Editors Preface.
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As a nineteen year old graduate Leonhard Euler wrote as his first publication a short paper on tautochrones in viscous media, that appeared in the Leibzig Acta eruditorum . Also during the following years he directed his efforts mainly to mechanics. Of the 28 papers, which the young scholar published in the period from the start of his appointment at the Petersburg Academy in the summer of 1727 to the end of 1734, nine are directly concerned with mechanics, and closely connected with it are 11 further papers, dealing with the summation of infinite series, integration of differential equations, shortest lines on curved surfaces and isoperimetric problems. As these publications show, Euler also studied in these first years in Petersburg the writings of Galileo, Huygens and Newton, in which the new science of motion is founded, developed and applied. In addition he made himself familiar with the works of Leibniz, Jacob and Johann Bernoulli, Hermann and others, who used the new infinitesimal calculus for the solution of problems in mechanics.

From all these efforts of Euler there eventually arose the plan for a textbook on the whole of mechanics. As Euler says in Mechanica Pt.1, § 98, " the different types of bodies that can undergo motion will guide us in choosing the main subdivisions of the work. In the first instance we shall consider infinitesimally small bodies, or such as can more or less be regarded as points. We shall then deal with bodies of finite size that are rigid, i.e. that do not permit any change in their shape. Next we shall discuss flexible bodies, followed by bodies that can expand or be compressed. Following this we shall examine the motion of separate bodies, that impede each other. Finally we shall discuss the motion of liquids".

That only a part of this great enterprise was completed is less due to Euler than to unfavourable external circumstances. At the end of 1734 he writes to Daniel Bernoulli "The first volume of Mechanica is quite complete, but I have little hope that it will be printed here".[1] The fact that the first and second parts of the *Mechanica sive motus scientica analytice exposita*, dealing with the kinetics of mass points, appeared towards the end of 1736 as supplementary volumes to the commemorative writings of the Petersburg Academy, is thanks to Baron J.A. v. Korff, who at that time managed the affairs of the Academy; to him Euler dedicated the work. The volume on the mechanics of rigid bodies was completed in 1760; but since at that time no publisher could be found, it only appeared in 1765 as *Theoria motus corporum solidorum seu rigidorum* published by Roese in Rostock and Greifswald. Instead of the volume on the mechanics of fluids four large scale papers were presented to the Petersburg Academy, and appeared in their publications between 1769 and 1772. Preliminary work for these and the other planned volumes are available in the form of numerous papers. In the *Opera omnia Leonhardi Euleri* they will be collected, ordered, and given new life; only this new publication will make it possible to have a clear picture of Euler's immense activity in the whole field of mechanics.

[1] Biblioth. Mathem. 7_3, 1906, p.139.

Mechanica, with which the second series of the *Opera omnia*, mechanics and astronomy, is opened, starts with an introduction, in which the concept of motion in general, and the action of forces on a mass point are discussed. The subsequent discussion concerning motion of a mass point is in two sections: free motion and constrained motion. In the case of free motion a distinction is made between whether the trajectory is straight or curved, and between whether the point moves in empty space or in a medium resisting motion; from these distinctions follow four types of motion, which are examined in succession. A first subdivision of constrained motion is concerned with whether the point is forced to follow a given curve, or whether it is constrained to remain on a given curved surface; secondly a distinction must be made between motion in empty space and in a medium resisting motion.

Whilst this clear subdivision constitutes a systematic progress, the real merit of Euler's work is the method by which he deals with the subject. As is explained both by the subtitle *Scientia motus analytice exposita* and in more detail by the preface to the first part, it was his intention to replace the cumbersome geometric-synthetic proofs of his predecessors by an easily understood analytic treatment; through this he hoped to enable also the less gifted to undertake independent research. The attempts in this direction by Newton, Johann Bernoulli, Varignon[2], and Hermann[3] are less successful than this new approach that we owe to Euler, and the *Mechanica* remains, as Lagrange put it in his *Mécanique analytique* (1788) " The first great work where Analysis has been applied to the science of movement". But it would be incorrect to believe that Euler's analytical method is merely a replacement of geometrical considerations by calculation. As important as the use of the infinitesimal calculus is also the "Ansatz" that leads to the differential equations of motion, and here Euler wins by introducing the moving coordinate system, consisting of tangent and normal to the trajectory, which accompanies the moving point. A discussion by Maclaurin in his *Treatise of fluxions* (1742), which is limited to a rigid coordinate system at rest, is a supplement to this. Later Euler has considered both systems of coordinates in his *Theoria motus.* Only by making clear these facts can we have a just appreciation of *Mechanics* .

Through writing *Mechanics*, Euler has placed himself among the classic authors in mathematics.
Nicolaus Fuss is right when he says in his eulogy on the master: " If clearness of concepts, accuracy of expression and well planned arrangement are the necessary characteristics of a classic work, then Euler's work on mechanics deserves this name to a high degree…..However not by far are these characteristics the most important ones of this work. It firmly establishes Euler's reputation and gives him a place amongst the foremost mathematicians alive; and this is saying a lot if one bears in mind that Johann Bernoulli was still alive. Only an extraordinary mind could make progress so rapidly as to overtake that still very active old man, who had both made and accepted so many

[2] Proceedings in the Pariser Memoiren, 1700 to 1711
[3] Proceedings in the Exercitationes subsecivae of the University of Frankfurt a.O.

challenges in the field of mathematics, and who had never left the arena without honour."[4]

Whilst Johann Bernoulli usually guarded his own reputation jealously, he always recognised Euler's achievements without envy. Soon after receiving both volumes of *Mechanica* he writes to him on 6. November 1737 to say that the work, which at this stage he has only superficially scanned, is full of the most sublime and difficult investigations, which do honour to the genius and acuity of mind of its author. Following this he makes several critical remarks. The title *Mechanica* he regards as not suitable, because this is traditionally assigned to the field of the "dead" forces (statics); following Leibniz ,one should refer to the field of the "living" forces as dynamics. He praises Euler himself, and he also praises Newton, but says nothing of the errors that he (Bernoulli) in the year 1713 had pointed to in Acta eruditorum. Finally he claims the solution of proposition 89 in the first part to be in error.[5] In his answer of 10. december 1737 Euler shows that Bernoulli has misunderstood that part of *Mechanica.* On the other hand he confesses to have made an error in proposition 78; he points out that when determining the motion of a point under the action of a central force, it is not, as he had maintained, necessary to assume that the initial direction of motion is perpendicular to the radius vector, so that consequently the corresponding scolium 2 has to be deleted.[6]

The first review of Mechanica was in Bliothèque germanique[7], which was published in Amsterdam by a group of Berlin scholars, headed by S. Formey. The author, a knowledgeable and well-meaning man, confines himself essentially to an exhaustive listing of the contents. He states that the material is by and large what is known, which was necessary since Euler intended an all embracing presentation of the whole of mechanics. However the analytical method he employs yields not only more satisfying proofs of old truths, but leads almost automatically to generalisations and new theorems.

Enthusiastic recognition is the tenor of a review published in 1738 in Nova Acta Eruditorum[8]. Here it says :"Until now there has never been a book containing such an abundance of wonderful and hidden things, taken from the innermost sanctum of mathematics". Although Chr.Wolf had discussed motion and equilibrium in his Elements under the title *Mechanica*, it was, in keeping with the plan of his work, only very briefly.[9] The only other works whose aim is to serve as textbooks for the theory of motion are Newtons *Principia* (1687) and Hermann's *Phoronomia* (1716); however these books are written in a too condensed form and follow the synthetic–geometric

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[4] *Leonardy Euleri Opera omnia,* series1, vol.1. p.LVIII
Leonardi Euleri Opera omnia II_1 Mechanica
[5] Biblioth.Mathem. 5_3, 1904, p.263.
[6] ibid. p. 269.
[7] Bibliothèque germanique, Anné 1737, T.39, p.93-108.
[8] Nova acta eruditorum 1738, p.113-133, 262-279.
[9] Chr. Wolf, Elementa matheseous universae. T.2, qui Mechanicam cum Statica,Hydrostaticam,Aerometriam atque Hydraulicam complectitur. Editio nova, Halae Magdeburgicae 1733.

method, used in the past. Euler has eliminated these shortcomings, has presented the voluminous material in a unified manner and added much that is new; whoever is reasonably familiar with higher mathematics, can read the work without difficulty and apply it.
Quite different sounds the review in the Paris Mémoires de Trévoux of 1740[10], authored by a Cartesian." From the renewal of physics", so the author says," to which Galileo without doubt has contributed much, has arisen the modern mechanics, that deals with motion, whilst the old mechanics was only statics. Mechanics is useful, even necessary for physics, however it is not physics itself, with which Newton confused it. Newton's physics is rather a kind of geometry, and far removed from being a true and pure physics. He has only had too many imitators, and geometry is set to destroy physics, and even more so the physicists; for only few of those inclined to physics reach their aim by way of conic sections, curves, quadratures, the infinitesimally small and the most impenetrable and arcane calculations." The long review ends with a conditional praise: " The work is very geometrical, but the geometry is deep and fine…Nothing is more thorny than all these problems of the author, who carefully, step by step, follows his path, the torch of analysis in his hands. It is easier to treat these speculations with disdain, than to follow them".

Finally we must report on an English publication of 1739, that contains a critique of *Mechanica*[11]. Its author, the well known ballistics expert Robins, can claim the merit of having uncovered some weaknesses and errors in *Mechanica*; however the impertinent way of his dealing with Euler is not to his credit. Soon after, Euler translated the "New principles of gunnery" of Robins into German, adding numerous supplements: the resulting work was published in Berlin in 1745 with the title " Neue Grundsaetze der Artillerie", and found much applause and wide distribution. Nicolaus Fuss recounts: "Whilst doing, wherever possible, justice to Mr. Robins in this translation, Mr. Euler, with rare modesty, corrects errors in the theory, and all revenge he takes against his opponent in relation to the old insult, consists in making his work more famous than it would have become without him".[12]

When Robins considers Euler's proofs in the introductory chapters on the foundations of mechanics to be inconclusive and untenable, then his criticism is directed less against Euler than against the metaphysical views of the time. It must also be emphasized that just here one finds thoughts of enduring value. This refers in particular to the discussion of relative motion and of the concept of a mass point (german: materieller Punkt). Actually the adjective *materiell* is not yet to be found in *Mechanica*, but Euler has recognised and used the " infinitely small body, that is more or less a point" as a concept required by analytical mechanics, independent of all philosophical and physical speculations.

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[10] Mémoires pour l'histoire des sciences et des beaux arts. Commencés d'être imprimés l'an 1701 à Trévoux. Paris 1740, p. 816-834, 1407-1422.
[11] B. Robins, Remarks on Mr. Euler's treatise of motion, Dr. Smith's complete system of opticks, and Dr.Jurin's essay upon distinct and indistinct vision. London 1739, p.1-29.
[12] Leonhardi Euleri Opera omnia, seriesI, vol.1, p.LXV.

Numerous criticisms by Robins refer to lack of care in the use of differential calculus, a shortcoming that in part is more a matter of form than of substance.

There is repeated criticism of unnecessary calculations, since the problem in hand could be solved in a much simpler and shorter way. Euler was well aware of this. His intention was to demonstrate the methods of solution, that had been developed in analytical generality, by means of easily solvable examples, and didactic considerations prevented him from employing the shortcuts possible in the particular examples.

The situation is different with regard to a remark Robins makes concerning the fifth chapter of the first part, which deals with the curvilinear motion of a free point in empty space. He thinks Euler would have done better, when discussing motion under a central force, to introduce first the theorems about equal areas being swept through in equal times and about equal velocities at equal distances from the centre, which would have resulted in considerable simplification. Instead he derived these theorems only by the way as corollaries of the 74. Proposition. Here he (Robins) hits on a real shortcoming of *Mechanica,* where the significance of the formulae is not always stressed. This is the more remarkable since, as his later works show, Euler had a very vivid conception of mechanical processes, and used such conception with success.

The main aim of Robins' criticism is directed at certain paradoxes that Euler encountered in the theory of motion under a central force. When a mass point moves rectilinearly towards a centre of force, the question arises as to what happens when it arrives there. Euler thought that on the basis of his formulae he had to conclude that for certain force laws the point would remain at the centre, where " it would in a sense be annihilated", but for other laws of force, for example that of Newton, it would be repelled from the centre, and then oscillate between the latter and a point of return. It is not clear how this could be possible, since, as it approaches the centre, the speed of the point rises without limit. Euler decides that " however this may be, we must here trust our calculation more than our power of judgement". Robins criticises this blind submission to calculation, which according to him has led to extraordinary errors and peculiar claims, and in reality, for the case of Newton's force law, the point executes oscillations about the centre.

That Euler's *Mechanics* has been read and used until the most recent times is shown by many papers in the literature, and in particular by text books and collections of problems in mechanics. A German translation has been published by J. Ph. Wolfers[13]. The fact that in the historical treatments by Montucla-Lalande, Dühring, Mach, Heller, Rosenberg and others, *Mechanica* is only mentioned in passing, is probably explained by the fact that in these works the emphasis is on the historical development of the principles of mechanics, for which *Mechanica* has little relevance. The situation would be quite different, if one wanted to write a summary of problems solved in the 18th century in the field of the mechanics of a mass point.

[13] Leonhard Eulers *Mechanic oder analytische Darstellung der Wissenschaft von der Bewegung.*With comments and explanations edited by Dr.J.Ph. Wolfers.Two parts, Greifswald 1848 and 1850.

In recent times the importance of *Mechanica* for the history of mathematics has begun to be appreciated, in connection with the growing realisation that an understanding of the development of mathematical thought remains incomplete, if, as was the case until now, one limits oneself to mathematical works in the more restricted sense, and that in fact publications from applied mathematics need also to be considered. [14]

As pointed out by Eneström[15],*Mechanica* is the first place where cyclometric functions appear. In the first part (§ 184) the symbol ***A*** .x is introduced for the inverse of tan (x). The same symbol serves in the second part (§ 305) for the inverse of sin (x), and later (§ 573), by way of distinction, the inverse of tan (x) is written as ***A***t.x . Moreover, in *Mechanica* (Part 2,§ 832) Euler was the first to use partial differential equations with homogeneous functions of two independent variables. Finally *Mechanica* is the first publication in which Euler used for the circumference of the circle of unit diameter and for the basis of the natural logarithms the symbols π and e respectively, symbols still in use today.

Mechanica should however also be mentioned in historical treatments of ordinary differential equations, in connection with integrating factors and singular solutions, and likewise in connection with the calculus of variations. The role it has played in the history of differential geometry should also be stressed. This is so firstly because of the previously mentioned coordinate system that accompanies a moving point, and secondly because here for the first time the normal to a curved surface is determined in general, and an analytic expression is derived for the curvature of an arbitrary plane section (Part II,§ 68).

The guidelines for the present new edition of *Mechanica* can be found in the general Editor's plan of the Euler edition. Only a few comments have been added. In particular it was decided not to add explanatory or critical comments to ideas that can only be understood in the context of their time; to some extent the present Preface may take their place.

At the proof stage the editor has had the support of the editorial staff of the Euler edition and of Prof. Dr. M. Winkelmann of Jena; all of them I give my heartfelt thanks. Likewise thanks are due to the firm of B.G. Teubner for their patience during the lengthy stage of print setting and for kindly meeting all wishes regarding the layout of the work.

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Paul Stäckel.

[14] See P. Stäckel, Biblioth. Mathem. 2_3, 1901, p. 120
[15] Biblioth.Mathem. 5_3, 1904, p.207,208,310; 6_3, 1905, p. 319.