reducere, ita ut formulae tantum ostendant motum respectivum, absoluto nullam plane mutationem passo. Atque hinc etiam istud Theorema ex praecedente ita demonstrari potest : concipiatue praeter corpora *A* et *B* punctum *O*, quod secundum directionem *Oo* parallelam illi, secundum quam corporibus novus motus imprimitur, uniformiter moveatur eadem celeritate, ita ut tempusculo *dt* percursurum esset spatium $Oo = Ap = Bq$ his parallelum, sed contra directum. Quoniam igitur ante demonstravimus motum respectivum corporum *A* et *B* respectu puncti *O* iisdem viribus atque absolutum determinari, evidens est hunc motum respectivum obtineri, si toti spatio cum corporibus motus aequalis et contrarius ei, quo punctum *O* movetur, imprimatur. Hoc autem modo punctum *O* ad quietem redigitur, corporibus *A* et *B* autem ipse ille motus secundum *Ap* et *Bq* imprimitur; et quia ea respectu puncti *O* eundem motum retinent, etiam inter se eundem motum relativum conservbunt.

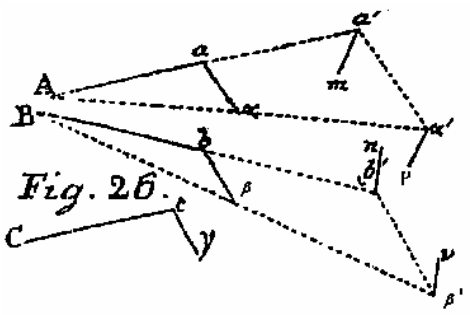
SCHOLIUM 2

249. Quaestio de motu quocunque respectivo seu apparente per calculum determinando eo redit, ut definiatur primo, qualis motus corpori insuper mente saltem imprimi debeat, deinde a qualibus viribus praeter eas, quibus actu urgetur, sollicitari sit intelligendum, ut, si hic motus tanquam absolutus tractetur et per formulas supra traditas exprimatur, ipse motus respectivus, qui desideratur, sit produturus. Evidens enim est semper tam in motu insito quam in viribus sollicitantibus eiusmodi mutationem concipi posse, ut motus hoc modo mutatus cum respectivo, quem quaerimus, conveniat. Totum ergo hoc negotium duplici mutatione, altera in motu insito, altera in viribus sollicitantibus facta absolvitur, quae autem utraque mente tantum instituitur; unde nulla difficultas ex eo nasci potest, quemadmodum corporibus *A* et *B* motus illi secundum *Ap* et *Bq* praeter eos motus, quibus iam feruntur, imprimi debeant. Sufficit enim declarasse hanc impressionem ita esse intelligendam, ut corpus *A* celeritate *Aa* latum, si ipsi insuper celeritas *Ap* tribuatur, motu per diagonalem $A\alpha$ expresso progredi sit censendum. Haec scilicet motus impressio seu potius additio conformis est regulis supra datis circa resolutionem motus in duos tresve laterales, quae etiam mente tantum instituitur. Talis motus impressio etiam ita referri solet, ut totum spatium cum corporibus in eo contentis motu quodam abripi concipiatur. Atque in priori quidem Theoremate vidimus, si punctum, cuius respectu motum aestimari oporteat, uniformiter in directum progrediatur, pro motu respectivo definiendo nihil in viribus sollicitantibus esse mutandum, sed tantum motum insitum ita mutari debere, ut insuper inprimatur motus aequalis et contrarius ei, quo punctum illud moveatur.

THEOREMA 8

250. Si corporpuscula *A, B C* utcunque moveantur a viribus quibuscunque sollicitata eaque insuper secundum directiones parallelas a viribus ipsorum massis proportionalibus sollicitentur, eorum situs relativus non turbabitur.

DEMONSTRATIO



Fuerint nunc corpuscula in *A, B, C,* quae tam ob motum insitum quam vires sollicitantes tempusculo *dt* pervenirent in *a, b, c*, quibus punctis iam eorum situs relativus definitur (Fig. 26). Concipiamus autem ea interea praeter istas vires sollicitari singula secundum directiones parallelas $a\alpha, b\beta, c\gamma$ a viribus, quae sint ipsorum massis proportionalis, eaque iam non in *a, b, c* reperientur, sed in α, β, γ ita ut spatiola $a\alpha, b\beta, c\gamma$ futura sint inter se parallela et aequalia; atque evidens est punctorum α, β, γ situm relativum inter se eundem fore ac punctorum *a, b, c*, ubi essent futura, si hae novae vires non accessissent.

COROLLARIUM 1

251. Si ergo corpuscla *A, B, C* quovis instanti praeter vires, quibus actu urgentur, a viribus ipsorum massis proportionalibus secundum directiones inter se parallelas sollicitentur, ad quodvis tempus eundem inter se situm relativum tenebunt, ac si istae novae vires abfuissent.

COROLLARIUM 2

252. Motus igitur solis ac planetarum relativus inter se non immutantur, si singula haec corpora praeter vires, quibus actu sollicitantur, a novis viribus ipsorum massis proportionalibus impelli concipiantur secundum directiones inter se parallelas.

COROLLARIUM 3

253. Si istae vires adiectae ita assumantur, ut ea, quae in corpusculum *A* agit, aequalis sit et contraria ei, qua actu sollicitatur, huius motus non immutabitur; quod si singulis momentis fieri concipiamus, corpusculum *A* in statu suo permanebit et uniformiter in directionem promovebitur.

EXPLICATIO

254. Dubium hinc oriri potest, an, etis puncta α et β eundem situm inter se teneant ac puncta *a* et *b*, deinceps non alius situs relativus sit proditurus? Ad quod diluendum seponamus primo vires, quibus haec corpuscula actu sollicitantur, ac remotis etiam viribus adiectis sequenti tempusculo corpuscula pervenirent in *a'* et *b'*, ut esset $aa' = Aa$ et $bb' = Bb;$ sin autem hae vires pro tempusculo praecedente *dt* admittantur, pervenient in α' et β', ut sit $\alpha\alpha' = A\alpha$ et $\beta\beta' = B\beta,$ sicque erit $b'\beta'$ aequalis et parallela ipsi $a'\alpha'$, ita ut situs relativus punctorum α', β' idem sit, qui punctorum *a'*, *b'*. Recte quidem hic obiiceretur spatiola $\alpha\alpha'$ et $\beta\beta'$ perperam ipsis $A\alpha$ et $B\beta$ aequali assumi, cum ob actionem virium celeritates sint mutatae, sed quia mutatio utrinque est similis, nihilominus spatiola $\alpha\alpha'$ et $\beta\beta'$ inter se manebunt aequalia et parallela, id quod sufficit, etiamsi non sint ipsorum $a\alpha$ et $b\beta$ praecise dupla. Quaecunque autem vires per alterum hoc tempusculum *dt* in ambo corpuscula agant, prius *A* aeque ex α' detorquebitur, atque ex *a'*, et posterius *B* aeque ex β', atque ex *b'*; sicque etiam, sive novae vires massis proportionales accesserint sives secus, idem adhuc situs relativus conservabitur. Ponamus enim a viribus propriis corpusculum *A* ex *a'* in *m* transferri, idemque ex α' in μ transferetur, ut sit *a'm* aequale et parallelum $\alpha'\mu$; simili modo, si corpusculum *B* a proprius viribus ex *b'* in *n* transfertur, idem ex β' in ν transferetur, ut sit $\beta'\nu$ aequale et parallelum ipsi *b'n*. Cum igitur puncta μ et ν eundem situm relativum teneant, quem puncta *m* et *n*, patet etiam temporis successu a viribus illis insuper adiectis situm relativum non mutari.

PROBLEMA 19

255. Si corpusculum B moveatur utcunque a viribus sollicitatum, eius respectu determinare motum respectivum corpusculi A, quod etiam a viribus quibuscunque sollicitatum utcunque moveatur (Fig. 25)

SOLUTIO

Imprimatur initio utrique corpori motus aequalis et contrarius ei, quo tunc corpusculum *B* fertur, ac primo saltem momento corpusculum in quietem redigetur; ambo autem corpora motu relativo inter se perinde incedent, ac si iste motus communis illis non fuisset impressus; quin etiam, cum tantum in statu initiali haec mutatio sit facta, utriusque motus subsequens iisdem formulis exprimetur. Corpusculum vero *B*, quatenus actioni virium est subiectum, deinceps quidem movebitur; verum si id continuo insuper a viribus his contrariis et aequalibus agitur, concipiamus, ut illarum effectus destruatur, id perpetuo in quiete perseverabit. Quare, ne motus relativus turbetur, concipiamus etiam corpusculo *A* singulis temporis momentis similes vires applicare, quae sint contrariae illis, quibus corpusculum *B*

sollicitatur, ad easque se habeant ut massa *A* ad massam *B*. Hoc modo corpusculum *B* plane ad quietem redigetur, motu alterius *A* respect huius non mutato, eritque ergo iste motus ipsius *A* eius motus respectivus, qualis spectatori in *B* constituto esset appariturus. Ad hunc igitur motum respectivum per calculum determinandum corpusculum *A* a duplicis generis viribus sollicitari est considerandum, primo scilicet ab iis ipsis viribus, quibus revera sollicitatur: deinde vires, quibus corpusculum *B* sollicitatur, in ratione massarum *B* ad *A* augeantur vel minuantur atque secundum directiones contrarias corpusculo *A* insuper applicatae intelligantur. Ex his viribus motus corpusculi *A*, quasi esset absolutus, per praecepta ante exposita determinetur atque obtinebitur eius motus respectivus quaesitus.

COROLLARIUM 1

256. Si ergo elapso tempore *t* corpusculum *A* sollicitetur a vi = *P*, corpusculum *B* vero a vi = *Q*, hinc capiatur vis = $\frac{AQ}{B}$, quae insuper corpusculo *A* applicetur in directione contraria ei, qua vis *Q* in corpusculum *B* agit.

COROLLARIUM 2

257. Quodsi ex his viribus corpusculo *A* quovis tempore applicatis formuale differentio-differentiales eius motum definientes colligantur, integratio ad statum initialem, qui ut cognitus spectatur, est accommodanda, dum scilicet constantes per integrationes ingressae ex hoc statu determinantur.

SCHOLION 1

258. Ope hius regulae motus lunae, qualis ex centro terrae spectaretur, definiri solet; etsi enim corpora coelestia ob vastam magnitudinem hinc excludi videntur, tamen infra docebitur ea perinde moveri, ac si eorum massae in cuiusque centro gravitatis esset collectae, ita ut instar punctorum considerari possint. Ad motum ergo hunc lunae apparentem definiendum non sufficit vires nosse, quibus luna continuo sollicitatur, sed etiam vires diligenter sunt inquirendae, quarum actioni ipsi terra subiicitur. Has vires deinde in ratione massae terrae ad massam lunae deminui oportet haeque insuper lunae in directionibus contrariis iis, quibus in terram agunt, applicatae concipi debent ; atque ex his viribus iunctim sumtis motus lunae respectivus, qualis spectatori in centro terrae constituto esset appariturus, determinari debet. Simili modo si centrum solis non quiescat motusque planetarum primariorum respectu centri solis sit definiendus, omnes vires, quas sol subit, praecepto modo insuper in planetas transferri debent. Unde patet usum hius problematis per universam Astronomiam Theoreticam esse amplissimum; verum etiam inde in investigationem aliorum motuum, ubi saepenumero motus respectivos nosse expedit, maxima subsidia redundant.

SCHOLION 2

259. Atque haec sunt, quibus ea, quae in superioribus libris de motu punctorum exposui, partim illustranda partim supplenda sunt visa, ubi equidem non solum motus principia clarius exposuisse et confirmasse videor, sed etiam eorum applicationem ad quosvis casus non mediocriter sublevavi reductionemque ad mensuras absolutas faciliorem reddidi. Tum vero etiam doctrinam de motu respectivo, in illis libris fere penditis neglectam, hic diligentius exponendam putavi, quoniam ea etiam in sequentibus uberrimum usum praestabit. Progredior itaque ad eas Mechanicae partes, quas in illis libris plane non attigeram, ac primo quidem occurrunt corpora rigida, quarum figura nullius mutationis est capax, quorum motus evolvi oportebit, tam quando sibi sunt relicta quam a viribus quibuscunque sollicitata. Tum vero demum licebit has investigationes ad motus corporum flexibilium, elasticorum atque adeo fluidorum prosequi; quorsum etiam referri debent motus ex concurso plurium corporum cuiusque indolis oriundi. Quae diversa genera si perpendamus, intelligemus in Mechanica amplissimum campum aperiri nostris studiis, cuius cultura largissimam messem polliceatur.